POWER UNIT

Circuits II Notebook
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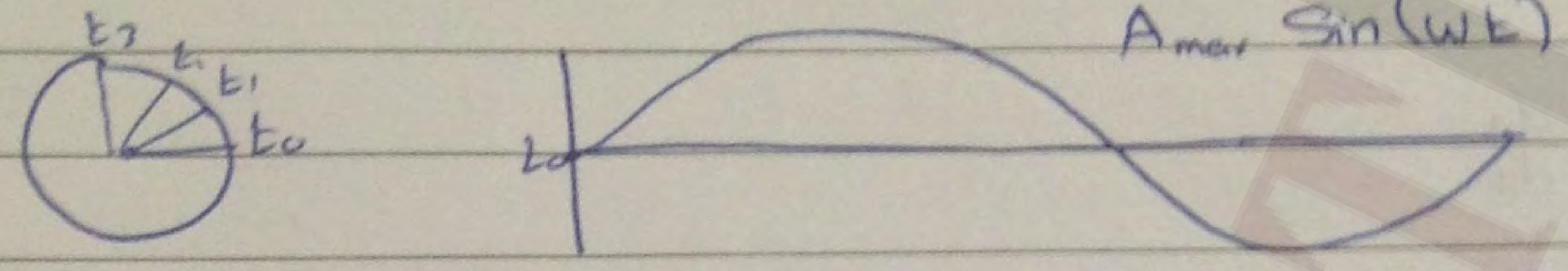
بأفكارنا_نُبدع

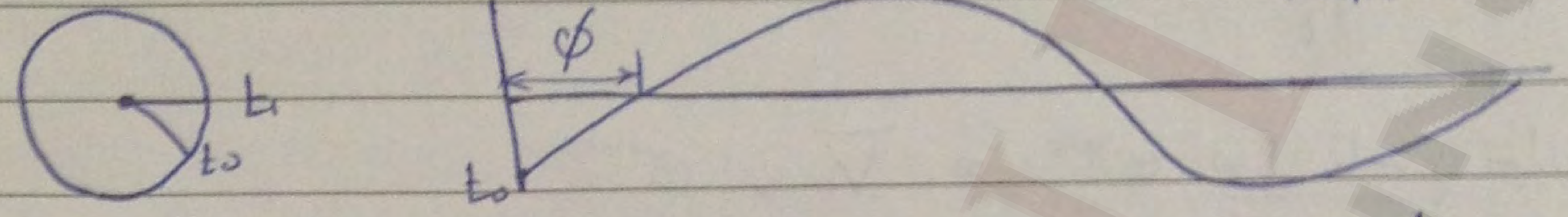
first exam material

Circuits(2)

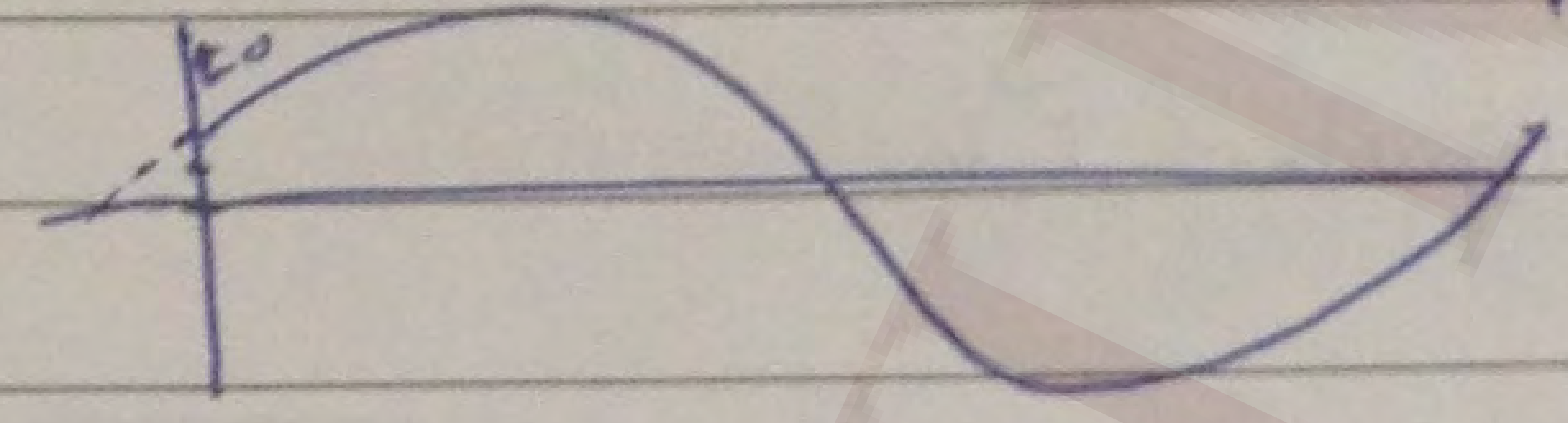
Sin



$A_{max}\ Sin(wt)$

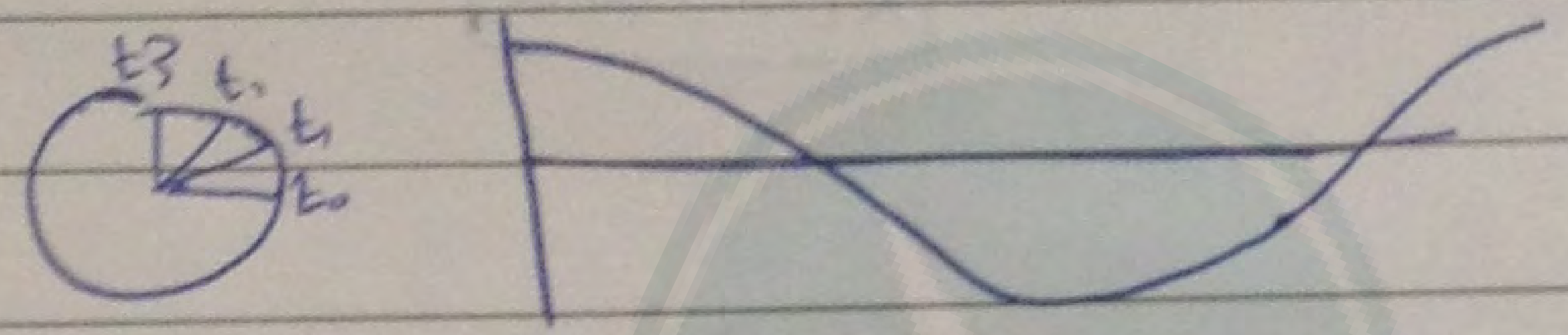


$A_m\ Sin(wt-\phi)$

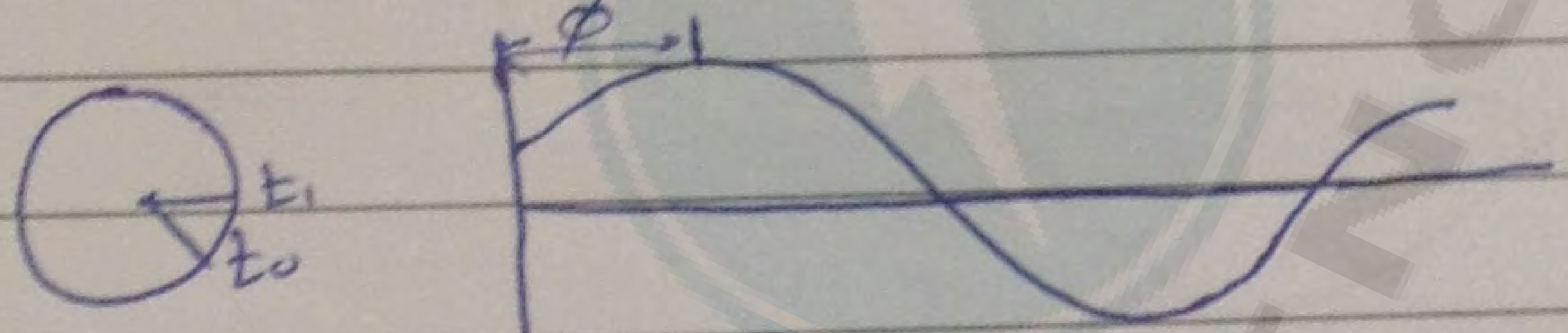


$A_m\ Sin(wt+\phi)$

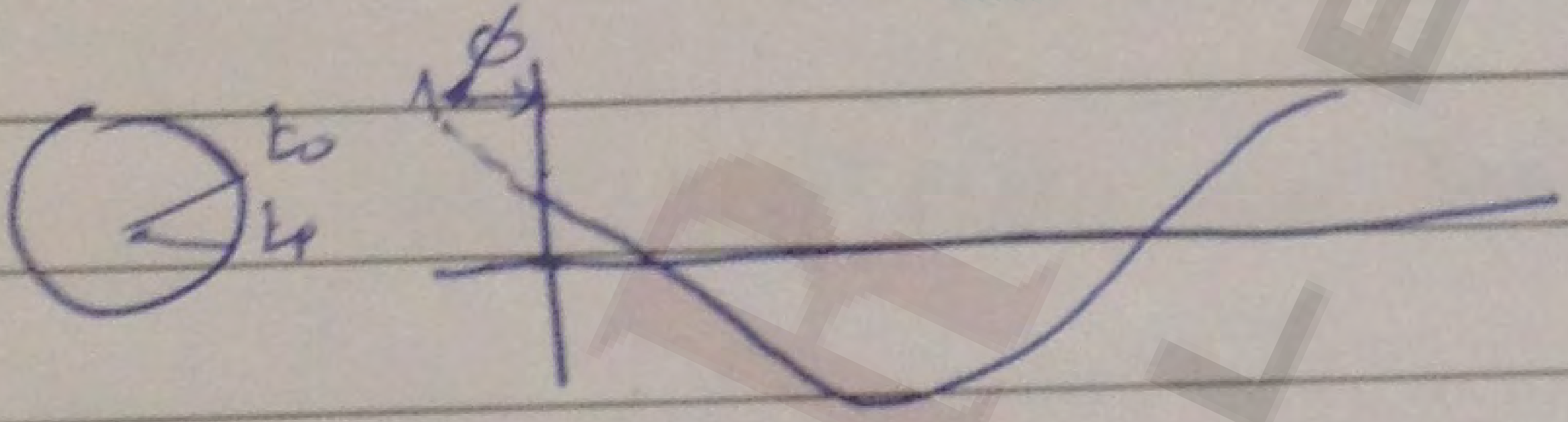
Cos



$A\ Cos(wt)$



$A\ Cos(wt-\phi)$



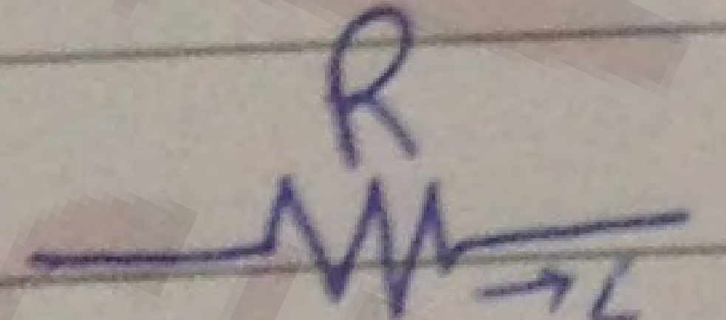
$A\ Cos(wt+\phi)$

$$A(t) = A\ Cos(wt+\phi) + j\ A\ Sin(wt+\phi)$$

$$= A_m\ e^{j(wt+\phi)} \quad = \underbrace{A\ e^{j\phi}}.e^{jwt}$$

$$= \vec{A}\ e^{jwt}$$

*



$i(t) = I_{max}\ Cos(wt+\phi)$

$v = R\ i$

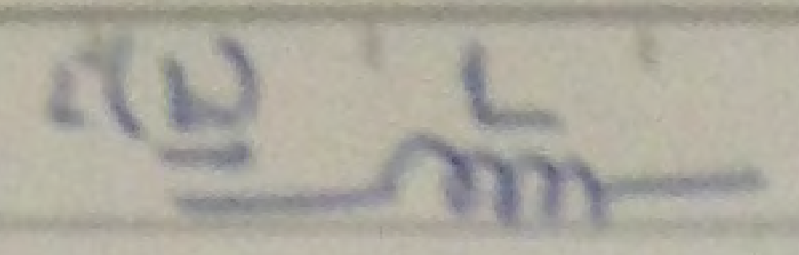
$v(t) = R\ I_{max}\ Cos(wt+\phi)$

$= V_{max}\ Cos(wt+\phi)$

$i(t) = I_{max}\ Cos(wt+\phi)$

$$i(t) = I_{max}\ Cos(wt+\phi) + j\ I_{max}\ Sin(wt+\phi)$$

$$= I_{max}\ e^{j\phi}\ e^{jwt} = I_{max}\ e^{jwt} = I_{max}\angle\phi$$



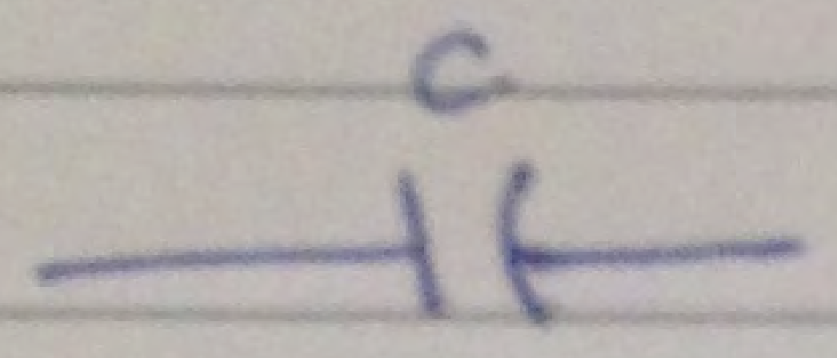
$$V = L\frac{di}{dt}$$

$$v(t) = L\frac{d}{dt}\,\vec{I}_m\,e^{jwt}$$

$$v(t) = \underbrace{jwL\,I_m}\,e^{jwt} = \vec{V}_{max}\,e^{jwt}.$$

$$\therefore \vec{V}_{max} = jwL\,\vec{I}_{max}$$

$$\vec{Z}_L = \frac{\vec{V}_{max}}{\vec{I}_{max}} = \frac{jwL\,\cancel{I_{max}}}{\cancel{I_m}} \Rightarrow \boxed{Z_L = jwL} \quad \boxed{X_L = wL}$$



$$i_c = C\frac{dv}{dt}$$

$$v(t) = V_{max}\,Cos(wt)$$

$$\vec{v}(t) = \vec{V}_{max}\,e^{jwt}$$

$$i = C\frac{d}{dt}\,\vec{V}_{max}\,e^{jwt}$$

$$\therefore \vec{I}_{max} = jwC\,\vec{V}_{max}$$

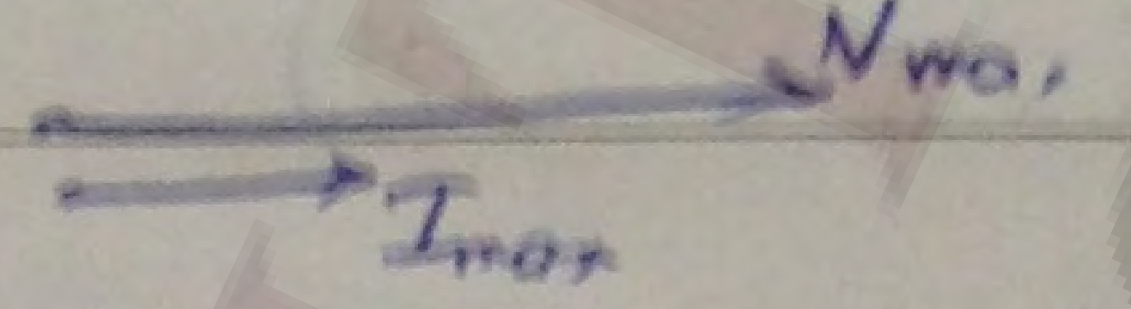
$$\boxed{\vec{Z}_C = \frac{1}{jwC}} \qquad \boxed{X_C = \frac{1}{wC}}$$

$$\Rightarrow V(t) = V_{max}\,Cos(wt + \phi_v) \xrightarrow{Phose} \vec{V}_{max} = |V_{max}|\,e^{j\phi_v}$$

$$i(t) = I_{max}\,(wt + \phi_i) \xrightarrow{Phose} \vec{I}_{max} = |I_{max}|\,e^{j\phi_i}$$

$$\vec{Z} = \frac{V_{max}}{I_{max}}$$

$\boxed{a}$ $R = \dfrac{V_m}{I_m}$

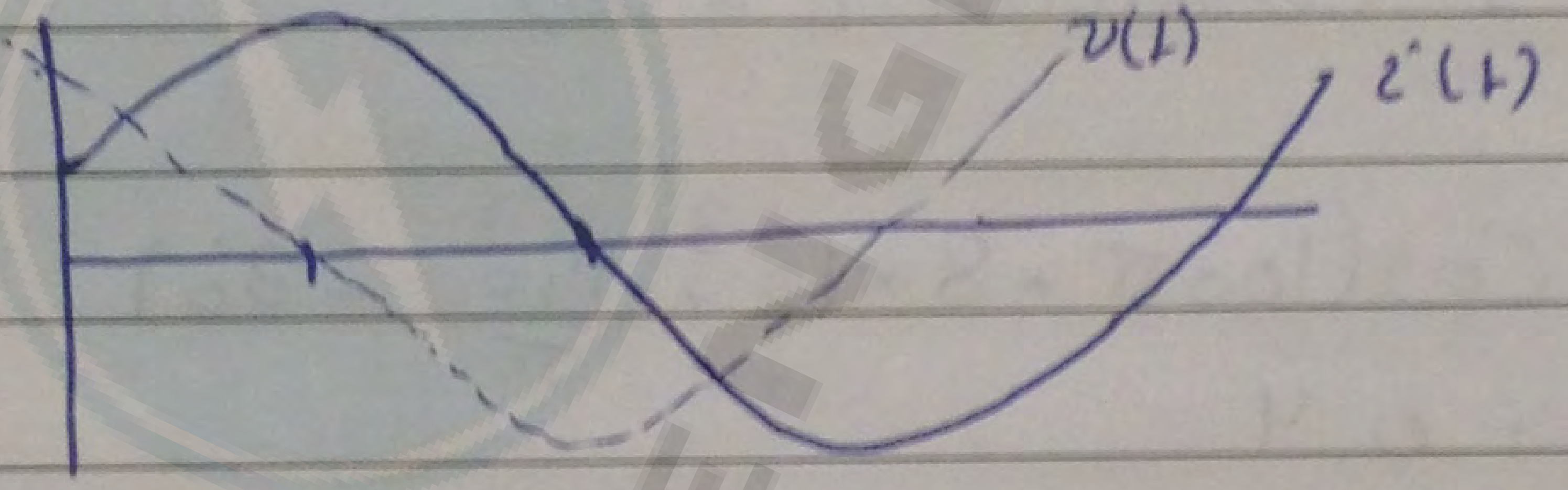


$\boxed{b}$ $L \longrightarrow Z = \dfrac{V_{max}}{I_{max}} = \dfrac{|V_m| \angle \phi_v}{|I_m| \angle \phi_i} = Z \angle \phi_v - \phi_i = \boxed{\omega L \angle 90°}$

$\boxed{c}$ $C \rightarrow Z = \dfrac{|V_m| \angle \phi_v}{|I_m| \angle \phi_i} = \dfrac{1}{j\omega c} = \boxed{Z \angle -90}$

exp $v(t) = 100 \cos(\omega t + 30) = 100 \angle 30°$

$i(t) = 5 \cos(\omega t - 60) = 5 \angle -60°$



$P(t) = v(t)\, i(t)$ ⤳ Instantanuous Power

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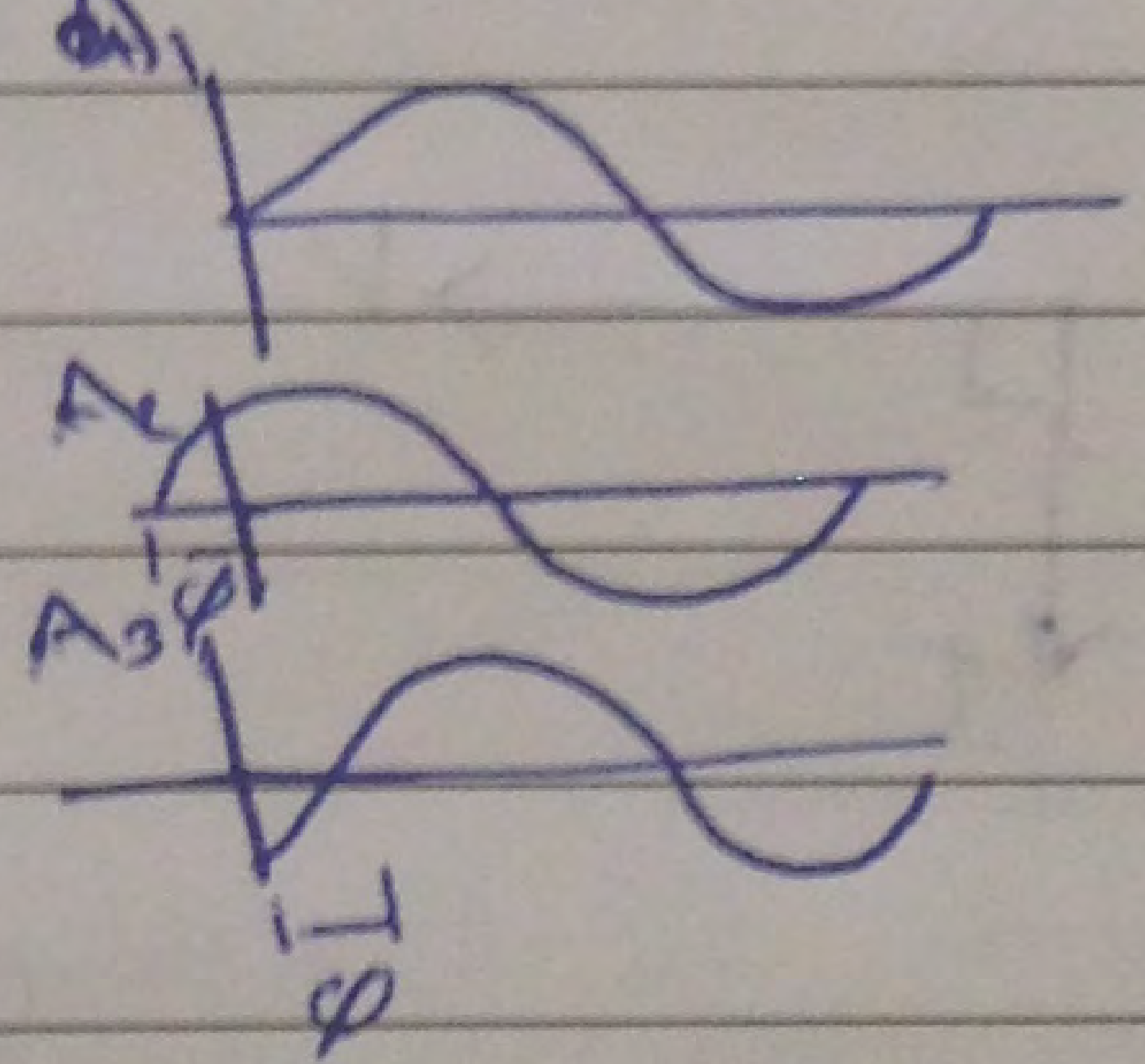
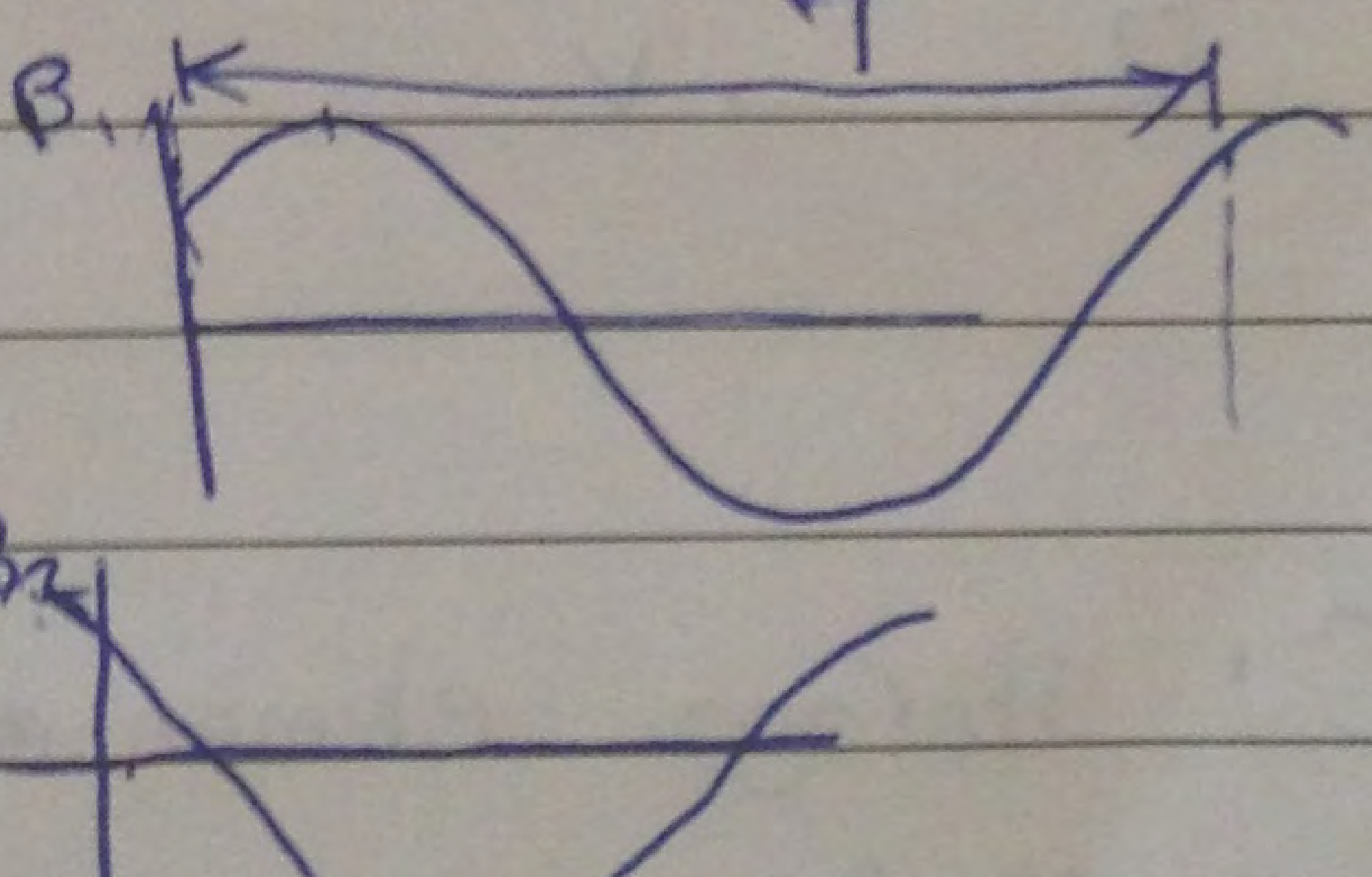
ex~~R~~ $A_1 = A_m \sin(\omega t)$

$A_2 = A_m \sin(\omega t + \phi)$

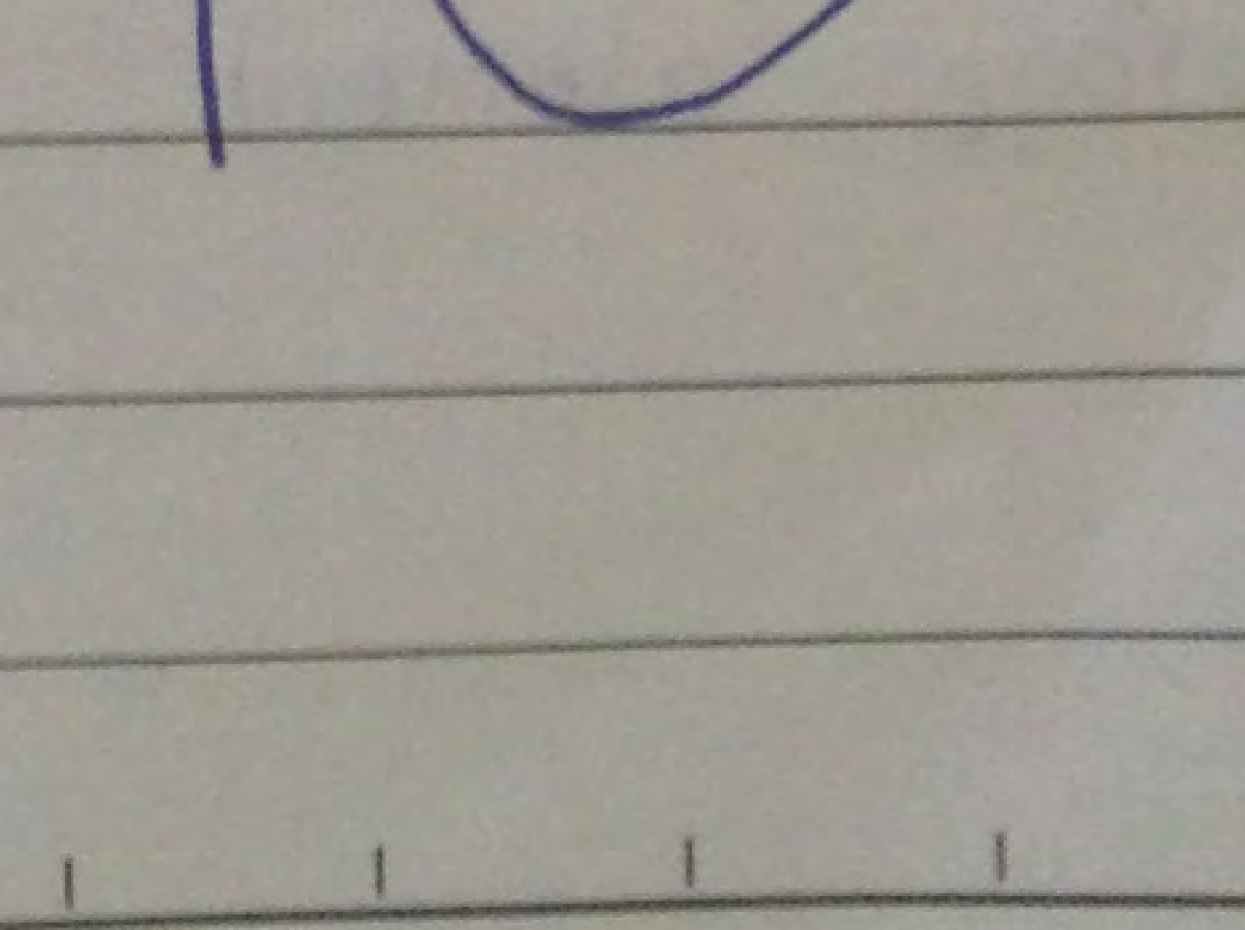
$A_3 = A_m \sin(\omega t - \phi)$





exp $B_1 = B_m \cos(\omega t - \phi)$

$B_2 = B_m \cos(\omega t + \phi)$



$T = 360^\circ = 2\pi$ rad (one Period)

$\Rightarrow A_1 = B_m \cos(2\pi t - \phi)$

$W = \frac{2\pi}{T} = 2\pi \longrightarrow T = 1$ sec

$\Rightarrow A_2 = B_m \cos(100\pi t - \phi)$

$W = \frac{2\pi}{100\pi} = \frac{1}{50}$ sec $= 20$ msec

$\Rightarrow A(8ms) = 1 \cos(100\pi t - 30^\circ)$

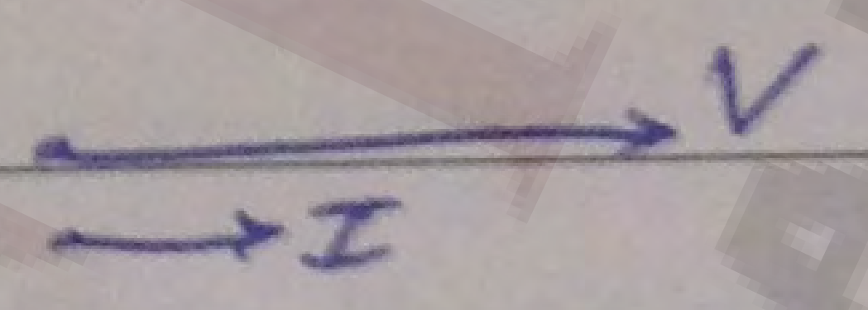
$= \cos(100\pi * 8 * 10^{-3} * \frac{180}{\pi} - 30^\circ)$

$= -0.4$

*

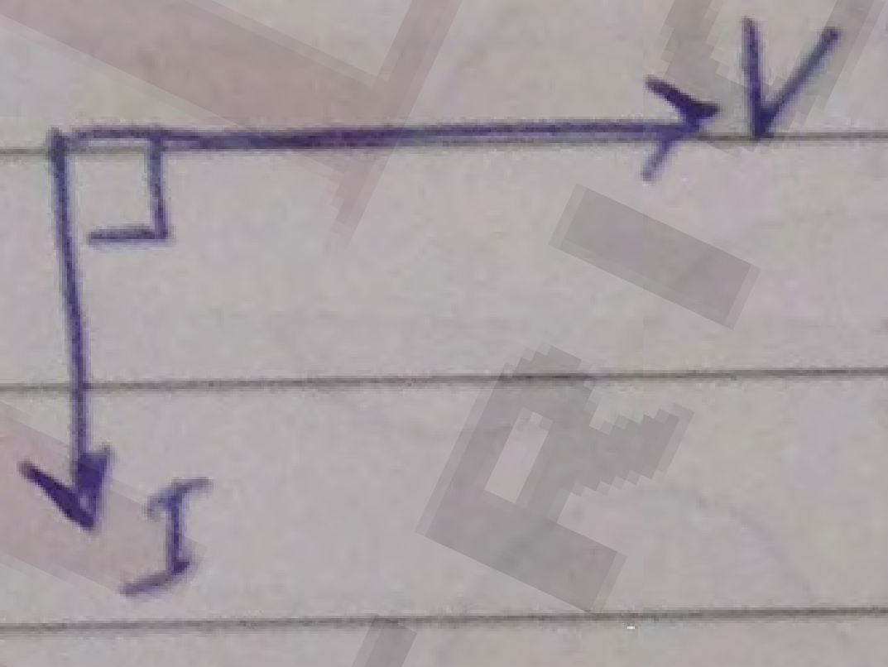
Pure

1) R



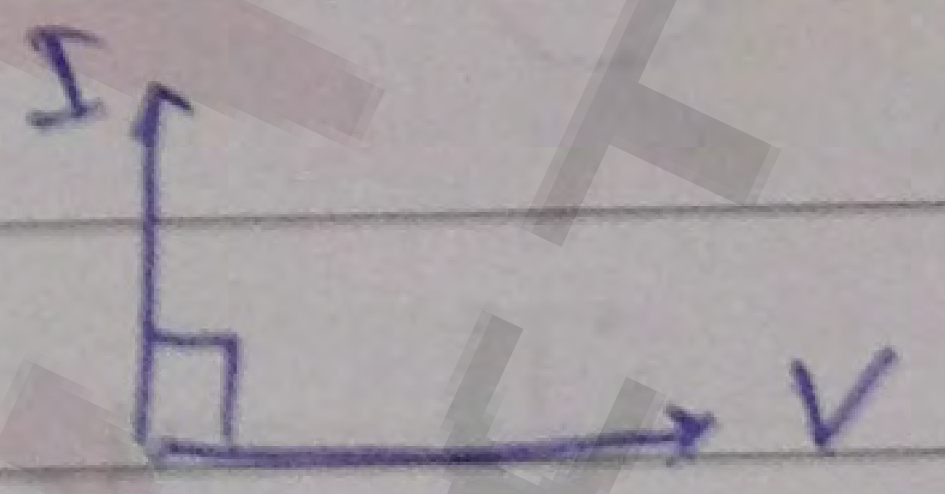
$\phi = \phi_v - \phi_i = 0$ (Unity PF)

2) Z_L



$\phi = \phi_v - \phi_i = 90$ (Zero PF lagging)

3) Z_C



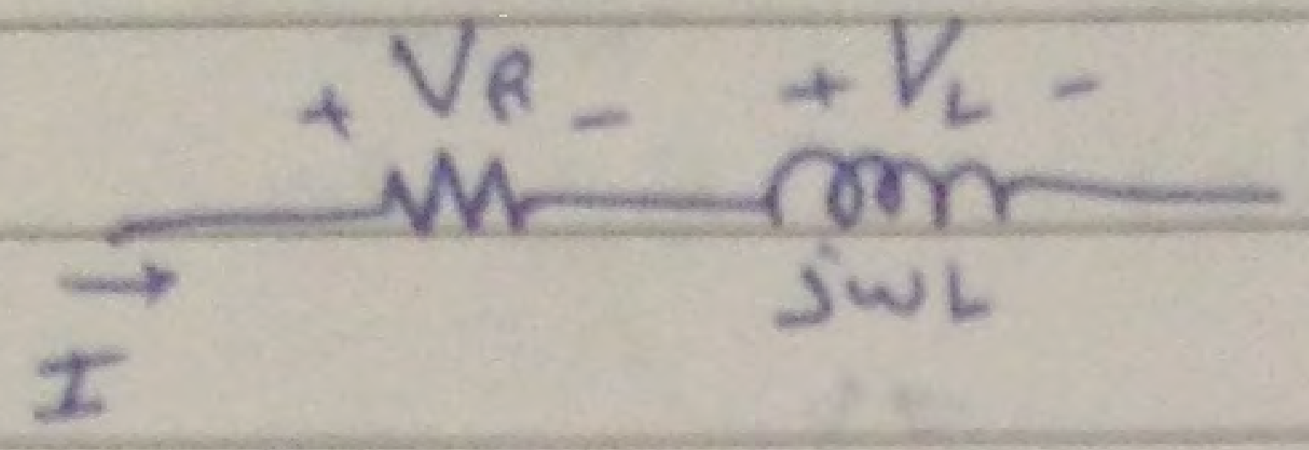
$\phi = \phi_v - \phi_i = -90$ (Zero PF leading)

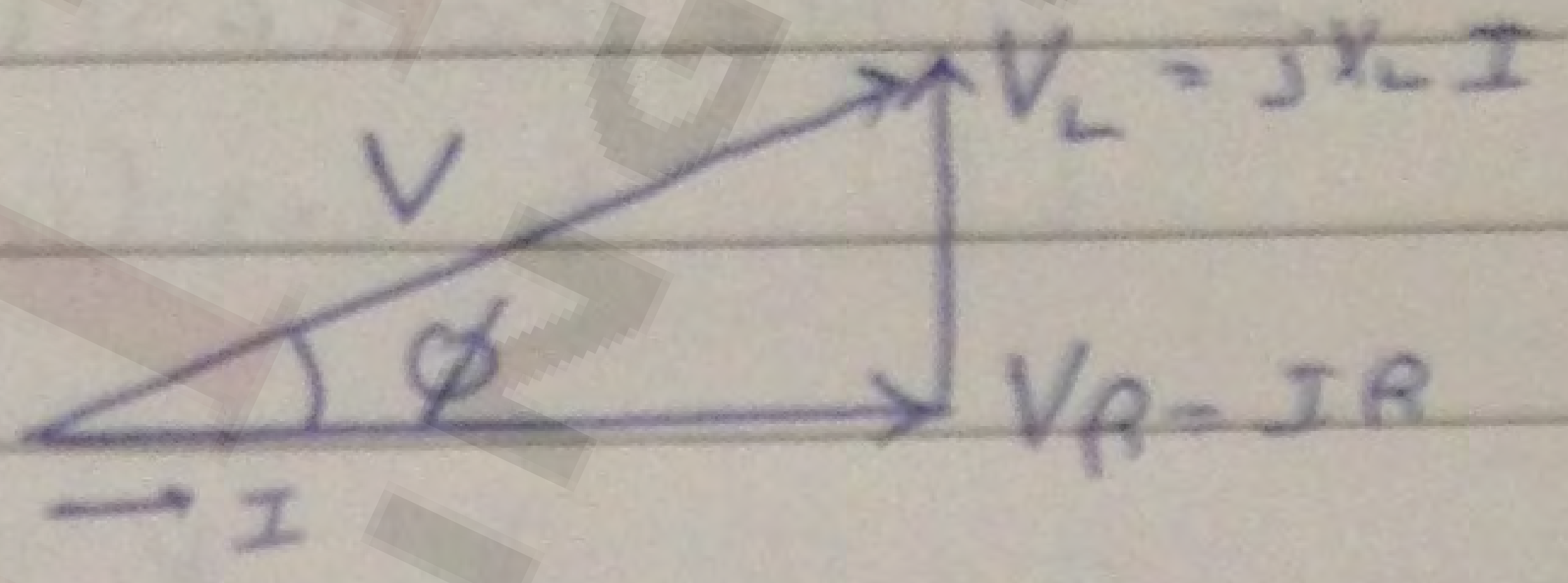
C : storage element in form of electric field.

L : storage element in form of magnetic field.

* Practical.

1) L : inductive (consists of inductor and R)

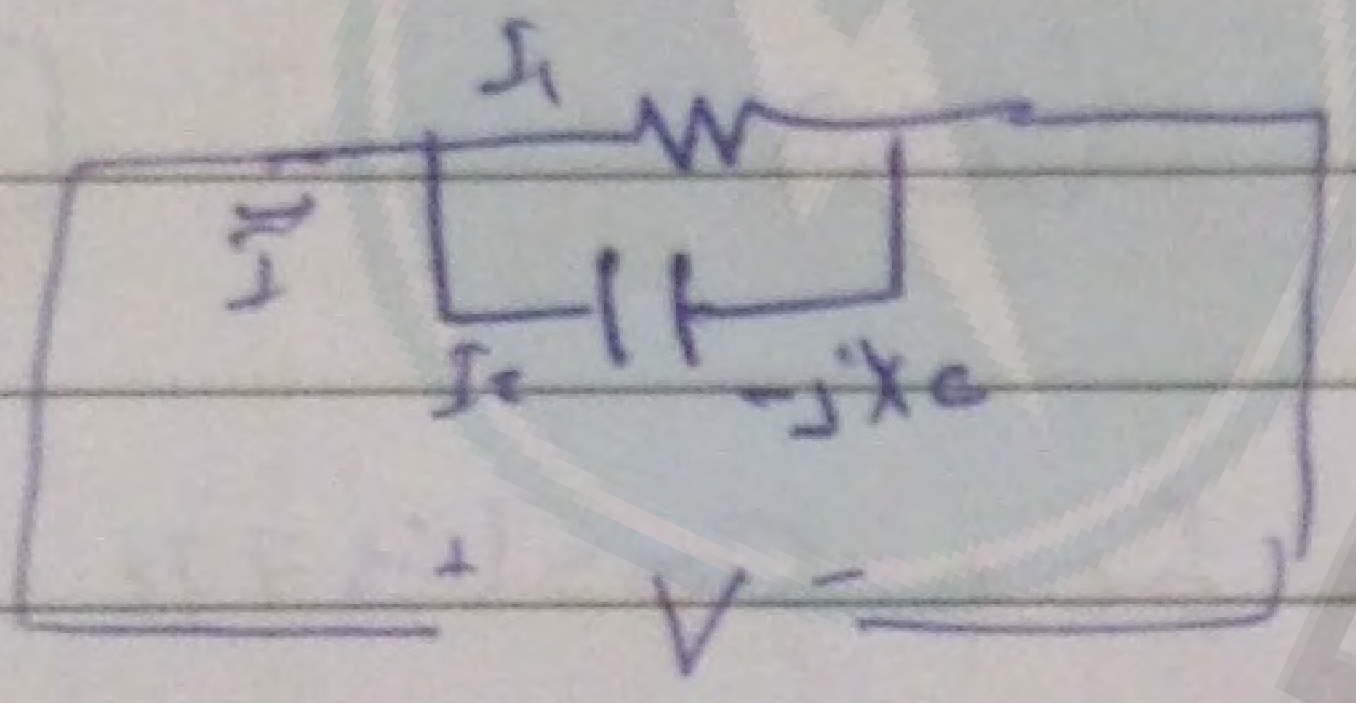


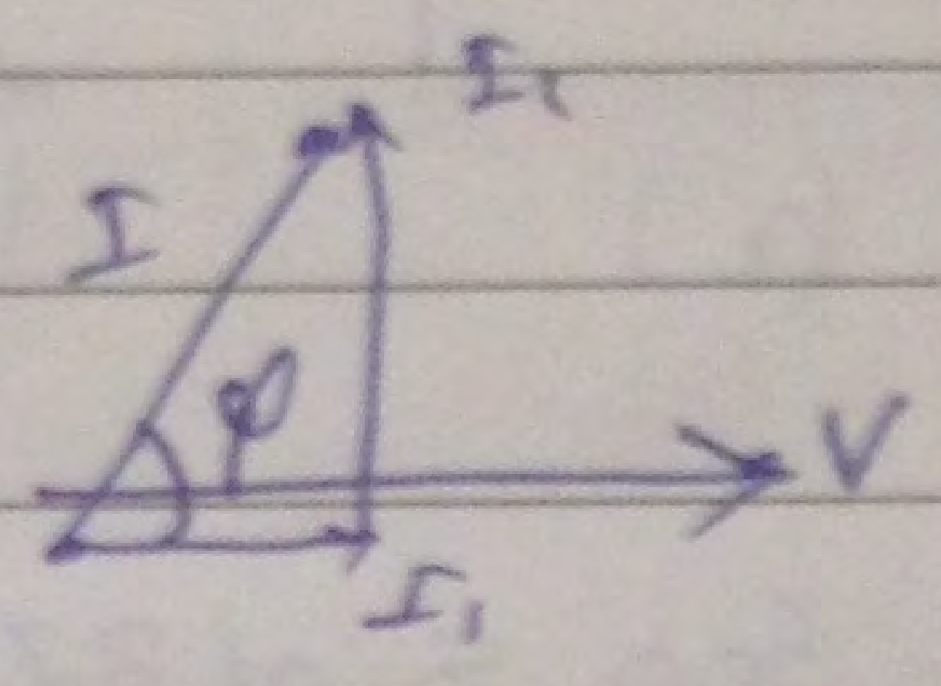


lagging PF

- Once I lags V the ϕ is +ve
- once I leads V the ϕ is -ve

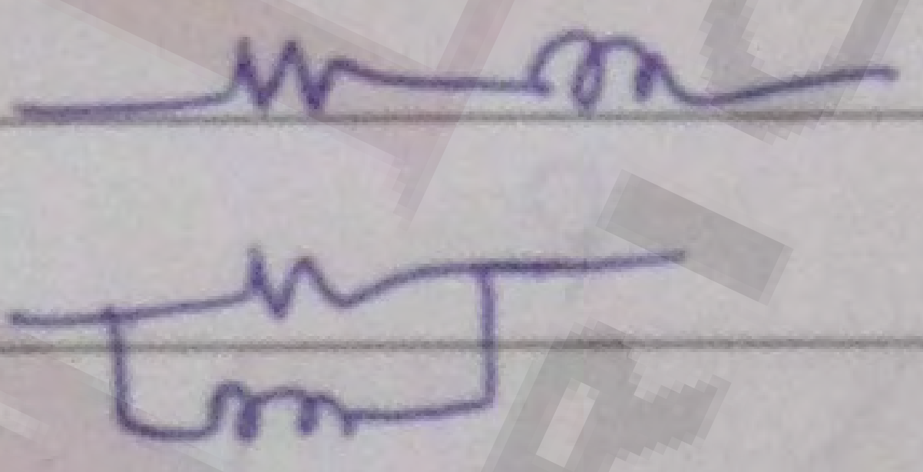
2) C : Capacitive



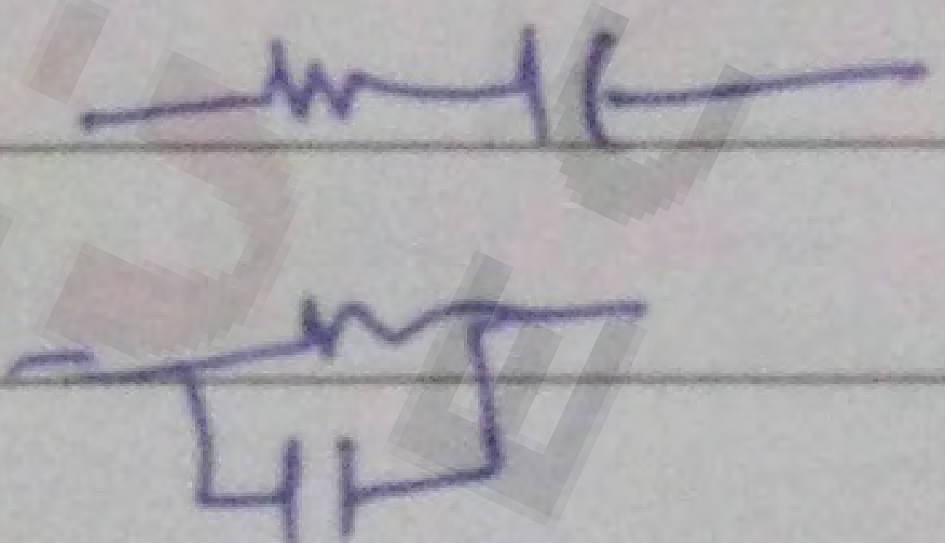


leading PF

* $0 < \phi < 90 \Rightarrow$ inductive



* $-90 < \phi < 0 \Rightarrow$ Capacitive

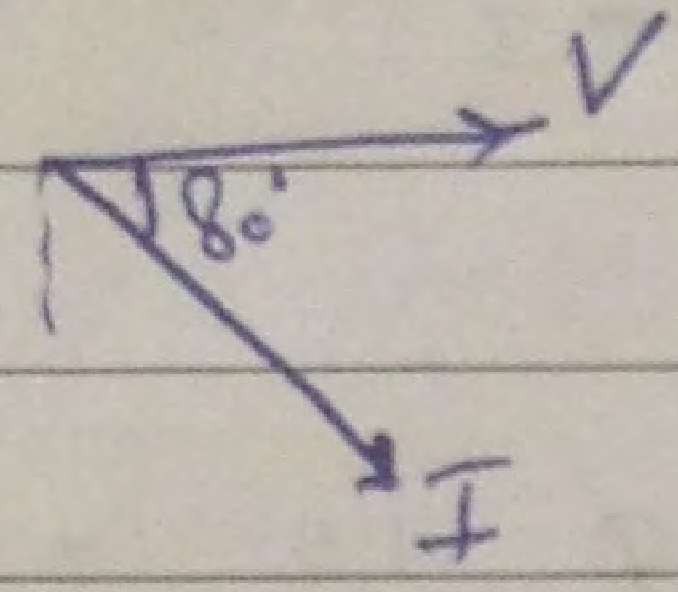


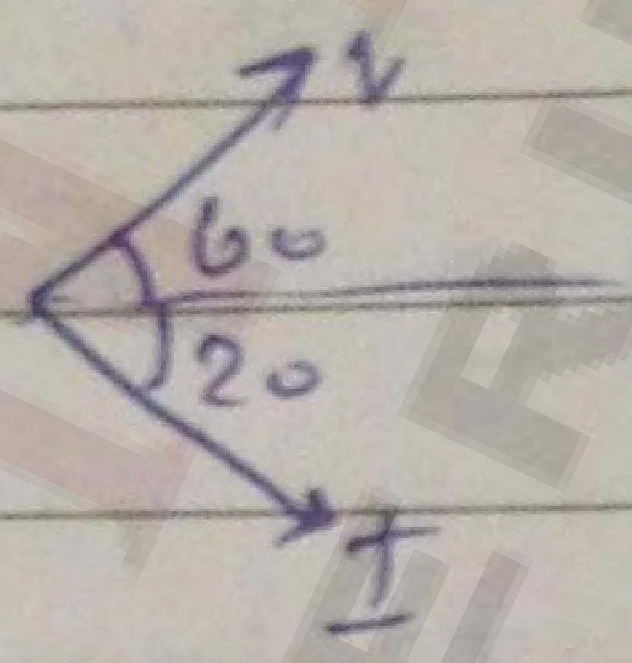
exp $v(t) = V_m \cos(wt + \phi_v)$

$i(t) = V_m \cos(wt + \phi_i)$

- $v(t) = 100 \cos(100\pi t + 60) = 100\angle 60$

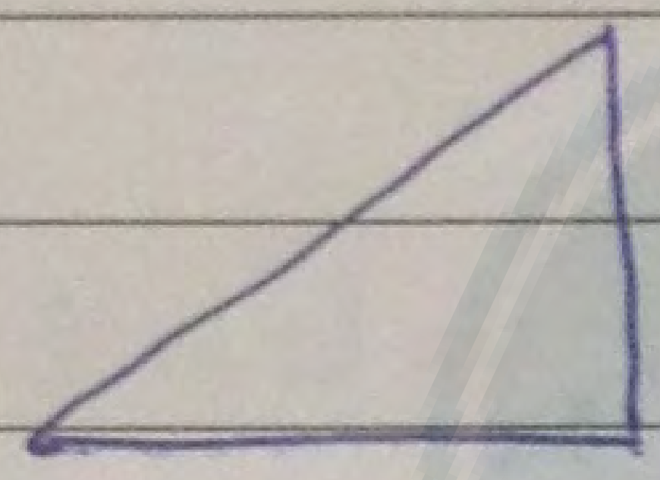
$i(t) = 5 \cos(100\pi t - 20) = 5\angle -20$



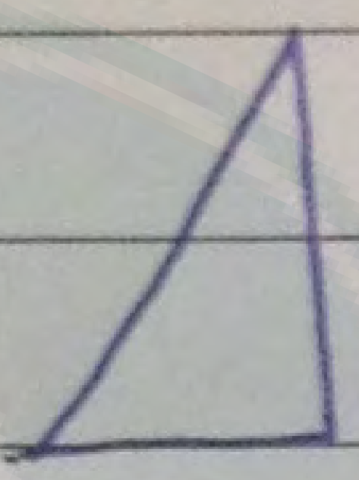


$\bar{Z} = \frac{100}{5}\angle 60+20 = 20\angle 80$ – impedance triogle

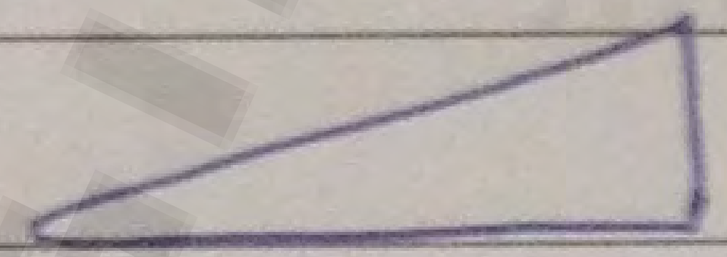
$= R + jX$



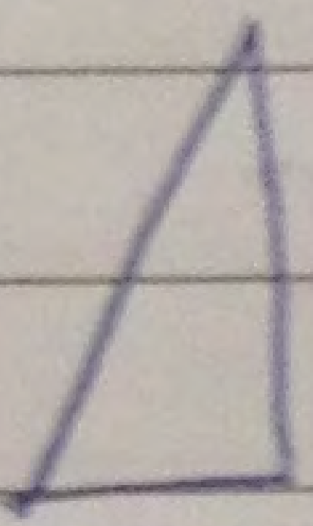
(a)



(b)



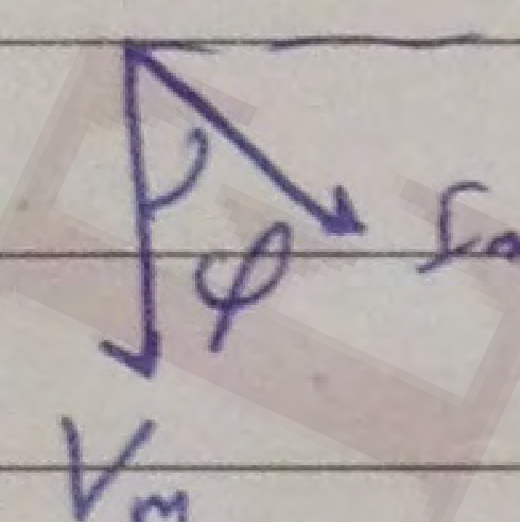
(c)



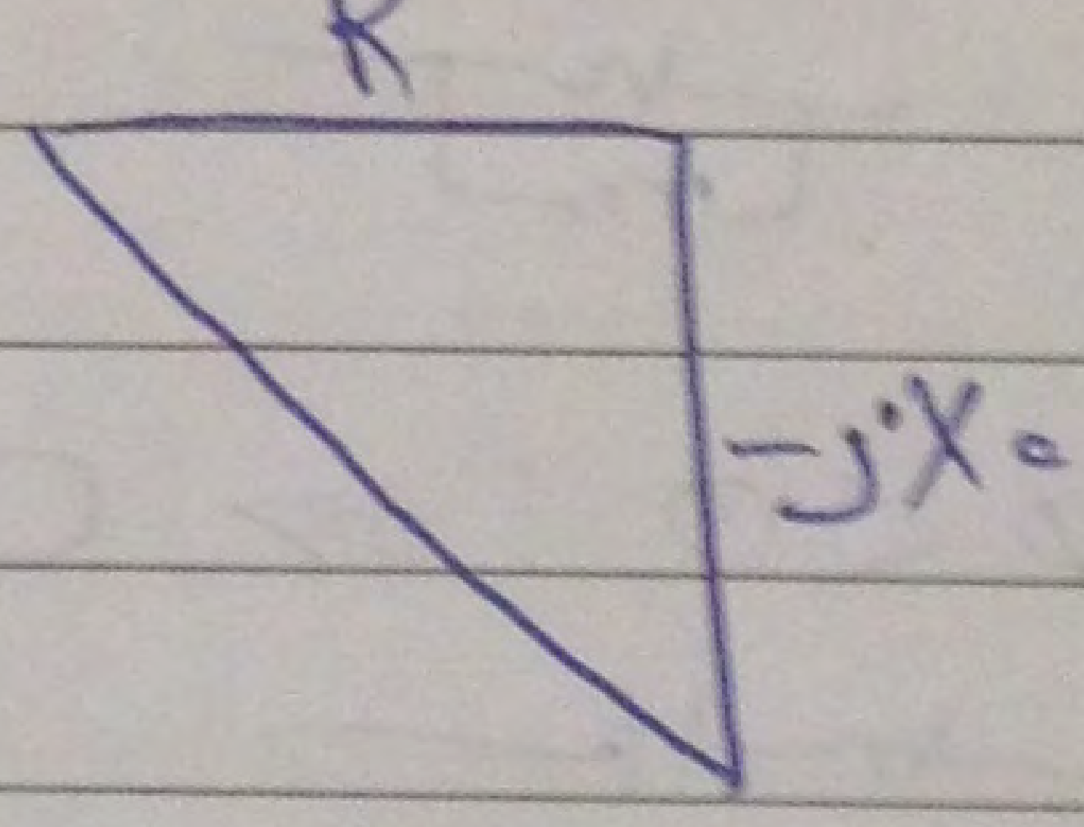
$\boxed{d}$

- $v(t) = 500 \sin(377t) = 500 \cos(377t - 90) = 500\angle -90$

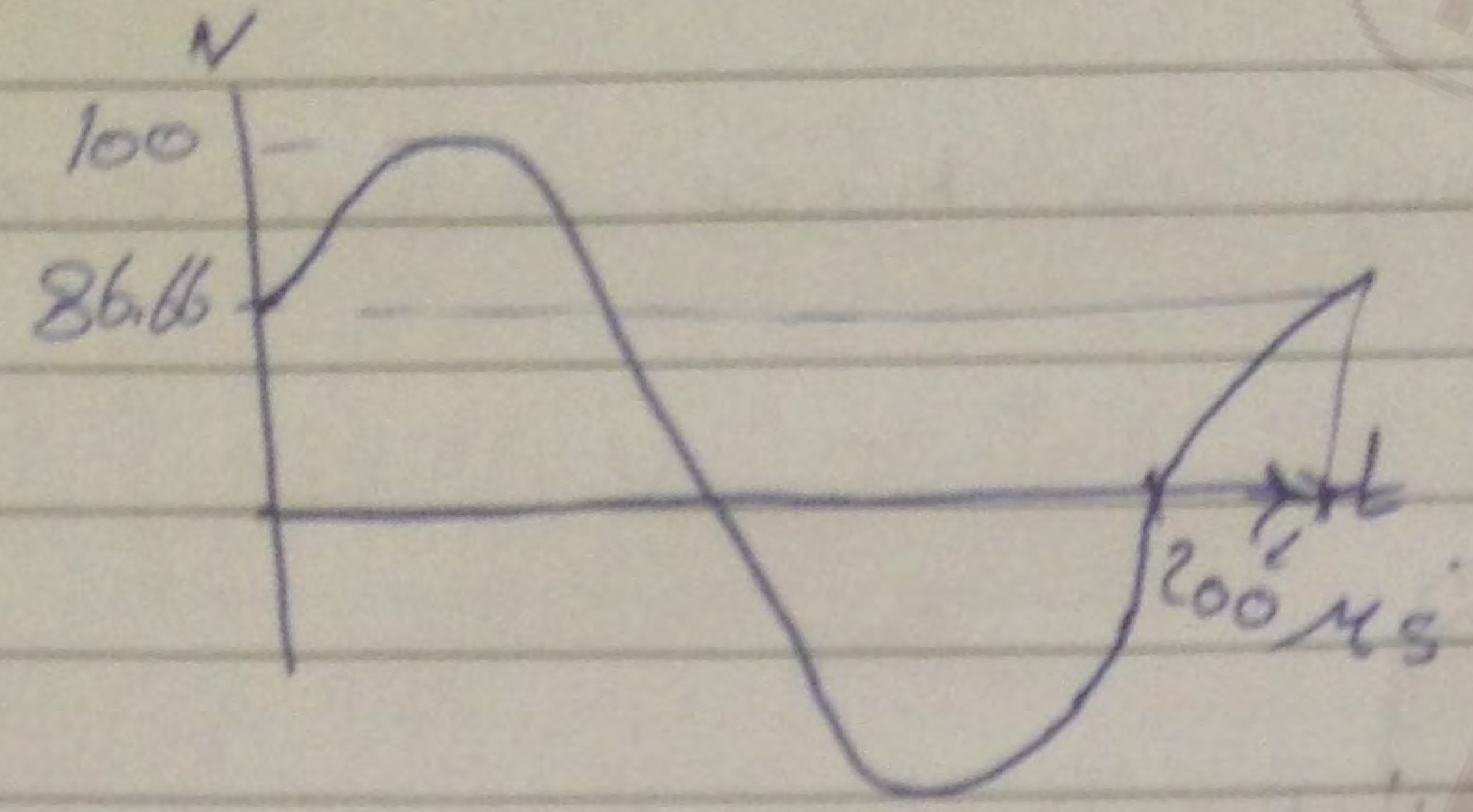
$i(t) = 10 \cos(377t - 30°) = 10\angle -30$



ϕ is -ve (Capacitive



exp



$T = \frac{2\pi}{200} = \frac{\pi}{100}$

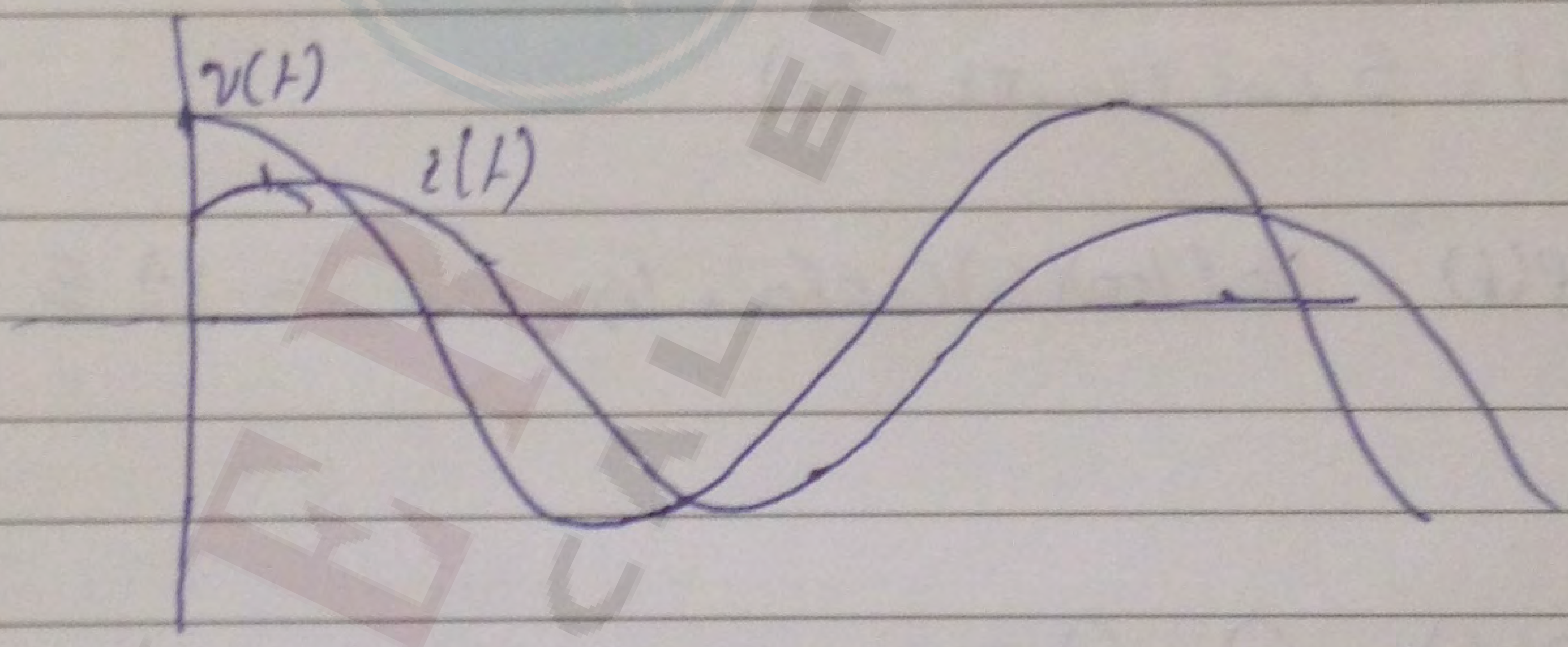
$x(t) = 100\cos(\omega t - 30)$ ~ $v(0) = 86.6 = 100\cos\phi \rightarrow \phi = 30^\circ$

$v(t) = 100\sin(10\pi t + \phi)$ ~ $v(0) = 86.6 = 100\sin\phi \rightarrow \phi = 60^\circ$

$= 100\sin(10\pi t + 60)$

exp $v(t) = V_m \cos(\omega t)$

$i(t) = I_m \cos(\omega t - \phi)$ lag

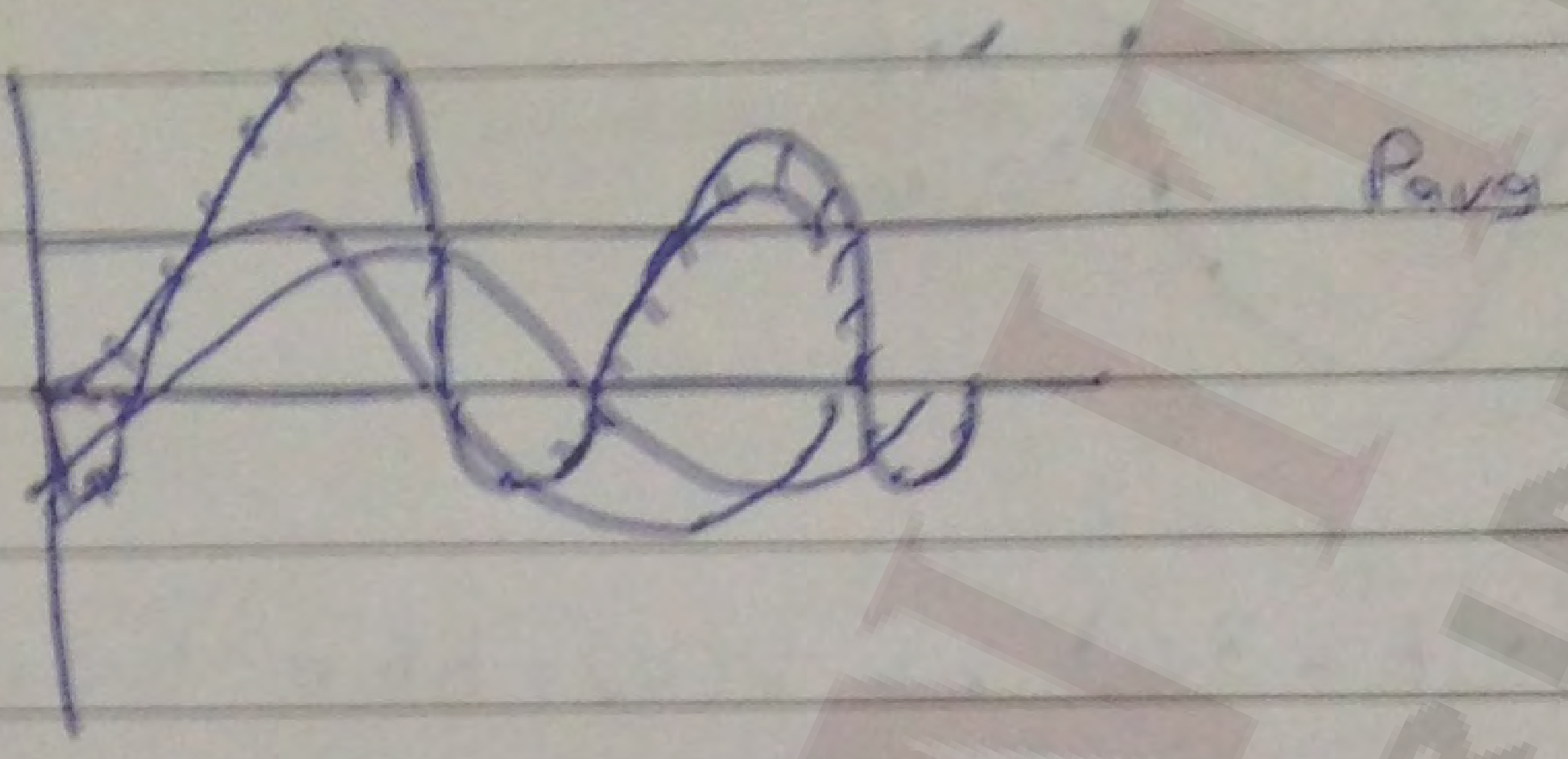


$$P(t) = \frac{1}{2} V_m I_m \left[\cos\phi + \cos(2\omega t - \phi)\right]$$

$$= \frac{1}{2} V_m I_m \cos\phi + \frac{1}{2} V_m I_m \cos(2\omega t - \phi)$$

$$= \frac{1}{2} V_m I_m \cos\phi + \frac{1}{2} V_m I_m \cos\phi \cos 2\omega t + \frac{1}{2} V_m I_m \sin 2\omega t \sin\phi$$

⇒ Instan. Power Frequency is twice either current or Voltage Freq.



$$P_{avg} = P_{max} - \frac{P_{max} - P_{min}}{2}$$

$$P_{avg} = P_{min} + \frac{P_{max} - P_{min}}{2}$$

exp $v(t) = 100 \cos 100\pi t$

$i(t) = 5 \cos(100\pi t - 60)$

$p(t) = \frac{1}{2}(100)(5)\cos 60 + \frac{1}{2} \ldots = 125$

$$P(t) = P + P\cos 2wt + Q \sin 2wt$$

$$P = 1/2\, V_m I_m \cos(\phi_v - \phi_i)$$

$$Q = 1/2\, V_m I_m \sin(\phi_v - \phi_i)$$

$$\vec{S} = P + jQ \quad \text{(Complex Power)}$$

$$= \frac{1}{2} V_m I_m (\cos\phi + j\sin\phi)$$

$$= 1/2\, V_m I_m\, e^{j\phi} = 1/2\, V_m I_m\, e^{j(\phi_v - \phi_i)}$$

$$S = 1/2\, V_m e^{j\varphi_v} \cdot I_m e^{-j\varphi_i}$$

$$\boxed{\bar{S} = 1/2\, \bar{V}_{max}\, \bar{I}_{max}^{*}}$$

$$|S| = \boxed{AP = 1/2\, |V_m|\,|I_m|}$$

$p(t)$ → Watt

P_{avg} → Wtt

Q → VAR

AP → VA

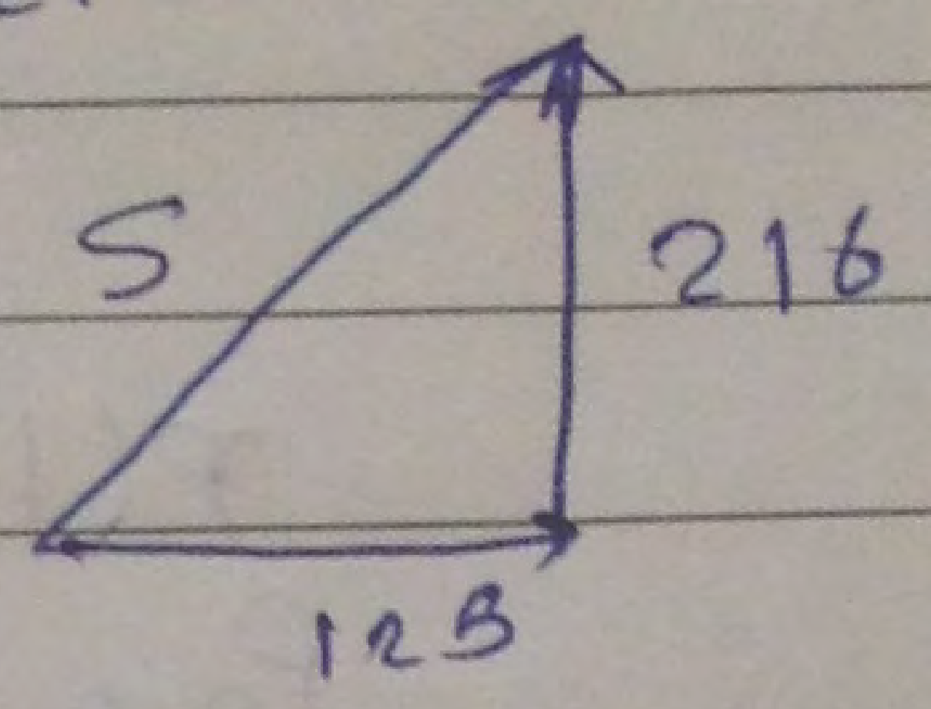
ex $\bar{V}_{max} = 100\angle 0$

$I_{max} = 5\angle -60$

$\bar{S} = 1/2\ V_m I_m^* = 1/2\ (100\angle 0)(5\angle 60) = 250 \cos 60 + j250 \sin 60$

$= 125 + j\ 250 * 0.866$

Power triangle



$P_{avg} = 125$ W

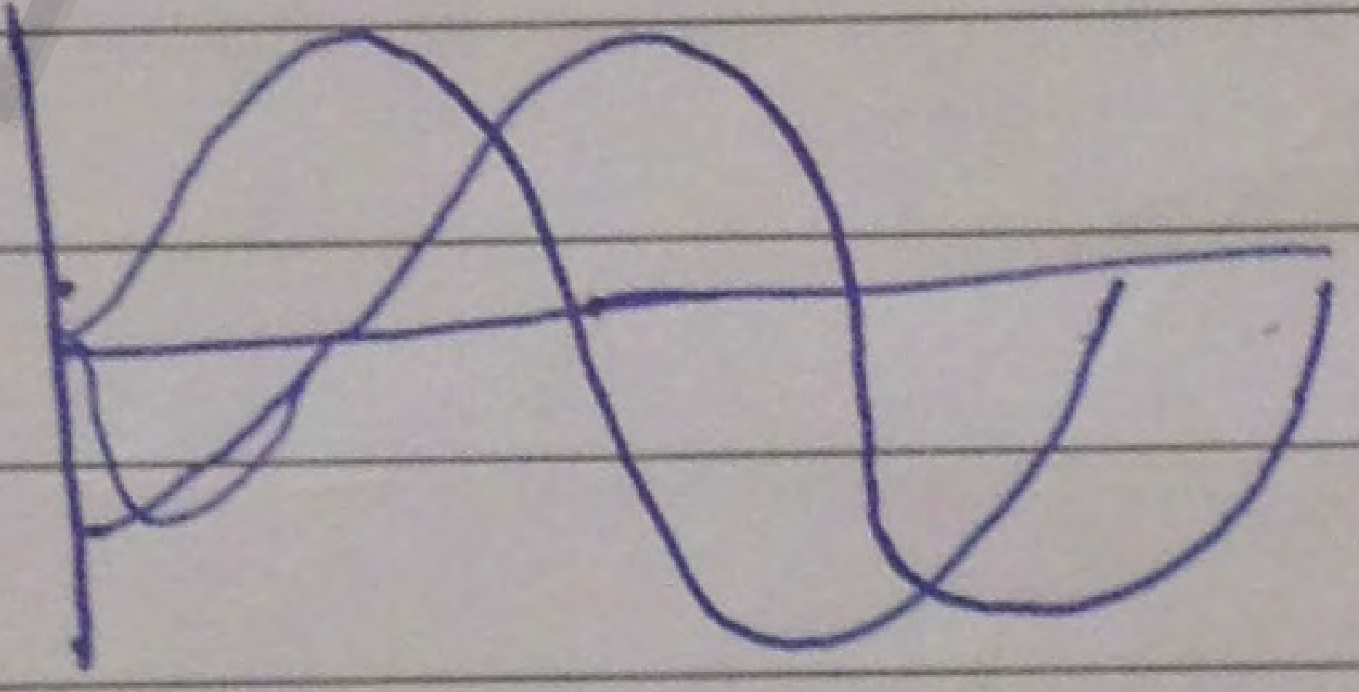
$Q = 216$ VAR

$|S| = 250$

$p(t) = 125 + 125 \cos 200\pi t + 216 \sin 200\pi t$

$= 125 + 250 \cos(200\pi t - 60)$

$P_{avg\,L} = 0$

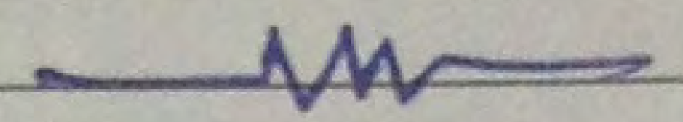


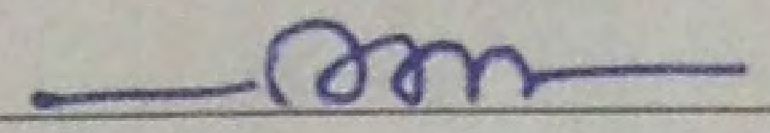
$p(t) = 0 + 250 \underbrace{\cos(200\pi t + 90)}$

$= \Phi$ P_{max}

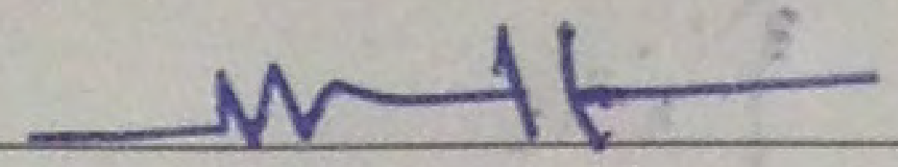
$= -1$ P_{min}

ex Draw the current , Voltage and instantanuous Power in the following elements :-

a) 

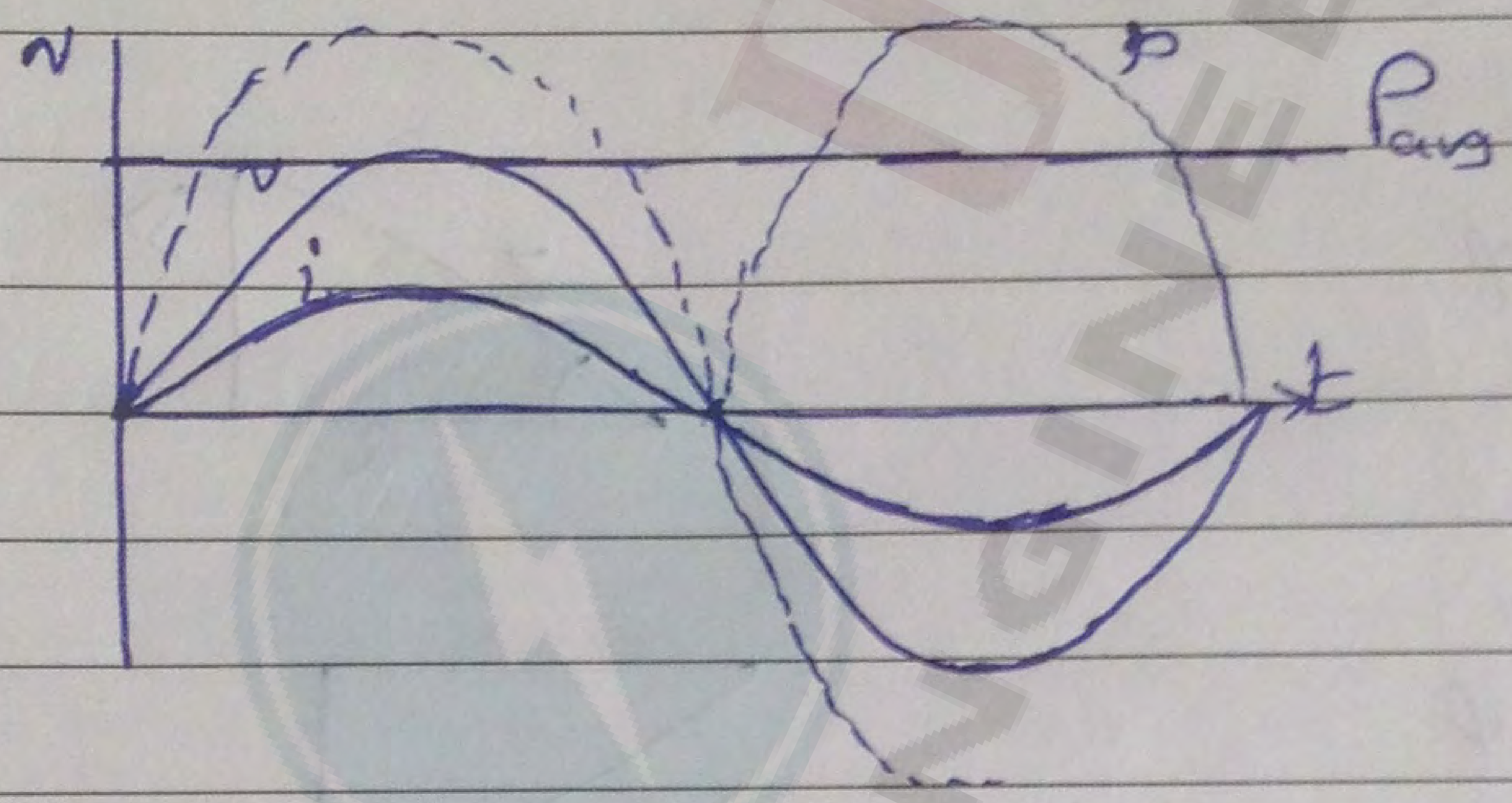
b)  c) 

d) 

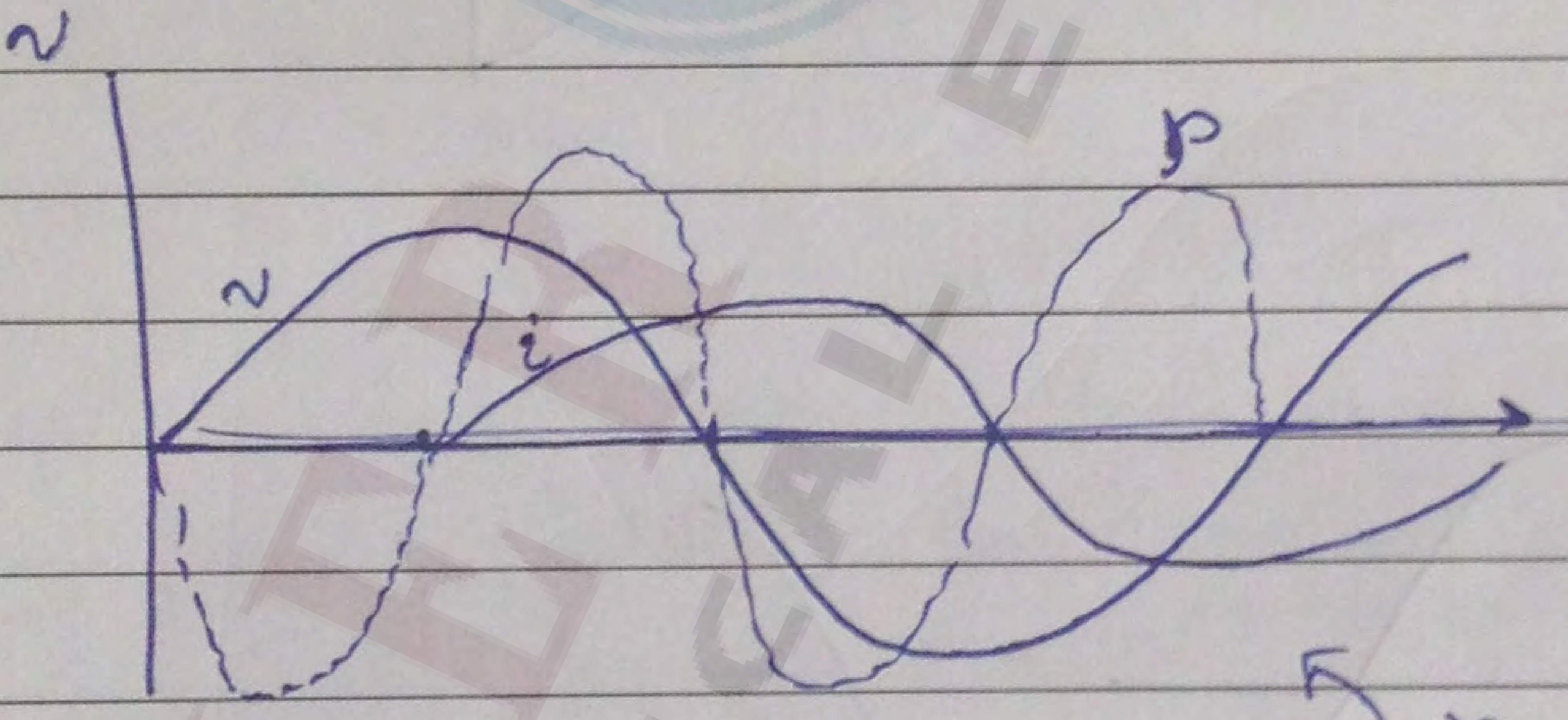
e) 

suppose $v(t) = V_m \sin \omega t$

a)



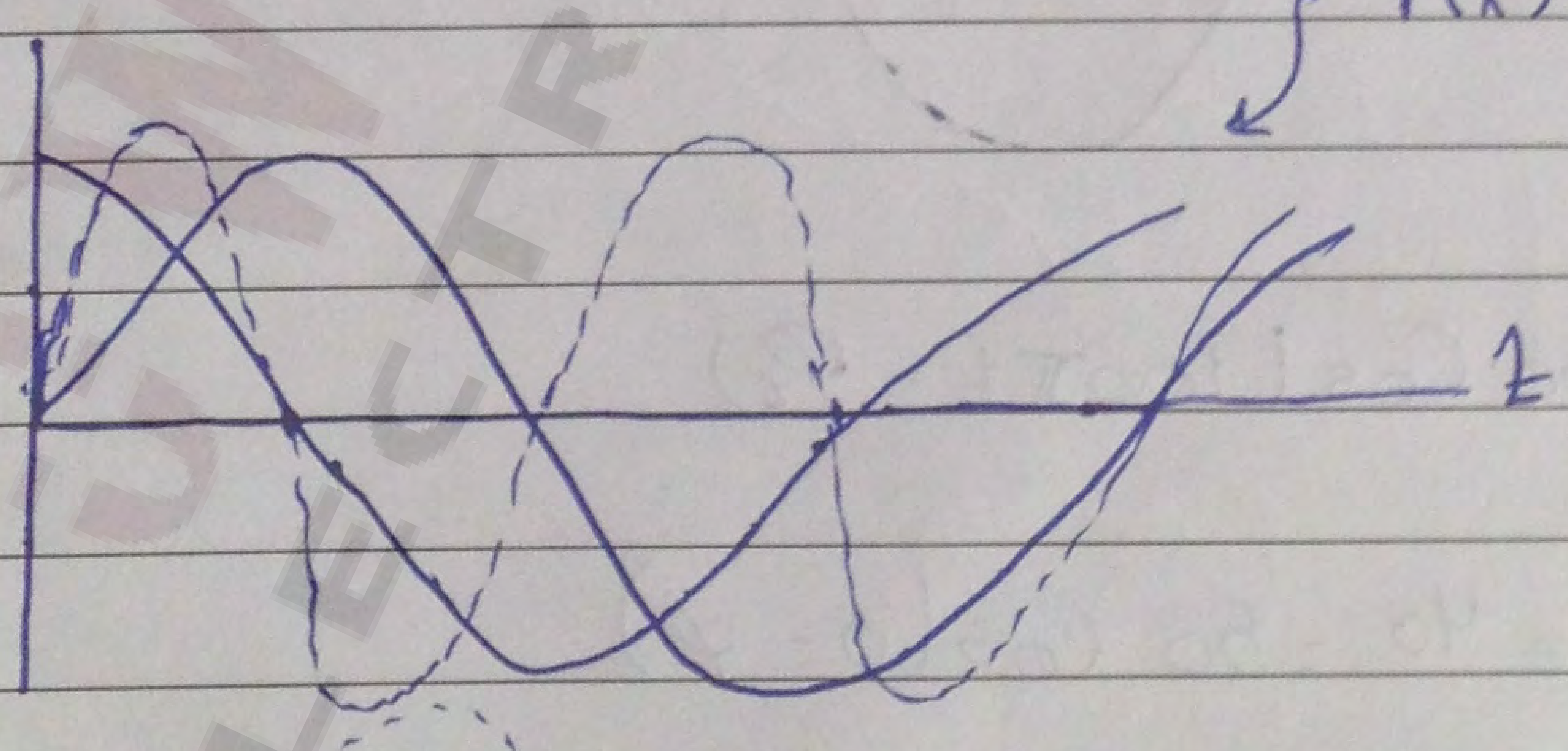
b)



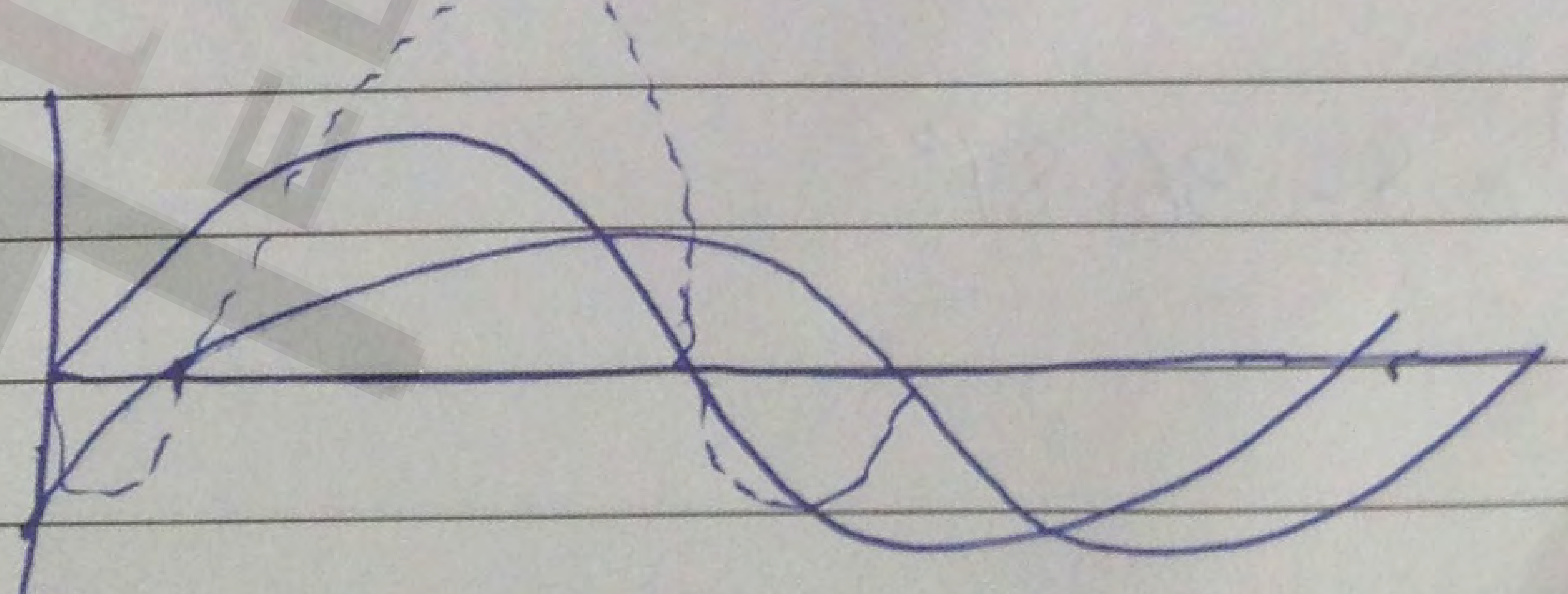
$P_{avg} = zero$

$P(x) = 0 + \frac{1}{2} V_m I_m \cos(2\omega t + 90)$

c)



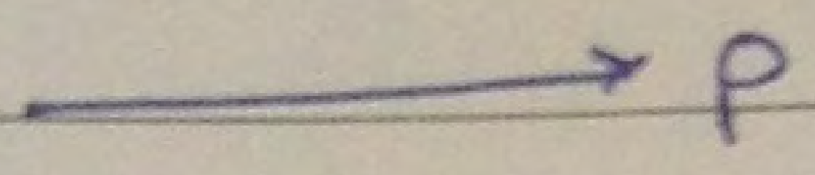
d)

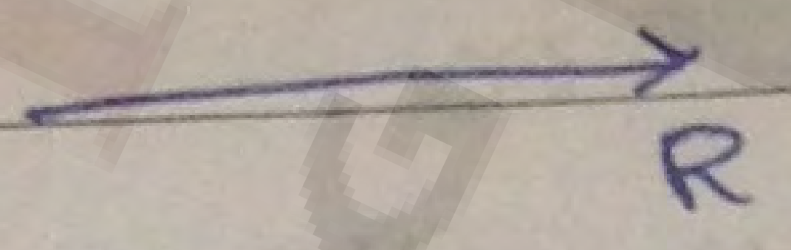


Power triangle

Imp. triangle

a)





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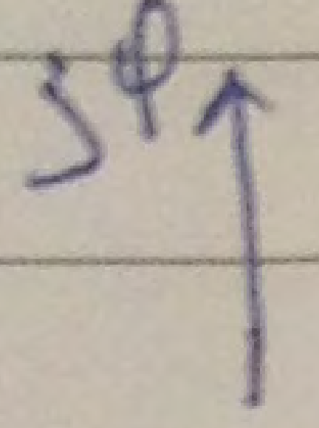
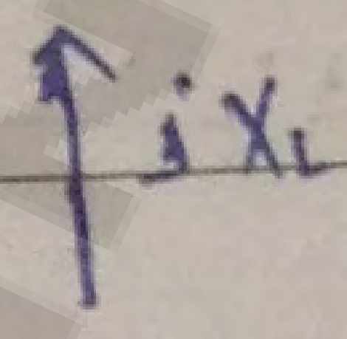
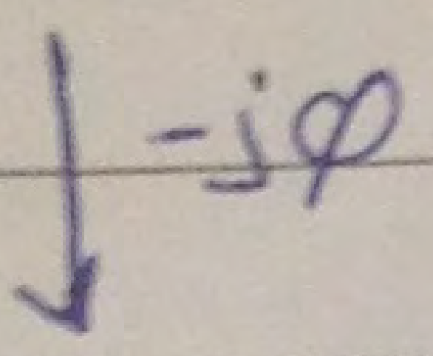
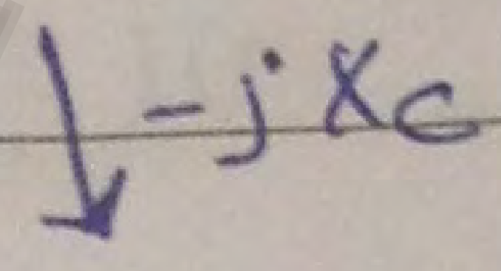


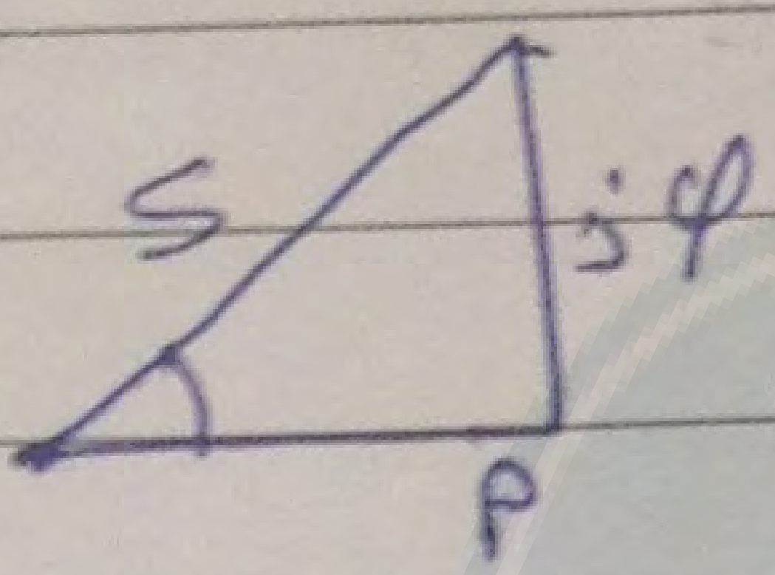


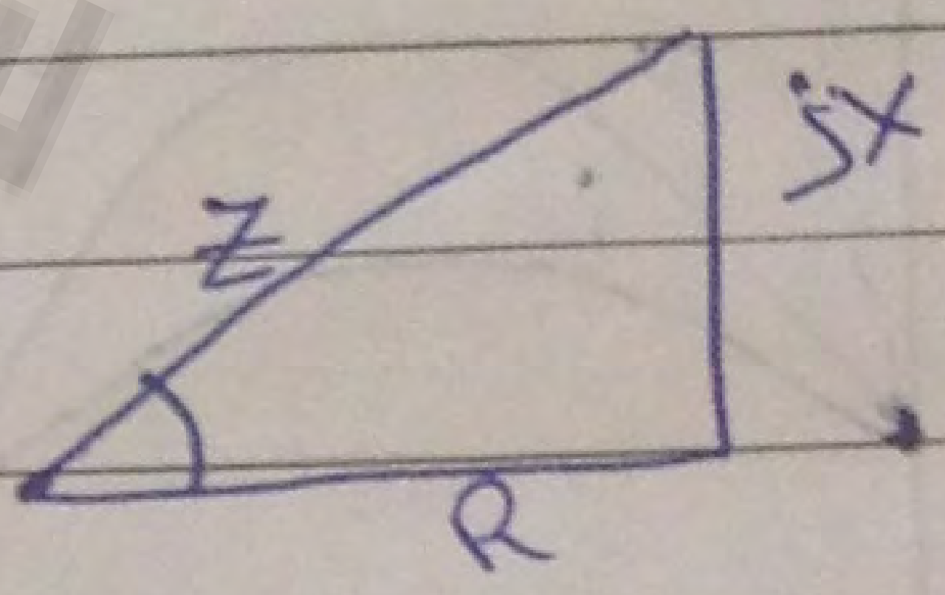
c)



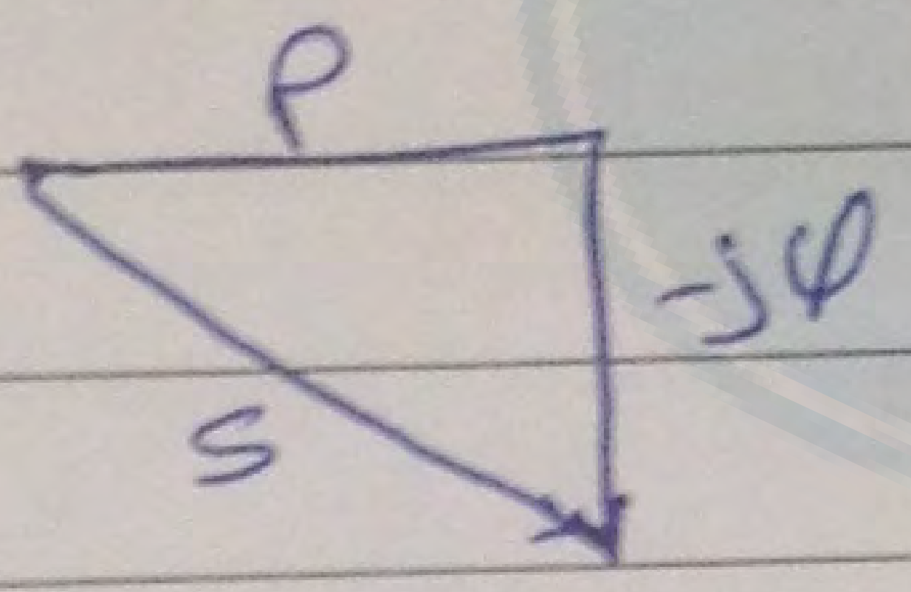


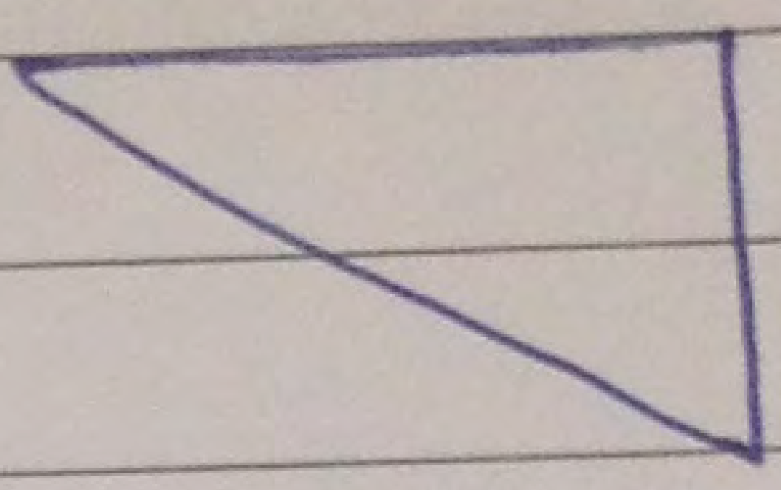
d)



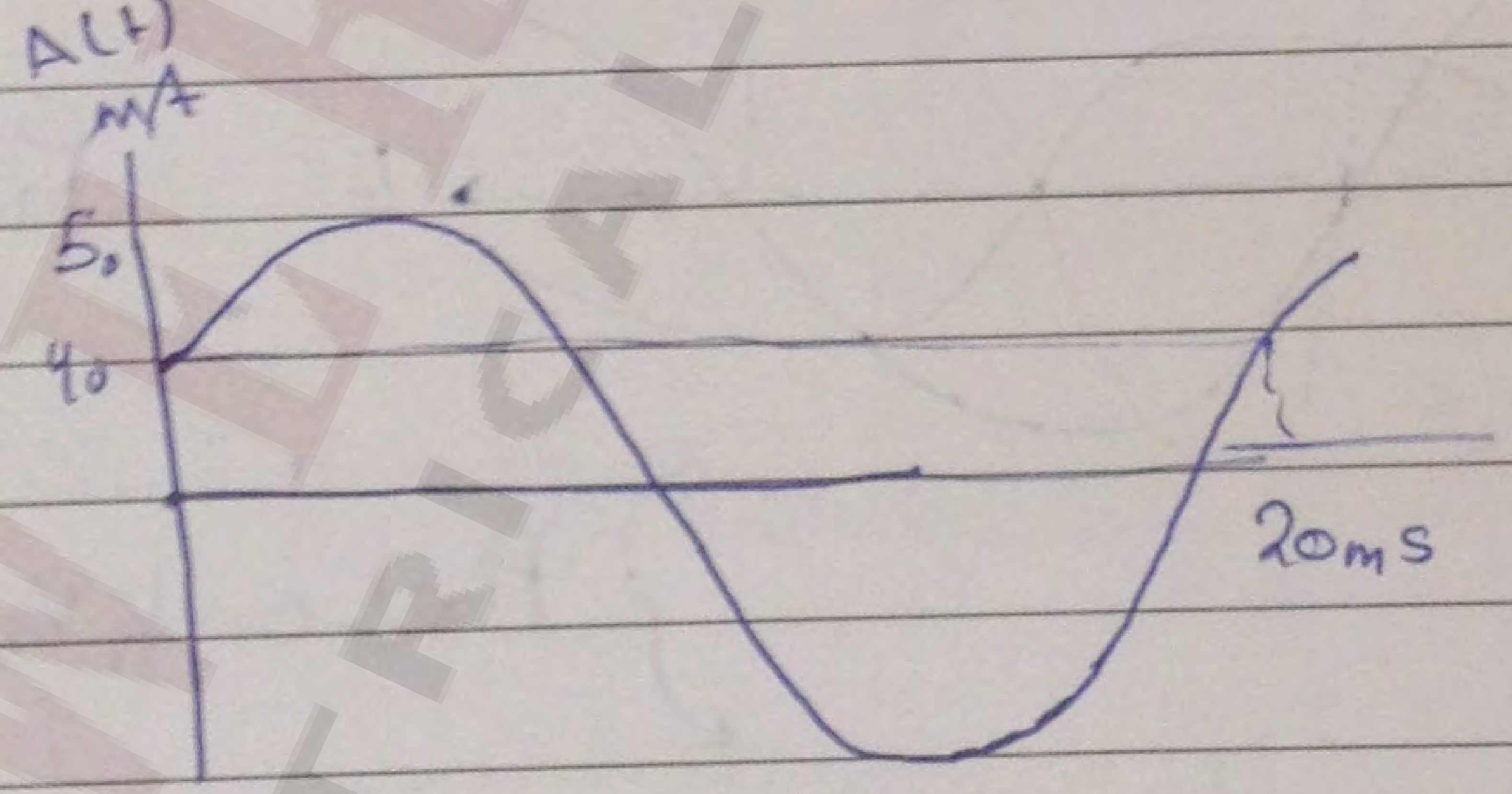


e)





Note

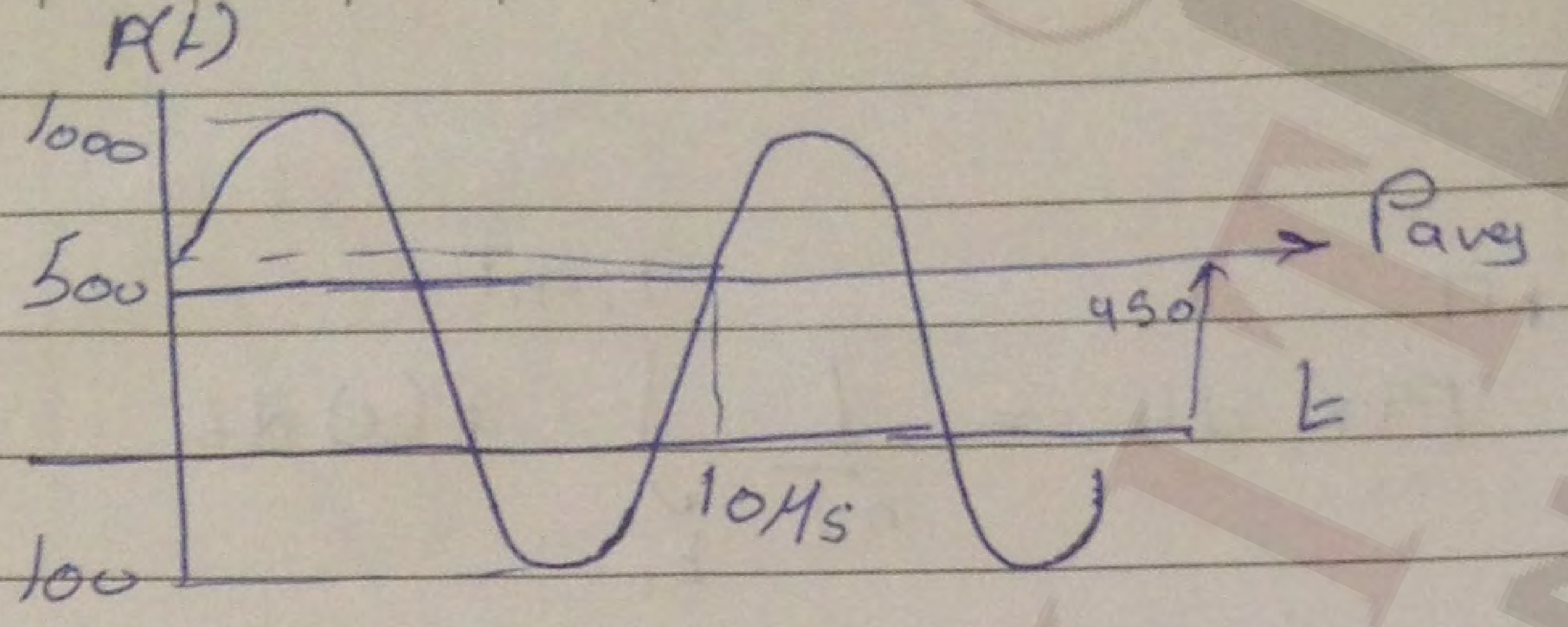


$$A(t) = 50 \cos(100\pi t - \varphi_c)$$

$$i(0) = 40 = 50 \cos(0 - \varphi)$$

$$\varphi_0 = \cos^{-1} 0.8 = 36.87^\circ$$

exam question



$$P_{av} = P_{max} - \frac{P_{max} - P_{min}}{2}$$

$$= 1000 - \frac{1000 - 100}{2} = 1000 - 550 = 450$$

$$p(t) = P_{max} \, Cos(\omega t - \phi_0)$$

$$50 = P(0) = 550 \, Cos(\omega t - \phi)$$

$$\phi = Cos^{-1} \frac{50}{550} = 84°$$

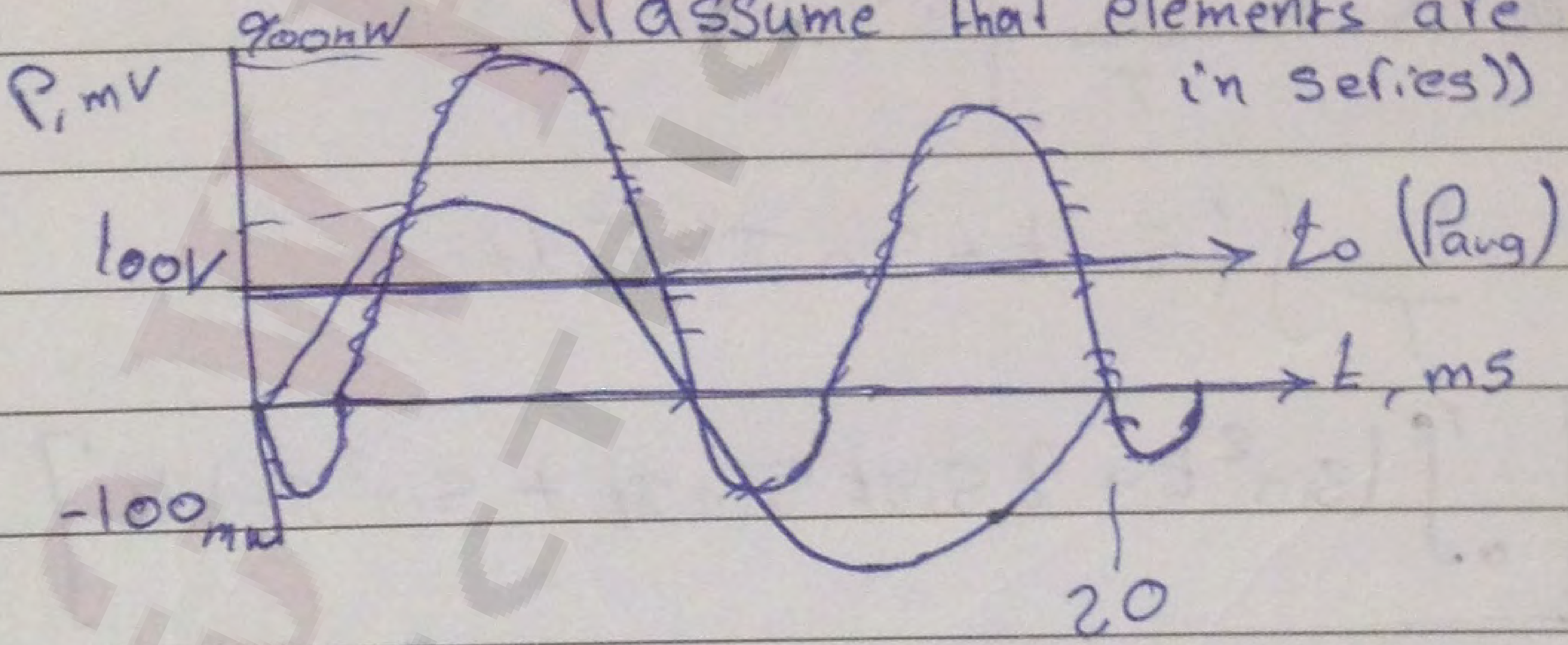
⇒ Suppose that given Voltage and P(t) find ① Current

((assume that elements are Connected in series))

② Pavg

③ Complex P

④ R, L or C



$$P_{DC} = I^2 R$$

$$P_{DC} = \frac{1}{T}\int_{t_1}^{t_1+T} P(x)\,dt = \frac{1}{nT}\int_{t_1}^{t_1+nT} P(x)\,dt$$

$$= \lim_{n\to\infty} \frac{1}{nT}\int_{-\frac{nT}{2}}^{\frac{nT}{2}} p(t)\,dt$$

$$P_{DC} = I^2 R = \frac{1}{T}\int_0^T R\, i^2(t)\,dt$$

$$\boxed{I = \sqrt{\frac{1}{T}\int_0^T i^2(t)\,dt}}$$ effective current (RMS)

~~R~~ $$\boxed{P = I_{eff}^2 R}$$

$\Rightarrow$ $i(t) = \sin t + \sin \pi t$

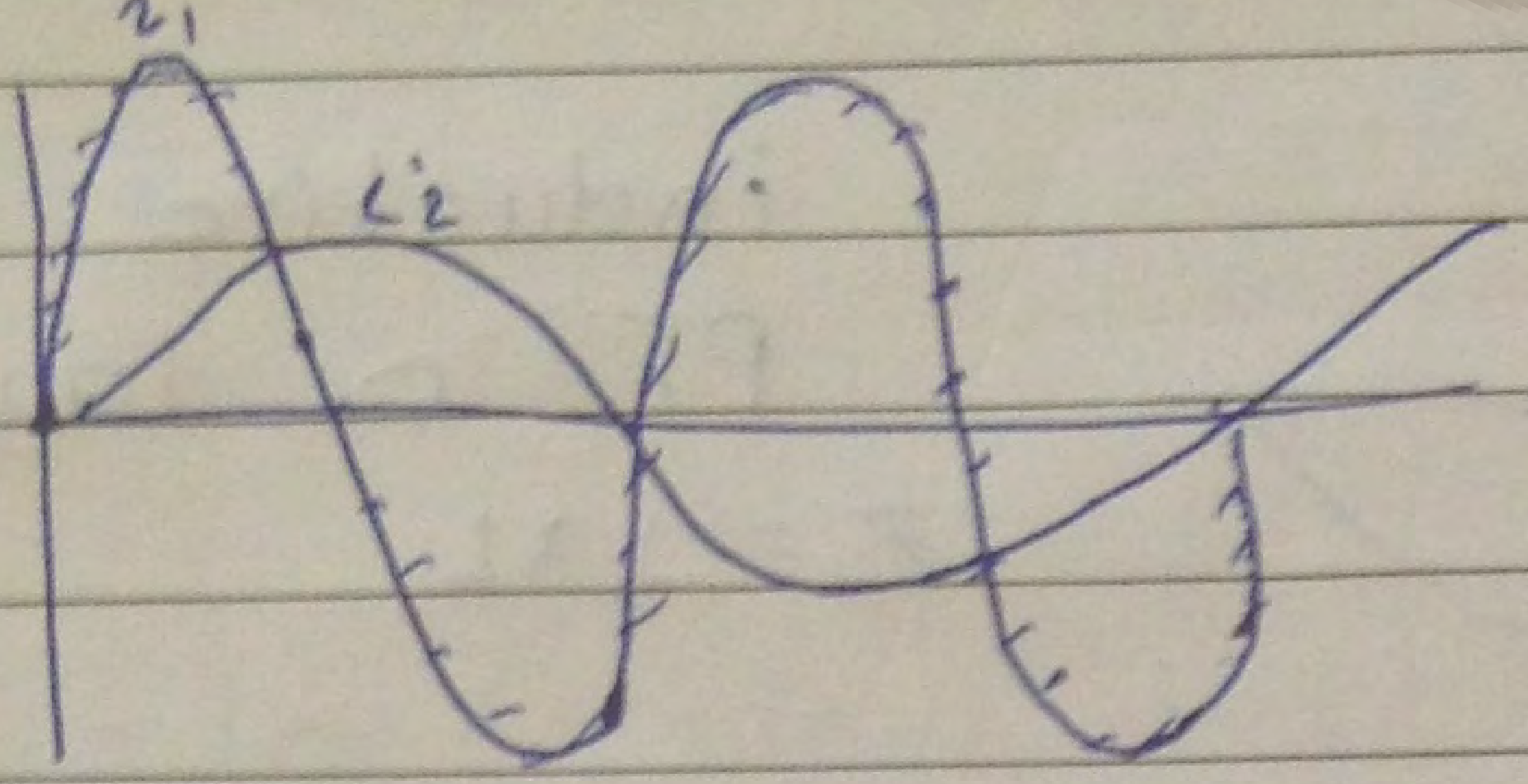
$$P = I_{eff}^2 R = \frac{R}{T}\int_0^T i^2(x)\,dt$$

$$I_{eff}^2 = \frac{1}{T}\int (\sin t + \sin \pi t)^2\,dt$$

$$= \frac{1}{T}\left[\int_0^T (\sin^2 t + 2\sin t \sin \pi t + \sin^2 \pi t)\,dt\right]$$

$$= \int_0^T \frac{1}{2}(1-\underbrace{\cancel{\cos 2t}}_{\to 0}) + \frac{1}{2}(1-\underbrace{\cancel{\cos 2t}}_{\to 0})$$

$$I_{rms} = \sqrt{I_{1eff}^2 + I_{2eff}^2 \sim}$$



$i(t) = i_1(t) + i_2(t)$

$$I_{eff} = \frac{1}{T}\int_0^T (I_m \, Sin(wt))^2 \, dt$$

$$= \frac{I_m^2}{T}\int_0^T \frac{1}{2}(1 - Cos 2wt)\, dt = \sqrt{\frac{I_n^2}{2}} = \boxed{\frac{I_{max}}{\sqrt{2}}}$$

$$\boxed{I_{rms} = \frac{I_{max}}{\sqrt{2}}}$$

* Since i_3 and i_2 have same frequency then they have to find the equivalent.

$i_1 = I_{m1} \, Cos(2wt + \phi_1)$

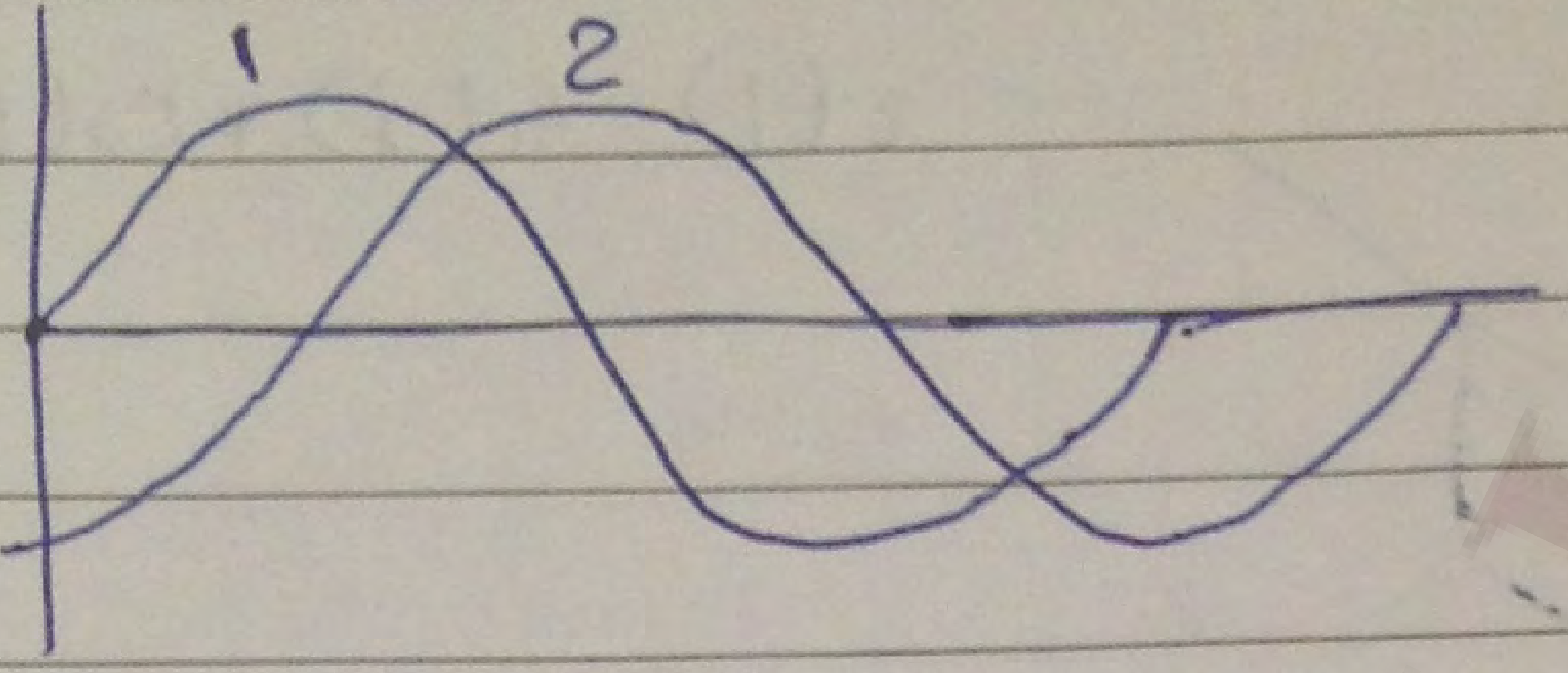
$i_2 = I_{m2} \, Cos(wt + \phi_2) \longrightarrow I_{m2} \angle \phi_2$

$i_3 = I_{m3} \, Cos(wt + \phi_3) \longrightarrow I_{m2} \angle \phi_3$

$I_{23} = i_2 + i_3 = I_{23(max)} \, Cos(wt + \phi_{23})$

1 → V

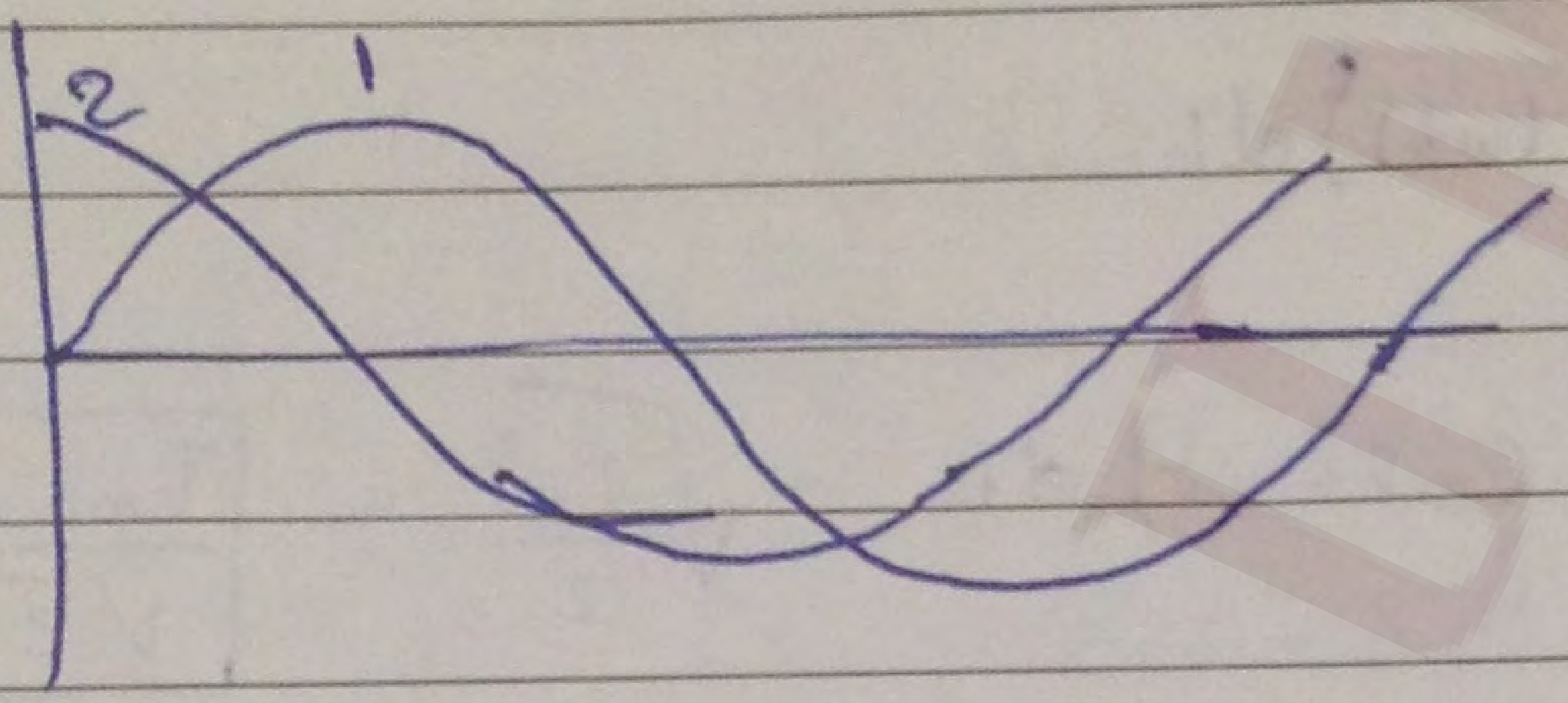
2 → I



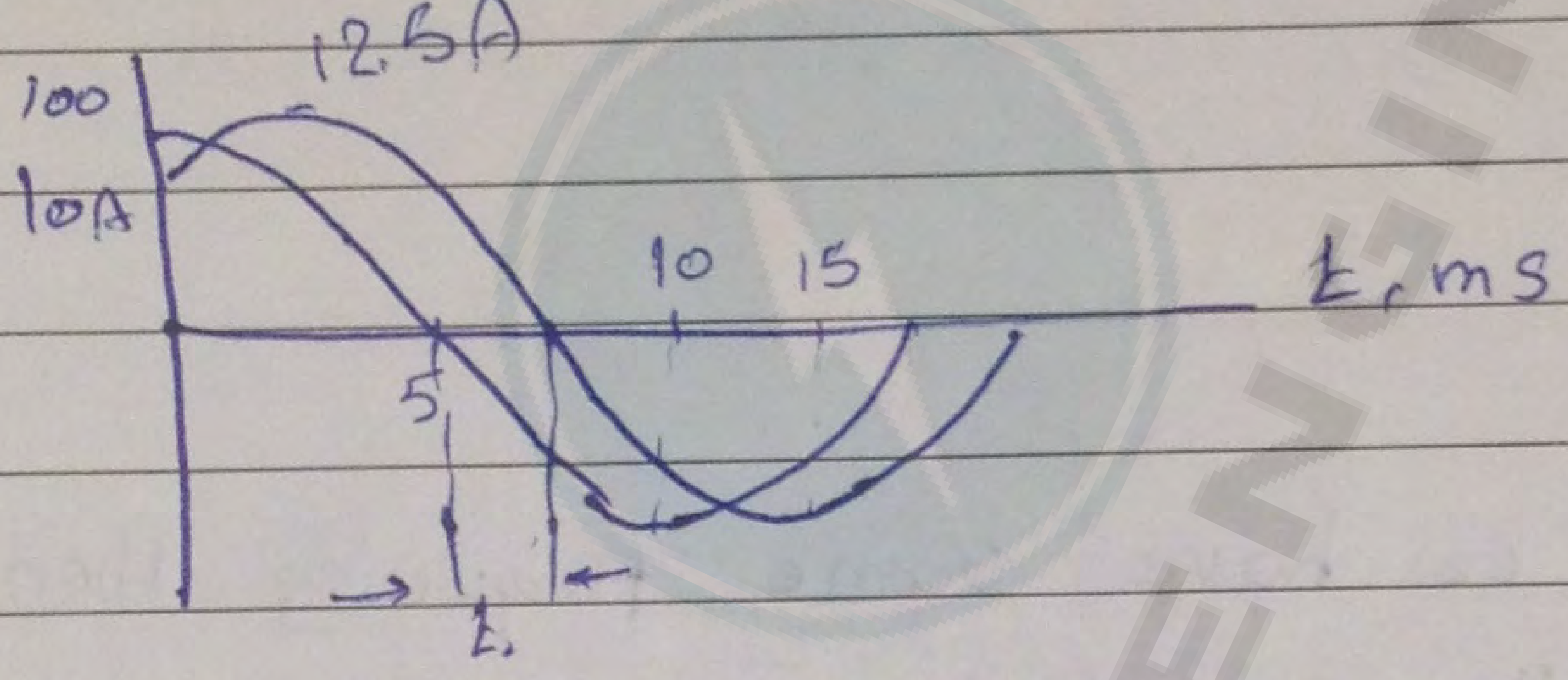
inductive

PF = 0 lagging

$Z_L = j X_L$



inductive (same as previous)



$10\ cm = 100\ V \rightarrow K_V = 10\ V/cm$

$10\ Cm = 12.5\ V \rightarrow K_i = 1.25\ A/cm$

$v(t) = 100\ Cos(100\pi t)$

$i(t) = 12.5\ Cos(100\ \pi t - Cos^{-1}\frac{10}{12.5})$

$T = 20\ ms = 360^\circ = 20\ cm$

$? = 2.5\ cm$

$$\phi_0 = \frac{2.5 \times 360^\circ}{20\ Cm} = 45^\circ$$

$I_1 = 100\angle 0$

$I_2 = 100\angle -36.87$

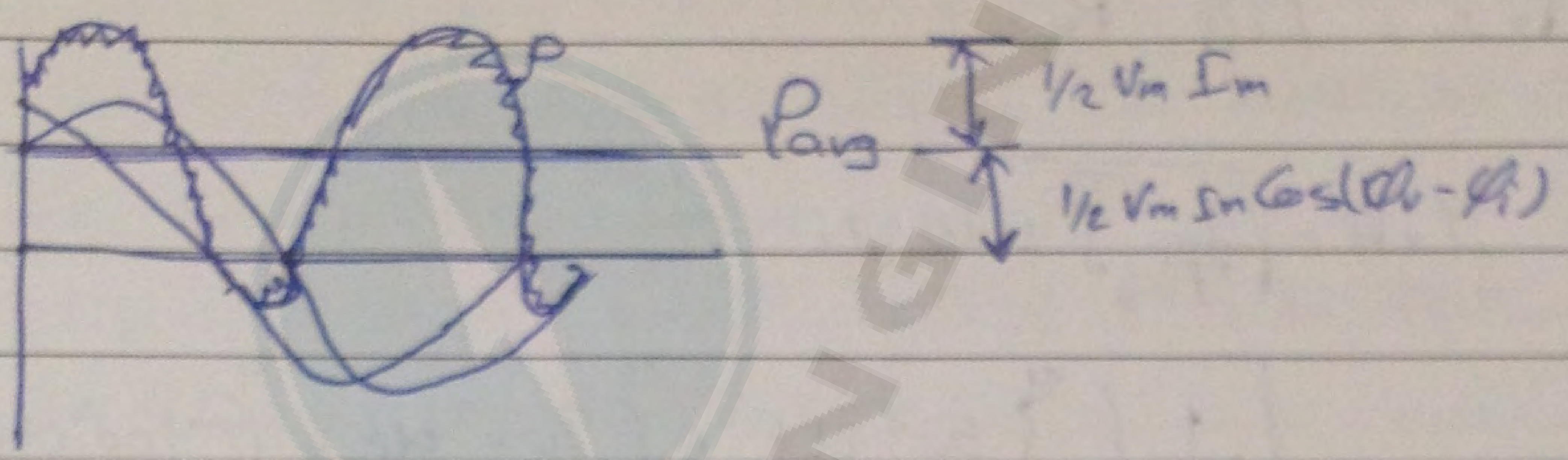
$I_{12} = 100\angle 0 + 100\angle -36.87$

$= 100 + 80 - j60$

$= 180 - j60 \quad = \sqrt{180^2 + 60^2}\ \angle -\tan^{-1}\left(\frac{60}{180}\right) = 190\angle -18$

$P = 1/2\ V_m I_m \cos 36.87$

$= 1/2\,(100)(12.5)\,0.8 = 500W$

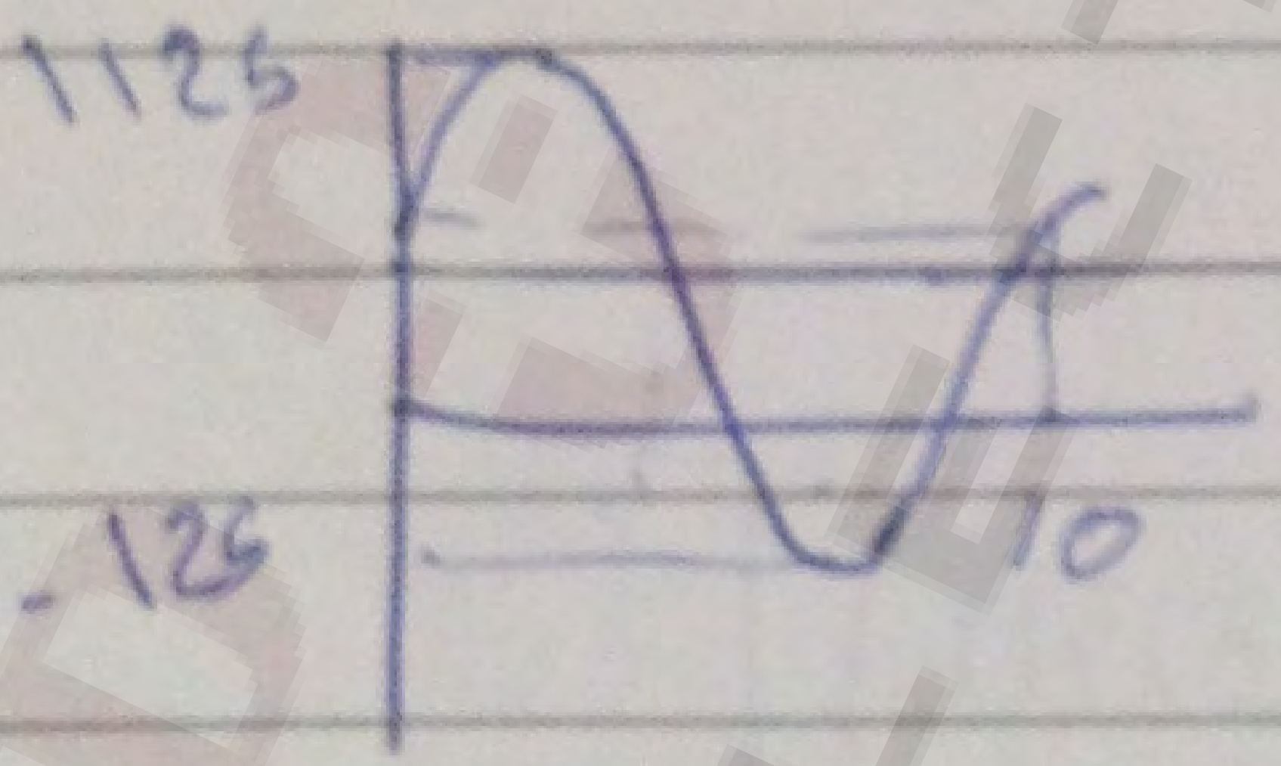


$P(t) = 500 + 625 \cos(200\pi t - 36.87)$

$= 1/2\ V_m I_m \cos\phi + 1/2\ V_m I_m\ (\cos 2wt + \phi)$

$\rightarrow P_{max} = 1/2\ V_m I_m \cos\phi + 1/2\ V_m I_m = 1/2\ V_m I_m\,(PF + 1)$

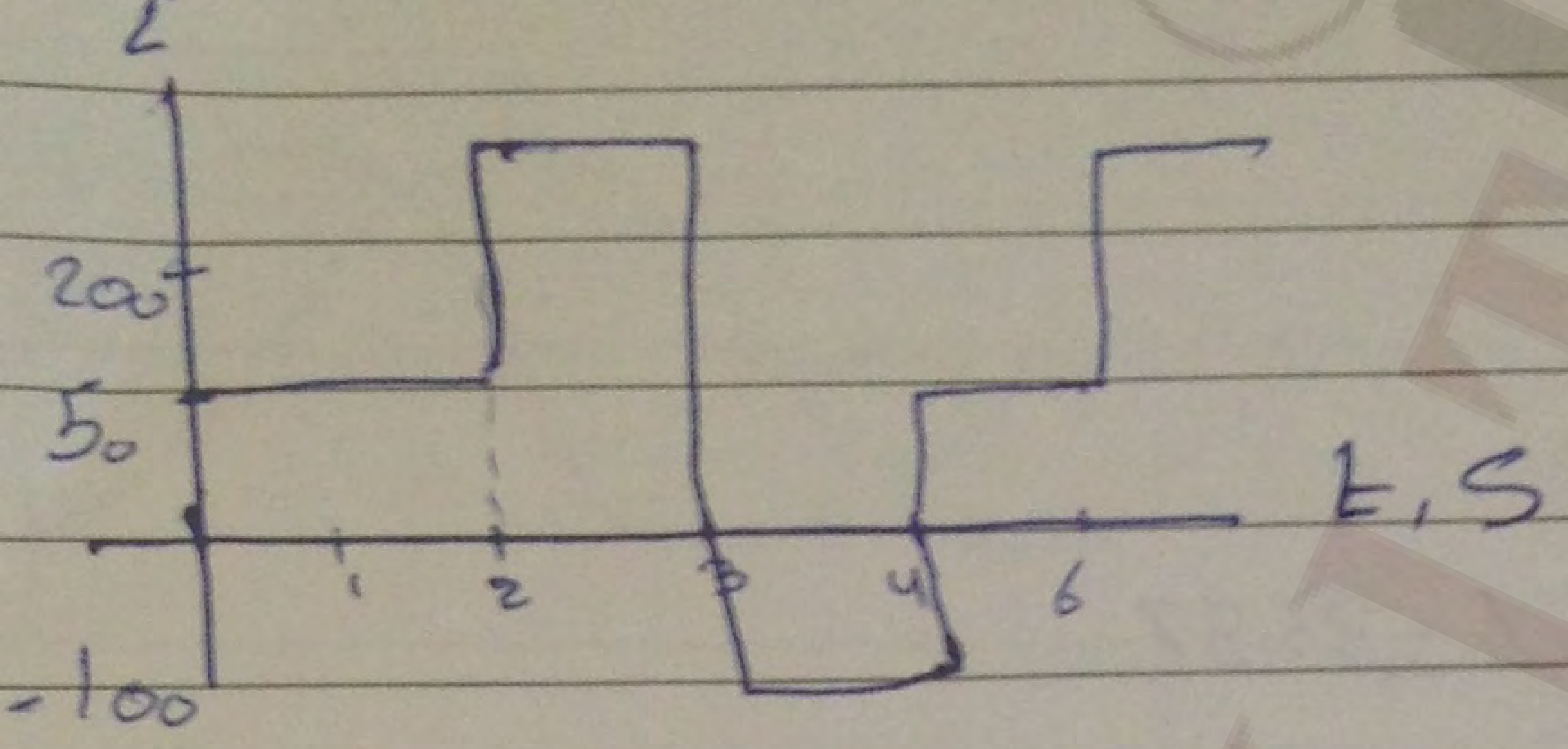
$\rightarrow P_{min}\quad 1/2\ V_m I_m \cos\phi + 1/2\ V_m I_m = 1/2\ V_m I_m\,(PF - 1)$

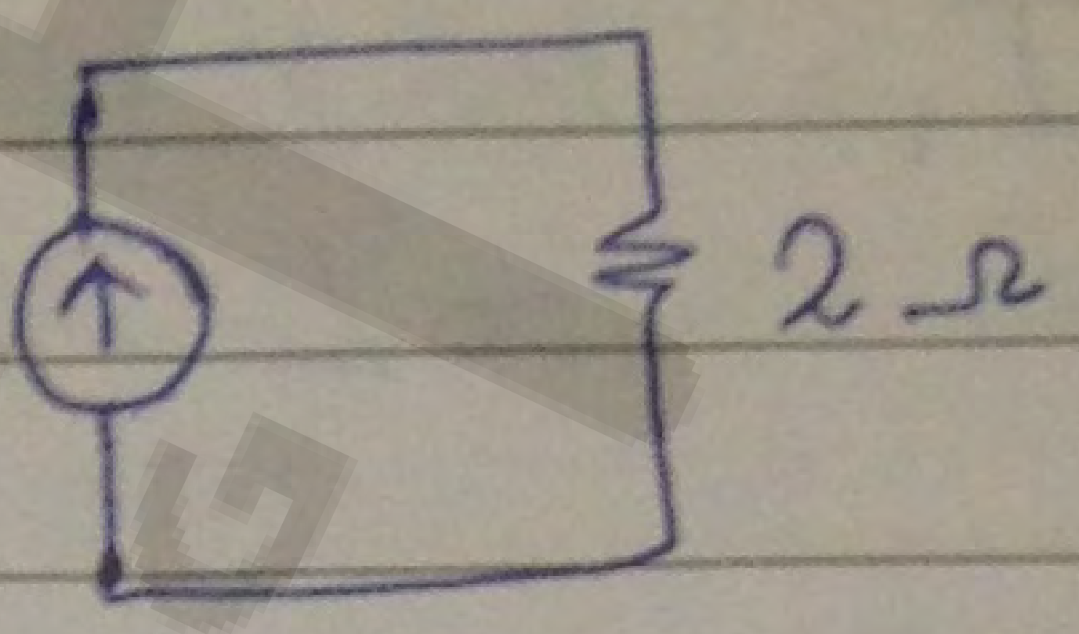


$P_{av} = P_{max} - \frac{\cancel{P_{av}} - P_{min}}{2} = 500W$

$P_{max} = S(PF + 1) \rightarrow 1125 = S\ (PF + 1)$

exp





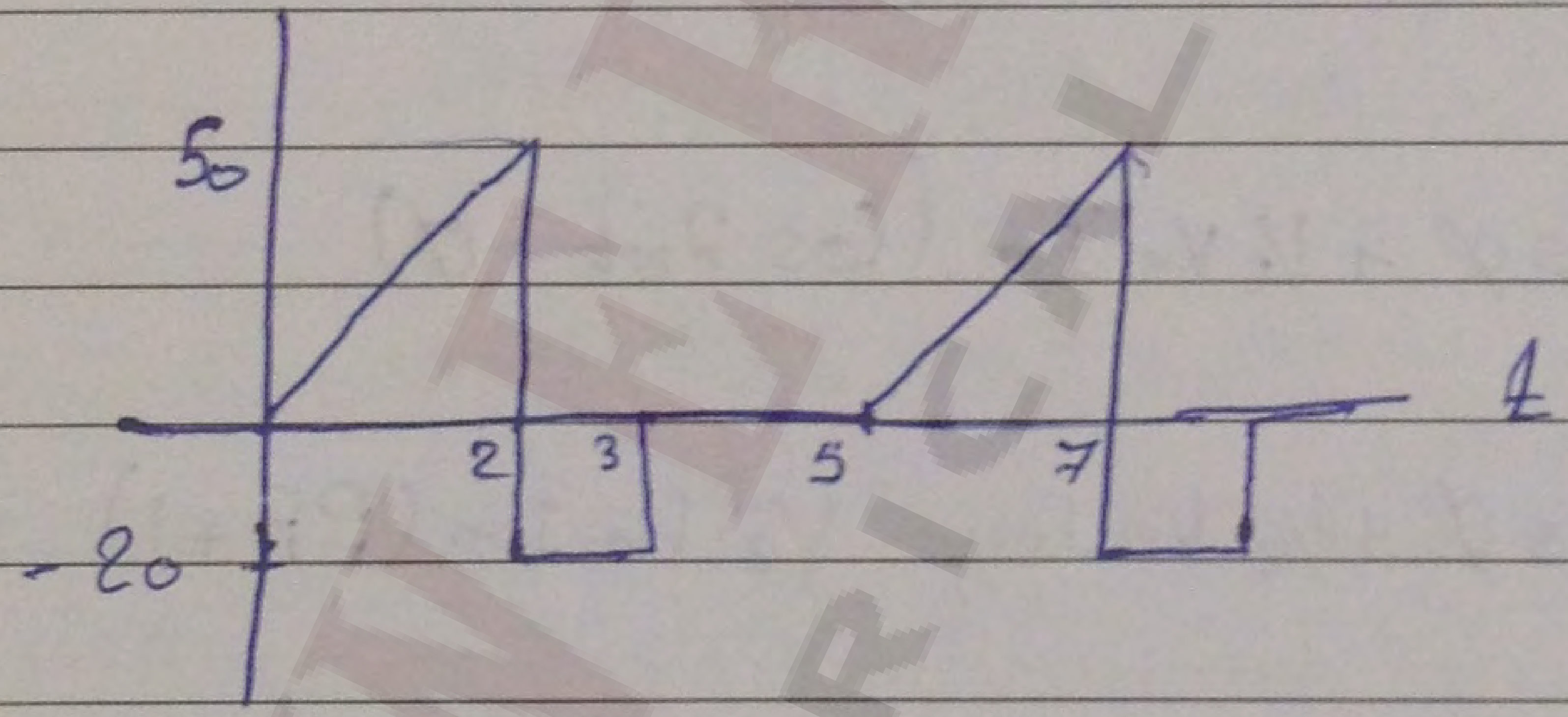
$$i(t) = \begin{cases} 50 & 0<t<2 \\ 200 & 2<t<3 \\ -100 & 3<t<4 \end{cases}$$

$$P_{2\Omega} = I_{eff}^2 \; R$$

$$I_{eff} = \sqrt{\frac{1}{T}\int_0^T i^2(t)\,dt}$$

$$= \left[\frac{1}{4}\left(\int_0^2 50^2\,dt + \int_2^3 200^2\,dt + \int_3^4 (-100^2\,dt)\right)\right]^{1/2}$$

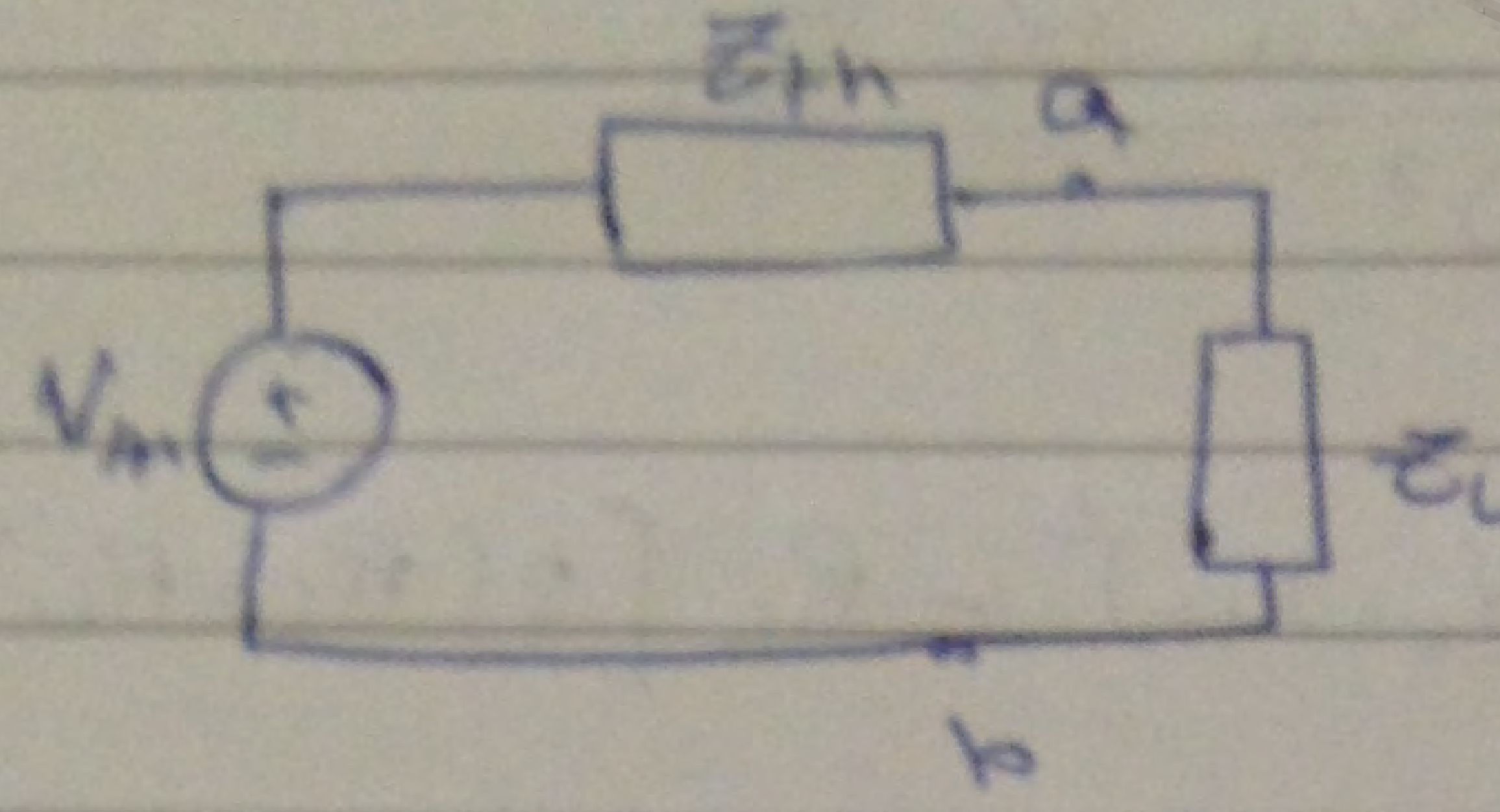
exp

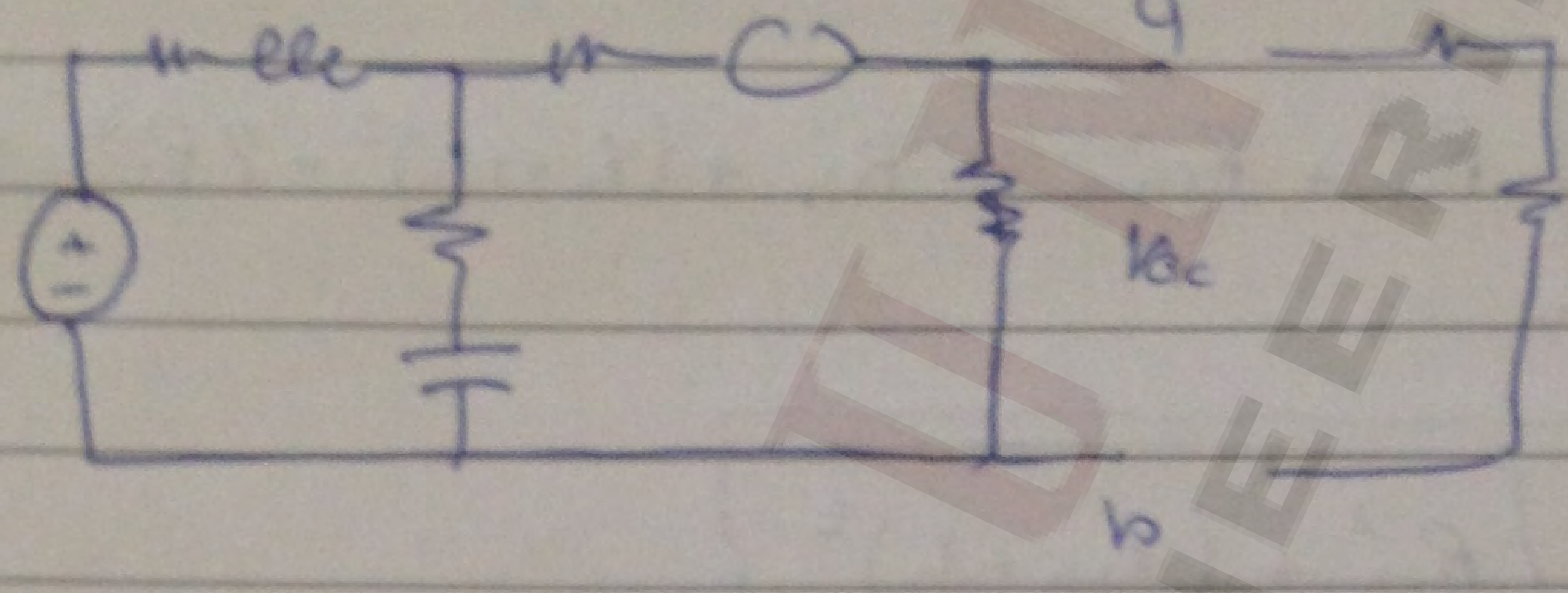


$$i = \begin{cases} 25t & 0<t<2 \\ -20 & 2<t<3 \end{cases}$$

$$I_{eff} = \left[\frac{1}{5}\left(\int_0^2 (25t)^2\,dt + \int_2^3 (-20)^2\,dt\right)\right]^{\frac{1}{2}}$$

- Maximum Power transfer:-





1) Remove the branch where we have to find I

2) Calc. $V_{oc} = V_{th}$

3) Calc. Z_{th} after killing the sources

4) reconnect the ckt.

$$\bar{I} = \frac{V_{th}}{Z_L + Z_{th}} = \frac{V_{th}}{(R_L + jX_L) + (R_{th} + jX_{th})} = \frac{V_{th}}{(R_L + R_{th}) + j(X_{th} + X_L)}$$

$$P_L = |I|^2 R_L = 1/2\, |I_m|^2 R$$

$$|I| = I_{eff} = I_{RMS}$$

$$|I| = I_{eff} = \frac{V_{th}}{\sqrt{(R_{th} + R_L)^2 + (X_{th} + X_L)^2}}$$

$$\boxed{P_L = \frac{V_{th}^2}{(R_{th} + R_L)^2 + (X_{th} + X_L)^2} \cdot R_L}$$

$$\frac{\partial P_L}{\partial R_L} = 0 = \frac{V_{th}^2 (D) - V_{th}^2 R_L \frac{\partial D}{\partial R_L}}{D^2}$$

$$= V_{th}^2 \left[(R_L + R_{th})^2 + (X_L + X_{th})^2 - R_L \left[2(R_L + R_{th}) \right] \right]$$

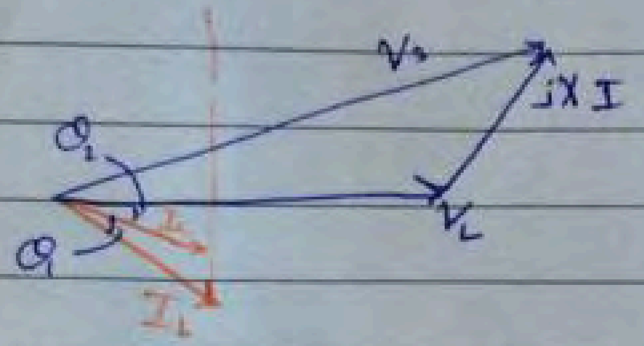
$$= V_{th}^2 \left[R_L^2 + R_{th}^2 + 2R_L R_{th} + (X_L + X_{th})^2 - 2R_L^2 - 2R_L R_{th} \right] = 0$$

$$\boxed{R_L = \sqrt{R_{th}^2 + (X_{th} + X_L)^2}}$$

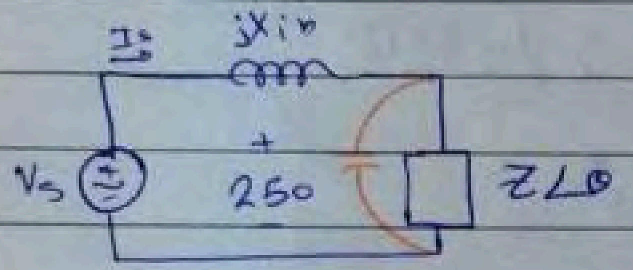
$$\frac{\partial P_L}{\partial X_L} = 0 = \frac{-R_L(2(X_L + X_{th}))}{D^2}$$

$$2R_L(X_L + X_{th}) = 0$$

$$\boxed{X_L = -X_{th}} \qquad \therefore \boxed{Z_L = R_{th} + j X_{th}}$$



exp



707 W

PF = 0.707

① increase the PF at the source to be 0.95 lag

② " " " " " load "

$|I_L| = \frac{|S|}{|V|}$

$I = \frac{S^*}{V^*}$

Note

$P = S\cos\theta$

$S = \frac{P}{\cos\theta} = \frac{P}{PF}$

$S = \frac{P}{PF} = 1000\ VA$

$\theta = \cos^{-1} PF = 45^\circ$ $S = 1000\angle 45^\circ$

$I = \frac{1000\angle 45}{250\angle 0} = 4\angle -45\ A$

$I_L = 4\angle -45\ A$

$S_{line} = V_{line}\ I^*_{line} = Z\ I_L\ I_L^*$

$= Z\ I_L^2$

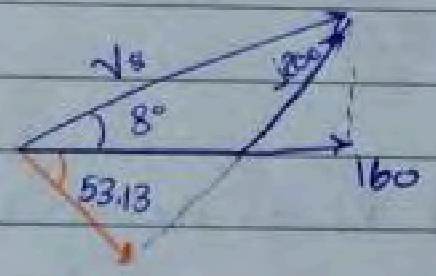
$= j5(4)^2 = j80\ VAR$

exp 500V 0.6 lag
1000 VA

$$|S| = |V|.|I|$$

$$|I| = \frac{1000}{500} = 2A$$

$$\bar{I} = 2 \underline{\angle -\cos^{-1} PF} = 2 \underline{\angle -53.13°}$$

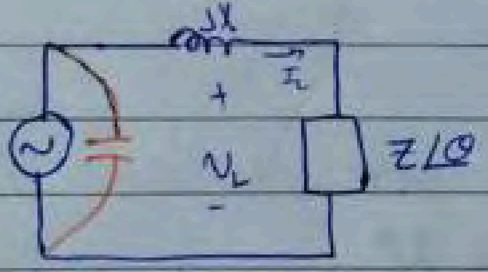


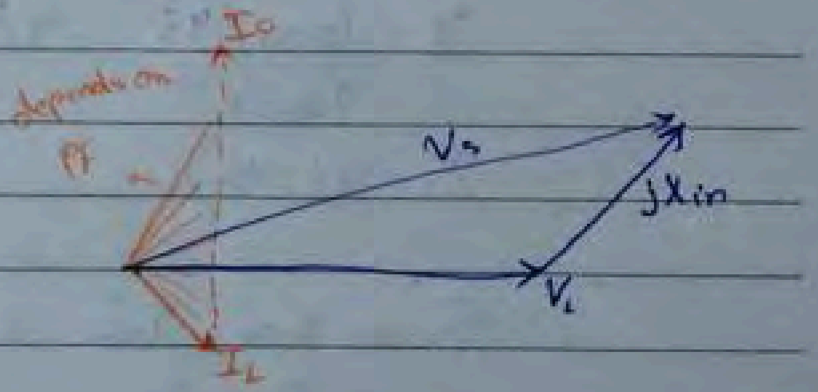
$$V_s = (500+1160)+j100$$
$$= \sqrt{660^2+100^2} = 667.5V$$

$$PF = \cos(\theta_{V_s} - \theta_{I_s}) = \cos(8-(-53.13))$$
$$= \cos 61.13 = \boxed{0.48} \text{ lag}$$

exp Improve the PF of the source to the value of 0.9 lag
Find the Qc, C...

$$\cos^{-1} 0.9 = 25.83°$$

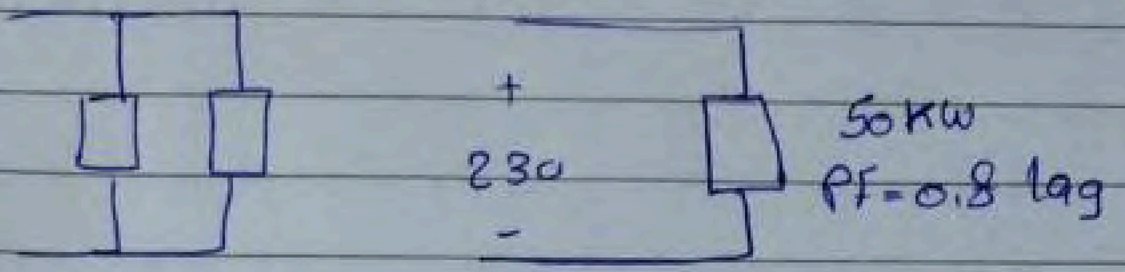




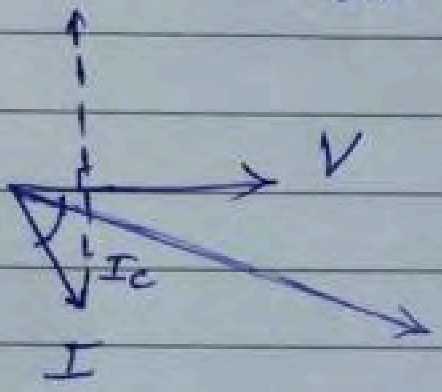
$$\theta_{V_s} - \theta_{I_s} = \theta_S$$

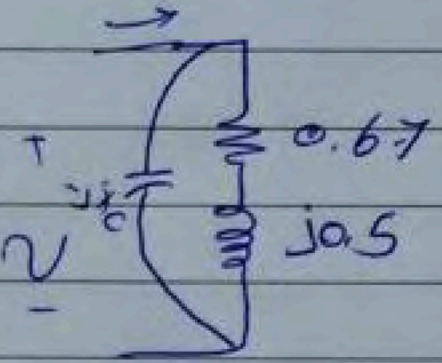
- required to decrease the angle btween Vs and Is to improve PF
- a Capacitor is Connected in parallel with the load.

exp



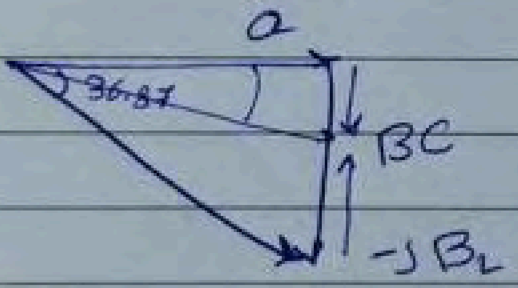
$$Z = \frac{V^2}{S^*} = \frac{230^2}{\frac{50}{0.8} - j\frac{50}{0.6}} = 0.841\angle 36.87^\circ$$
$$= 0.67 + j0.6$$





$$\cos(\phi_V - \phi_I) = PF = 0.95$$

$$Y_L = \frac{1}{Z} = \frac{1}{0.84}$$

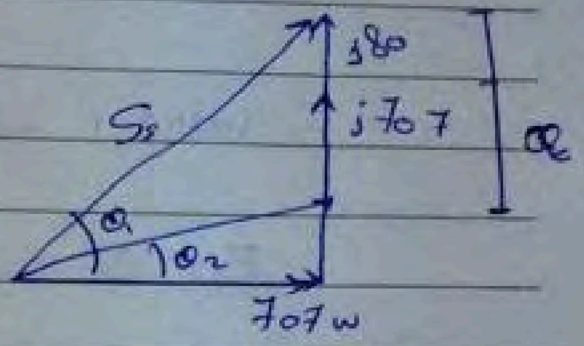


$$\frac{I}{V} = \omega C = B_c = B_1 - G\tan\phi_2$$

$$X_c = \frac{1}{B_c}$$

$S_s = \sqrt{707^2 + 787^2}$

$= 1057 \angle 48$



$Q_c = Q_1 - Q_{new}$

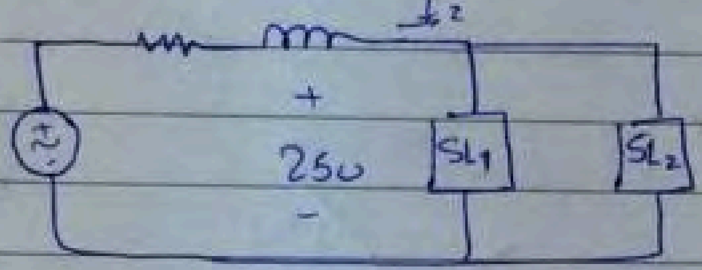
$= P\tan\theta_1 + P\tan\theta_2$

$= P(\tan\theta_1 - \tan\theta_2)$

$= \omega_c V_c^2 = P(\tan\theta_1 - \tan\theta_2)$

---

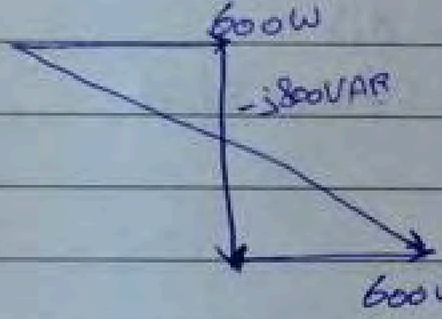
exp



PF load ?

$L_1 = 1000$ w
PF = 0.6 leading

$L_2 = 600$W



$S_L = 1200 - j800 = 1442 \angle -33.7°$

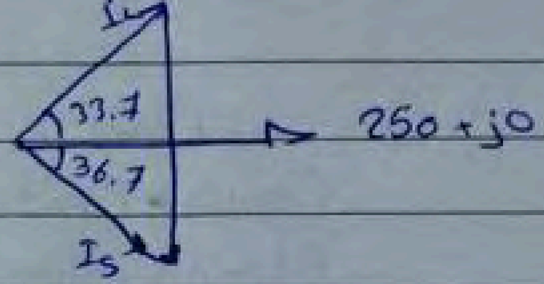
$I_L = \frac{1442}{250} = 5.8A$ $I_L = 5.8 \angle 33.7°$

Why the angle of the load curr. is +ve? inductive

$S = V . I^* = \frac{V^2}{Z^*}$

$\therefore Z = \frac{V^2}{S} = \frac{250^2}{1442 \angle 33.7} = 36.5 - j24$, $PF_{load} = \cos 33.7$ lead
$= 0.83$ leading

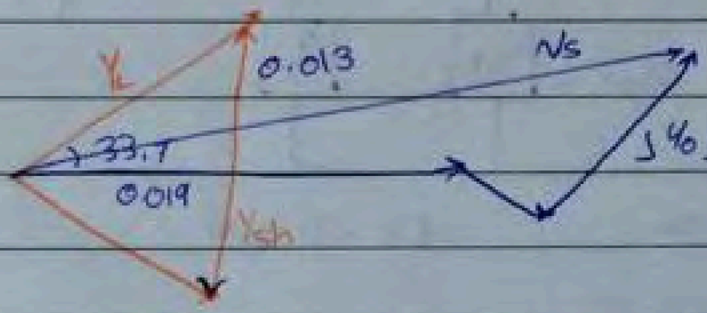
exf Required to find the element that have been Connected in parallel with load to obtain 0.8PF lagging

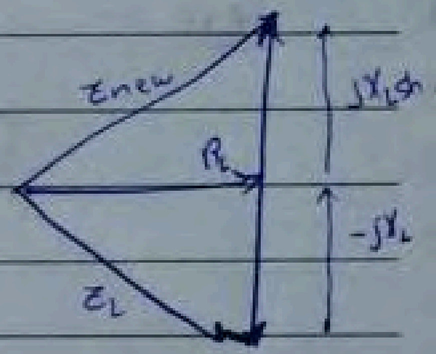


inductor ─∿∿─ (jX_{Lsh})

$Z_L = 36.05 - j24$

$Y_L = \frac{1}{Z_L} = 0.019 + j0.013$





$Y_{sh} = G\tan 33.7 + G\tan\cos^{-1}0.8$

$= G(\tan\theta_1 + \tan\theta_2)$

$\frac{1}{\omega L} = 0.019(\tan 36.87 + \tan 33.7)$

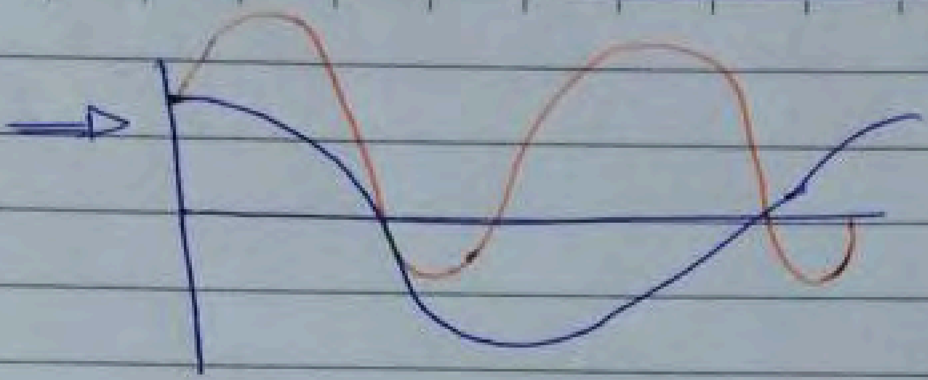
Note: $Y = G \mp jB$

$= \frac{1}{R} \mp j(\omega c - \frac{1}{\omega L})$

$= \frac{1}{R} + j(B_c - B_L)$

H.W Prove $\Sigma P = 0$

$\Sigma Q = 0$



$p(t) = V_m \cos(\omega t + \theta_v) I_m \cos(\omega t + \theta_i)$

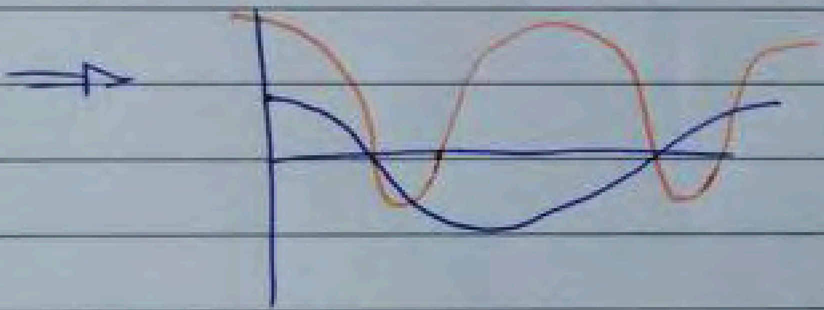
$p(t) = 1/2 \, V_m I_m [\cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v + \theta_i)]$

$P_{avg} = 900 - \frac{900 - 100}{2} = 500$

$P = P + |S| \cos(2\omega t + (\theta_v + \theta_i))$

$\theta_v + \theta_i = -\cos^{-1} \frac{400}{500} = -36.87°$

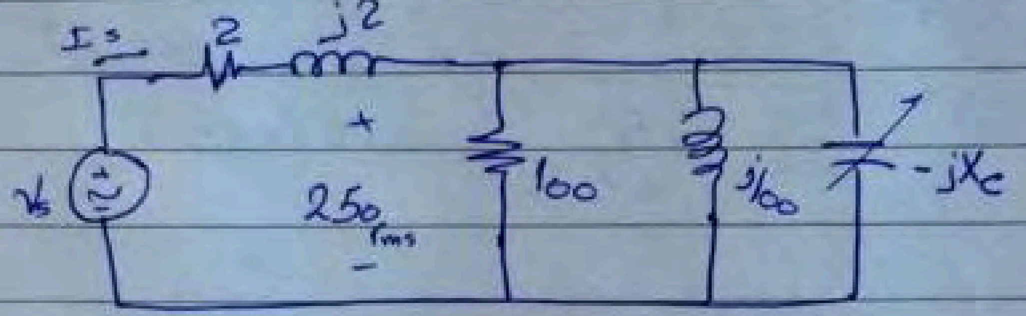
$\therefore \theta_i = -36.87°$

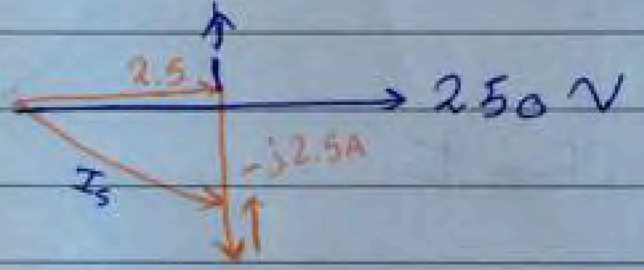


$P + \frac{P}{PF} (\cos(2\omega t + \theta_v + \theta_i)$

$\theta_i = 36.8°$

exp X_C is adjusted in such a way that the current I_s is minimum.
whats the value of X_c and the Complex value delivered by the source.





$V_s = 250 + 2.5(2 + j2.5)$
$= 255 + j5 = 255.1\angle 1.1^\circ$

$S = -(255.1\angle 1.1)(2.5) = -637.7\angle 1.1$
$= -637.6 + j12.2$

$P = -637.7$ w → delivered
$Q = -12.2$ VAR → delivered.

prove $\Sigma P = 0$

$P_{delv} = 637$
$P_{absorb} = (2.5)^2 * 100 + (2.5)^2 * 2 = 637.5$

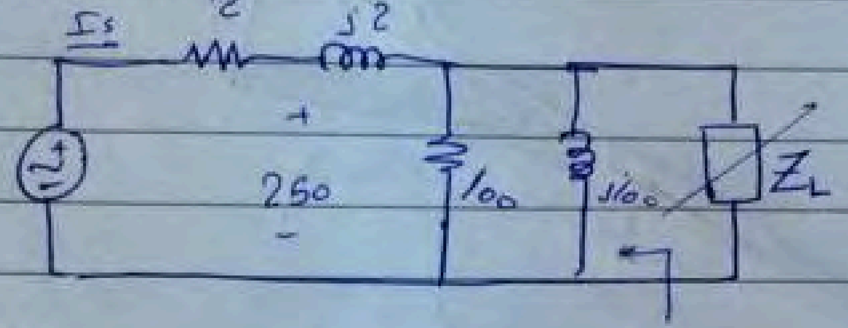
$\underline{\Sigma Q = 0}$

$Q_{abs} = (2.5)^2 * j2 + (2.5)^2 * j100 = j637.5$

$Q_{delv.} = (2.5)^2 - j100 = -j62.5 - j12.2$

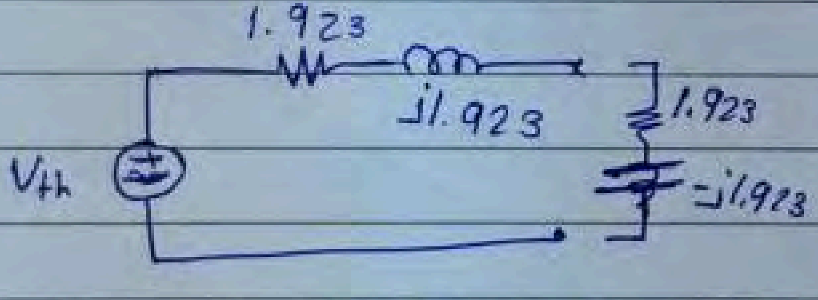
---

exp Whats the Value of Z_L that the avg. Power tranfer to it equal to its maximum Value.



$$Z_L = \frac{1}{Z_{qc}} = \frac{1}{\frac{1}{100} + \frac{1}{j100} + \frac{1}{2+j2}} \Rightarrow Z_{oc} = Z_{th} = 2.719\angle 45$$

$$Z_L = Z_{th}^* \Rightarrow Z_L = 2.719\angle -45$$



$\boxed{V_{th} = 250}$

$$P_L = |I|^2 R_L = \left(\frac{V_{th}}{2 \times 1.923}\right)^2 \cdot 1.923 = 8125 \text{ w}$$

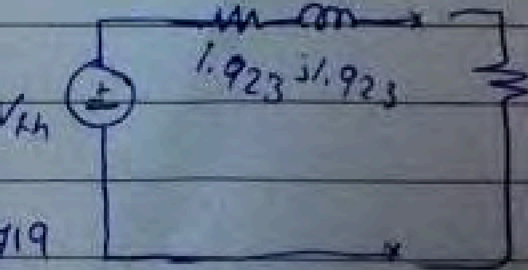
⇒ If we replaced Z_L with variable R.

$R_L = \sqrt{R_{th} + (X_L + X_{th})^2}$

$|Z_{th}| = 2.719\ \Omega$

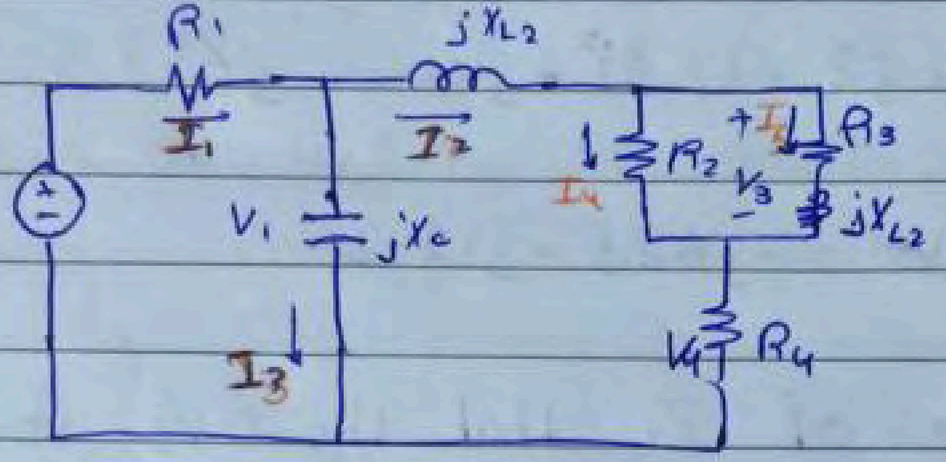
$P = |I|^2 R = \frac{250^2}{(1.923 + 2.719)^2 + 1.923^2} * 2.719$

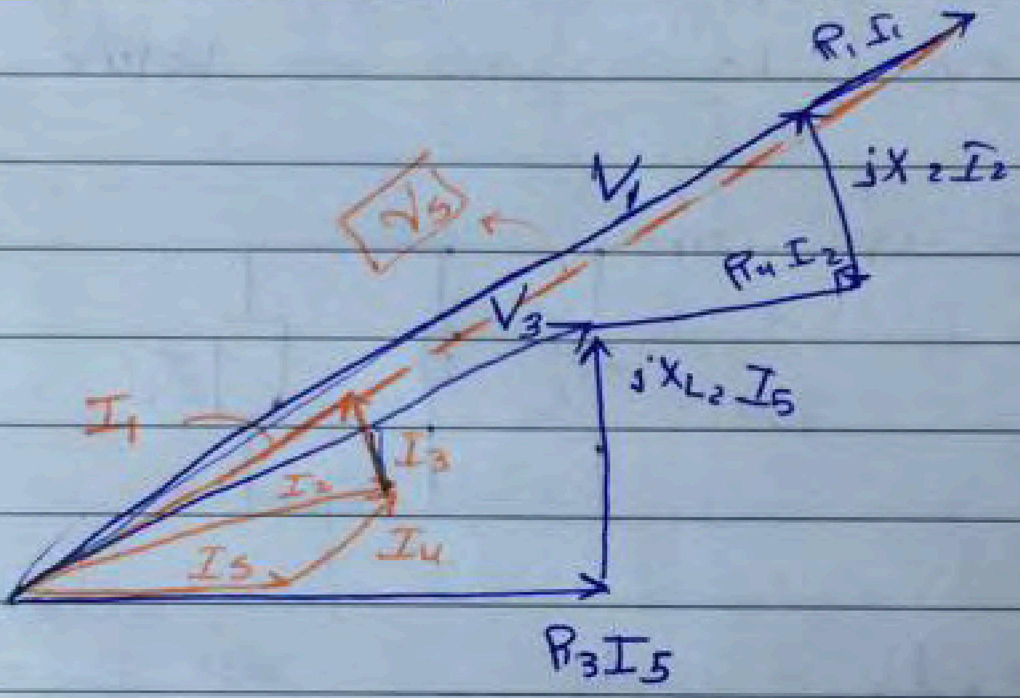
$= \boxed{6722}$ w

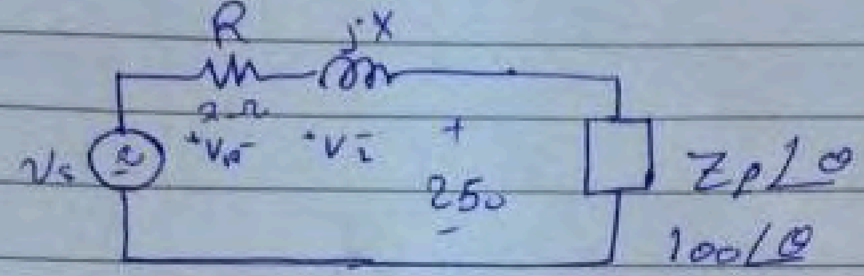


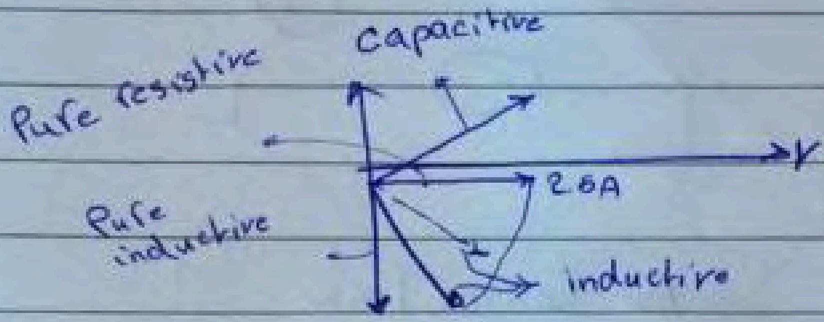
* P less than P when the load is Capacitor.

exp Draw phasor diagram.





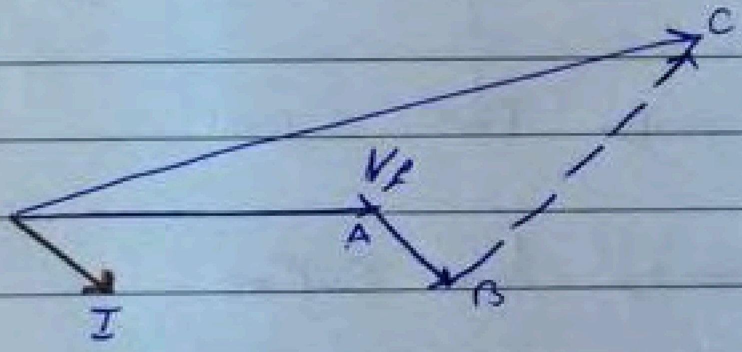




$$\frac{1}{R \pm jX} = \frac{R}{R^2+X^2} - j\frac{X}{R^2+X^2} = G \mp jB$$

inductive
capacitive

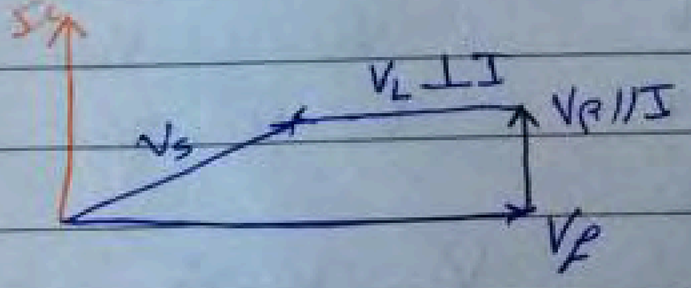
inductive
Capacitive



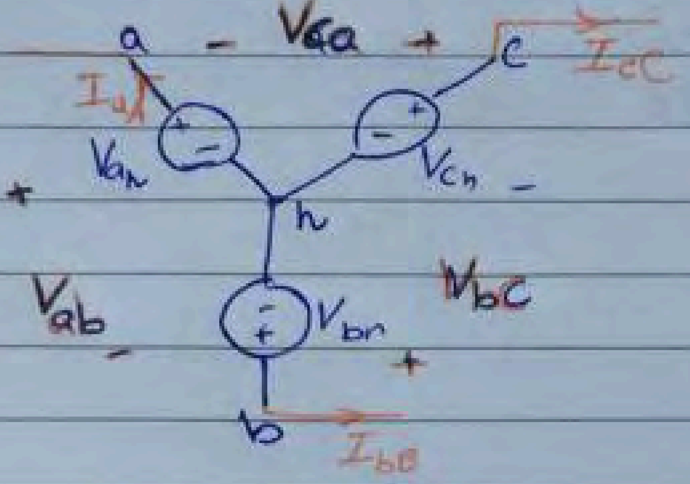
$V_R = RI$, $V_R \parallel I$

$V_L = jXI$, $V_L \perp I$

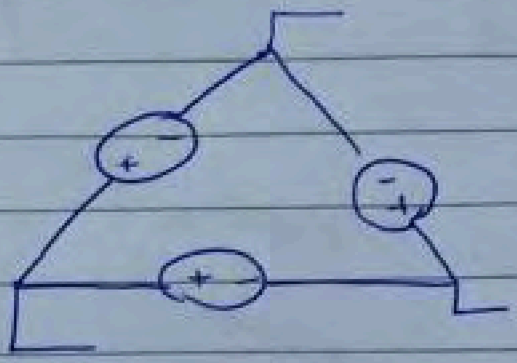
⇨



Three Phase :-

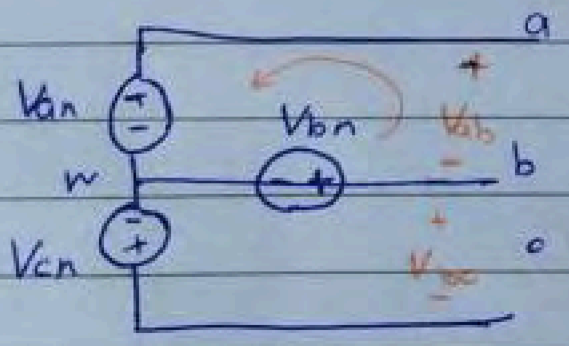


Y- Connection



Δ-Connection

- V_{ab} , V_{bc} , V_{ca} ⟹ Line Voltage
- V_{an} , V_{bn} , V_{cn} ⟹ Phase Voltage
- I_{an} , I_{bn} , I_{cn} ⟹ Phase current
- I_{aA} , I_{bB} , I_{cC} ⟹ Line Current

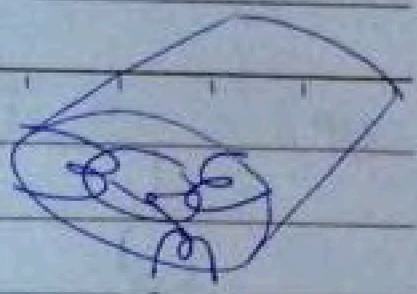


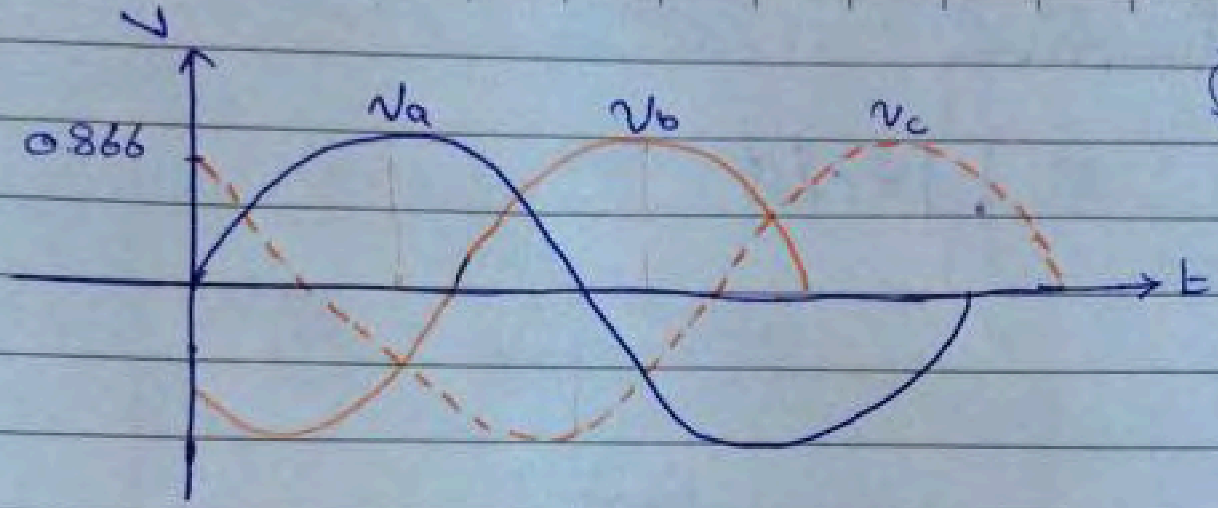
$+V_{an} - V_{bn} - V_{cb} = 0$

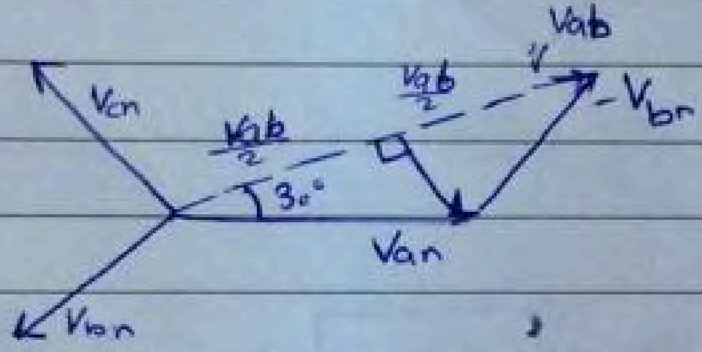
$V_{ab} = V_{an} - V_{bn}$

$V_{bc} = V_{bn} - V_{cn}$

$V_{ca} = V_c - V_a$



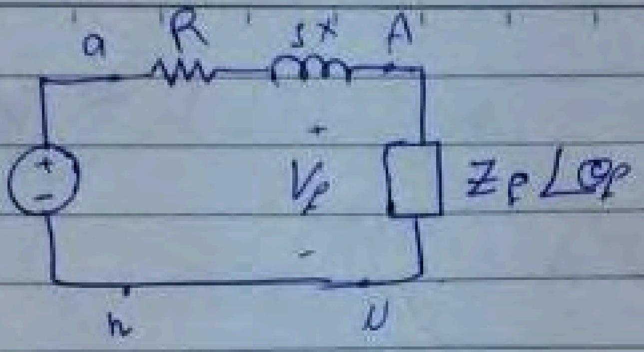


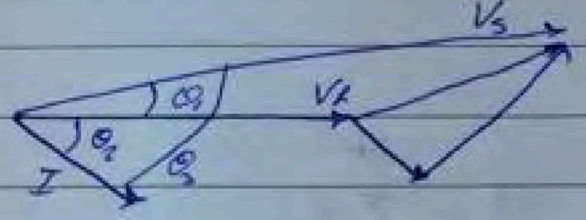


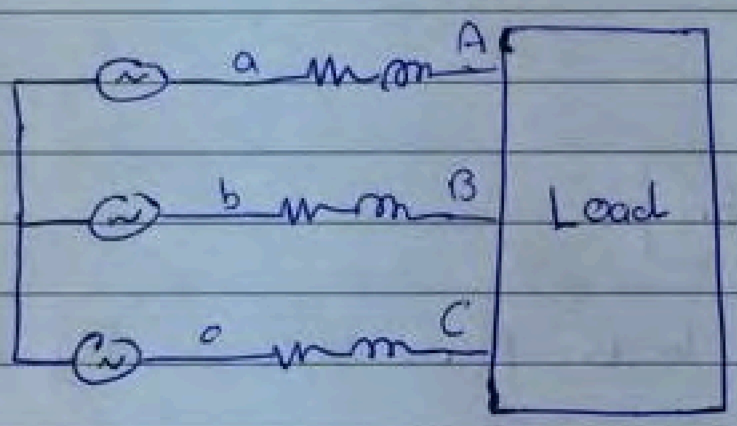
$$\boxed{V_{ab}=\sqrt{3}\,V_{an}}$$

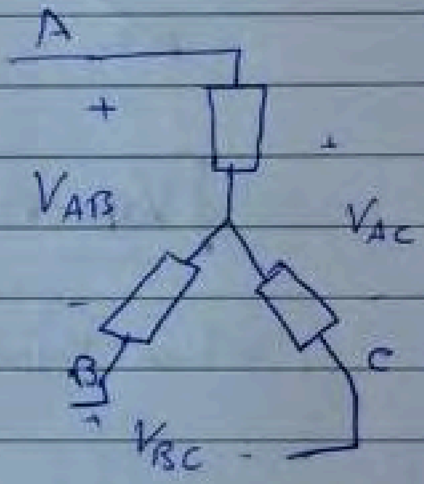
* line Voltage leads Phase Voltage by 30°

exp









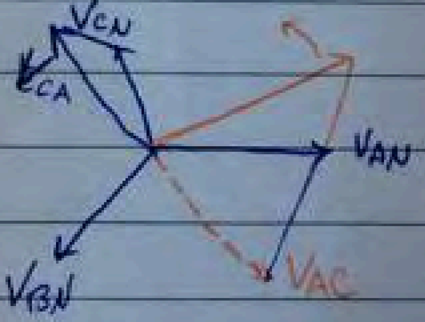
$$V_{AB} = \sqrt{3}\ V_{AN} \angle 30^\circ$$

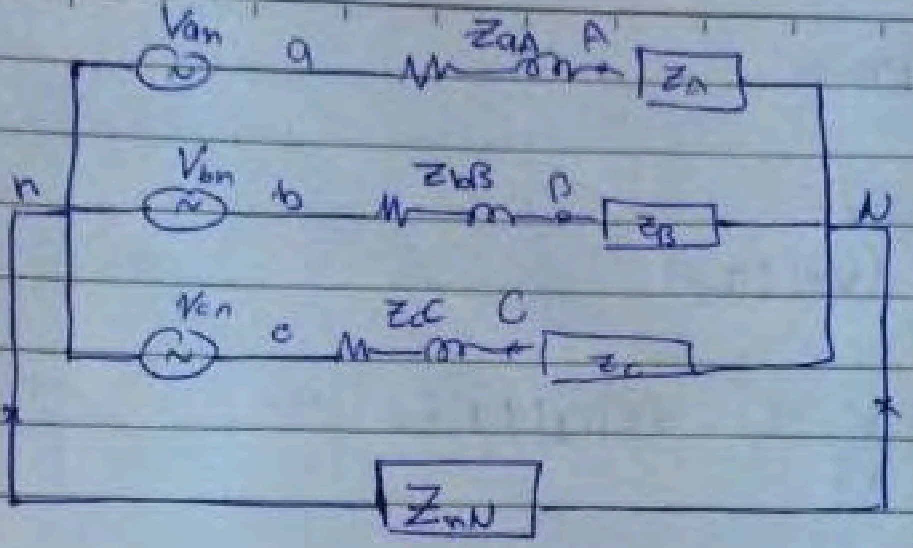
$$V_{CA} = \sqrt{3}\ V_{CN} \angle 30^\circ = \sqrt{3}\ V_{BN} \angle -90$$

$$V_{BC} = \sqrt{3}\ V_{BN} \angle 30^\circ$$

$$V_{CA} = \sqrt{3}\ V_{AN} \angle 150^\circ$$

$$V_{AC} = \sqrt{3}\ V_{AB} \angle -60$$





$$\frac{V_N - V_{an}}{Z_A + Z_{aA}} + \frac{V_N - V_{bn}}{Z_B + Z_{bB}} + \frac{V_N - V_{cn}}{Z_C + Z_{cC}} + \frac{V_N}{Z_{nN}} = 0$$

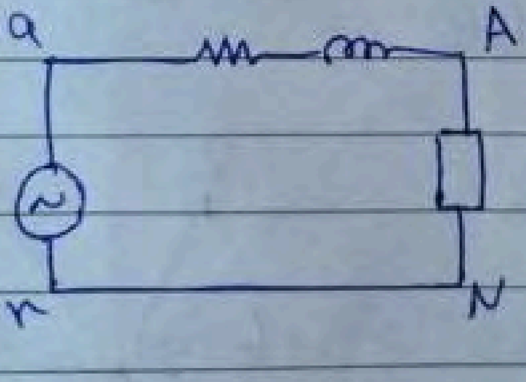
$\Rightarrow Z_\phi = Z_A + Z_{aA}$

$$Z_A + Z_{aA} = Z_\phi = Z_B + Z_{bB} = Z_C + Z_{cC}$$

$$\left(\frac{1}{Z_\phi} + \frac{1}{Z_N} + \frac{1}{Z_\phi}\right) V_N = \frac{V_{an} + V_{bn} + V_{cn}}{Z_\phi}$$

Since $V_{an} + V_{bn} + V_{cn} = 0$

$\therefore V_N = 0$



- Power in 3-phase

$$S_{3\phi} = 3 S_{1\phi}$$

$$= 3 |V_\phi| |I_\phi|$$

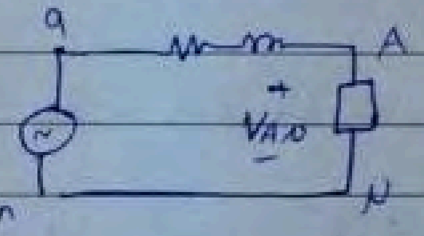
$$= 3 \frac{V_L}{\sqrt{3}} I_L = \sqrt{3} |V_L| |I_L|$$

$$\Rightarrow \boxed{S_{3\phi} = \sqrt{3} |V_L| |I_L|}$$

exp: $V_{AB} = 415$ V

30kVA

$Pf = 0.8$



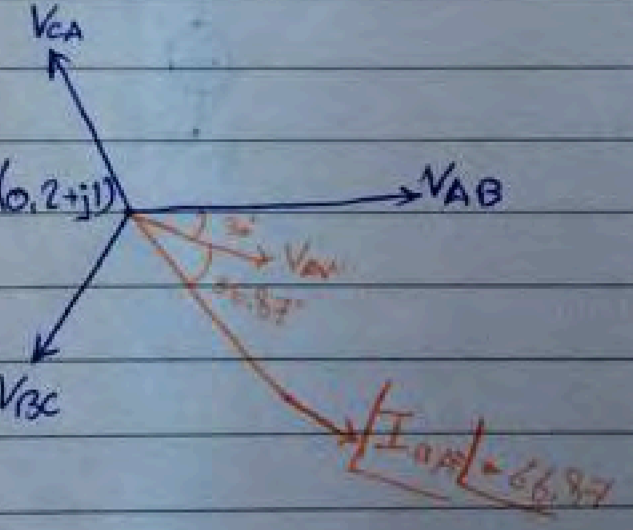
$$V_{AB} = \sqrt{3}\ V_{AN} \angle 30 \longrightarrow V_{AN} = \frac{V_{AB}}{\sqrt{3}} \angle -30$$

$$V_{AN} = \frac{415}{\sqrt{3}} \angle -30°$$

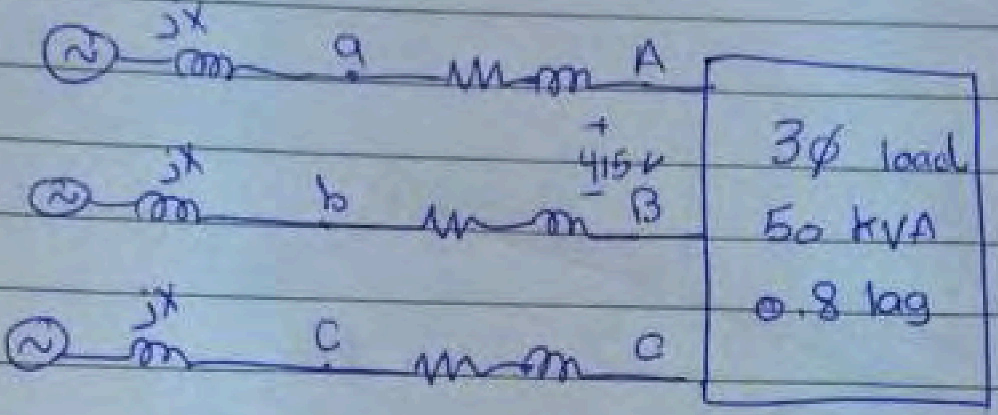
$$|I_L| = \frac{S_{3\phi}}{\sqrt{3} V_L} = \frac{50000}{\sqrt{3}(415)} = \boxed{69.5}$$

$$I_{aA} = 69.5 \angle -66.87°$$

$$V_{an} = \frac{415}{\sqrt{3}} \angle -30 + 69.5 \angle -66.87° (0.2 + j1)$$



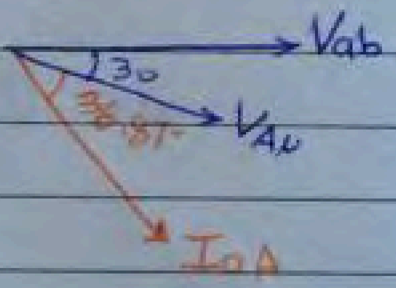
exp



V_{ab}, V_{an}, S_0 ?

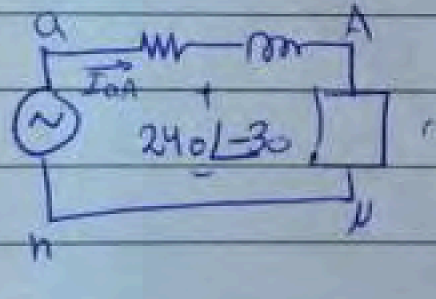
$$|I_L| = \frac{|S_{3\phi}|}{\sqrt{3}\,|V_L|} = \frac{50\,000}{\sqrt{3}(415)} = 69.5\text{ A}$$

∴ Since PF is lag by 80%
then the phase current has to be
lag by 36.87° phase v



$I_{aA} = 69.5\angle -66.87$

→ Use 1ϕ ckt to phase a



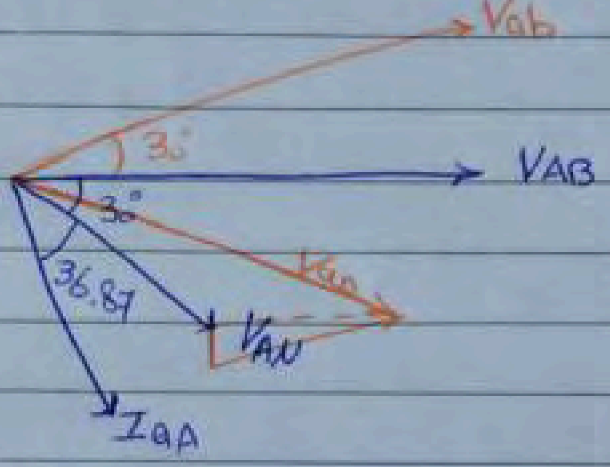
The phase Voltage of Phase - a
is:

$$\frac{V_{ab}}{\sqrt{3}}\angle -30 = \frac{415\angle 0}{\sqrt{3}}\angle -30 = 240\angle -30^{\circ}$$

$$V_{an} = 240\angle -30 + 69.5\angle -6687\,(0.2+j1) = 296\angle -20$$

$$V_{ab} = \sqrt{3}\,(296\angle -20)\angle 30 = 513.7\angle 9.1$$

Phase-A :-

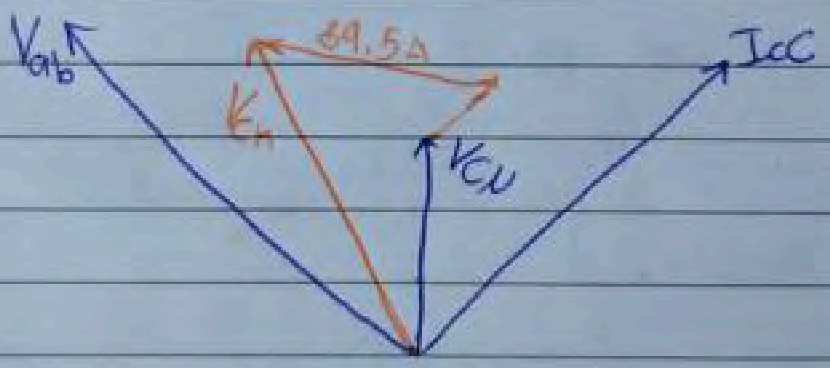


$V_{BC} = 415\angle -120$

$V_{BN} = 240\angle -150$

$I_{bB} = 69.5\angle -116.8$

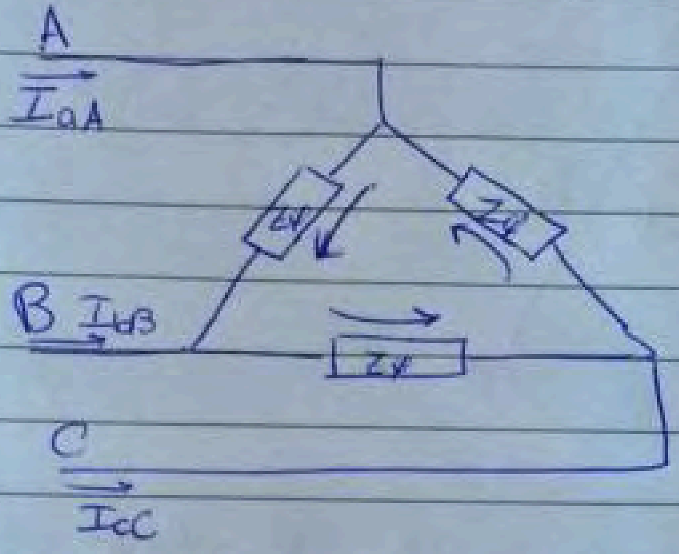
Phase-C :-



$S = V_{an} I_{aA}^* + V_{bn} I_{bB}^* + V_{cn} I_{cC}^*$

$= \sqrt{3} V_L I_L = \sqrt{3} (513.7)(69.5) = 61.8 kVA$

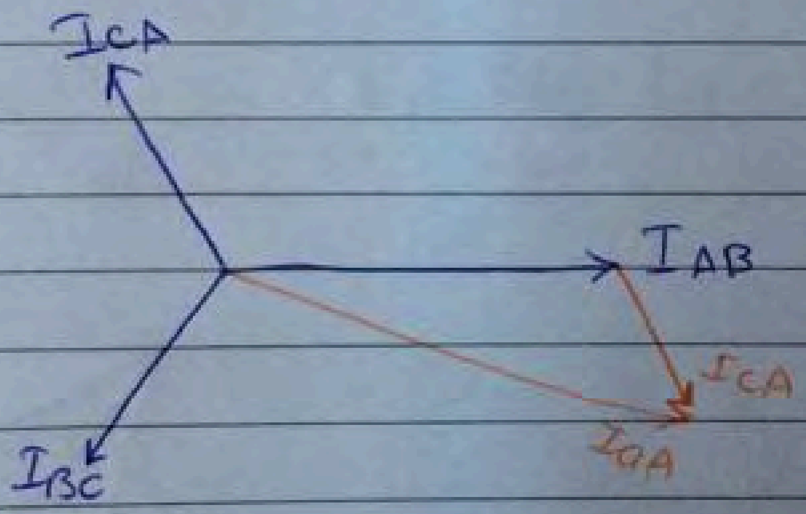
Δ- load



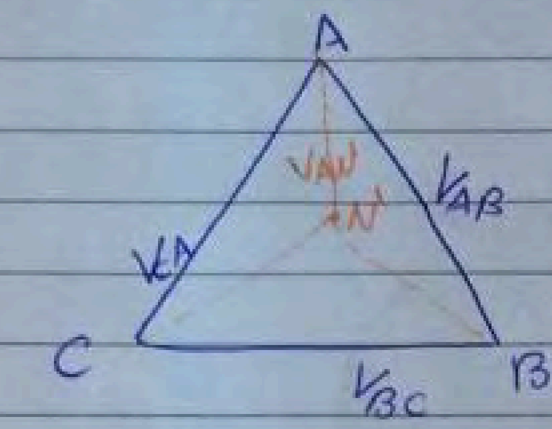
KCL at A

$$\left.\begin{aligned} I_{aA} &= I_{AB} - I_{CA} \\ I_{cC} &= I_{CA} - I_{BA} \\ I_{bB} &= I_{BC} - I_{AB} \end{aligned}\right] \text{+ve sequence}$$

$$\begin{aligned} S_\Delta &= 3\, S_{1\phi} \\ &= 3\, V_\phi I_\phi \\ &= 3\, V_\phi \frac{I_L}{\sqrt{3}} \quad = \underline{\underline{\sqrt{3}\, V_L\, I_L}} \end{aligned}$$

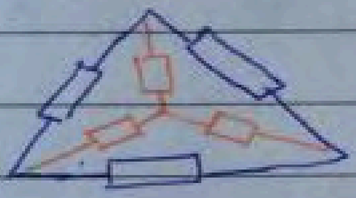


$\Delta \xrightarrow{\text{To}} 1\phi$ equivalent ckt.



$\Rightarrow$ Keep in mind $\frac{V_L}{\sqrt{3}} \angle -30 = V_{ph}$

$\Rightarrow Z_Y = \frac{Z_\Delta}{3}$

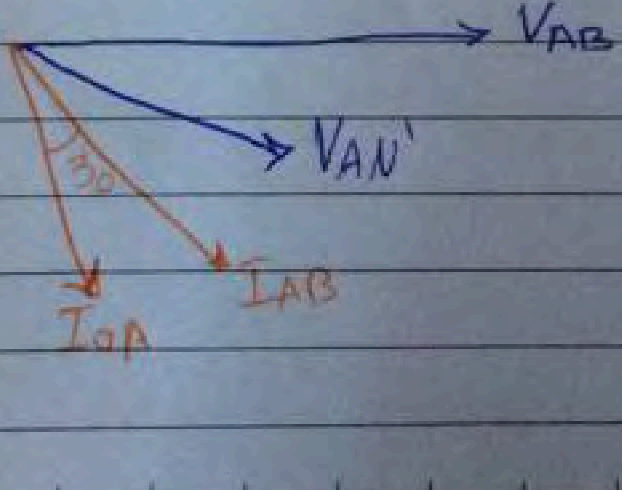


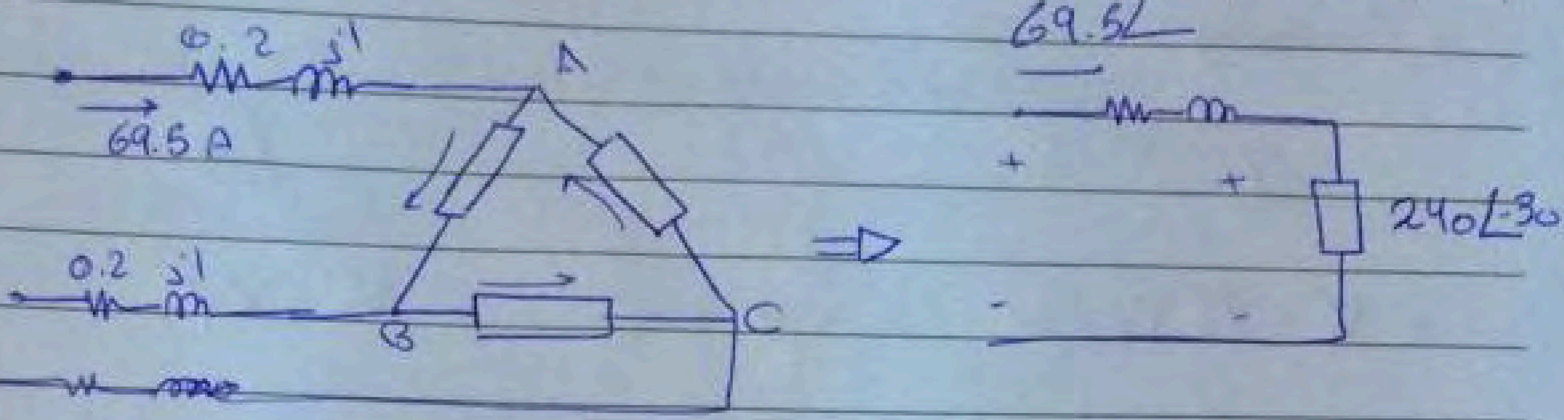
exp: Same as previous question but the load :-

50 KVA
0.8 lag
Δ

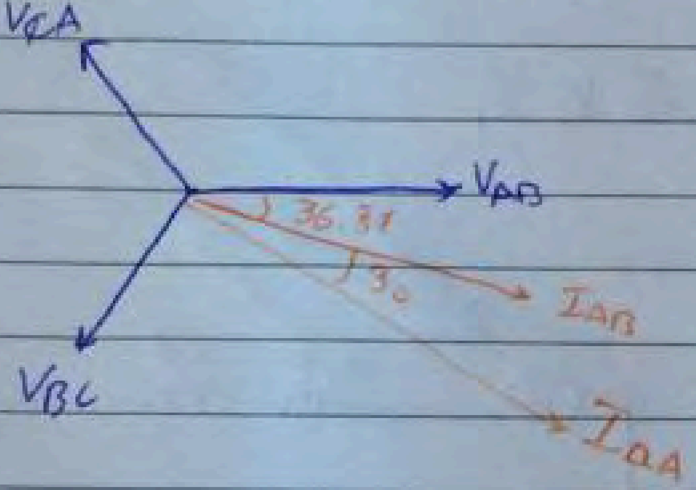
$S_\Delta = 50\,000 = \sqrt{3}\, V_L I_L$

$I_L = 69.5\,A$





$|I_L| = 69.5 A$



$\phi = \varphi_{V_{AB}} - \varphi_{I_{AB}}$

$V_{AN} = 296\angle -20$

$V_{ab} = \sqrt{3}(296\angle -20)\angle 30 = 512\angle 10$

Z_Δ ?

$S_{3\varphi} = 3V_\varphi I_\varphi^* = 3V_\varphi \frac{V_\varphi^*}{Z_\varphi^*}$

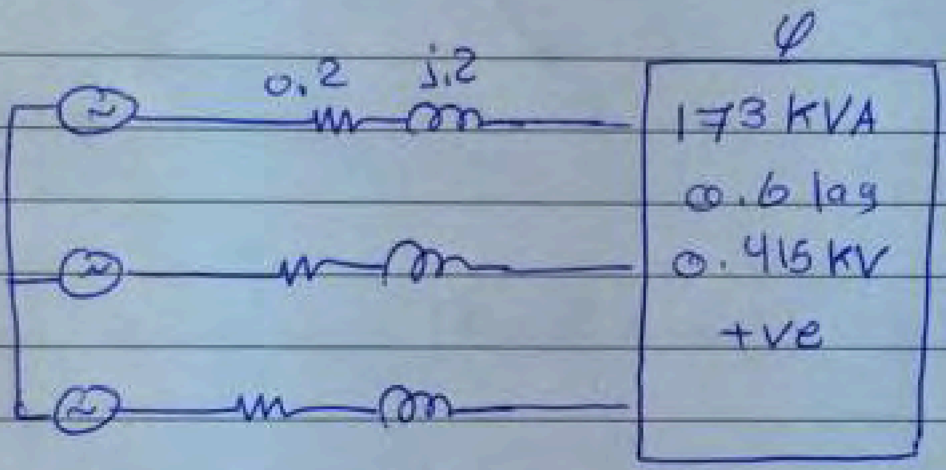
$Z_\varphi = \frac{3V_\varphi^2}{S_{3\varphi}^*} = \frac{3\left(\frac{V_L}{\sqrt{3}}\right)^2}{S_{3\varphi}^*} = \frac{V_L^2}{5\to} = \frac{415^2}{50k} = 3.5\ \Omega$

- Summary :-

$\underline{Y}$		$\underline{\Delta}$
• $V_L = \sqrt{3} V_\varphi \angle 30$	+ve seq.	• $V_L = V_\varphi$
• $I_L = I_\varphi$		• $I_L = \sqrt{3} I_\varphi \angle -30$
• $S = \sqrt{3} V_L I_L$		• $S = \sqrt{3} V_L I_L$

---

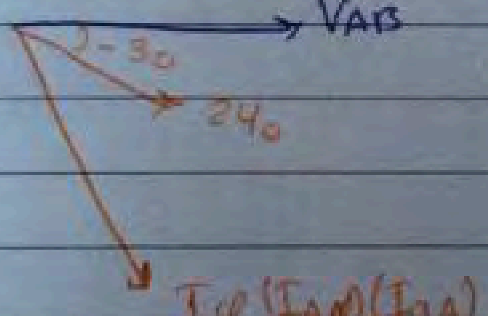
ex

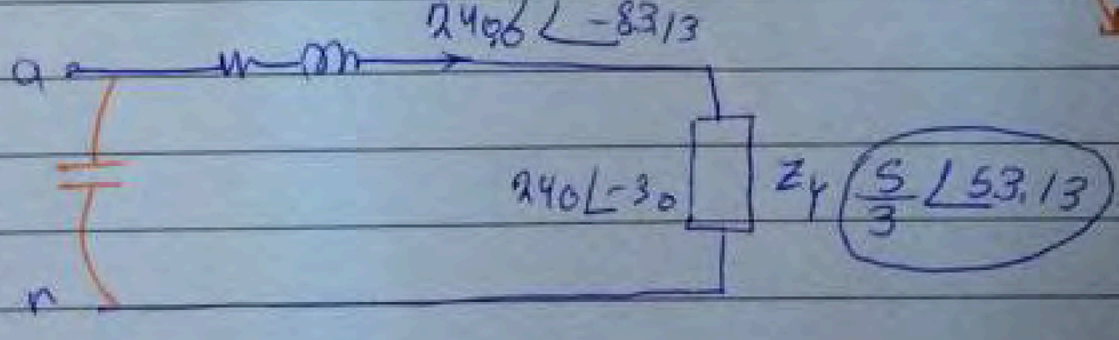


Required to increase the PF to the value of 0.9 lag at source

$I_L = \dfrac{173}{0.415\sqrt{3}} = 240.6$

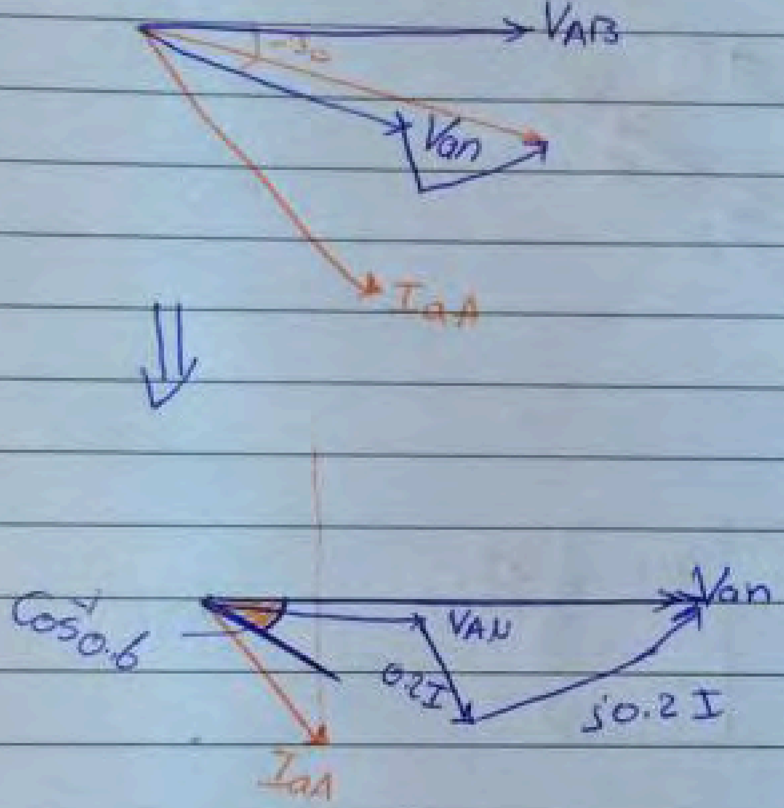
$V_\varphi I_\varphi = \dfrac{173}{3}$





$V_{an} = 240\angle -30 + (240.6 \angle -83.13)(2+j.2)$

$= 307 \angle -31$

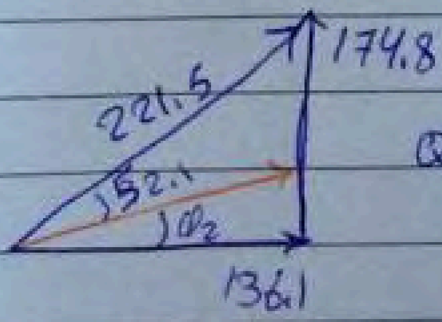


$S_s = 3 V_\varphi I_\varphi = 307 \times 240.6 \times 3$

$= 221.5$ kVA

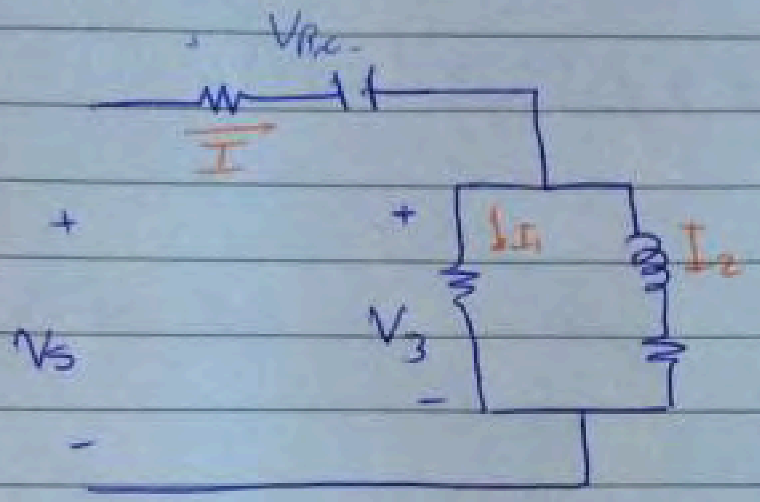
$S = 221.5 \angle 52.1$

$\varphi_s = \varphi_{VS} - \varphi_{IS} = -31 - (-83.13) = 52.1°$

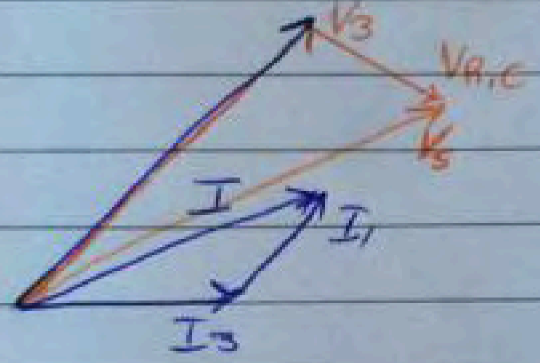


$Q_C = 174.8 = 136.1 \tan \cos^{-1} 0.9$

exp

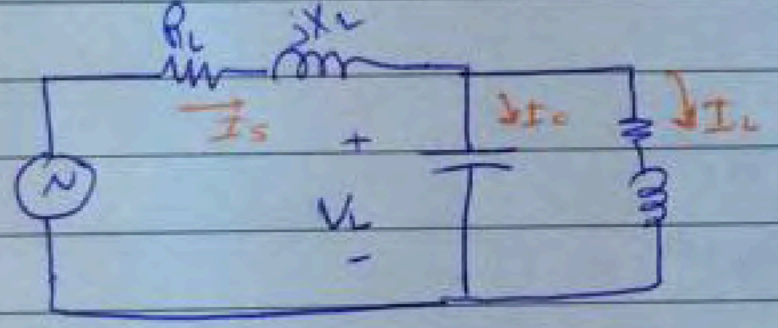


V_{R,C}
I
+
+
jI₁
I₂
Vs
V₃
-
-

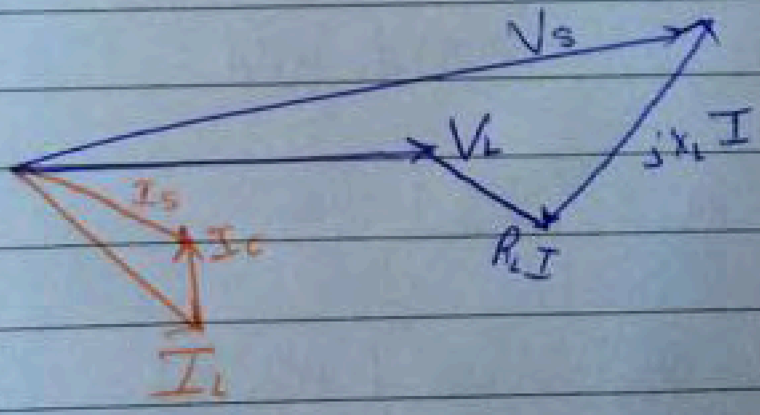


V₃
V_{R,C}
Vs
I
I₁
I₃

exp

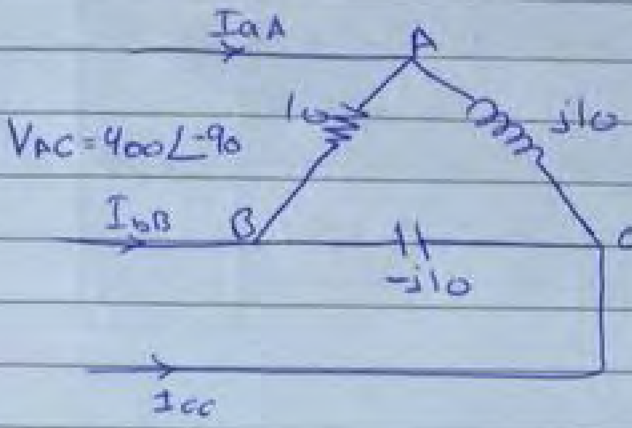


R_L
jX_L
I_s
+
jI_c
I_L
V_L
-



Vs
V_L
jX_L I
R_L I
I_s
I_c
I_L

exp



Voltage balanced

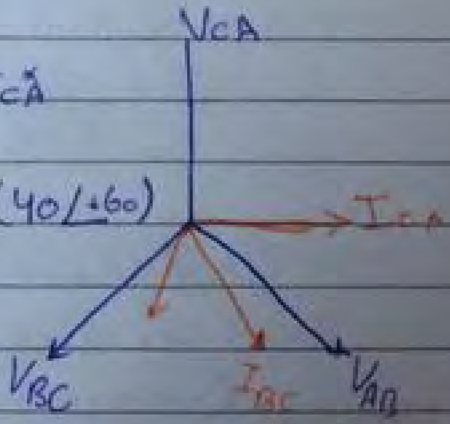
load Ab "

Balanced = $|V|$ $\qquad \angle \theta = 120$

$$I_{aA} = I_{AB} - I_{CA}$$

$$= \frac{V_{AB}}{Z_{AB}} - \frac{V_{CA}}{Z_{CA}} = \frac{400\angle -30}{10} - \frac{400\angle +90}{j10}$$

$$= 40\angle -30 - 40\angle 0 = 20.7\angle -105$$



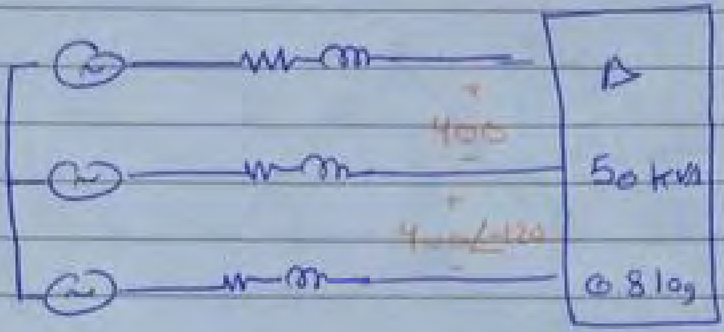
$$S = V_{AB} I_{AB}^* + V_{BC} I_{BC}^* + V_{CA} I_{CA}^*$$

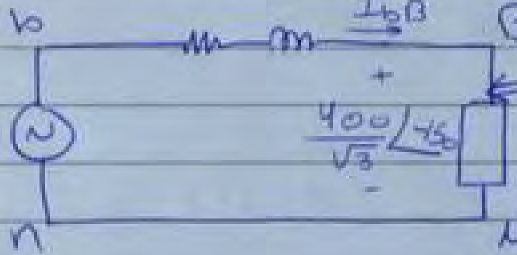
$$= 400\angle -30 (40\angle 30) + 400\angle -150 (40\angle +60)$$
$$+ 400\angle 90 (40\angle 0)$$

$$P_{3\phi} = \frac{400^2}{10}$$

$$Q = \frac{V_L^2}{Z^*} = \frac{400^2}{-j10} = -j1600$$

exp





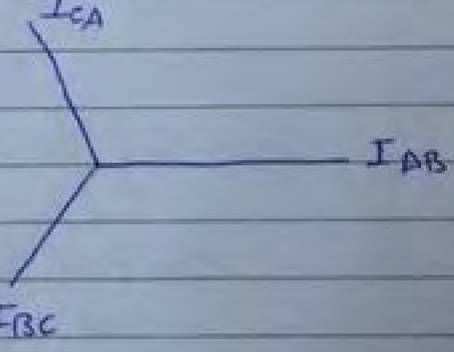
$$(400\angle -120)\left(\frac{1}{\sqrt{3}}\angle -30\right)$$
$$= \frac{400}{\sqrt{3}}\angle -150$$

$$|I_{bB}| = |I_L|$$

$$I_L = \frac{|S_{3\phi}|}{\sqrt{3}\,V_L} = \frac{50k}{\sqrt{3} \cdot 400} = 69.6A$$

The magnetude of the line cullent f

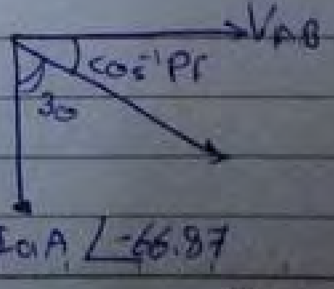
$$I_L = \frac{S_{3\phi}}{\sqrt{3}V_L}$$



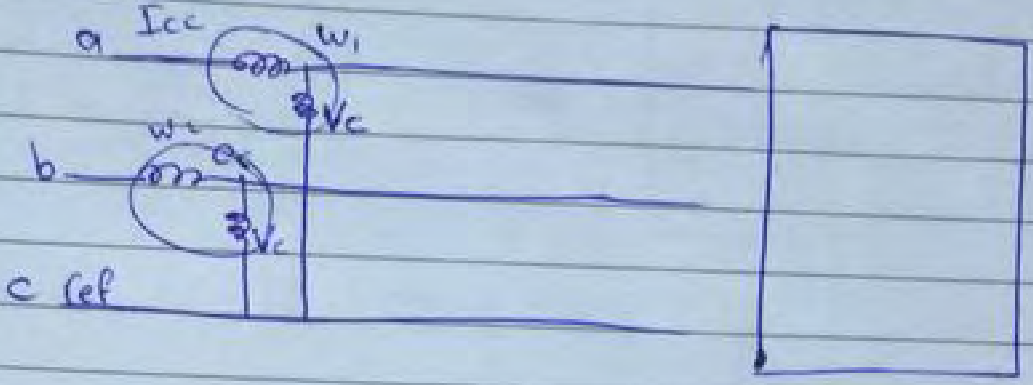
To detelmine the angle of the line cullent

Δ-Connect $V_\phi = V_L$

I_ϕ (lag or lead)



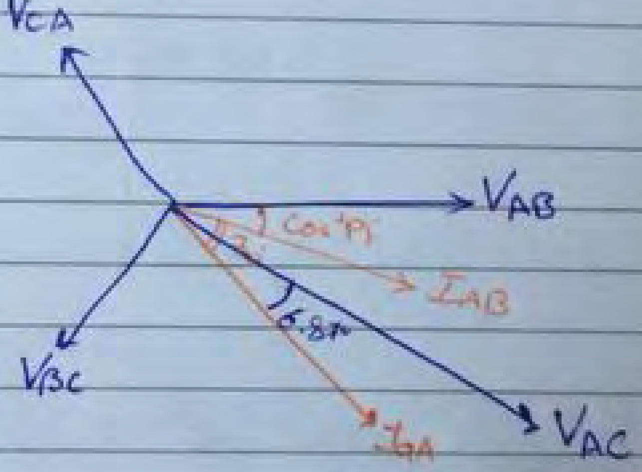
- Wattmeter:-



$W_1 = |V|_{AC} \, |I|_{aA} \cos\theta$, $\theta = \phi_{V_{AC}} - \phi_{I_{aA}}$

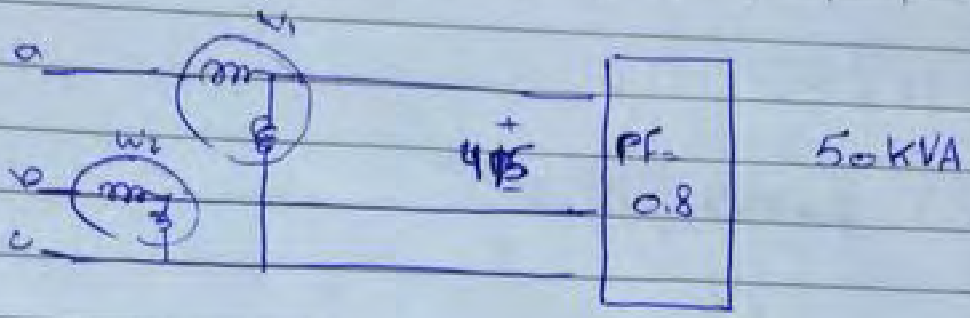
$W_2 = |V|_{BC} \, |I|_{bB} \cos\theta$, $\theta = \phi_{V_{BC}} - \phi_{I_{bB}}$

exp



$W_1 + W_2 = \sqrt{3} \, V_L I_L \cos\theta = S$

$(W_1 - W_2 = V_L I_L \sin\theta) = \frac{Q}{\sqrt{3}}$



$w_1 = |V_{AC}|\,|I_{aA}|\,\cos\theta_1$

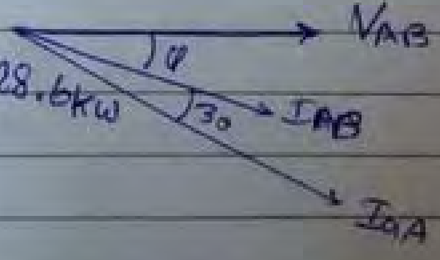
$w_2 = |V_{BC}|\,|I_{bB}|\,\cos\theta_2$

$S = \sqrt{3}\,|V_L|\,|I_L|$
$P = \sqrt{3}\,|V_L|\,|I_L|\,\cos\varphi$
$Q = \sqrt{3}\,|V_L|\,|I_L|\,\sin\varphi$

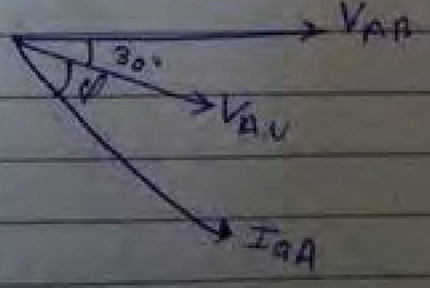
$$I_L = \frac{50000}{\sqrt{3} \times 400} = 69.5A$$

Pf determine the angle btw V_p and I_p

$W_1 = (415)(69.5)\cos(30 - 36.87) = 28.6kw$



$w_2 = (415)(69.5)\cos(30 + \varphi)$
$= 28KW$



⇒ Sum of $w = w_1 + w_2$
should = 40K
(50K×0.8 = 40KVA)

$W_1 - W_2 = 17.6$ kW

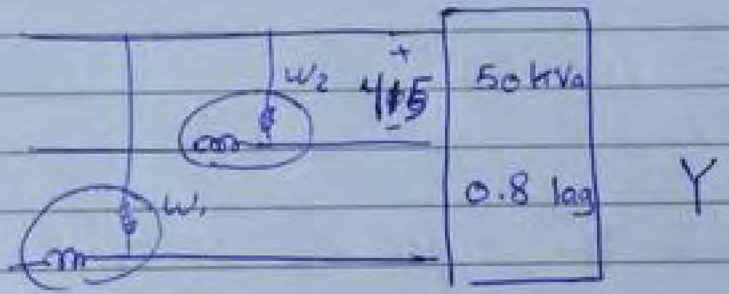
$\sqrt{3}(W_1 - W_2) = 30$ kVA $= Q_{3\varphi}$

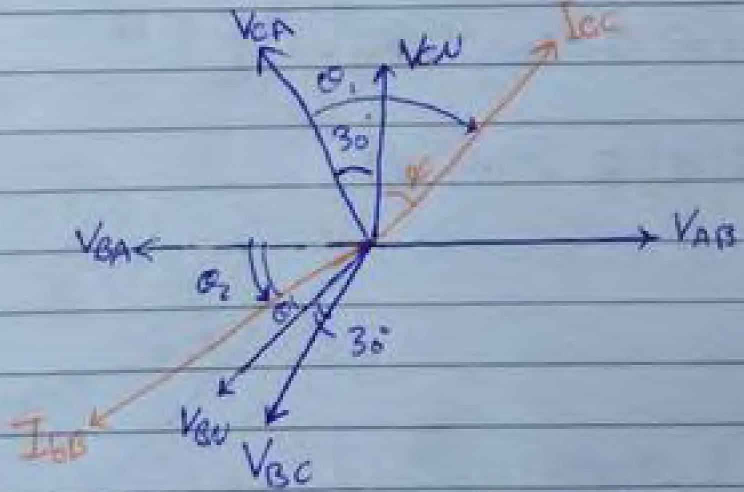
$$\frac{W_1 - W_2}{W_1 + W_2} = \frac{V_L I_L \sin\theta}{\sqrt{3} V_L I_L \cos\theta}$$

$$= \frac{\tan\theta}{\sqrt{3}}$$

$$\varphi = \tan^{-1}\frac{(W_1 - W_2)\sqrt{3}}{W_1 + W_2} = 36.87°$$

exp

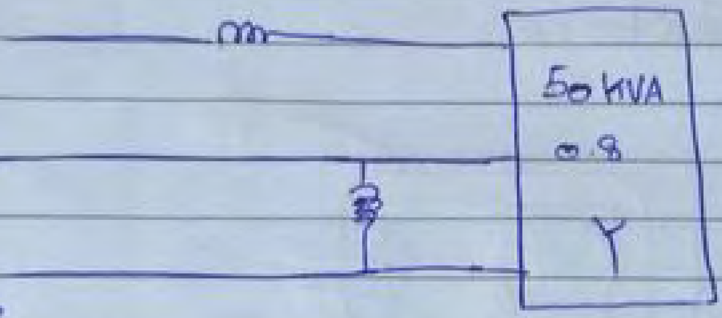




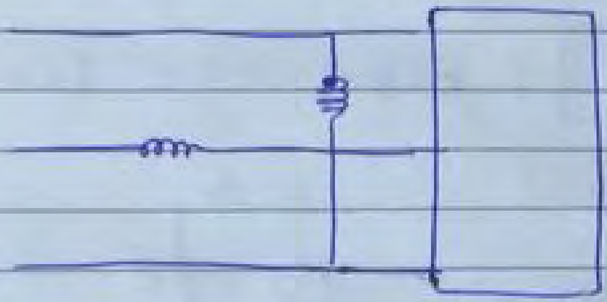
$W_1 = |V_{CA}|\,|I_{cc}| \cos(30 + \varphi)$

$W_2 = |V_{BA}|\,|I_{bB}| \cos(30 - \varphi)$

exp



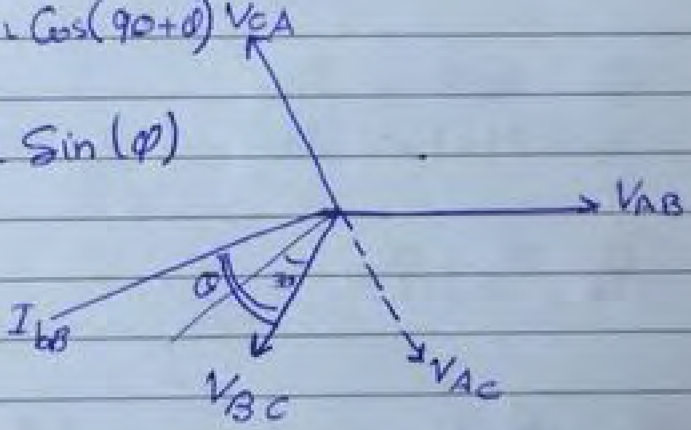
or

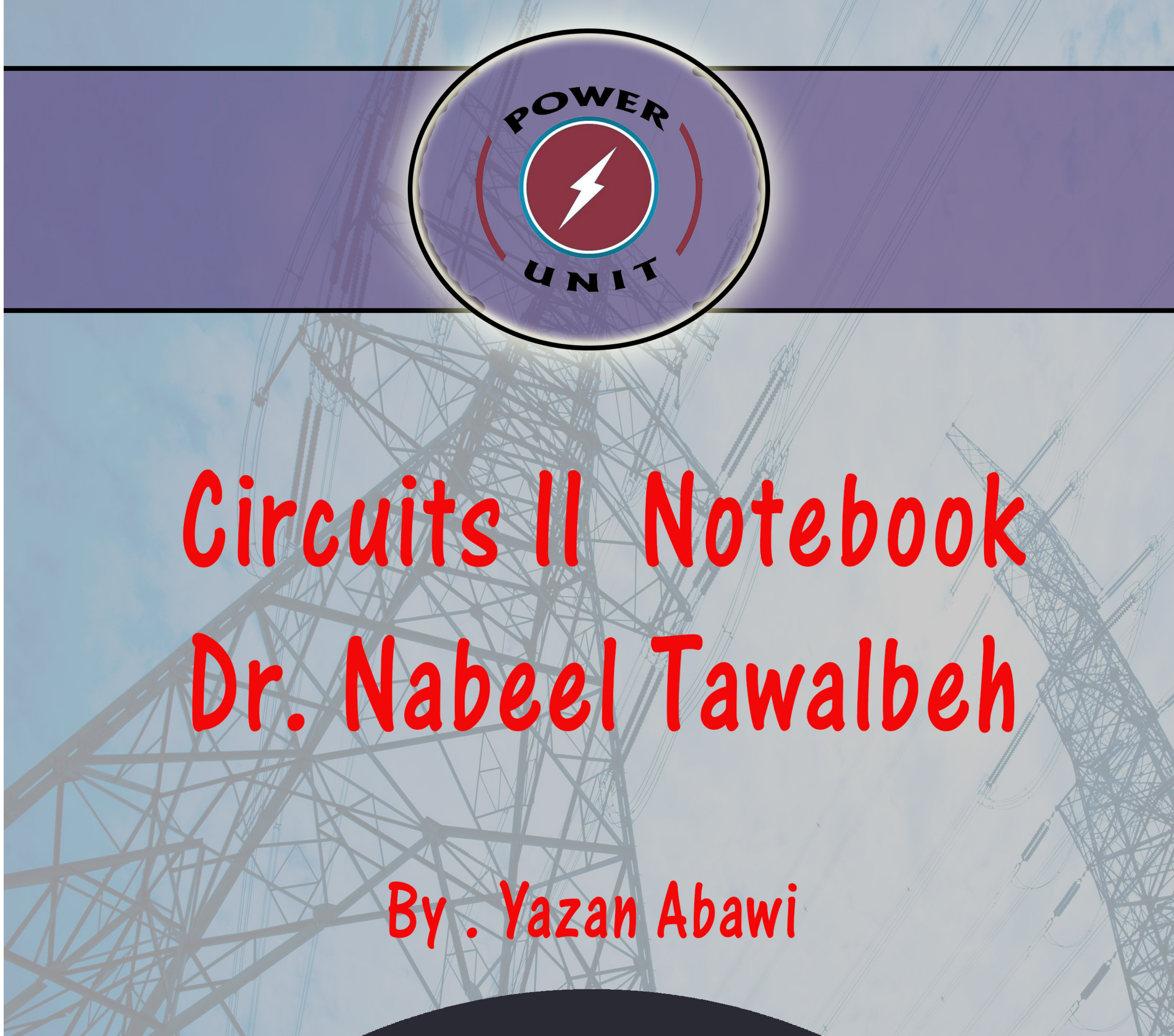


$W = V_{AC} I_{bB} \cos\theta$

$= V_L I_L \cos(90+\theta)$

$= -V_L I_L \sin(\varphi)$





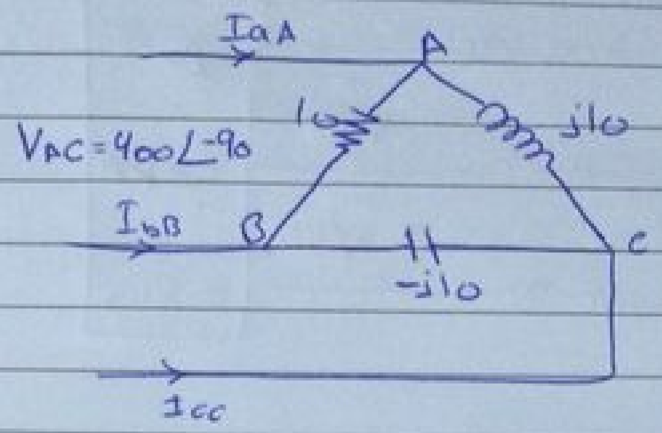
Circuits II Notebook
Dr. Nabeel Tawalbeh

By . Yazan Abawi

بأفكارنا_نُبدع

4th week

exp



Voltage balanced

load Ab

Balanced = |V| $\angle\theta = 120$

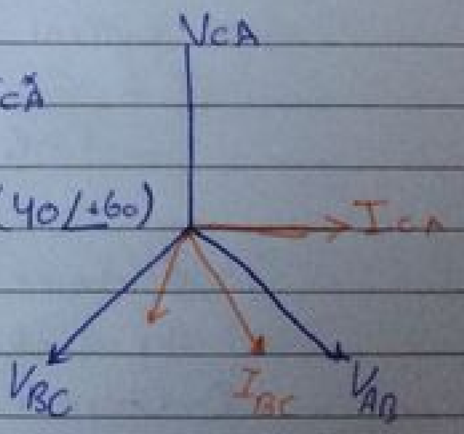
$I_{aA} = I_{AB} - I_{CA}$

$$= \frac{V_{AB}}{Z_{AB}} - \frac{V_{CA}}{Z_{CA}} = \frac{400\angle -30}{10} - \frac{400\angle +90}{j10}$$

$$= 40\angle -30 - 40\angle 0 = 20.7\angle -105$$

$$S = V_{AB} I^*_{AB} + V_{BC} I^*_{BC} + V_{CA} I^*_{CA}$$

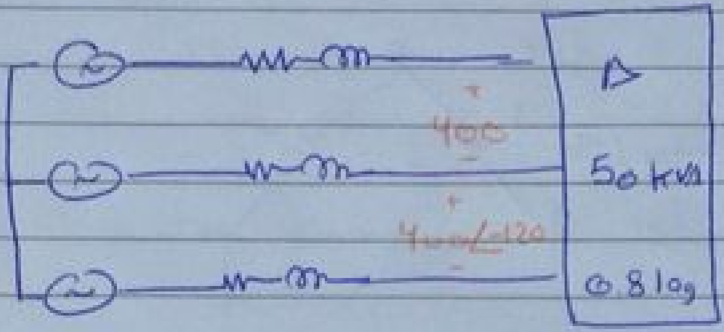
$$= 400\angle -30 (40\angle 30) + 400\angle -150 (40\angle +60)$$
$$+ 400\angle 90 (40\angle 0)$$

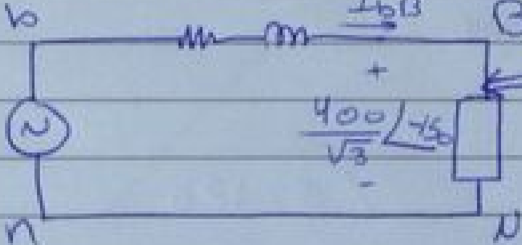


$$P_{3\phi} = \frac{400^2}{10}$$

$$Q = \frac{V_L^2}{Z^*} = \frac{400^2}{-j10} = -j1600$$

exp





$(400\angle{-120})(\frac{1}{\sqrt{3}}\angle{-30})$

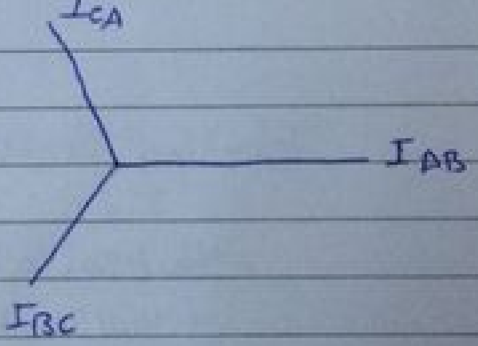
$= \frac{400}{\sqrt{3}}\angle{-150}$

$|I_{bB}| = |I_L|$

$I_L = \frac{|S_{3\phi}|}{\sqrt{3}\,V_L} = \frac{50k}{\sqrt{3}*400} = 69.5A$

The magnetude of the line current f.

$I_L = \frac{S_{3\phi}}{\sqrt{3}V_L}$



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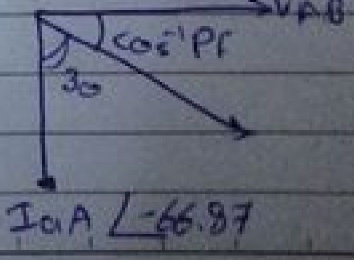
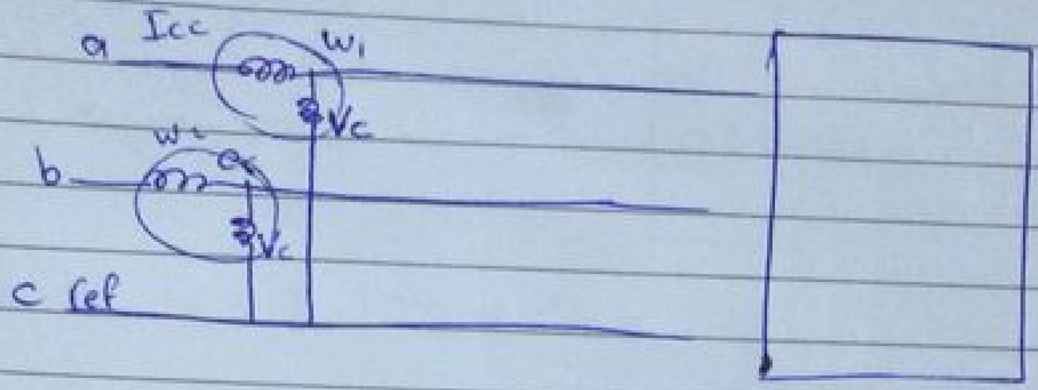


-Wattmeter:-



$W_1 = |V|_{AC} \, |I|_{aA} \cos\theta$, $\theta = \phi_{V_{AC}} - \phi_{I_{aA}}$

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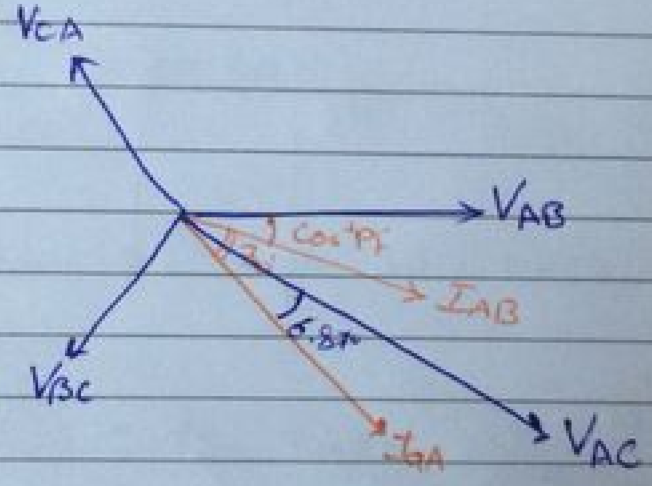
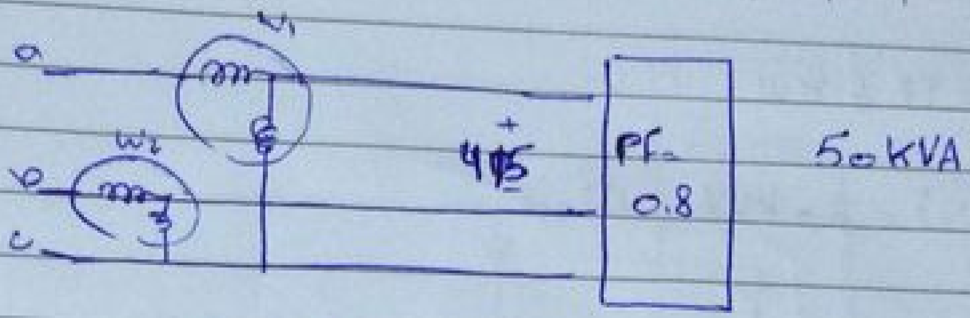
exp



$$W_1 + W_2 = \sqrt{3} \, V_L I_L \cos\theta = S$$

~~[illegible]~~

$$(W_1 - W_2 = V_L I_L \sin\theta) = \frac{Q}{\sqrt{3}}$$



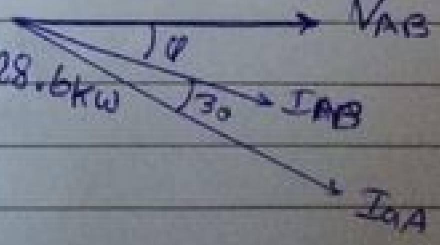
$W_1 = |V_{AC}|\,|I_{aA}|\cos\theta_1$

$W_2 = |V_{BC}|\,|I_{bB}|\cos\theta_2$

$S = \sqrt{3}\,|V_L||I_L|$
$P = \sqrt{3}\,|V_L|\,|I_L|\cos\varphi$
$Q = \sqrt{3}\,|V_L||I_L|\sin\varphi$

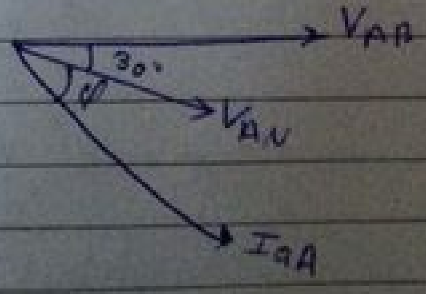
$I_L = \frac{50000}{\sqrt{3}\times 400} = 69.5A$

PF determine the angle btw V_p and I_p



$W_1 = (415)(69.5)\cos(30-36.87) = 28.6kW$

$W_2 = (415)(69.5)\cos(30+\varphi)$
$= 11 kW$



⇒ Sum of $W = W_1 + W_2$
should = 40K
$(50K\times 0.8 = 40kVA)$

$W_1 - W_2 = 17.6 \text{ KW}$

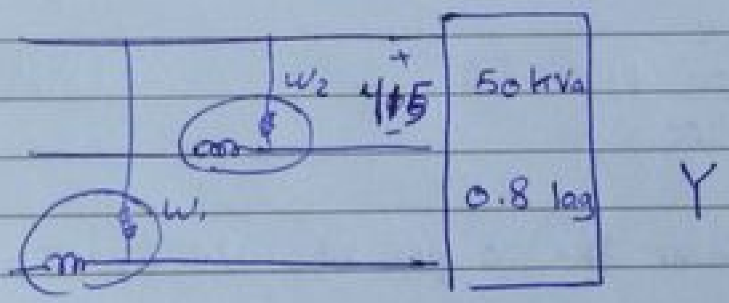
$\sqrt{3}(W_1 - W_2) = 30 \text{ kVA} = Q_{3\varphi}$

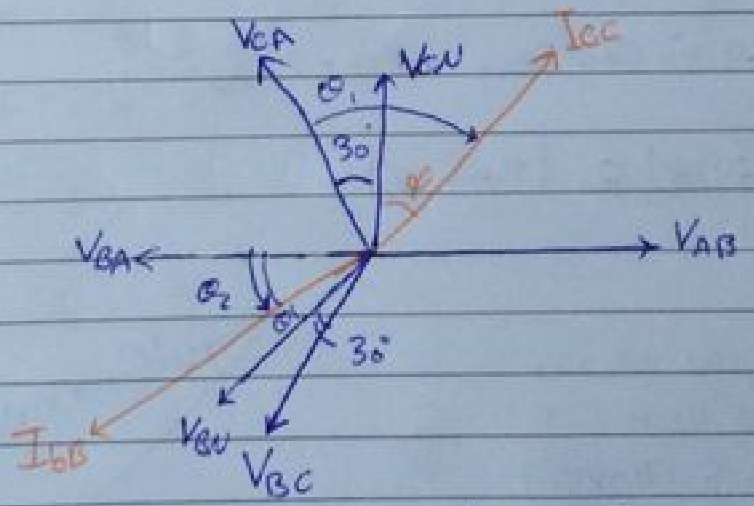
$$\frac{W_1 - W_2}{W_1 + W_2} = \frac{V_L I_L \sin\theta}{\sqrt{3}\, V_L I_L \cos\theta}$$

$$= \frac{\tan\theta}{\sqrt{3}}$$

$$\varphi = \tan^{-1}\frac{(W_1 - W_2)\sqrt{3}}{W_1 + W_2} \qquad = 36.87°$$

<u>exp</u>

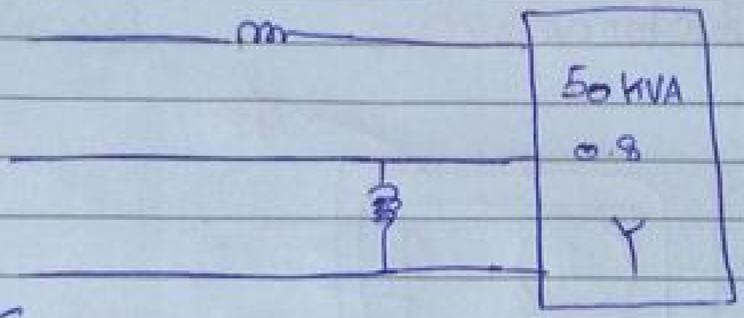




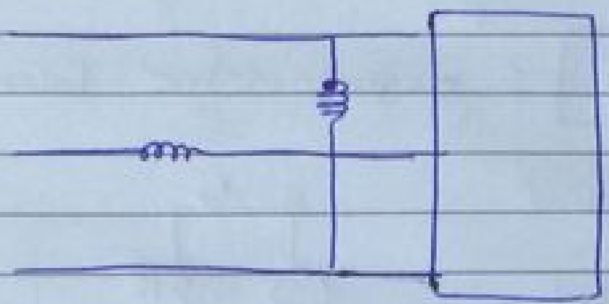
$W_1 = |V_{CA}|\,|I_{CC}|\cos(30+\varphi)$

$W_2 = |V_{BA}|\,|I_{BB}|\cos(30-\varphi)$

exp



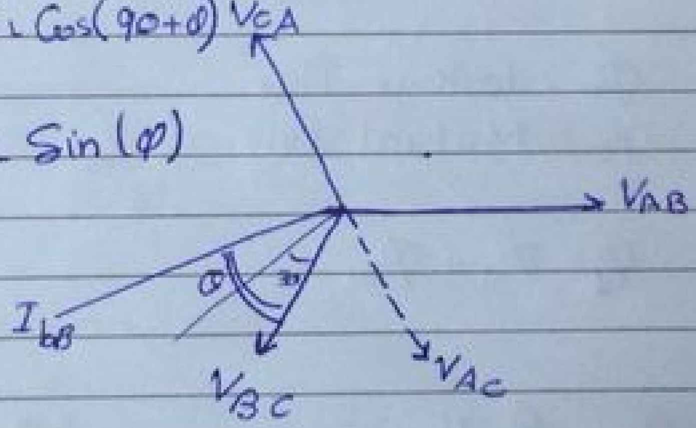
or



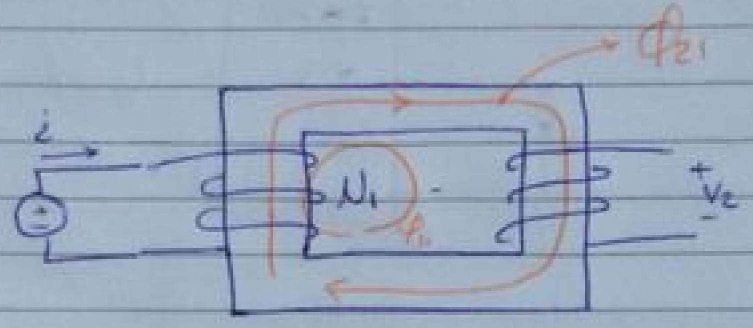
$W = V_{AC} \, I_{bB} \cos\theta$

$= V_L I_L \cos(90+\phi)$

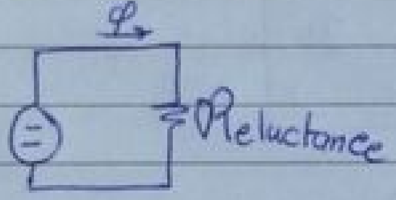
$= -V_L I_L \sin(\phi)$



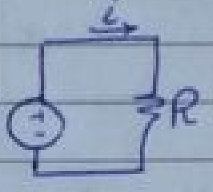
Mutual Conductance :-



$\boxed{i N = mmF}$: magnetomotive Force



MMF



EMF

ϕ_{11} : leakage flux
ϕ_{21} : Mutual flux

$$\phi_1 = \phi_{11} + \phi_{21}$$

$$\phi_{11} = P_{11} N_1 i_1$$

P_{11} : Permeance of the material occupied by P_{11}

$$\phi_{21} = P_{21} N_1 i_1$$

$$\phi_1 = P_1 N_1 i_1$$

$$\therefore \boxed{P_1 = P_{11} + P_{21}}$$

$V = \frac{\partial \phi}{\partial t}$ ψ ϕ : Flux linkage

$\psi = N\phi$

$v = \frac{\partial N\phi}{\partial t} = N\frac{\partial \phi}{\partial t}$

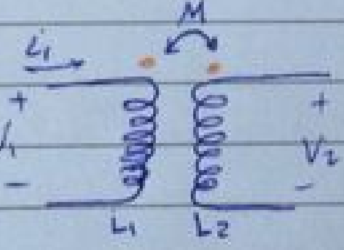
$v_1 = N_1 \frac{\partial \phi}{\partial t} = N_1 \frac{\partial}{\partial t} P_1 N_1 i_1 = N_1^2 P_1 \frac{d}{dt} i_1 = L \frac{di}{dt}$

$\boxed{V_1 = L_1 \frac{di}{dt}}$ L_1 : Self Voltage

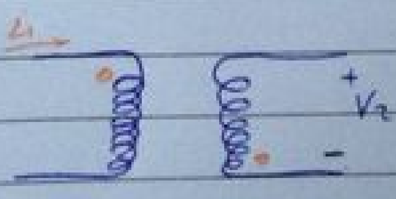
$V_2 = N_2 \frac{d\phi_{21}}{dt} = N_2 \frac{d}{dt}(P_{21} N_1 i_1) = N_2 N_1 P_{21} \frac{di_1}{dt}$

$\boxed{V_2 = M_{21} \frac{di}{dt}}$ M_{21} : mutual Voltage

- Dot Convention:-



$V_2 = M \frac{di_1}{dt}$



$V_2 = -M \frac{di_1}{dt}$

$L_1 = P_1 N_1^2$

$L_2 = P_2 N_2^2$

$$L_1 L_2 = P_1 N_1^2 N_2^2 P_2 = (P_{11} + P_{21}) N^2 \cdot (P_{22} + P_{12}) N_2^2$$

$$= N_1^2 P_{21} \left(1 + \frac{P_{11}}{P_{12}}\right) N_2^2 P_{12} \left(1 + \frac{P_{22}}{P_{12}}\right)$$

$$= N_1^2 N_2^2 P_{12}^2 \left(1 + \frac{P_{11}}{P_{12}}\right)\left(1 + \frac{P_{22}}{P_{12}}\right)$$

$$= N_1^2 N_2^2 P_{12}^2 \frac{1}{k^2} = M^2$$

$L_1 L_2 = M^2 \frac{1}{k^2}$

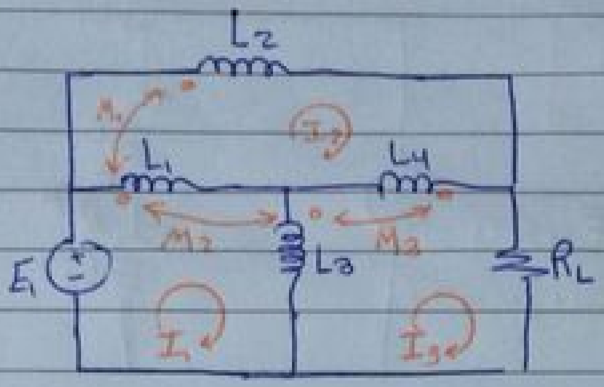
$\boxed{M = k\sqrt{L_1 L_2}}$ k: Coupling Factor

$\rightarrow$ at $P_{11} = 0 \longrightarrow k = 1$

$\rightarrow$ at $P_{12} = 0 \longrightarrow k = 0$

ⓐ

exp

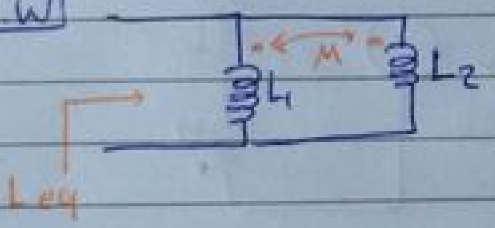


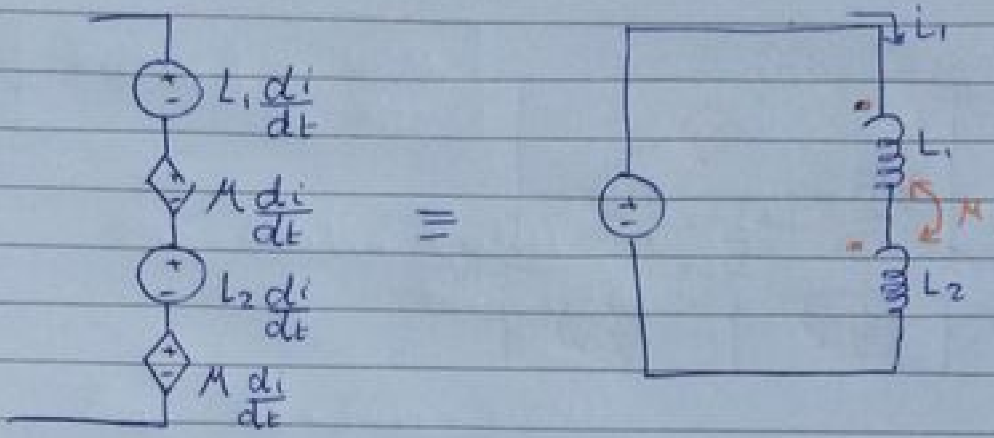
$$E_1 = L_1 \frac{d}{dt}(i_1 - i_2) + L_3 \frac{d}{dt}(i_1 - i_3) \quad \Big] \text{ Self Voltage}$$

$$\left.\begin{aligned} & M_1 \frac{d}{dt} i_2 + M_2 \frac{d}{dt}(i_1 - i_3) \\ & + M_3 \frac{d}{dt}(i_1 - i_3) + M_3 \frac{d}{dt}(i_2 - i_3) \end{aligned}\right] \text{ Mutual Voltage}$$

⋮

H.W

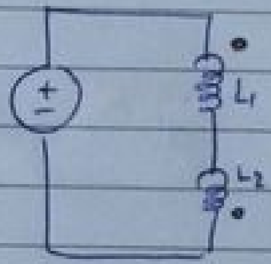




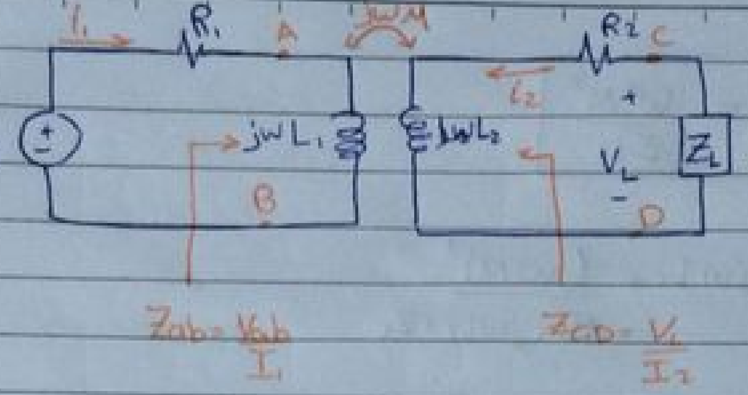
$$E_1 = \left(L_1 \frac{di}{dt} + L_2 \frac{di}{dt} \ldots + M \frac{di}{dt} + M \frac{di}{dt} \right)$$

$\boxed{L_{eq} = L_1 + L_2 + 2M}$ في حالة الSeries

$\boxed{L_{eq} = L_1 + L_2 - 2M}$ ⇒



ex



$$V_S = (\underbrace{R + jwL_1}_{Z_{11}}) I_1 + jwMI_2$$

$$0 = (\underbrace{R_2 + jwL_2 + Z_L}_{Z_{22}}) I_2 + jwMI_1$$

$$V_S = Z_{11} I_1 + jw\,MI_2$$

$$0 = jwMI_1 + Z_{22} I_2$$

$\therefore$ Z_{11} loop impedence in ① - forward
Z_{22} loop impedence in ② - backward

$$I_1 = \frac{\begin{vmatrix} V_s & jwM \\ 0 & Z_{22} \end{vmatrix}}{\begin{vmatrix} Z_{11} & jwM \\ jwM & Z_{22} \end{vmatrix}} = \frac{Z_{22}}{Z_{11} Z_{22} + w^2 M^2} V_s$$

$$Z_{in} = \frac{V_s}{I_1} = \frac{Z_{11} Z_{22} + w^2 M^2}{Z_{22}} = \boxed{Z_{11} + \frac{w^2 M^2}{Z_{22}}}$$

$$wM = jwM = |Z_M|$$

$$Z_M = jwM = jX_m$$

⑫

$$Z_{in} = Z_{f.w} + Z_{\text{Reflected}}.$$

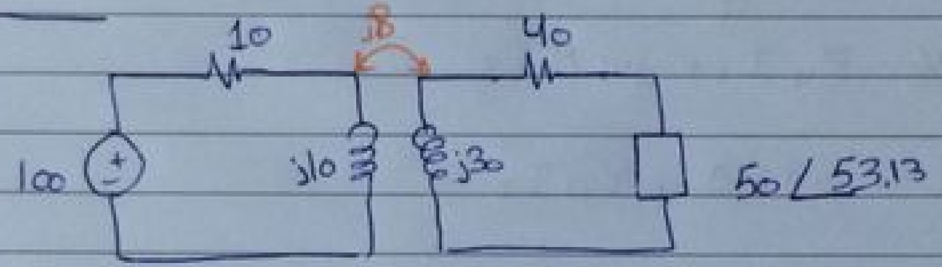
$$\boxed{Z_{reflected} = \frac{w^2 M^2}{Z_{22}}}$$

$$Z_{AB} = jwL_1 + \frac{(wM)^2}{R_2 + jwL_2 + Z_L}$$

$$Z_{CD} = R_2 + jwL_2 + \frac{(wM)^2}{R_1 + jwL_1}$$

* All power independent sources are Zeroing

exp



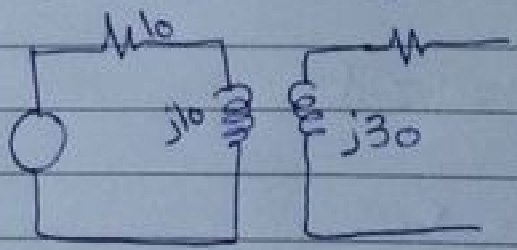
Find I_1, I_2 ?

$$I_1 = \frac{100}{Z_s} = \frac{100}{10 + j10 + \frac{64}{70 + j70}} = \boxed{7.1\angle -42}$$

$$0 = (70 + 70j) I_2 = j8(I_1)$$

$$I_2 = \boxed{5.7\angle 2.6}$$

Find I_2 Without using Mesh (use Thevinen)



Kill the souce

$$Z_{th} = 40 + j30 + \frac{64}{10 + 10j}$$

$$V_{th} = V_{o.c} = j8\, I_1 = j8\left(\frac{100}{10 + j10}\right) = \boxed{56.5 \angle 45^\circ}$$

$$I_2 = \frac{V_{th}}{Z_L + Z_{th}} = \frac{56.5 \angle 45^\circ}{50 \angle 5213 + 50.8 \angle 31} = \boxed{0.57 \angle 2.6}$$

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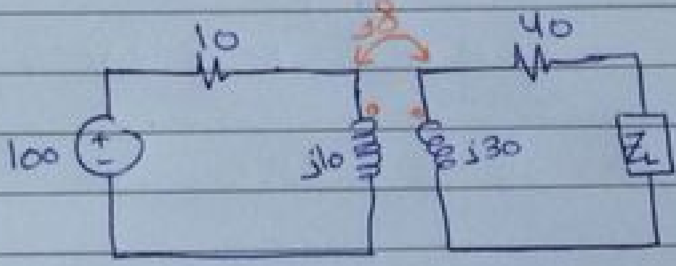
* Take home exam



Find the Z looking by the Souce if the Z_L is adjusted that make maximum Power

$$Z = 10 + j10 + \frac{64}{40 + j30 + Z_{th}^*} \longrightarrow \text{seen by } Z_L \text{ terminal}$$

$$Z_{th} = 40 + j30 + \frac{64}{10 + j10} = 50 \angle 32$$

$$Z = 10 + j10 + \frac{64}{40 + j30 + 50 \angle -32}$$

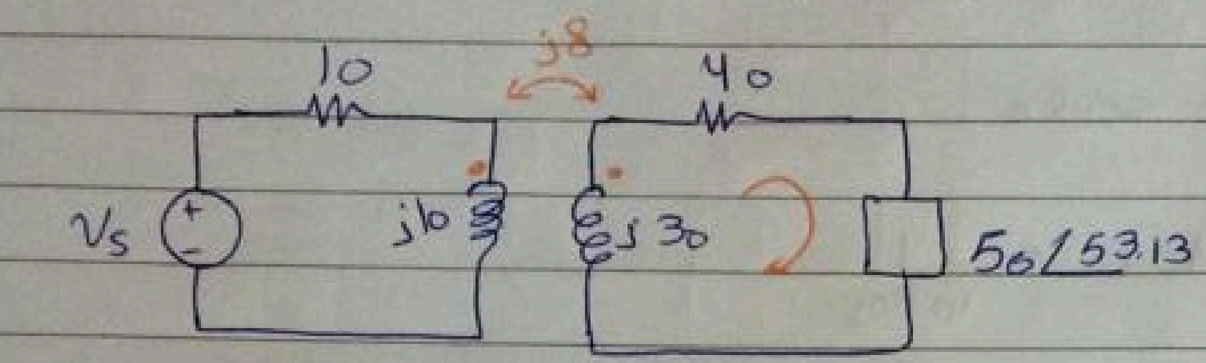
Circuits II Notebook
Dr. Nabeel Tawalbeh

By . Yazan Abawi

بأفكارنا_نُبدع

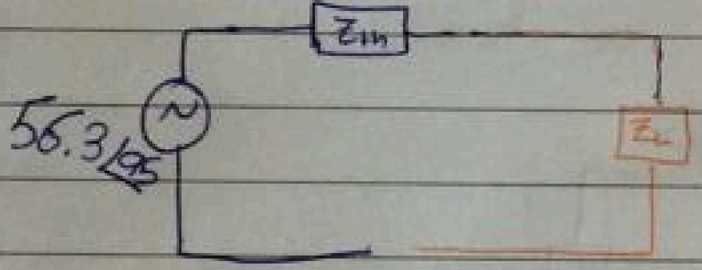
5th week

Week(5)



- Find Z seen by the source terminal if Z_L is adjusted

$Z_{th} = 50.8\angle -31$



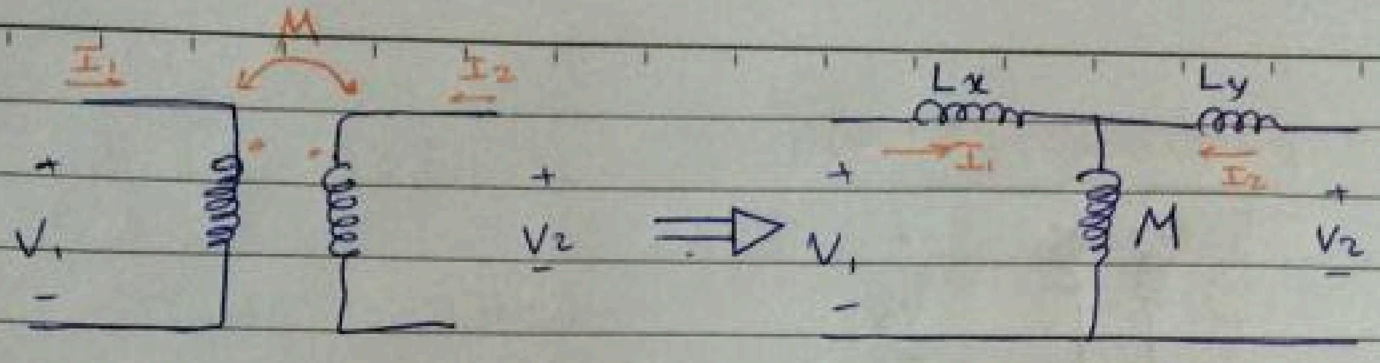
$$Z_s = 10 + j10 + \frac{64}{40 + j30 + 50.8\angle -31}$$

$$R_o = \left| 40 + 30j + \frac{64}{10 + j10} \right| = |50.8\angle 31| = 50.8$$

$$Z_s = 10 + j10 + \frac{64}{40 + j30 + 50.8}$$

ⓒ

exp ⇒



$* \; V_1 = jwL_2 I_1 + jw M I_2$

$* \; V_2 = jw L_2 I_2 + jw M I_1$

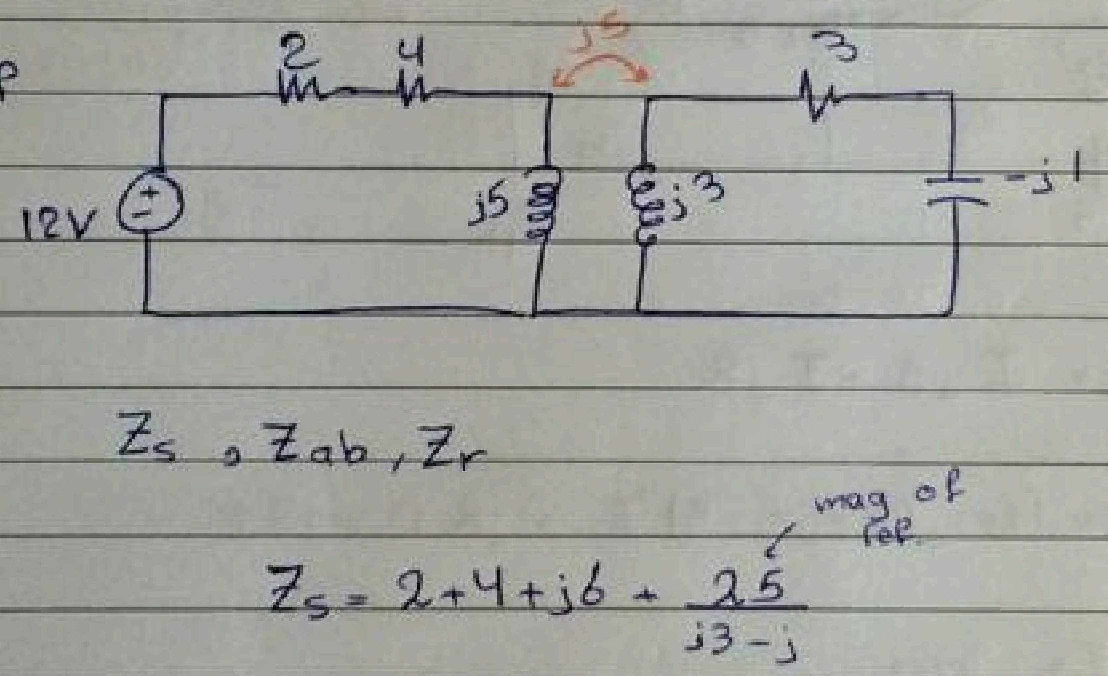
$* \; V_1 = jw(L_x + L_M) I_1 + jwM I_2$

$* \; V_2 = jwL_m I_1 + jw M(L_M + L_y) I_2$

$\Rightarrow L_M = M$

$L_x + M = L_1 \longrightarrow L_x = L_1 - M$

exp



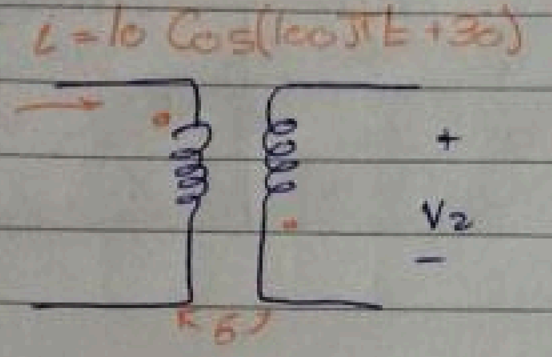
Z_s , Z_{ab} , Z_r

$$Z_s = 2+4+j6 + \frac{\overset{\text{mag of ref.}}{25}}{j3-j}$$

$$Z_{th} = 3+j3 + \frac{25}{6+j6} \quad , \quad V_{th} = \left(\frac{12}{6+j6}\right) \cdot j5$$

$$Z_{ab} = Z_s - 2$$

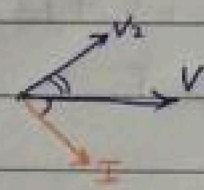
$$Z_r = \frac{25}{3+j2}$$



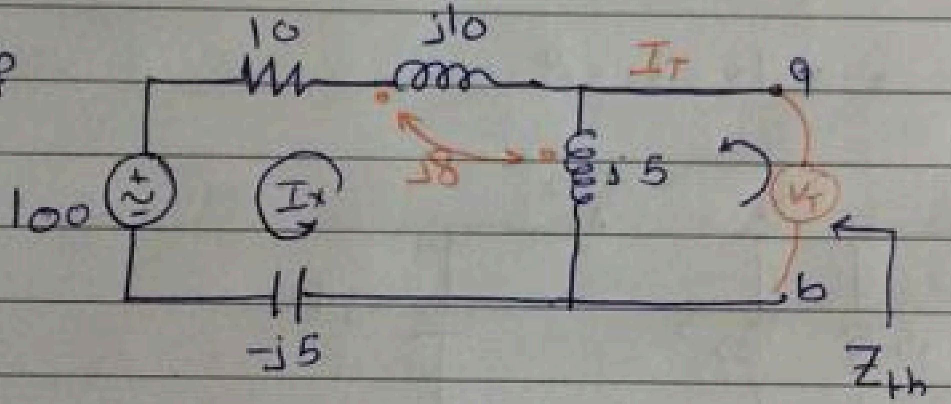
$$V_2 = -M \frac{d}{dt} 10 \cos(100 wt - 30)$$

$$= +5 * 10 * 100\pi \cos(100\pi t + -120)$$

$$V = 500\pi \angle 60^\circ$$



exp



$$\Rightarrow V_{ab} = \left(\frac{100}{26j+10}\right)(j13)$$

$$V_{ab} = I\,j5 + I\,j8$$

$$100 = (10 + j10 + j5 - 5) I + j8 I + j8 I$$

$$I = \frac{100}{10 + 26j}$$

$$V_T = j5 I_T - j5 I_x - j8 I_x$$

$$0 = (10 + j10) I_x - j5 I_T - j8(I_T - I_x) - j8 I_x$$

$= 5 I_T - j13 I_v = V_T$

$-j13 I_T + (10+j18) I_x = 0$

$$I_T = \frac{\begin{vmatrix} V_T & -j13 \\ 0 & 10+j18 \end{vmatrix}}{\begin{vmatrix} j5 & -j13 \\ -j13 & +j18 \end{vmatrix}} = \frac{(10+j18)}{T} V_T$$

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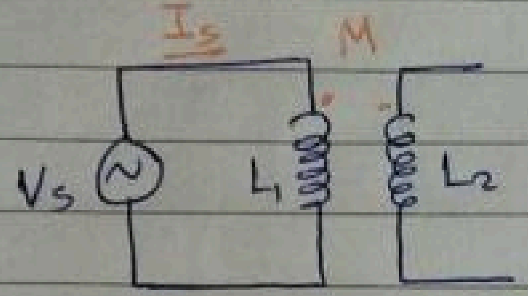


$$w_1 = \int_0^{i} P\,dt = \int_0^{I_1} L_1 i_1\, di_1 = \boxed{\frac{1}{2} L\, i_1^2}$$

$$\boxed{P_1 = V_1 i_1} = L_1 \frac{di_1}{dt} I_1$$

$$P_2 = V_2 i_2$$

$$w_2 = \int P\,dt = \int L_2 \frac{di_2}{dt} i_2 = \int_0^{I_2} L_2\, di_2$$

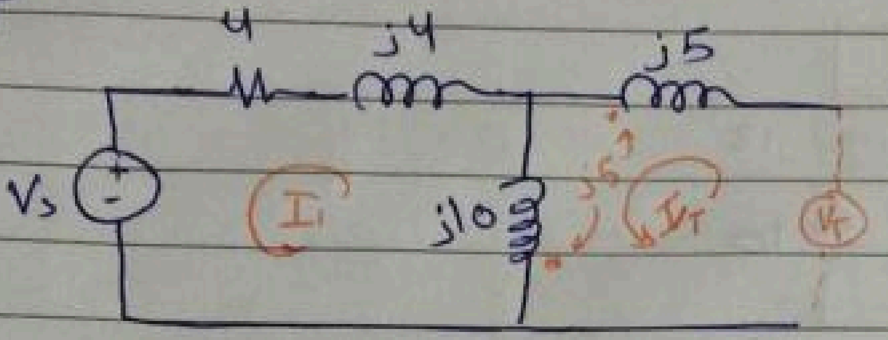
$$V_2 = i_2 \frac{di_2}{dt} \quad , \quad V = M \frac{di_1}{dt}$$

$$I_1 V_2 = i_1 M \frac{di_2}{dt}$$

$$W = \int_0^{I} i_1 M \frac{di_2}{dt} \partial t = M I_1 I_2$$

$$\boxed{w = \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 + M I_1 I_2} \longleftarrow \text{Energy}$$

exp



$V_T = (j5+j10)\, I_T - j10\, I_1 + j5(I_1 - I_T) - j5\, I_T$

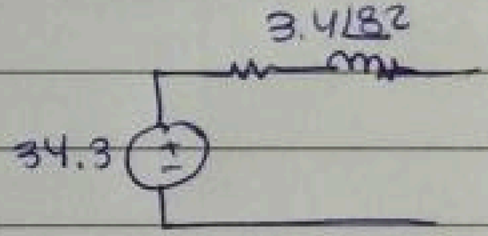
$\boxed{V_T = j5\, I_T - j5\, I_1}$ --- ①

$-100 = (4+j4) I_1 - j10\, I_T + j5\, I_T$

$\boxed{-100 = -j I_T + (4+j14) I_1}$ -- ②

$$I_T = \frac{\begin{pmatrix} V_T & -j5 \\ 0 & (4+j14) \end{pmatrix}}{\begin{pmatrix} j5 & -j5 \\ -j5 & 4+j14 \end{pmatrix}} = \frac{(4+j14)\, V_T}{(-70+j20)+25}$$

$$Z_{th} = \frac{V_T}{I_T} = \frac{-45+j20}{4+j14} = 3.4\angle 82$$



$$I_1 = \frac{100}{4+j14} = 6.9\angle{-74}$$

$i_1(t) = 6.9\sqrt{2}\, \cos(100\pi t - 74)$

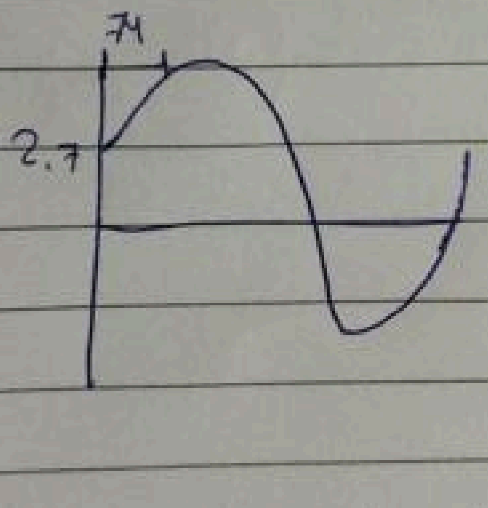
$i_1(\frac{1}{4}) = i(5\,ms) = 4.38\ A$

$W = \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 - M i_1 i_2$

$L_1 = \frac{X}{\omega} = \frac{10}{100\pi}$

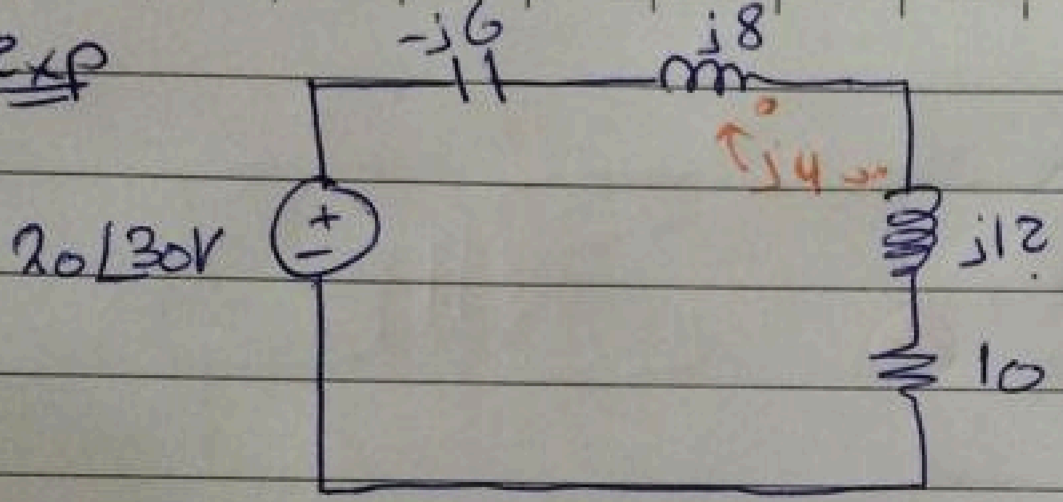
$L_2 = \frac{X}{\omega} = \frac{5}{100\pi}$

$M = \frac{5}{\omega}$



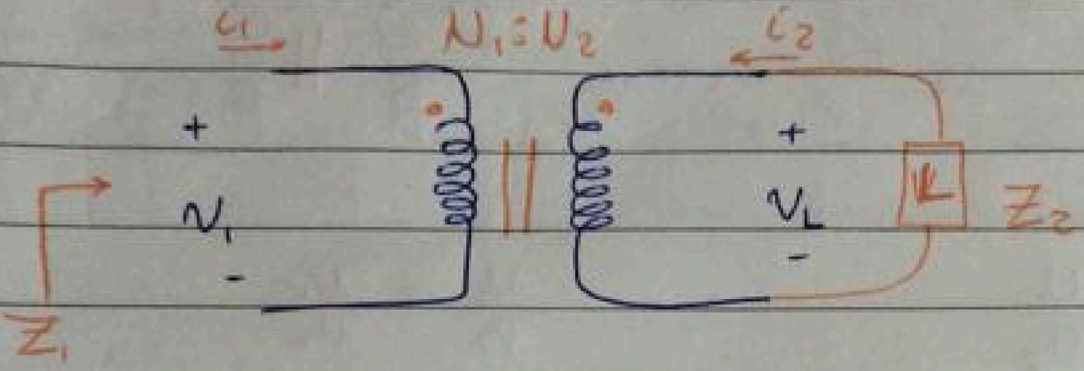
22/7

exp



⑦

Ideal Transformer :-



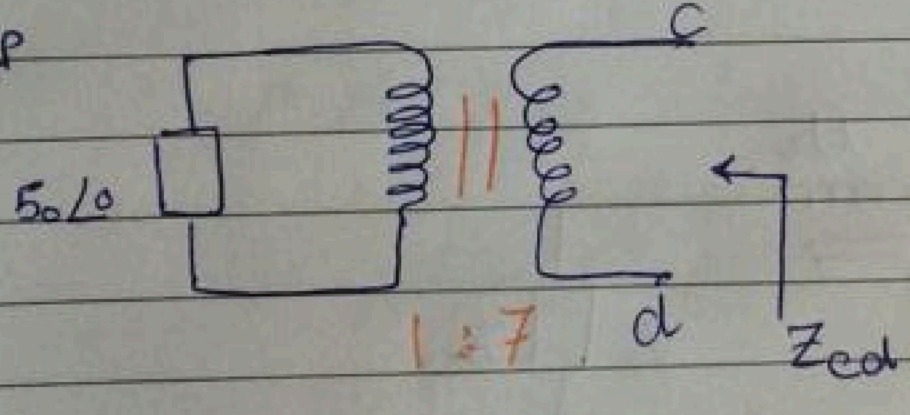
$$\boxed{\frac{N_1}{N_2} = \frac{V_1}{V_2} = \frac{I_2}{I_1}}$$

$$\Rightarrow N_1 I_1 = N_2 I_2$$

$$Z_1 = \frac{V_1}{I_1} = Z_2' = \frac{N_1 V_2 N_1}{N_2 N_2 I_2} = \left(\frac{N_1}{N_2}\right)^2 \cdot \frac{V_2}{I_2} = \left(\frac{N_1}{N_2}\right)^2 Z_2$$

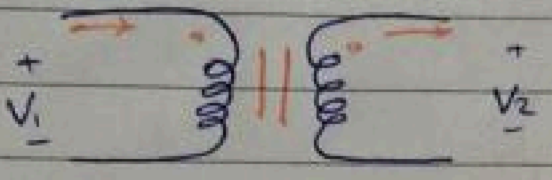
$Z_2' \Rightarrow$ الممانعة في الطرف الثاني
منسوبة للطرف الاول

exp



$$50\angle 0 \left(\frac{7}{1}\right)^2 = Z_{cd}$$

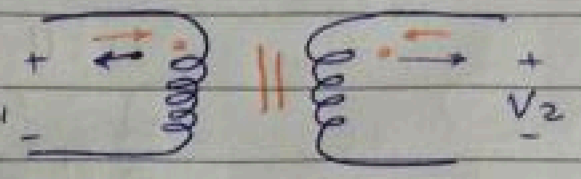
*



$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$

$$N_1 I_1 = N_2 I_2$$

*

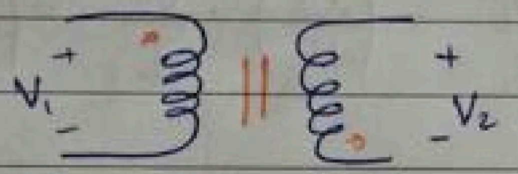


$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$

$$N_1 I_1 = -N_2 I_2$$

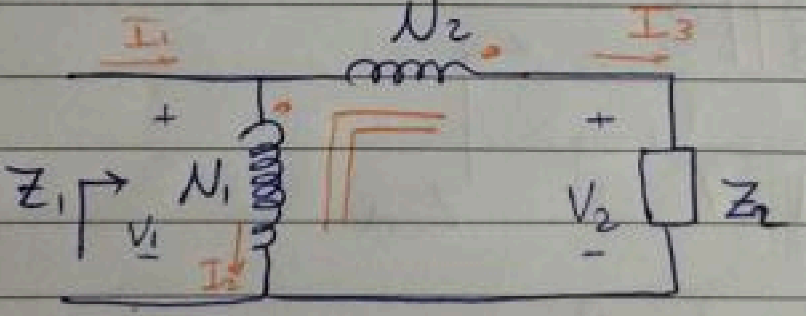
(كلا السهمين داخلين
او خارجين -ve)

*



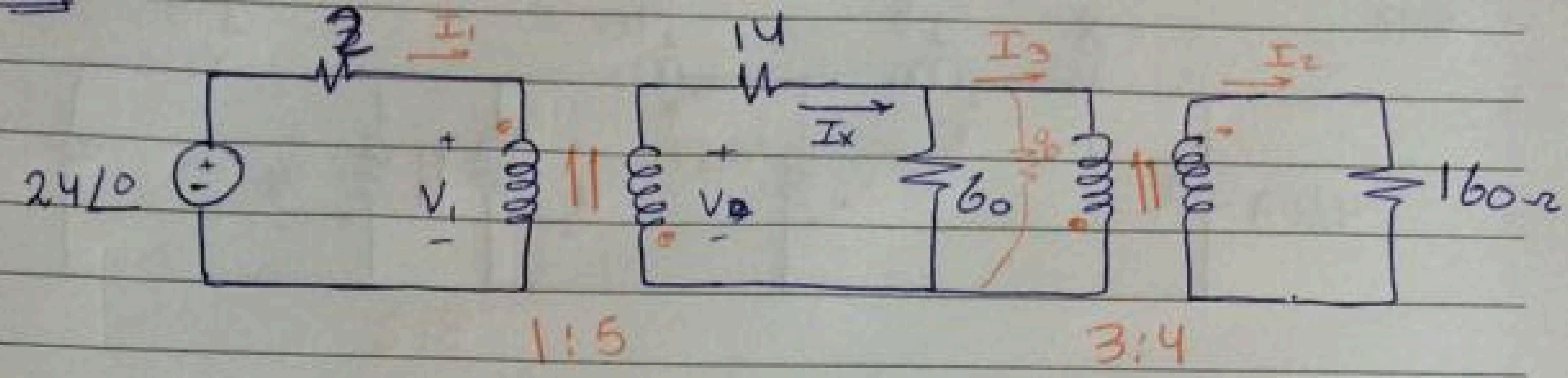
$$\frac{V_1}{V_2} = -\frac{N_1}{N_2}$$

H.W



Prove that $Z_1 = Z_2 \left(\frac{N_2}{N_2 + N_1}\right)^2$

exp



I_1, I_2, I_3 and V_0

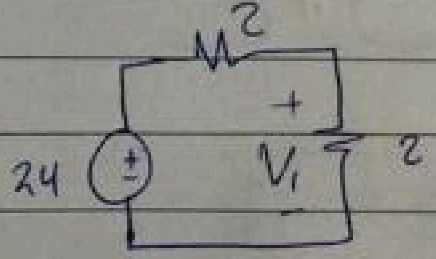
Sol: $Z_1 = \frac{V_1}{I_1} \longrightarrow I_1 = \frac{V_1}{Z_1} = \frac{24}{4} = \boxed{\cancel{4.8}}\text{A}\ \boxed{6}\text{A}$

$$\left[\left\{\left[\left(160\left(\frac{3}{4}\right)^2\right) // 60\right] + 14\right\} \times \left(\frac{1}{5}\right)^2\right] + 2 = \boxed{\cancel{6}}\ \boxed{4}$$

$I_x = -6\left(\frac{1}{5}\right)$

$I_3 = \left(\frac{-6}{5}\right)\frac{60}{150} = \left(\frac{-6}{5}\right) \times 0.4 = -0.48\text{ A}$

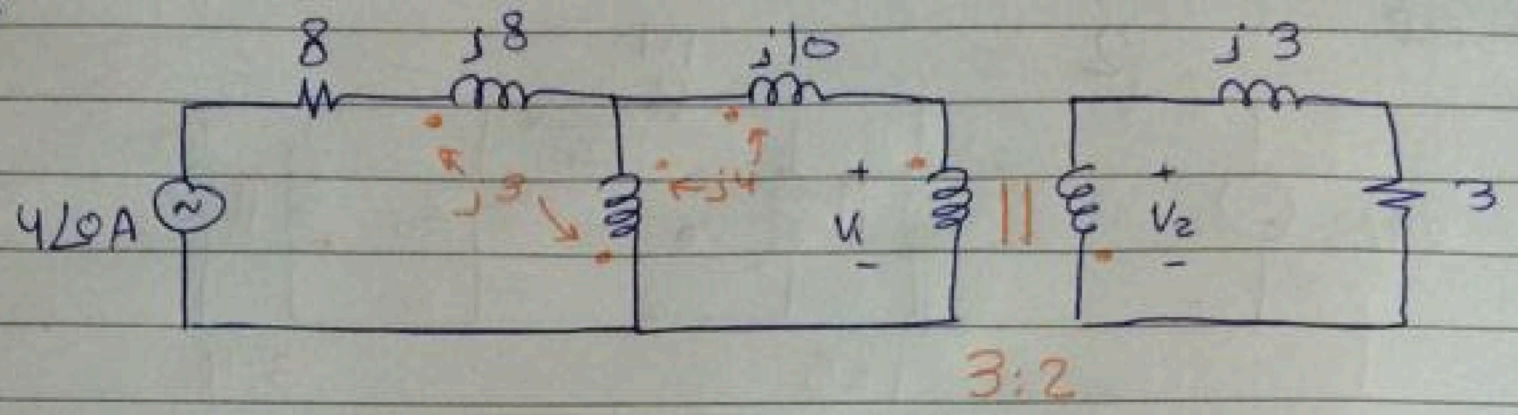
$I_2 = -\left(\frac{-6}{5}\right)(0.4) \times \frac{3}{4} = 0.36\text{ A}$



$V_1 = 24 \times \frac{2}{4} = \boxed{12}\text{ V}$

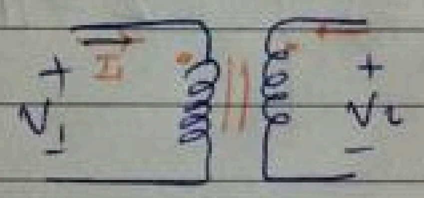
$V_0 = \boxed{-60}\text{ V}$

exp



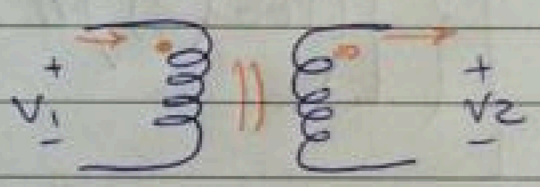
Find V_2 and I_2

$$Z' = (3+j3)\left(\frac{3}{2}\right)^2$$



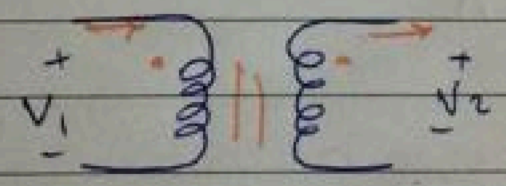
$$\frac{V_1}{N_1} = \frac{V_2}{N_2}$$

$$N_1 I_1 = -N_2 I_2$$



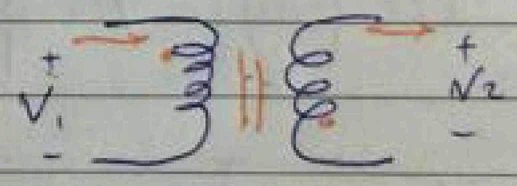
$$\frac{V_1}{N_1} = \frac{V_2}{N_2}$$

$$N_1 I_1 = N_2 I_2$$



$$\frac{V_1}{N_1} = \frac{V_2}{N_2}$$

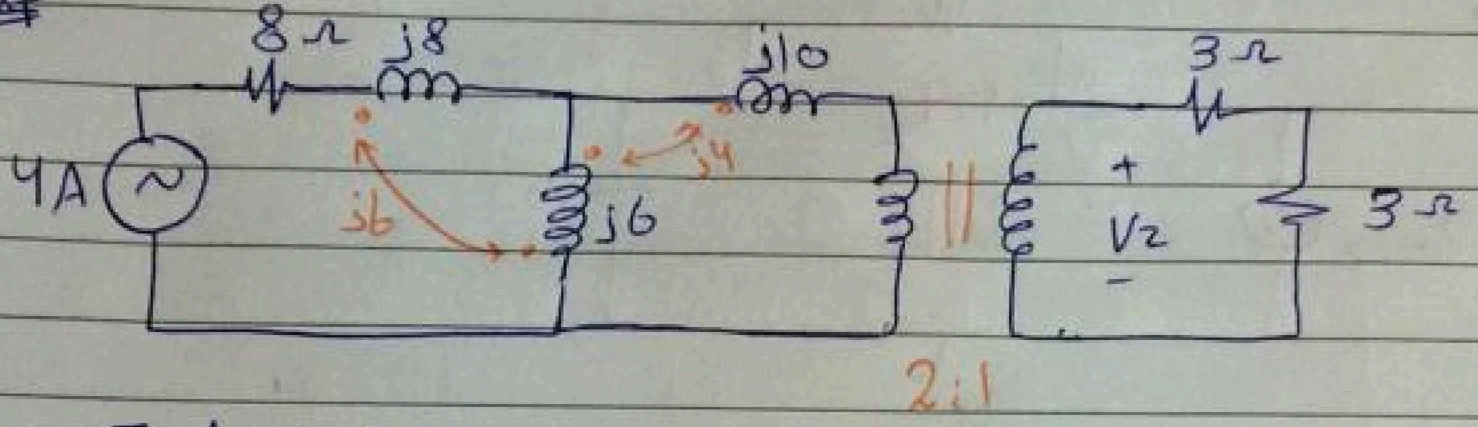
$$N_1 I_1 = N_2 I_2$$



$$\frac{V_1}{N_1} = \frac{-V_2}{N_2}$$

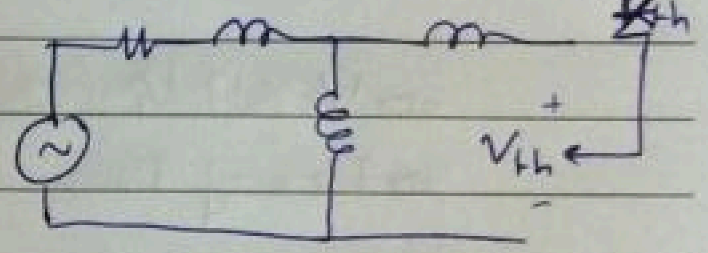
$$I_1 N_1 = -I_2 N_2$$

exp



Find V_2 ?

$6\left(\frac{2}{1}\right)^2 = \boxed{24}$



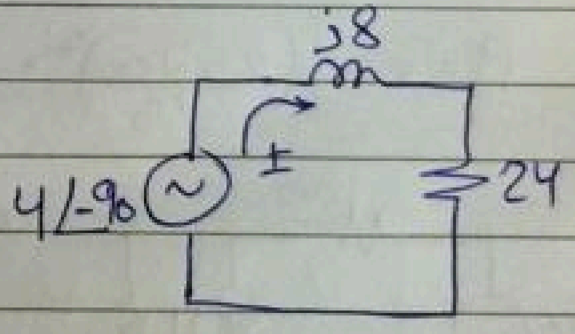
$V_{th} = [(-j4) + j6 - j3] = -j4$

$= 4\angle -90$

$Z_{th} = j8$

$V_{j10} = -j4(4)$

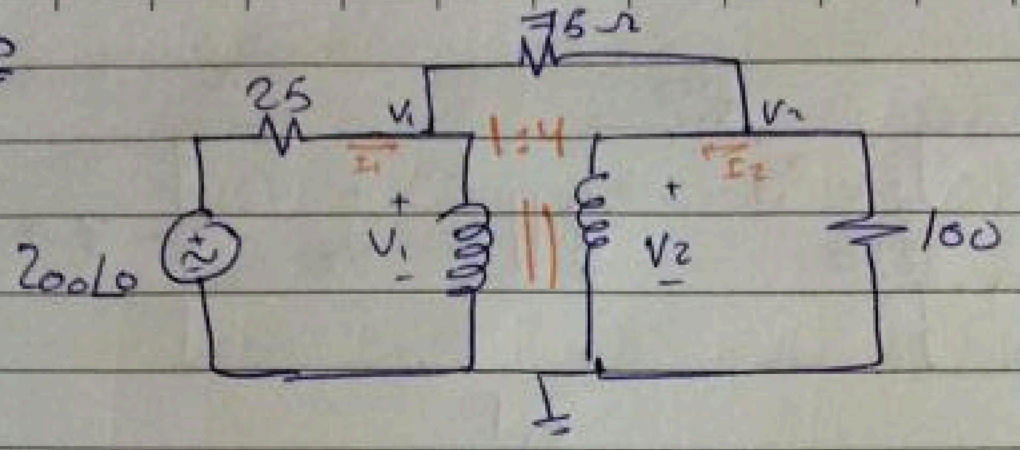
$V_{j6} = (j6(4)) - j3(4)$



$\left(\left(\frac{4\angle -90}{24+j8}\right) * \frac{2}{1}\right) * 3$

$I = \frac{4\angle -90}{24+j8}$

exp



$$\frac{V_1-200}{25}+\frac{V_1-V_2}{75}+I_1=0$$

$\Rightarrow V_2=4V_1$

$\Rightarrow I_2=\frac{-1}{4}I_1$

$$\frac{V_2-V_1}{75}+\frac{V_2}{100}+I_2=0$$

$V_1=25$

$V_2=100$

$$P_{25\Omega}=\left|\left(\frac{25-100}{75}\right)^2\right|\times 75$$

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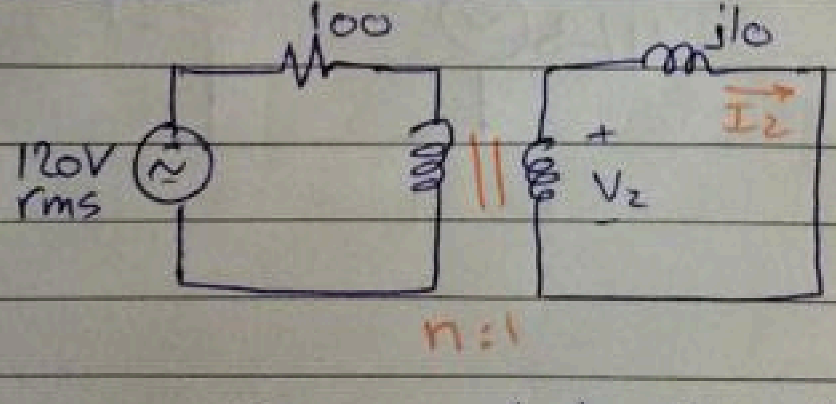
exp



if $|I_2|=1$ A

Find n

$120=100\left(\frac{1}{n}\right)+(j10I_2)n$ $V_2=j10I_2$

or

$Z_{in}=100+j10\left(\frac{n}{1}\right)^2$

$\frac{120}{1/n}=100+j10(n)^2$

exp Linear Transformer $\omega = 5000$ rad/sec

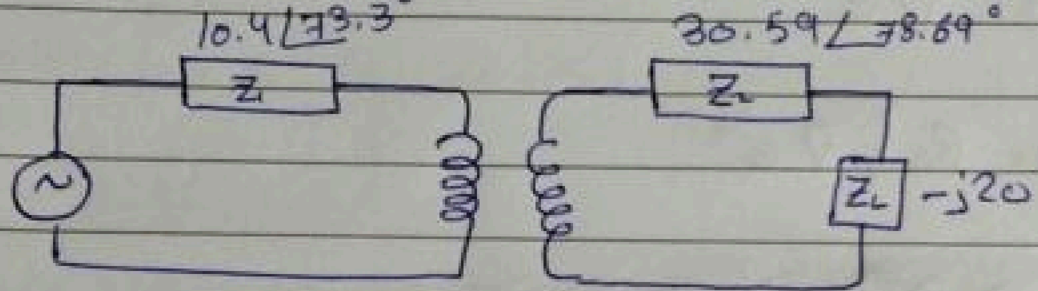
$Z_1 = 10.4\angle 73.3^\circ$ (Primary imp.)

$Z_2 = 30.59\angle 78.69^\circ$ (secondary imp)

reflected impedence magnetude $= 13.08\ \Omega$

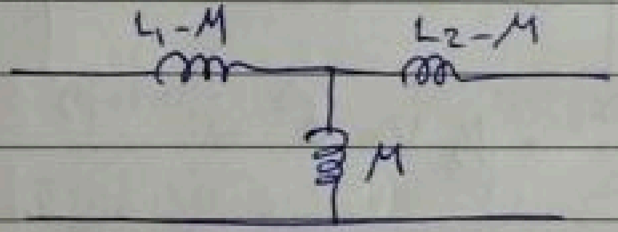
$Z_L = -j20$ (load imp)

Find Teq?



$$\Rightarrow \left|\frac{\omega^2 M^2}{Z_{21}}\right| = 13.08\ \Omega$$

$$Z_{in} = Z_{11} + Z_r$$



$$Z_{11} = R + j\omega L_1$$

Complex Frequency :-

$$v(t) = V_m \, e^{st} \, \mathrm{Cos}(wt - \varphi)$$

$$v(t) = V_m \, \mathrm{Cos}(wt) = \mathrm{Reu}\{V(t)\} = \mathrm{Re}\{\bar{V} e^{jwt}\}$$

$$V(t) = \bar{V} e^{jwt}$$

$$v(t) = \mathrm{Rs}\{ \bar{V} e^{st}$$

~~$\Rightarrow e^{j\theta} = \mathrm{Cos}\theta + j\mathrm{Sin}\theta$~~

$$\Rightarrow \sin x = \frac{e^{jx} - e^{-jx}}{j2}$$

$$v(t) = V_m \, e^{\sigma t} \, \mathrm{Cos}(wt - \varphi)$$

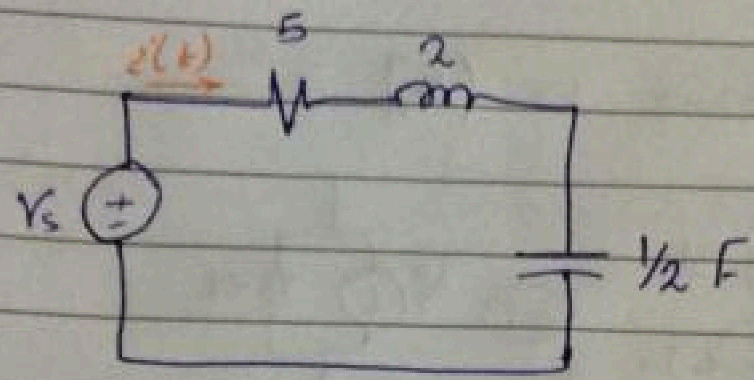
$$= \tfrac{1}{2} V_m \, e^{\sigma t} \left[e^{j(wt+\varphi)} + e^{-j(wt+\varphi)} \right]$$

$$= \tfrac{1}{2} V_m \, e^{\sigma t} \cdot e^{j\theta} + \tfrac{1}{2} V_m \, e^{\sigma t} \, e^{-j(wt+\varphi)}$$

$$= \underbrace{\tfrac{1}{2} V_m \, e^{j\theta}}_{} \, e^{(\sigma + jw)t} + \underbrace{\tfrac{1}{2} V_m \, e^{-j\theta}}_{} \cdot e^{(\sigma - jw)t}$$

$$= \boxed{\bar{V} e^{S_1 t} + \bar{V}^{*} e^{S_1^{*} t}}$$

Exp



$$V(t) = 100\, e^{-3t} \cos(4t - 50)$$

$$\bar{V} = 100\angle -50$$

$$S = -3 + j4$$

$$V_s(t) = 5\, i(t) + 2 \frac{d}{dt} i(t) + 2 \int_0^t i(t)\,dt$$

$$= 5\bar{I}\cancel{e}^{st} + S2\bar{I}\cancel{e}^{st} + \frac{2}{s} I \cancel{e}^{st} = V\cancel{e}^{st}$$

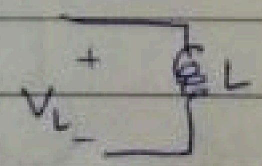
$$\bar{I} = \left. \frac{100\angle -50}{5 + 2S + \frac{2}{S}} \right|_{s=-3+j4} = \boxed{12.9\angle +149}$$

$$i(t) = \bar{I} \cdot e^{st}$$

$$= I e^{-st} \cos(4t \pm \phi_i)$$

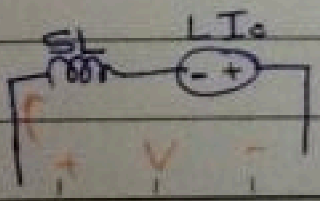
$$i(t) = 12.9\, e^{-st} \cos(4t - 149^{\circ})$$

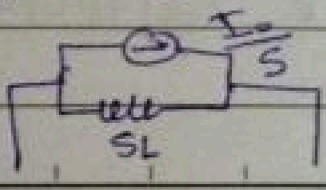
In Laplace



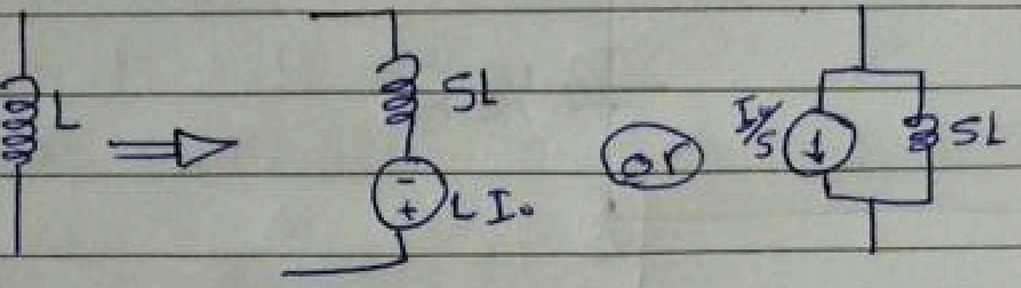
① $V_L = L \frac{di}{dt} \Big| \mathcal{L}$

$$V(s) = \mathcal{L}\{v(t)\} = \mathcal{L}\left\{L \frac{di}{dt}\right\} = L\left[S I(s) - I_0\right] = SL\, I(s) - L I_0$$





⇒ Convert (L) from time to (S). domain

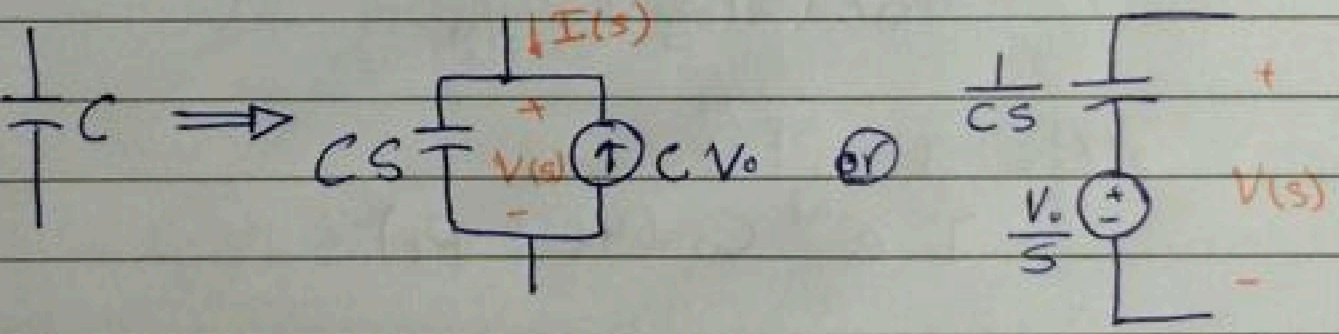


② 

$$i = C\frac{dv}{dt} \Big| \mathcal{L}$$

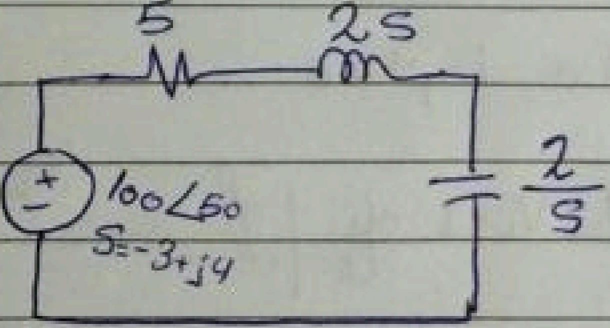
$$\mathcal{L}\{i(t)\} = I(s) = \mathcal{L}\{C\frac{dv}{dt}\} = C\left[sV(s) - V_0\right]$$

⇒ Convert to S domain



* Back to the example :-

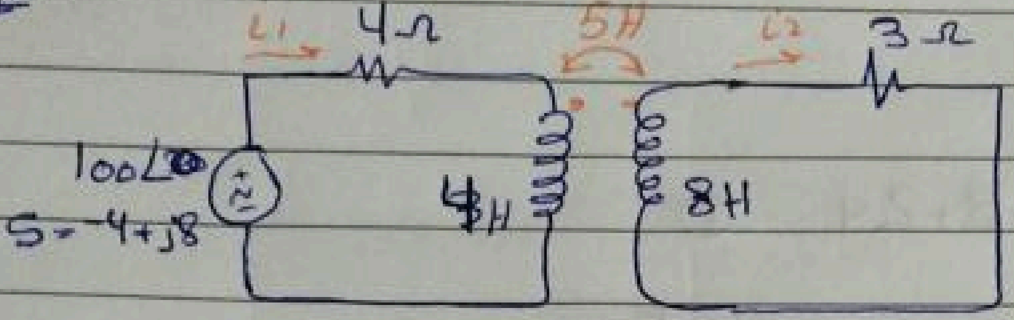
Convert to S domain



$$\bar{I} = \frac{100\angle -50}{5+2S+\frac{2}{s}} = 12.9\angle -149$$

$$i(t) = 12.9\, e^{-3t} \cos(4t-149)$$

exp:-



$i_1(0) = i_2(0) = 4A$

$100\angle$ [illegible] $= (4+3s)\, I_1(s) - 5s\, I_2$ ... ①

$0 = 5s I_1 + (3+8s) I_2$ ... ②

$$I_2 = \frac{\begin{pmatrix} 4+4s & 100 \\ -5s & 0 \end{pmatrix}}{\begin{pmatrix} 4+4s & -5s \\ -5s & 3+8s \end{pmatrix}} = \frac{500s}{7s^2+44s+12}$$

$$= \frac{500\,S}{7(S+\frac{2}{7})(S+6)}$$

$$\bar{I}_2(s) = \left.\frac{\frac{500}{7}S}{(S+\frac{2}{7})(S+6)}\right|_{s=-4+j8} = 8.8\angle -74$$

- Poles: القيم التي تجعل المقام صفر
- Zeros: البسط صفر " " "

$i_{2n}(t) = A e^{-\frac{2}{7}t} + B e^{-6t}$ (natural)

$i_2(t) = i_{2n}(t) + i_p(t)$

$$\boxed{i_2(t) = Ae^{-\frac{2}{7}t} + Be^{-6t} + 8.8\, e^{-4t} \cos(8t - 74°)}$$

TO Find A, B

$i_2(0) = 4 = A + B + 2.4$ -- ①

~~$\frac{d\,i_2(t)}{dt} = -\frac{2}{7}A - 6B - 18.16$~~

$$\frac{\partial i_2(t)}{dt} = -\frac{2}{7}A - 6B - 18.16$$

$$\frac{\partial i_2(t)}{\partial t} = ??$$

$$\Rightarrow 100\angle 30 = (4+4s)\,I_1(s) - 5s\,I_2$$

$$= 4i_1(0) + 4\,\cancel{s}\frac{i_1(0)}{dt} - 5\,\cancel{s}\,\frac{di_2(0)}{dt}$$

$$= 4i_1(0) + 4\,\frac{di(0)}{dt} - 5\,\frac{di_2(0)}{dt}$$

$$② = -5\,\frac{di_1(0)}{dt} + 3\,i_2(0) + 8\,\frac{di_2(0)}{dt}$$

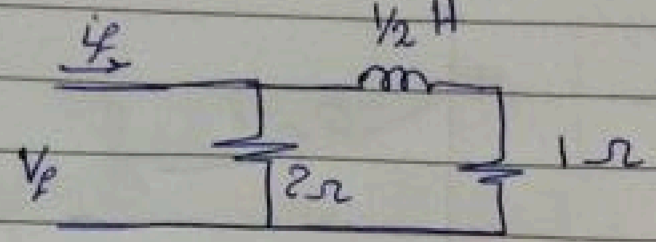
Circuits II Notebook
Dr. Nabeel Tawalbeh

By . Yazan Abawi

بأفكارنا_نُبدع

6th week

exp



① Find the forced current response at input if the Voltage $V_f(t) = 2\,e^{-t}$ for all t?

$$H(s) = \frac{\text{output}}{\text{input}} = \frac{I(s)}{V(s)} = Y(s)$$

$$I_f(s) = H(s) \cdot V(s) = Y(s) \cdot V(s)$$

$$i_f(t) = I_f \cdot e^{-t}$$

$$I_f(s) = V \cdot Y(s) = 2 \cdot \left.\frac{2+\left(1+\frac{s}{2}\right)}{\left(1+\frac{1}{2}s\right)\cdot 2}\right|_{s=-1} = 2\,\frac{2+1+-\frac{1}{2}}{\left(1-\frac{1}{2}\right)\cdot 2} = 5$$

$$\boxed{i_f(t) = 5e^{-t}}$$

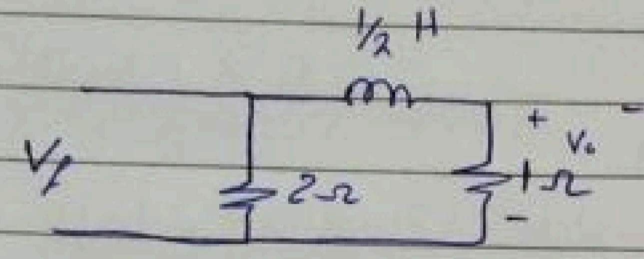
② Find the forced Voltage response at input if the current $i_f(t) = 1.5\cos(2t + 42^\circ)$ for all t?

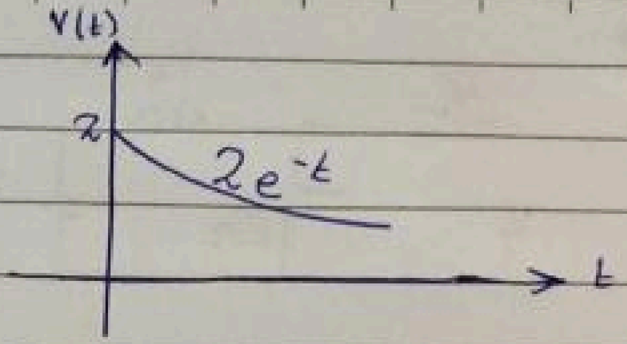
$$H(s) = \frac{V(s)}{I(s)} = H(s) = Z(s)$$

$$V_f(s) = H(s)\, I(s)\Big|_{s=j2} = 1.5\,\left.\frac{(2+s)\,2}{(6+s)}\right|_{s=j2} = 1.34\angle 68.6^\circ$$

$$\boxed{V_f(t) = 1.34\cos(2t + 68.6^\circ)}$$

exp





Find $i(t)$?

$$i(t) = i_n(t) + i_f(t)$$
$$= Ae^{P_1 t} + Be^{P_2 t} + \dots + I_f e^{st}$$

$$Z(s) = \frac{2(s+2)}{s+6}$$

$$I(s) = \frac{V_s}{Z(s)} = \left.\frac{2(s+6)}{2(s+2)}\right|_{s=-1} = 5$$

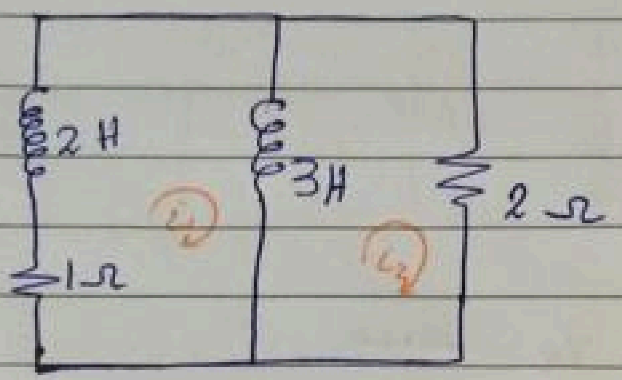
$$I(s) = \frac{2(s+6)}{2(s+2)} \Rightarrow i_n(s) = Ae^{-2t}$$

$$i(t) = 3e^{-t} + Ae^{-2t}$$

$$i(0) = 5 + A = 1 \Longrightarrow \boxed{A = -4}$$

$$\boxed{i(t) = 3e^{-t} - 4e^{-2t}}$$

exp



Find $i_1(t)$, $i_2(t)$ if $i_1(0) = i_2(0) = 11$ A .

$V_s = (1+5s) I_1 - 3s I_2$ -- ①

$0 = -3s I_1 + (2+3s) I_2$ -- ②

$$I_1 = \frac{\begin{pmatrix} V_s & -3s \\ 0 & 2+3s \end{pmatrix}}{\begin{pmatrix} 1+5s & -3s \\ -3s & 2+3s \end{pmatrix}} = \frac{(2+3s)\ V_s}{6(s^2 + \frac{13}{6} s + \frac{2}{6})}$$

$$I(s) = \frac{3(s+\frac{2}{3})}{6(s+2)(s+1/6)}$$

[Zeros : $-2/3$

Poles : $-2, -1/6$

$i_n(t) = A e^{-1/6 t} + B e^{-2t}$

$i_1(0) = 11 = A + B$ -- $\boxed{1}$

$$0 = 1\, i_1(0) + 5 \frac{d}{dt} i_1(0) - 3 \frac{d}{dt} i_2(0)$$

$$0 = -3 \frac{d}{dt} i_1(0) + 2\, i_2(0) + 3 \frac{d\, i_2(0)}{dt}$$

$5X_1 - 3X_2 = -11$

$-3X_1 + 3X_2 = -22$

$X_1 = \cancel{[illegible]}$, $X_2 = \cancel{[illegible]}$

-16.5 $\qquad$ -23.8

$$\frac{di_1(0)}{dt} = -16.5 = -\frac{1}{6}A - 2B \;\text{--}\; \boxed{2}$$

$A = 3$

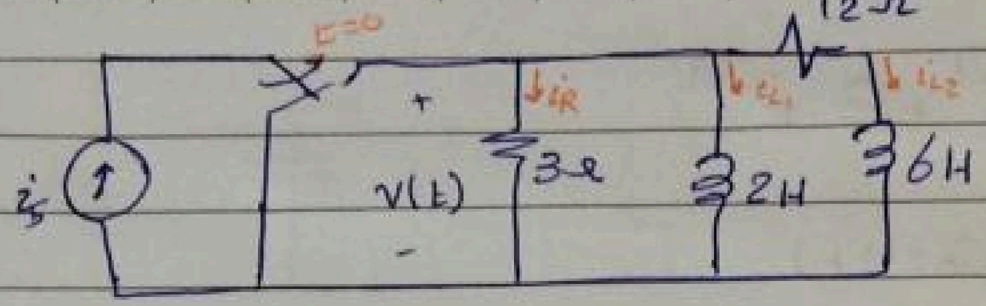
$B = 8$

$$\boxed{i_{1n}(t) = 3e^{-1/6\,t} + 8e^{-2t}}$$

$$\frac{di_2(0)}{dt} = -23.8 = -\frac{1}{6}A - 2B \;\text{--}\; \boxed{3}$$

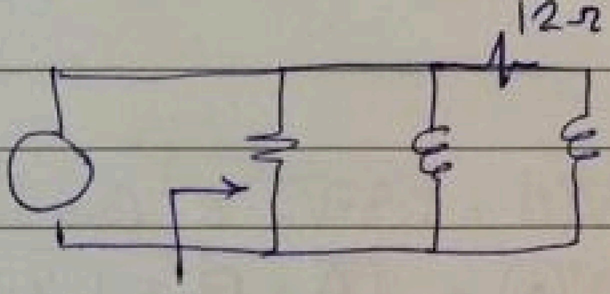
$$\boxed{i_2(t) = -0.98e^{-1/6\,t} + 11.98e^{-2t}}$$

exp



$i_s(t) = e^{-t} \cos 2t$

Find $v(t)$



$V = Z . I$

$$V(s) = \frac{1}{Y} \cdot I = \frac{I(s)}{\frac{1}{3} + \frac{1}{2s} + \frac{1}{6s+12}}$$

$$\frac{V(s)}{I(s)} = H(s)$$

$$V(s) = H(s)\ I(s)$$

$$\cancel{H(s) = \frac{36s^2}{12\left(s^2 + \frac{18}{12}s + \frac{6}{12}\right)}}$$

$$v(s) = \frac{6s(12+6s)}{12s^2 + 24s + 36 + 18s + 6s} = \frac{36\left(s^2 + \frac{72}{36}s\right)}{12\left(s^2 + 4s + 3\right)}$$

$$= \frac{3s(s+2)}{(s+1)(s+3)}$$

Zeros: 0, -2

Poles: -1, -3

$$v_n(t) = A e^{-t} + B e^{-3t}$$

$$v_f(t) = \overline{V_f}\, e^{-t} \cos(2t)$$

$$v_f(s) = I(s) * H(s) \Big|_{s=-1+j2} = \frac{3s(s+2)}{(s+1)(s+3)} \Big|_{s=-1+j2} = 1.87\sqrt{2} \angle 45^\circ$$

$$\Longrightarrow V(t) = 1.875\sqrt{2}\, e^{-t} \cos(2t+45) + A e^{-t} + B e^{-3t}$$

$$V(0) = \boxed{A + B + 1.875 = 3} \; -- \boxed{1}$$

$$V_R = R \cdot i_s$$

$$i(0) = 1 \Longrightarrow V_R = V(t) = 3V$$

$$V(t) = 3 i_R = 3 i_s - 3 i_{L_1} - 3 i_{L_2}$$

$$\frac{dv(t)}{dt}(0) = 3\frac{di(0)}{dt} - 3\frac{di_{L_2}}{dt} - 3\frac{di_{L_2}}{dt}$$

$$\Longrightarrow \frac{dv(0)}{dt} = 1.875\sqrt{2}\left(-e^{-t}\cos(2t+45) - 2e^{-t}\sin(2t+45)\right)\Big|_{t=0} - A - 3B$$

$$-A - 3B - 3 * 1.875 = \frac{dv}{dt}(0)$$

$$\frac{dv(t)}{dt} = -3 \cancel{*} - 3 \times \frac{3}{2} - 3 \times \frac{1}{2} = \boxed{-9}$$

$$* \; 3 = L_1 \frac{di_{L_1}}{dt}$$

$$\boxed{A + 3B = 3.375} \; -- \boxed{2}$$

$$\frac{di_{L_1}}{dt} = 3/2$$

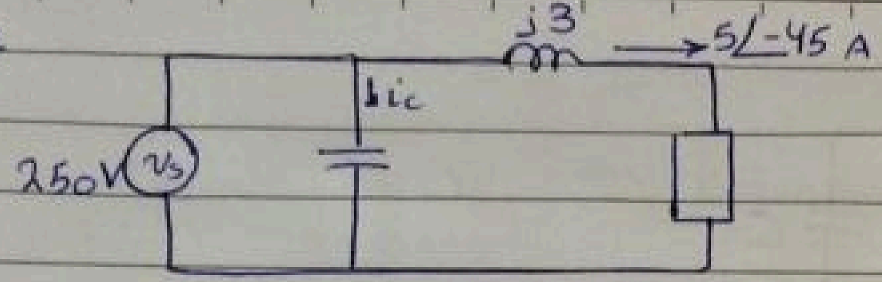
$$A = 0$$

ⓑ $B = 1.125$

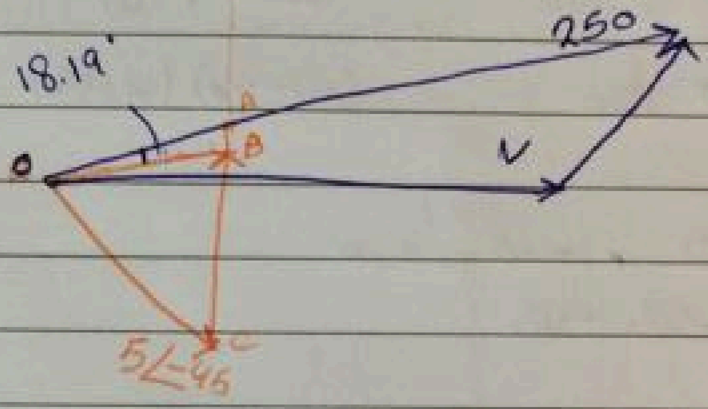
$$* \; 3 = L_2 \frac{di_{L_2}}{dt}$$

Quiz

[1]



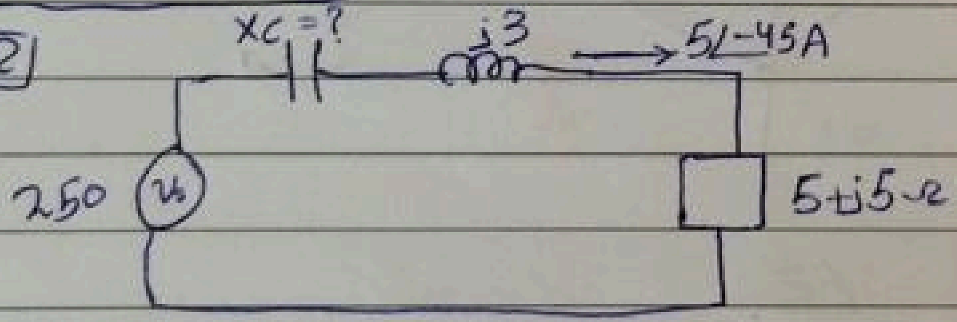
Find the value of ic that inprove the PF to 0.95 lagging using the phasor diagram.



$BC = I_c = 0A(\tan \varphi_1 - \tan \varphi_n)$

$Req\{I\}(\tan \varphi_1 - \tan \varphi_n)$

[2]



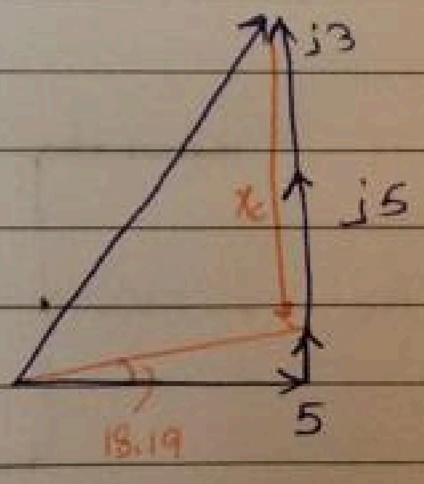
PF = 0.95

$X_c = ?$

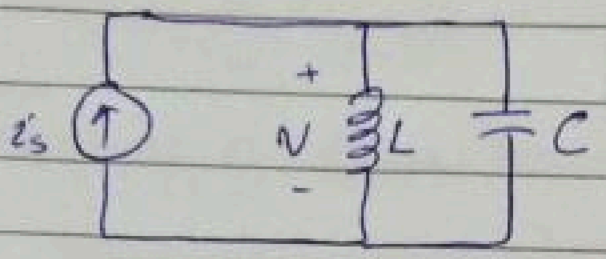
$X_c = j(8 - 1.6)$

$= -j6.4$

$Z_{in} = 5 + j1.6 \Rightarrow \theta = \tan^{-1}\left(\frac{1.6}{5}\right)$



exp



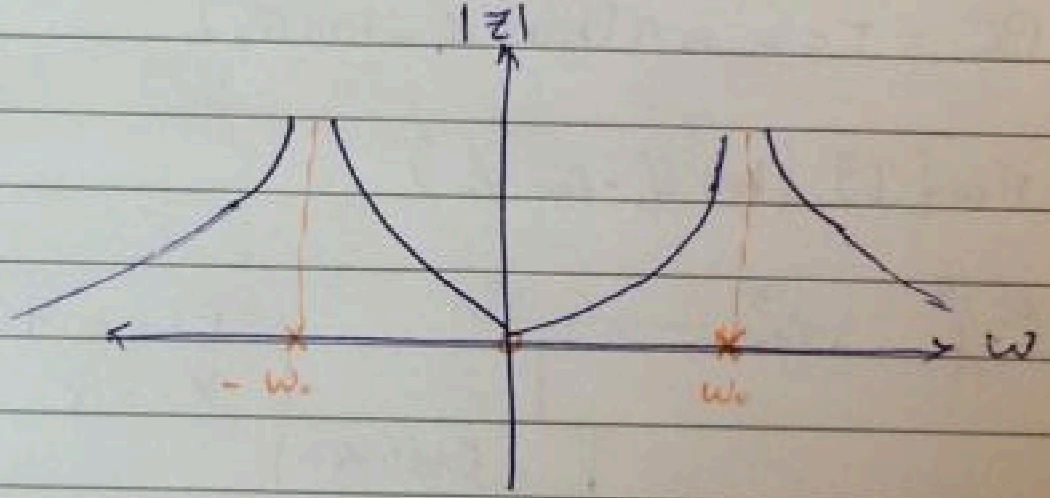
|Z| v.s. w

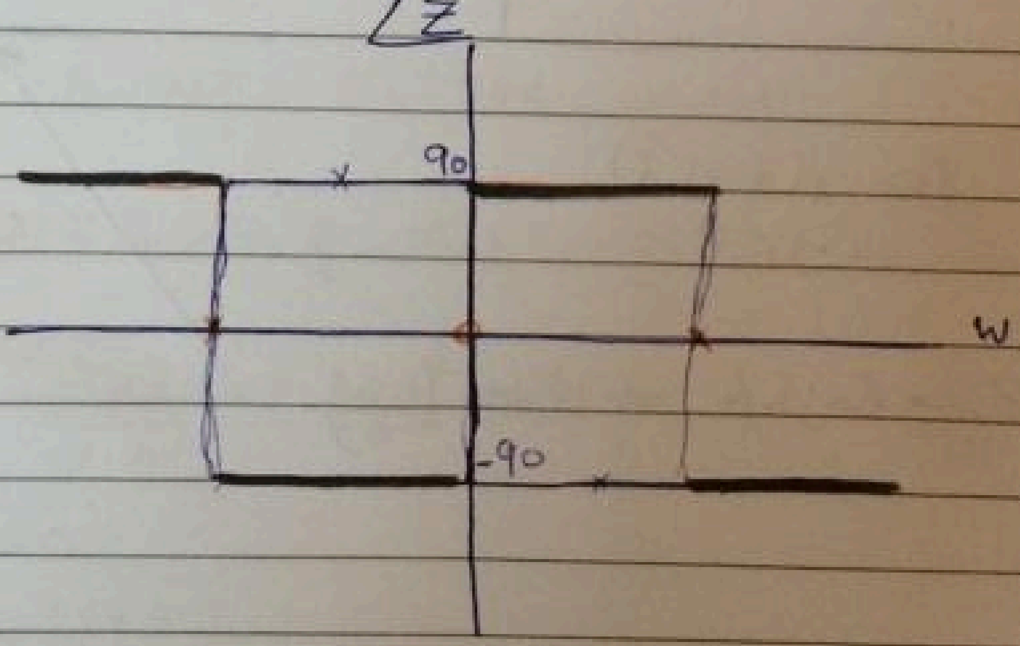
$$Z = \frac{\frac{1}{jwc} * jwc}{jwc + \frac{1}{jwc}} = \frac{\frac{1}{C} \times w}{(w-w_0)(w+w_0)}$$

Zeros: 0 , $-\infty$, $+\infty$

Poles: $-w_0$ $+w_0$

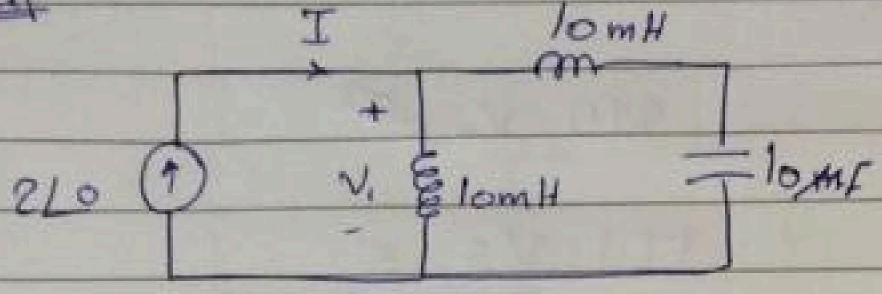
$v_0 = \frac{1}{\sqrt{LC}}$





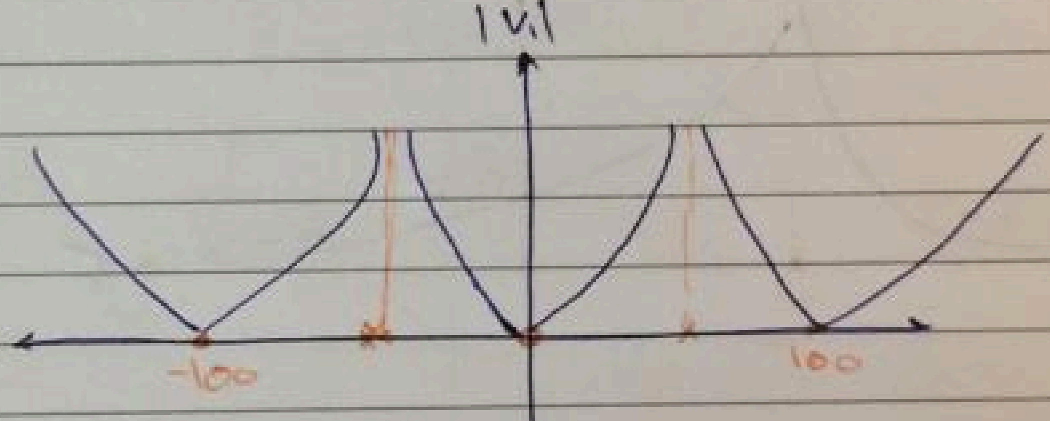
∞

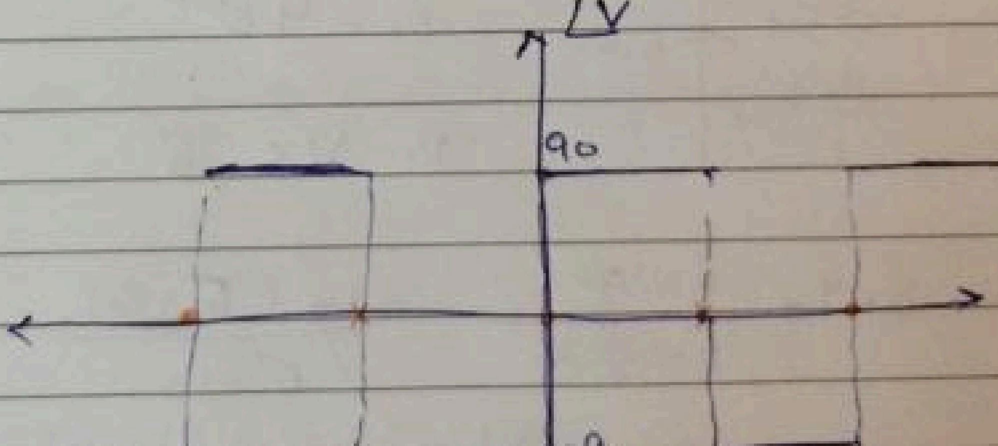
exp



$$V_1 = I . Z = I\left(j\omega 10mH + \frac{1}{j\omega 10\mu F} \parallel j\omega 10mH\right)$$

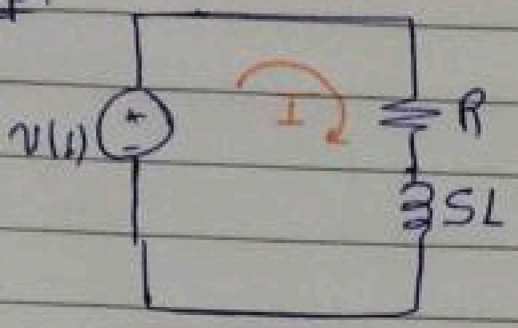
$$= \frac{j2\omega(\omega^2-1000)}{200(\omega^2-5000)}$$





ⓐ

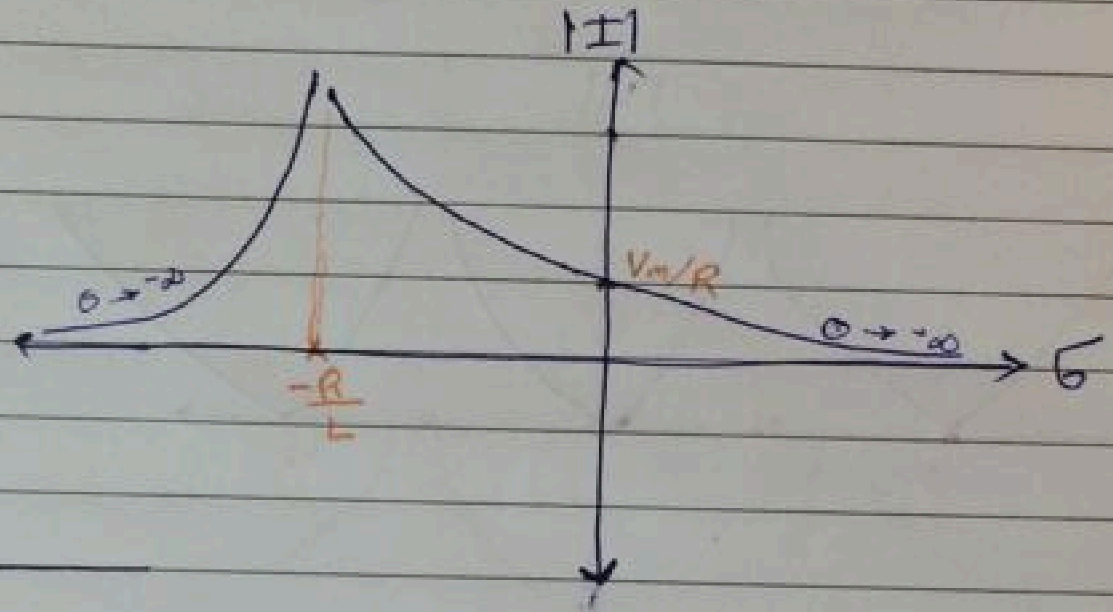
exp:



$v(t) = V_m e^{\sigma t}$

$|I|$ V.S. σ

$$I = \frac{V_m}{R + \sigma L} = \frac{V_m / L}{\left(\frac{R}{L} + \sigma\right)}$$



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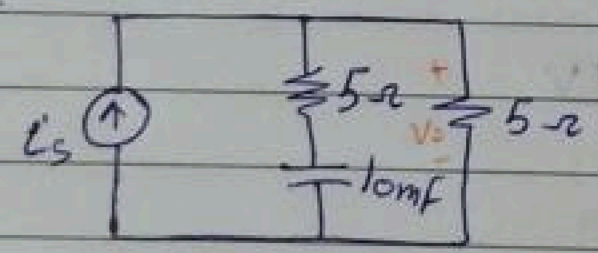
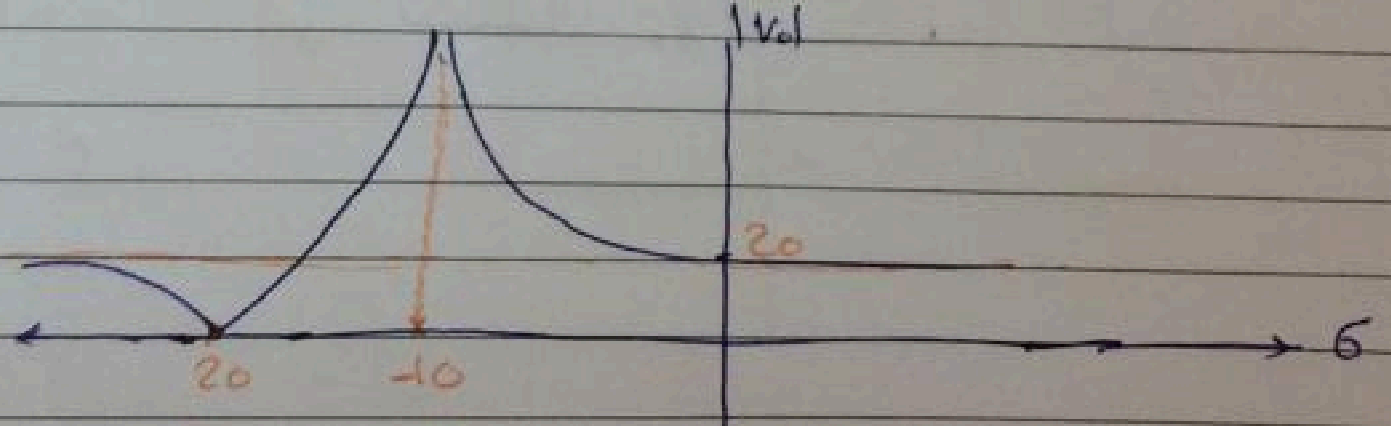
exp:



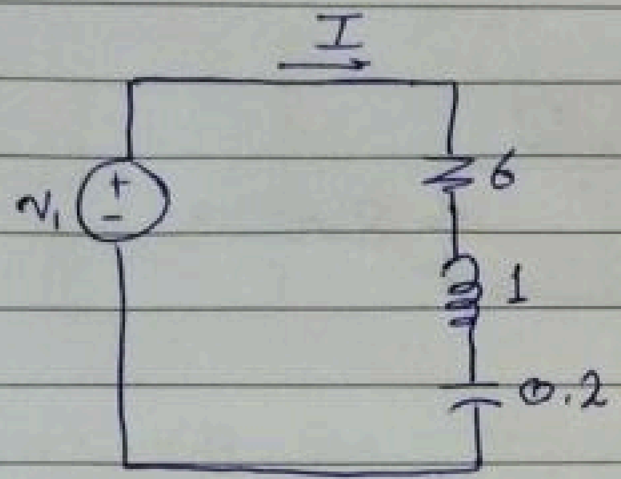
$i_s = 4 e^{\sigma t}$

v_o V.S. σ

$$v_o = 5 \times 4 \times \frac{5 + \frac{100}{\sigma}}{5 + \frac{100}{\sigma} + 5} = \frac{10(\sigma + 20)}{\sigma + 10}$$



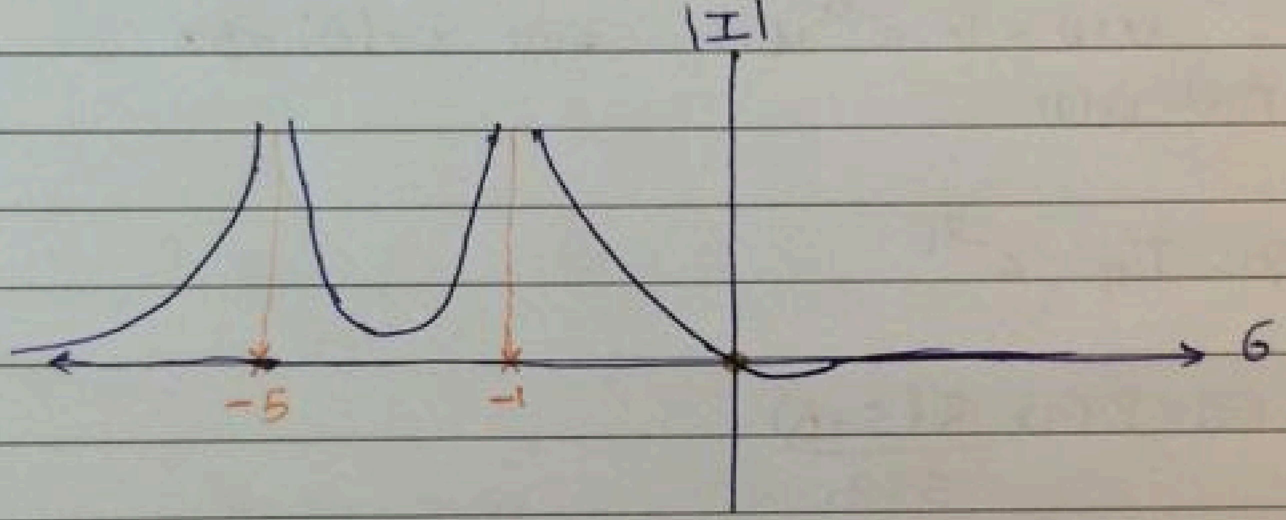
H.W



$V_1 = 100\, e^{\sigma t}$

$|I|$ v.s σ

$$I = \frac{100}{6 + \sigma + \frac{5}{\sigma}} = \frac{100\,\sigma}{(\sigma+1)(\sigma+5)}$$



Critical Points (minimum and maximum)

$$\Rightarrow \frac{\partial I}{\partial \sigma} = \frac{(\sigma+1)(\sigma+5) - \sigma(2\sigma+6)}{(\qquad)^2}$$

$$-\sigma^2 + 5 = 0$$

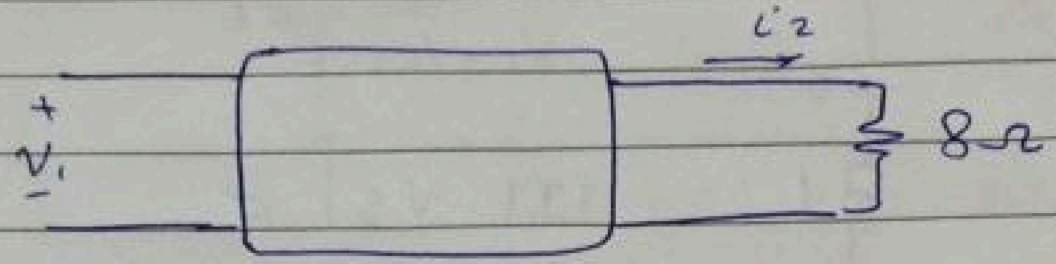
$$\boxed{\sigma = \pm\sqrt{5}}$$

$I(\sqrt{5}) = 9.55$ maximum

$I(-\sqrt{5}) = 5.45$ minimum

$I(\sqrt{3}) = 9.42$

Quiz



$$\frac{I_2(s)}{V_1} = \frac{8(s+5)}{s+20}$$

a) if $v_1(t) = 10\,e^{-8t}$, Find $i_{2f}(t)$

b) " $v_1(t) = 20$, " "

c) " $v_1(t) = 10\,e^{-8t}u(t)$ and $i_2(0) = 50$

Find $i_2(t)$

a) $i_{2f} = I_{2f}\,e^{-8t}$

$$I_2(s) = V_1(s)\,\frac{8(s+5)}{s+20}$$

$$I_{2f} = 10 \times \frac{8(s+5)}{s+20}\Bigg|_{s=-8} = \cancel{40}\ -20$$

$$\boxed{i_{2f}(t) = -20\,e^{-8t}}$$

b) $\boxed{I_{2f} = 40}$

c) $i_2(t) = i_{2n}(t) + i_{2f}(t)$

$$i_2(t) = A\,e^{-20t} - 20\,e^{-8t}$$

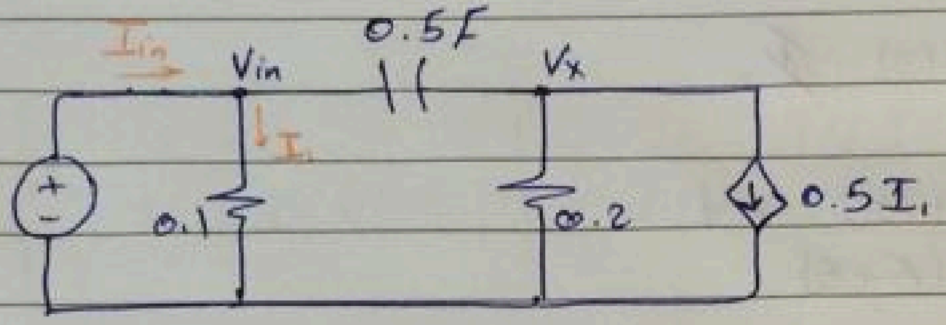
$i_2(0) = 50 = A - 20$

$\boxed{A = 70}$

⑫

$$i_2(t) = \left(70\,e^{-20t} - 20\,e^{-8t}\right)u(t).$$

exp



a) write an expression of $Z(s) = \frac{V_{in}}{I_{in}}$

b) If V_{in} is replaced by a current source
$i_{in} = 4e^{\sigma t}$, Plot $|V_{in}(\sigma)|$ v.s. σ

ⓐ $Z(s) = \frac{V_{in}}{I_{in}} = H(s)$

$\frac{V_{in} - V_x}{2/s} + \frac{V_{in}}{0.1} = I_{in}$ -- node ①

$\frac{V_x - V_{in}}{2/s} + \frac{V_x}{0.2} + 0.5 I_1 = 0$ --- node ②

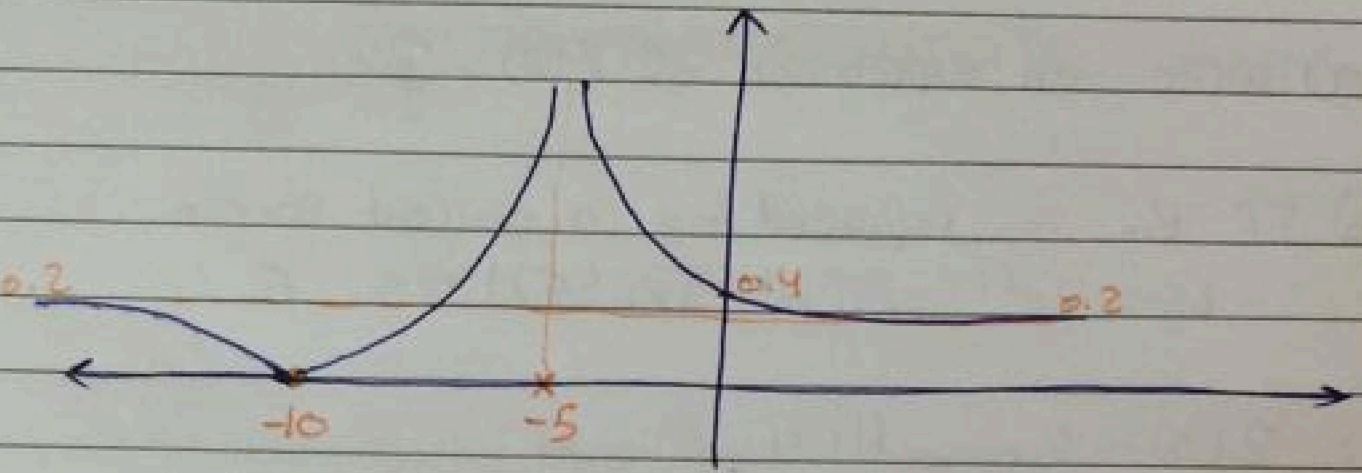
$\left(\frac{s}{2} + 10\right) V_{in} - \frac{2}{s} V_x = I_{in}$

$\left(-\frac{s}{2} + 5\right) V_{in} + \left(\frac{s}{2} + 5\right) V_x = 0$

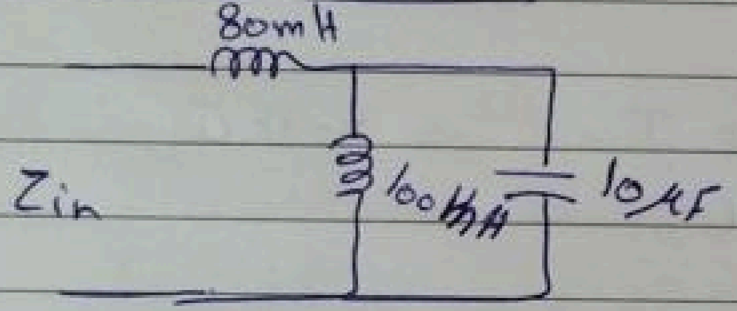
$$\boxed{H(s) = \frac{V_1}{I} = \frac{s+10}{20(s+5)}}$$

ⓑ $V_{in} = H(S)\ I_{in}$

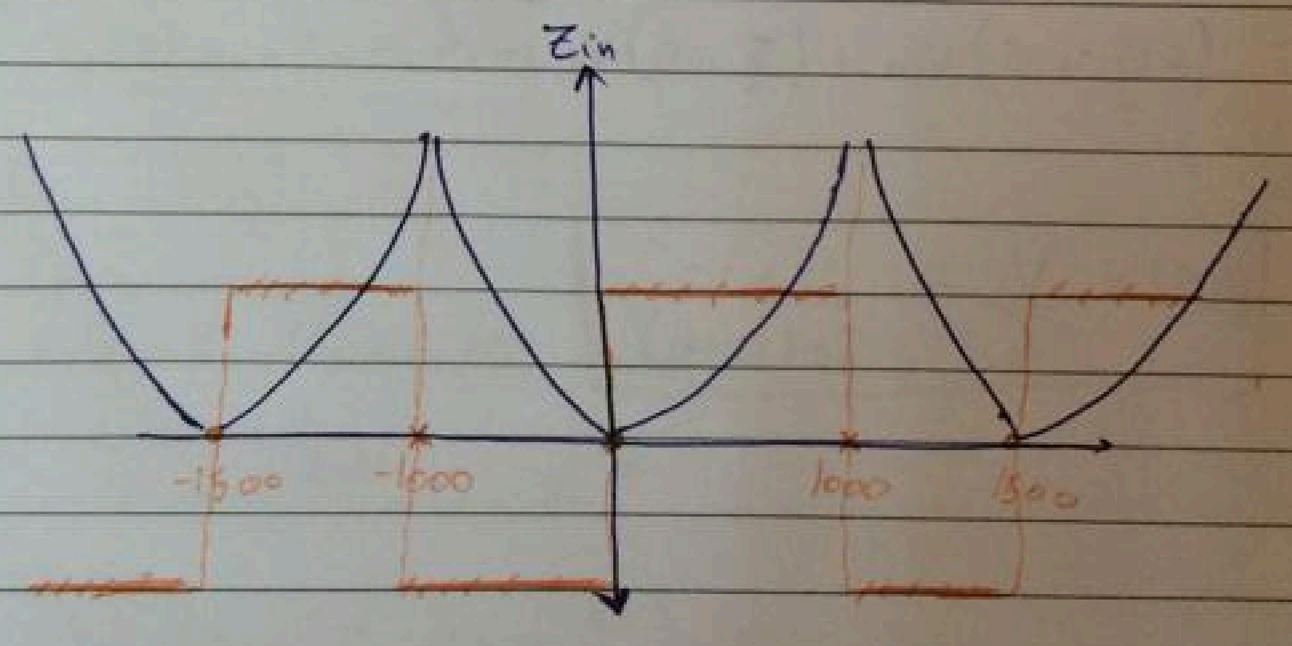
$$= \frac{(6+10)}{20(6+5)} * 4$$



exp



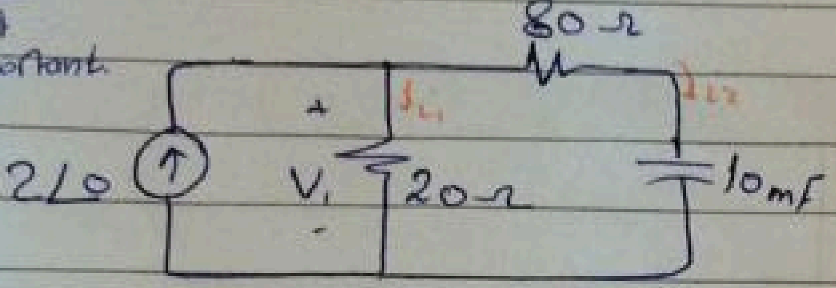
$$Z_{in} = \frac{j0.08w(w-1500)(w+500)}{(w-1000)(w+1000)}$$



⑭

exp

not important.



|V_1| vs. w

∠V_1 v.s w

$$V_1 = 32\sqrt{\frac{w^2+1.5625}{w^2+1}} = \frac{32(w-j1.25)}{w-j1}$$

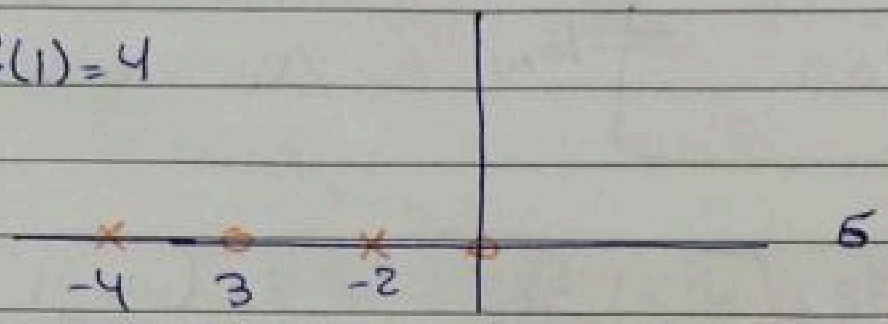
$$\angle V_1 = \tan^{-1}\frac{1.25}{w} - \tan^{-1}\frac{1}{w}$$

w	V_1	∠V_1	j_2
0	40	0	
1.25	39.6	-2.7	
2	33.8	-5.4	
∞	32	0	

exp

Find the H(s)

$G(1) = 4$



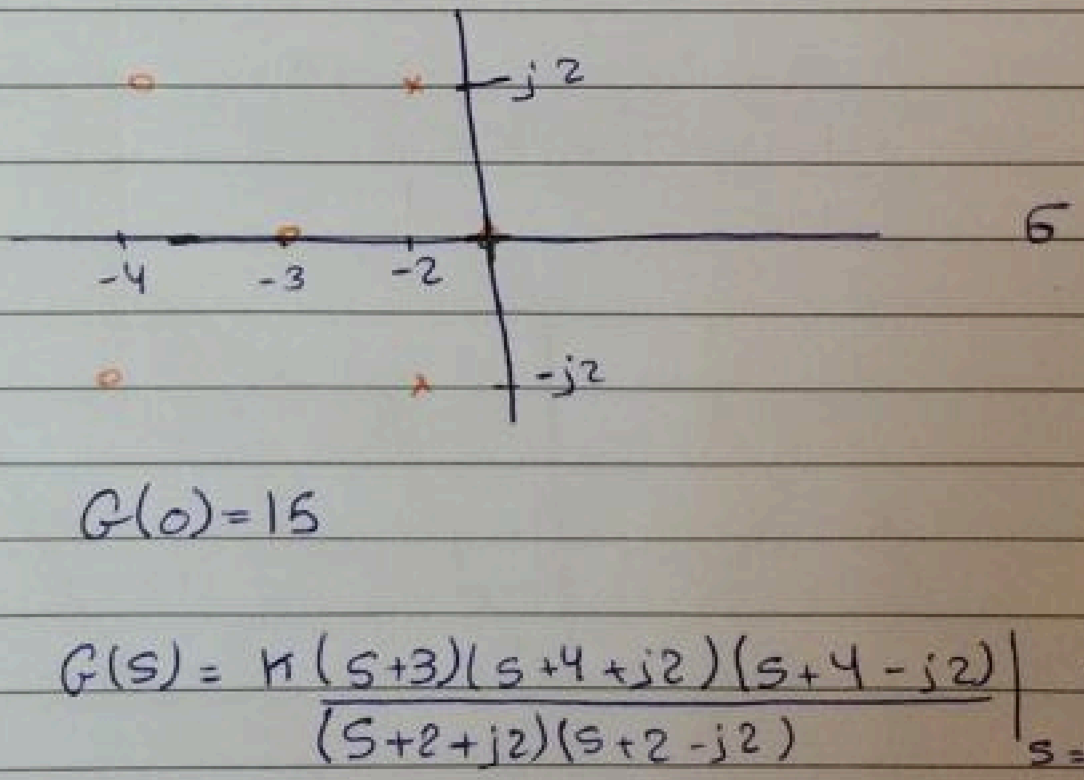
$$H(s) = K\,\frac{(s+Z_1)(s+Z_2)\ -}{(s+P_1)(s+P_2)\ \sim}$$

$$G(1) = 4 = K\,\frac{s(s+3)}{(s+2)(s+4)}\Bigg|_{s=1} = 15$$

$$G(s) = 15\,\frac{s(s+3)}{(s+2)(s+4)}$$

---

exp



$G(0) = 15$

$$G(s) = K\,\frac{(s+3)(s+4+j2)(s+4-j2)}{(s+2+j2)(s+2-j2)}\Bigg|_{s=0} = 15$$

$\boxed{K=2}$ $\boxed{G(s) = \frac{2s^3+22s^2+88s+120}{s^2+4s+8}}$

exp Find Complex Frequency.

① $(2e^{-100t} + e^{-200t}) \sin 2000t$

$-200 + j2000 \,,\; -100 + j2000$

$-200 + j2000 \,,\; -200 - j2000$

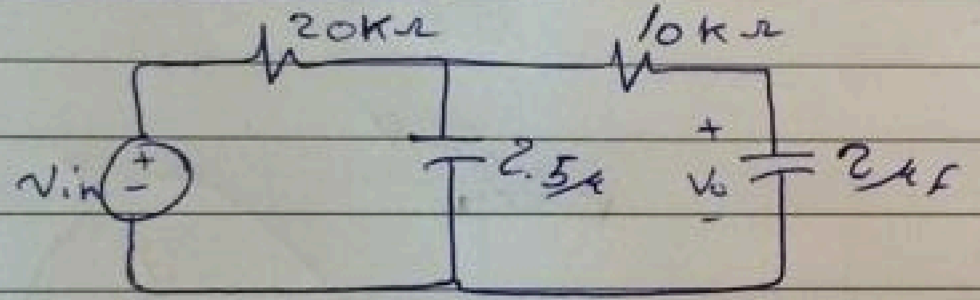
② $(2 - e^{-10t}) \cos(4t + \phi)$

$j4 \,,\; -j4$

$-10 + j4 \,,\; -10 - j4$

---

exp



a) Plot $Z_{in}(\sigma)$

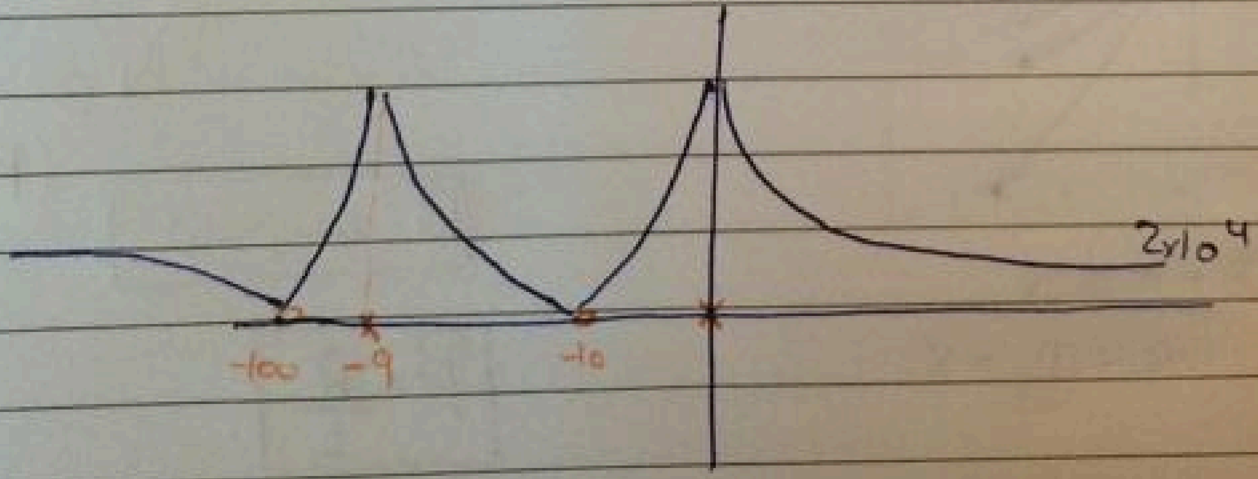
b) critical freq.

c) evaluate

$Z(-10), Z(-95)$

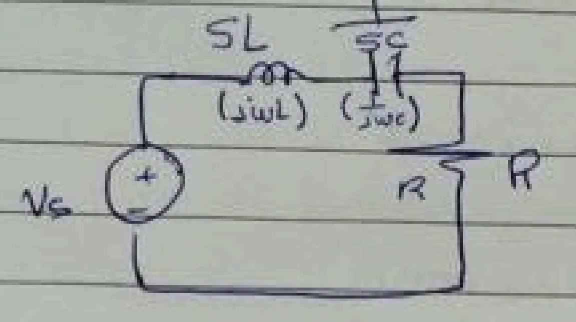
$Z(-50), Z(10\,\frac{Np}{s})$

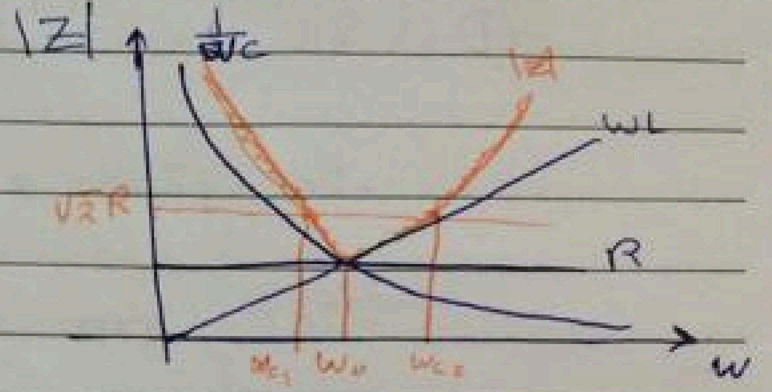
ⓐ $Z_{in} = 2\times10^{4} \dfrac{(\sigma+10)(\sigma+100)}{\sigma(\sigma+90)}$



Resonance:-

R, L, C, Series and Parallel





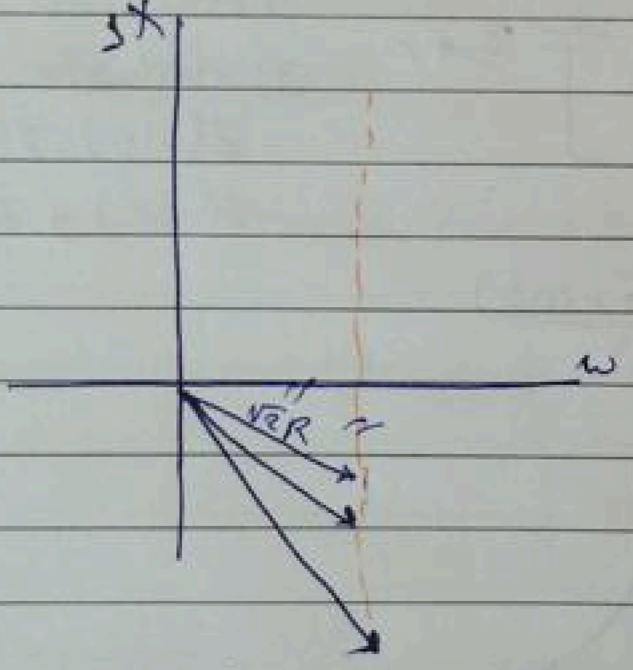
$$Z = R + j\left(wL - \frac{1}{wC}\right)$$

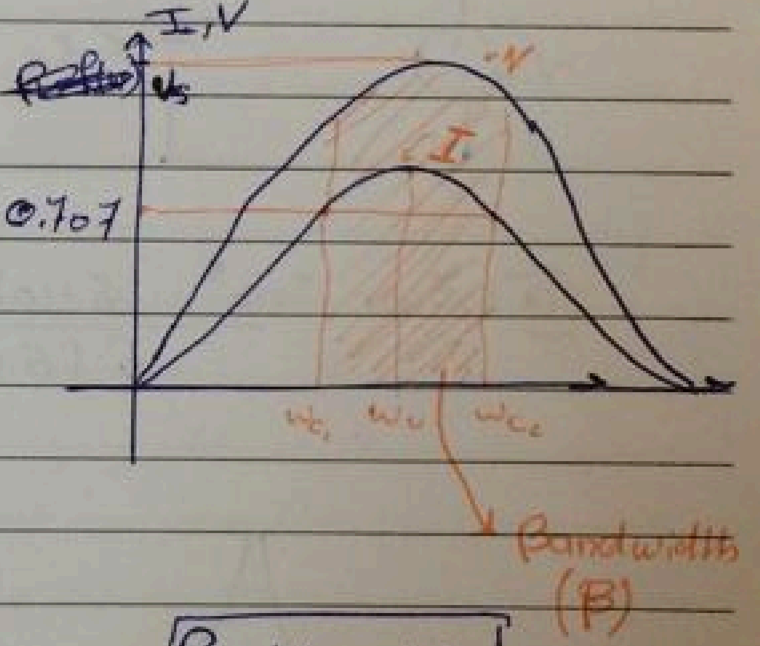
$$|Z| = \sqrt{R^2 + \left(wL - \frac{1}{wC}\right)^2}$$

w_{c_1}: Lower cut-off freq.

w_{c_2}: upper " " "

$$wL - \frac{1}{wC} = 0 \Longrightarrow w = \frac{1}{\sqrt{LC}}$$





$$\boxed{\beta = w_{c_2} - w_{c_1}}$$

$$\boxed{\beta = \frac{R}{L}}$$

$$Z = R \pm jX$$

$$wL - \frac{1}{wC} = X$$

$$w^2LC - 1 = wCX$$

$$w^2LC - CXw - 1 = 0$$

$$w^2 - \frac{CX}{LX}w - \frac{1}{LC} = 0$$

$$w^2 - \frac{X}{L} w - \frac{1}{LC} = 0$$

$$\boxed{w = \sqrt{\left(\frac{X}{2L}\right)^2 + \frac{1}{LC}} \pm \frac{\cancel{X}}{2L}}$$

$R = \pm X$

$$\boxed{w_{c_1,c_2} = \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \pm \frac{R}{2L}}$$

* Quality Factor :-

$$Q_0 = \frac{w_0 L}{R} = \frac{1}{w_0 C R} = \frac{\sqrt{L}\sqrt{L}}{R\sqrt{L}\sqrt{C}} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

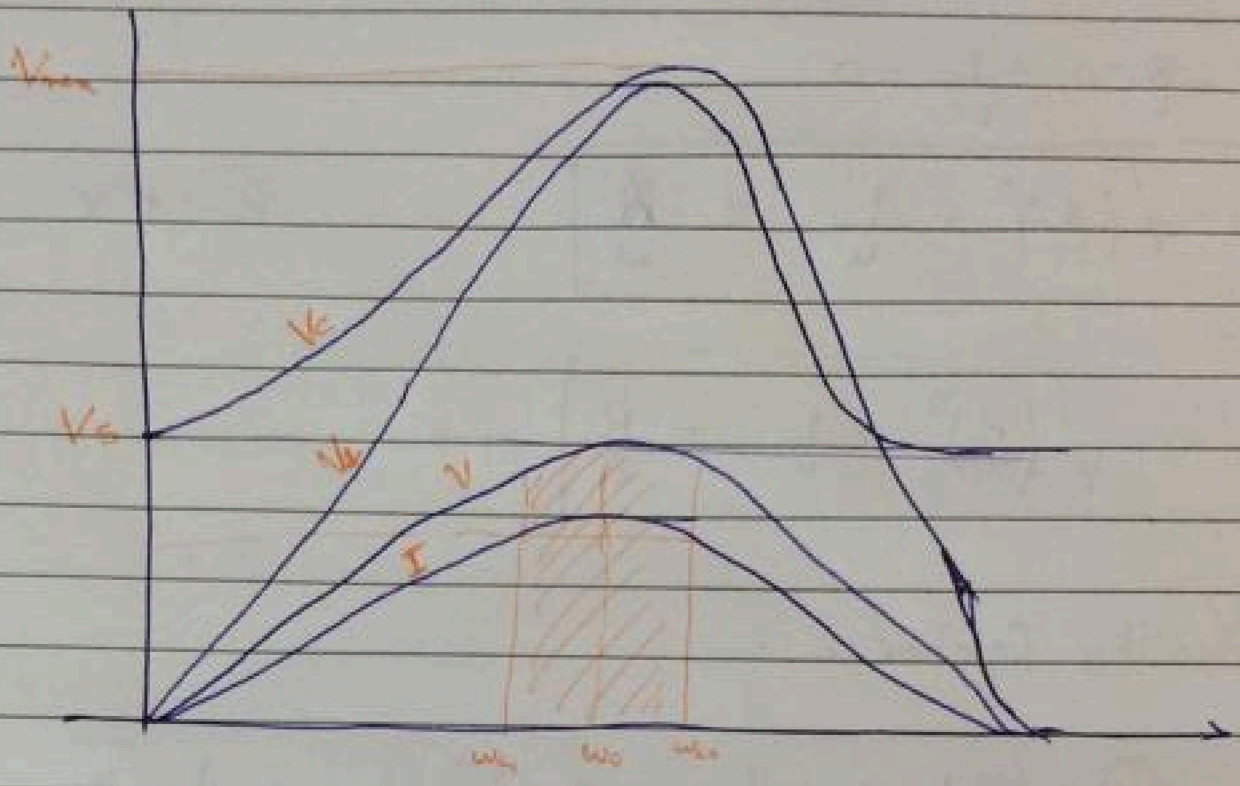
$$\boxed{Q_0 = \frac{w_0}{\beta}}$$

$\Longrightarrow V_L = V_s \frac{wL}{Z}$

- $V_L(w_0) = V_s \frac{w_0 L}{R} = Q_0 V_s$
- $V_L(w_{c_1,c_2}) = V_s \frac{w_{c_1} L}{\sqrt{2}\, R} = 0.707\ V_{max}$
- $V_L(w=\infty) = V_s$
- $V_L(0) = 0$

$\Longrightarrow V_C = V_s \frac{1}{wCZ}$

- $V_L(w_0) = V_s \frac{1}{w_0 C R} = Q V_s$
- $V(w_{c_1}) = 0.707\ V_{max}$
- $V(0) = V_s$
- $V(\infty) = 0$



$$Q_s = \frac{X_s}{R_s} \quad , \quad Q_p = \frac{R_p}{X_p}$$

<u>proof</u> $Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$

$$\frac{Z}{R} = \left(1 + j\left(\frac{\omega L}{R} + \frac{1}{\omega C R}\right)\right) * \frac{\omega_0}{\omega_0}$$

$$= 1 + j\left(\frac{\omega_0 L}{R} \cdot \frac{\omega}{\omega_0} - \frac{1}{\omega_0 C R} \cdot \frac{\omega_0}{\omega}\right)$$

$$= 1 + jQ\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)$$

$$= 1 + jQ\,\frac{\omega^2 - \omega_0}{\omega_0 \omega} = 1 + jQ\,\frac{(\omega - \omega_0)(\omega + \omega_0)}{\omega_0 \omega}$$

$\boxed{\omega \simeq \omega_0}$

$$= 1 + jQ\,\frac{(\omega - \omega_0)\,2\omega_0}{\omega_0^2} = 1 + jQ\,2\frac{(\omega - \omega_0)}{\omega_0}$$

⑳

$$\frac{Z}{R} = 1 + j\,\frac{\omega - \omega_0}{B/2} = 1 + jN$$

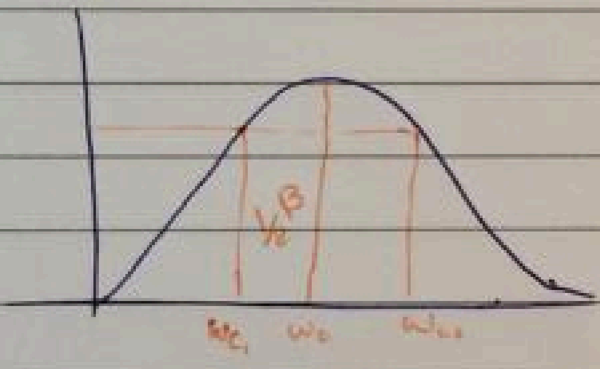
$$W_{c_1,c_2} = \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \mp \frac{R}{2L}$$

$$= \sqrt{\left(\frac{W_0}{2Q}\right)^2 + W_0^2} \mp \frac{\beta}{2}$$

$$W_{c_1,c_2} = W_0\sqrt{\left(\frac{1}{2Q}\right)^2 + 1} \mp \frac{\beta}{2}$$

$\underline{\underline{Q \geq 5}}$

$$W_{c_1,c_2} = W_0 \mp \frac{\beta}{2}$$



$$Z(s) = R + SL + \frac{1}{SC}$$

$$= \frac{RLS + S^2LC + 1}{SC} = \frac{LC\,s^2 + RL\,S + \frac{1}{LC}}{CS}$$

$$= \frac{\left(S^2 + \frac{R}{L}S + \frac{1}{LC}\right)L}{CS}$$

POWER UNIT

Circuits II Notebook
Dr. Nabeel Tawalbeh

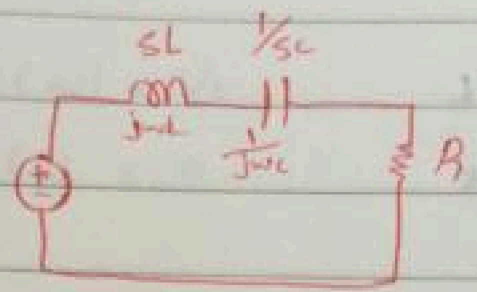
By . Yazan Abawi

بأفكارنا_نُبدع

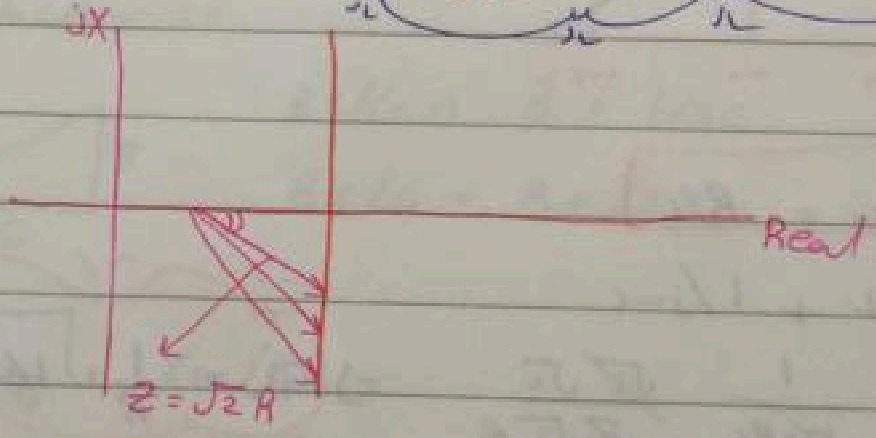
* Resonance Frequency *

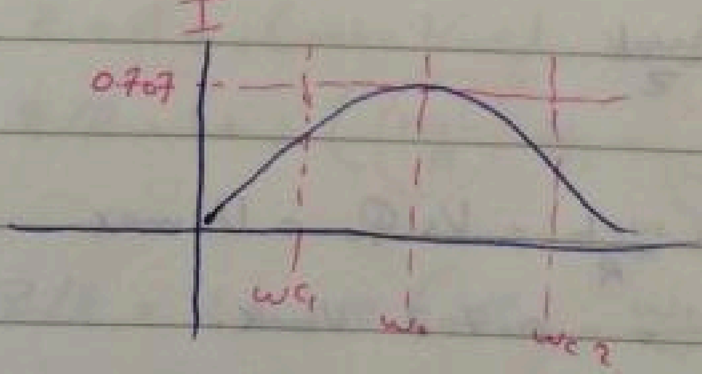
عنوان الدرس Lecture 1 اليوم التاريخ / / ١٤

* RLC series & Parallel 8>



* $Z = R + j(wL - \frac{1}{wC})$
* $R, \; wL, \; \frac{1}{wC}$
* $Z = \sqrt{R^2 + (wL - \frac{1}{wC})^2}$
* $wL - \frac{1}{wC} = 0 \Rightarrow \boxed{w_0 = \frac{1}{\sqrt{LC}} \Rightarrow \text{resonant frequency}}$





1 at $0, \infty = 0.C \Rightarrow 0$

$I_{max} = V/R$

$Z = R \pm jX \qquad wL - \frac{1}{wC} = X$

$w^2LC - wCXw - 1 = 0$

$w^2 - \frac{CX}{LC}w - \frac{1}{LC} = 0$

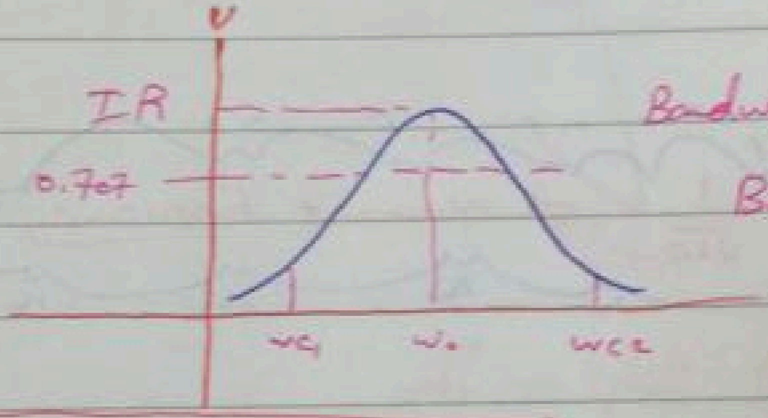
$w^2 - \frac{X}{L} w - \frac{1}{LC} = 0 .$

$w = \sqrt{(X/2L)^2 + 1/LC} \quad + X/2L$

$X = -R , +R \Rightarrow R = \pm X .$

if $X = -R$

$w_{c_1, c_2} = \sqrt{(R/2L)^2 + 1/LC} \mp \overset{\text{lower -ve freq}}{\underset{\text{upper +ve freq}}{R/2L}}$



Bandwidth (B) = $w_{c_2} - w_{c_1}$

Band Pass filter.

* $\boxed{B = w_{c_2} - w_{c_1} = R/L}$

* $Z = R + jwL + 1/jwC$

* $Q_0 = \frac{w_0 L}{R} = \frac{1}{w_0 R C} \quad \frac{\cancel{\sqrt{L}}\sqrt{L}}{\cancel{\sqrt{L}}\sqrt{C}\,R} \Rightarrow Q_0 = \frac{1}{R}\sqrt{LC}$

$Q_0 = \frac{w_0}{B}$ $\qquad V_L = V_S - \frac{w_0 L}{Z} .$

at resonance $V_L(w_0) = V_s \frac{w_0 L}{R} = V_s Q_0 = V_L max$

$V_L(w_{c_1}, c_2) = V_s \frac{w_0 L}{\sqrt{2} R} = 0.707 \; V_{max} .$

$V_L(w = \infty) = V_s \qquad V_L(w = 0) = 0$

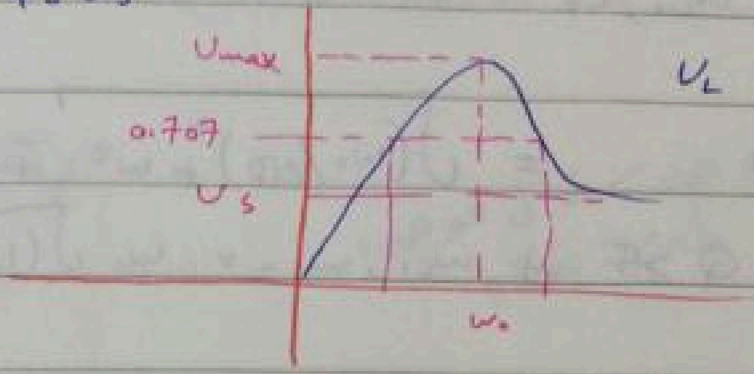
at 0 $\Rightarrow V_C = 0 \qquad V_C = V_s . \frac{1}{w_0 L Z}$

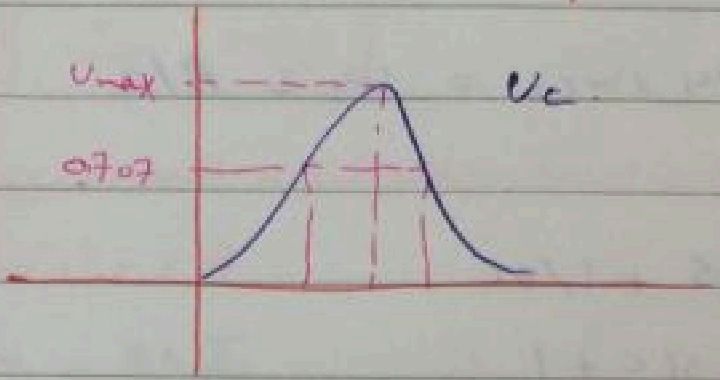
عنوان الدرس Continued. اليوم التاريخ / / ١٤

$V_c(\omega) = V_s * \frac{1}{\omega_o C R} = Q_o V_s$.

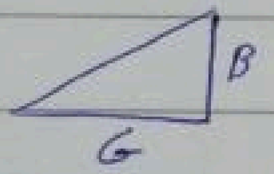
$V_c(\infty) = V_s$

$V_L(0) = 0$.





* Capacitor :=



$BC/G \Rightarrow B = 1/X_C$.

$BC/G = R_P/X_P$.

$Q_{series} = X_s/R_s$.

$Q_{Parallel} = R_P/X_P$.

$Z = R + j(\omega L - 1/\omega C) = R\left(1 + j\left[\frac{\omega L}{R} - \frac{1}{\omega C R}\right]\right)\frac{\omega_o}{\omega_o}$.

$Z/R = 1 + j\left(\overset{Q_o}{\boxed{\frac{\omega_o L}{R}}} \cdot \frac{\omega}{\omega_o} - \overset{Q_o}{\boxed{\frac{1}{\omega_o R C}}} \cdot \frac{\omega_o}{\omega}\right)$

$Z/R = 1 + jQ\left(\omega/\omega_o - \frac{\omega_o}{\omega}\right) = 1 + jQ\,\frac{\omega^2 - \omega_o^2}{\omega_o \omega}$.

$= \frac{1 + jQ\,(\omega - \omega_o)(\omega + \omega_o)}{\omega_o \omega}$.

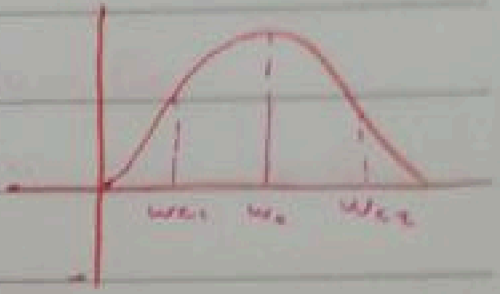
* $\omega = \omega_o \Rightarrow 1 + jQ^2\,\frac{(\omega - \omega_o)}{\omega_o} = 1 + j\,\frac{\omega - \omega_o}{B/2} = 1 + jN$.

$$w_{c1,c2} = \sqrt{(R/2L)B + 1/LC} \mp R/2L.$$

$$= \sqrt{(w_0/2Q)^2 + w^2} \mp B/2.$$

$$Q \geq 5 \Rightarrow w_{c1}, w_{c2} = w_0\sqrt{(1/2Q)^2 + 1} \mp B/2$$

$$Q \geq 5 : w_{c1} / w_{c2}' = w_0 \mp B/2$$



$w_{c1}, w_{c2} = w_0 \mp \frac{1}{2} \beta.$

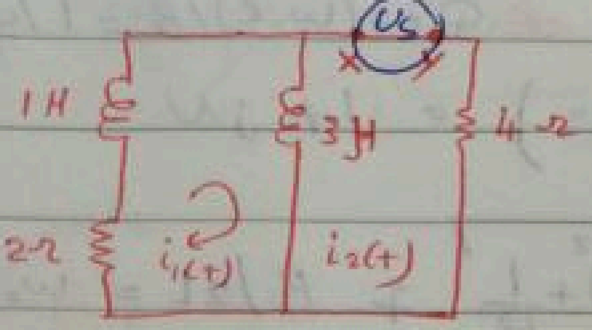
$$Z(s) = R + Ls + 1/sC$$

$$Z(s) = \frac{RLs + sLC + 1}{sC}$$

$$= \frac{LCs^2 + RLs + 1}{sC} \begin{matrix} \div LC \\ \div LC \end{matrix}$$

$$= \frac{s^2 + \frac{R^2}{C} + \frac{1}{LC}}{Cs/L}.$$

* سؤال امتحان مادة Complex Frequency



* هاد سؤال موجود بأسئلة الكتاب

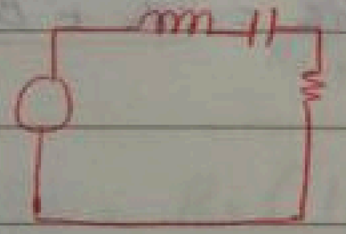
* evaluate $i_2(t)$ by install a suitable source across Points x, y $i_1(0) = i_2(0) = 1A$.

$U_s = (4+3s) I_2 - 3s I_1$

$0 = -3s I_1 + (2+4s) I_1$.

$I_2 = \frac{(2+4s) U_s}{3(s^2 + \frac{22}{3}s + 8/3}$ = ----

* * * * * *

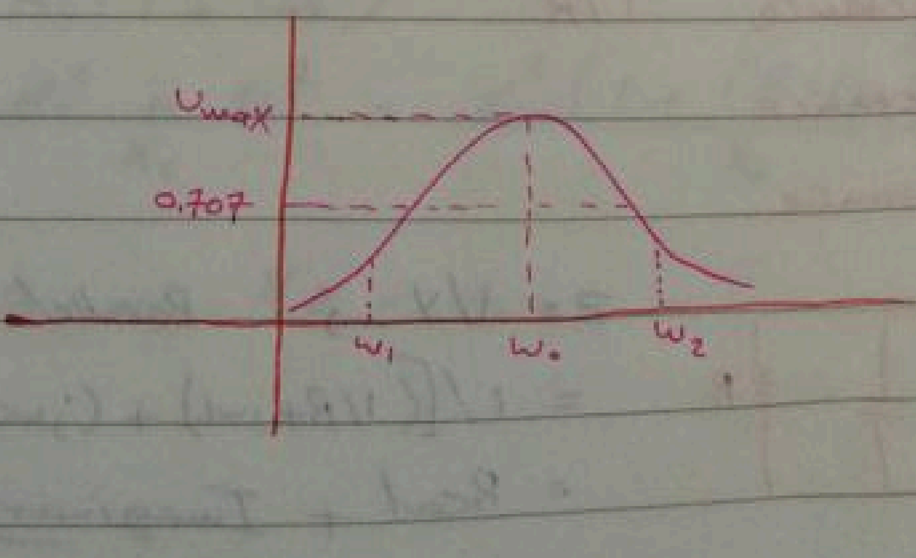


$U_c = U_s * (X_c / Z)$

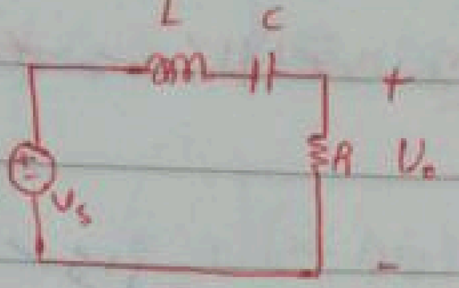
$U_c(w_0) = U_s * [(1/w_0 c)/R] = Q_0 U_s$.

$U_c(0) = U_s$ $U_c(\infty) = 0$

$U_c(wc_1) = 0.707\ U_{max}$



عنوان الدرس Continued اليوم التاريخ / / ١٤



Series.

* $Z = R + j(wL - 1/wc)$

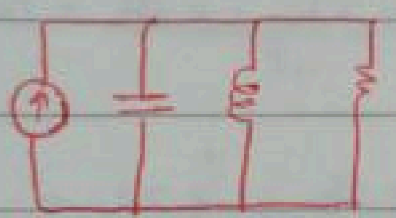
* $Q_{0s} = (w_0 C)/R = 1/w_0 C R = \frac{1}{R}\sqrt{\frac{L}{C}}$

* $\frac{Z}{\text{[illegible]}} = 1 + j\left(\frac{w - w_0}{B/2}\right) = 1 + jN$

* $w_{c_1, c_2} = \sqrt{(R/2L)^2 + \frac{1}{LC}} \mp R/2L = w_0 \mp B/2$

* $B = R/L = w_0/Q_0$

Parallel



$G + jBc - jBL$

$Y = G + j(wc - 1/wL) = 1 + jQ\left(\frac{w}{w_0} - \frac{w_0}{w}\right).$

$Q_{0P} = w_0 C R = \frac{R}{w_0 L} = R\sqrt{C/L} \Rightarrow$ series عكس قانون ال *

$w_{c_1, c_2} = \sqrt{(1/2RC)^2 + 1/LC} \mp 1/2RC = \boxed{w_0 \mp B/2}$

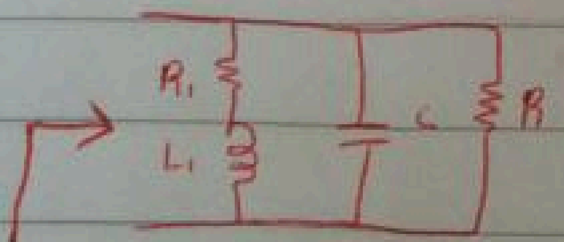
$= w_0\sqrt{1 + (1/2Q)^2} \mp 1/2Q$

$B = \frac{1}{RC} = \frac{w_0}{Q_0}.$

* G → Conductance 1/R.

* B → Susbetance

* Y → Admittance.



$Z = 1/Y \rightarrow$ Parallel باعتبار

$= 1/\left[(1/R + jwL) + (jwc) + (1/R)\right]$

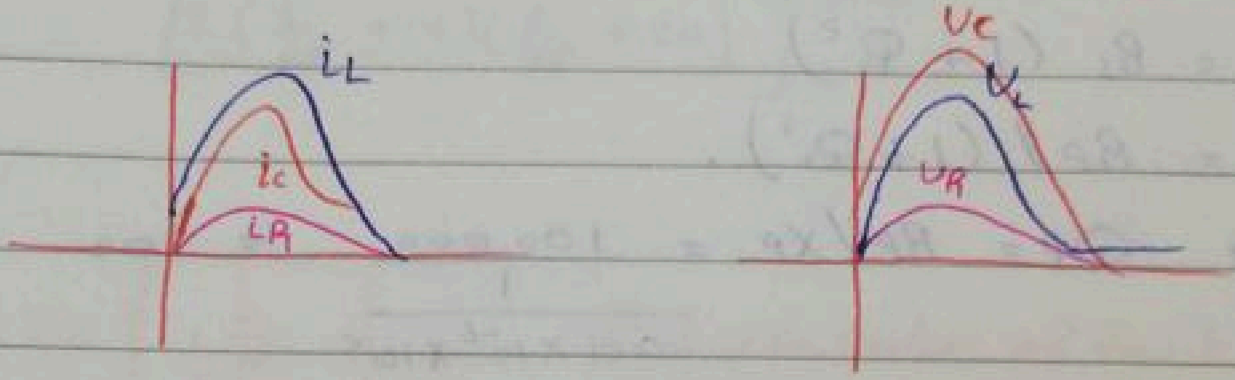
$=$ Real + Imaginary

عنوان الدرس Continued اليوم التاريخ / / ١٤

$I_{mg} = 1/R_2 + j\omega C + 1/(R + j\omega L)$.

Bep

$W_0 = \sqrt{\frac{1}{LC} - \left(\frac{R_1}{c}\right)^2}$ Pure RLC الكويز مو *



$Z_s \rightarrow Z_p$. للتحويل من $P \rightarrow S$ وبالعكس

* $Z_s = R_s + jX_s \xrightarrow{Y \text{ بدي احولها}} Y$

* $Y = 1/Z = 1/(R_s + jX_s) = R_s/(R_s^2 + jX_s) - j\frac{X_c}{R_s^2 + X_s^2}$

* $Y = G_p - jB_p = \frac{1}{R_p} - j\frac{1}{X_p}$.

* $1/R_p = \frac{R_s}{R_s^2 + X_s^2} \Rightarrow R_p = \frac{R_s^2 + X_s^2}{R_s} = \frac{R_s^{\cancel{2}}\left(1 + (X_s/R_s)^2\right)}{\cancel{R_s}}$

$\boxed{R_p = R_s(1 + Q_0^2)}$ هاي اهم معادلة موضوعي الاشغاله *

for $Q_1 \geq 5 \rightarrow R_p = R_s \cdot Q_s^2$

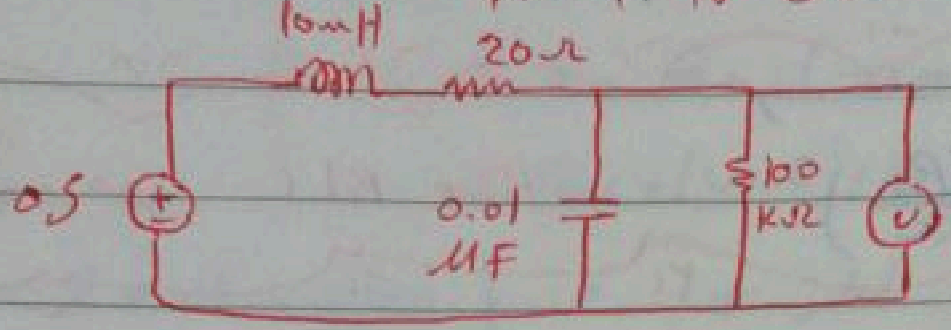
$R_p = R_s(1 + Q_L^2)$

$X_p = \frac{R_s^2 + X_s^2}{X_s} = \frac{X_s^2\left(1 + (R_s/X_s)^2\right)}{X_s} = X_s\left(1 + \frac{1}{Q^2}\right)$

* $Q \geq 5 \Rightarrow X_p = X_s$

عنوان الدرس Continued اليوم .... التاريخ / / ١٤

سؤال مهم جداً جداً



* شو قراءة ال V ؟؟

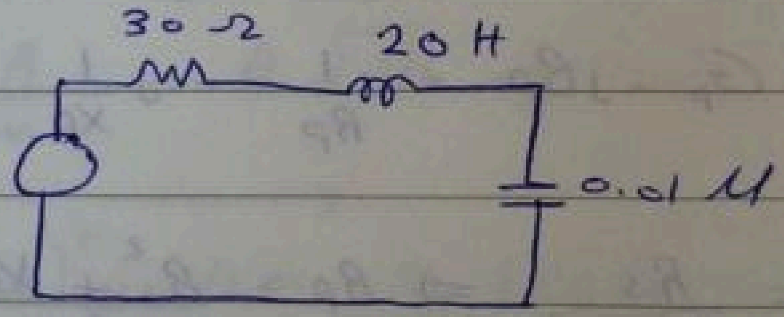
$$R_p = R_s \, (1 + Q_o^2)$$

$$R_s = R_p / (1 + Q_o^2).$$

$$Q_p = R_p / X_p = \frac{100\,000}{\frac{1}{0.01 \times 10^{-6} \times 10^{5}}} = 100.$$

$$w_o = \frac{1}{\sqrt{0.01 \times 1.0 \times 10^{-9}}} = 10^5.$$

بعد التحويلة

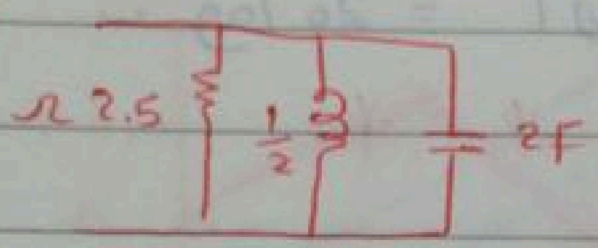


* Scalling :-

$$H(S) = \frac{2S}{2(s+10)(s+20)}$$ ⇒ اضرب ب Km كل ال Func.
واقسم كل S على Kf

$$H(s) = \left[\frac{2(S/kf)}{2(\frac{S}{kf}+10)(\frac{S}{kf}+20)}\right] Km.$$

*



find the value of each elemen
after scaled by $kf = 5 \times 10^6$
$Km = 200$

$R_{new} = 500$ $\quad L_{new} = 200\,\mu H$ $\quad C_{new} = 200\,pF$

$$= L\,\frac{Km}{kf}. \qquad = \frac{2}{Km\,kf}.$$

* Bode Plot *

$$H(s) = K\,\frac{(S+Z_1)}{(S+P_1)(S+P_2)}$$

$$H(jw) = K\,\frac{(jw+Z_1)}{(jw+P_1)(jw+P_2)} = K\,\frac{Z_1(1+jw/z_1)}{P_1P_2\,(1+jw/P_1)(1+jw/P_2)}$$

$$= K\,\frac{Z_1}{P_1P_2}\cdot\frac{|(1+w/z_1)|}{|1+w/P_1|\,|1+w/P_2|}$$

$$|H(jw)| = K\cdot\frac{|1+jw/z_1|}{|1+jw/P_1|\,|1+jw/P_2|}$$

$$H_{dB} = 20\log|H(j\omega)| = 20\log k + 20\log|1+j\frac{\omega}{z}| - 20\log|1+j\omega/p_1| - 20\log|1+j\omega/p_2|.$$

$$\angle H(j\omega) = \tan^{-1}\omega/z_1 - \tan^{-1}\omega/p_1 - \tan^{-1}\omega/p_2$$

الزاوية مو داخلة

$H(j\omega) = j\omega.$

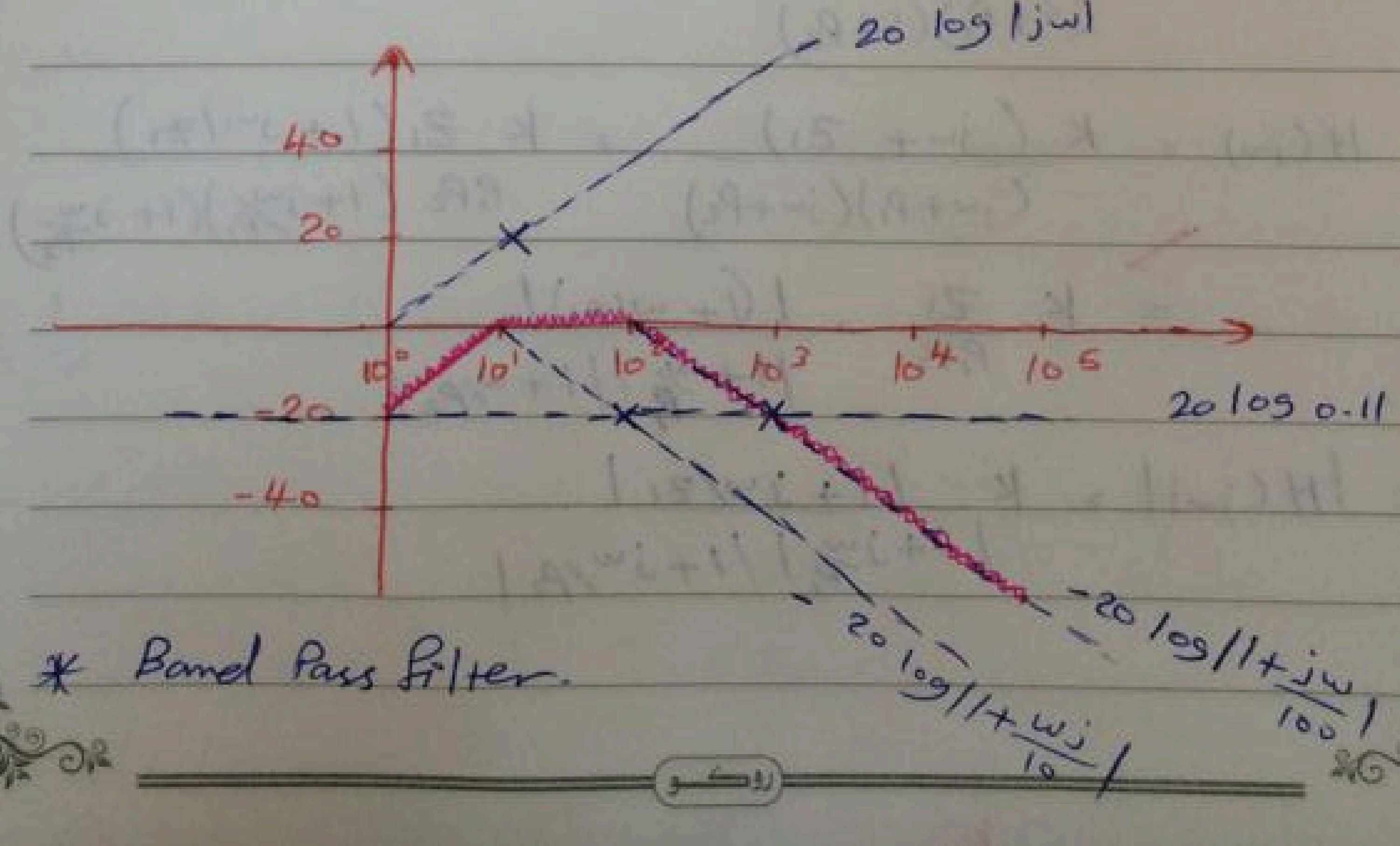
$H_{dB} = A_{dB} = 20\log|H(j\omega)| = 20\log\omega.$

* * * * * * *

$$H(s) = \frac{110s}{s^2+110s+1000} = \frac{110s}{(s+10)(s+100)}$$

$$H(s) = \frac{110s}{100*10(1+\frac{s}{10})(1+\frac{s}{100})} \Rightarrow \text{standard form.}$$

$$= 0.11\frac{s}{(1+\frac{s}{10})(1+s/100)}$$



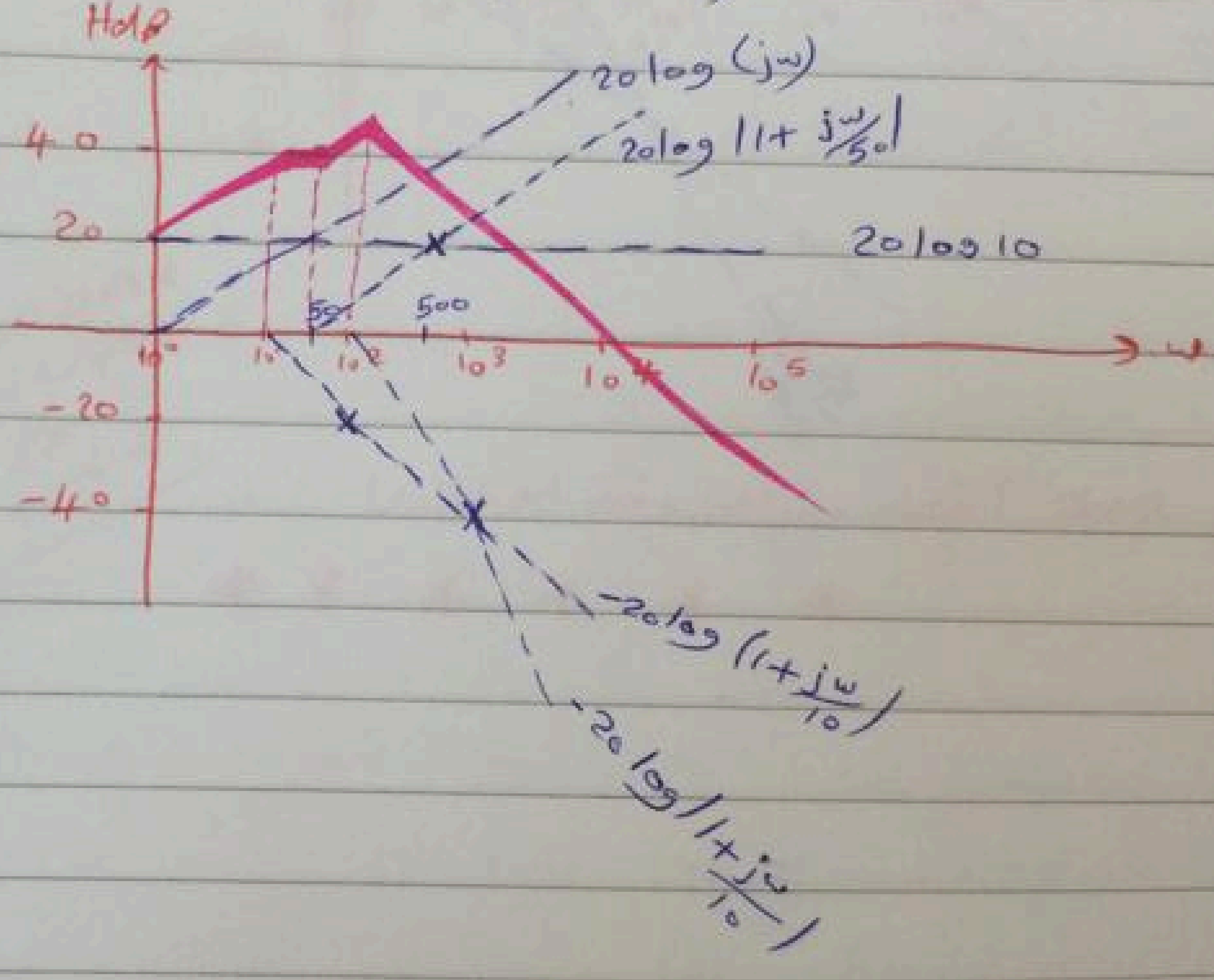
* Band Pass filter.

عنوان الدرس Continued اليوم التاريخ / / ١٤

$$H(s) = \frac{2\times10^4\ s\ (s+50)}{(s+100)(s^2+110\ s+1000)}$$

$$= \frac{2\times\cancel{10^4}\times 5\cancel{0}\ s\ (1+s/50)}{\cancel{100*10*100}\ (1+ s/_{100})(1+s/_{10})}$$

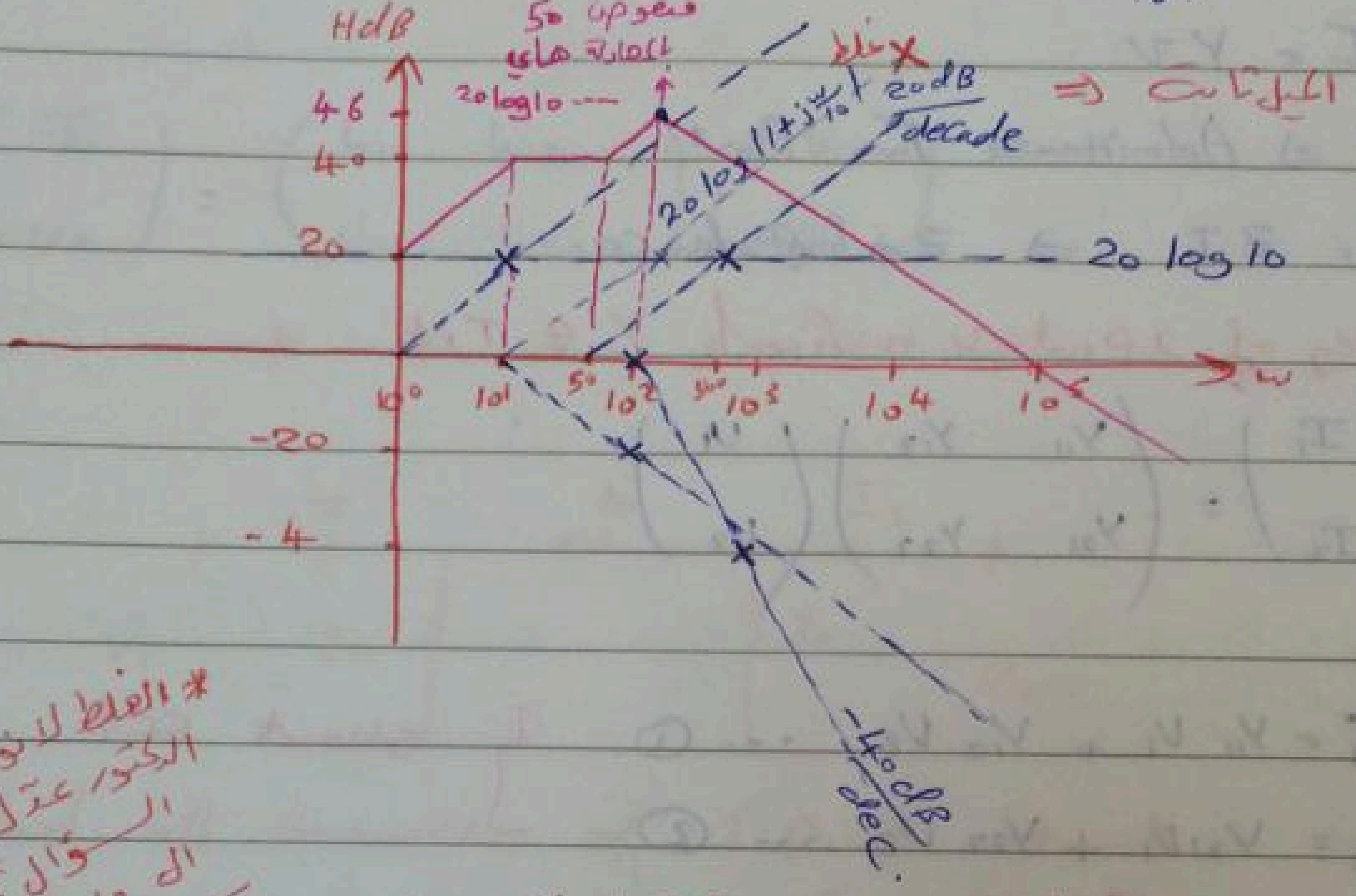
$$= 10\ \frac{(s+s/50)}{(1+s/_{100})(1+s/_{10})}$$



عنوان الدرس Lecture 2 .... اليوم .... التاريخ / / ١٤

$$H(s) = \frac{10 S \, (1+S/50)}{(1+S/10)(1+S/100)^2} = 20\log 10 + 20\log(jw) + 20\log(1+j\frac{w}{50}$$

$$- 20\log\left|1+j\frac{w}{10}\right| - 2\times 20\log\left|1+j\frac{w}{1}\right.$$



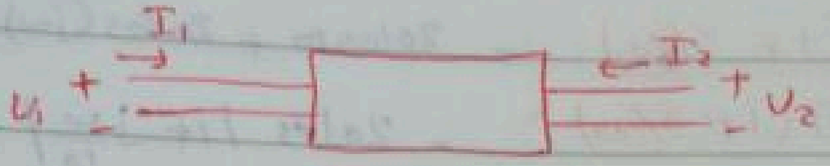
* الغلط لانو رجع
الدكتور عدل عم
السؤال مكان
ال ١٠ ـ 50

* ال phase أجّلناه وازا ضل وقت اخدناها.

* * * * * *

* 2- Port Network *

عنوان الدرس Lecture 2 اليوم ...... التاريخ / / ١٤



* $I = Y V$.

* $Y \Rightarrow$ Admittance Two Port.

* $V = Z I \Rightarrow Z \Rightarrow$ impedance.

* Key of 2-port is to find Y, Z, T, h.

$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

* $I_1 = Y_{11} V_1 + Y_{12} V_2$ ... ①
* $I_2 = Y_{21} V_1 + Y_{22} V_2$ ... ②.

} short cct

* $SC_{\text{Ⅱ}}^{\text{output}}$, i.e $V_2 = 0 \longrightarrow Y_{11} = I_1 / V_1$ $Y_{21} = I_2 / V_1$.
* $SC_{\text{Ⅰ}}$ Input, i.e $V_1 = 0 \longrightarrow Y_{22} = I_2 / V_2$ $Y_{12} = I_1 / V_2$.

* for impedance :- Ⅱ ⇐ $I_2 = 0 \Rightarrow$ O.C

- $V_1 = Z_{11} I_1 + Z_{12} I_2$ ... ①
- $V_2 = Z_{21} I_1 + Z_{22} I_2$ ... ②

$Z_{11} = V_1 / I_1$, $Z_{21} = V_2 / I_1$

$Z_{22} = V_2 / I_2$, $Z_{12} = V_1 / I_2$

* for Transfer 2-Port :- $I = I_1 = 0 \Rightarrow$ O.C.

$$\begin{pmatrix} V_1 \\ I_1 \end{pmatrix} = \begin{pmatrix} \overset{A}{t_{11}} & \overset{B}{t_{12}} \\ \overset{C}{t_{21}} & \overset{D}{t_{22}} \end{pmatrix} \begin{pmatrix} V_2 \\ -I_2 \end{pmatrix}$$

$V_1 = t_{11} V_2 - t_{12} I_2$ ... ①

$I_1 = t_{21} V_2 - t_{22} I_2$ ... ②

$OC_{\text{Ⅱ}}$, $I_2 = 0$ $\longrightarrow$ $\overset{A}{t_{11}} = \frac{V_1}{V_2}$, $\overset{C}{t_{21}} = \frac{I_1}{V_2}$

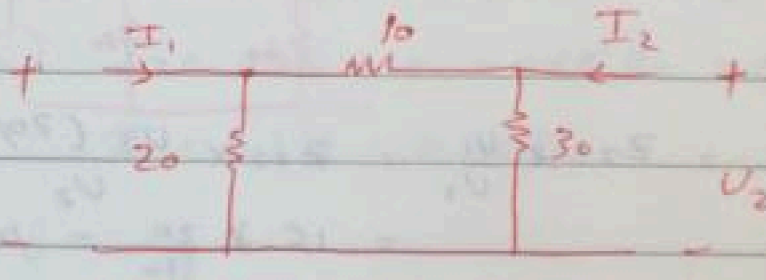
$SC_{\text{Ⅱ}}$, $V_2 = 0$ $\longrightarrow$ $D = -I_1 / I_2$, $B = -\frac{V_1}{I_2}$

$* \quad V_1 = h_{11} I_1 + h_{12} V_2$

$I_2 = h_{21} I_1 + h_{22} V_2$

$$* \begin{pmatrix} I_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix} \begin{pmatrix} V_1 \\ I_2 \end{pmatrix}$$

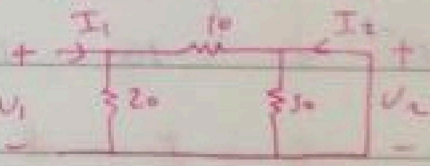
* * * * * *



* find all Parameters of 2-Port network

$V_2 = 0 \rightarrow Y_{11} = I_1 / V_1$

$\searrow Y_{21} = I_2 / V_1$



$Y_{11} = I_1 / V_1 = 1 / (10 // 20)$ ⇐ sc مع parallel لأنها راحت 30 ال *

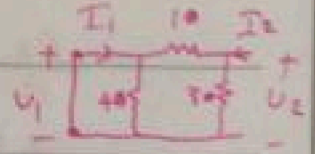
$Y_{21} = \frac{I_2}{V_1} * \frac{I_1}{I_1} = Y_{11} \cdot I_2 / I_1$

$= Y_{11} * \frac{-I_1 (20/30)}{I_1} = -2/3 \; Y_{11}.$

اعتبرناه الرئيسي لانو I_2 ← s.c والسالب
لانو اتجاهه عكسي

$V_1 = 0 \rightarrow Y_{22} = I_2 / V_2 = 1 / (10 // 30)$

$\searrow Y_{12} = I_1 / V_2 = I_1 / V_2 * \frac{I_2}{I_2}$



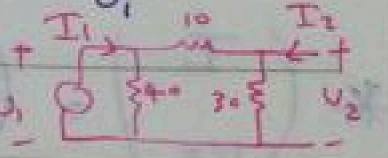
$= Y_{22} \; \frac{-I_2 (30/40)}{I_2} = -3/4 \; Y_{22}.$

$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} 3/20 & -1/10 \\ -1/10 & 4/30 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} \Rightarrow Z = Y^{-1}$$ طريقة ①

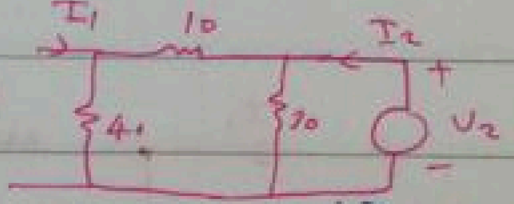
$* \ Z_{11} = U_1 / I_1 \Big|_{I_2=0} = 40/20 = 80/6$

$* \ Z_{21} = U_2 / I_1 \Big|_{I_2=0} = \frac{U_2}{I_1} * \frac{U_1}{U_1} = Z_{11} * \frac{U_2}{U_1}$

$= Z_{11} * \frac{U_1 (30/40)}{U_1}$



$* \ Z_{22} = \frac{U_2}{I_2} \Big|_{I_1=0} = 30 // 30 = 15 \Omega$



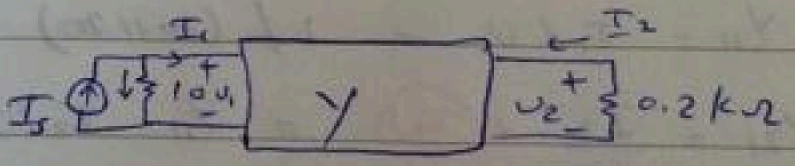
$* \ Z_{12} = \frac{U_1}{I_2} * \frac{U_2}{U_2} = Z_{22} * \frac{V_1}{U_2} = Z_{22} \times \frac{U_2 \ (20/30)}{U_2}$

$= 15 * \frac{20}{30} = 10 \Omega.$

$* \ Z = Y^{-1} = \begin{pmatrix} 80/6 & 10 \\ 10 & 15 \end{pmatrix}$

* * * * * *

$Y = \begin{pmatrix} .2 & .05 \\ 0.04 & 0.06 \end{pmatrix}$



Find $G_U = U_2 / U_1$

$G_i = I_2 / I_1$

$\frac{P_2}{P_1} = G_P$

Z_{in}, Z_{out}

$Z_{in} = \frac{U_1}{I_1} \qquad Z_{out} = \frac{U_2}{I_2}$

عنوان الدرس Continued اليوم .... التاريخ / / ١٤

$I_1 = 0.2\, V_1 - 0.05\, V_2$ .... ①

$I_2 = 0.04\, V_1 + 0.06\, V_2$ .... ②.

$I_2 = -\frac{V_2}{200} = -0.005\, V_2$ ... ③ Condition جبناها من ال *

عال Load ← output

$I_1 = I_s - 0.1\, V_1$ ... ④ ..→ KCL Loop.

* ③ بعوضها ب ② و ④ ب ①.

$3 \to 2$ $I_2 = 0.04\, V_1 + 0.065\, V_2 = 0$ ⟵ نقلناها للطرف الثاني

$4 \to ①$ $I_s = 0.3\, V_1 - 0.05\, V_2$

$$V_1 = \frac{\begin{pmatrix} 0 & .. \\ I_s & .. \end{pmatrix}}{\begin{pmatrix} .. & .. \\ .. & .. \end{pmatrix}} = \ldots\ V_s$$

* استخدمي Mode 5-1

بال Calculator وشاطرين.

$V_2 = -80/43\ V_s$

$V_1 = 130/43\ V_s \Rightarrow \frac{V_2}{V_1} = \frac{-80 * 43}{43 * 130} = -8/13.$

$I_1 = 0.2\, V_1 - 0.05\, V_2$

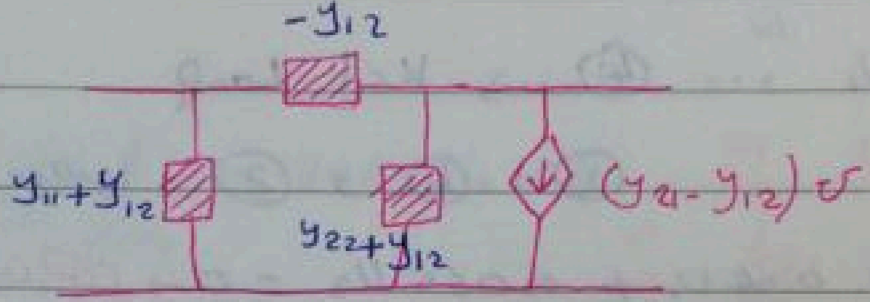
$I_2 = -V_2/0.2 = -80/-43 * 1/0.2 =$

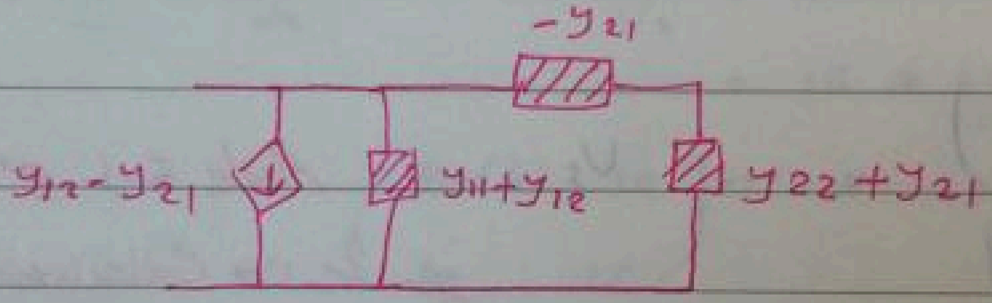
$I_1 = I_s - 0.1\, V_1.$

$I_1 = y_{11} V_1 + y_{12} V_2$ ---- ①

$I_2 = y_{21} V_1 + y_{22} V_2 + \underbrace{y_{12} V_1 - y_{12} V_1}_{\text{ما بتأثر اضافتها.}}$

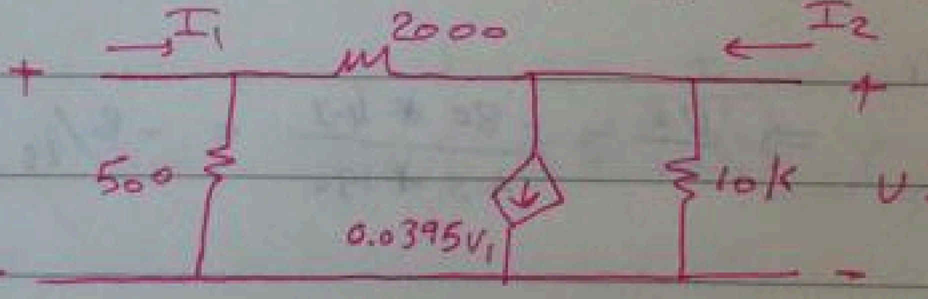
$I_2 - (y_{21} - y_{12}) V_1 = y_{12} V_1 + y_{22} V_2$ --- ②





هاي عكسها.

* * * * * *



$y_{12} = -1/2000 = -0.5 \ s$

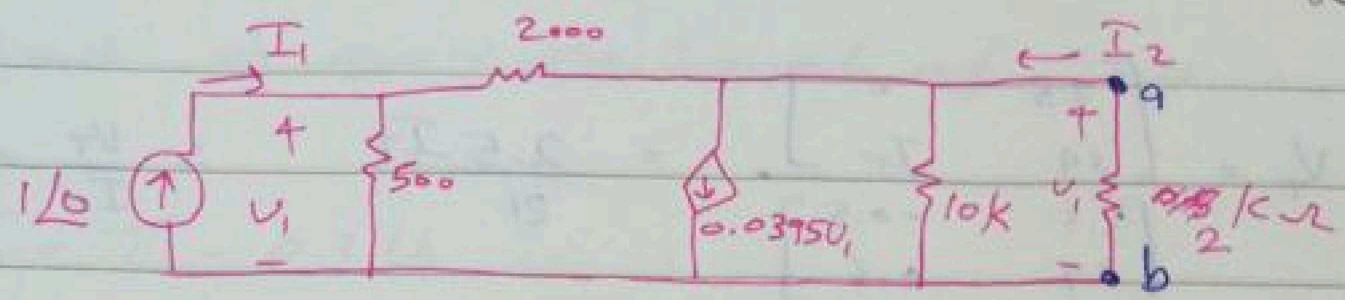
$y_{11} + y_{12} = 1/500 \Rightarrow y_{11} = 1/500 - (-0.5) = 2.5 \ ms$

$y_{22} + y_{12} = 1/10000 \Rightarrow y_{22} = 1/10000 - (-0.5) = 0.6m$

$(y_{21} - y_{12}) \cancel{V_1} = 0.0395 \cancel{V_1}$

$y_{21} = 0.0395 + y_{12} = 0.0395 * 1000 - 0.5 = 39 \ ms.$

عنوان الدرس Continued اليوم ...... التاريخ / / ١٤



هاد نفس سؤال المرة الماضية بس طالب فيه .. Zout, Zin, Gi, ..

$I_1 = 2.5V_1 - 0.5V_2$ --- ①

$I_2 = 39V_1 + 0.6V_2$ ---- ②

$I_1 = 1\,mA\,V_1$ --- ③

$I_2 = -2V_2$

$2.5V_1 - 0.5V_1 = 1$ .... ⑤

$39V_1 + 2.6V_2 = 0$ ... ⑥ استخدمي mode 5-1

$V_1 = 0.1 \quad V_2 = -1.5\,V$

$G_V = -15$

$G_i = \frac{-2(-1.5)}{1} = 3$

$G_p = G_V G_i = 45$

$Z_{in} = V_1/I_1 = 0.1\,K\Omega$.

* مشان اطلع Zout بعمل V_{th} ⇐ a-b → o.c

بعدين بعمل s.c مشان اطلع $I_{s.c}$.

$I_1 = 2.5V_1 - 0.5V_2$

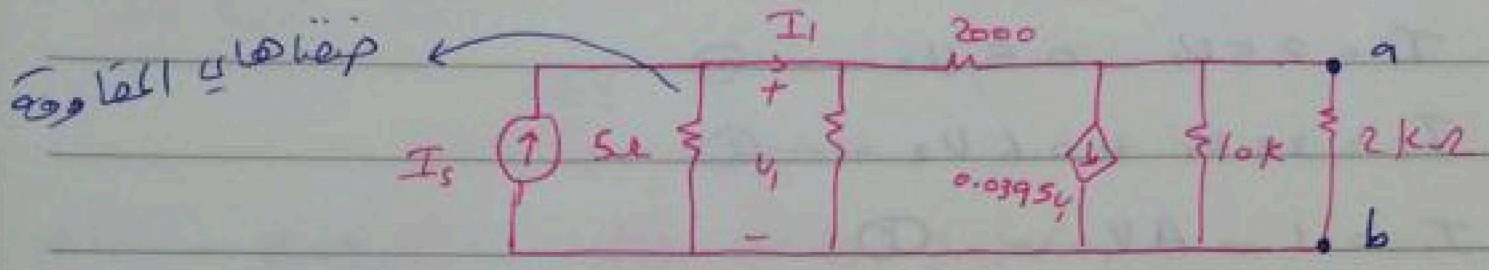
$I_2 = 39V_1 + 0.6V_2$

$2.5V_1 - 0.5V_T = 0$

$39V_1 + 0.6V_T = I_T$

عنوان الدرس Continued اليوم .... التاريخ / / ١٤

$$V_T = \frac{\begin{bmatrix} 2.5 & 0 \\ 39 & I_T \end{bmatrix}}{\begin{bmatrix} 2.5 & -0.5 \\ 39 & 0.6 \end{bmatrix}} = \frac{2.5\, I_T}{21} \quad \therefore \frac{V_T}{I_T} = 119\,\Omega$$



$I_1 = I_s - 0.2\,V_1$ ... ①

$I_2 = -2\,V_2$ ... ②

$2.7\,V_1 - 0.5\,V_2 = I_s$ ... ③

$39\,V_1 + 2.6\,V_2 = 0$ ... ④

$V_1 = 0.09\,I_s$

$V_2 = -1.47\,I_s$

* نعوض قيمة $I_1 = 1$ بالاعتماد ابطلع
معي الجواب بدلالة I_s

$G_V = -1.47\,\cancel{I_s} / 0.09\,\cancel{I_s} = -16.3$

$I_1 = 0.98\,I_s$

$I_2 = 2.94\,I_s.$

$G_i = 3.1$

$G_p = -(-16.3)(3.1) = 50.53.$

$Z_{in} = 0.09\,\cancel{I_s} / 0.98\,\cancel{I_s} = 0.091$

Z_{out} ⇒ remove the branch of a - b

$I_1 = I_s - 0.2 U_1 \quad \dots 1$

for Z_{out} ⇒ $I_1 = -0.2 U_1$

نعوض ع المعادلات الاتيه.

$I_1 = 2.5 U_1 - 0.5 U_2$

$I_2 = 39 U_1 + 0.6 U_2$

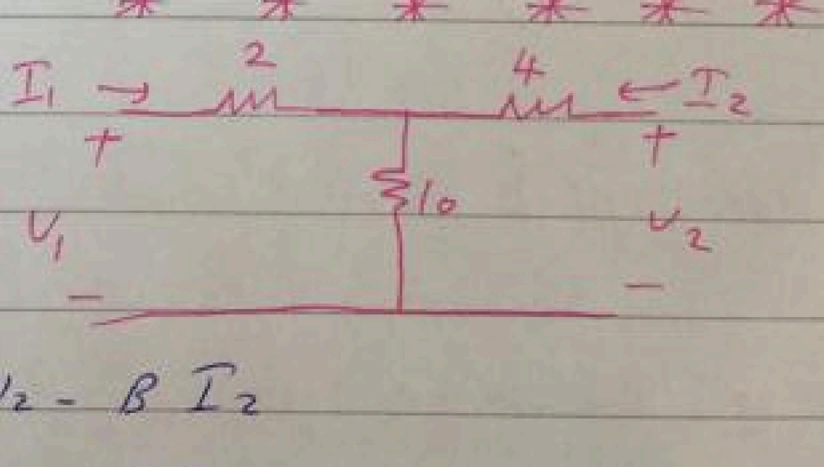
$2.7 U_1 - 0.5 U_T = 0$

$39 U_1 + 0.6 U_T = I_T$

$Z_{out} = \left(\frac{352}{45}\right) = -1.47 I_s.$

تفصيل ⇒ $\frac{2.7}{(524/25)}$

* * * * * *



$U_1 = A U_2 - B I_2$

$I_1 = C U_2 - D I_2$.

$* \; O.C_{II} \rightarrow A = U_1 / U_2$ $\qquad$ $* \; S.C_{II} \rightarrow D = \frac{-I_1}{I_2}$

$\searrow C = I_1 / I_2$ $\qquad$ $\searrow B = -U_1 / I_2$

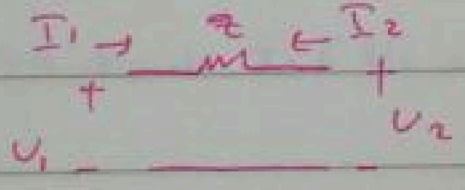
First → 15 , final → 30
second → 5

عنوان الدرس Cascading اليوم .... التاريخ / / ١٤

$$\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 1/10 & 1 \end{pmatrix}\begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1.2 & 2 \\ 0.1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1.2 & 6.8 \\ 0.1 & 1.4 \end{pmatrix}$$

$$\begin{pmatrix} V_1 \\ I_1 \end{pmatrix} = \begin{pmatrix} 1.2 & 6.8 \\ 0.1 & 1.4 \end{pmatrix}\begin{pmatrix} V_2 \\ -I_2 \end{pmatrix}$$



$V_1 = V_2 - zI_2$

$I_1 = -I_2$

$T = \begin{pmatrix} 1 & z \\ 0 & 1 \end{pmatrix}.$

شرح.

* في سؤال كبير بالفاينال عن كل الحكي هاد ← شوفوه من الكتاب الشرح التفصيلي.

* شوفي علاقات الـ Parameters مع بعضهم من الكتاب