

$$V_1 = \sqrt{\frac{L_1}{L_2}} \cdot V_2, \quad V_1 = \frac{N_1}{N_2} V_2.$$

$\frac{V_2}{V_1} = \frac{N_2}{N_1} = n$

→ Turns ratio

if $N_2 > N_1$ → step-up transformer

if $N_2 < N_1$ → step-down transformer.

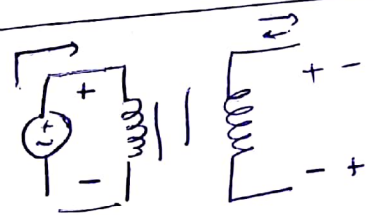
* For a lossless transformer :-

$$S_1 = S_2.$$

$$|V_1| |I_1| = |V_2| |I_2|$$

$$\frac{|V_2|}{|V_1|} = \frac{|I_1|}{|I_2|} = \frac{N_2}{N_1} = n.$$

$\frac{I_1}{I_2} = \frac{N_2}{N_1} = n.$



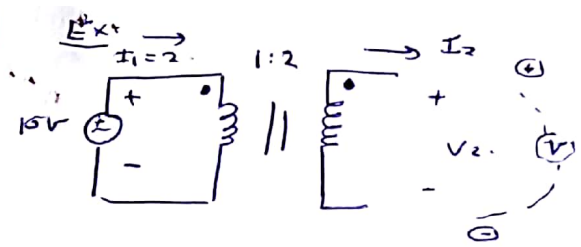
* Two Rules to define voltage polarities & current directions:

1

if V_1 & V_2 one both +ve or both -ve at the dotted terminals, use +n for $\frac{V_2}{V_1}$, otherwise, use -n.

2

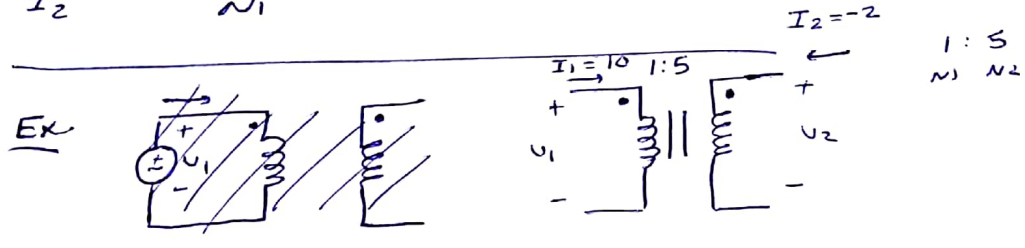
if I_1 & I_2 both enters or both leaves the dotted terminal, use -n, otherwise, use +n.



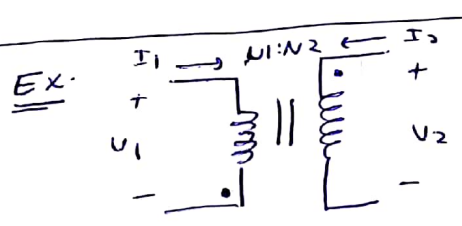
V_2 will be 20 volt, but the polarity?

$\frac{V_2}{V_1} = + \frac{N_2}{N_1} = +n$: the voltmeter will read +20. volt

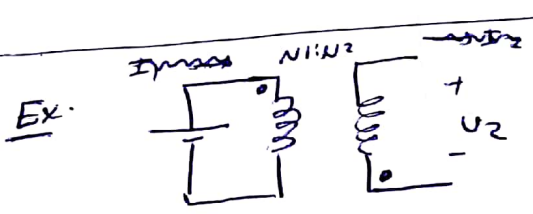
$\frac{I_1}{I_2} = + \frac{N_2}{N_1} = +n$, $i_2 = +1A$.



$\frac{V_2}{V_1} = + \frac{N_2}{N_1} = +n$, $\frac{I_1}{I_2} = - \frac{N_2}{N_1} = -n$.

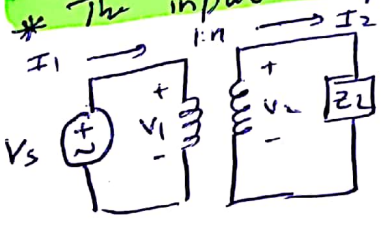


$\frac{V_2}{V_1} = - \frac{N_2}{N_1} = -n$, $\frac{I_1}{I_2} = + \frac{N_2}{N_1} = +n$.
 so, if $V_1 = 100V \rightarrow V_2 = -100V$



$V_2 = 0$, DC source (No change in flux)

*** The input impedance:**



$V_2/V_1 = n$.
 $I_1/I_2 = n$.

$Z_{in} = \frac{V_s}{I_1} = \frac{V_1}{I_1} = \frac{V_2/n}{n I_2} \rightarrow Z_{in} = \frac{1}{n^2} Z_L$
 = source reflected impedance.
 (ZL الـ منبع الـ سوي)

Ex: an ideal TR is rated $\frac{2400}{120}$ Volt, (9.6) KVA & has 50 turns on the secondary side, find: [3]

S (apparent power)
 primary \rightarrow
 secondary \rightarrow
 (No losses).
 $(S_1 = S_2 = S)$

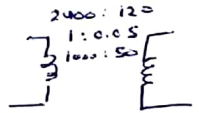
[1] turn ratio,

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = n = \frac{120}{2400} = .05$$

[2] turns on the primary side.

$$\frac{N_2}{N_1} = n \rightarrow \frac{50}{N_1} = .05$$

$$\rightarrow N_1 = 100 \text{ turns.}$$



[3] the current ratings for the primary & secondary coils.

$$S_1 = V_1 I_1$$

$$9600 = 2400 * I_1 \rightarrow I_1 = 4 \text{ A.}$$

$$S_2 = V_2 I_2$$

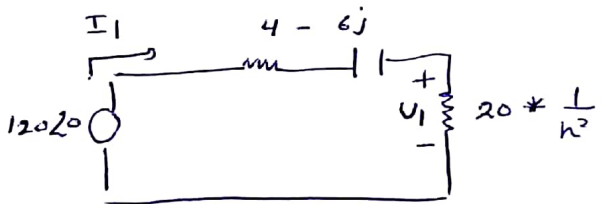
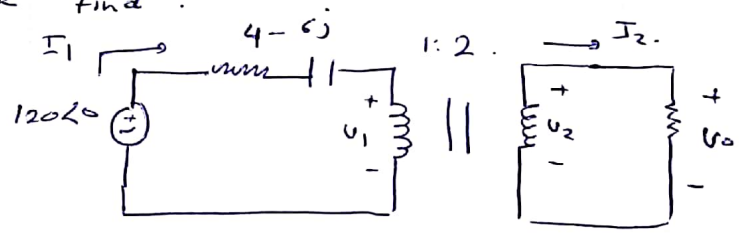
$$9600 = 120 I_2 \rightarrow I_2 = 80 \text{ A OR } I_2 = \frac{I_1}{n} = 80 \text{ A.}$$

Ex. for the shown ideal TR find:

[1] the source current (I_1)

[2] the output voltage (V_o)

[3] P_c complex power supplied by the source. (S')



[1] $Z_{in} = 9 - j6$

$$I_1 = \frac{V_s}{Z_{in}} = \frac{120 \angle 0}{9 - j6} = 11.09 \angle 33.7$$

[2] $\frac{I_1}{I_2} = -n = -2$

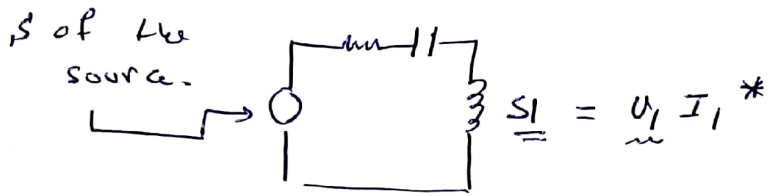
$$\frac{11.09 \angle 33.7}{I_2} = -2, \quad I_2 = -5.545 \angle 33.7$$

$$I_2 = +5.545 \angle 213.7$$

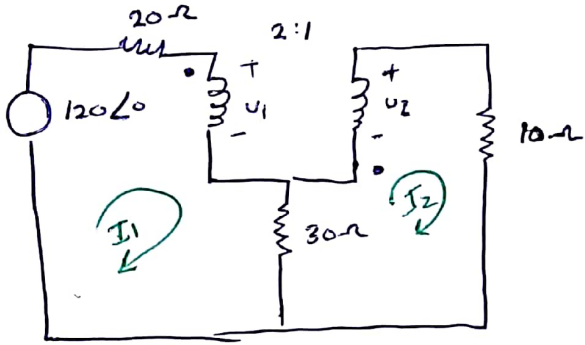
$$V_o = 20 * I_2 = 110.9 \angle 213.7 \text{ volt.}$$

OR $V_1 = 5 I_1$ (or voltage division)
 $= 55.45 \angle 33.7 \text{ V}$
 $\frac{V_2}{V_1} = -n = \frac{V_2}{55.45 \angle 33.7}$
 $V_2 = 110.9 \angle 213.7 \text{ volt.}$

$S = V_s I_1^* = 120 \angle 0^\circ * 11.09 \angle -38.7^\circ$



EX. Calculate the power supplied to the 10Ω resistor.



Use ideal transformer
 Reflection Mesh (KVL)

→ Mesh analysis:-

① Mesh #1.

$-120 \angle 0^\circ + 20 I_1 + V_1 + 30(I_1 - I_2) = 0$

$\frac{V_2}{V_1} = -\frac{1}{2} \rightarrow V_2 = -\frac{1}{2} V_1$

② Mesh #2.

$10 I_2 + 30(I_2 - I_1) - V_2 = 0$

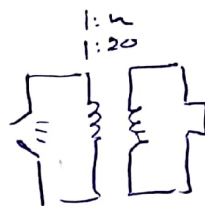
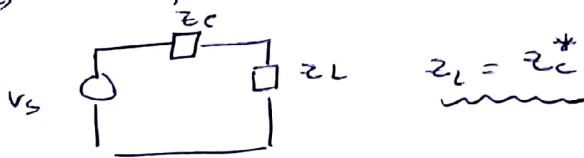
$\frac{I_1}{I_2} = -\frac{1}{2} \rightarrow I_1 = -\frac{1}{2} I_2$

تساوی معادلات (معادله معادلات)
 (معمولاً)

→ $I_2 = -0.7272 \text{ A}$

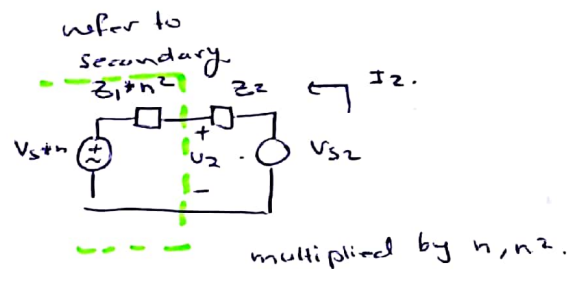
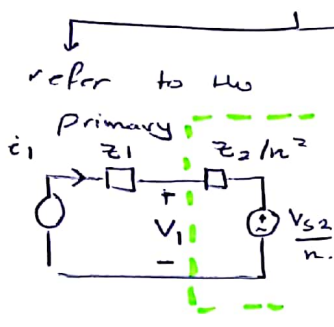
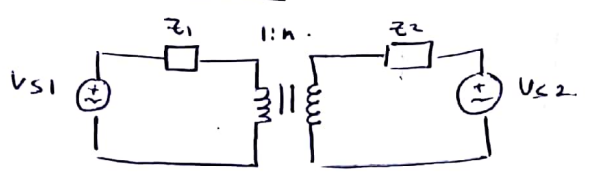
$P = I_2^2 R = (-0.7272)^2 * 20 = 5.3 \text{ W}$

→ Transformers are used for impedance matching



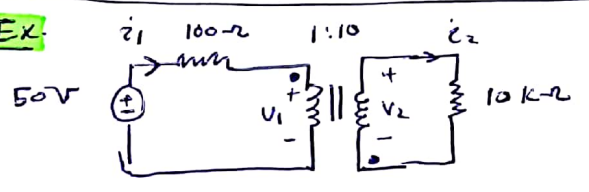
$\frac{1}{n^2} Z_L = 10 \rightarrow \frac{1}{20^2} * 4000 = 10 \rightarrow n = 20$

Equivalent ckt:



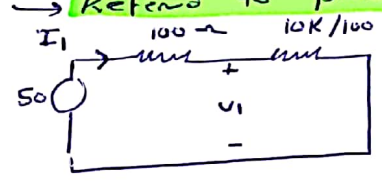
* keep the primary ckt as it for secondary:
 divide each source by n
 & each impedance by n^2

Ex.



← i_1, i_2 استة اقاوم كالتة
 انا بفرسهم كالتة

Refered to primary:



$$I_1 = \frac{50 \angle 0}{200} = 0.25 \angle 0^\circ = 0.25$$

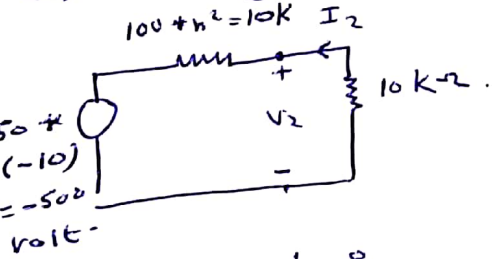
$$V_1 = 25$$

* to find V_2 & I_2 → Ratio, (don't forget dot conv.)

$$\rightarrow \frac{V_2}{V_1} = -10 \rightarrow V_2 = 250 \angle 180^\circ$$

$$\rightarrow \frac{I_1}{I_2} = -10 \rightarrow I_2 = 0.025 \angle 180^\circ$$

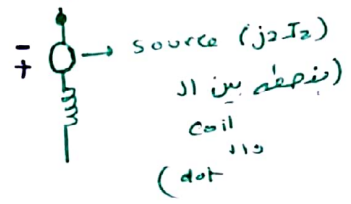
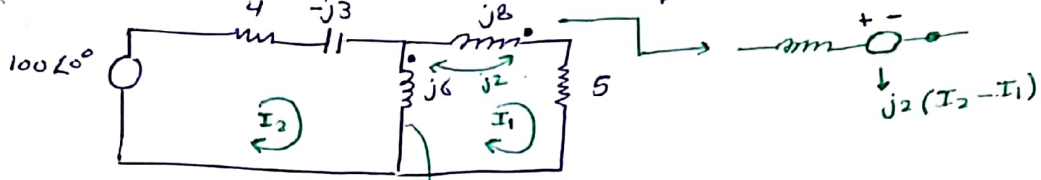
Referring to secondary:



$$\rightarrow \frac{V_2}{V_1} = -10, I_2 = \frac{500 \angle 180}{20 \times 10^3} = 0.025 \angle 180^\circ$$

$$\rightarrow = 500 \angle 180^\circ$$

Ex: Write down the mesh eq. for:-



mesh #1

$$-100 \angle 0 + (4 - j3)I_1 + j6(I_1 - I_2) - j2I_2 = 0$$

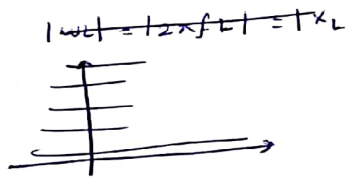
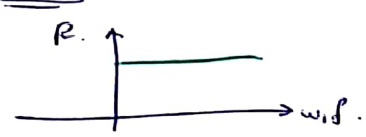
mesh #2

$$5 + j8I_2 + j6(I_2 - I_1) + j2I_2 + j2(I_2 - I_1) = 0$$

* Frequency Response *

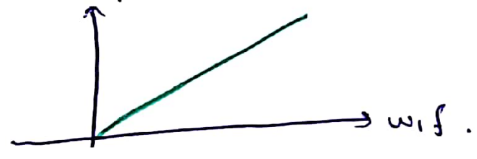
→ what if we change the frequency?

* Resistor :-



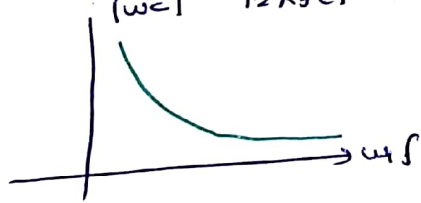
* Inductor

$$|w| = |2\pi f L| = |X_L|$$



* Capacitor

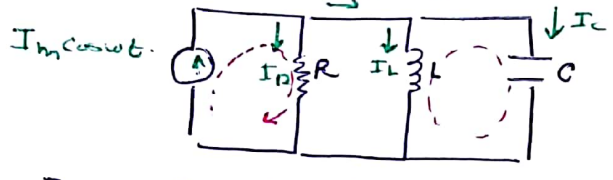
$$|w| = \frac{1}{|2\pi f c|} = |X_C|$$



* Resonance: a ckt with L and C where the voltage & current are in-phase.

Parallel
Series.

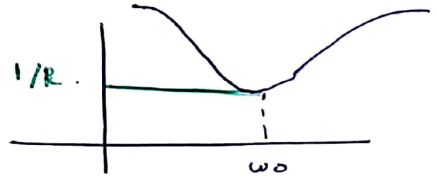
→ Parallel resonance :-
 $I_C + I_L = 0$



$$Z_{eq} = R \parallel j\omega L \parallel 1/j\omega C$$

$$Y_{eq} = \frac{1}{R} + \frac{1}{j\omega L} + j\omega C \quad (\text{in moh})$$

$|Y_{eq}|$



at $\omega = 0 \rightarrow |Y_{eq}| = \infty$

at $\omega = \infty \rightarrow |Y_{eq}| = \infty$

$$Y_{eq} = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

↳ at point (ω_0) they will be equal.

* at ω_0 :-

$$\omega C - \frac{1}{\omega L} = 0 \rightarrow \omega_0 = \frac{1}{\sqrt{LC}} \rightarrow f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$|Y_{eq}(\omega_0)| = \frac{1}{R}$$

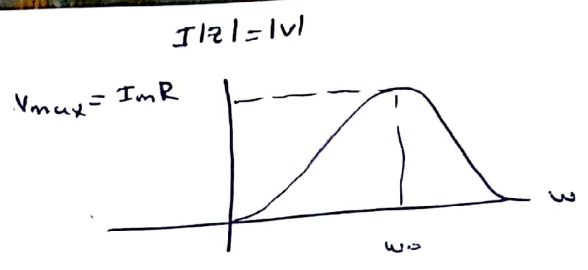
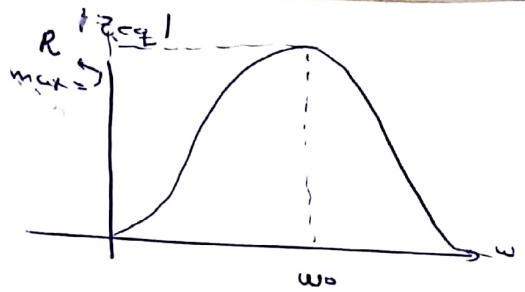
[1] at resonance, Inductor & Capacitor will cancel each other effect.

[2] at Resonance, $I_R = I_m \cos \omega_0 t$.

$$\begin{matrix} I_L \neq 0 \\ I_C \neq 0 \end{matrix} \rightarrow I_L + I_C = 0$$

R is in series with source (L & C)

(L & C are in parallel)



- when we have parallel resonance: the voltage is maximum.
- there is a current in L & C; but they are equal & opposite.
- Y is minimum = $\frac{1}{R}$, Z is maximum = R.

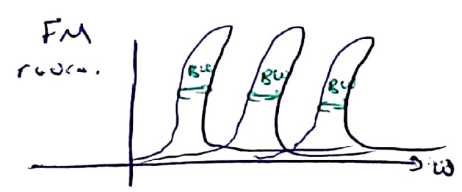
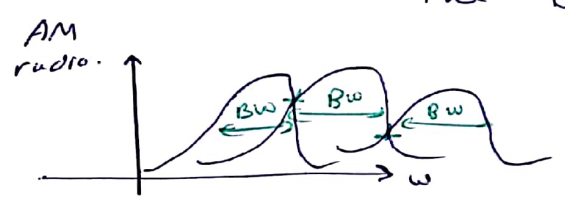
$$\rightarrow |I_{L10}| = \frac{V_{L10}}{|X_L|} = \frac{I_m * R}{|\omega_0 L|} = -|I_{C10}|$$

at resonance

$$|I_{C10}| = \frac{V_{C10}}{|X_C|} = \frac{I_m * R}{|\frac{1}{\omega_0 C}|} = |I_m R \omega_0 C| = -|I_{L10}|$$

* ~~$|I_{L10}| + |I_{C10}|$~~ → $I_{L10} + I_{C10} = 0$

* Quality factor: a parameter that represents the BW of a CRT.



Quality factor = $2\pi \frac{\text{maximum stored energy}}{\text{total energy lost per period}}$

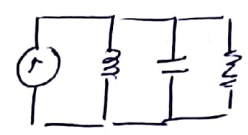
$$Q_0 = \frac{2\pi [w_L(t) + w_C(t)]}{Q_R * T}$$

at resonance

energy stored in C = $\frac{1}{2} C V^2 = \frac{I_m^2 R^2 \cos^2 \omega_0 t}{2}$

energy stored in L = $\frac{1}{2} L i^2 = \frac{1}{2} L \left(\frac{1}{L} \int \dots \right)^2$

$$Q_0 = 2\pi \frac{\text{maximum energy stored}}{\text{total energy lost per period.}}$$



$$= 2\pi \frac{[w_L(t) + w_C(t)]_{\text{max}}}{P_P T}$$

$$i(t) = I_m \cos(\omega t)$$

$$v(t) = R I_m \cos(\omega t)$$

$$w_C(t) = \frac{1}{2} C v^2 = I_m^2 R^2 C \cos^2 \omega t$$

$$w_L(t) = \frac{1}{2} L i^2(t) = \frac{1}{2} L \left(\int v_C(t) \cdot dt \right)^2 = \frac{I_m^2 R^2 C \sin^2(\omega t)}{2}$$

$$w_L(t) + w_C(t) = \frac{I_m^2 R^2 C}{2}$$

$$P_R = \frac{1}{2} I_m^2 R$$

$$P_R T = \frac{T I_m^2 R}{2} = \frac{I_m^2 R}{2 f_0}$$

$$Q_0 = 2\pi \cdot \frac{I_m^2 R^2 C / 2}{I_m^2 R / 2 f_0} = 2\pi R f_0 C = \omega_0 R C \Rightarrow \frac{1}{\sqrt{LC}}$$

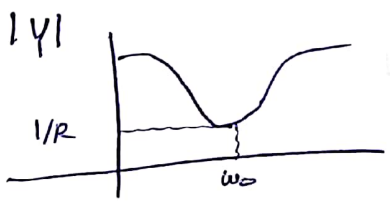
$$Q_0 = \omega_0 R C = R \sqrt{\frac{C}{L}} = \frac{R}{|X_{C0}|} = \frac{R}{|X_{L0}|}$$

Ex. parallel RLC ckt, $L = 2\text{mH}$, $Q_0 = 5$, $C = 10\text{nF}$, find R .
 & the magnitude admittance at $0.9\omega_0$, ω_0 , $1.1\omega_0$.

$$Q_0 = R \sqrt{\frac{L}{C}}$$

$$5 = R \sqrt{\frac{10^{-8}}{2 \times 10^{-3}}} \Rightarrow R = 2.236 \text{ k}\Omega$$

admittance at ω_0 ?



$$Y(\omega = \omega_0) = \frac{1}{R} = \frac{1}{2.236 \times 10^3} = 4.422 \times 10^{-4} \text{ S}$$

$$Y(\omega = 0.9\omega_0) \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}} = 223.6 \text{ krad/sec.}$$

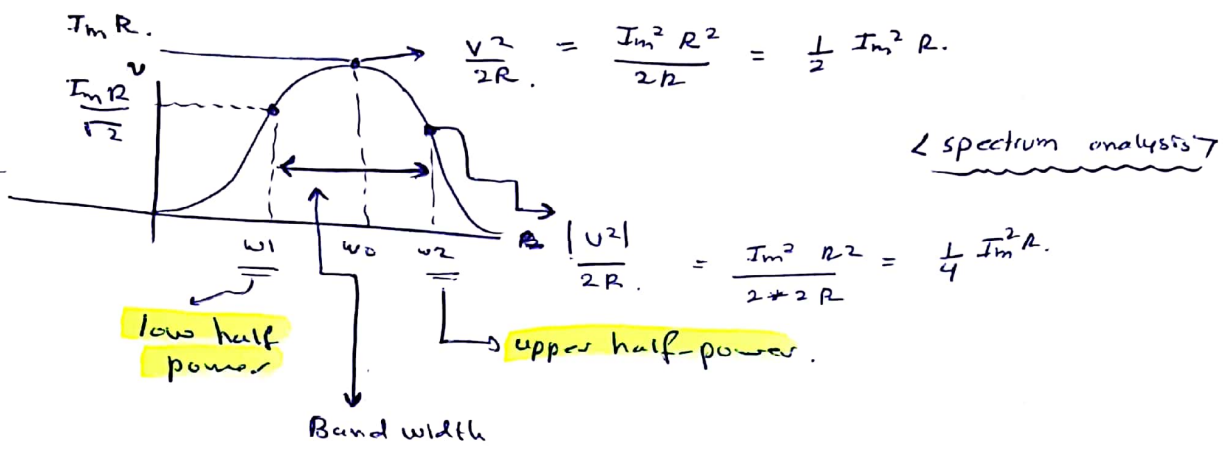
$$Y(0.9 \times 223.6 \text{ k}) = \frac{1}{R} + j\omega C + \frac{1}{j\omega L}$$

$$= \frac{1}{2.236 \times 10^3} + j(0.9 \times 223.6 \text{ k}) \times 10^{-8}$$

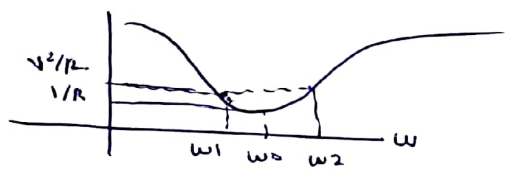
$$+ j \frac{1}{(0.9 \times 10^3 \times 223.6) \times 2\text{m}}$$

$$Y [0.9\omega_0] = 6.504 \times 10^{-4} \text{ s}^{-1}$$

$$Y [1.1\omega_0] = \frac{1}{R} + j(\omega_0 * 1.1)C + \frac{1}{j(1.1 * \omega_0) * L} = 6.182 \times 10^{-4} \text{ s}^{-1}$$



$|Y|$



* How to find the BW:

$$Y = \frac{1}{R} + j(\omega C - \frac{1}{\omega L}) + \frac{R}{R} = \frac{1}{R} + j\frac{1}{R}(\omega CR - \frac{R}{\omega L}) * \frac{\omega_0}{\omega_0}$$

use $Q_0 = \omega_0 RC = \frac{R}{\omega_0 L}$

$$Y = \frac{1}{R} + j \frac{1}{R} \left(\frac{\omega CR \omega_0}{\omega_0} - \frac{\omega_0 R}{\omega_0 \omega L} \right) = \frac{1}{R} + j \frac{1}{R} Q_0 \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right)$$

$$|Y(\omega_1)| = \sqrt{2} |Y(\omega_0)| = \frac{\sqrt{2}}{R}$$

$$|Y(\omega_2)| = \sqrt{2} |Y(\omega_0)| = \frac{\sqrt{2}}{R}$$

$$|Y(\omega_0)| = 1/R$$

follow the same processor for ω_2

$$\omega_2 = \omega_0 \left[\sqrt{1 + \left(\frac{1}{2Q_0}\right)^2} + \frac{1}{2Q_0} \right]$$

$$\frac{1}{R} \sqrt{1 + Q_0^2 \left(\frac{\omega_1}{\omega_0} - \frac{\omega_0}{\omega_1} \right)^2} = \frac{\sqrt{2}}{R}$$

$$Q_0^2 \left(\frac{\omega_1}{\omega_0} - \frac{\omega_0}{\omega_1} \right)^2 = 1 \rightarrow Q_0 \left(\frac{\omega_1}{\omega_0} - \frac{\omega_0}{\omega_1} \right) = 1$$

$$\omega_1 = \omega_0 \left[\sqrt{1 + \left(\frac{1}{2Q_0}\right)^2} - \frac{1}{2Q_0} \right] \quad \text{solve for } \underline{\omega_1}$$

$$B \triangleq \omega_2 - \omega_1$$

$$B = \frac{\omega_0}{Q_0}$$

$$\omega_0 = \sqrt{\omega_1 \omega_2} \rightarrow \text{Geometric mean.}$$

* Approximation for high Q_0 Ckt:-

for $Q_0 > 5$ "high"

$$\omega_1 \approx \omega_0 \left[1 - \frac{1}{2Q_0} \right]$$

$$\omega_2 \approx \omega_0 \left[1 + \frac{1}{2Q_0} \right]$$

$$\omega_0 = \frac{\omega_1 + \omega_2}{2} \rightarrow \text{Arithmetic mean.}$$

Ex. Parallel RLC Ckt with $R = 400 \Omega$.
 $L = 1 \text{ H}$, $C = \frac{1}{64} \mu\text{F}$, find the location of
 two half power frequencies (ω_1/ω_2) & find
 the approximation value of admittance
 at $\omega = 8200 \text{ rad/sec}$.

→ Solution.

$$Q_0 = \omega_0 RC, \quad \omega_0 = \frac{1}{\sqrt{LC}} = 8 \text{ k rad/sec.}$$

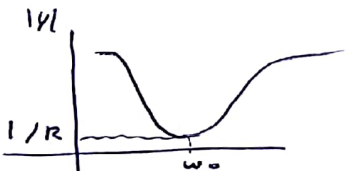
$$= 8 \times 10^3 \times 400 \times 10^{-6} \times \frac{1}{64} \times 10^{-6} = \underline{\underline{5}}$$

so, I can use approximation.

→ exact solution:

$$Y(\omega = 8200) = \frac{1}{R} + j \left[\omega C - \frac{1}{\omega L} \right] = \frac{1}{400 \times 10^3} + j \left[\frac{1}{64} \times 10^{-6} \times 8200 - \frac{1}{8200 \times 1} \right]$$

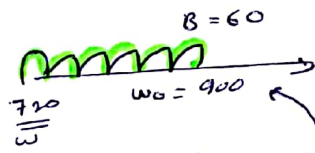
→ approximation solution.



$$Y = \frac{1}{R} [1 + jN]$$

$$N = \frac{\omega - \omega_0}{1/2 B}$$

→ no. of half BW off resonance.
 $B = 60 \text{ rad/sec.}$



$$\frac{720 - 900}{30} = -6$$

$\omega_0 = 1000 \text{ rad/sec.}$

for N @ 1200 rad/sec.

$$N = \frac{1200 - 1000}{300} = \underline{\underline{4}}$$

→ In our example:-

$\omega_0 = 8000 \text{ rad/sec.}$

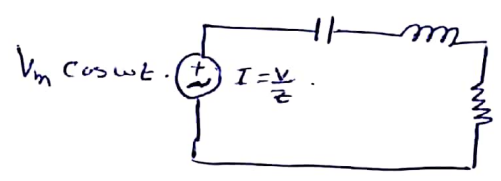
$\omega = 8200 \text{ rad/sec.}$

$B = 1600 \text{ rad/sec.}$

$N = \frac{8200 - 8000}{200} = \frac{1}{4} = 0.25.$

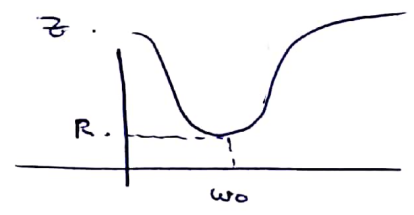
$Y[8200] = \frac{1}{400 \times 10^3} [1 + j0.25].$

* Series resonance:-



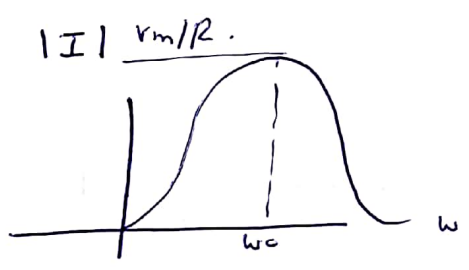
$Z = R + j\omega L + \frac{1}{j\omega C} = R + j[\omega L - \frac{1}{\omega C}]$

→ there is ω_0 when a effect equal & opposite to L effect.

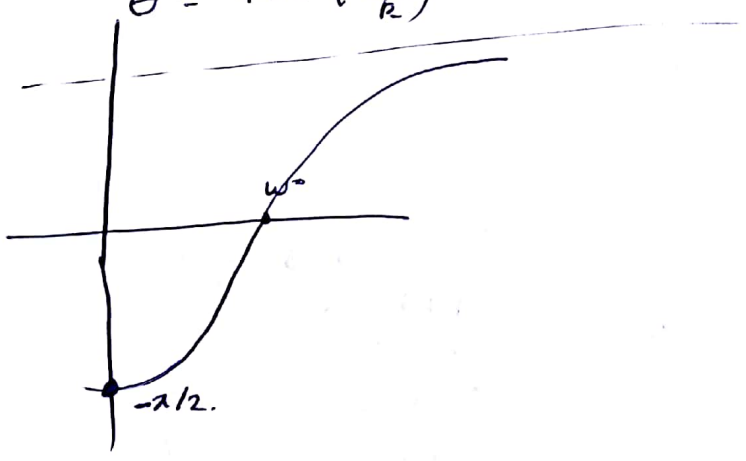


(a) $\omega_0 \frac{1}{\omega_0 C} = \omega_0 L \rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$

$f_0 = \frac{1}{2\pi\sqrt{LC}}$

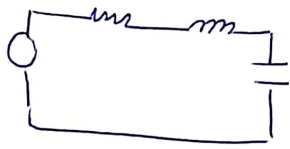


$\theta = \tan^{-1}(\frac{Z}{R})$



E
C
B
Z
I

Series resonance.



$$Z = R + j\omega L + \frac{1}{j\omega C} = R + j \left[\omega L - \frac{1}{\omega C} \right]$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$Q_0 = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC} = \frac{\omega_0}{B}$$

BW: $\omega_1, \omega_2 \rightarrow$ half-power frequency

$$Z(\omega_1) = \sqrt{2} Z(\omega_0)$$

$$Z(\omega_2) = \sqrt{2} Z(\omega_0)$$

$$B = \omega_2 - \omega_1 = \frac{\omega_0}{Q_0}$$

$$\rightarrow \omega_{1,2} = \omega_0 \left[\sqrt{1 + \left(\frac{1}{2Q_0}\right)^2} \mp \frac{1}{2Q_0} \right]$$

\rightarrow for high Q_0 [$Q_0 \geq 5$].

$$\omega_{1,2} = \omega_0 \mp \frac{1}{2} B = \omega_0 \mp \frac{1}{2Q_0}$$

$$\rightarrow Z = \frac{R}{1 + jN}, \quad N = \frac{\omega - \omega_0}{1/2 B}$$

Ex. $V_s = 100 \cos \omega t$ mV, for series RLC ckt,

$$R = 10 \Omega, \quad C = 200 \text{ nF}, \quad L = 2 \text{ mH}.$$

calculate $|I|$ if $\omega = 48 \text{ krad/sec}$.

$$Q_0 = \frac{1}{\omega_0 RC} = \underline{50} \checkmark \quad \text{use approximation.}$$

$$\rightarrow \omega_0 = \frac{1}{\sqrt{LC}} = 50 \text{ krad/sec.}$$

$$\omega_1 = \omega_0 - \frac{1}{2} B$$

$$\omega_2 = \omega_0 + \frac{1}{2} B.$$

$$B = \frac{\omega_0}{Q_0} = 5 \text{ krad/sec}$$

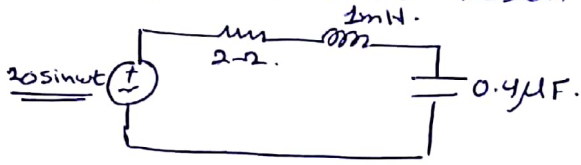
$$Z = R [1 + jN] = R + j\omega L + \frac{1}{j\omega C}$$

$$\rightarrow N = \frac{\omega - \omega_0}{1/2 B} = \frac{48 * 10^3 - 50 * 10^3}{1/2 * 5 * 10^3} = -0.8.$$

$$Z(48 \text{ k rad/sec}) = 10(1 - j0.8) = 12.81 \angle -36.66^\circ$$

$$|I| = \frac{|V|}{|Z|} = \frac{100 \times 10^{-3}}{12.81} = 7.806 \text{ mA}$$

EX. For a series Resonance Res. CKT shown:



1] find $\omega_1, \omega_2, \omega_0$.

$$Q_0 = \frac{1}{\omega_0 RC} = \frac{\sqrt{L}}{RC} = 25 \rightarrow \omega_0 = 50 \text{ k rad/sec}$$

$$\omega_1 = 49 \text{ k rad/sec}$$

$$\omega_2 = 51 \text{ k rad/sec}$$

~~$\omega_0 = 50 \text{ k rad/sec}$~~

2] find β & Q_0 .

$$\beta = \frac{\omega_0}{Q_0} = \frac{50 \times 10^3}{25} = 2 \text{ k rad/sec}$$

3] Find the magnitude of current at: $\omega_0, \omega_1, \omega_2$.

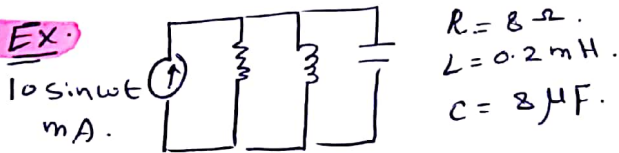
$$|Z(\omega_0)| = R = 2 \Omega \quad (\text{L just } \underline{\text{cancel}} \text{ c } \underline{\text{cancel}})$$

$$|I(\omega_0)| = \frac{20}{10} = 2 \text{ A}$$

$$|Z(\omega_1)| = \sqrt{2} R = 2\sqrt{2} \Omega = |Z(\omega_2)|$$

$$|I_{\pm}(\omega_1)| = \frac{20}{2\sqrt{2}} = 7.07 \text{ A} = |I(\omega_2)|$$

EX.



1] find $\omega_0, \omega_1, \omega_2, \beta, Q_0$.

2] power dissipated at $\omega_0, \omega_1, \omega_2$.

solution-

$$1] \omega_0 = \frac{1}{\sqrt{LC}} = 25 \text{ k rad/sec}$$

$$Q_0 = \omega_0 RC = \frac{R}{\omega_0 L} = \underline{\underline{1600}} \text{ we approximate.}$$

$$B = \frac{\omega_0}{Q_0} = 15.625 \text{ rad/sec.}$$

$$\rightarrow \omega_1 = \omega_0 - \frac{1}{2}B = 24992 \text{ rad/sec.}$$

$$\rightarrow \omega_2 = \omega_0 + \frac{1}{2}B = 25008 \text{ rad/sec.}$$

[2] power only diss. by the resistor.

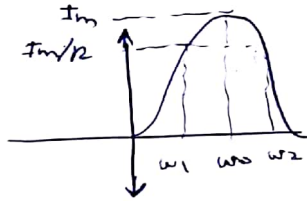
$$P = \frac{1}{2} I_m^2 R$$

$\xrightarrow{\quad}$ I_R (only).

$$P(\omega_0) = \frac{1}{2} I_m^2 R = \frac{1}{2} (10^{-2})^2 \times 8 \times 10^3 = 0.4 \text{ W}$$

$$P(\omega_1) = \frac{1}{2} \left(\frac{I_m}{\sqrt{2}} \right)^2 R = 0.2 \text{ W}$$

$$P(\omega_2) = \frac{1}{2} \left(\frac{I_m}{\sqrt{2}} \right)^2 R = 0.2 \text{ W}$$



→ Replacing the current source by voltage source of $10 \sin(\omega t)$ V

- * the voltage on R won't change
- * the magnitude of I on R won't change.

$$* P(\omega_0) = \frac{1}{2} I_m^2 R.$$

Other resonance forms:-

→ Determine the resonant frequency.

Key solution:-

$$I_m[Y] = 0$$

$$I_m[Z] = Z.$$

$$Y(\omega) = \frac{1}{R_2} + j\omega C + \frac{1}{j\omega L + R_1} \frac{(R_1 - j\omega L)}{(R_1 - j\omega L)}$$

$$= \frac{1}{R_2} + j\omega C + \frac{R_1 - j\omega L}{(R_1)^2 + (\omega L)^2}$$

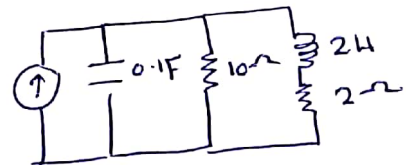
$$I_m[Y] = \frac{j\omega C - \frac{j\omega L}{(R_1)^2 + (\omega L)^2}}$$

$$I_m[Y(\omega_0)] = \frac{j\omega_0 - \frac{j\omega_0 L}{R_1^2 + (\omega_0 L)^2}}$$

$$\omega_0 = \sqrt{\frac{1}{LC} - \left(\frac{R_1}{L}\right)^2}$$

Ex. Find ω_0 .

$$\omega_0 = \sqrt{\frac{1}{LC} - \left(\frac{R_1}{L}\right)^2}$$



continue.

* frequency response: (to find transfer function of frequency).
 <that relates input to output>

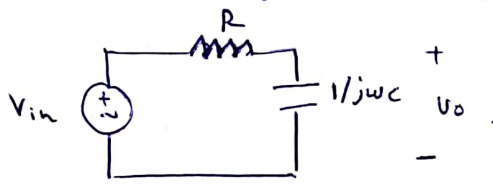
CKT
 ω
 $\frac{V_o}{V_i}$
 $\frac{I_o}{I_i}$

T.F = $H_V(\omega) = \frac{V_o(\omega)}{V_{in}(\omega)}$



T.F = $H_I(\omega) = \frac{I_o(\omega)}{I_{in}(\omega)}$

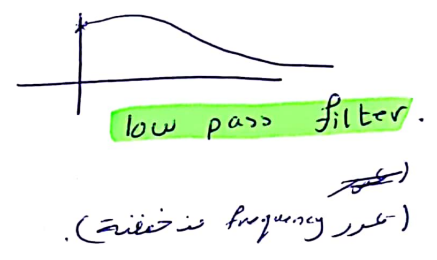
Ex. for the voltage shown, find the voltage transfer function.



$$H_V(\omega) = \frac{V_o(\omega)}{V_{in}(\omega)} = \frac{I * \frac{1}{j\omega C}}{I [R + \frac{1}{j\omega C}]} = \left(\frac{1/j\omega C}{R + 1/j\omega C} \right) * \frac{j\omega C}{j\omega C} = \frac{1}{1 + j\omega RC}$$

$$|H_V(\omega)| = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

$H_V(0) = 1$
 $H_V(\infty) = 0$
 in low frequency
 $\omega \rightarrow \infty$



* low frequency signals & blocks high frequency signals

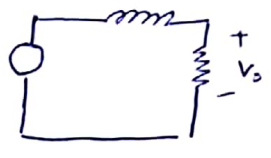
→ Ideal low pass filter , $\omega_c = \frac{1}{RC}$.

* cut off frequency is defined as the frequency

at which gain = $\frac{1}{\sqrt{2}} H_{w(\nu)}_{max}$, $|H_V(\omega)| = \frac{1}{\sqrt{2}}$

$$\frac{1}{\sqrt{1+(RC)^2}} = \frac{1}{\sqrt{2}} \Rightarrow \omega_c = 1/RC$$

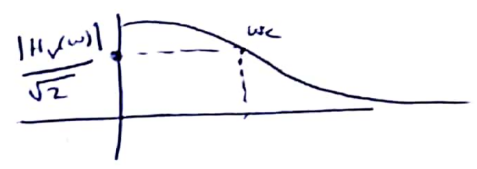
* other circuit :-



$$H_V(\omega) = \frac{R}{R + j\omega L} \quad , \quad |H_V(\omega)| = \frac{R}{\sqrt{R^2 + (\omega L)^2}}$$

$$H(0) = 1$$

$$H(\infty) = 0$$

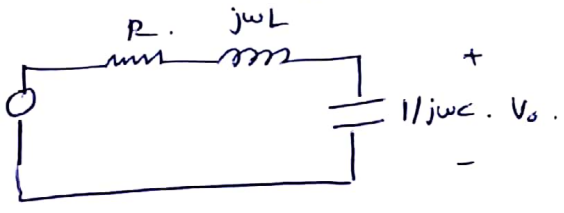


@ ω_c

$$|H_V(\omega_c)| = \frac{1}{\sqrt{2}} = \frac{R}{\sqrt{R^2 + (\omega_c L)^2}}$$

$\omega_c = \frac{R}{L}$ \equiv cut off frequency

Ex. find the voltage. T.F (gain) for shown CKT.

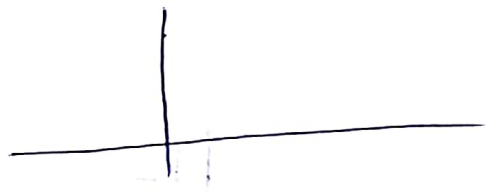


$$H_V(\omega) = \frac{V_o(\omega)}{V_{in}(\omega)} = \frac{1/j\omega C}{R + j\omega L + 1/j\omega C} = \frac{1}{1 + j\omega RC - \omega^2 LC}$$

$$|H_V(\omega)| = \frac{1}{\sqrt{1 + (\omega RC)^2 + (\omega^2 LC)^2}}$$

$$H(0) = 1$$

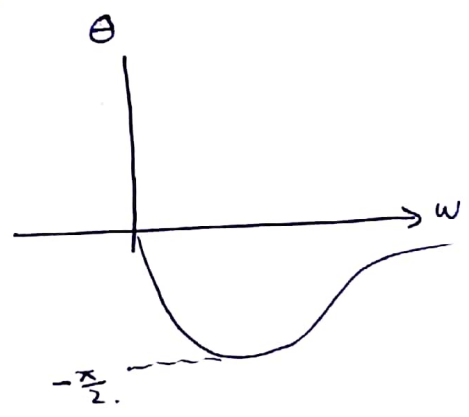
$$H(\infty) = \infty$$



LPF.

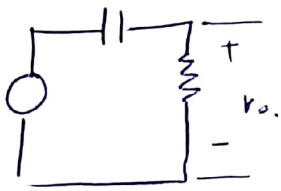
$$\angle H_V(\omega) = -\tan^{-1}\left(\frac{\omega RC}{1 - \omega^2 LC}\right)$$

Phase angle is negative



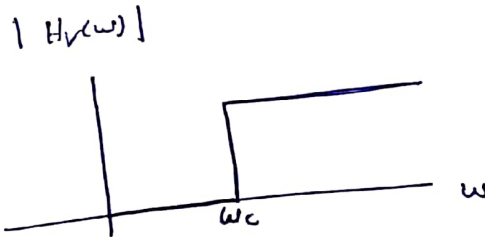
* High pass filter: is one that passes the high frequencies & reject low frequencies.

Ex.



$$H_V(\omega) = \frac{V_o}{V_{in}} = \left(\frac{R}{R + 1/j\omega C} \right) * \frac{j\omega C}{j\omega C} = \frac{Rj\omega C}{1 + j\omega RC}$$

$$|H_V(\omega)| = \frac{\omega RC}{\sqrt{1 + (\omega RC)^2}}$$

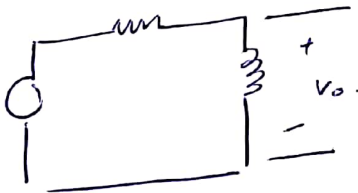


ω_c : is the frequency where:-

$$H_V(\omega_c) = \frac{1}{\sqrt{2}} |H(\omega)|_{max}$$

$$H_V(\omega_c) = \frac{1}{\sqrt{2}} = \frac{\omega_c RC}{\sqrt{1 + (\omega_c RC)^2}} \Rightarrow \boxed{\frac{1}{RC} = \omega_c}$$

Ex.

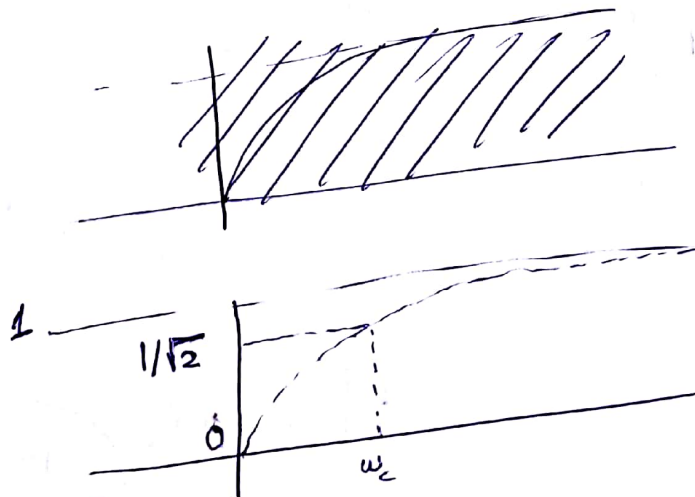


$$H_V(\omega) = \frac{V_o}{V_{in}} = \frac{j\omega L}{R + j\omega L}$$

$$|H_V(\omega)| = \frac{\omega L}{\sqrt{R^2 + (\omega L)^2}}, \quad H(0) = 1, \quad H(\infty) = 0$$

$$\frac{1}{\sqrt{2}} = H_V(\omega) = \frac{\omega_c L}{\sqrt{R^2 + (\omega_c L)^2}}$$

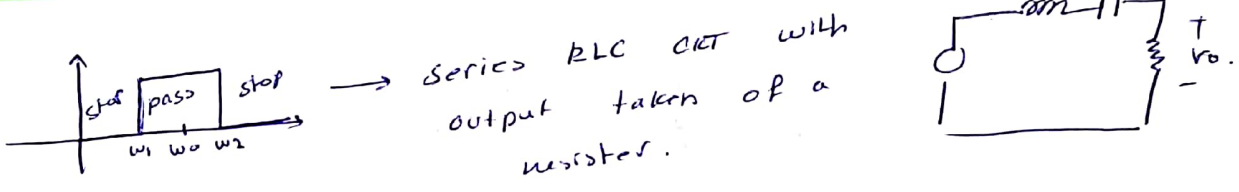
$$\boxed{\omega_c = \frac{R}{L}}$$



Types of filters :-

- [1] LPF. ✓ (low pass filter)
- [2] HPF. ✓ (High pass filter)
- [3] band pass filter.
- [4] band stop filter.

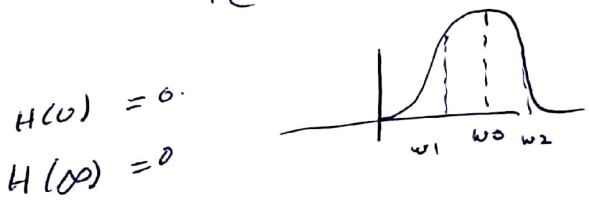
* band pass filter.



$$H_v(\omega) = \left(\frac{R}{R + j\omega L + 1/j\omega C} \right) * \frac{j\omega C}{j\omega C} = \frac{j\omega RC}{j\omega RC - \omega^2 LC + 1}$$

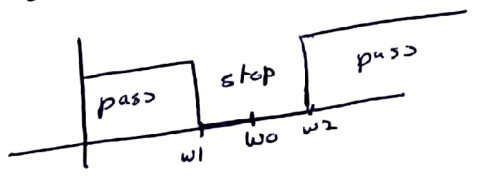
\downarrow Im.
 \downarrow Real.

$$|H_v(\omega)| = \frac{\omega RC}{\sqrt{(1 - \omega^2 LC)^2 + (\omega RC)^2}}$$



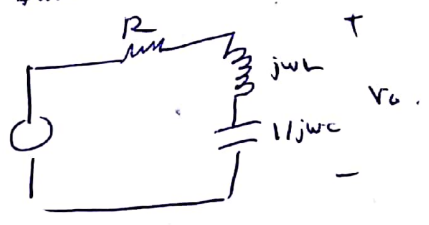
$w_1, w_2, w_0 \rightarrow$ can be calculated based on series resonance expression.

* band stop filter:-

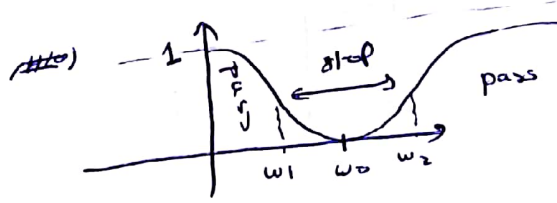


\rightarrow RLC Series ckt with the output taken at L & C.

$$H_v(\omega) = \frac{j\omega L + 1/j\omega C}{R + j\omega L + 1/j\omega C} = \frac{1 - \omega^2 LC}{1 + j\omega RC - \omega^2 LC}$$



$$|H_v(\omega)| = \frac{1 - \omega^2 LC}{\sqrt{(1 - \omega^2 LC)^2 + (\omega RC)^2}}$$

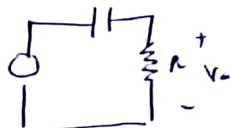


w_0, w_1, w_2 can be found by series resonance expression.

Ex. Design a high pass filter with a corner frequency of 3kHz . المصفاة بتردد زاوية

→ Choose an RC circuit with output on R.

$$H_V(\omega) = \frac{R}{R + 1/j\omega C} = \frac{|V_o(\omega)|}{|V_{in}(\omega)|}$$



$$|H_V(\omega)| = \frac{\omega RC}{\sqrt{1 + (\omega RC)^2}}$$

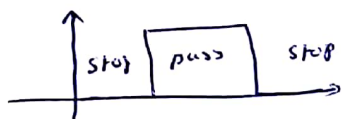
Take:-

$$R = 4.7 \text{ k}\Omega$$

$$C = 11.29 \text{ nF}$$

قاسم الجهد
مقاومة

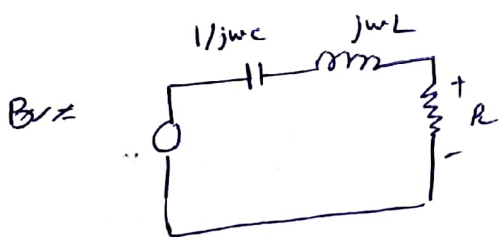
Ex. Design a band pass filter characterized by a $BW = 1\text{MHz}$.
& upper half BW frequency of 1.1MHz . Bandwidth.



$$\omega_2 = 1.1 \times 2\pi \times 10^6 = 6.912 \text{ Mrad/sec.}$$

$$\omega_1 = \omega_2 - B = 0.1 \times 2\pi = 628.3 \text{ krad/sec.}$$

$$\omega_0 = \sqrt{\omega_1 \omega_2} = 2.084 \times 10^6 \text{ rad/sec.}$$



$$\Rightarrow B = \frac{\omega_0}{Q} = \frac{\omega_0}{\omega_0 L/R} = \frac{R}{L} = 6.28 \times 10^6 \quad [1]$$

$$\Rightarrow \omega_0 = \frac{1}{\sqrt{LC}} = 2.084 \times 10^6 \quad [2]$$

Take:-

$$L = 50 \text{ mH.}$$

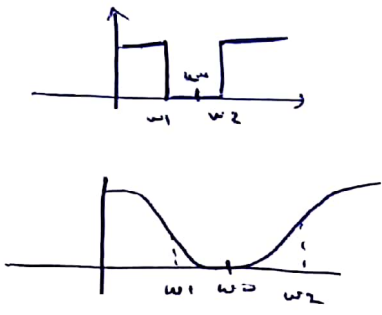
$$C = 4.6 \text{ pF.}$$

$$R = 314 \text{ k}\Omega$$

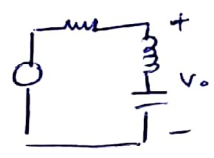
Ex.

∴ a band stop filter is designed to reject a $\frac{200 \text{ Hz}}{\text{center freq.}}$ frequency while passing other frequencies.

Calculate L & C : take : $R = 150 \Omega$, $B = 100 \text{ Hz}$.



→ ckt →



$$B = 2\pi * 100 = 200 \frac{\text{rad/sec}}{\text{rad/sec.}} \text{ rad/sec.}$$

$$B = \frac{\omega_0}{Q_0} = \frac{\omega_0}{\omega_0 L/R} = \frac{R}{L}$$

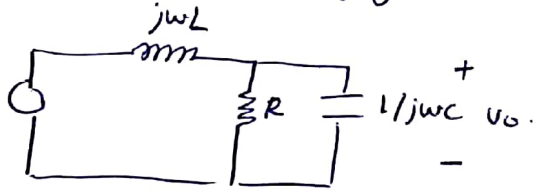
$$L = \frac{R}{B} = \frac{150}{200} = 0.2327 \text{ Hz}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 400 \text{ rad/sec}$$

$$\Rightarrow C = 2.65 \mu\text{F}$$

Ex.

Determine what type of filter is shown in figer.



∴ input & output filter ∴ $H_v(\omega)$ ∴

$$H_v(\omega) = \left(\frac{R \parallel 1/j\omega C}{j\omega L + R \parallel 1/j\omega C} \right) = \frac{\frac{R}{1 + j\omega RC}}{j\omega L + \frac{R}{1 + j\omega RC}} = \frac{R}{R + j\omega L + \frac{R}{1 + j\omega RC}}$$

$$H_v(\omega) = \frac{\frac{R}{1 + j\omega RC}}{\frac{R}{1 + j\omega RC} + j\omega L} = \frac{R}{R + j\omega L - \omega^2 LRC}$$

Real parts.

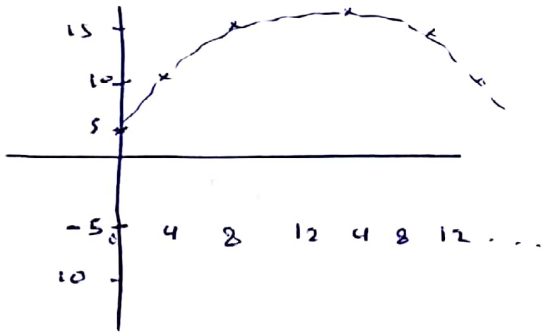
$$|H_v(\omega)| = \frac{R}{\sqrt{(R - \omega^2 LC)^2 + (\omega L)^2}}$$

$$H(0) = L$$

$$H(\infty) = 0$$



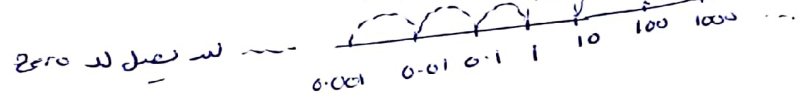
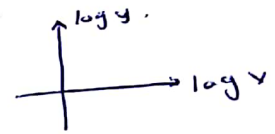
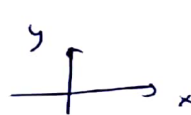
The Decibel scale:-



$$a: [1, 2] \rightarrow a: [0, \log 2]$$

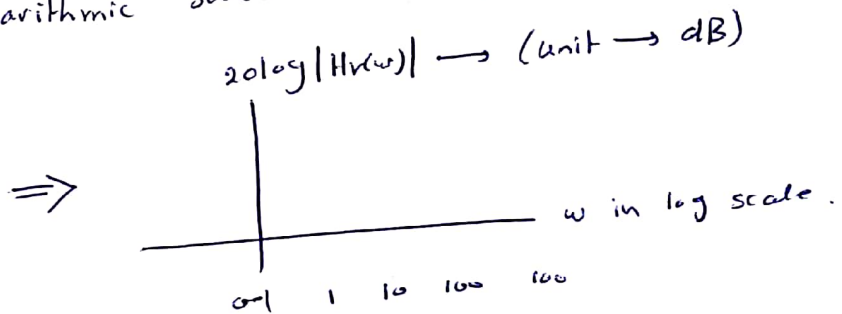
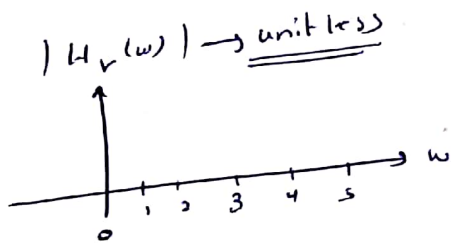
$$b: [10^{-5}, 10^{-7}] \rightarrow b: [-5, -7]$$

$$c: [10^3, 10^5] \rightarrow c: [3, 5]$$

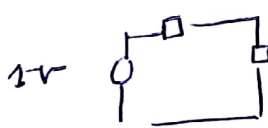


The Bode plots: (draw by hand accurate).

→ Bode plots is away to plot magnitude in decibels & phase vs. logarithmic scale.



previously.



$$|H_v(\omega)| = 1 \Rightarrow 20 \log |H_v(\omega)| = 0$$

$$1 \Rightarrow 0 \text{ dB.}$$

$$|H_v(\omega)| = 10 \Rightarrow 20 \log 10 = 20$$

$$10 \text{ unitless} = 20 \text{ dB.}$$

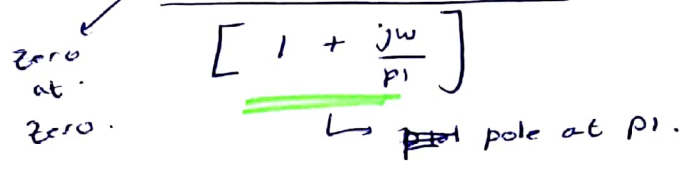
$$|H_v(\omega)| = 0.1 \Rightarrow 20 \log 0.1 = -20$$

$$0.1 \Rightarrow -20 \text{ dB.}$$

* How to construct Bode plots:

→ write your gain function in standard form:-

$$\frac{N(\omega)}{D(\omega)} = H(\omega) = K (j\omega)^{z_1} \left[1 + \frac{j\omega}{z_1} \right] \rightarrow \text{zero at } z_1$$

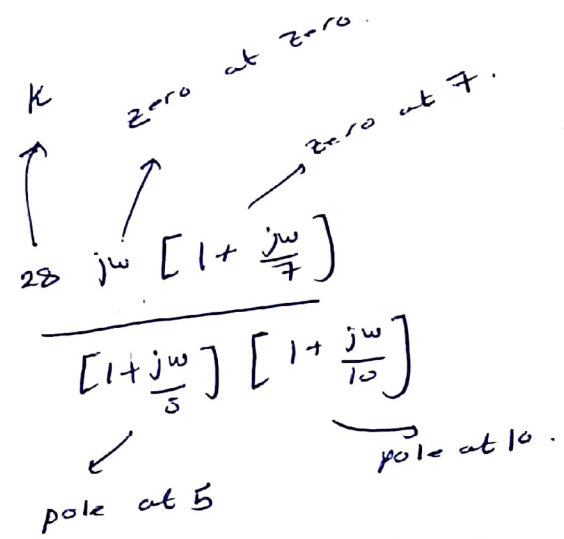


zero. ← $\frac{1}{s}$
pole. ← s

$$H(\omega) = \frac{200 j\omega [j\omega + 7]}{[j\omega + 5] [j\omega + 10]}$$

① write in standard form.

$$H(\omega) = \frac{200 j\omega 7 \left[\frac{j\omega}{7} + 1 \right]}{5 \left[1 + \frac{j\omega}{5} \right] 10 \left[1 + \frac{j\omega}{10} \right]}$$



② take $20 \log |H_v(\omega)|$.

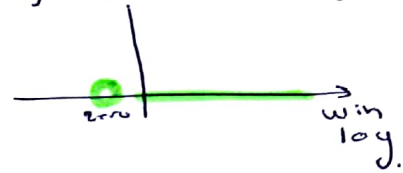
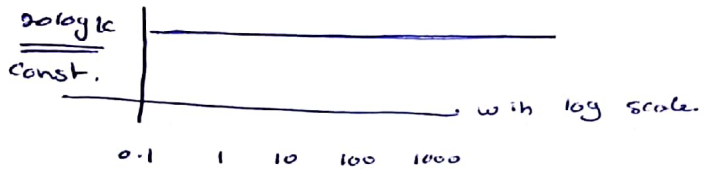
$$20 \log |H_v(\omega)| = 20 \log 28 + 20 \log \left| 1 + \frac{j\omega}{7} \right| + 20 \log |j\omega| - 20 \log \left| 1 + \frac{j\omega}{5} \right| - 20 \log \left| 1 + \frac{j\omega}{10} \right|$$

بجانب ماس تعین کرتے ہیں

$$|H_v(\omega)| = \frac{K (j\omega)^{z_1} \left(1 + \frac{j\omega}{z_1} \right)}{\left(1 + \frac{j\omega}{p_1} \right)}$$

$$|H_v(\omega)| = \underbrace{20 \log |K|}_{(1)} + \underbrace{20 \log |j\omega|}_{(2)} + \underbrace{20 \log \left| 1 + \frac{j\omega}{z_1} \right|}_{(3)} - \underbrace{20 \log \left| 1 + \frac{j\omega}{p_1} \right|}_{(4)}$$

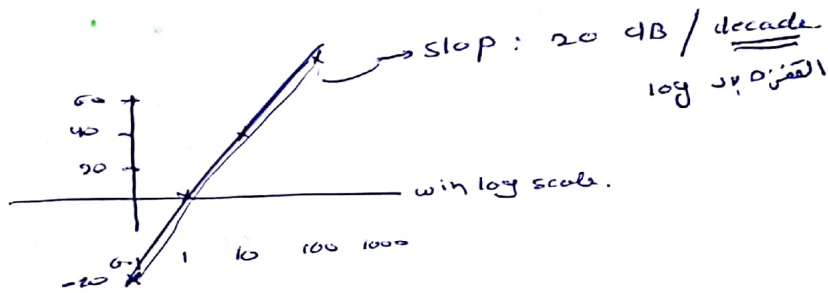
① $20 \log |k| = 20 \log k. \rightsquigarrow H_v(\omega) = k \ (\theta = 0) \quad \theta = \tan^{-1} \left(\frac{Im}{Real} \right)$



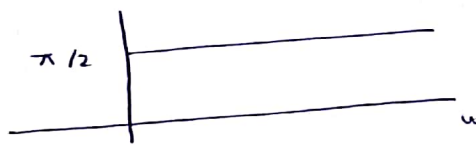
② $20 \log |j\omega|$

$H_v(\omega) = j\omega.$

$|H_v(\omega)| = \omega.$



$\theta = \tan^{-1} \left(\frac{Im}{Re} \right) = \pi/2.$



③ $H_v(\omega) = \frac{1}{j\omega}$

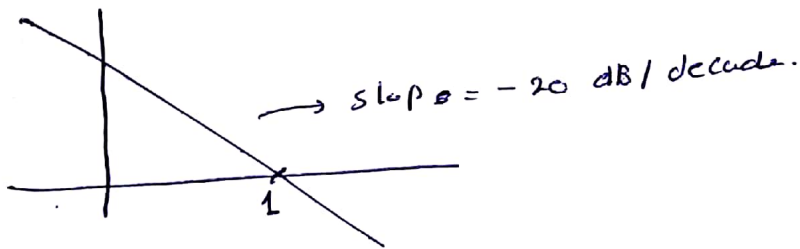
$|H_v(\omega)| = \left| \frac{1}{j\omega} \right|$

$20 \log |H_v(\omega)| = 20 \log \left| \frac{1}{j\omega} \right| = -20 \log |\omega|$

نفس الركة السابقة

منزب (-1)

$20 \log |j\omega|$



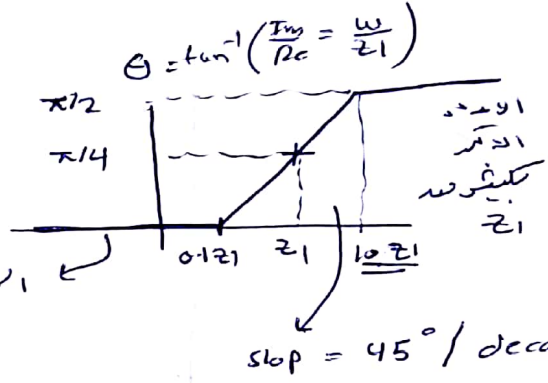
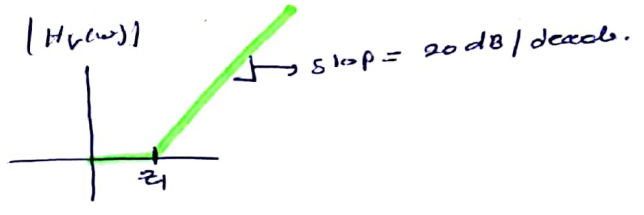
G



$H_V(\omega) = 1 + \frac{j\omega}{z_1}$

$20 \log |H_V(\omega)| = 20 \log |1 + \frac{j\omega}{z_1}|$

$|H_V(\omega)| = \sqrt{1 + (\frac{\omega}{z_1})^2}$ if $\omega \ll z_1$
 $|H_V(\omega)| = 1$
 $\log 1 = 0$



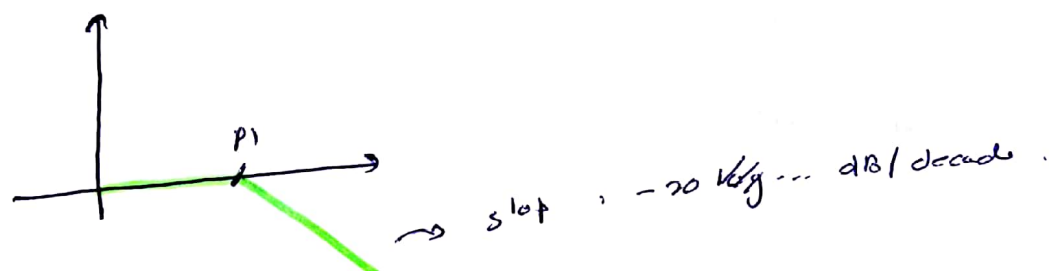
الاعداد الاقل بكثير من z1

(Simple pole).

$H_V(\omega) = \frac{1}{1 + \frac{j\omega}{p_1}} = (1 + \frac{j\omega}{p_1})^{-1}$

$20 \log |H_V(\omega)| = 20 \log |(1 + \frac{j\omega}{p_1})^{-1}| = -20 \log |1 + \frac{j\omega}{p_1}|$

نقطة انعطاف سالبة
 (-1) *



$\theta = \tan^{-1}(\frac{\omega}{p_1})$

