POWER UNIT

Circuits I Notebook
Dr. Ra'ed Alzo3bie

By : Marah Alomarie

بأفكارنا_نُبدع

Full Material

16/9/

***units**

International system of units (SI)

=> 7 basic units:

meter	m
kilogram	kg
Second	s
Ampere	A
kelvin	K
mole	mol
Candela	cd

***other units**

=> unit of work or energy is Joule (J) = kgm^2/s^2

=> unit of power is watt (w) = J/s

***S I prefixes**

10^{-3}	milli (m)
10^{-6}	micro (μ)
10^{-9}	nano (n)
10^{-12}	pico (p)

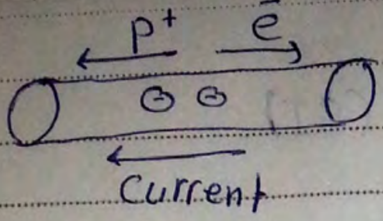
*** Charge** → negative (electron)

-1.602×10^{-19} C → Coulomb

→ postive (proton) $+1.602 \times 10^{-19}$ C

16/9/

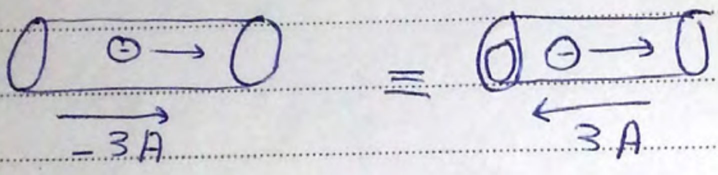
Subject:........................



=> The direction of the current is opposite to direction of negative charges..

* Current

إذا عكسنا الاتجاه



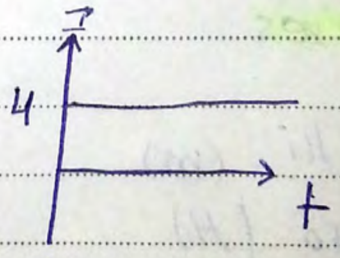
* types of current

① DC (Direct current) :

A current that is Constat with time

e.g => $I = 4$ A

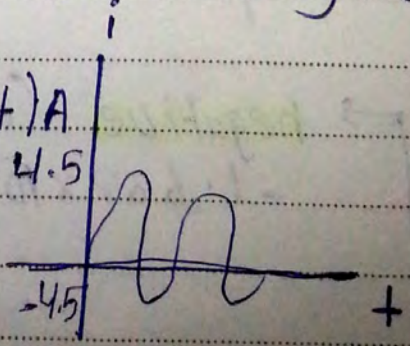
$I = -2.3$ A



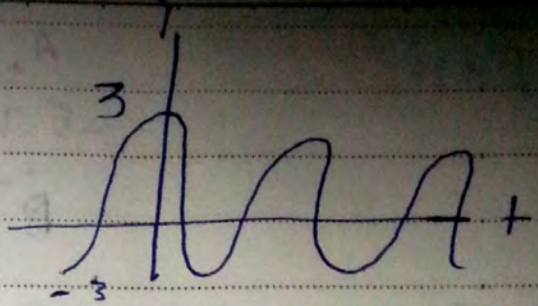
② AC (Alternating Current) :

a current that changes sinusoidally with time

e.g => $i(t) = -4.5 \sin(200\pi t)$ A



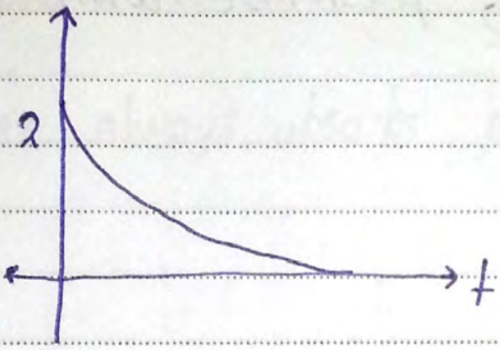
eg=> $i(t) = 3\cos(100\pi t)$ A



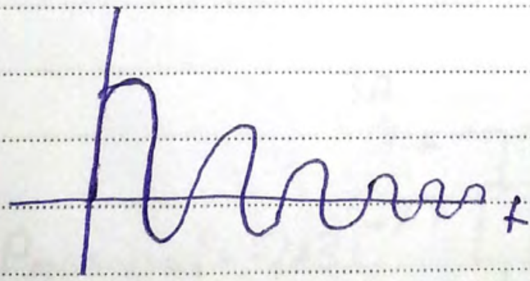
③ othe types :

=> exponantial current

$j(t) = 2e^{-3t}$ A



=> damped sinusoidal current

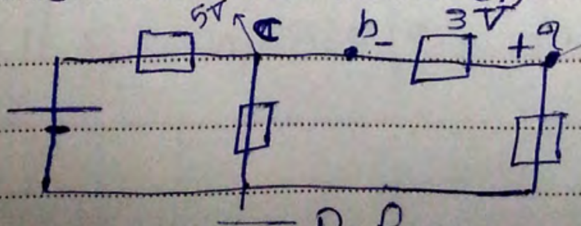


* يكون بيها مفتاح

* Voltage :

=> unit => Volt (V)

=> it is measured between any two points in the circuit



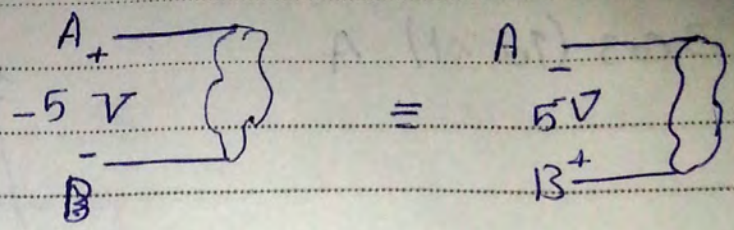
Referance Ground

Terminal a is 3V positive with respect to terminal b

Subject:......................

16/9/

Ex :



* Terminal A is 5V negative with respect to Terminal B

* Terminal B is 5V positive with respect to Terminal A

18/9/

* power :

unit : watt (w)

$p(t) = i(t)v(t)$

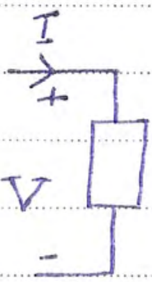
$P = I*V$

=> power could be Generated (Supplied) or absorbed (consumed)

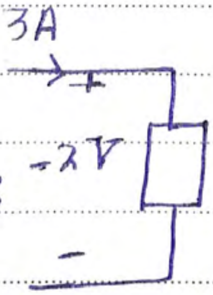
=> Resister always absorbs power

*

$P_{absorbed} = I \cdot V$



Ex:



Find $P_{absobed} = -2*3$
$= -6$

$P_{generated} = +6$ W

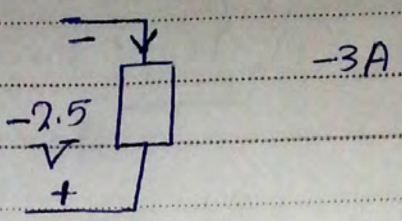
* هنا يستحيل ان تكون
مقاومة لأن Pgenerated
+ موجبة

=> it is inpossible to be resister.

Subject:...................... 18/9/

H:

Ex :

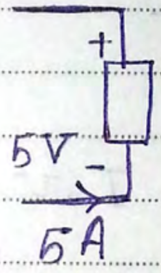


$P_{absorbed} = -2.5 * 3$

$= -7.5$ w

$P_{generated} = 7.5$ w

Ex :



$P_{generated} = 25$ w

$P_{absorbed} = 5 * -5 = -25$ w

$0 = \sum P_{absorbed}$ *

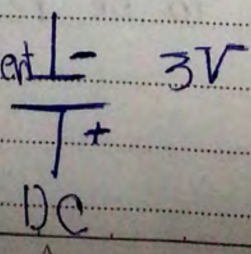
$\sum p_{generated} = 0$

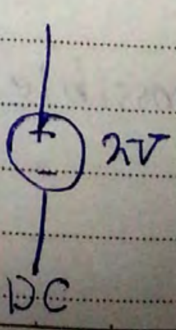
$\sum p_{absorbed} = \sum p_{generated}$

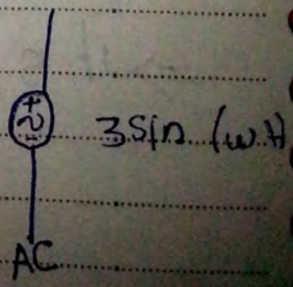
*Soures :

① voltage Soures

└→ independent

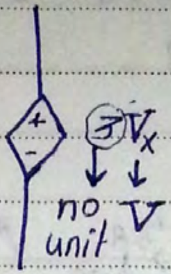


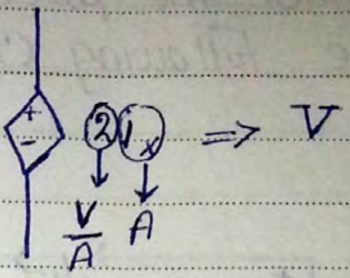




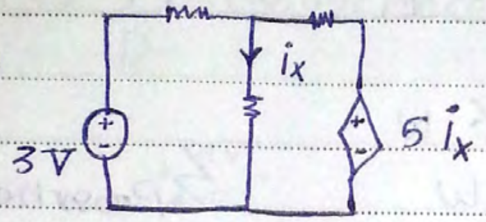
18/9/

* dependent



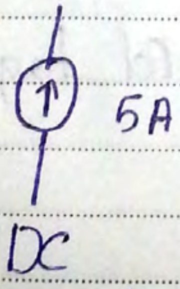


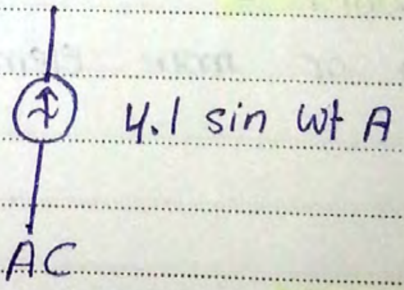
Ex 3:



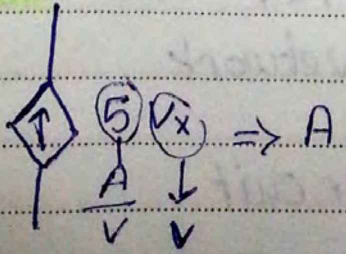
② Current Sources

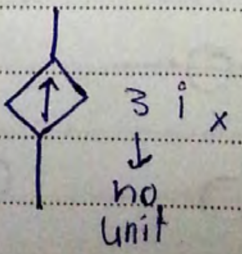
→ independent





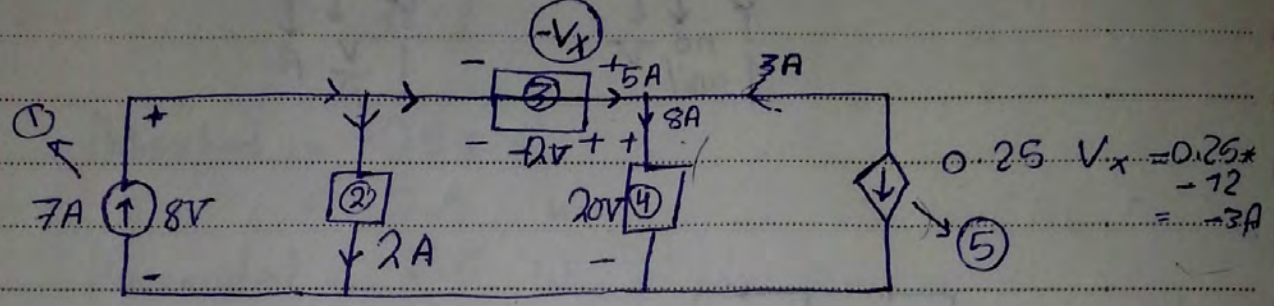
→ dependent





18 / 9 /

Ex : Find the pwer absorbed by each element in the Following Circuit :



Pabsorbed (1) = 8* -7 = -56 W

Pabsobed (2) = 8 * 2 = 16 W

Pabsorbed(3) = 12*-5 = -60 W

Σ Pabsorbed = 0

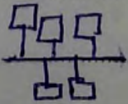
Pabsorbed (4) = 20* 8 = 160 W

Pabsorbed (5) = 20 *-3 = -60 W

* Network :-

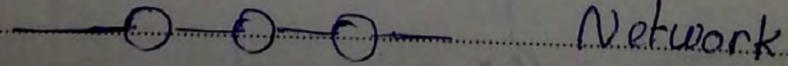
Two or more elements concteted togthor

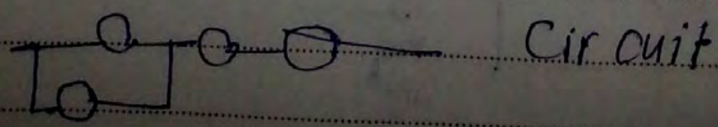
P 3



* Circuit

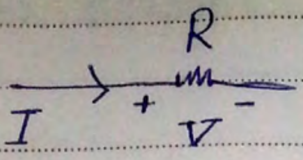
a Network contains at lest one Cloesed path





18/9/

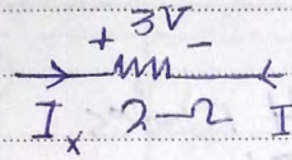
*Ohm's law



=> $I = \frac{V}{R}$ => R: resistance (Ω)

=> $G = \frac{1}{R}$ conductance (℧ mho)
(S siemens)

Ex :



find I_x :

$I_x = \frac{3}{2} = 1.5$ A

$I_y = \frac{-3}{2} = -1.5$A

$$P_{absorbed(R)} = IV$$
$$= I.I.R$$
$$= \frac{V^2}{R}$$

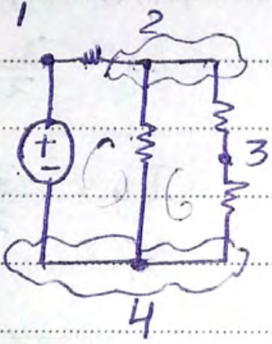
$$P_{absorbed} = IV$$
$$= I^2R$$
$$= \frac{V^2}{R}$$

↓

$\boxed{+ve}$

Ch 3 : KVL, KCL, Series and parallel connection

=> **Node** : a point at which two or more elements are connected



=> 4 Nodes
=> 3 loops
=> 5 branches.

=> **Path** : a set of elements and nodes passed. (one element should passed one time) From node to node

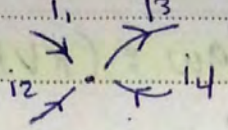
=> **loop** : a closed path.

=> **Branch** : element

* Kirchhoff current law (KCL)

=> charge conservation

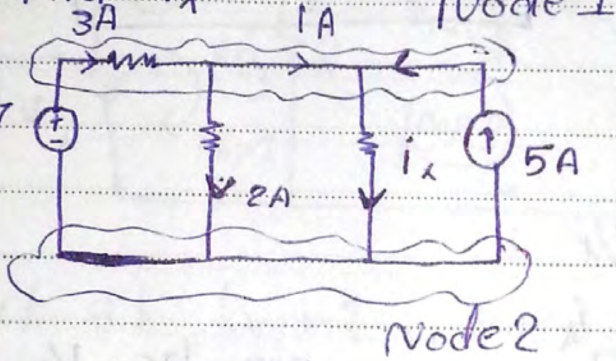
=>



$$\sum i_{in} = \sum i_{out}$$

$$i_1 + i_2 + i_4 = i_3$$

Ex Find i_x.



$i_x =$

KCL at Node 1:

$3 + 5 = 2 + i_x$

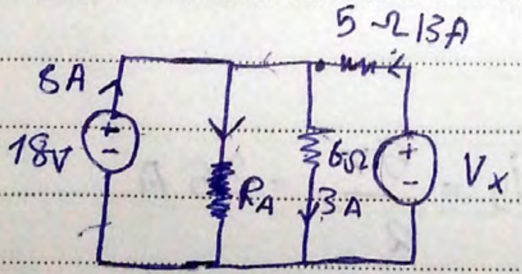
$i_x = 6$ A

KCl at Node 2

$2 + i_x = 5 + 3$

$i_x = 6$ A

Ex



Find R_A = $8 + 13 = 3 + j_A$

$j_A = 18$

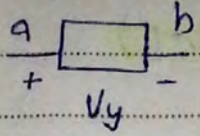
$R = \frac{18}{18} = 1\Omega$

$U_{5\Omega} = 13 * 5 = 65$ V

$65 = V_x - 18$

$\Rightarrow V_x = 83$ V

Subject: 21/9/2014

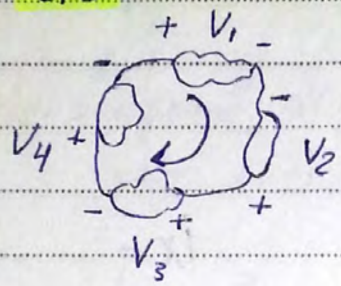
* 

$\Rightarrow V_y = V_a - V_b$

* Kirchhoff Voltege law : (KVL)

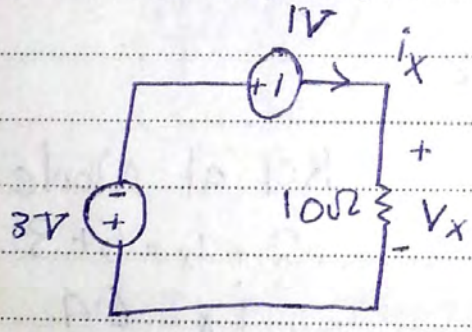
⇒ energy Conservation

⇒ The sum of Voltages over aclosed path is zero.



$$\boxed{V_1 - V_2 + V_3 + V_4 = 0}$$

Ex: Find i_x and V_x

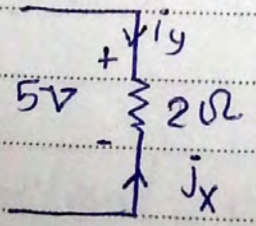


$3V + 1V + V_x =$

$$\boxed{V_x = -4V}$$

$$i_x = \frac{-4}{10} = \boxed{-0.4A}$$

Ex



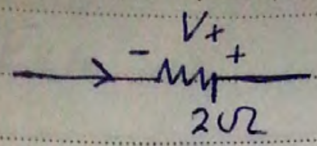
$$i_y = \frac{5V}{2} = 2.5A$$

$$i_x = -2.5A$$

⇐ قانون أوم يحسب التيار اللي داخل من الـPositive

Subject:.................... 21/9/2014

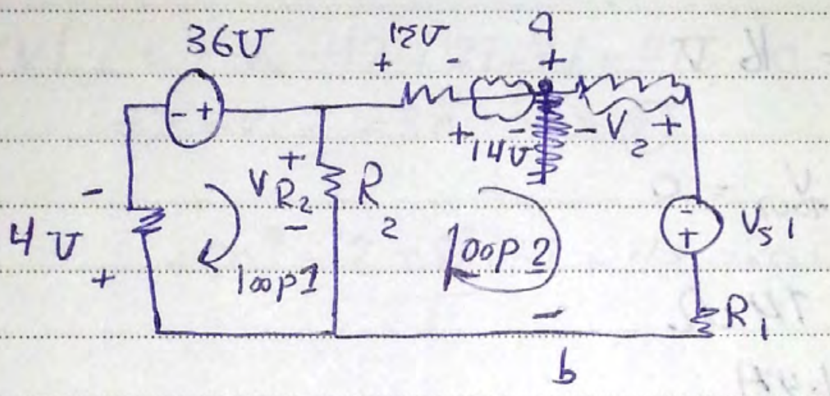
*



$i = 3A$

$V_x = -6V$

Ex Find V_{R_2} and V_x



kvl at loop 1 : $4 - 36 + V_{R_2} = 0$

$V_{R_2} = 32 V$

kvl at loop2 : $-32 + 12 + 14 + V_x = 0$

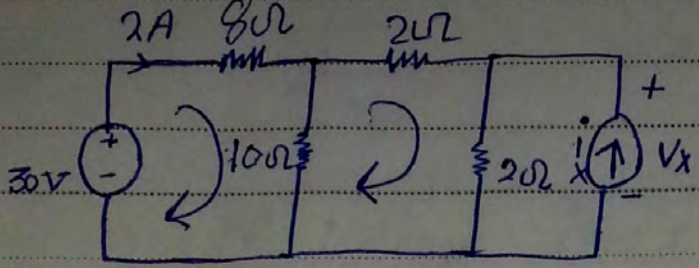
$V_x = 6 V$

kvl at loop 3 : $4 - 36 + 12 + 14 + V_x = 0$

$V_x = 6 V$

23/9 /2014

Ex :



Find V_x and i_x :   * 4 Nodes

=> $V_{8\Omega} = 2*8 = 16$ V

=> $-30 + 16 + V_{10\Omega} = 0$

$V_{10\Omega} = 14 \Omega$

=> $I_{10\Omega} = \frac{14}{10} = 1.4A$

=> $I_{2\Omega} = 2 - 1.4 = 0.6 A$

=> $V_{2\Omega} = 0.6 * 2 = 1.2V$

=> $-14 + 1.2 + V_x = 0$

$V_x = 12.8$ V

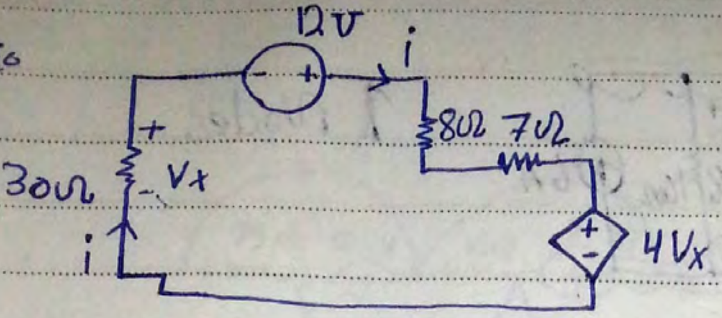
=> $i_{2\Omega} = \frac{V_x}{2} = \frac{12.8}{2} = 6.4 A$

$i_x + 0.6 = 6.4$

$i_x = 5.8A$

Subject:...................... 23 / 9 / 2014

Ex Eo



Find V_x

KVL : $-V_x - 12 + 8i + 7i + 4V_x = 0$ => ①

=> $+V_x = -30i$ => ②

$+30i - 12 + 8i + 7i + 4 * -30i =$

$i = -0.16\,A$

$V_x = -30 * 0.16 = 4.8\,V$

* Calculate the absorbed power by each element

$P_{absorbed\ 30\Omega} = I^2 R = (0.16)^2 * 30\Omega = 0.768\ W$

$P_{absorbed\ 12V} = 12 * +0.16 = 1.92\ W$

$P_{absorbed\ 8\Omega} = I^2 R = (0.16)^2 * 8 = 0.2048\ W$

$P_{absorbed\ 7\Omega} = I^2 R = 0.1792\ W$

$P_{absorbed\ (4V_x)} = 4V_x * i$

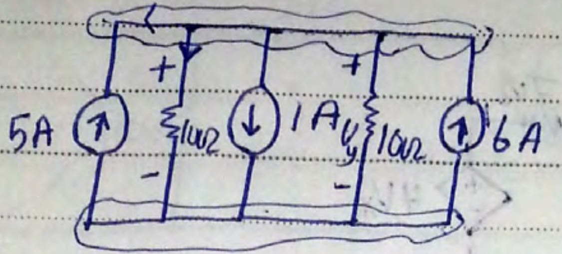
$= 4 * (4.8) * -0.16) = \underline{-3.072}\ W$ ↗ generated

$\sum P_{absorbed} =$ Zero

Subject:.........................

23/9/2014

Ex:



2 Nodes

Find V_y

KCl Node I => $5+6 = 1 + \frac{V_y}{10} + \frac{V_y}{10}$

$$10 = \frac{2V_y}{10}$$

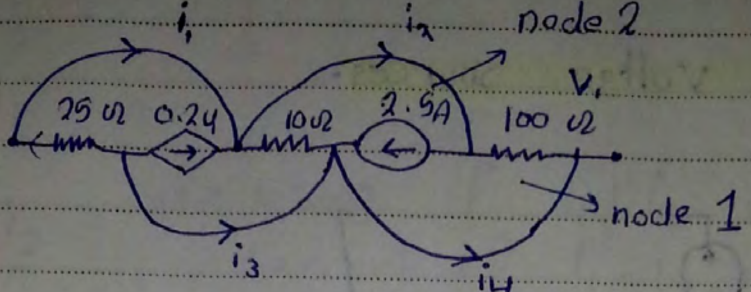
$$\frac{100}{2} = V_y$$

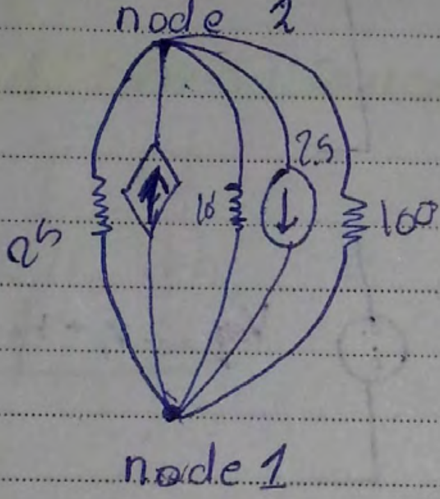
$$50 = V_y$$

$P_{generated\ 5A} = 250$

$P_{absorbed} = I \cdot V = 50 * -5 = -250$

Ex





$i_1 = -2A$

$i_2 = 3A$

$i_3 = -8A$

$i_4 = -0.5A$

KCl => at Node 2

$$0.2\,V_1 = \frac{V_1}{25} + \frac{V_1}{10} + 2.5 + \frac{V_1}{100}$$

$$\Rightarrow V_1 = 50v$$

$$i_1 = \frac{-V_1}{25} = \frac{-50}{25} = -2A$$

$$i_3 + 0.2V_1 + i_1 = 0$$

$$i_4 = \frac{-V_1}{100} = -0.5A$$

$$i_3 = -8A$$

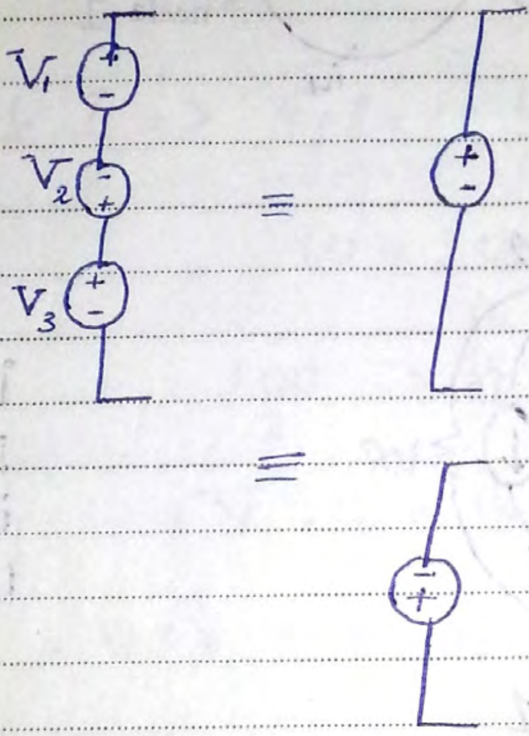
$$i_2 + i_4 = 2.5$$

$$i_4 = 3A$$

Subject:.......................... 25/9/2014

* series and parallel connections:

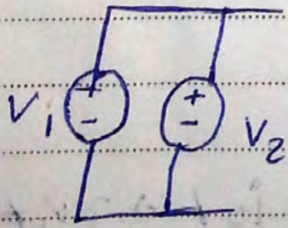
=> series voltge sources.



$V_{eq} = V_1 - V_2 + V_3$

$V_{eq} = -V_1 + V_2 - V_3$

=> parallel voltge sources.

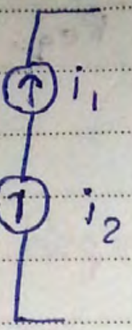


=> wrong connection if $V_1 \neq V_2$
but it is correct if $V_1 = V_2$

KVL: $-V_1 + V_2 = 0$
$V_1 = V_2$

Subject: 25/9/2014

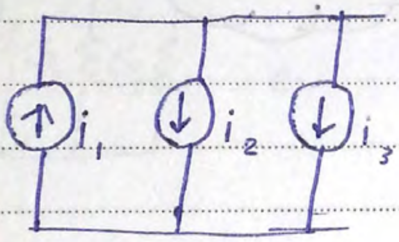
=> Series ~~circuit~~ current Source :



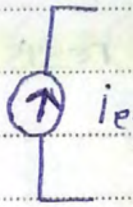
=> wrong if $i_1 \neq i_2$
Correct if $i_1 = i_2$

KCL => $i_2 = i_1$

=> parallel current sources :

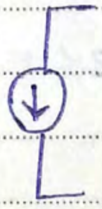


≡



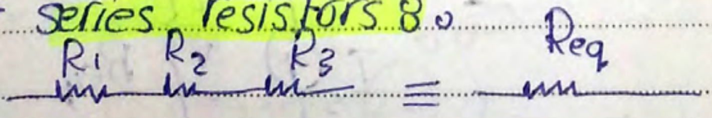
$i_{eq} = i_1 - i_2 - i_3$

|||

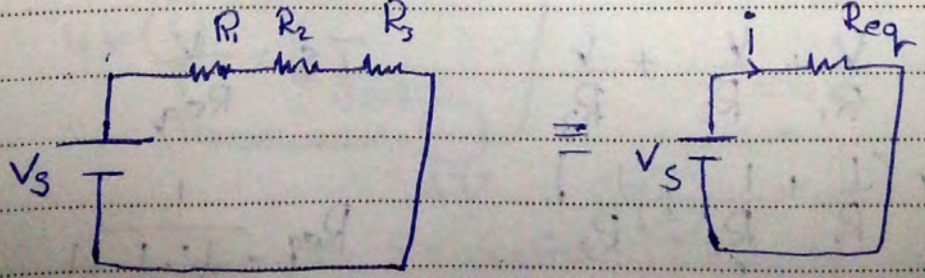


$i_{eq} = -i_1 + i_2 + i_3$

* series resistors :



$R_{eq} = R_1 + R_2 + R_3$



25/9/2014

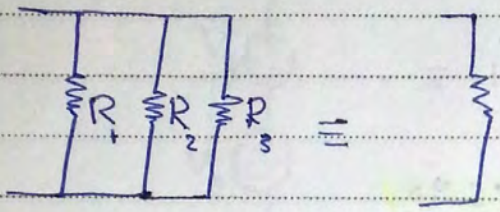
KVL : $-V_s + iR_1 + iR_2 + iR_3 = 0$

$$i = \frac{V_s}{R_1+R_2+R_3}$$

$-V_s + iR_{eq} = 0$

$$i = \frac{V_s}{R_{eq}}$$

* parrallel resistor

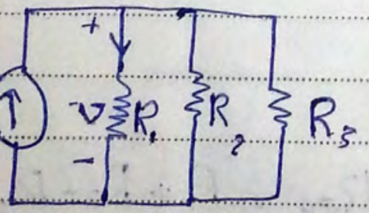


$$= \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}$$

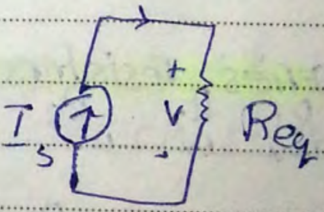
=> Special case of two resistors

$$R_{eq} = \frac{R_1 * R_2}{R_1 + R_2}$$

=> if $R_1 = R_2$ => $\boxed{R_{eq} = \frac{R_1}{2}}$



≡



KCl : $I_s = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$

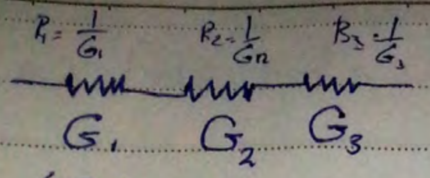
$Is = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$

$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$ =>

$Is = \frac{V}{R_{eq}}$

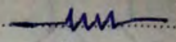
$R_{eq} = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}$

25/9/20



(ʊ)

(S)



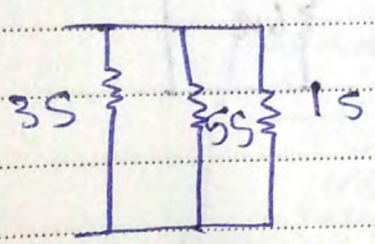
$$= \quad G_{eq} = \frac{1}{\frac{1}{G_1}+\frac{1}{G_2}+\frac{1}{G_3}}$$

$$R_{eq} = \frac{1}{G_1} + \frac{1}{G_2} + \frac{1}{G_3}$$

$$G_{eq} = \frac{1}{\frac{1}{G_1}+\frac{1}{G_2}+\frac{1}{G_3}}$$

$$G_{eq} = G_1 + G_2 + G_3$$

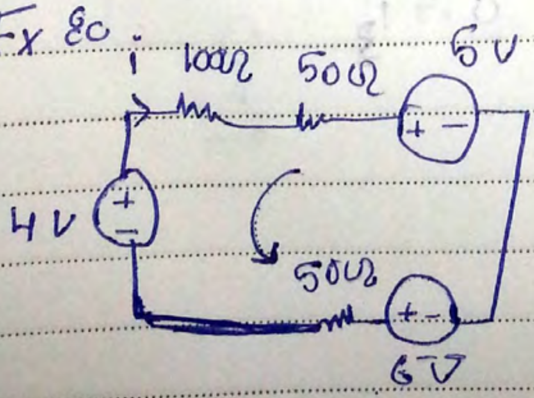
Ex Eo



=> Find Req =>

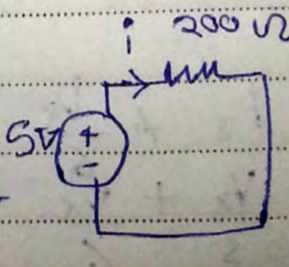
$$G_{eq} = 3+5+1 =>$$
$$= 9ʊ$$
$$=> R_{eq} = \frac{1}{9}\,\Omega$$

Ex Eo :



Find i

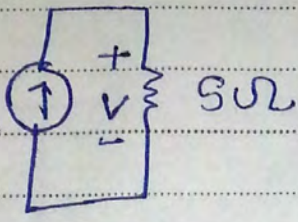
$4 - 5 + 6 = 5V$



$i = \frac{5}{200} =$ [illegible] $25\ mA$

25/9/2014

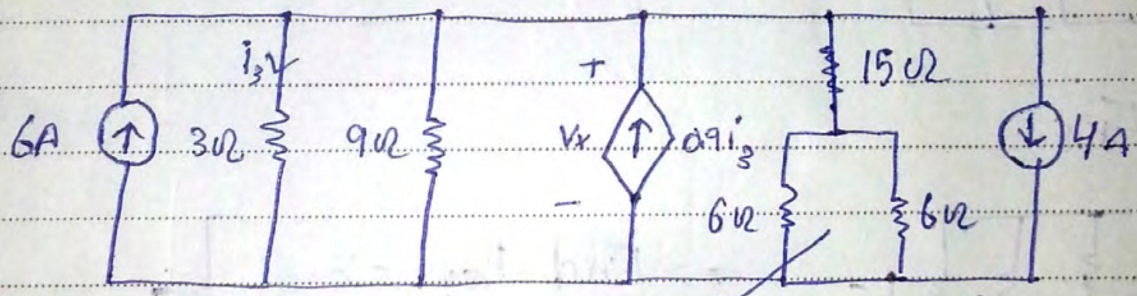
Ex:



5-1+6=10A

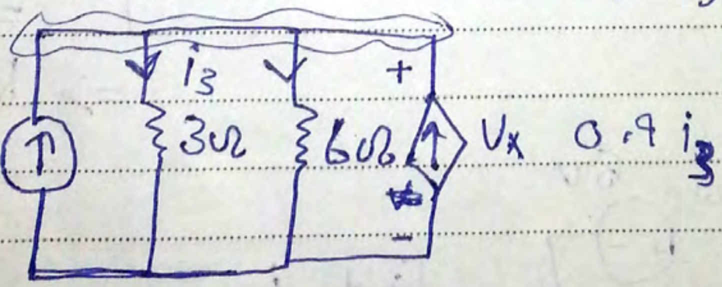
$V = 10 * 5 = 50V$

ExE: Calculate the voltage and the absorbed power of the dependent source.



6//6 =3Ω

node 1



6-4= 2A

Kcl at Nod 1

$$2 + 0.9 i_3 = i_3 + \frac{V_x}{6}$$

$$2 + 0.9 * \frac{V_x}{3} = \frac{V_x}{3} + \frac{V_x}{6} \Rightarrow V_x = 10 V$$

Subject:

25/9/2014

$$\Rightarrow P_{absorbed} = 0.9 * -i_3 * V_x = -0.9 * \frac{10}{3} * 10 = \underline{-30\,W}$$

28/9/2014

Ex:

20Ω//5Ω
4Ω

10Ω // 15Ω => 6Ω
6Ω = 14Ω => 20Ω

$\Rightarrow$

$\Rightarrow$

$-4+1-6=-9A$

$$kcl: -9 + \frac{V_x}{20} + \frac{V_x}{10} = 0$$

$$V_x = 60V$$

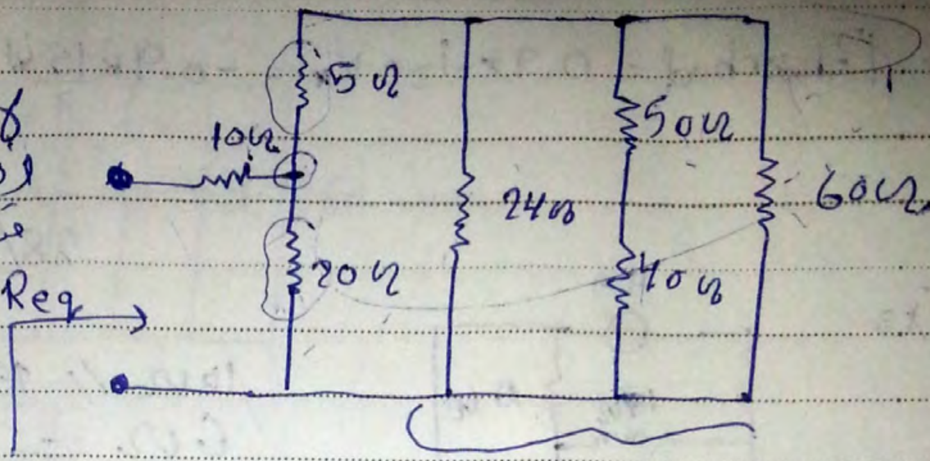
$$i_x = \frac{-60}{10} = -6A$$

Driven RL and Rc Circuit

Subject:.................... 28 / 9 / 2014

Ex:

لا توالي ولا توازي
إنه هناك مقاومة
مترحة

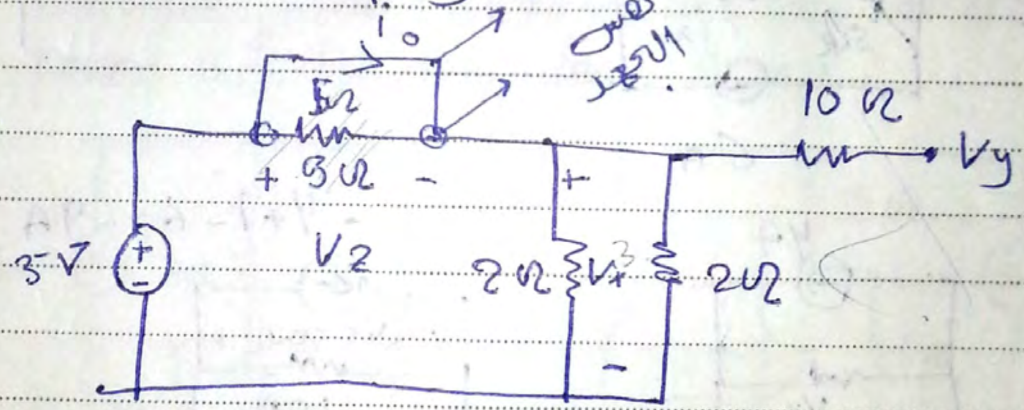


~~24 // 90 // 60~~

Find $R_{eq} = 10 + 20 // (24 // 90 // 60 + 5)$

$= 9.85$

Ex:



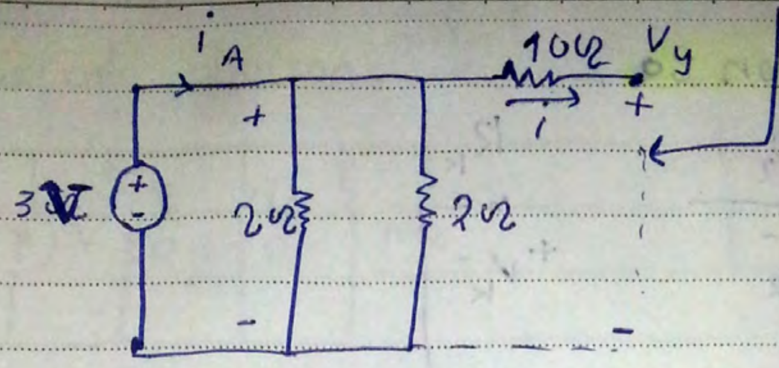
Find i_o, $i_{5Ω}$, V_x, V_y, V_z

⇒ short ~~Circ~~ circuit ⇒ $i_{5Ω} = $ zero

$V_z = $ Zero

$i_A = i_o$

Subject: 28/9/2014



لو المنطقي
الجهة نحسب
ال 10Ω

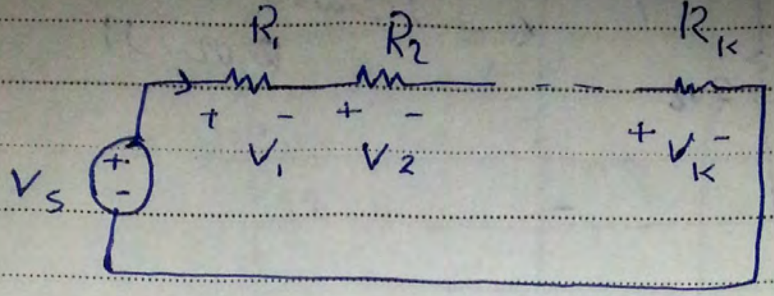
$I_{10\Omega} = Zero$

$V_{10\Omega} = Zero \Rightarrow V_y = V_x = 3V$

$i_A = \frac{3}{1\Omega} = 3A$

Subject:

*Voltage Division :



$$V_k = V_s \frac{R_k}{R_1 + R_2 + \cdots R_k}$$

$\Rightarrow$ KVL $\Rightarrow$ $-V_s + IR_1 + IR_2 + \cdots + IR_k = 0$

$$I = \frac{V_s}{R_1 + R_2 + \cdots + R_k}$$

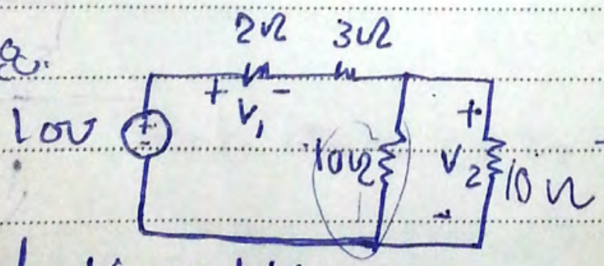
$$V_1 = I R_1$$

$$= V_s \frac{R_1}{R_1 + R_2 + R_k}$$

$$\frac{1}{\frac{1}{10} + \frac{2}{10}}$$

$\frac{10}{5}$

Ex :



= 5Ω.

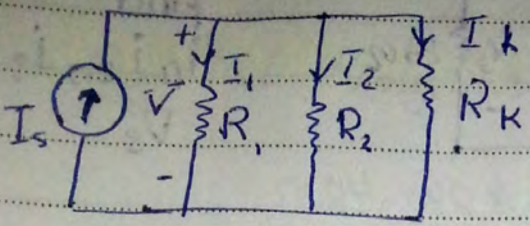
Find V_1 and V_2

$$V_1 = 10 * \frac{2}{2+3+5} = 2V$$

$$V_2 = 10 * \frac{5}{2+3+5} = 5V$$

Subject: 30/9/2014

* Current divison



KCL ⇒ at node one 1 :

$$I_s = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_K}$$

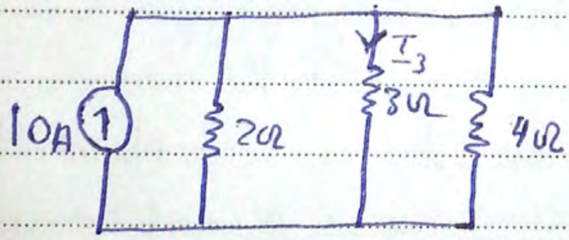
$$I_s = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_K}\right)$$

$$V = \frac{I_s}{\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_K}\right)}$$

$$I_k = \frac{V}{R_K}$$

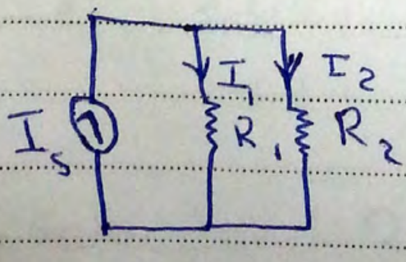
$$\boxed{I_k = I_s \frac{\frac{1}{R_K}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_K}}}$$

Ex



$$I_{3\Omega} = 10 * \frac{\frac{1}{3}}{\frac{1}{2} + \frac{1}{3} + \frac{1}{4}}$$

* Special case : two resistors



$$\Rightarrow I_1 = I_s \frac{R_2}{R_1 + R_2}$$

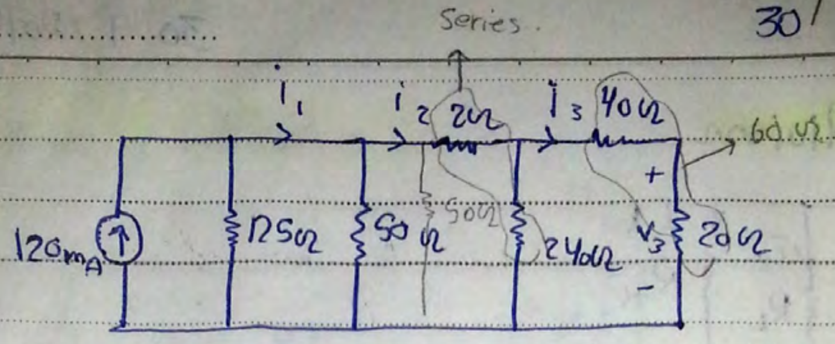
$$\Rightarrow I_2 = I_s \frac{R_1}{R_1 + R_2}$$

Driven RL and RC Circuit

Subject:.......................

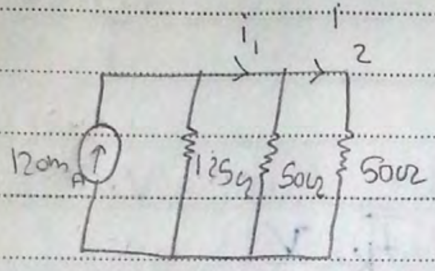
30/9/2014

Ex

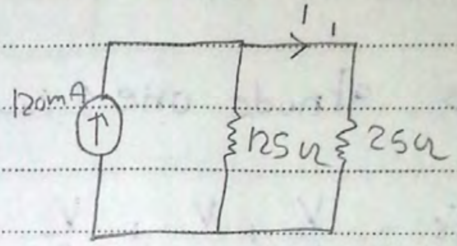


Find i_1, i_2, i_3 and V_3

60 // 240 = 48 Ω



=>



=> $i_1 = 120 * \frac{125}{125+25}$ = 100 mA

=> $i_2 = i_1 * \frac{50}{50+50}$

= 50 mA

=> $i_3 = i_2 * \frac{240+}{240+60}$ = 40 mA

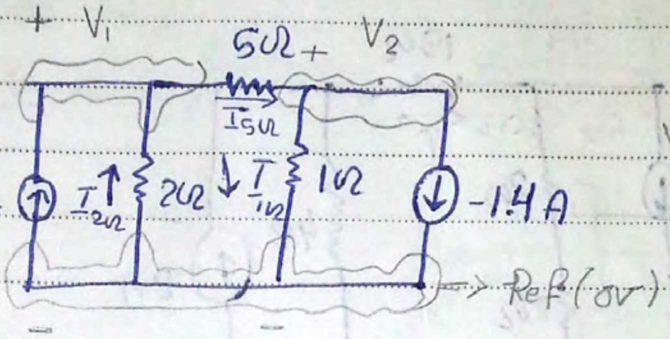
=> $V_3 = 20 * i_3 = 0.8$ V

30/9/2014

* Chapter 4

* Nodal Analysis: to Find the voltage at each node in the Circuit.

Ex:



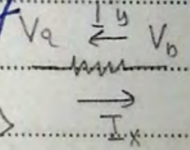
نطبق معادلات KCL على كل node
نختار الـ node الي عندها اكثر branch

Steps of Solution: 1- locate all the nodes in the Circuit
2- select one node as a referance node ($V=0$) (the node that has the highest number of Connection)
3- locate V_1, V_2 ...

KCL at node 1: ($\sum i_{out} = 0$)

$$-3.1 + \frac{V_1 - 0}{2} + \frac{V_1 - V_2}{5} = 0 \quad \text{①}$$

note ⇒



$$I_x = \frac{V_a - V_b}{R}$$

$$I_y = \frac{V_b - V_a}{R}$$

KCL at node 2:

$$-1.4 + \frac{V_2 - 0}{1} + \frac{V_2 - V_1}{5} = 0 \quad \text{②}$$

⇒ Solu 1 and 2:

$V_1 = 5V$ $V_2 = 2V$

Subject:...................... 30/9/2014

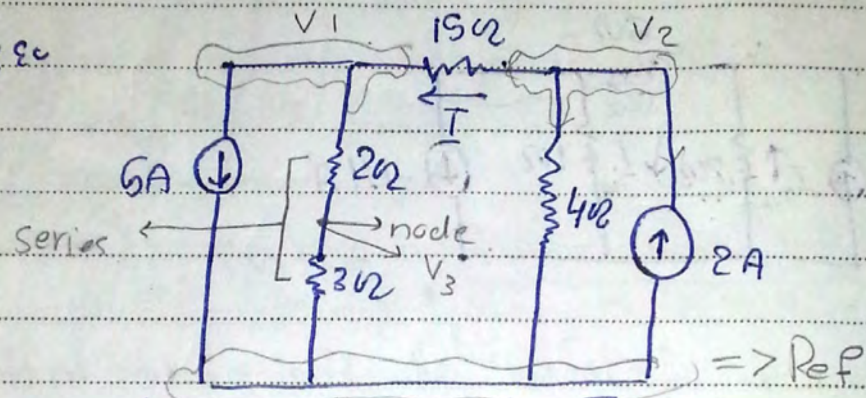
$$I_{1\Omega} = \frac{V_2 - 0}{1\Omega} = 2A$$

$$I_{5\Omega} = \frac{V_1 - V_2}{5} = \frac{3}{5}\ A$$

$$I_{2\Omega} = \frac{0 - V_1}{2} = -\frac{5}{2}\ A$$

عدد المعادلات هو عدد node-1

Ex:



Use nodal analysis to find I :

KCL at node 1 :

$$5 + \frac{V_1 - 0}{5} + \frac{V_1 - V_2}{15} = 0 \quad --- ①$$

kcl at node 2 :

$$-2 + \frac{V_2 - 0}{4} + \frac{V_2 - V_1}{15} = 0 \quad -- ②$$

$$\Rightarrow V_1 = \frac{-145}{8}\ V$$

$$\Rightarrow V_2 = \frac{5}{2}\ V$$

S T A R S Notebook

$\Rightarrow I = \frac{V_2 - V_1}{15} = 2.57\ A$

$V \Rightarrow$ 1 voltage Divison

2

kcl at node 3 :

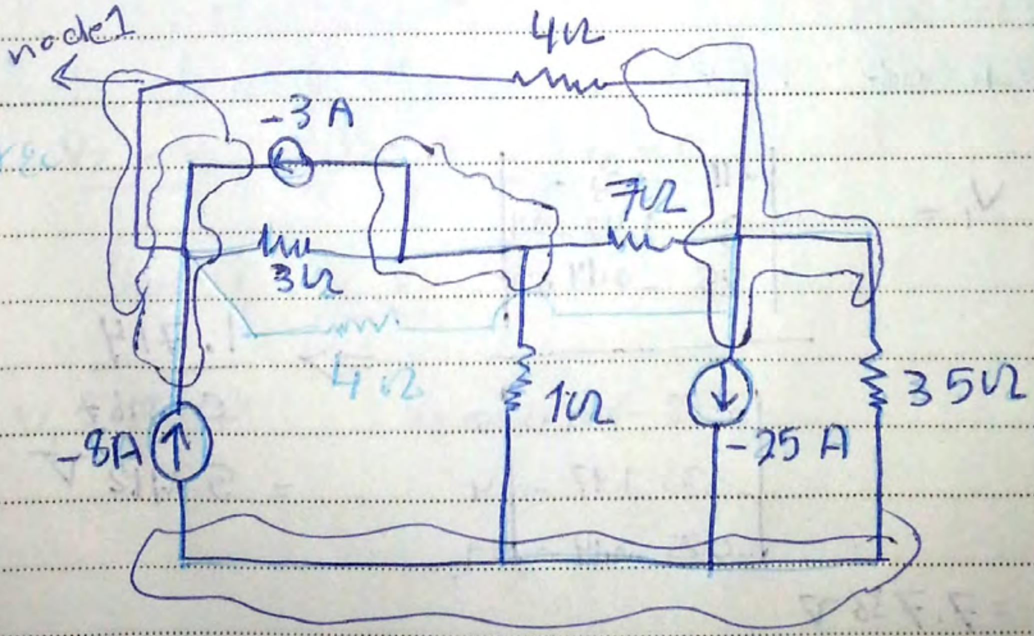
$$\frac{V_3 - V_1}{2} + \frac{V_3 - 0}{3} = 0$$

$$V_3 = \frac{-145}{8} * \frac{3}{5}$$

CH2 ⇒ 20, 23, 26, 33, 34, 36.

CH3 ⇒ 13, 14, 15, 26, 27, 29, 30, 35, 36, 37, 38,
42, 50.

Exe



Q apply the nodal analysis to analysis this circuit

Subject:…………………… 2/10/2014

kcL at node 1:

$$8+3+\frac{V_1-V_2}{3}+\frac{V_1-V_3}{4}=0 \quad -- ①$$

kcl at node 2:

$$-3+\frac{V_2-V_1}{3}+\frac{V_2-V_3}{7}+\frac{V_2-0}{1}=0 \quad -- ②$$

kcL at node 3:

$$-25+\frac{V_3-V_2}{7}+\frac{V_3-V_2}{7}+\frac{V_3-V_1}{4}+\frac{V_3-0}{5}=0 \quad -- ③$$

$$0.58\,V_1-0.33\,V_2-0.25\,V_3=-11 \quad -- ①$$
$$-0.33\,V_1+1.47\,V_2-0.14\,V_3=3 \quad -- ②$$
$$-0.25\,V_1-0.14\,V_2+0.59\,V_3=25 \quad -- ③$$

By ~~circuit~~ Cramer's rule:

$$V_1=\frac{\begin{vmatrix}-11 & -0.33 & -0.25\\ 3 & 1.47 & -0.4\\ 25 & -0.14 & 0.59\end{vmatrix}}{\begin{vmatrix}0.58 & -0.33 & -0.25\\ -0.33 & 1.47 & -0.4\\ -0.25 & -0.14 & 0.59\end{vmatrix}} \Longrightarrow \frac{1.714}{0.3167}=5.412\,V$$

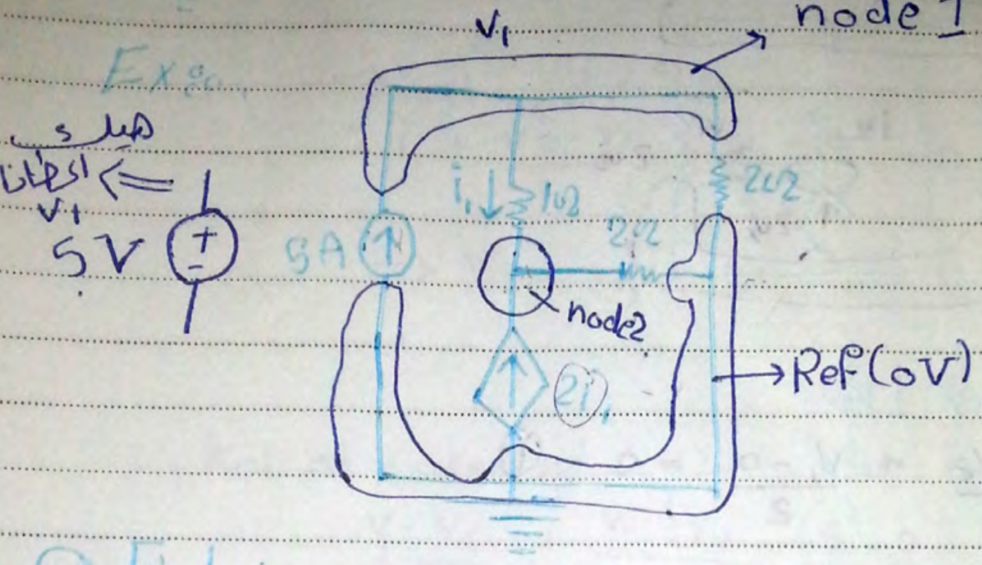
$V_2=7.736\,V$

$V_3=46.32\,V$

Subject: 2/10/2014

⏚ ⟹ Ref (oV)

Ex:



Q Find i_1 using nodal analysis:

* KCL at node 1:

$$-5 + \frac{V_1 - V_2}{1} + \frac{V_1 - 0}{2} = 0 \quad ---- ①$$

* يجب وضع يساوي صفر

* KCL at node 2:

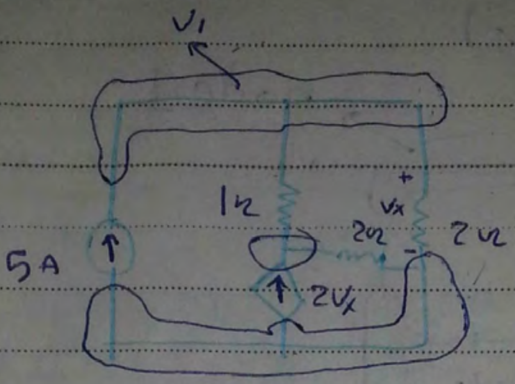
$$\frac{V_2 - V_1}{1} + \frac{V_2 - 0}{2} - 2i_1 = 0$$

but $i_1 = \frac{V_1 - V_2}{1}$

$V_1 = \frac{70}{9}$ V , $V_2 = \frac{20}{3}$ V

Subject:.......................... 2/10/2014

Ex:



kcl at node 1:

$$-5+\frac{V_1-V_2}{1}+\frac{V_1-0}{2}=0 \quad \text{--} ①$$

kcl at node 2:

$$-2V_x+\frac{V_2-V_1}{1}+\frac{V_2-0}{2}=0 \quad \text{--} ②$$

but $V_x = V_1 - 0$

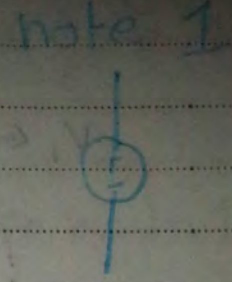
$V_1 = -10V$

$V_2 = -20V$

Subject:........................ 2 / 10 /2014

=> at node 1 :- $V_1 - 0 = 5$

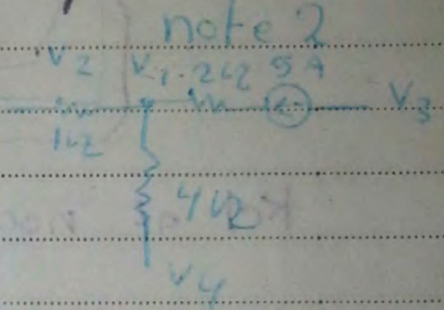
$V_1 = 5\,\bar{V}$



=> at node 2 :-

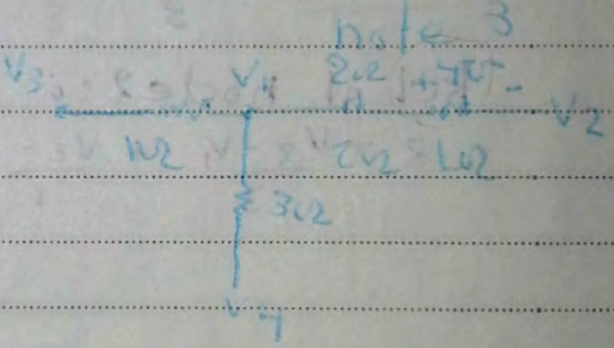
$$-2 i_1 + \frac{V_2 - V_1}{1} + \frac{V_2 - 0}{2} = 0$$

$$\Rightarrow V_2 = 30/7\,\bar{V}$$



KCl at node I :-

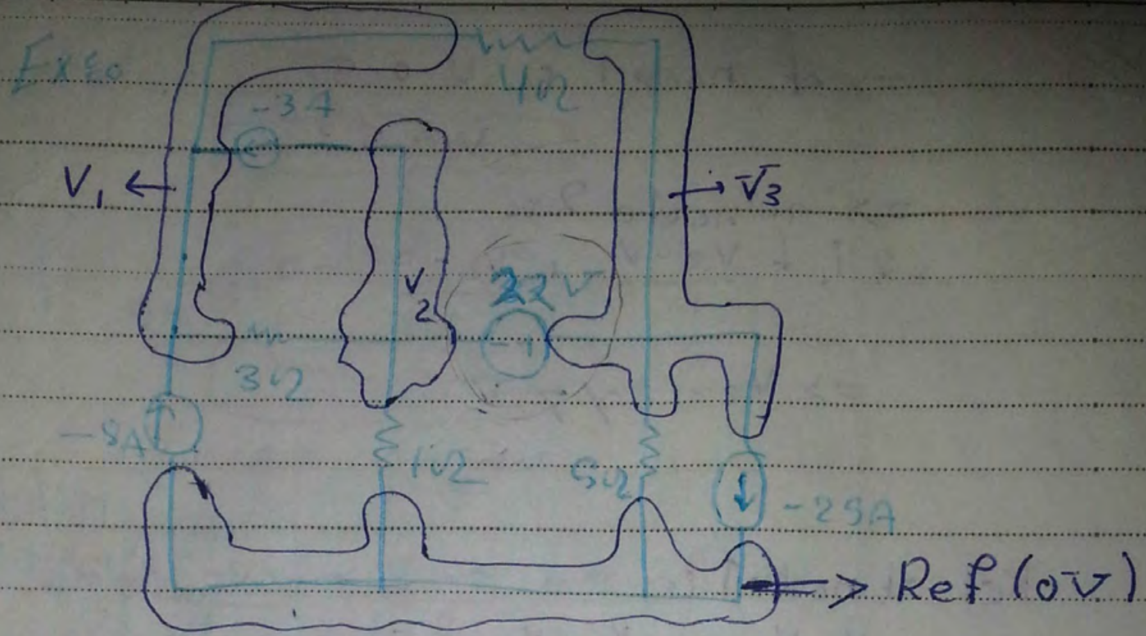
$$\frac{V_1 - V_2}{1} + \frac{V_1 - V_4}{4} - \boxed{5} = 0$$



KCL at node 1 :-

$$\frac{V_1 - V_3}{1} + \frac{V_1 - V_4}{3} + \frac{4}{1} = 0$$

Subject:.......................... 2/10/2014



Kcl at node 1:-

$$8+3+\frac{V_1-V_2}{3}+\frac{V_1-V_3}{4}=0 \quad ①$$

Kcl at node 2:-

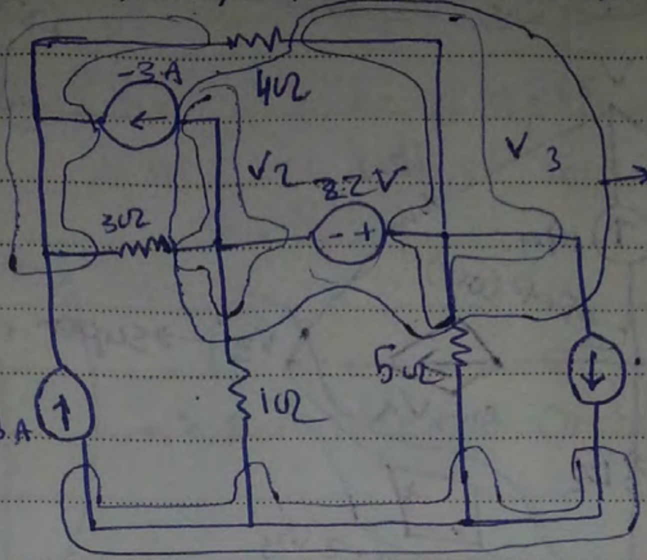
$$-3+\frac{V_2-V_1}{3}+\frac{V_2-0}{1}+22 \quad X$$

Subject:......................... 9/10/2014

7th edition

CH3: 56, 57, 59, 60, 67, 68, 69, 72, 74, 78, 80

Ex:



→ super node 1

✱ KCL at Supernode 1:

$$-25 + \frac{V_3 - 0}{5} + \frac{V_2 - 0}{1} + \frac{V_2 - V_1}{3} - 3 + \frac{V_3 - V_1}{4} = 0 \text{ --} ②$$

✱ inside super node 1:

$V_3 - V_1 = 22$

$V_1 = 1.071$ V

$V_2 = 10.5$ V

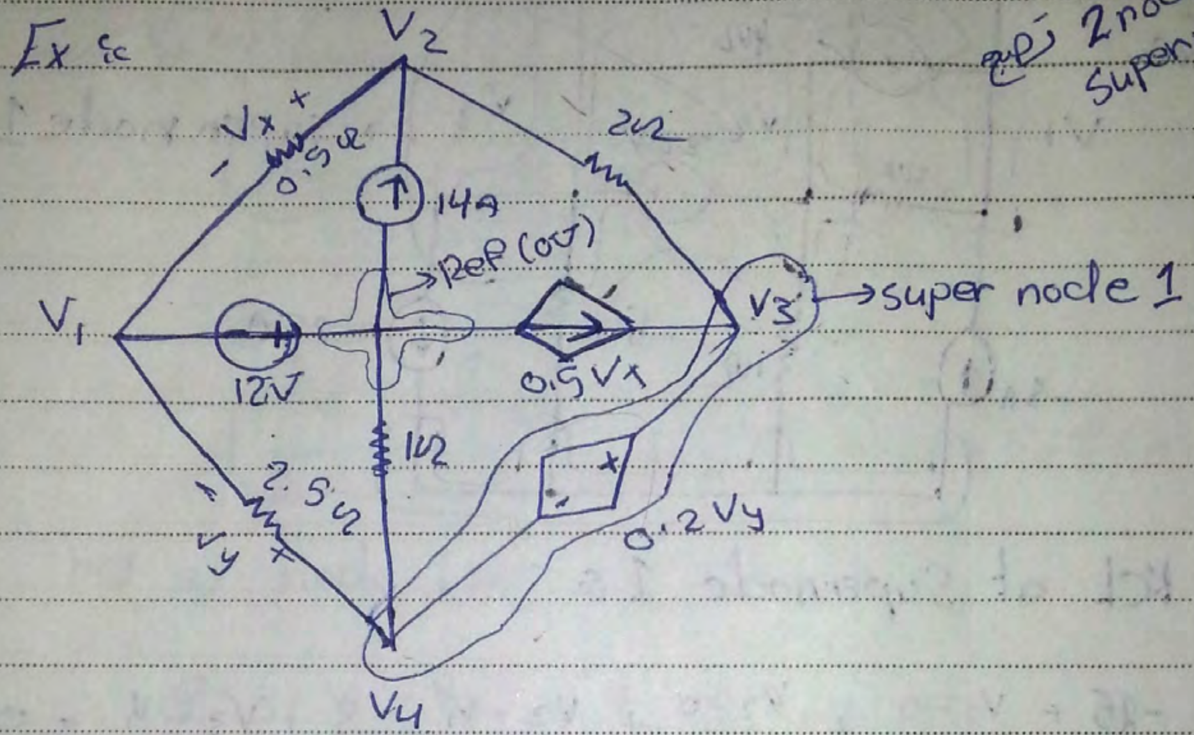
$V_3 = 32.5$ V

9/10/2014

Super node

إذا كان مصدر جهد
بين 2nodes نجمع
supernode

Ex :



KCL at node 1 :

$\Rightarrow V_1 = -12V$

Kcl at node 2 :

$\Rightarrow -14 + \frac{V_2 - V_1}{0.5} + \frac{V_2 - V_3}{2} = 0$

Kcl at super node 1 :

$\Rightarrow \frac{V_4 - V_1}{2.5} + \frac{V_4 - 0}{1} - 0.5V_x + \frac{V_3 - V_2}{2} = 0$

Inside super node 1 :

$V_3 - V_4 = 0.2V_y$

but $V_x = V_2 - V_1 = V_2 + 12$

$V_y = V_4 - V_1 = V_4 + 12$

$V_1 = -12V$

$V_2 = -4V$

$V_3 = 10^{-14}V \approx 0$

$V_4 = -2V$

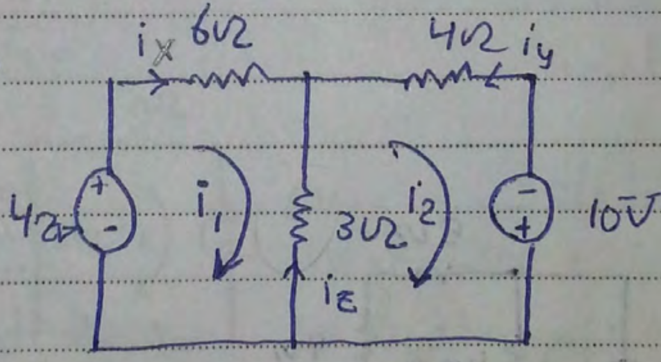
✱ Mesh Analysis :

⇒ it is used if the circuit is a planar circuit
(a circuit that can be drawn on a plane surface)

⇒ non plannar circuit

mesh : it is a loop that does not contain any other loop

Ex :



✱ دائماً مع عقارب الساعة.

$i_x = 6A$

$i_y = -4A$

$i_z = i_2 - i_1$
$= -2A$

kvl For mesh 1 :

$$-42 + 6i_1 + 3(i_1 - i_2) = 0 \text{ --- ①}$$

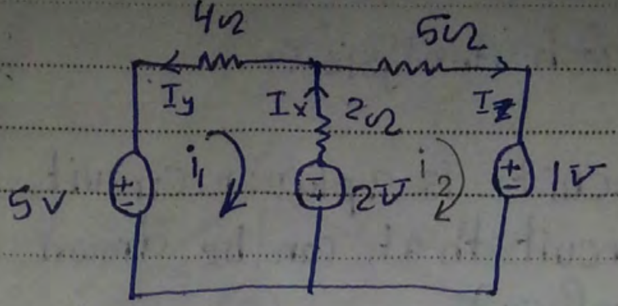
kvl For mesh 2 :

$$-10 + 3(i_2 - i_1) + 4i_2 = 0 \text{ --- ②}$$

$i_1 = 6A$ $i_2 = 4A$

12/10/2014

Ex



Find I_x, I_y, I_z using mesh analysis:

=> KVL of mesh 1 :-

$-5 + 4i_1 + 2(i_1 - i_2) - 2 = 0$ ①

=> KVL of mesh 2 :-

$5i_2 + 1 + 2 + 2(i_2 - i_1) = 0$ ②

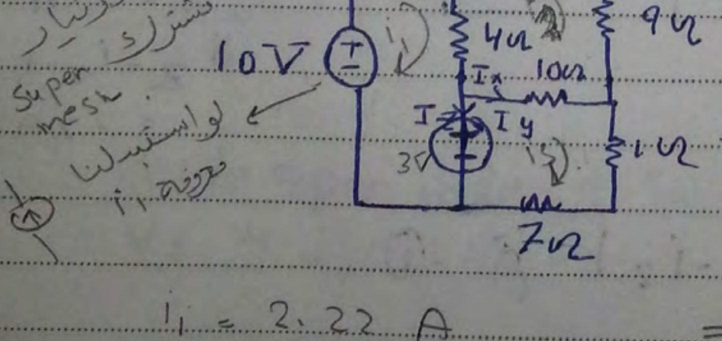
$i_1 = \frac{43}{38}$ A $i_2 = \frac{2}{19}$

$I_y = -i_1$

$I_x = i_2 - i_1$

$I_z = i_2$

Ex



=> KVL at mesh 1 :-

$-10 + 4(i_1 - i_2) + 3 = 0$ ①

=> KVL of mesh 2 :-

$4(i_2 - i_1) + 5i_2 + 9i_2 +$
$10(i_2 - i_3) = 0$ ②

=> KVL of mesh 3 :-

$-3 + 10(i_3 - i_2) + i_3 +$
$7i_3 = 0$ ③

$i_1 = 2.22$ A

$i_2 = 0.47$ A

$i_3 = 0.427$ A

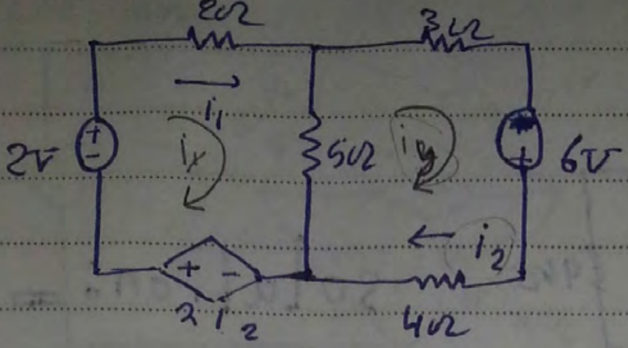
$i_x = i_2 - i_1$

$i_y = i_3 - i_2$

$I_z = i_1 - i_3$

Subject:.......................   12/10 /2014

Ex



$*$ Find i_1 & i_2 using the mesh analysis :

$i_2 = i_y$

kvl at mesh 1 :

$-2 + 2i_x + 5(i_x - i_y) - 2(i_2) = 0$ --- ①

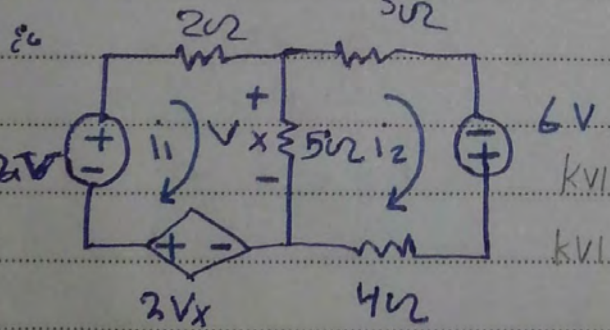
kvl at mesh 2 :

$3i_y + -6 + 4i_y + 5(i_y - i_x) = 0$ -- ①

$i_1 = i_x = \frac{66}{44}$ A

$i_2 = i_y = \frac{52}{44}$ A

Ex :



solution :-

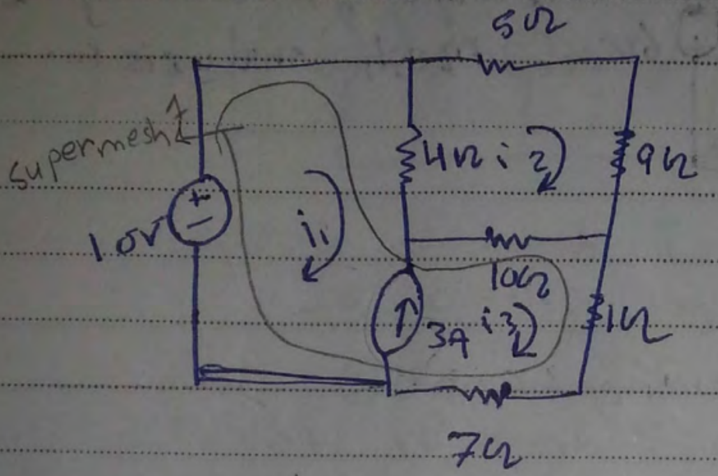
① kvl => $-2V + 2i_1 + 5(i_1 - i_2) - 2V_x = 0$

② kvl => $3i_2 - 6 + 4i_2 + 5(i_2 - 1) = 0$

but $V_x = 5(i_1 - i_2)$

12/10/2014

Ex: super mesh



solution: — JoJo

=> mesh 2:

$5i_2 + 9i_2 + 10(i_2 - i_3) + 4(i_2 - i_1) = 0$ -- ①

=> super mesh 1:

$-10 + 4(i_1 - i_2) + 10(i_3 - i_2) + 1i_3 + 7i_3 = 0$ -- ②

=> inside supermesh:

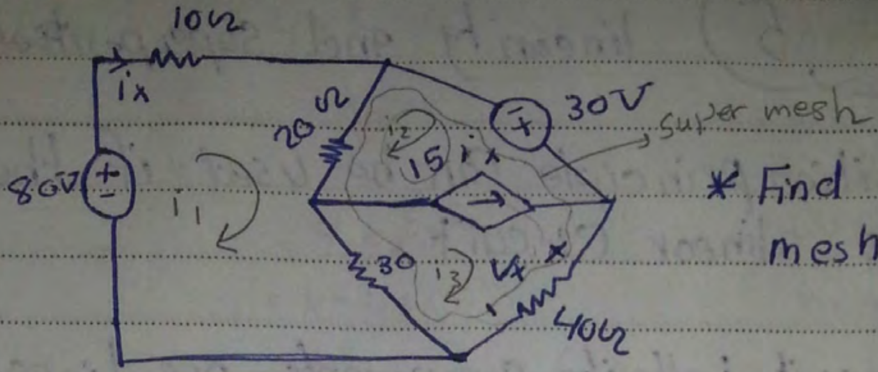
$i_3 - i_1 = 3$ -- ③

$i_1 = -\frac{29}{15}$ A

$i_3 = \frac{16}{15}$ A

$i_2 = \frac{11}{105}$ A

Subject:........................ 12/10/2014



* Find i_x and V_x using mesh analysis εc

mesh 1 εc

$-80 + 10 i_1 + 20(i_1 - i_2) + 30(i_1 - i_3) = 0$ -- ①

KVL For Supermesh 1 εc

$-30 + 40 i_3 + 30(i_3 - i_1) + 20(i_2 - i_1) - 0$ -- ②

inside Supermesh 1 :

$i_3 - i_2 = 15 i_x$

$i_1 = i_x = \frac{87}{149}$ A $\qquad i_2 = \frac{-917}{149}$ A

$i_3 = \frac{388}{149}$ A $\qquad V_x = 40 i_3 = 104.16$ V

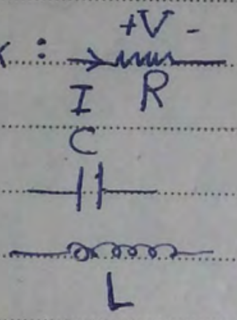
* Chapter 5 linearity and superposition.

- Super position principle Can be used if the Circuit is alinear Circuit.

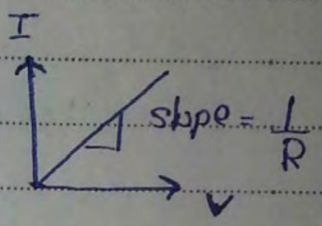
* linear circuit : all its components are linear element.

* linear element : the relation ship between Volage and Current is linear.

=> Ex :

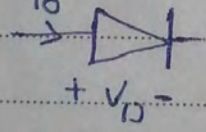


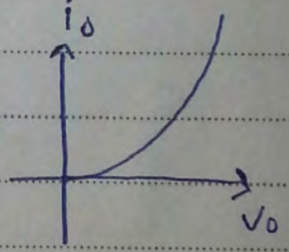
=> $I = \frac{V}{R}$



=> non-linear elements

Diode





*To Check any relation Ship

=> Methode 1 :

$y = 3x$ => $x = kx$ → if $y_{new} = ky$, then it is linear relationship.

$y_{new} = 3(kx)$

$y_{new} = k\,3x$

$y_{new} = ky$

Ex: $y = 4x^2$

$x_{new} = kx$

$y_{new} = 4x^2$

$= 4 * k^2 * x^2$

$= k^2 \, 4 * x^2$

$y_{new} = k^2 y$ non-linear

methode 2 :

$y = 4x^2$

$y = 4(x_1 + x_2)^2$

$y = 4(x_1^2 + 2x_1x_2 + x_2^2)$

non - linear

Methode 2 :

Ex: $y = 3x$

$x_{new} = x_1 + x_2$

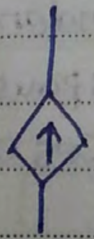
$y_{new} = 3\,x_{new}$

$= 3(x_1 + x_2)$

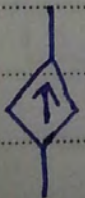
$y_{new} = 3x_1 + 3x_2 \Rightarrow y_{new} = y_1 + y_2$ linear

Ex:

$i_2 = k\,i_x$ (linear)



$i_2 = k\,i_x^2$ (non linear)

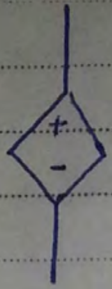


$i_2 = 2 i_x V_x$ (non-linear)

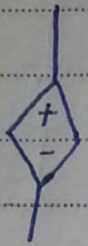
Subject:

14/10/2014

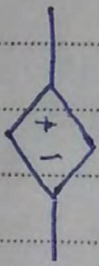
Ex :



$V_2 = 3\,i_x$ (linear)



$V_2 = 7 i_x - 5 V_x$ (linear)



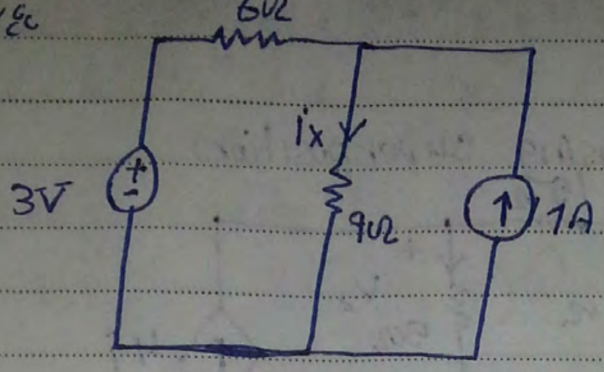
$V_2 = V_x^2 + 1$ (non-linear)

* Super position => Can be used to Find Current or Voltage but not power.

* Super position technique => The $\overset{\text{Voltage or current}}{\text{respouse}}$ in a linear Circuit having more than one indpendent Source Can be obtained by adding the respouses Caused by the separate independent sources acting alone.

14/10/2014

Ex:

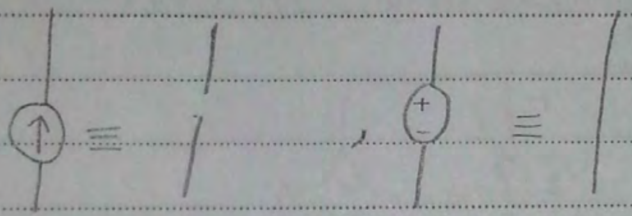


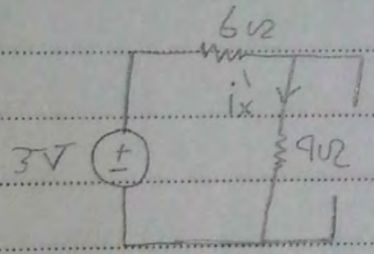
a) Find I_x using super position

b) Find the absorbed power by 9Ω

=> i_x due to 3V only (i_x')

kill all other independent source =>

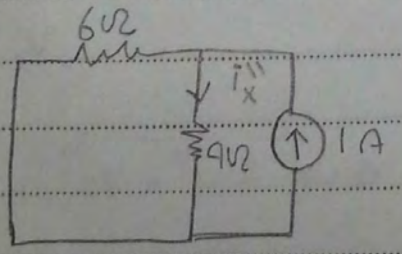




$i_x' = \frac{3}{15} = 0.2$ A

=> i_x due to 1A only (i_x'')

→ Kill all other independent source =>



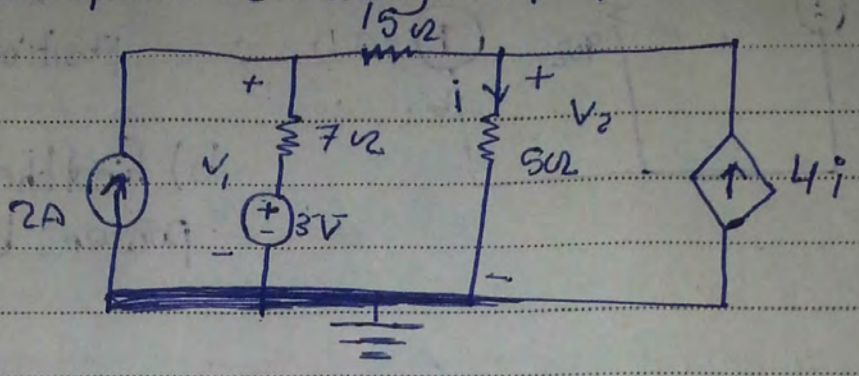
$i_x'' = 1 * \frac{6}{15} = 0.4$ A

$i_x = i_x' + i_x'' = 0.2 + 0.4 = 0.6$ A

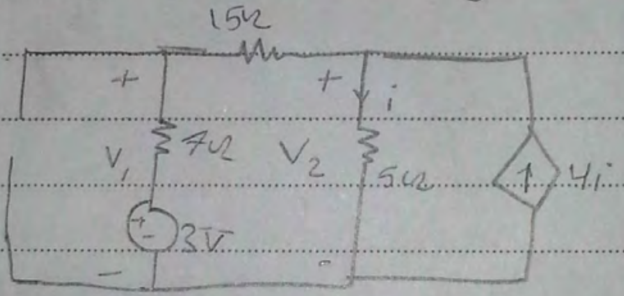
Subject: 14/10/2014

ⓑ $P_{absorbed} = i_x^2 * 9 = (0,6)^2 * 9 = 3.24W$

Ex 2: Find V_1 and V_2 using superposition



=> V_1 and V_2 due to 3V only (V_1', V_2')



kcl at node 1 :-

$$\frac{V_1'-3}{7} + \frac{V_1'-V_2'}{15} = 0 \quad --①$$

kcl at node 2

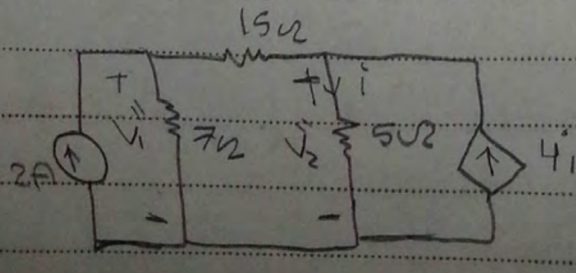
$$\frac{V_2'-0}{5} + \frac{V_2'-V_1'}{15} - 4\overset{\frac{V_2'-0}{5}\nearrow}{\textcircled{i}} = 0 \quad --②$$

Solve 1 & 2

$V_1' = 1.967 V$

$V_2' = -0.245 V$

=> V_1 and V_2 due to 2A only (V_1'' + V_2'')



14/10/2014

=> KCl at node 1 :

$$-2 + \frac{V_1'' - 0}{7} + \frac{V_1'' - V_2''}{15} = 0 \quad -- ①$$

=> KCl at node 2 :

$$\frac{V_2'' - 0}{5} + \frac{V_2'' - V_1''}{15} - 4\textcircled{i} = 0 \quad \textcircled{i} \rightarrow \frac{V_2'' - 0}{5}$$

$V_1'' = 9.18 V$

$V_2'' = -1.148$

$V_1 = V_1' + V_1''$

$= 11.197 V$

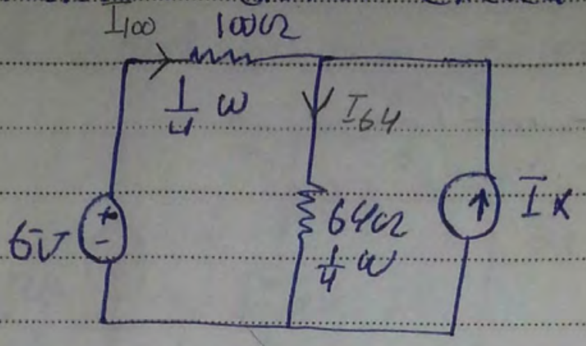
$V_2 = V_2' + V_2''$

$= -1.394 V$

Subject:

16/10/2014

Ex: Find the maximum Current I_x such that the resistors donot exceed their power rating



Sol:

=> Power rating : maximum absorbed power

$$P_{max}(100\Omega) = \frac{1}{4} = w$$

$$I^2_{(100)max} R = \frac{1}{4}$$

$$I^2_{(100)max} * 100 = \frac{1}{4}$$

$$\sqrt{I^2_{(100)max}} = \sqrt{\frac{1}{400}} \Rightarrow I_{(100)max} = 50 \text{ mA}$$

$$P_{max}(64\Omega) = \frac{1}{4} \text{-} w$$

$$I^2_{64} = \frac{1}{64*4} = I_{64} = 62.5 \text{ mA}$$

$$I_{(100\Omega)} \leqslant I_{100(Max)}$$

$$I'_{100} + I'' \leqslant 50 \text{ MA}$$

$\downarrow$ $\downarrow$

due to 6V only

16 / 10 / 2014

$$\frac{6}{164} + \left(-I_x \frac{64}{64+100}\right) \leq 50 \text{ mA}$$

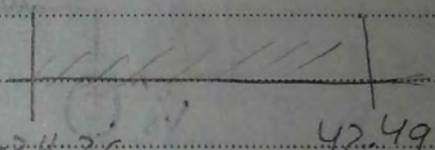
$$\Rightarrow \boxed{I_x \geq -34.36 \text{ mA}}$$

$$I_{64} < I_{64(max)}$$

$$I'_{64} + I''_{64} \leq 62.5 \text{ mA}$$

$$\frac{6}{64+100} + I_x \frac{100}{100+164} \leq 62.5 \text{ mA}$$

$$I_x \leq 42.49 \text{ mA}$$

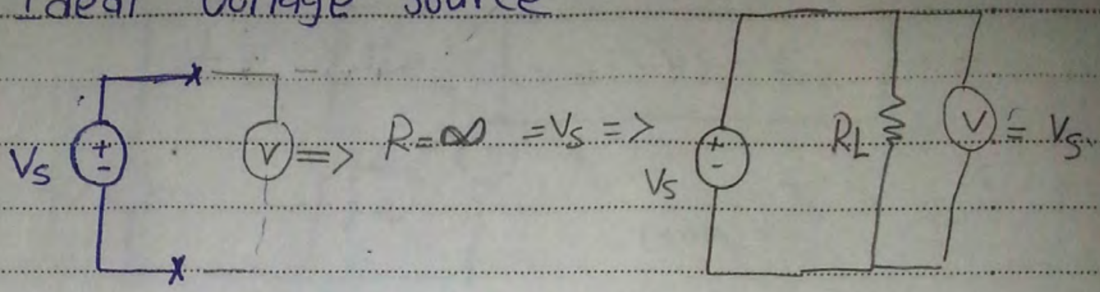


(First Exam)

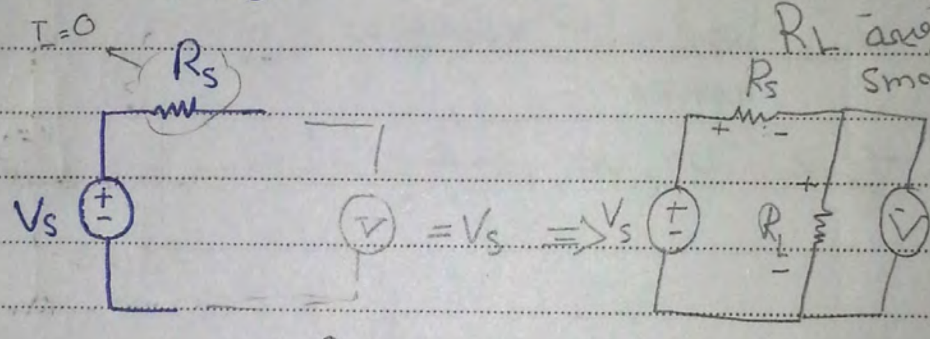
Subject: 16/10/2014

* ~~par~~ractical and Ideal Sources:

- Ideal voltage Source



_ practical Voltage Source



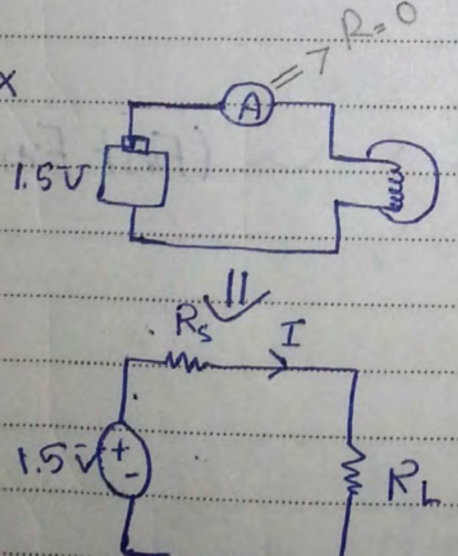
$= V_s \frac{R_L}{R_s+R_L}$

كلما زادت قيمة RL
كلما كان small loading effect

loading effect

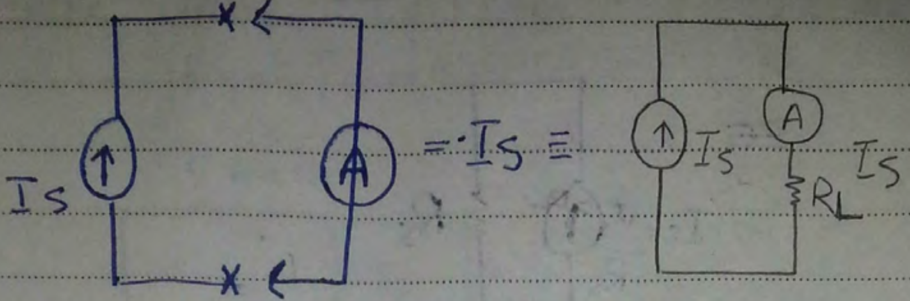
as $R_L \uparrow$ = small loading effect

Ex

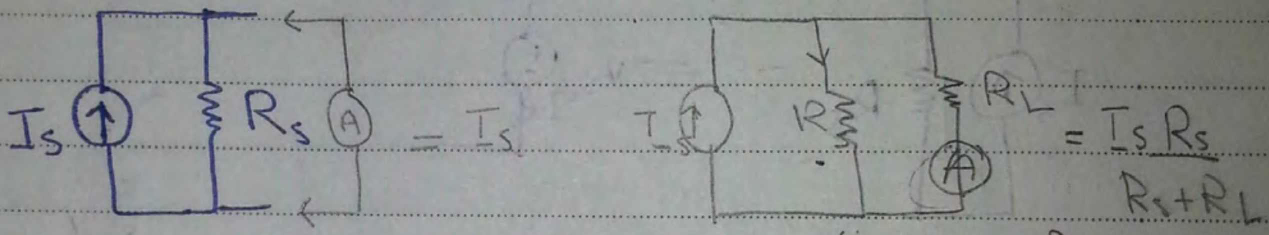


Subject:.................... 16/10/2014

* Ideal Current Source Eq



* Practical Current Source Eq.



$= \frac{I_s R_s}{R_s+R_L}$

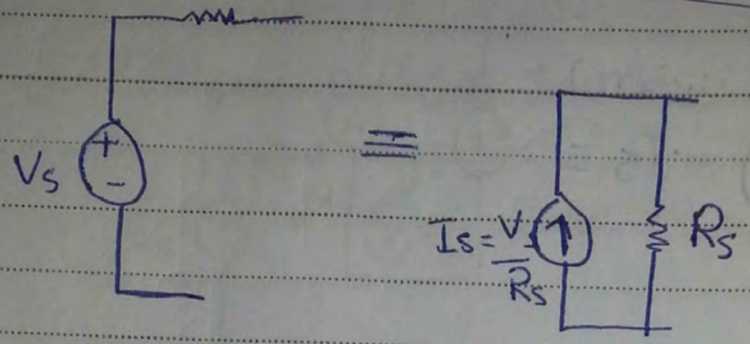
(loading effect)

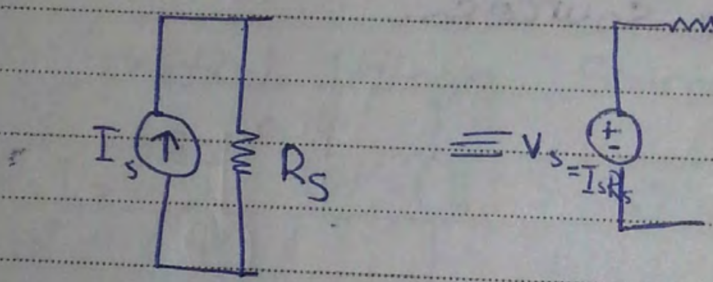
$R_L \downarrow$ small loading effect

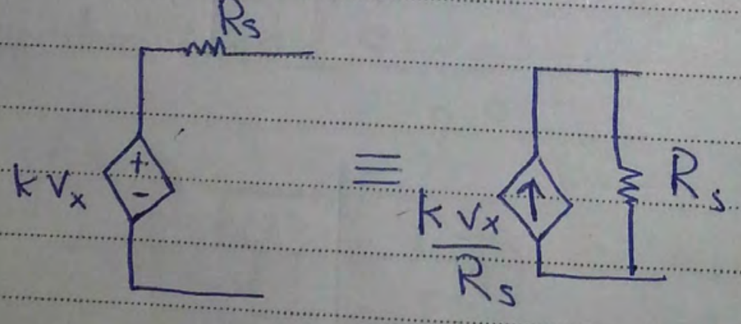
Subject:

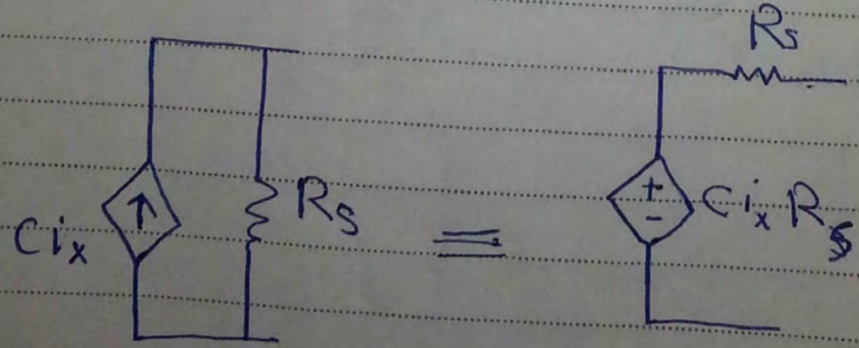
19/10/2014

Source transformation 3>



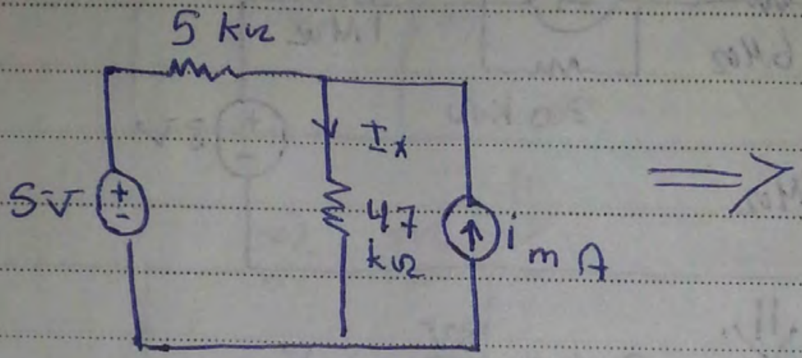


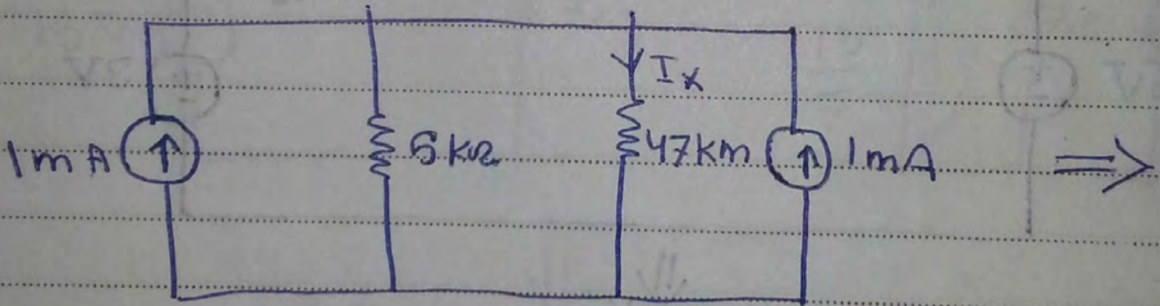


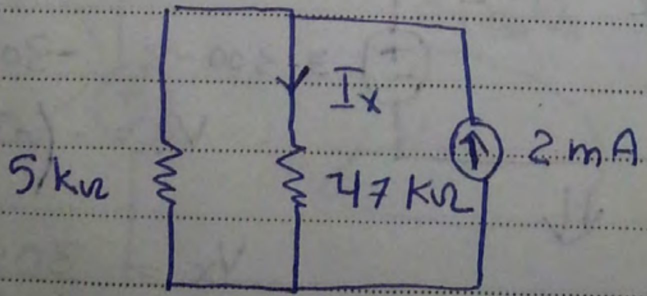


Subject:......................   19/10/2014

Ex Ec Find I_x (use in your solution Source transformation)







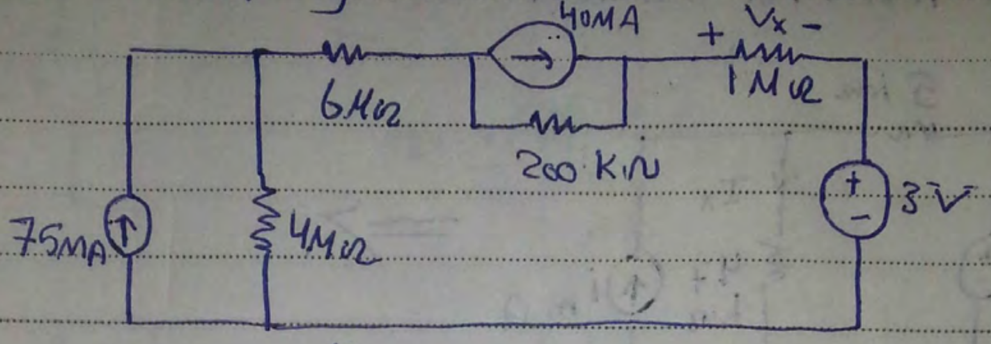
$$I_x = 2 * \frac{5}{5+47}$$

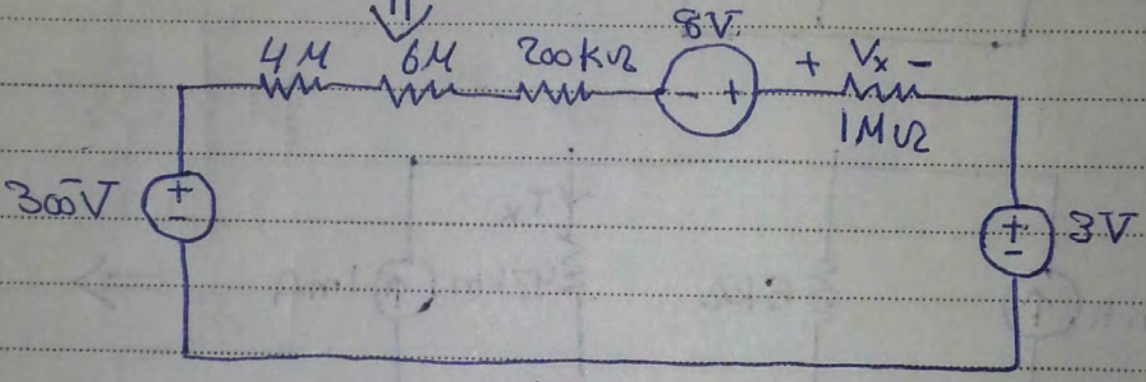
$= 192$ MA

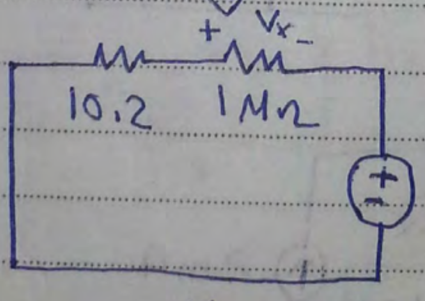
19/10/2014

Source transformation :

Ex: simplify this circuit and find Vx



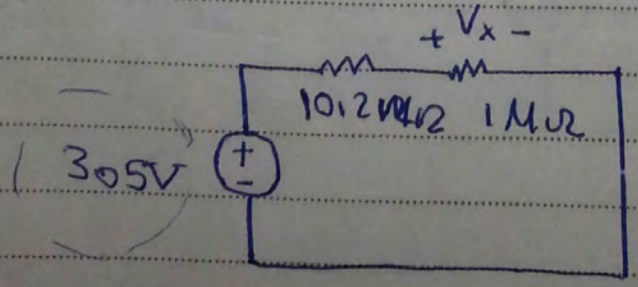




$3-300-8 = -305\ V$

$$V_x = -\left(-305 * \frac{1}{1+10.2}\right)$$

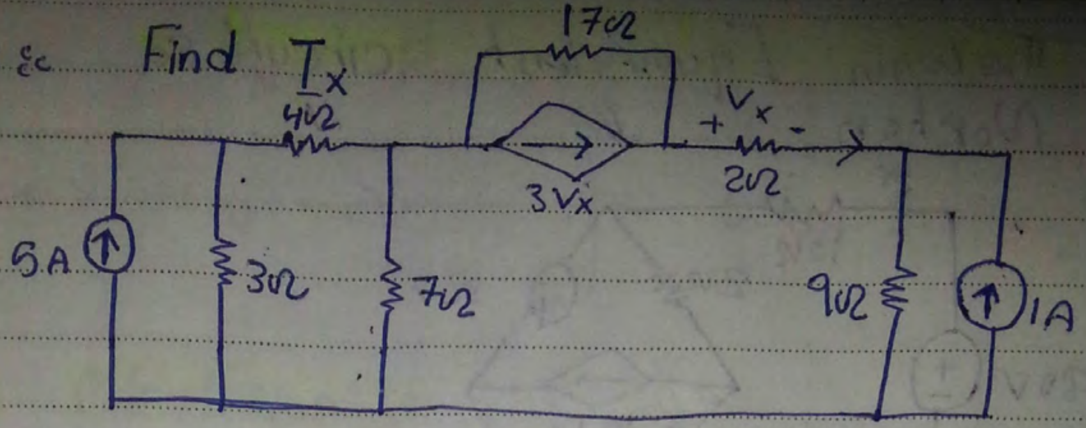
$$V_x = \frac{305}{11.2} = 27.23V$$



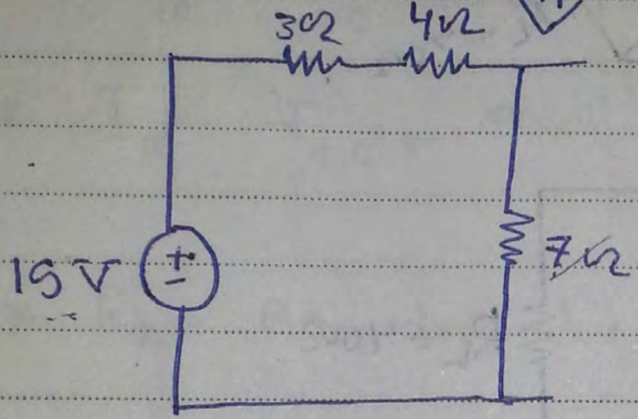
Subject: ....

19/10/2014

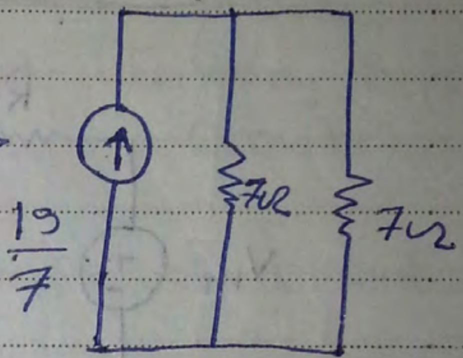
Ex.ec Find I_x



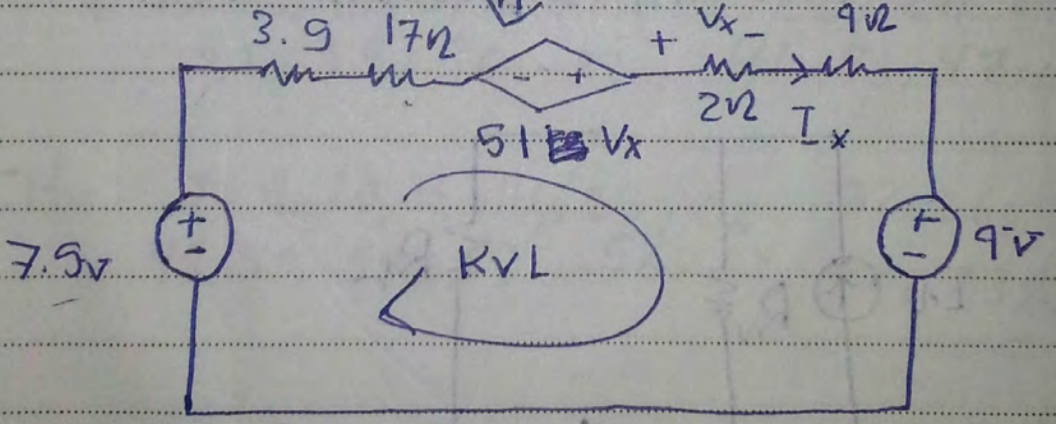
⇓



=>



⇓



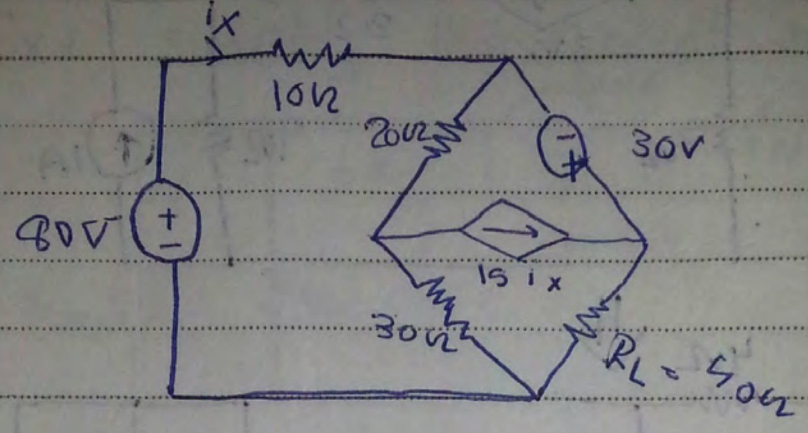
$V = 3V_x + 17$

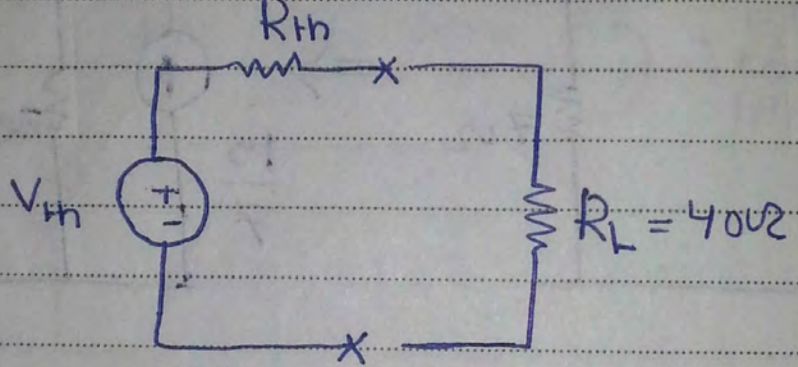
$$-7.5 + 31.5\, I_x - 51\, V_x + 9 = 0$$

$$\Rightarrow I_x = 21.276\, MA$$

19/10/2014

* Thevenin Equavalent circuit Ec.
* Norton " "





* Thevenin Equavalent Circuit

* Norton equavalent circuit

Subject: 19/10/2014

$$\left.\begin{aligned} &\Rightarrow R_N = R_{th} \\ &\Rightarrow I_N = \frac{V_{th}}{R_{th}} \end{aligned}\right\} \Rightarrow$$ using source transformation

* Calculations:
* $V_{th} = V_{o.c}$ ← open circuit (For any type of circuits)
* $I_N = I_{s.c}$ ← short circuit (For any type of circuits)
* R_{th}: According to the type of circuit.

Ch4 => 8, 9, 10, 11, 16, 18, 22, 24, 29, 30, 31,
32, 35, 36, 39, 41, 43, 47, 52, 53

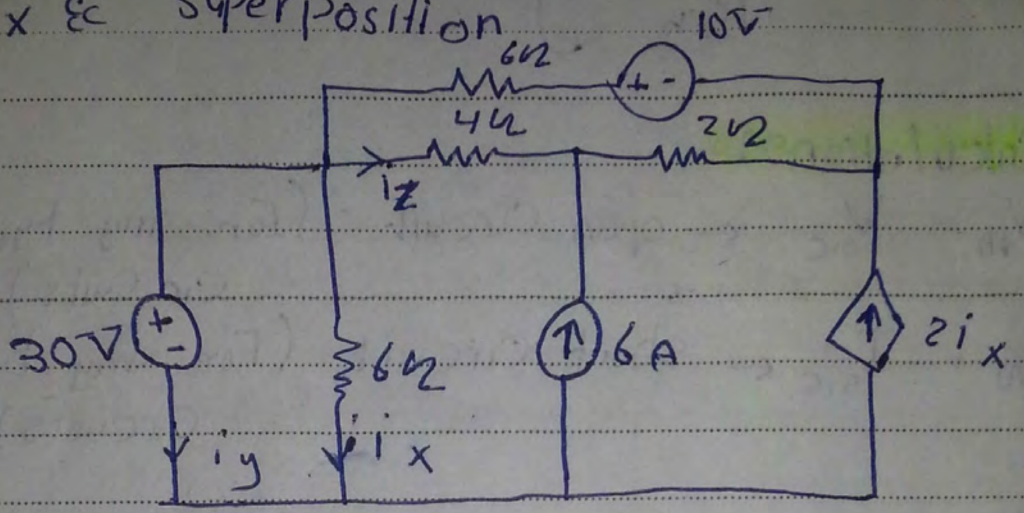
CH5: 7, 9, 10, 12, 14, 25, 26, 32, 36, 39, 47,
48, 49, 54, 55, 56, 63, 72, 73, 75

After Rev ?

Subject: ....   21 / 10 / 2014

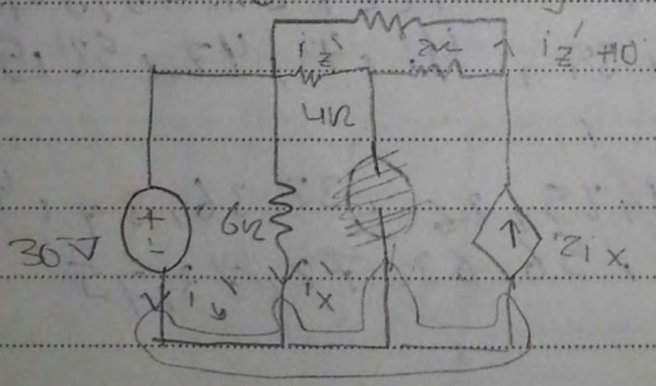
Rev

Ex & Superposition



Find ix, iy iz

due to 30V only



KCL

$i_x' = \frac{30}{6} = 5A$

$i_y' + i_x' = 2i_x' \Rightarrow i_y' = 5A$

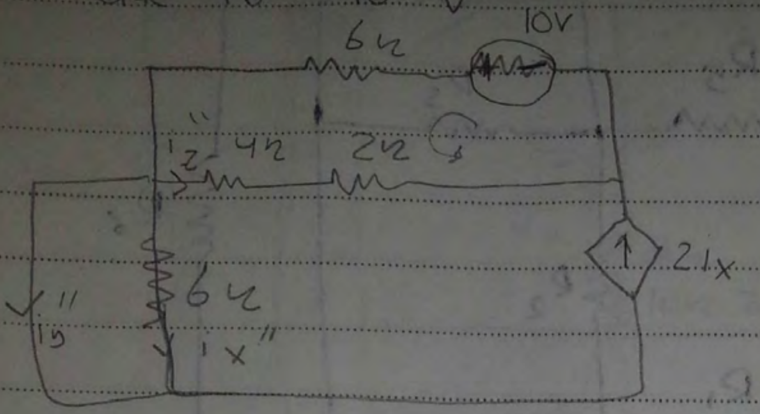
KVL $4i_z' + 2i_z' + 6(i_z' + 10) = 0$

$12 i_z' = -60$

$i_z' = -5A$

Subject:........................ 21 / 10 / 2014

due to 10 V



$i_x'' = \frac{0}{6} = 0\ A \Rightarrow 2i_x'' = 0$

kcl $I_y'' + \cancel{i_x''} = \cancel{2i_y''}$

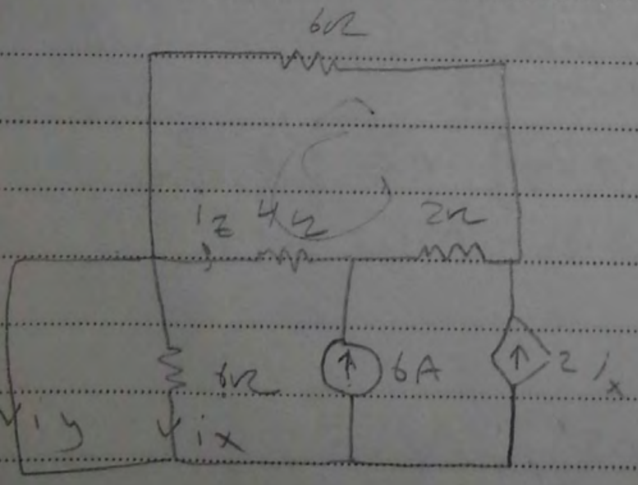
$I_y'' = 0\ A$

$4I_z'' + 2i_z'' - 10 + 6i_z'' = 0$

$12i_z'' = 10$

$i_z'' = \frac{10}{12}$

due to 6A



$i_x''' = 0$

Kcl

$i_y''' + i_x''' = 6 + 2i_x'''$

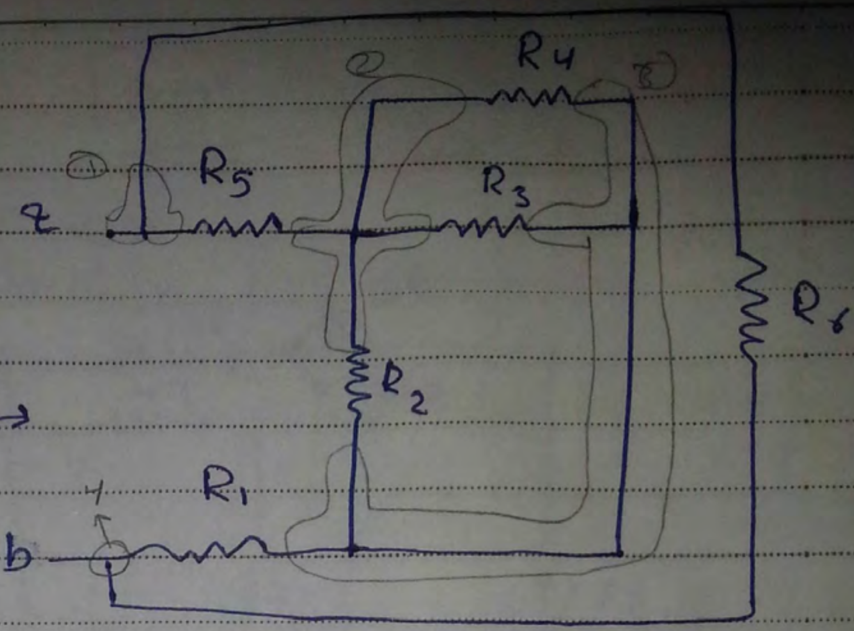
$\boxed{i_y''' = 6A}$

$kv = 4i_z''' + 2(6 + i_z'' + 6(6 + i_z''') = 0$

$i_z'''' = \frac{-48}{12}\ A$

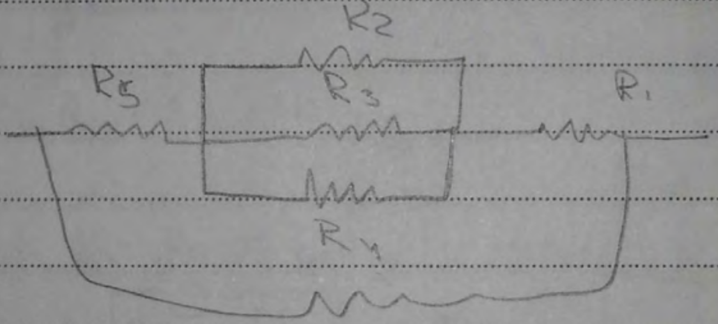
25 / 10 / 2014

Ex



Req ↳

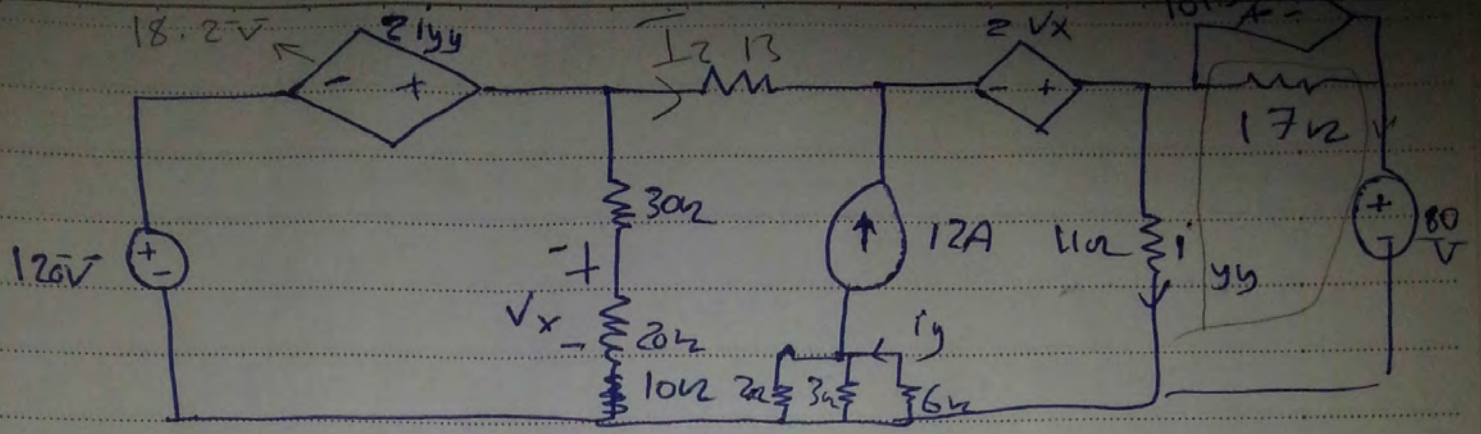
Req = ?



$$Req = \left[R5 + \left[R_2 \,//\, R_3 \,//\, R_4\right] + R_1\right] // R_6$$

بسم الله الرحمن الرحيم

Subject:........................ 21 / 10/ 2014



Sol :

$$i_y = 12 * \frac{\frac{1}{6}}{\frac{1}{2}+\frac{1}{3}+\frac{1}{6}} = 2A \longrightarrow 10 i_y = 20 V$$

KVL $11 i_{yy} - 80 - 20 = 0$

$$\boxed{I_{yy} = \frac{100}{11} A}$$

KVl $-120 - 18.2 + V_+ = 0$

$V_+ = 138.2 V$

$V_x = V_+ \frac{20}{20+3+10}$ $\boxed{= 46.1 V}$

Kvl $-120 - 18.2 + 13 * I_x - 2 * 46.1 + 20 V + 80 = 0$

$\boxed{I_x = 10.02 A}$

$I = I_2 - I_{yy}$

$= I_x + 12 - I_{yy}$

$= 10.02 + 12 - \frac{100}{11}$

$= 12.92 A$

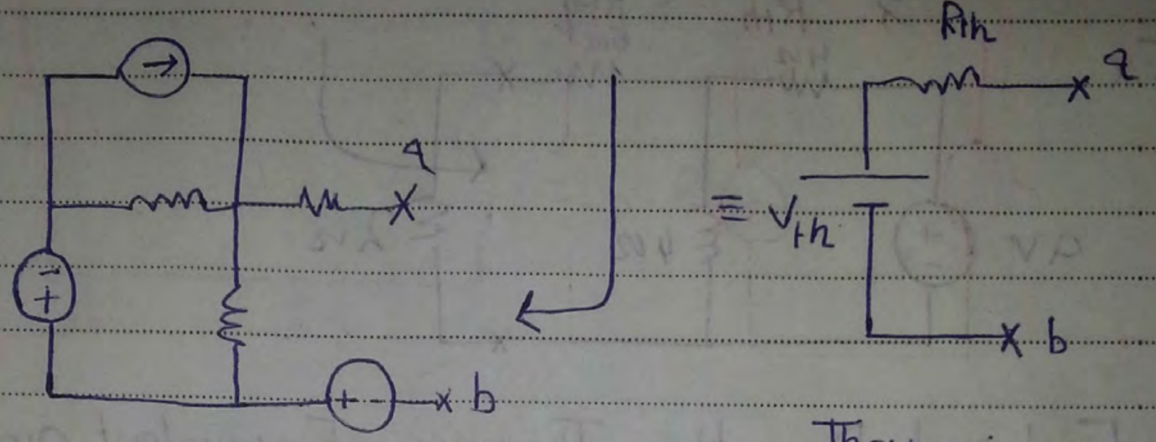
Pabsorbed (80 V) = I * V

= 80 * 12.92 A

P = 1033.6 W

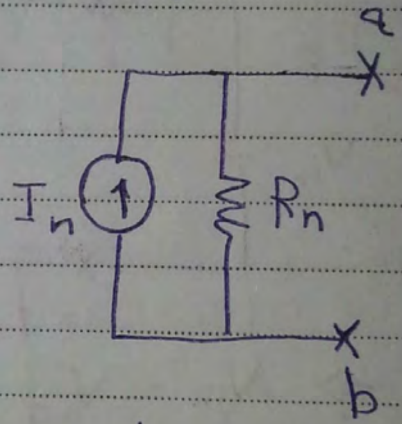
26/10/2014

Thevenin and Nortan Circuit :



Thevenin eq. circuit

|||



nortan eq. circuit

① Type 1 : The circuit has only independent Source.

② Type 2 : The circuit has dependent and independent Sources.

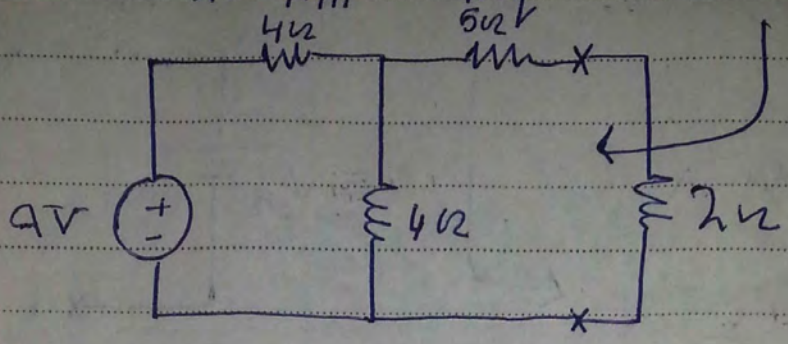
③ Type 3 : The circuit has only dependent Source.

Subject:.......................   26/10/2014

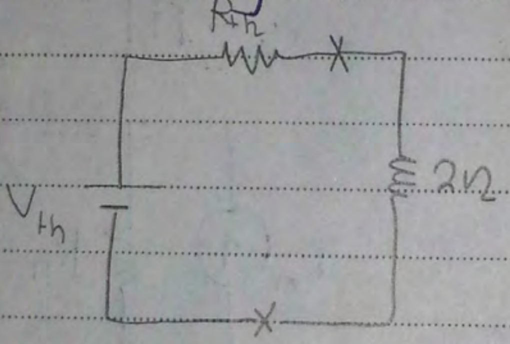
R_{th} => 1. kill all the source.

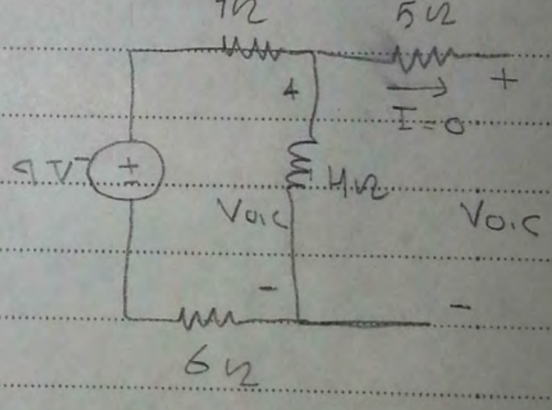
2. $R_{th} = R_{eq}$

Ex



Find and Draw the Thevenin Equivalent circuit seen by 2Ω.





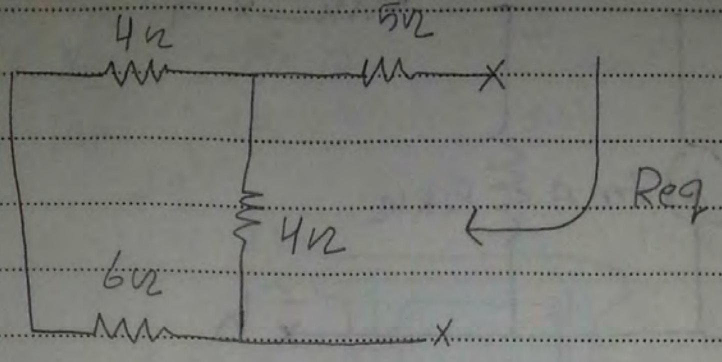
=> $V_{th} = V_{o.c}$

=> $V_{o.c} = \frac{9 * 4}{4+4+6}$

Ty be 1

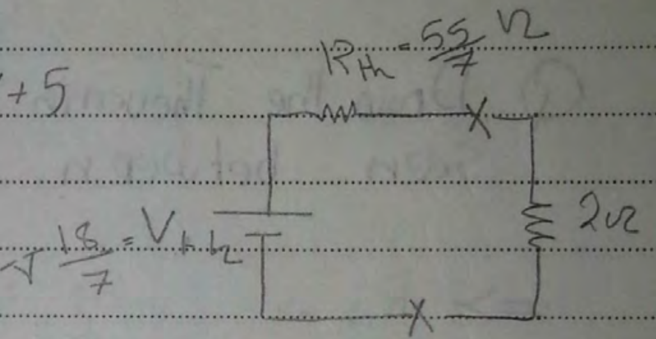
$V_{th} = V_{o.c} = \frac{18}{7}$ V

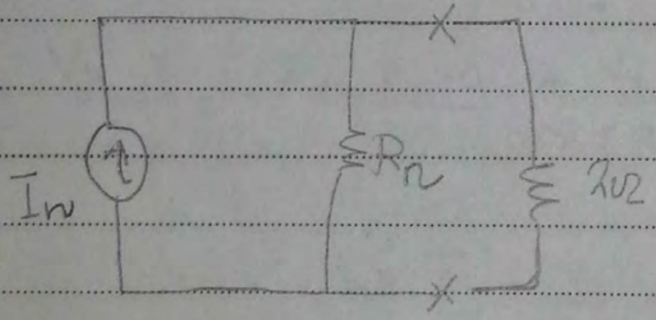
Rth ??



$R_{th} = Req = (6+4) // 4+5$

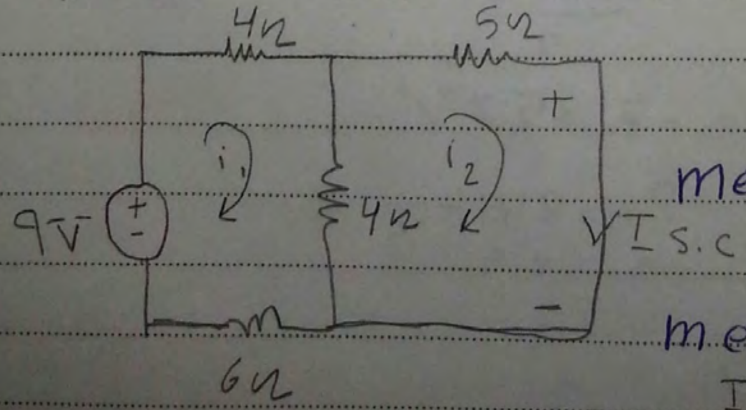
$R_{th} = \frac{55}{7}\ \Omega$





$\Rightarrow R_n = R_{th} = \frac{55}{7}\ \Omega$

$\Rightarrow I_N = I_{s.c}$ or $I_n = \frac{V_{th}}{R_{th}} \Rightarrow \frac{\frac{18}{7}}{\frac{55}{7}} = \frac{18}{55}\ A$



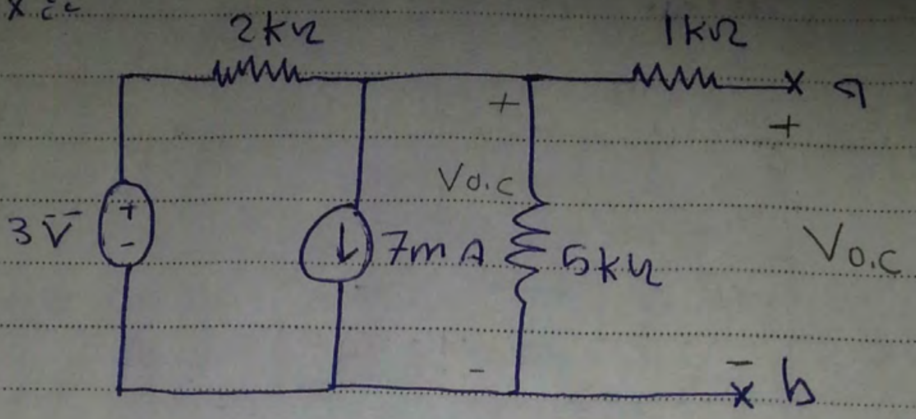
mesh₁: $-9 + 4I_1 + 4(I_1 - I_2) + 6I_1 = 0$

mesh₂: $5I_2 + 4(I_2 - I_1) = 0$

$I_{s.c} = I_2 = \frac{18}{55}\ A$

Subject: 26/10/2014

Ex:



Q Draw the Thevenin and Norton Equivalent Circuits seen between a & b

=> R_{th} => - kill all Sources
- Req =

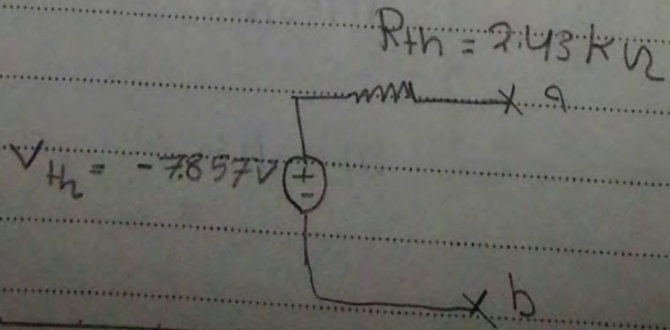
* Req = 2 // 5 + 1 = 2.43 kΩ

=> V_{th} = $V_{o.c}$

kcl at $V_{o.c}$:

$$\frac{V_{o.c}-0}{5} + 7 + \frac{V_{o.c}-3}{2} = 0$$

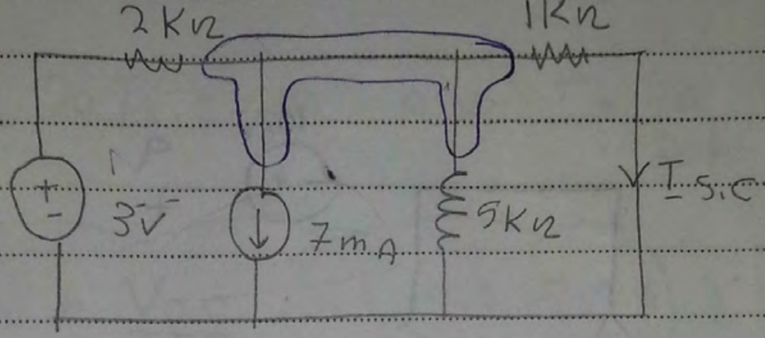
$V_{o.C} = V_{th} = -7.857\,V$



26/10/2014

$R_N = R_{th} = 2.45\ k\Omega$

$I_n = \frac{V_{th}}{R_{th}} = \frac{-55}{17} A$ or $I_n = I_{s.c}$



kcl at y εc

$$\frac{V_y - 3}{2} + 7 + \frac{V_y - 0}{5} + \frac{V_y - 0}{1} = 0$$

$$V_y = V \Rightarrow I_y = \frac{V_y}{1} = \frac{-55}{17}\ mA.$$

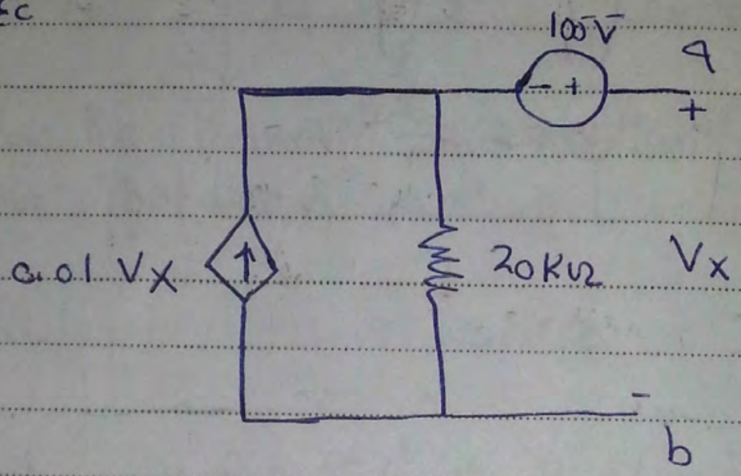
Subject:...................... 26/10/2014

Type 2: The circuit has independent and dependent sources.

$$R_{th} = \frac{V_{o.c}}{I_{s.c}}$$

* نقدر ان نطبقها على Tybe 1

Ex:



Q: Draw the Thevenin and norton equvelant circuits seen between a & b

=> Tybe 2:

=> $V_{o.c} = V_x$

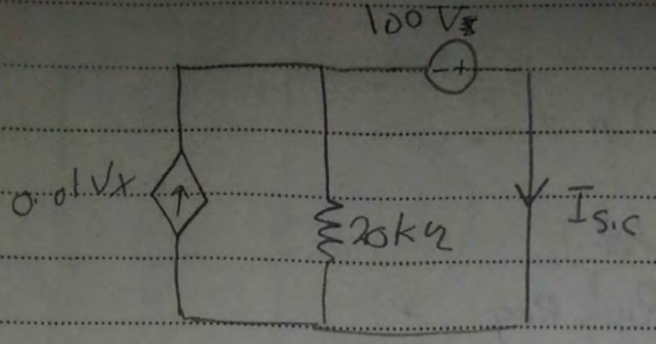
KVL => $-V_x + 100 + 20(0.01V_x) = 0$

$V_x = -0.503\ V$

$= V_{th} = V_{o.c}$

=> $I_{s.c}$ => ??

26/10/2014

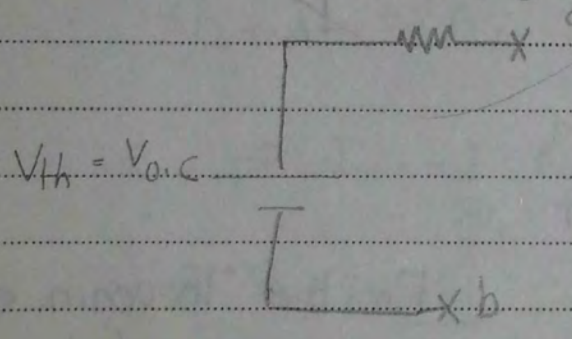


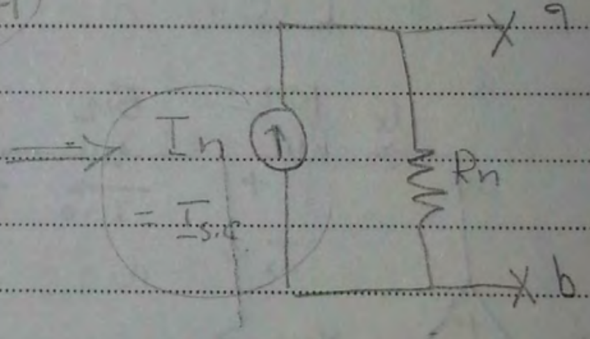
short circuit => $V_x = 0$ =>
0.01 $V_x = 0$ open circuit

KVL Eq. $20 I_{s.c} - 100 = 0 \Rightarrow I = \frac{100}{20k} = 5\ mA$

$$\boxed{\Rightarrow R_{th} = \frac{V_{o.c}}{I_{s.c}} = -100.5\ \Omega}$$

$= -100.5\Omega$





Subject:.................. 28/10/2014

$V_{th} = V_{o.c}$ $I_n = I_{s.c}$

$R_{th} =$

① Tybe 1 : Ⓐ kill

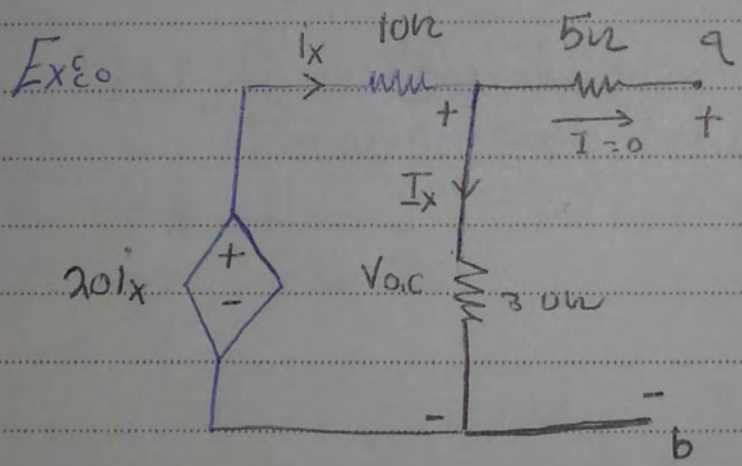
Ⓑ $R_{th} = R_{eq}$

② Tybe 2 : $R_{th} = \frac{V_{o.c}}{I_{s.c}}$

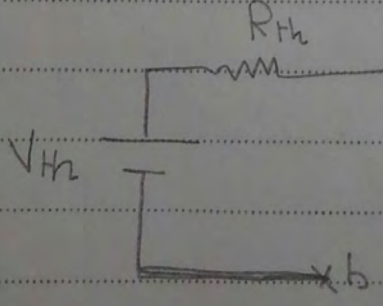
③ Tybe 3 : the circuit has only dependent source.

$R_{th} = \frac{\cancel{V_{test}}^{=1\bar{V}}}{I}$ or $R_{th} = \frac{V}{\cancel{I_{test}}^{=1A}}$

Ex:



Find the Thevenin eq Circuit seen betwen a & b



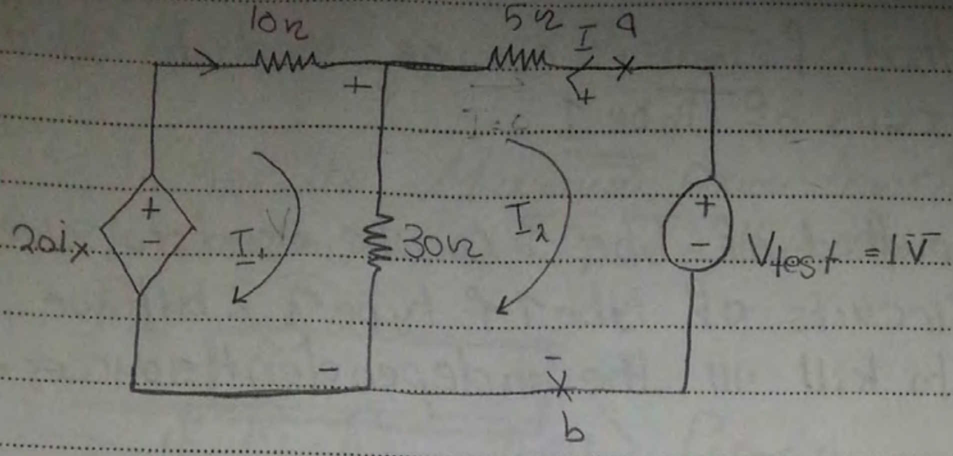
$V_{th} = V_{o.c} = 30 i_x$

KVL : $-20 i_x + 10 i_x + 30 i_x = 0$

$\Rightarrow i_x = 0 A$

$V_{th} = 0\bar{V} \leftarrow$

28/10 /2014



Tybe 3

$R_{th} = \frac{V_{test}}{I} \Rightarrow$

$(I_1 - I_2)$

$-20 i_x + 10 i_1 + 30 (I_1 - I_2) = 0$ ---①

$5 i_2 + 1 + 30 (I_2 \; I_1) = 0$ --②

$\Rightarrow I_2 = -\frac{1}{20}$ A $\qquad I \Rightarrow -I_2 = \frac{1}{20}$ A

$* \; R_{th} = \frac{\overset{1}{V_{test}}}{\frac{1}{20} \leftarrow I} = 20 \Omega$

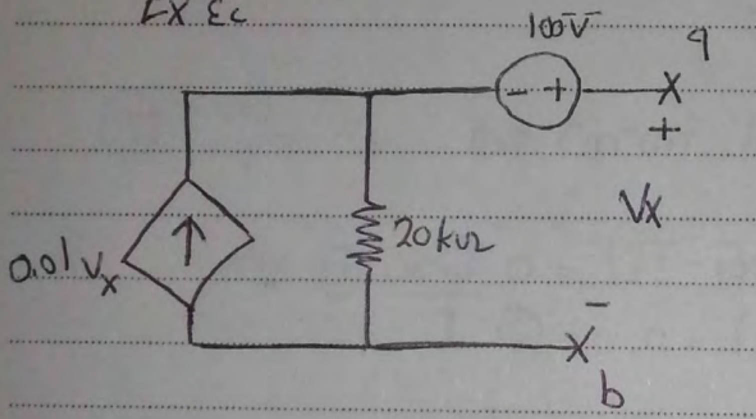
Subject:

28/10/2014

Note : ① The method of Tybe 2 can be used to solve the circuits of Tybe 1.

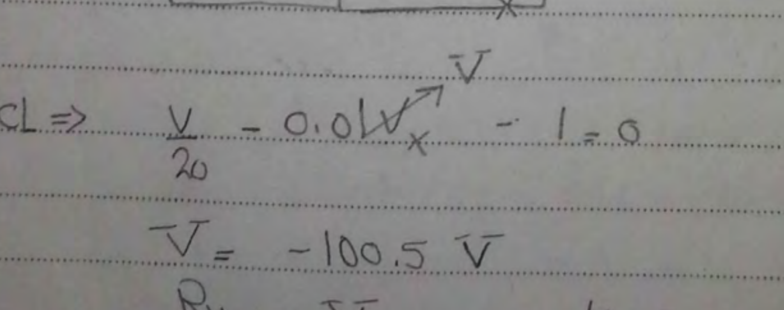
② The method of Tybe 3 can be used to solve the circuits of tybe 1 & tybe 2. but we have to kill all the independent sources.

Ex :



This circuit is tybe 2 circuit but, we can solve it using the method of tybe 3 :

⟹ kill all independent source



$R_{th} = \frac{V}{I}$ = 1A test

$$KCL \Rightarrow \frac{V}{20} - 0.01\cancel{V_x}^{\;V} - 1 = 0$$

$$V = -100.5\ V$$

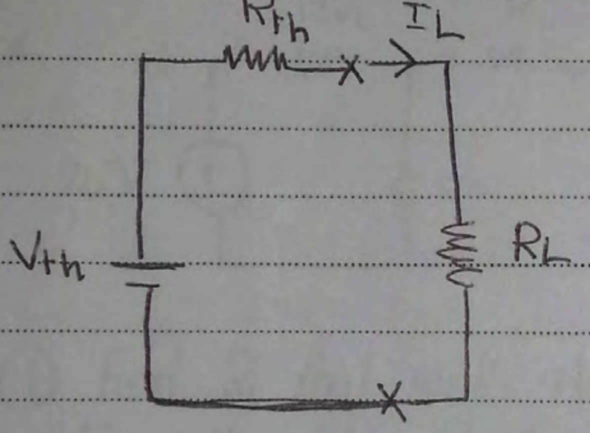
$$R_{th} = \frac{V}{\underset{1}{\cancel{I}}_{tes}} = -100.5\ \Omega$$

28/10 /2014

✱ Maximum power transfer:

To get the Maximum power transfer From the circuit to a certain load (R_L) in this circuit, R_L must equal R_{th}

⇒ if $R_L = R_{th}$, then $\overset{\text{we get}}{\nwarrow}$ $P_{L(max)}$ why ??!:



$$P_L = I_L^2 R_L$$

$$= \left(\frac{V_{th}}{R_{th} + R_L}\right)^2 R_L$$

$$= \frac{V_{th}^2 R_L}{(R_{th} + R_L)^2}$$

$$\frac{dP_L}{dR_L} = \frac{V_{th}^2 (R_{th} + R_L)^2 - 2(R_{th} + R_L) V_{th}^2 R_L}{(R_{th} + R_L)^4}$$

$$\Rightarrow \cancel{V_{th}^2}(R_{th} + R_L)^2 = 2(R_{th} + R_L)\cancel{V_{th}^2} R_L$$

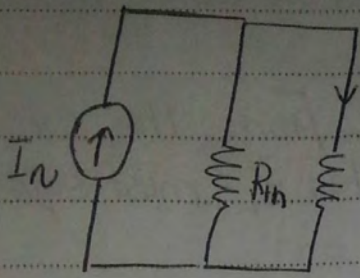
$$R_{th} + R_L = 2R_L$$

$$R_{th} = R_L \quad ✱$$

$$P_{L(max)} = \left(\frac{V_{th}}{R_{th} + R_{th}}\right)^2 R_{th}$$

$$P_{L(max)} = \frac{V_{th}^2}{4 R_{th}}$$

28/10/2014



$I_L = \frac{1}{2} I_n$

$R_L = R_{th} \Rightarrow P_{L(max)} = I_L^2 R_L$ *

$P_{L\,max} = \frac{I_N^2}{4} R_{th}$

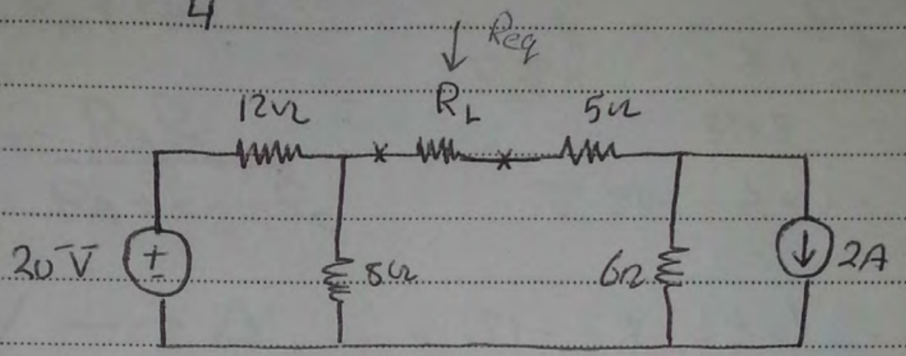
30/10/2014

$\Rightarrow P_{max}$ if $R_L = R_{th}$

$$\Rightarrow P_{max} = \frac{V_{th}^2}{4R_{th}}$$

$$= \frac{I_N^2 \; R_{th}}{4}$$

Ex

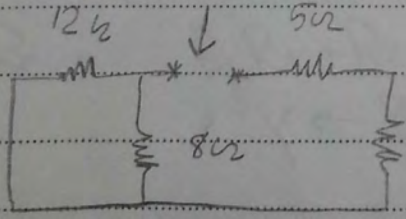


① Find R_L that will absorbs the maximum power From the circuit (P_{max})

② Find P_{max}

Sol:

$R_L = R_{th}$



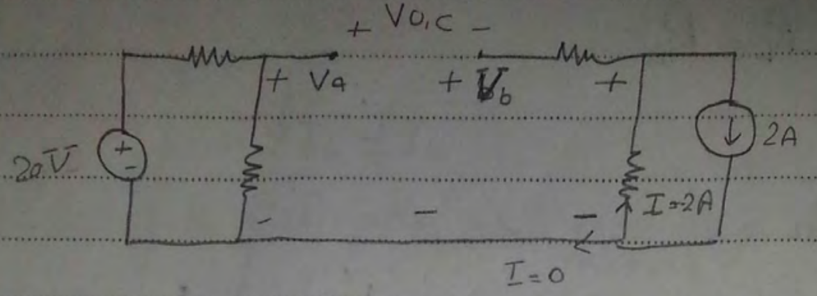
$R_{eq} = (12 // 8) + 5 + 6$

$= 15.8\ \Omega$

$$* \; P_{max} = \frac{V_{th}^2}{4R_{th}}$$

$V_{th} = V_{o.c}$

Subject:.......................... 30/10/2014



short circuit مع التوازي
I=0

$V_{o.c} = V_a - V_b$

$$V_a = \frac{20 * 8}{8+12} = 8\ V$$

$$V_b = -2 * 6 = -12\ V$$

$$V_{th} = V_{o.c} = 8 - -12 = 20\ V$$

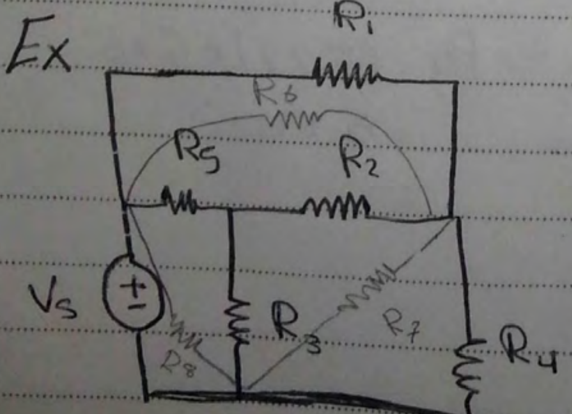
$$P_{max} = \frac{20^2}{4*15.8} = \underline{\underline{6.329\ W}}$$

$\Delta \rightarrow Y \ , \ Y \rightarrow \Delta$

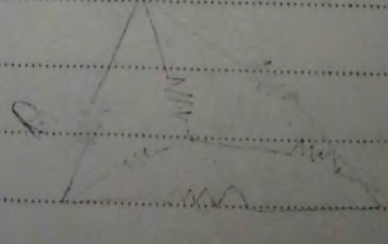
Y => ثلاث مقاومات مشبوكين
نقطة وحدة Node.

✱ Conversions 3)

Ex



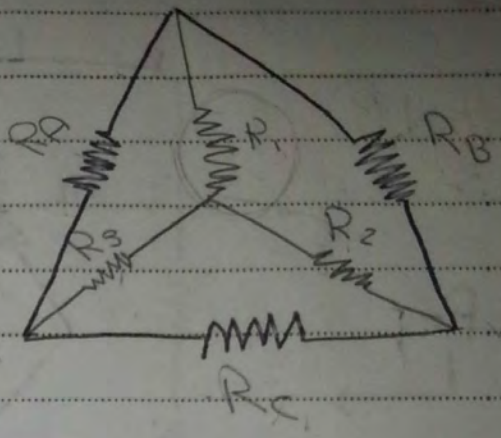
$\Delta \rightarrow Y$:



* No Parallel
* No series

Subject:........................ 30/10/2014

$\Delta \rightarrow Y$



$$R_1 = \frac{R_A R_B}{R_A + R_B + R_C}$$

$$R_2 = \frac{R_B R_C}{R_A + R_B + R_C}$$

$$R_3 = \frac{R_A R_C}{R_A + R_B + R_C}$$

$Y \longrightarrow \Delta$

$$R_A = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

$$R_B = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

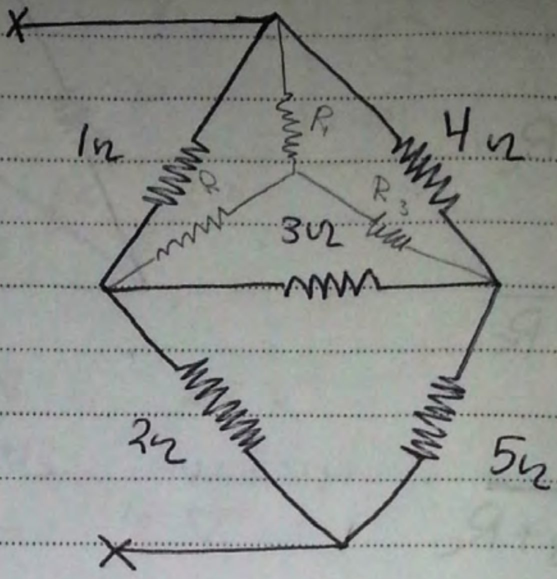
$$R_C = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

Subject:........................ 30 / 10/ 2014

Ex

Method ①

Req ↳



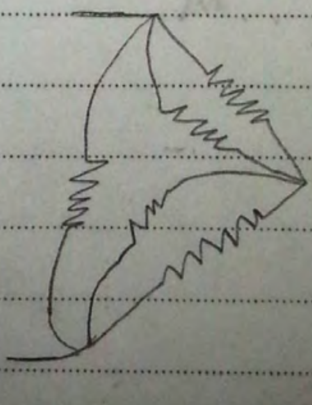
$$R_1 = \frac{4*1}{1+3+4} = \frac{4}{8}\,\Omega$$

$$R_2 = \frac{3*1}{1+3+4} = \frac{3}{8}\,\Omega$$

$$R_3 = \frac{4*3}{4+1+3} = \frac{12}{8}\,\Omega$$

$$R_{eq} = \left[(R_2+2) // (R_3+5)\right] + R_1$$

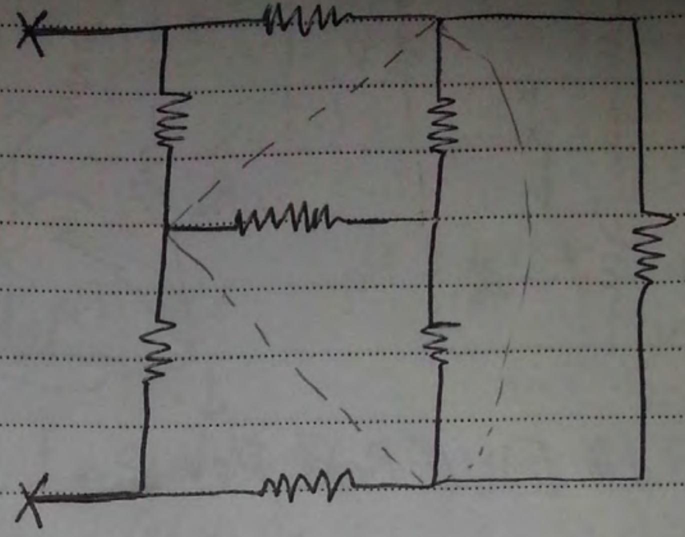
Method ②



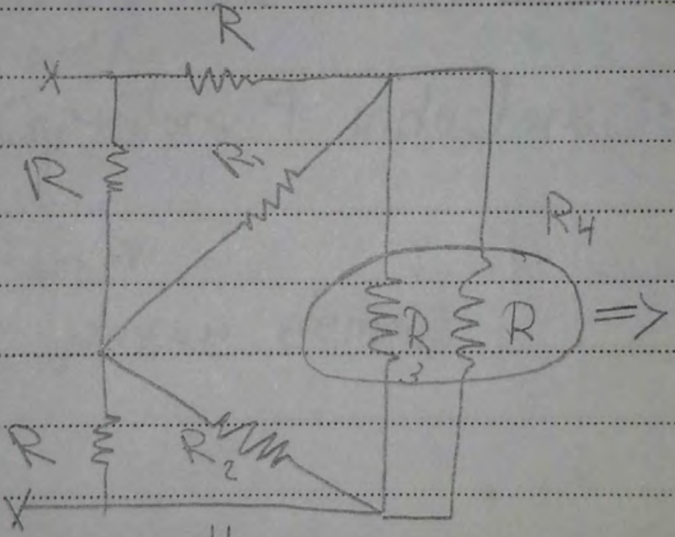
30/10/2014

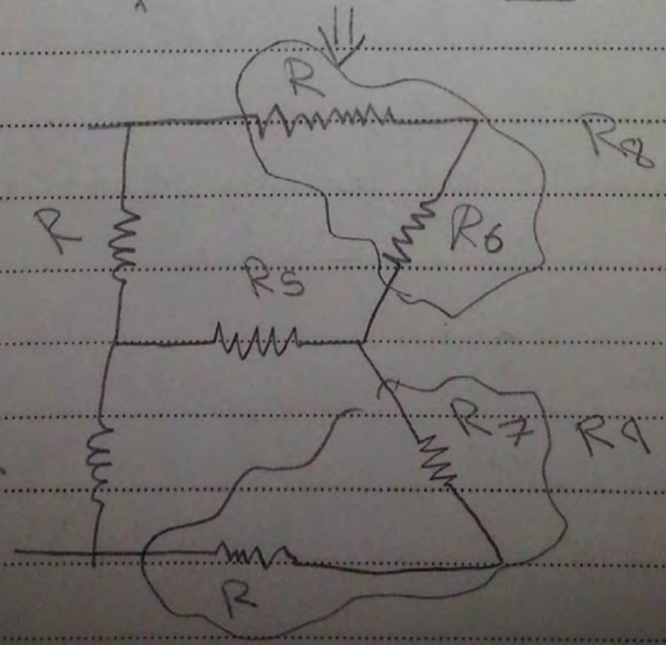
Exe

Req ↳



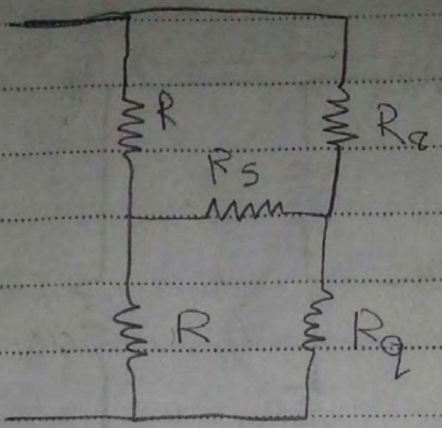
answer : $R_{eq} = \frac{8}{7} R$



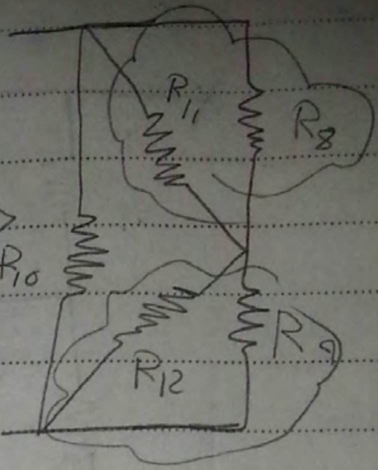


30/10/2014

=>

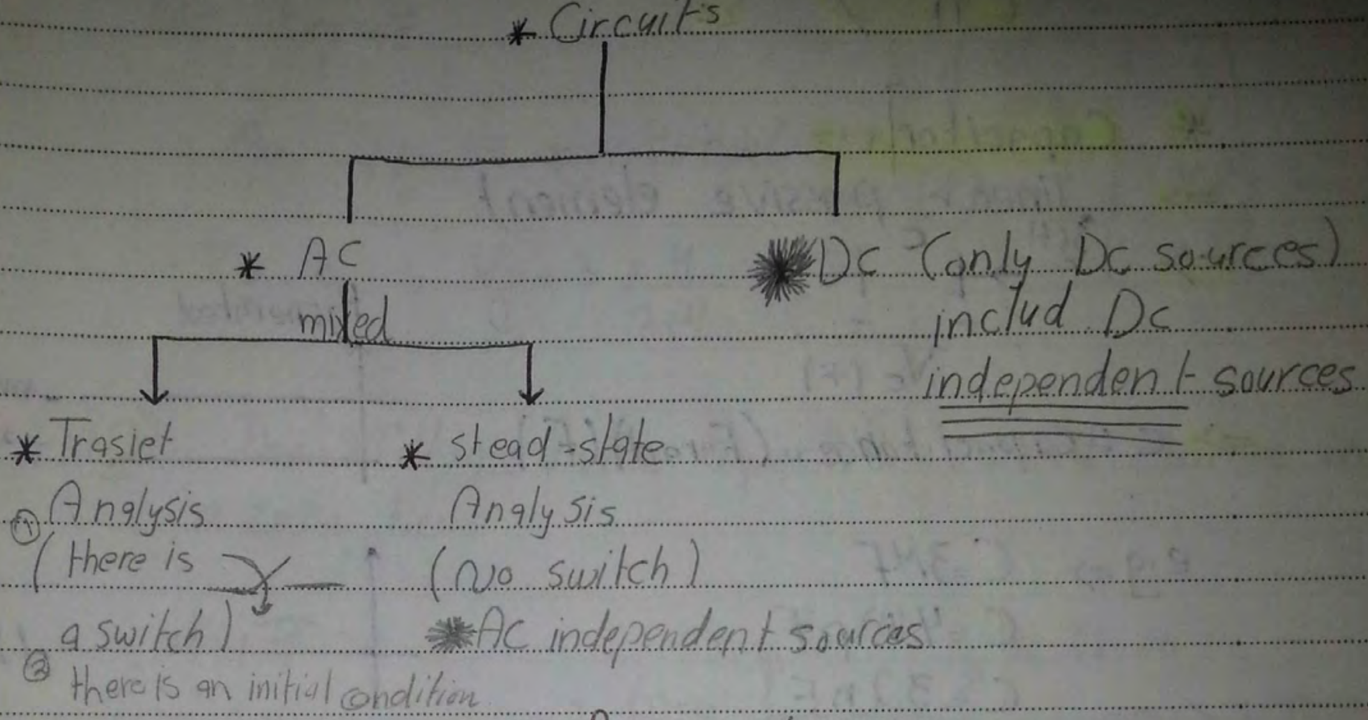


=>



$R_{eq} => R_{10} // [R_{11} // R_8 + R_9 // R_{12}]$

30 / 10/2014



* Circuits

* AC
mixed

* Dc (only Dc sources)
includ Dc
independent sources

* Trasiet
Analysis
① (there is a switch)
② there is an initial condition

* stead-state
Analysis
(No switch)
* AC independent sources

* CH 7: Capacitors & inductors :

* Capacitor :
=> linear passive element

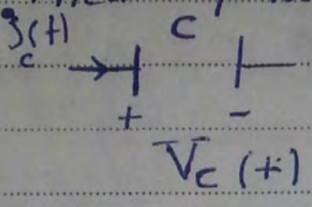
2/11/2014

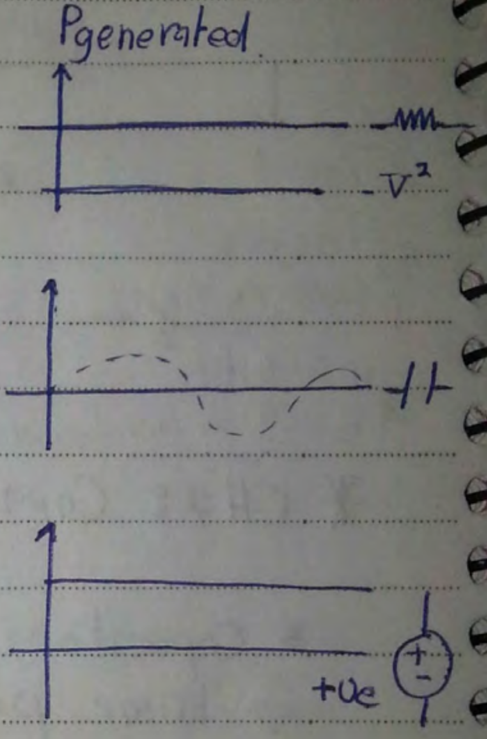
CH 7 Ɛ:

* Capacitor Ɛ:-

⇒ linear passive element

⇒





⇒ C : Capacitance (Farad)(F)

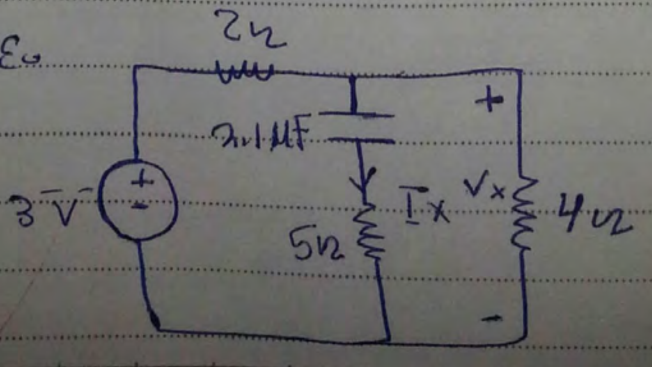
e.g ⇒ C=3μF
C=4.1 pF
C=3.2nF
C= 1F

$$\Rightarrow I_c(t) = C\frac{dv_c(t)}{dt}$$

* مهم Notes Ɛ:

① The capacitor in the Dc circuits is considered as open circuit, why ?!
Since $V_c(t)$ = Constant ⇒ $I_c(t) = C\frac{dV_c(t)}{dt} = 0A$

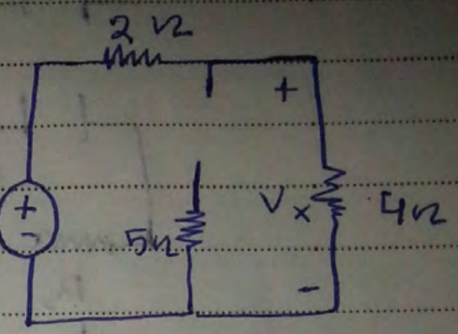
* مهم Ex Ɛ:



⇒

2/11/2014

DC circuit

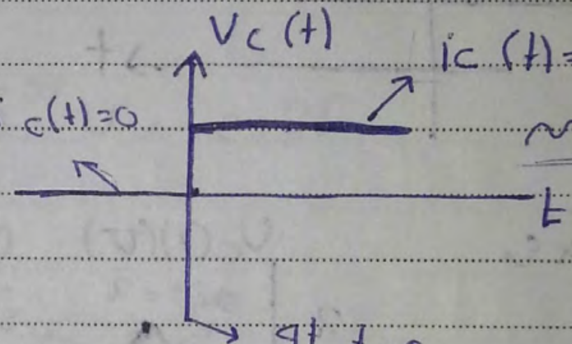


So $\dashv\vdash$ ≡ open circuit ≡

$I_x = 0$, $V_x = 3 * \frac{4}{2+4} = 2\bar{V}$

② The Voltage on the capacitor cann't change in a zero time

Why ?



ic (t) = 0 ~~> wrong

$i_c(t) = c\,\frac{dv(t)}{dt}$

slop

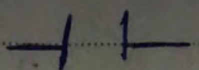
at t = 0

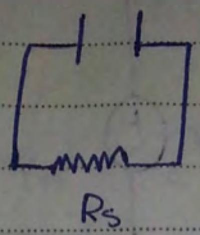
slop = $\frac{3}{0}$ = ∞

$i_c(t) = \infty$ => impossible

$V_c(0^-) = \bar{V}_c(0^+) = V_c(0)$

3 /11 /2014

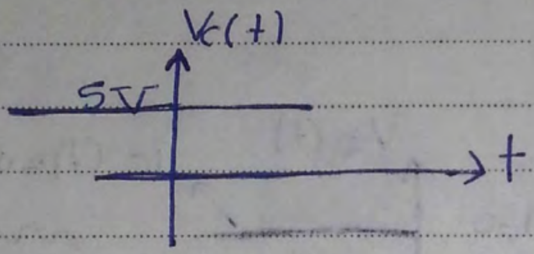
*  => ideal ✓



=> practical

↓
internal resistance ≈ ∞

Ex: find $i_c(t)$, if $C = 2F$

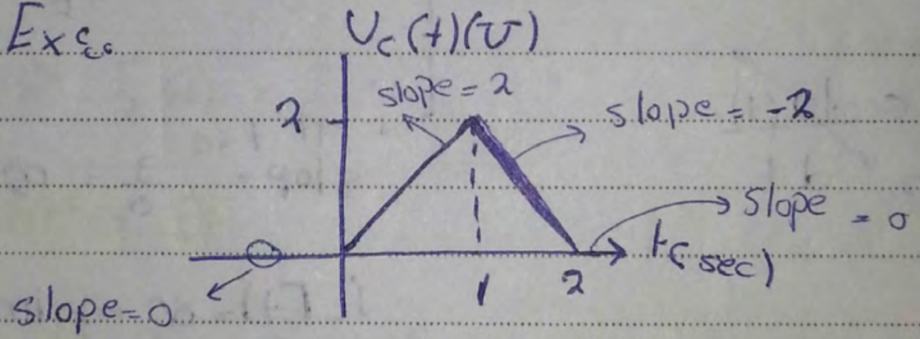


$i_c(t) = C \frac{dv_c(t)}{dt}$ → slope

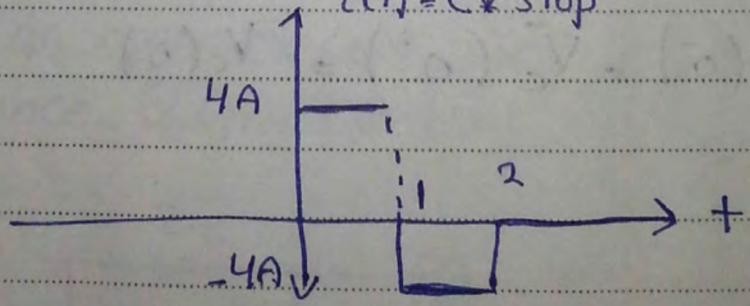
$= 0$

Ex:

$C = 2F$



$i_c(t) = C \times \text{slop}$



Ex: If $v_c(t) = 5\sin(\pi t)$

Sol: $C = 2F$, Find $i_c(t)$

$$i_c(t) = C \frac{dv_c(t)}{dt}$$

$$= 2 * 5\pi\cos(\pi t)$$

$$i_c(t) = 10\underset{\underset{\frac{22}{7}}{\Downarrow}}{\pi}\cos(\pi t) \Leftarrow$$

Ex: $V_c(t) = 2e^{-5t}U$, $C = 2F$

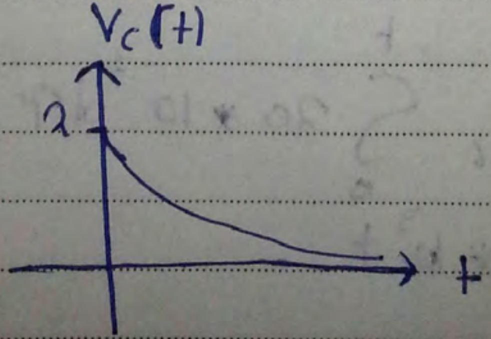
$I_c(t) = ??$

Sol:

$$I_c(t) = C * \frac{dV_c(t)}{dt}$$

$$I_c(t) = 2*(-2*5e^{-5t})$$

$$I_c(t) = -20e^{-5t}$$ ⇒ discharging



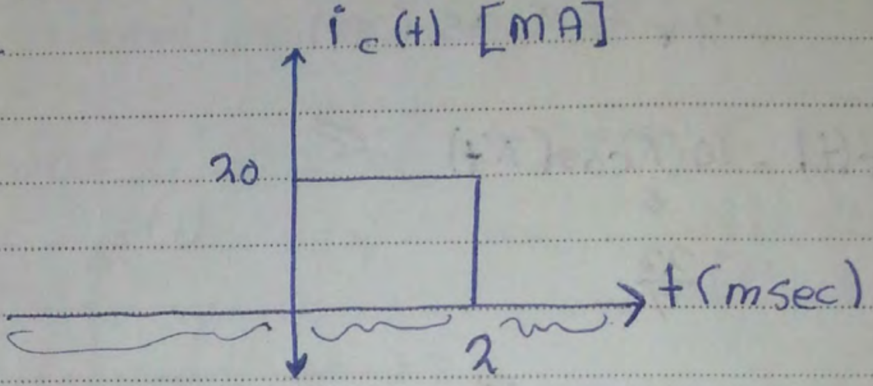
Subject: 2/11/2014

$$\Rightarrow \quad V_c(t) = \left[\frac{1}{C}\int_{t_0}^{t} i_c(\tau)\,d\tau\right] + V_c(t_0)$$

After we put them in the Circuit

initial value of $V_c(t)$
zero غالباً

مهم Ex ع



$C = 5 \mu F$ Draw $V_c(t)$

① نقسم الرسمة لفترات

For $-\infty \le t \le 0$ msec.

$$V_c(t) = \frac{1}{C}\int_{-\infty}^{t} i_c(\tau)\,d\tau + \overset{0}{\cancel{V_c(-\infty)}}$$

$$V_c(t) = 0\,V$$

For $0 \le t \le 2$

$$V_c(t) = \frac{1}{5*10^{-6}}\int_{0}^{t} 20*10^{-3}\,d\tau + \overset{0}{\cancel{V_c(0)}}$$

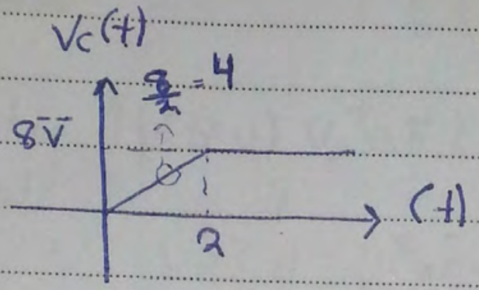
$$V_c(t) = 4*10^{3}\,t$$

Subject:........................ 2/11/2014

✱ For $2 \leq t \leq \infty$

$$V_c(t) = \cancelto{0}{\frac{1}{C}\int_{2}^{t} i_c(\tau)\, d\tau} + \cancelto{8\bar{V}}{V_c(2)}$$

$$V_c(t) = 8\bar{V}$$



4/11/2014

* Stored Energy in acapacitor :

$$W_c(t) = \frac{1}{2} C \left[V_c^2(t) - V_c^2(t_0) \right] + W_c(t_0) \quad (\text{Joule})$$

$V_c^2(t_0)$ ↓ initial value — $W_c(t_0)$ ↓ initialvalue — Joule ↓ J

⇒ if $V_c(t_0) = 0$ & $W_c(t_0) = 0$

$$W_c(t) = \frac{1}{2} C V_c^2(t) \quad (J)$$

* $P_c(t) = i_c(t)\, V_c(t)$ W (watt)

$$P_c(t) = \frac{dW_c(t)}{dt} \quad W$$

* فترة معينة

* energy with time

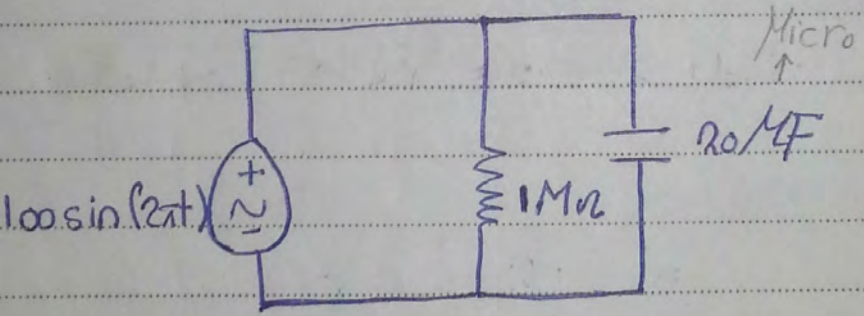
* كلما قل watt تكون استهلاكها اقل

* لمبات التوفير غالباً بتكون 23 watt

* اللمبات العادية بتكون 75 watt

$\overset{\uparrow}{P}$ – $\overset{\uparrow}{I}$ – V ثابت

Ex :



* Find the Maximum energy stored in Capacitor ove the interval $0 \leq t \leq 0.5$ sec. also Find the dissipated energy in the resisto during the same interval ?!

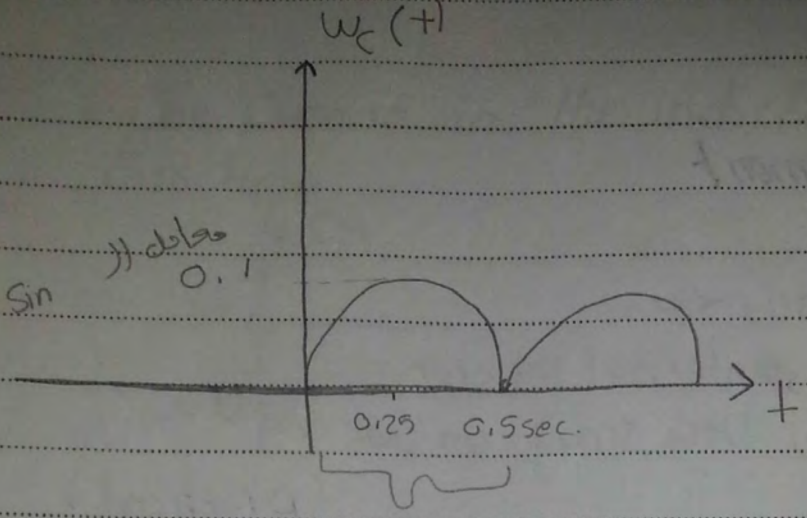
Sol ⇒

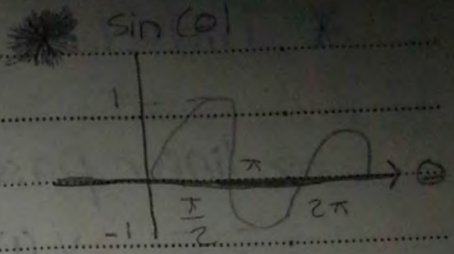
$$W_c(t) = \frac{1}{2} C \, V_c^2(t)$$

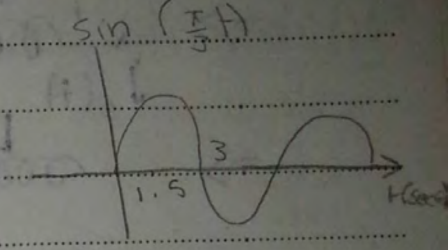
$$= \frac{1}{2} * 30 * 10^{-6} * (100 \sin(2\pi t))^2 \Rightarrow$$

$$= 0.1 \sin^2(2\pi t) \quad J$$

Subject:.......................   4/11/2014







$\frac{\pi}{3}t = \pi$

$t = 3sec.$

$\Rightarrow P_R(t) = i_R(t) \cdot V_R(t)$

$= \frac{V_R^2(t)}{R} = \frac{(100 \sin(2\pi t))^2}{1 * 10^6}$

$= 0.01 \sin^2(2\pi t)$ W

$\Rightarrow W_R = \int_0^{0.5} P_R(t)\, dt = \int_0^{0.5} 0.01 \sin^2(2\pi t)\, dt =$

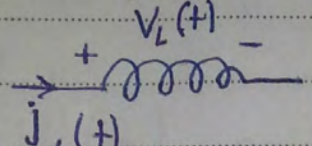
$0.01 \int_0^{0.5} \frac{1}{2} [1 - \cos(4\pi t)]\, dt$

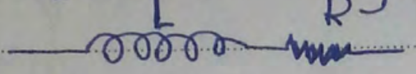
$= 2.5$ mJ

4/11/2014

* Inductors:

=> linear passive element

=> 

=> 

→ * internal resistance
Very small value ≈ 0Ω
(practical)

=> $V_L(t) = L \frac{di_L(t)}{dt}$

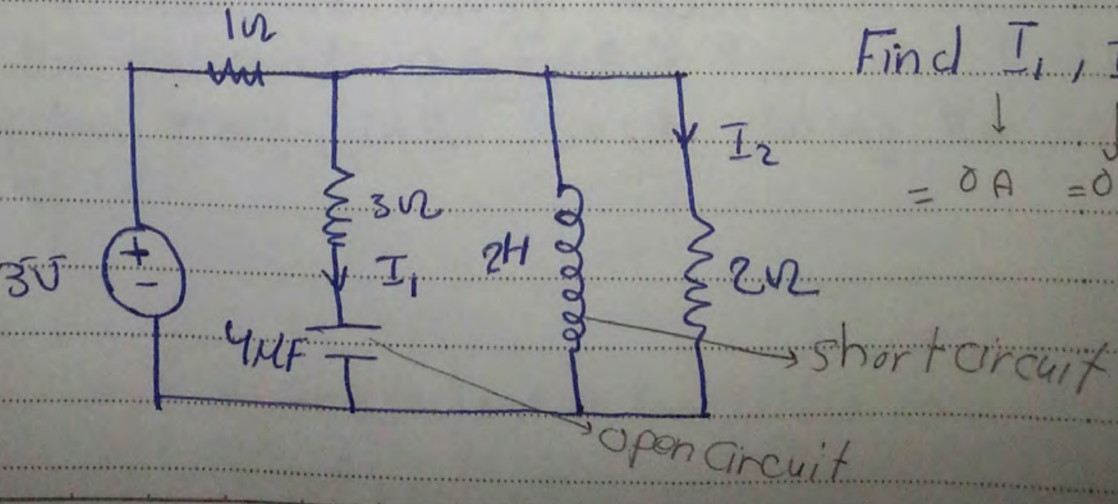
=> $i_L(t) = \frac{1}{L}\int_{t_0}^{t} v_L(\tau)\,d\tau + i_L(t_0)$

=> L; inductance [Henry] => H

* In Dc circuit, we consider the inductor as Short - circuit

=> $U_L(t) = L \frac{\overset{\text{constant}}{\cancel{di_L(t)}}}{dt} = 0$ => Shortcircuit

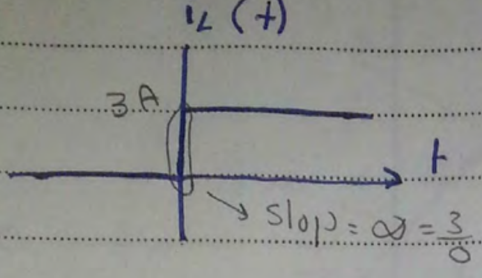
Ex:



Find I_1 , I_2
↓ ↓
= 0A = 0A

4/11/2014

* The Current in the inductor Cannot chang in a Zero time



=> wrong, because $V_L(t) = \frac{di_L(t)}{dt}$

↓

Slop

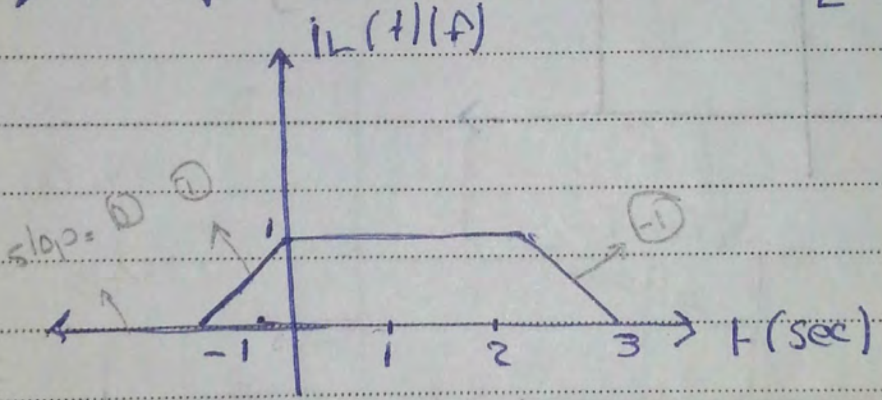
↓

∞

$V_L(t)$. → Impossible

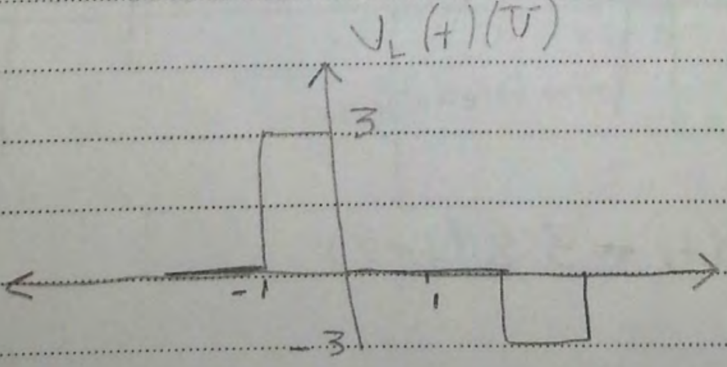
6/11/2014

Ex: if L = 3H , Find $V_L(t)$



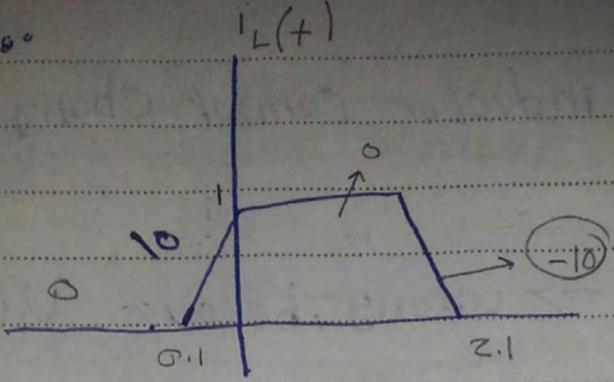
$V_L(t) = L\frac{di_L(t)}{dt}$

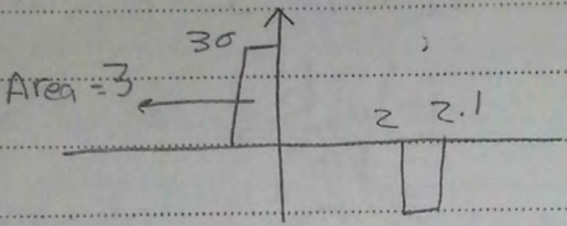
= L * Slop



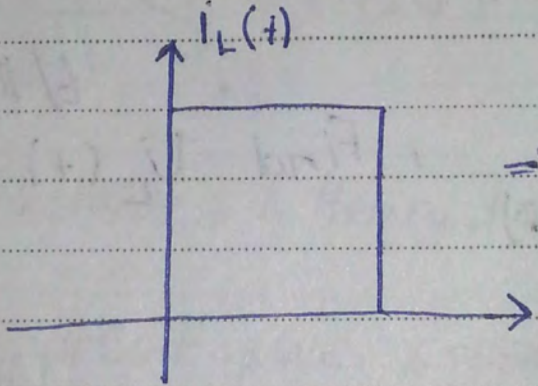
Subject:........................ 6 / / 2014

Ex :

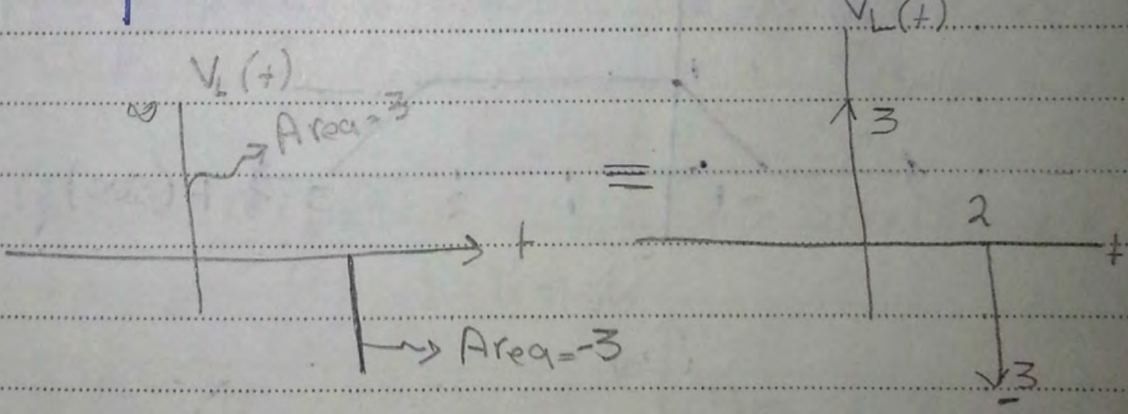




Ex ε.



=> wrong



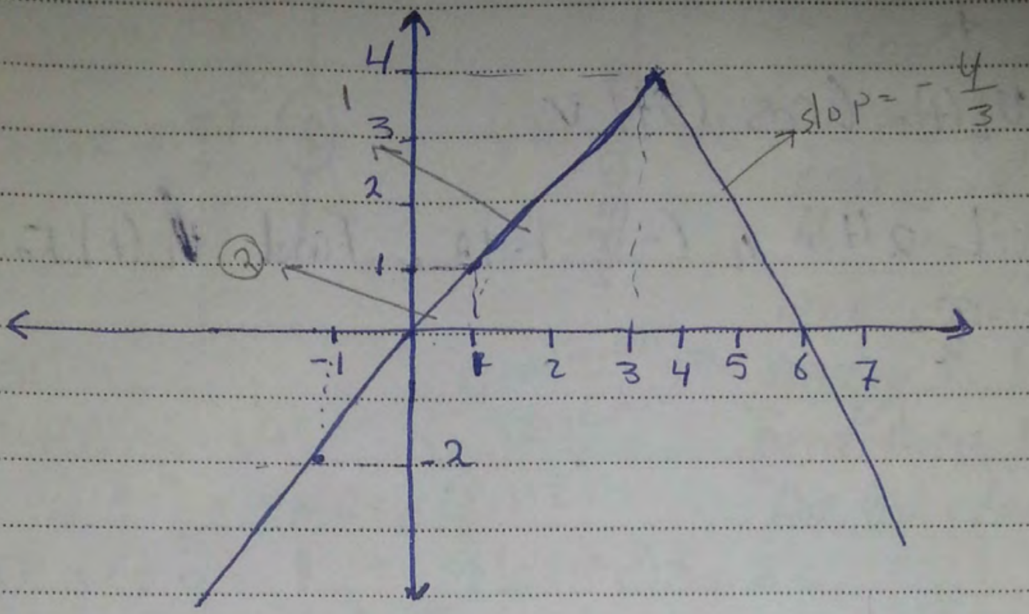
$v_L(t) = 3\,\delta(t) - 3\,\delta(t-2)$

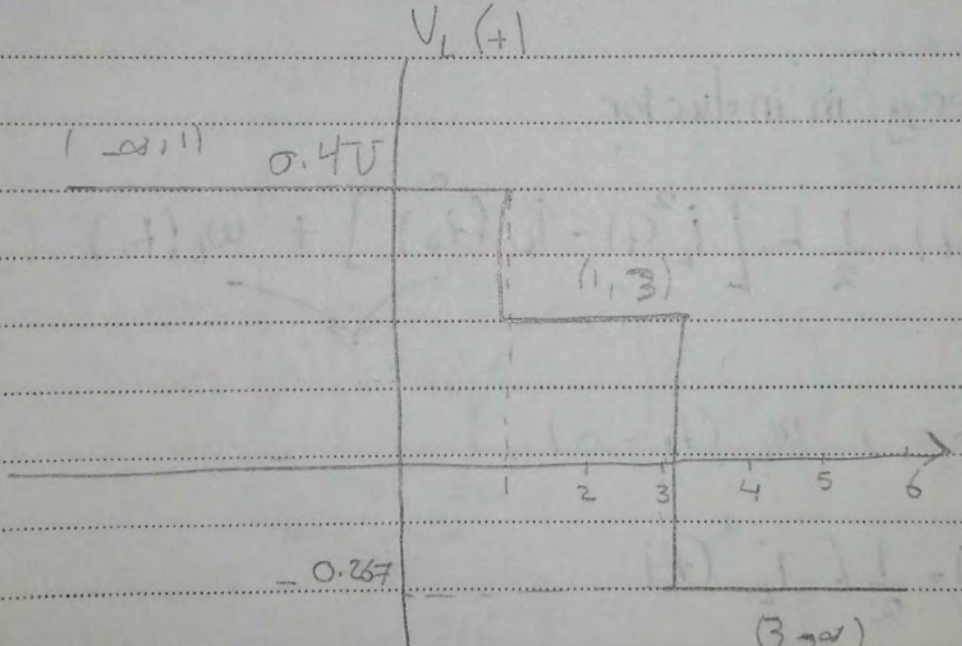
↙

impulse Function

6/11/2014

Ex if $L = 200\ mH$, Find $V_L(t)$





Subject:........................ 6/11/2014

$$* \quad i_L(t) = \frac{1}{L}\int_{t_0}^{t} v_L(\tau)\,d\tau + i_L(t_0)$$

Ex: if $v_L(t) = 6\cos(5t)\ V$,

and $L = 2H$, $i_L(-\frac{\pi}{2}) = 1A$, Find $i_L(t)$ Fo $t \geq -\frac{\pi}{2}$

Sol: $t_0 = -\frac{\pi}{2}$

$$\Rightarrow i_L(t) = \frac{1}{2}\int_{-\frac{\pi}{2}}^{t} 6\cos(5\tau)\,d\tau + 1 = 0.6\sin 5t + 1.6A$$

* stored energy in inductor

$$w_L(t) = \frac{1}{2}L\left[i_L^2(t) - i_L^2(t_0)\right] + w_L(t_0) \quad (J)$$

initial values

if $i_L(t_0) = 0$, $w_L(t_0 = 0)$

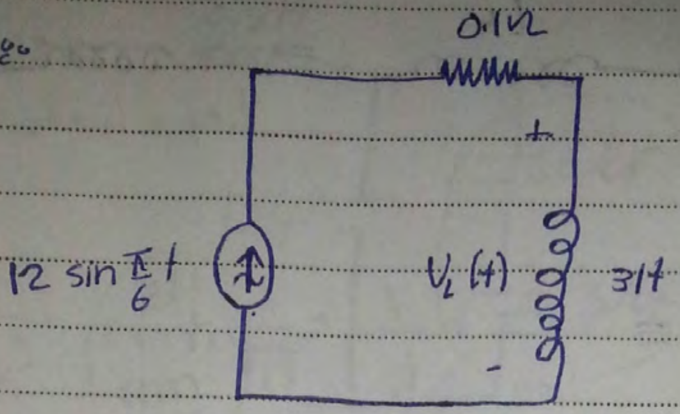
then

$$w_L(t) = \frac{1}{2}L\, i_L^2(t)$$

$$* \quad p_L(t) = \frac{dw_L(t)}{dt} = i_L(t)\, v_L(t)$$

Subject:.......................   6/11/2014

Ex:



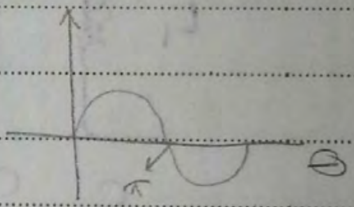
ⓐ Find the maximum energy stored in the inductor and ⓑ the energy dissipated in the resistor in the time during which the energy is being stored and then recovered from the inductor.

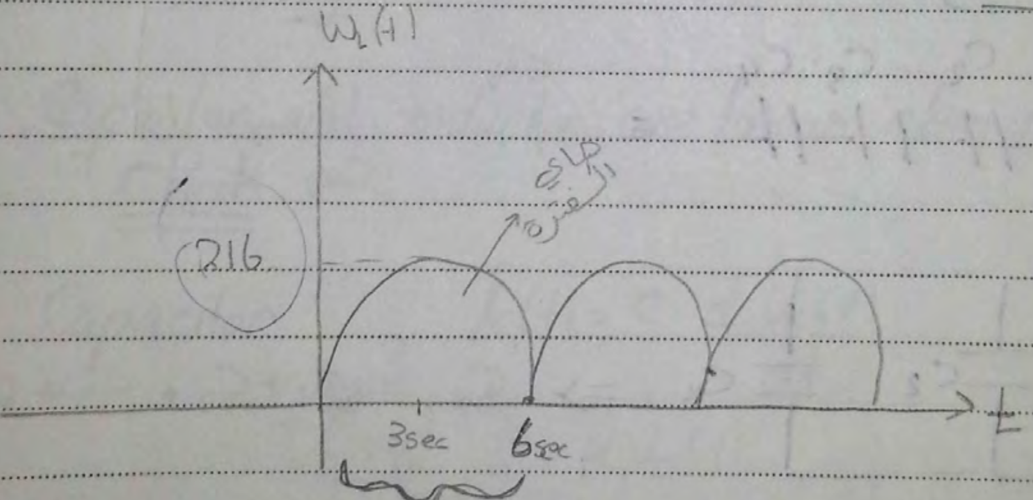
Sol:

$W_L(t) = \frac{1}{2} L\, i_L^2(t)$ ← الفترة اللي شحن الطاقات وفقدها

$= \frac{1}{2} * 3 * (12 \sin(\frac{\pi}{6} t))^2$

$= 216 \sin^2(\frac{\pi t}{6})$ J





ⓐ $W_{L(maximum)} = 216$ J

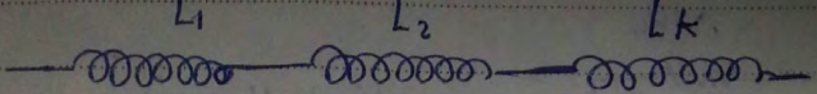
ⓑ $P_R(t) = I_R^2(t) R$

$= (12 \sin(\frac{\pi}{6} t))^2 * 0.1$

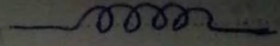
$= 14.4 \sin^2(\frac{\pi}{6} t)$ W

$W_R(t) = \int_0^6 14.4 \sin^2(\frac{\pi}{6} t)\, dt \Rightarrow 43.2$ J

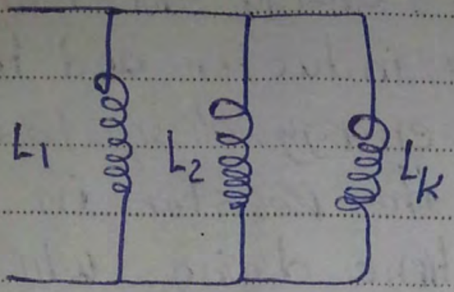
Subject: 6/11 2014



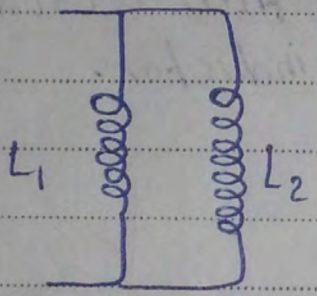
≡



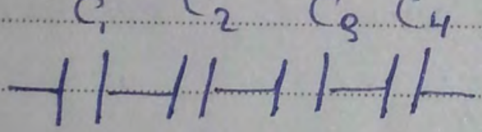
$$L_{eq} = L_1 + L_2 + \cdots + L_k$$



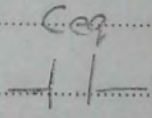
$$\equiv \frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_k}$$



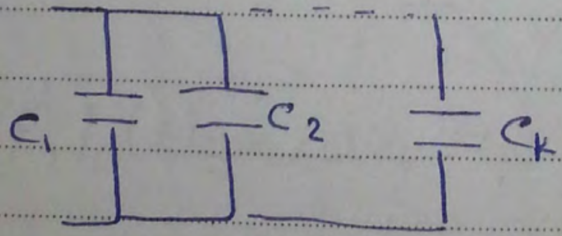
$$\Rightarrow L_{eq} = \frac{L_1 \times L_2}{L_1 + L_2}$$



≡



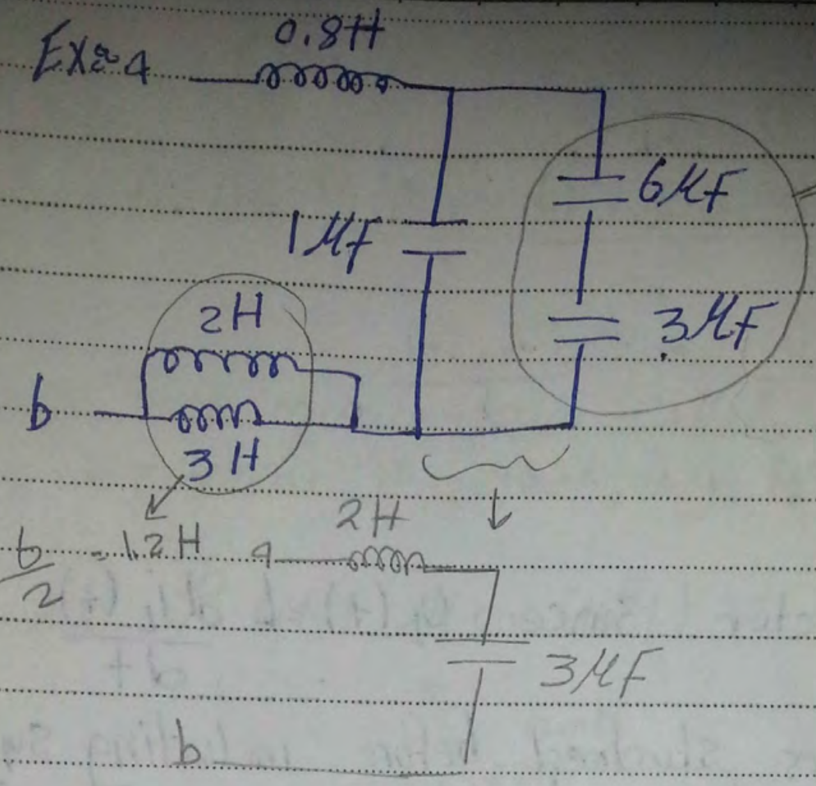
$$\equiv \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots$$



$$\Rightarrow C_{eq} = C_1 + C_2 + \cdots + C_k$$

9/11/2014

Ex: 4



$\frac{3\times6}{3+6} = 2\mu F$

* Linearity

Capacitor and inductor are linear elements.

Check

Capacitor $i_c(t) = C \frac{d V_c(t)}{dt}$

$V_{c_{new}}(t) = K V_c(t)$

$i_{c_{new}}(t) = C \frac{d V_{c_{new}}(t)}{dt}$

$= \frac{C d K V_c(t)}{dt}$

$i_{c_{new}}(t) = K C \frac{d V_c(t)}{dt}$

$\boxed{i_{c_{new}}(t) = K i_c(t)}$

∴ linear relatio

OR $\Rightarrow$

Subject:

9 / 11 / 2014

$$V_{C_{new}}(t) = V_{C_1}(t) + U_{C_2}(t)$$

$$i_{C_{new}}(t) = C\frac{d\left(U_{C_1}(t) + U_{C_2}(t)\right)}{dt}$$

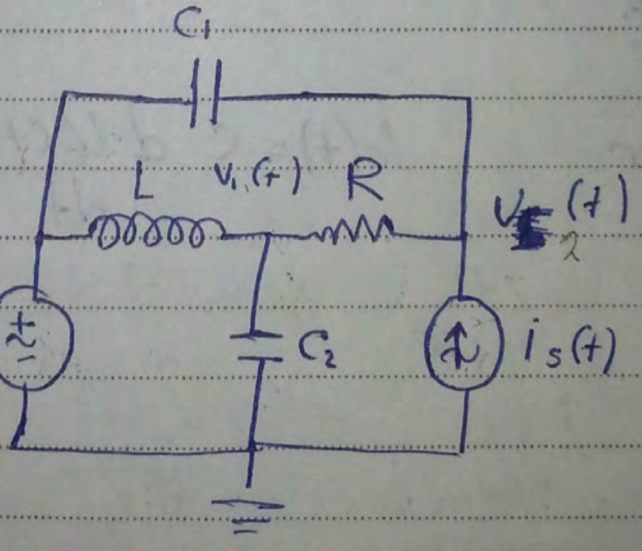
$$= C\frac{dU_{C_1}(t)}{dt} + C\frac{dU_{C_2}(t)}{dt}$$

$$i_{C_{new}}(t) = i_{C_1}(t) + i_{C_2}(t)$$

* Same For inductor Since $U_L(t) = L\frac{di_L(t)}{dt}$

* all the techniges studied before including super position can be applied to analyze circuits that have Capacitors and inductors.

Ex: write down the nodal equations:



Sol:

node 1 (KCL):

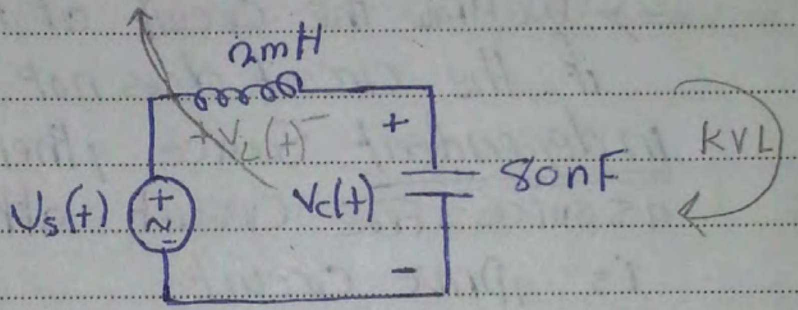
$$\frac{V_1(t) - V_2(t)}{R} + C_2\frac{d(U_1(t) - 0)}{dt} + \frac{1}{L}\int_{t_0}^{t}(U_1(\tau) - U_s(\tau))\,d\tau + i(t_0) = 0$$

node 2 (Kcl):-

$$-i_s(t) + \frac{v_2(t) - v_1(t)}{R} + C_1 \frac{d\left(v_2(t) - v_s(t)\right)}{dt} = 0 \quad ②$$

=> These equations are called Integro-differential equations it is not easy to solve them

Ex: if $v_c(t) = 4\cos(10^5 t)$ v, Find $V_s(t)$.



sol:-

$$i_s(t) = C\frac{dv_c(t)}{dt} = 80\times10^{-9} * (-4\times10^{5}\sin(10^{5}t)) = -32*10^{-3}\sin(10^{5}t)\ A$$

$$V_L(t) = L\frac{d\,\overset{i_c(t)}{\cancel{i_L}}(t)}{dt} = 2*10^{-3}(-32*10^{-3}*10^{5}\cos(10^{5}t)) = -6.4*\cos(10^{5}t)\ v$$

$$V_s(t) = V_L(t) + V_c(t)$$
$$= -6.4\cos(10^5 t) + 4\cos(10^5 t)$$
$$V_s(t) = -2.4\cos(10^5 t)\ v$$

Driven RL and RC Circuit

Subject: 9 / 11 / 2014

* CH8 : Transient Analysis :

نفس الطريقة

→ Source Free (RL, RC, RLC) circuits ✓

بعد switching ما فيها independent

نفس الطريقة

→ Driven (RL, RC, RLC) circuits ✓

بعد switching فيها independent

⟹ we draw the circuit at $t = \infty$, then if the circuit does not have any <u>independent</u> Source, then we it is a source - Free circuit otherwise, it is a Drive circuit.

$t = \infty$

$t = 0^-$

$t = 0^+$

* Source Free RL or RC circuits :

Step 1 : at $t = \infty$

⟹ Find the Type of the circuit

⟹ Find the Time Constant τ.

$$\tau = \frac{L_{eq}}{R_{eq}} \quad (\text{for RL circuit})$$

$$\tau = R_{eq} C_{eq} \quad (\text{For RC circuit})$$

⟹ solution

any voltage or Current on the Circuit ←

$$v(t) = v(0^+) e^{-t/\tau}$$

$$i(t) = i(0^+) e^{-t/\tau}$$

Subject:......................   9/11/2014

step2 :: at $t = 0^-$

=> Find the Voltage on the capacitor $V_c(0^-)$ => For RC circuit
=> Find the current on the inductor $i_L(0^-)$ => For RL circuit
=> Find the required Voltage or current.

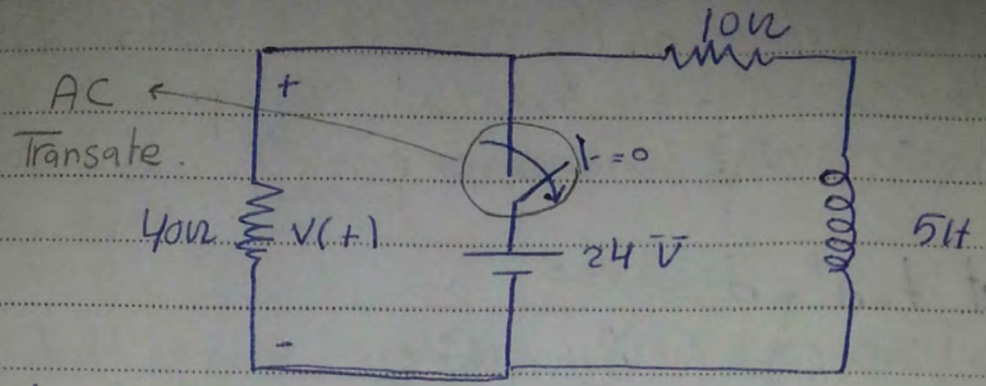
step3 :: at $t = 0^+ = 0$

=> replace the capacitor by a voltage source that has a value of $V_c(0^-)$

=> Replace the inductor by a current source that has a value of $i_L(0^-)$

=> Find the required Voltage or current $V(0^+)$ or $i(0^+)$
↑ ↑
0 0

Exc. Find $V(t)$ at $t = 200$ msec :



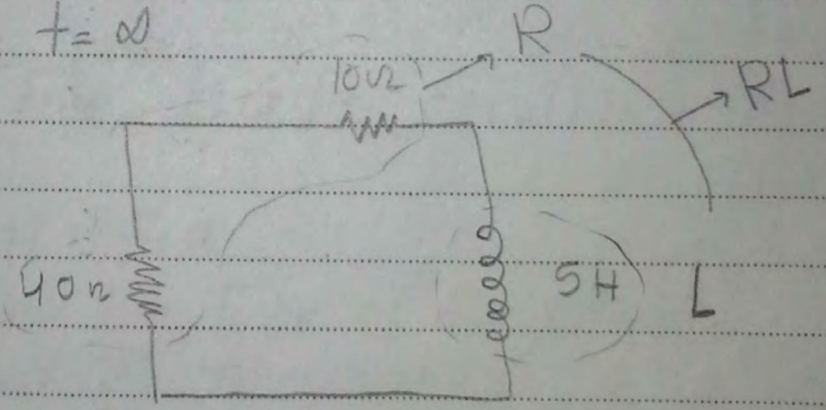
Sol:

$V(t)$ as a function of time After we substuation $t = 200$ msec.

3 step Solution

Step 1: $t = \infty$

independent لما
Source
Source Free [illegible]



=> Typ: Source Free RL Circuit

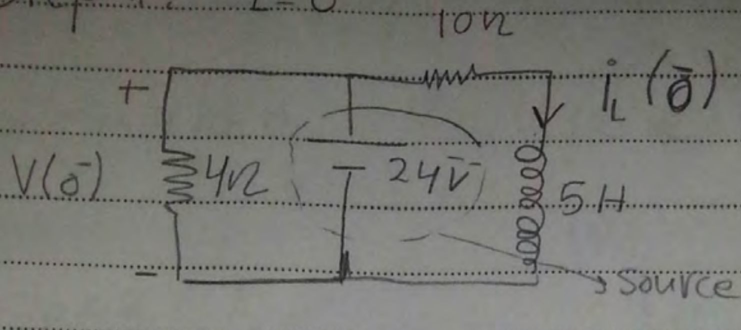
$$\Rightarrow \tau = \frac{L_{eq}}{R_{eq}} = \frac{5}{40+10} = \frac{5}{50} = 0.1 \text{ sec}.$$

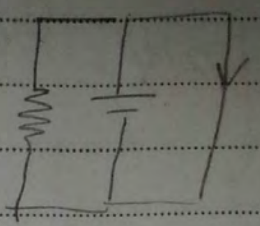
$$\Rightarrow \text{Solution: } V(t) = V(\underset{\uparrow}{\overset{0^+}{\,}}0)\, e^{-t/\tau} \quad \sim \quad (t \geq 0)$$

[illegible]

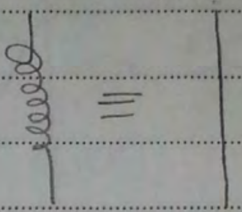
Subject:......................... 11/11/2014

Step 2: $t = 0^-$





DC circuit



$\Rightarrow V(0^-) = 24 V$

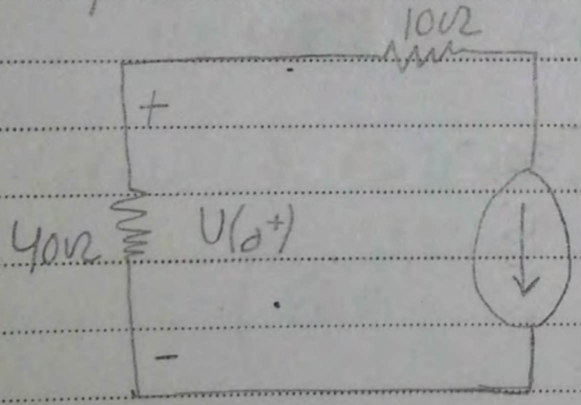
$\Rightarrow i_L(0^-) = \frac{24}{10} = 2.4 A$

يحول الملف الى مصدر تيار عكس المواسع

عند المواسع الجهد ثابت

عند الملف التيار ثابت

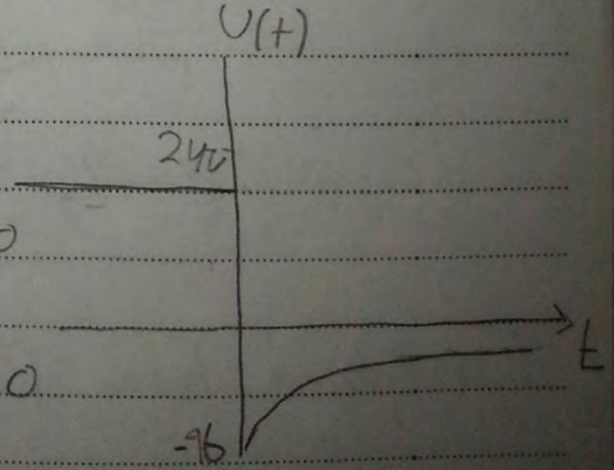
Step 3: $t = 0^+ = 0$



$(2.4A)\ i_L(0) = i_L(0^+) = i_L(0^-)$

$V(0^+) = -40 * 2.4 = \boxed{-96 V}$

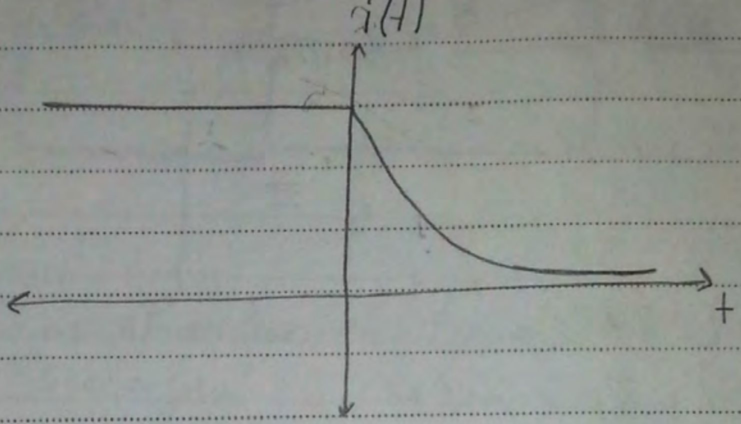
$$V(t) = \begin{cases} 24 V & , t < 0 \\ -96 e^{-t/0.1} V & , t \geq 0 \end{cases}$$



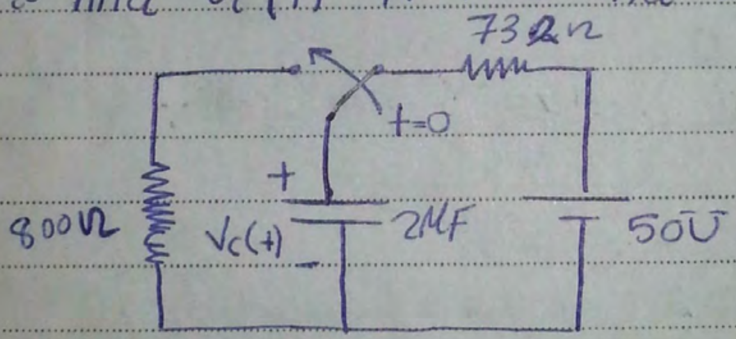
S T A R S Notebook

11/11 2014

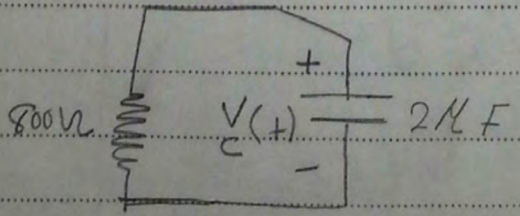
$$i_L(t) = \begin{cases} 2.4 & , t<0 \\ 2.4 e^{-t/0.1} & , t \geq 0 \end{cases}$$



Ex:- Find $V_c(t)$ at $t=0$ and $t=2$ msec :-



Sol:- Step 1 : $t=\infty$



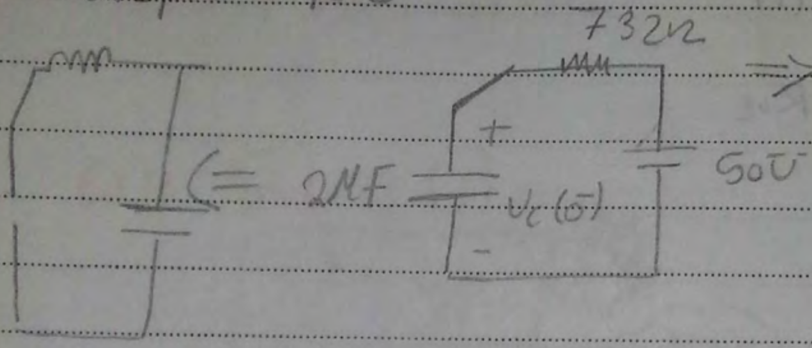
Tybe: Source Free RC Circuit

Subject:...................... 11/11/2014

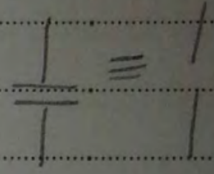
=> $\tau = R_{eq} C_{eq} = 800 * 2\mu F = 1.6$ msec

=> solution : $V_c = V_c(\overset{0^+}{0})\, e^{-t/\tau}$
(t ≥ 0)

step2 : $t = 0^-$



=> DC circuit ≡

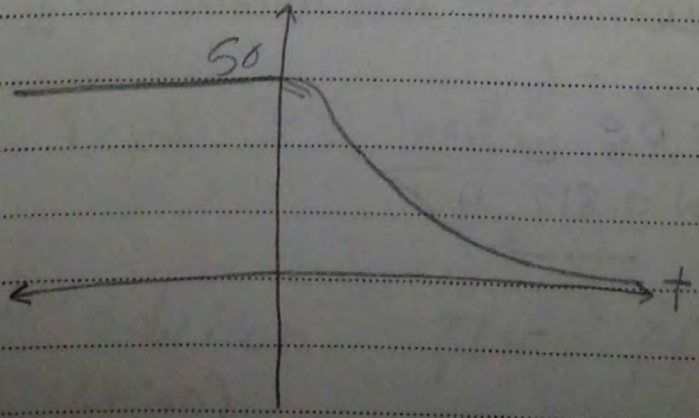


$V_c(0^-) = 50V$

step 3 : $t = 0^+$
we know that $V_c(0^+) = V_c(0^-) = 50$ V

$$V_c(t) = \begin{cases} 50V & , t < 0 \\ 50e^{-t/1.6\times10^{-3}} & , t \geq 0 \end{cases}$$

*
يعني نحوله

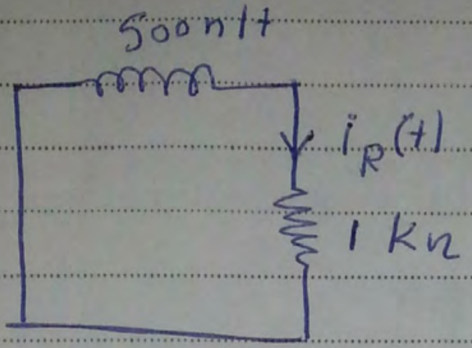


Subject: .... 11 / 11 / 2014

Ex: Find $i_R(t)$ at $t = 1n\sec$

ماكو) if $i_R(0) = 6A$

tep2 or 3



* switch
or initial condition
transit analysis.

=> step 1 : $t = \infty$

Tybe => Source Free RL circuit

$$\Rightarrow \tau = \frac{L_{eq}}{R_{eq}} = \frac{500 * 10^{-9}}{1000} = 0.5 \text{ nsecond}$$

$$\Rightarrow \text{solution : } i_R(t) = i_R(\overset{0^+}{\uparrow}\!\!\!\!0)\, e^{-t/\tau}$$

$$\boxed{i_R(t) = 6\, e^{-t/0.5*10^{-9}} \quad t \geq 0}$$

$$= 6e^{-\frac{1}{0.5}}$$

$$= 0.812 \text{ A}$$

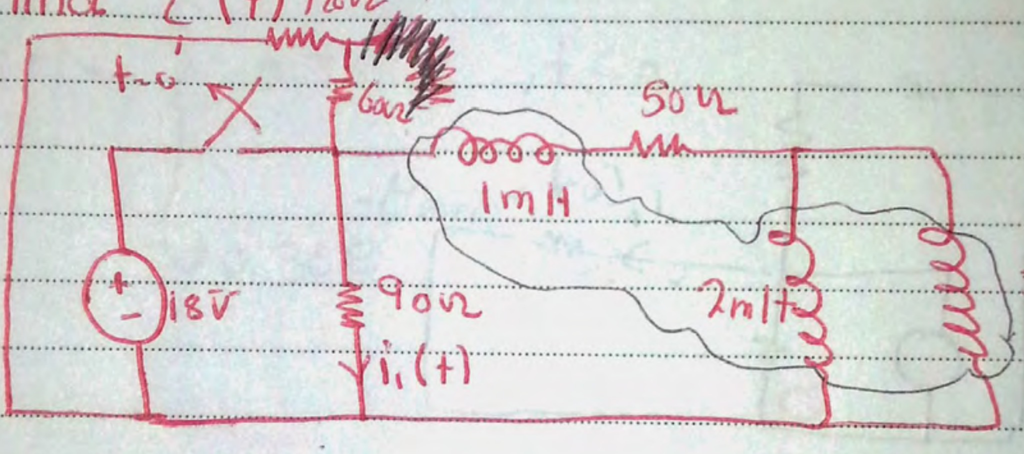
Subject: 13/11/2014

CH7: 1, 10, 15, 24, 27, 28, 35, 36, 48, 49, 50

CH8: 6, 9, 10, 24, 35, 37, 39, 40, 42, 43, 44, 57

✱ Mid-term exam to the end of Source-Free RL & RC circuit.

ex: Find $i(t)$



$\Rightarrow 2 // 3 + 1 = 2.2 mH$

Sol:

Req = (60 + 120)//90 + 50 = 110Ω

uns

Step 1: at = ∞

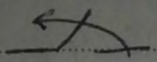
Tybe : Source Free RL Circuit

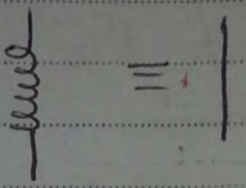
Find $\tau = \frac{Leq}{Req} = \frac{2.2}{110} = 20\ \mu sec$

$\Rightarrow$ solution $(t \geq 0)$ $i(t) = \overset{\overset{\circ}{\uparrow}}{i(0^+)}\, e^{-t/\tau}$

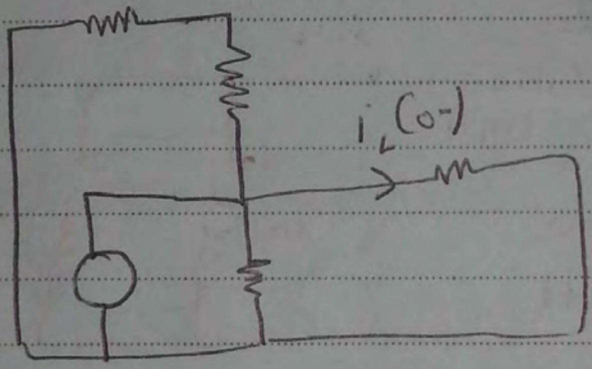
Subject:........................ 13/11/2014

step 2: $t=0^-$

 is closed

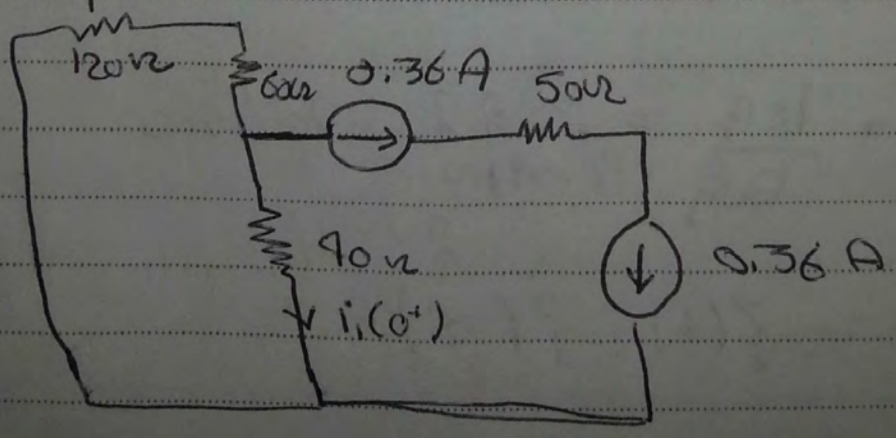
⇒ DC circuit ⇒ 

⇒ $i_1(0^-) = \frac{18}{90} = 0.2$ A



⇒ $i_L(0^-) = \frac{18}{50} = 0.36$ A

*step 3 $t=0^+=0$



$$i_1(0^+) = \frac{-0.36 * 180}{180+90}$$

$$= -0.24\,A$$

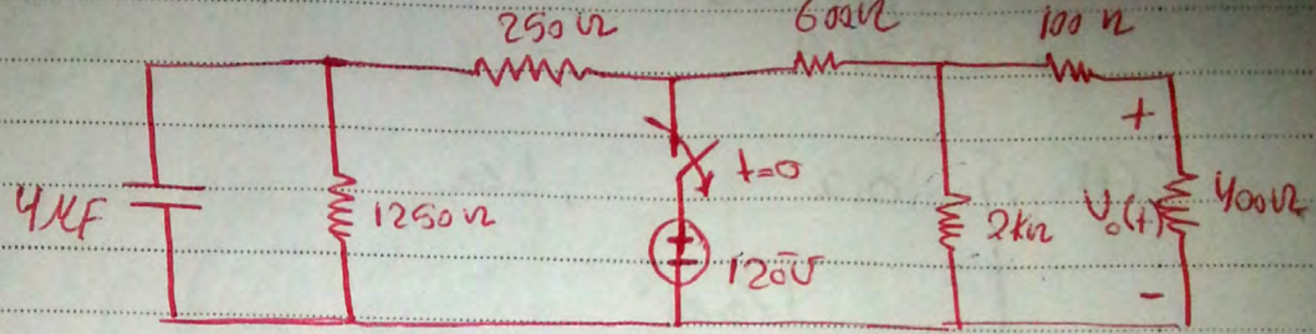
$$i_1(t) = \begin{cases} 0.2 & , t<0 \\ -0.24\, e^{-t/20\times10^{-6}}\ A & , t\geq 0 \end{cases}$$

$$i_{L_1}(t) = \begin{cases} 0.36 & , t<0 \\ 0.36\, e^{-t/20\times10^{-6}} & , t\geq 0 \end{cases}$$

Driven RL and Rc Circuit

Subject:.......................   13/11 /2014

ex: Find $V_o(t)$



Sol:

Step 1: $t=\infty$

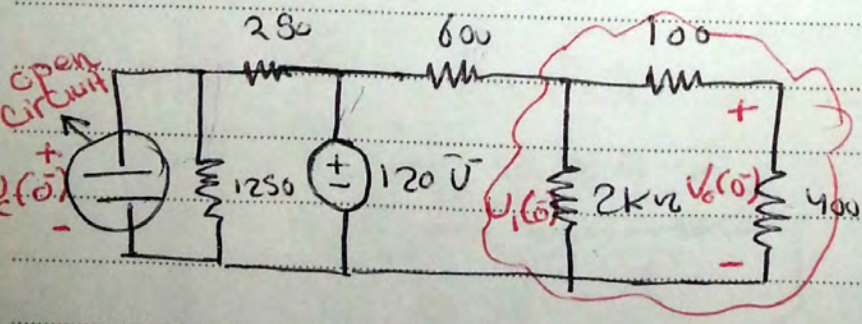
$Req = ((100+400)//2 + 600$
$\cancel{\$} + 250)//1250 =$
625Ω

Type ⇒ Source - Free RC circuit

$\Rightarrow \tau = Req\ Ceq = 625 * 4 * 10^{-6} = 2.5$ msec

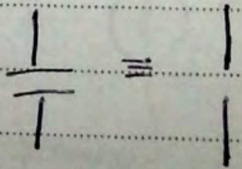
Solution $V_o(t) = V_o(0^+)\ e^{-t/\tau}$

Step 2: $t = 0^-$



⇒ Voltage division :

Dc circuit ⇒

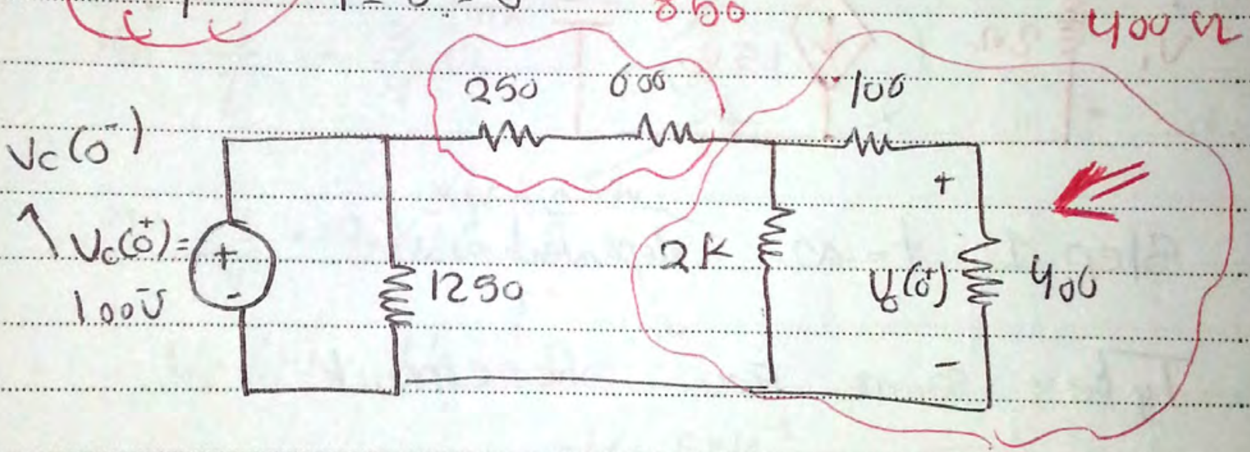


$$\Rightarrow V_c(0^-) = \frac{120 * 1250}{1250 + 250} = 100\bar{V}$$

$$V_1(0^-) = 120 \frac{400}{400+600} = 48\ \bar{V}$$

$$V_o(0^-) = V_1(0^-) * \frac{400}{400+100} = 38.4\ \bar{V}$$

Step 3 $t = 0^+ = 0$



$$V_o(0^+) = V_1(0^+) * \frac{400}{400+100}$$

$$V_1(0^+) = 100 * \frac{400}{400+850}$$

$$V_o(0^+) = V_o(0) = 25.6\ \bar{V}$$

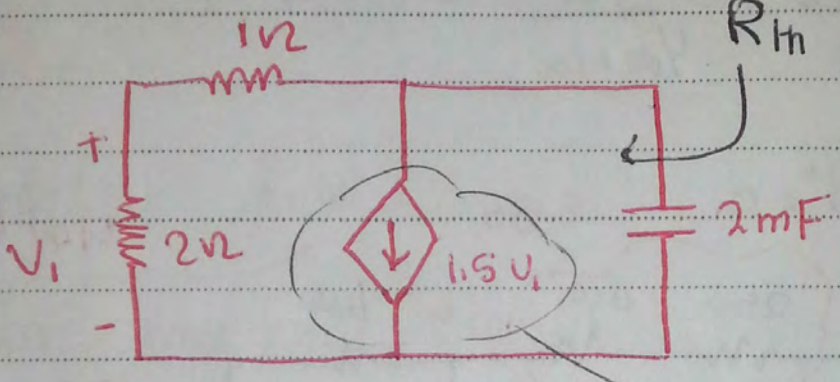
$$V_o(t) = \begin{cases} 38.4\ \bar{V} & t < 0 \\ 25.6\, e^{-t/2.5*10^{-3}} & t \geq 0 \end{cases}$$

Subject: 13/11/2014

$$v_c(t) = \begin{cases} 100 & t<0 \Rightarrow 0 \\ 100\, e^{-t/2.5*10^{-3}} & , t\geq 0 \Rightarrow C* \text{(المشتقة)} \end{cases}$$

$$i_c(t) = C\frac{dv_c(t)}{dt}$$

ex: Find $v_c(t)$, if $v_c(0^-) = 11\,V$ → initial condition so transiat Analysis



ما تعتبره مصدر
dependent نية

Step 1 $t=\infty$

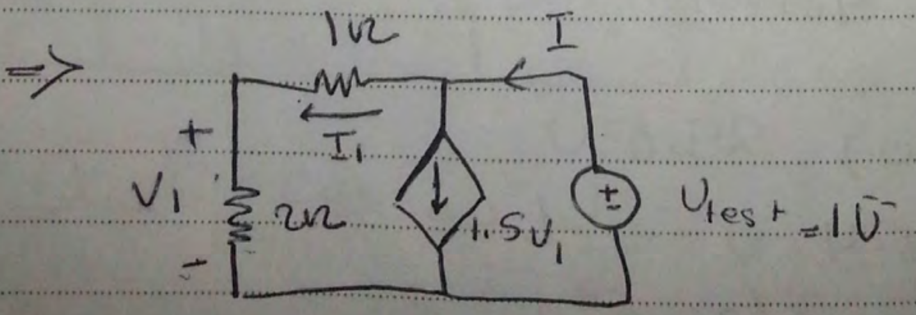
Tybe: source Free Rc circuit

$\Rightarrow \tau = \text{Req}\ \text{Ceq}$

Req ↓ Rth seen by C

Ceq ↓ 2mF

دائماً Rth عن جهة الكاباسيتر



13 / 11 / 2014

$$R_{th} = \frac{U_{test}}{I} = \frac{1}{I}$$

$$I = I_4 + 1.5 v_1$$

$$= \frac{\cancelto{①}{U_{test}} - U_1}{1\Omega} + 1.5\left(\cancelto{①}{U_{test}} * \frac{2}{2+1}\right)$$

$$= \frac{4}{3} A$$

$$R_{th} = \frac{1}{I} = \frac{3}{4} \Omega$$

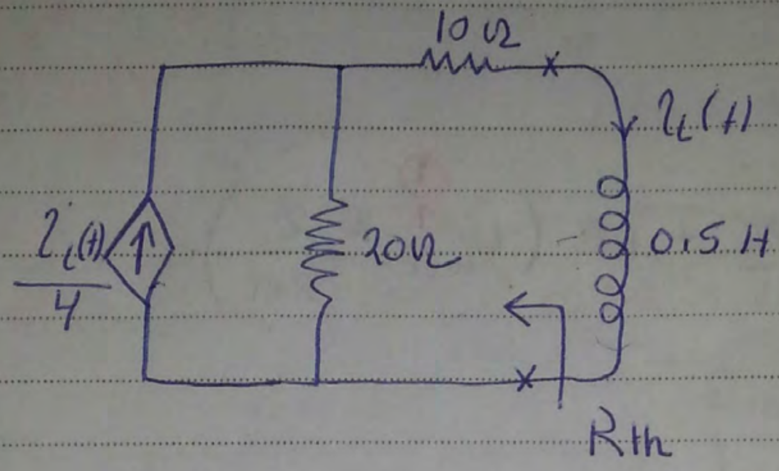
$$\tau = \frac{3}{4} * 2\times10^{-3} = 1.5 \ m\,sec$$

$$U_c(t) = U_c(\overset{0^+}{0}) e^{-t/\tau}$$

$$U_c(t) = \begin{cases} 11 e^{-t/1.5*10^{-3}} , & t \geq 0 \\ 11 , & t < 0 \end{cases}$$

16/11/2014

Ex: Find $i_L(t)$, if $i_L(0) = 10A$



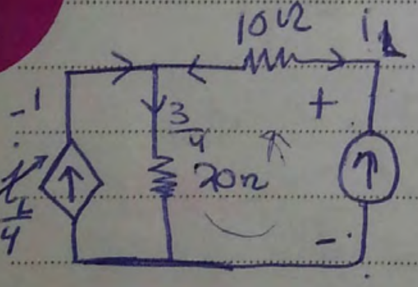
* general case
Req = Rth

Tybe 1
* parallel
series.

Step1 t = ∞

Tybe Source Free RL Circuit

$$\Rightarrow \tau = \frac{L_{eq}}{R_{eq}} = \frac{0.5}{R_{th}}$$



$I_{test} = 1A$

$$R_{th} = \frac{V}{I_{test}} = \frac{V}{1}$$

KJL $\frac{3}{4} * 20 - V + 1 * 10 = 0$

$$\boxed{V = 25 V}$$

16/11/2014

∴ $\tau = \frac{0.5}{25} = 0.02$ sec

⇒ $$i_L(t) = \underset{10}{\cancel{i_L(0^+)}}\, e^{-t/\overset{0.02}{\cancel{\tau}}}$$

$(t \geq 0)$

$$i_L(t) = \begin{cases} 10 & , t < 0 \\ 10e^{-t/0.02} & , t \geq 0 \end{cases}$$

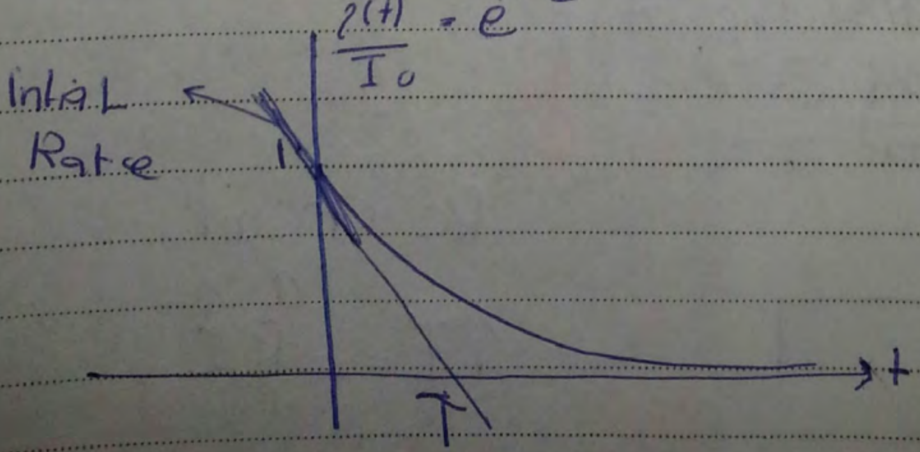
* Time constant (τ)

it is the time requircal For a response (current or voltage) to reach Zero if it decreses at its initial decaying rate

⇒ For source Free RL(circui) :

$$i(t) = I_0 e^{-t/\tau}$$

$$\frac{i(t)}{I_0} = e^{-t/\tau}$$



Subject: .... 16 / 11 / 2014

$$\text{initial rate} = \frac{d\,\frac{i(t)}{I_0}}{dt}\Bigg|_{t=0} = -\frac{1}{\tau}\, e^{t/\tau}\Bigg|_{t=0} = \boxed{\frac{-R}{L}}$$

initial Rate slope.

$$-\frac{R}{L} = \text{slope} = \frac{1-0}{0-\tau}$$

$$-\frac{R}{L} = \frac{1}{-\tau} \Rightarrow \tau = \frac{L}{R}$$

* بستفيد منو
لأعرف اي زائره
discharge بسرعه

$i_L(\tau) = 36.8\%\ I_0$

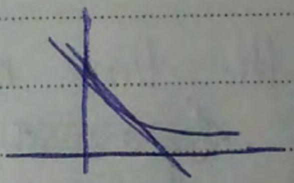
$i_L(2\tau) = 13.53\%\ I_0$

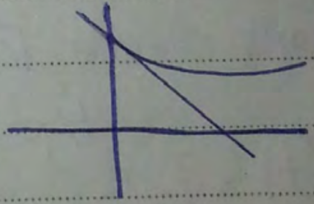
$i_L(3\tau) = 4.9\%\ I_0$

$i_L(4\tau) = 0.832\%\ I_0$

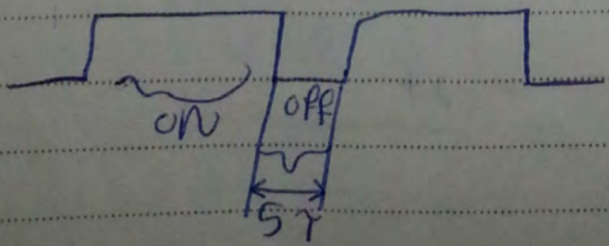
$i_L(5\tau) = 0.6738\%\ I_0$

we can say that the Current is almost Zero





Ex غاز السياره



RC circuit

$$\uparrow \tau = \overset{\uparrow}{R}\,\overset{\uparrow}{C}$$

Driven RL and Rc circuit

step1 = $t = \infty$

=> Tybe : if there is an independent source, then Driven circuit

Find $\tau = \frac{L_{eq}}{R_{eq}}$ / $\tau = R_{eq} C_{eq}$

=> solution -> $i(t) = \underbrace{i(\infty)}_{\text{Forced response}} + \underbrace{(i(0) - i(\infty)) e^{-t/\tau}}_{\text{natrual response}}$

=> find $i(\infty)$

step2 : $t = 0^-$

Same as For Source - Free circuit

step 3 : $t = 0^+$

Same as For Source - Free circuit.

Zero

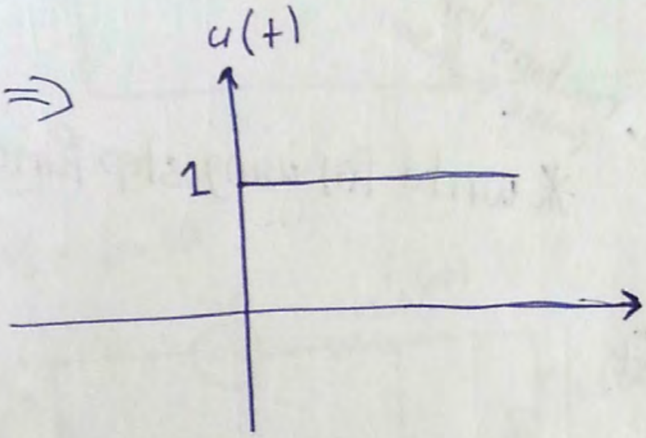
* Singularity Functions:

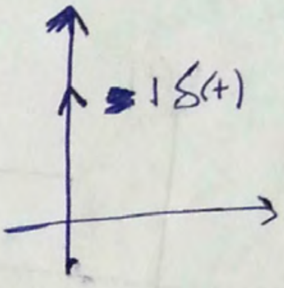
Function that have discontinous derivatives.

① unite step function $u(t)$

② unite impulse Function $\delta(t)$

$u(t) \Rightarrow$

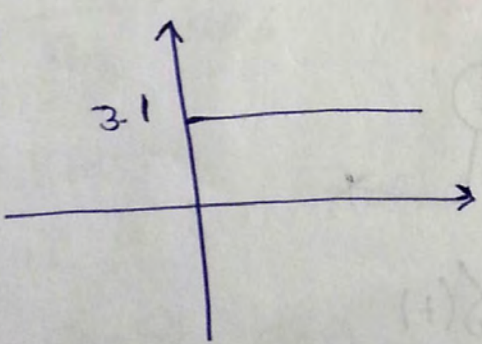


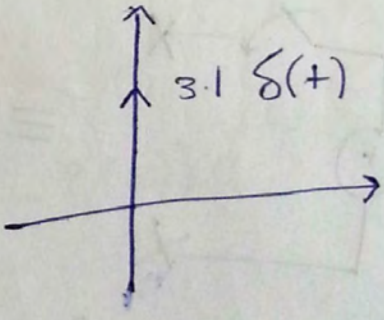


$$\Rightarrow u(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \\ \text{undifined}, & t = 0 \end{cases}$$

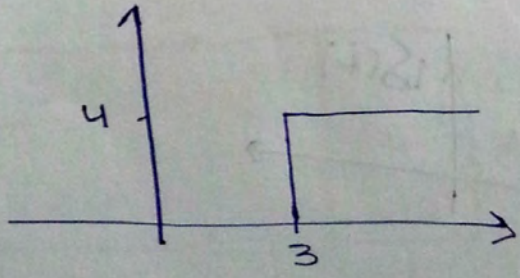
Ex:

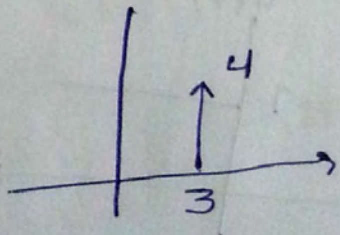
$3.1\, u(t)$



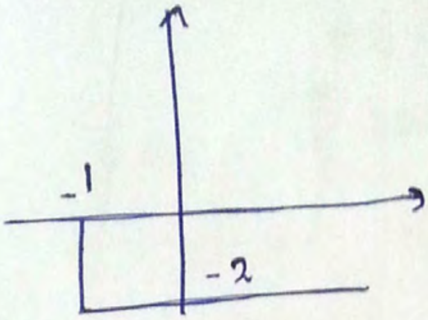


Ex: $4\, u(t-3)$

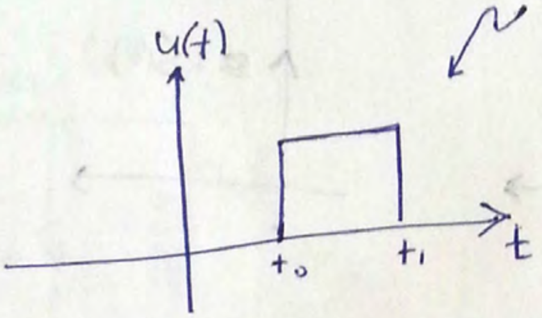




Ex :- $-2u(t+1)$



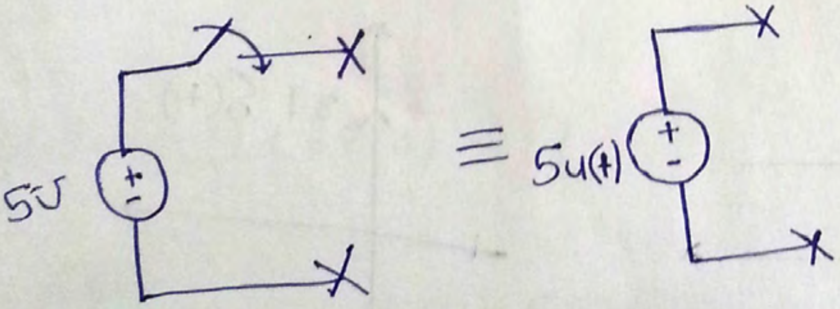
Ex



rectangular pulse function

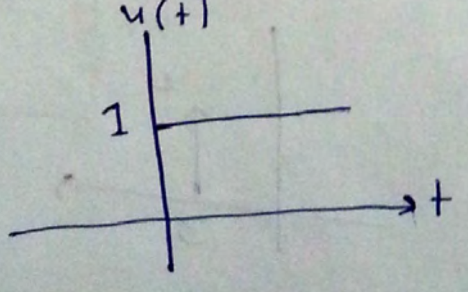
* write f(t) using step function?

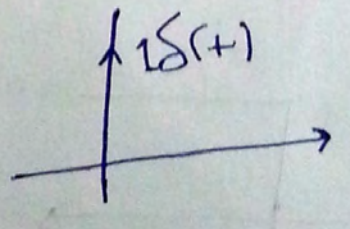
Sol :- $f(t) = U_m u(t - t_0) - U_m u(t - t_1)$



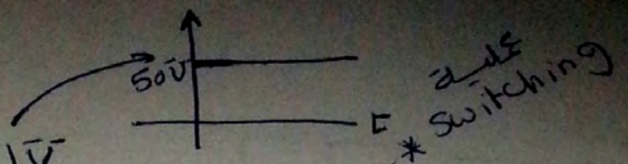
② unit impulse function $\delta(t)$

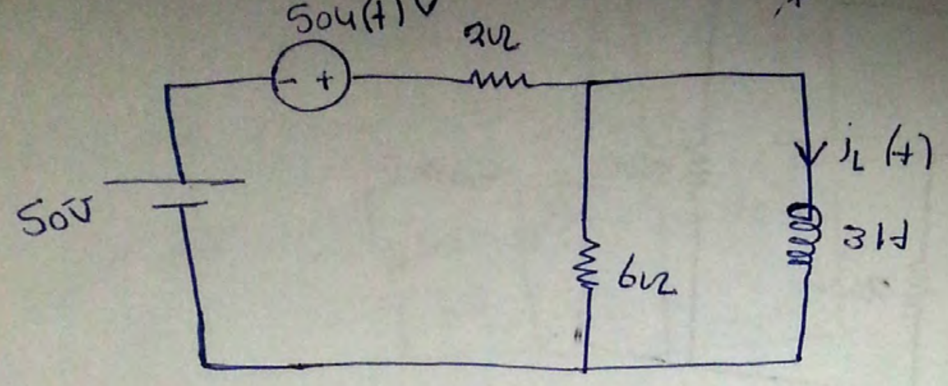
$$\delta(t) = \frac{du(t)}{dt}$$





Ex: Find $I_L(t)$

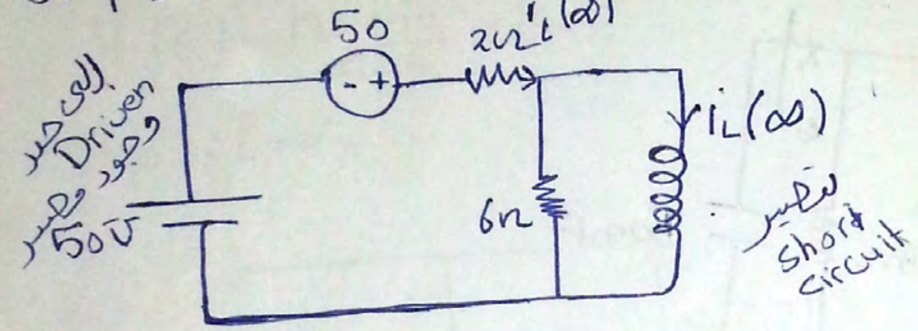




50u(t) في عند الصفر - تكون short circuit وبعد الصفر 50 volt

Sol:

step 1: t = ∞



⟹ Tybe: Driven RL Circuit

$$\Rightarrow \tau = \frac{L_{eq}}{R_{eq}} = \frac{3}{2//6} = \frac{3\times 8}{12} = 2\,\text{sec}$$

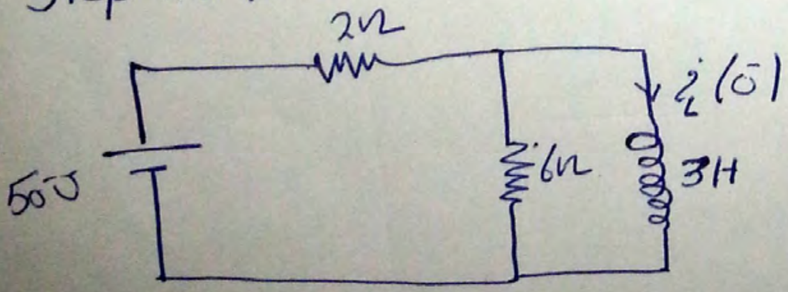
⟹ Solution:

$$i_L(t) = i(\infty) + \left(i(\overset{0^+}{\bar{0}}) - i(\infty)\right)e^{-t/\tau}$$

$(t \geq 0)$

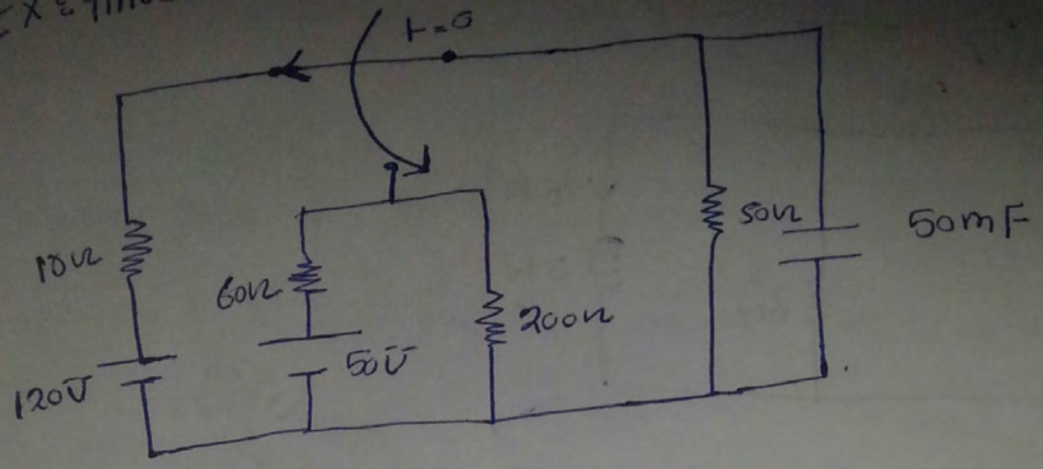
$$i_L(\infty) = \frac{50+50}{2} = 50A \qquad i_L(0^-) = \frac{50}{2} = 25A = i_L(0^+)$$

Step 2: $t = 0^-$

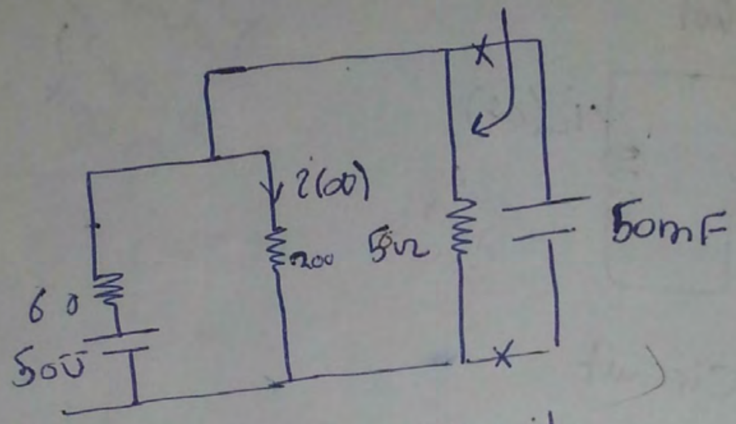


$$i_L(t) = \begin{cases} 25 & , t<0 \\ 50+(25-50)e^{-t/2} & , t\geq 0 \end{cases}$$

Ex: find $i(t)$



* step 1: $t = \infty$



=> Tybe : Driven RC circuit

=> $\tau = R_{eq} C_{eq} = (60 // 200 // 50) * 50 = 1.2$ sec

=> solution:

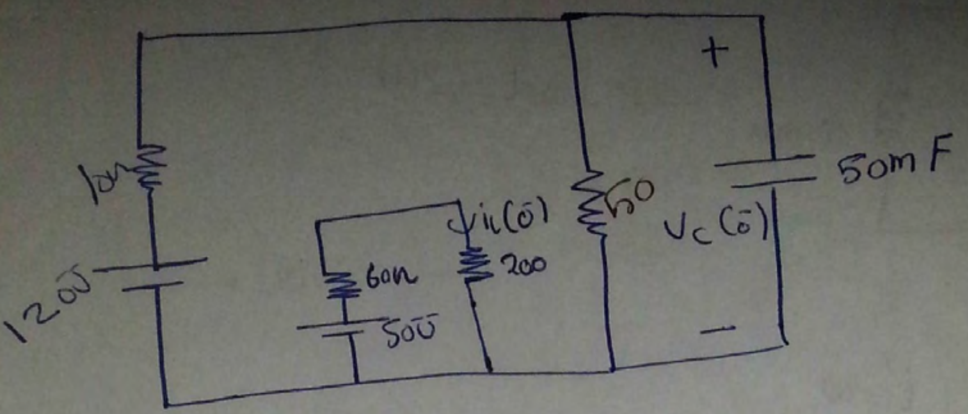
$$i(t) = i(\infty) + (i(0) - i(\infty)) e^{-t/\tau}$$

=> $i(\infty) = 22$

$$V_{200\Omega} = 50 * \frac{(50 // 200)}{(50 // 200 + 60)}$$

$$i(\infty) = \frac{V_{200\Omega}}{200} = 0.1 A$$

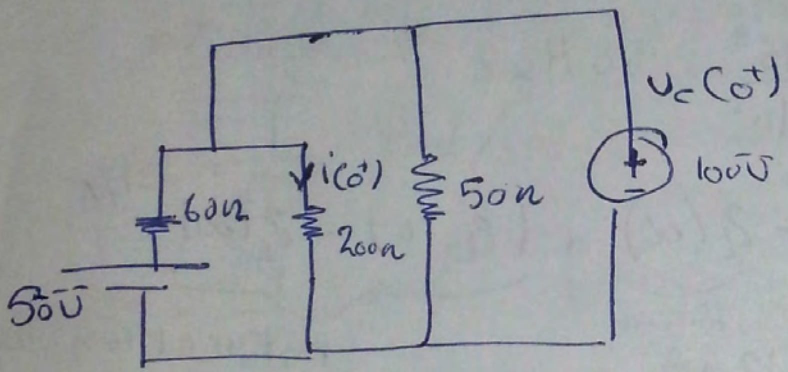
Step: $t=0^-$



$$i(0^-) = \frac{50}{60+200} = 0.1923A$$

$$V_c(0^-) = \frac{120*50}{50+10} = 100V$$

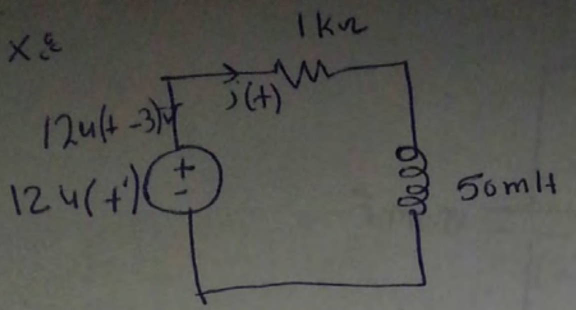
Step 3: $t=0^+=0$



$$i(0^+) = \frac{100}{200} = \frac{1}{2}A$$

$$i(t) = \begin{cases} 0.1923A & , t<0 \\ 0.1+(0.5-0.1)e^{-t/1.2} & , t\leq 0 \end{cases}$$

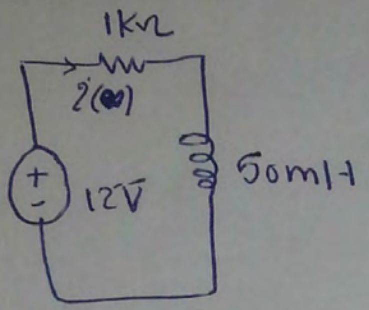
Ex:



Find i(t)

=> $t' = t-3$

=> step : at $t' = \infty$



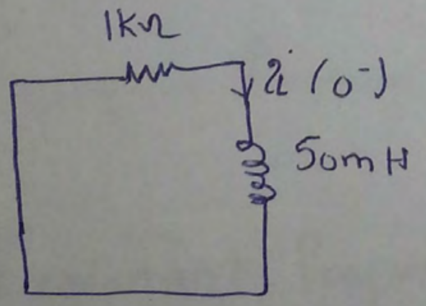
=> Type: Driven RL circuit

=> $\tau = \frac{leq}{Req} = \frac{50\times10^{-3}}{1\times10^{3}} = 50\,\mu sec$

=> solution $i_L(t) = \underbrace{i_l(\infty)}_{\text{Forced}} + \underbrace{(i_L(0) - i_L(\infty)}_{\text{natural response}} e^{-t/\tau}$

$i_l(\infty) = \frac{12}{1k\Omega} = 12nA$

* step 2 $t' = 0^-$



$i_L(0^-) = 0A$

$i_L(0^-) = i(0^+)$

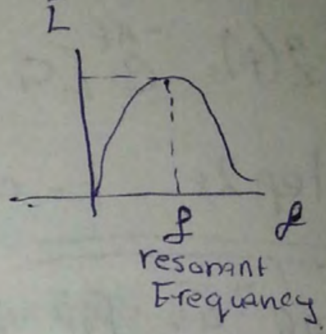
$i(t) = \begin{cases} 0 & , t<0 \\ 12-12e^{-t'/\tau} & , t' \geq 0 \end{cases}$

$i_l(t) = \begin{cases} 0 & , t<3 \\ 12-12e^{-t-3/50\times10^{-6}}\,mA & , t\geq3 \end{cases}$

* RLC circuit :

- → Source - Free
  - parrallel
  - series
- → Driven
  - → parrallel
  - → series.

* Source - Free RLC :



Step1 ⇒ $t = \infty$

⇒ Tybe :

⇒ Find The Following parameter :

ثوابت، ثابتة مقدار السؤال.

* $\omega_0 = \frac{1}{\sqrt{LC}}$ rad/sec (resonant Frequancy) → ظاهرة الرنين
  * response Max للي

* $\alpha = \frac{1}{2RC}$ sec^{-1} (parallel) ⇒ (Nepor Frequancy) or (exponential damping coefficient)
  $\alpha = \frac{R}{2L}$ sec^{-1} (series)

* $S_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$
* $S_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$

} complex frequancy

* $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$ natural resonant Frequancy

* $\zeta \Rightarrow \frac{\alpha}{\omega_0}$ damping factor
  Zeta

⇒Solution: $t \geq 0$

* if $\alpha > w_o$, then overdamped

$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

* if $\alpha = w_o$, then critical damping

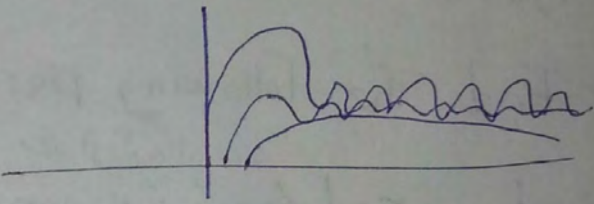
$$i(t) = B_1 t e^{-\alpha t} + B_2 e^{-\alpha t}$$

* if $\alpha < w_o$, then underdamped

$$i(t) = e^{-\alpha t} [C_1 \cos(w_d t + C_2 \sin(w_d t)]$$

* step 2: $t = 0^-$

same as for RL circuit
or RC



* step3 = $t = 0^+$

same as for RL circuit
or RC

Ex: aparallel RLC circuit
Given $L = 10 mH$, $C 100 \mu F$, Find R such that:

① over damping response.

② under damping

$\Rightarrow \omega_0 = \frac{1}{\sqrt{Lc}} = 10^3$ rad/sec

$\Rightarrow$ over dampied $\alpha > \omega_0$

$$\frac{1}{2Rc} > 10^3$$

$$R < 5\,\Omega$$

$\Rightarrow$ underdamping

$\Rightarrow \alpha < \omega_0$

$\frac{1}{2RC} < 10^3 \Rightarrow R > 5\,\Omega$

$\Rightarrow$ ciritical damp

$$R = 0$$

Exe if $R = 100\,\Omega$, $\alpha = 1000\ \text{sec}^{-1}$ (parallel RLC)

$\omega_0 = 800$ rad/sec, find L, C, S_1, S_2

$\Rightarrow \underset{1000}{\alpha} = \frac{1}{2RC} \Rightarrow C = 5\,\mu F$

$\underset{800}{\omega_0} = \frac{1}{\sqrt{L\underset{5}{C}}} \Rightarrow L = 312.5\ \text{mH}$

$\Rightarrow S_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$

$= -400\ \text{sec}^{-1}$

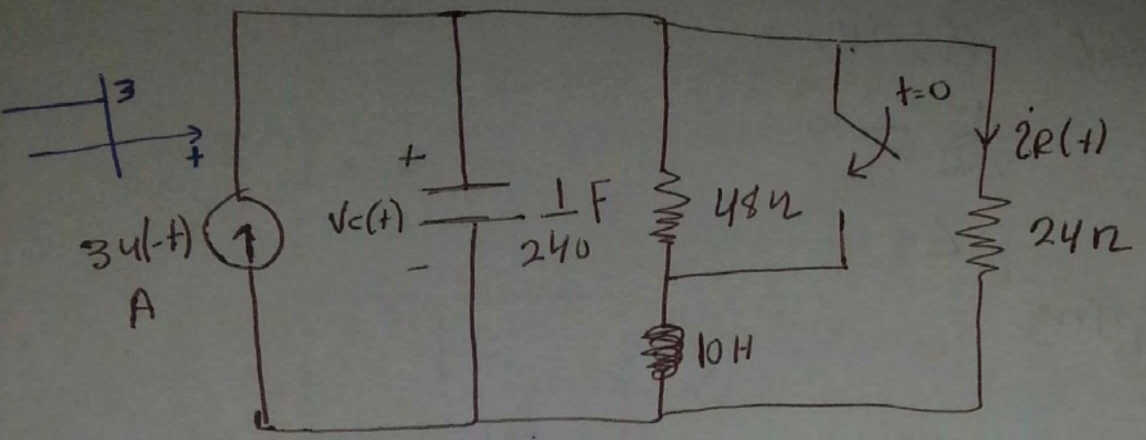
$\Rightarrow S_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$

$= -1600\ \text{sec}^{-1}$

ε→ overdamping ⇒ S_1 and S_2 are negative real number

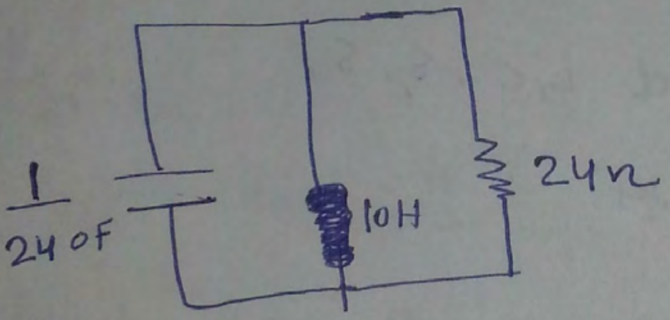
ε→ critical damping ⇒ $S_1 = S_2 = -\alpha$

Ex: Find $v_c(t)$ and $i_R(t)$.



Sol: Transient analysis => 3 steps solution

step 1 t = ∞



=> Tybe => Source - Free parallel RLC circuit

=> parameters

$$W_0 = \frac{1}{\sqrt{LC}} = \sqrt{24} \text{ rad/sec}$$

$$\alpha = \frac{1}{2RC} = 5 \sec^{-1}$$

$$S_1 = -\alpha + \sqrt{\alpha^2 - w_0^2} = -4 \sec^{-1}$$

$$S_2 = -\alpha - \sqrt{\alpha^2 - w_0^2} = -6 \sec^{-1}$$

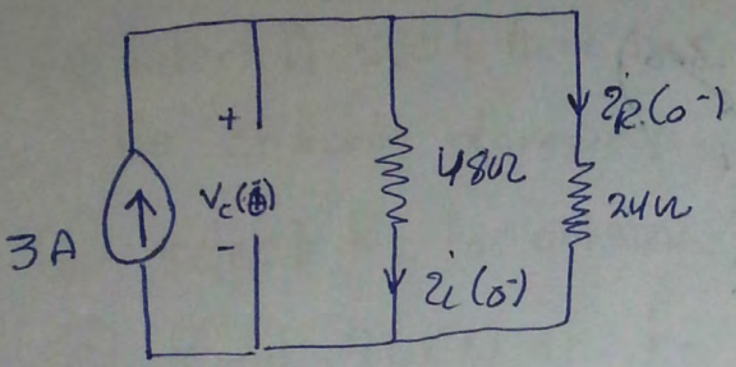
⇒ Solution :- $\alpha > \omega_0$ ⇒ over damping

$t \geq 0$

$$v_c(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$i_R(t) = B_1 e^{s_1 t} + B_2 e^{s_2 t}$$

Step 2 $t = 0^-$

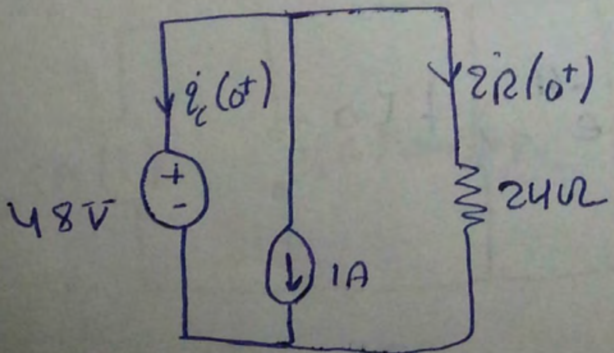


$$\Rightarrow i_L(0^-) = \frac{3 * 24}{24+48} = 1A$$

$\Rightarrow i_R(0^-) = 2A$

$\Rightarrow V_c(0^-) = 48 \times 1 = \underline{48 V}$

Step 3 $t = 0^+ = 0$



$$i_R(0^+) = \frac{48}{24} = 2A$$

$$\boxed{i_c(0^+) = -3A}$$ *

$$v_c(t) = A_1 e^{-4t} + A_2 e^{-6t}$$

$$V_c(\underset{0^+}{\underset{\downarrow}{0}}) = A_1 + A_2$$

$$\boxed{48 = A_1 + A_2} \text{ -- } ①$$

$$\Rightarrow i_c(0^+) = C \frac{dV_c(t)}{dt}\Big|_{t=0}$$

$$-3 = \frac{1}{240}\left(-4A_1 \underset{1}{e^{\cancel{-4\times0}}} - 6A_2 \underset{1}{e^{\cancel{-6\times0}}}\right),$$

$$\boxed{-3 = \frac{1}{240}\left(-4A - 6A_2\right)} \text{ -- } ②$$

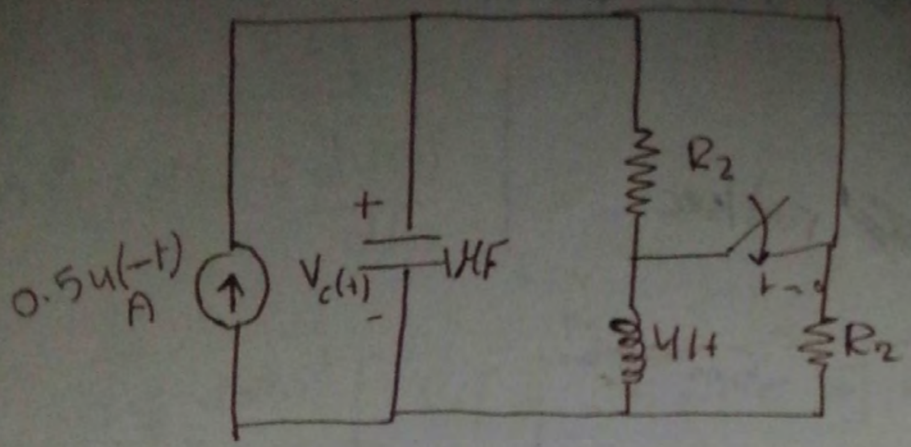
$*A_1 = -216$

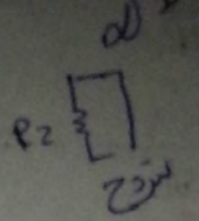
$*A_2 = 264$

$$V_c(t) = \begin{cases} 48\,V & t < 0 \\ -216e^{-4t} + 264e^{-6t} & t \geq 0 \end{cases}$$

$$i_R(t) = \frac{V_c(t)}{24} \Rightarrow \begin{cases} 2\,A & , t < 0 \\ -9e^{-4t} + 11e^{-6t} & , t \geq 0 \end{cases}$$

Ex:-





ⓐ select R_1 such that the response after $t=0$ will be critically damped

ⓑ select R_2 to obtain $v_c(0) = 100\,V$

ⓒ Find $v_c(t)$ at $t = 1$ msec.

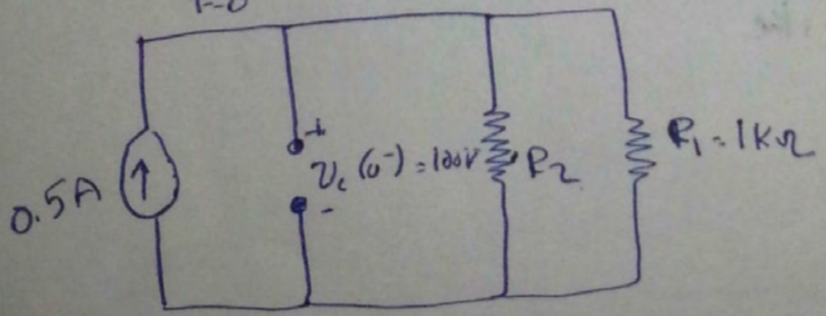
Sol:-

ⓐ $\alpha = \omega_0$

$$\frac{1}{2R_1C} = \frac{1}{\sqrt{LC}} \Rightarrow R_1 = 1k\Omega.$$

ⓑ $v_c(\overset{0^-}{\overset{\uparrow}{\overset{0^+}{\overset{\uparrow}{0}}}}) = 100\,V$

$t = 0^-$



$$I_{R_1} = \frac{100}{1k} = 0.1\,A$$

$I_{R_2} = 0.4\,A = \frac{100}{\;} \Rightarrow R_2 = 250$

ⓒ step1 : $t = \infty$

⇒ Type :. Source - Free Parallel RLC.

$$\alpha = \frac{1}{2R_1C} = 500 \text{ sec}^{-1}$$

$$w_0 = \frac{1}{\sqrt{LC}} = 500 \text{ rad/sec}$$

$$S_1 = -\alpha + \sqrt{\alpha^2 - w_0^2} = -500 \text{ sec}^{-1}$$

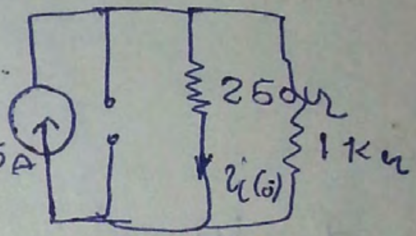
$$S_2 = -\alpha + \sqrt{\alpha^2 - w_0^2} = -500 \text{ sec}^{-1}$$

Solution :. $\alpha = w_0$ ⇒ critical damping

$$V_c(t) = B_1 t e^{-\alpha t} + B_2 e^{-\alpha t}$$

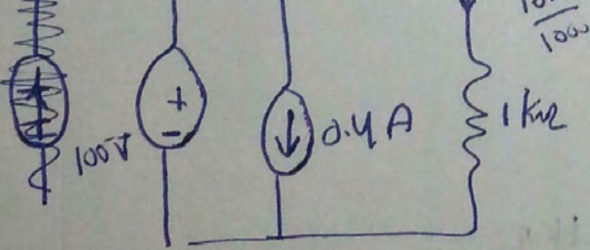
step 2 :. $t = 0^-$

we know that $V_c(0^-) = 100V$,



$$i_L(0^-) = \frac{0.5 * 1k\Omega}{250 + 1k\Omega} = 0.4A$$

step 3 : $t = 0^+$



$\frac{100}{1000} = 0.1A$

$$i_c(0^+) = -0.5A$$

$$\Rightarrow V_c(t) = B_1 t e^{-500t} + B_2 e^{-500t}$$

$$\Rightarrow \boxed{100 = B_2}$$

$$\Rightarrow i_c(0^+) = C \frac{dv_c(t)}{dt}\Bigg|_{t=0}$$

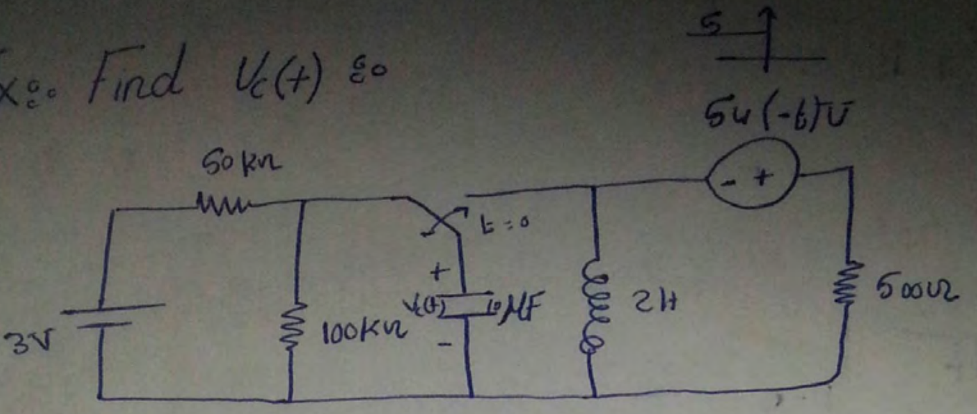
$$-0.5 = 1\times10^{-6}\left[B_1\left[e^{-500t} - 500te^{-500t}\right] - 500\underset{100}{\cancel{B_2}}e^{-500t}\right]$$

$$\Rightarrow B_1 = -4.5\times10^{6}$$

$$\Rightarrow V_c(t) = \begin{cases} 100 & t<0 \\ -4.5\times10^{6}te^{-500t} + 100e^{-500t} & t\geq 0 \end{cases}$$

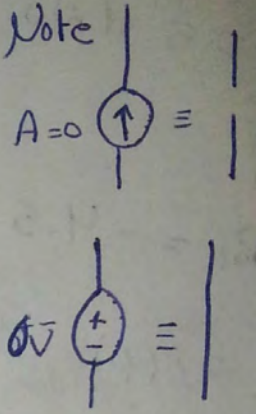
$$V_c(t=1msec) = -212\,V$$

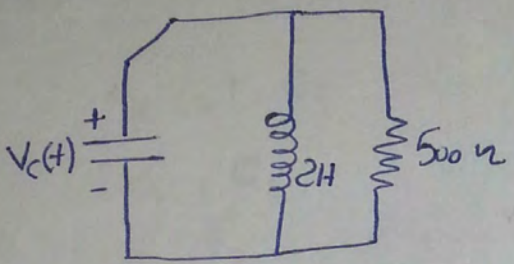
Ex: Find $V_c(t)$:



⇒ step 1 at $t = \infty$ → Switch ⇒ Transient analyisis → 3 step soL

※ Note





⇒ Tybe 1 : Source Free parallel RLC-Circuit

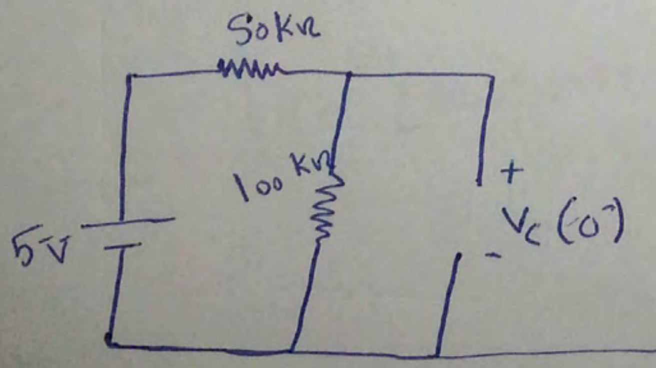
⇒ parameter : $w_o = \frac{1}{\sqrt{LC}} = 223.6$ rad/sec

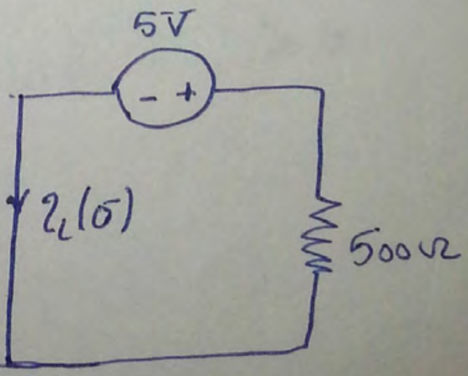
$$\alpha = \frac{1}{2RC} = \frac{1}{2*500*10\mu F} = 100 \ sec^{-1}$$

* لأنوا under damping ماتهمنا S

$$Wd = \sqrt{w_o^2 - \alpha^2} = 200 \text{ rad/sec}$$

⇒ Step2 : at $t = 0^-$

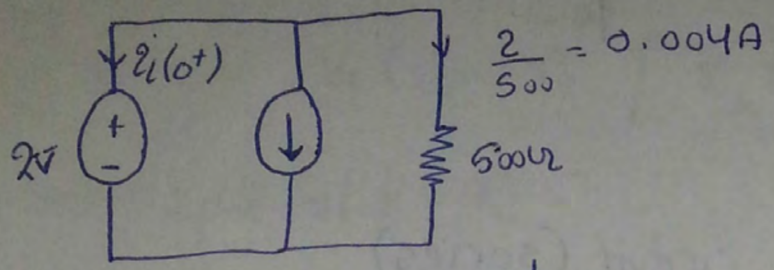




$V_c(0^-) = 3 * \frac{100}{150} = 2\bar{V}$

$i(0^-) = \frac{-5}{500} = -0.01A$

⇒ step 3 at $t = 0^+ = 0$



$\alpha > \omega_0$ under damping

⇒ solution : $V_c(t) = e^{-at}\left[C_1 \cos(w_d t) + C_2 \sin(w_d t)\right]$

$V_c(0) = C_1$

$2 = C_1$

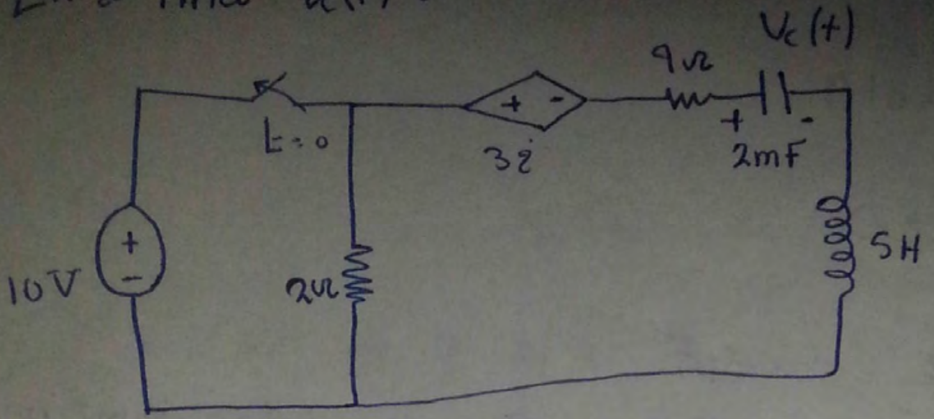
$i_c(\overset{0^+}{0}) = C\frac{dv_c(t)}{dt}$

$0.006 = 10*10^{-6}[-\alpha e^{-\alpha t}[C_1\cos(w_d t) + C_2\sin(w_d t)] + e^{-\alpha t}[-C_1 w_d \sin(w_d t) + C_2 w_d \cos(w_d t).]$

$C_2 = 4$

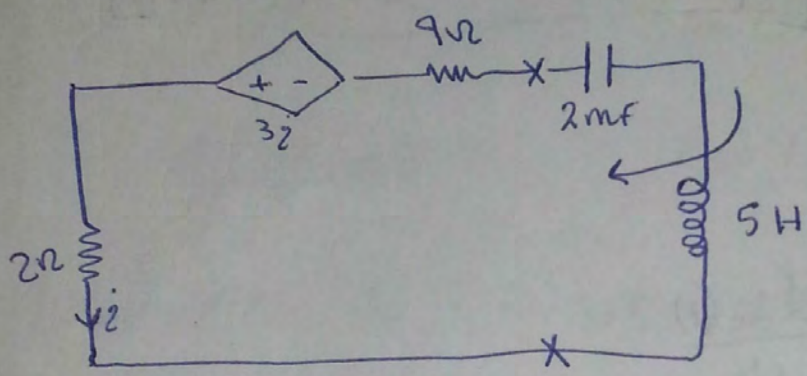
$$V_c(t) = \begin{cases} 2\bar{V} & , t < 0 \\ e^{-100t}[2\cos(200t) + 4\sin(200t), & t \geq 0 \end{cases}$$

Ex: Find $V_c(t)$:



Step 1 at $t = \infty$

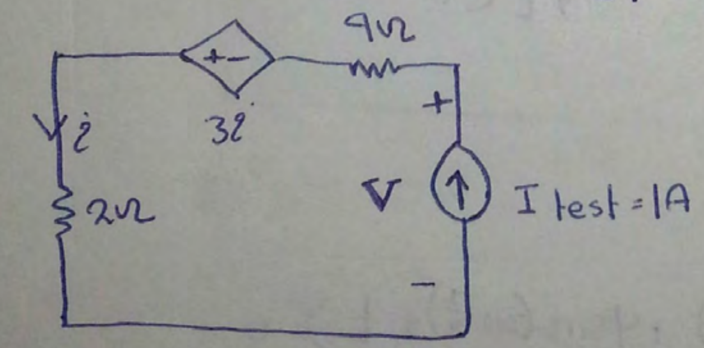
=> Type: Source - Free - RLC - circuit (series)



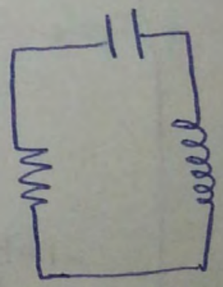
=> parameters: $\omega_0 = \frac{1}{\sqrt{LC}} = 10$ rad/sec

$\alpha = \frac{R_{th}}{2L} = 0.8 \text{ sec}^{-1}$

R_{th} => Type 3



=



KVL: $-V - 3*1 + 2*1 + 9*1 = 0$

$V = 8V \Rightarrow R_{th} = \frac{8}{1} = 8\Omega$

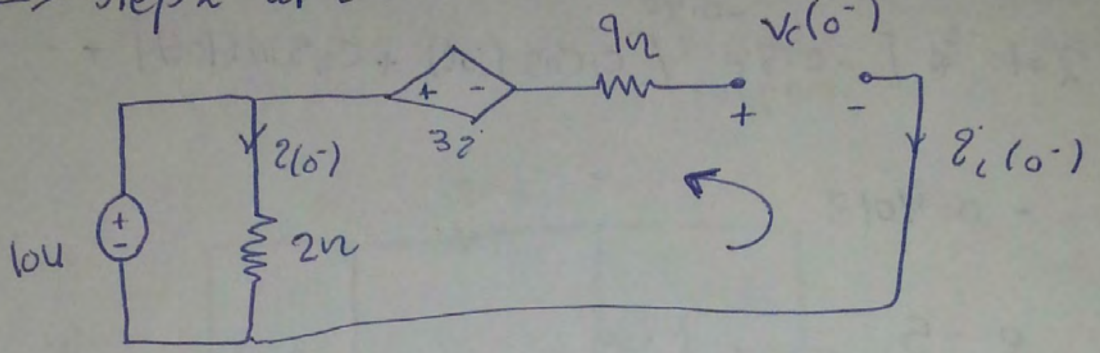
$w_d = \sqrt{w_o^2 - \alpha^2} = 9.96$ rad/sec

Solution :- underdamping $\alpha > w_o$

$t \geq 0$

$$V_c(t) = e^{-\alpha t}\left[C_1 \cos(w_d t) + C_2 \sin(w_d t)\right]$$

⇒ step 2 at $t = 0^-$

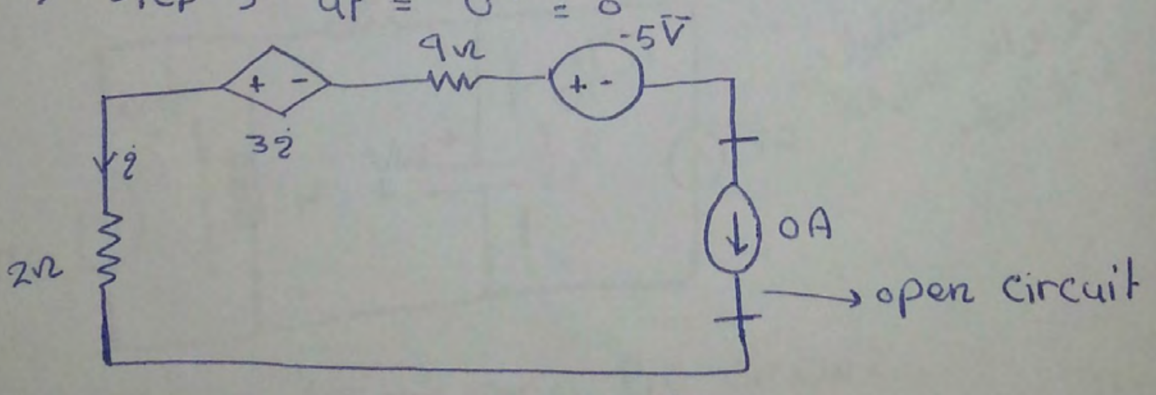


$i_L(0^-) = 0$ A $\qquad i(0^-) = \frac{10}{2} = 5$ A

KVL :- $-V_c(0^-) - 3*5 + 10 = 0$

$V_c(0^-) = -5\bar{V}$

⇒ step 3 at $= 0^+ = 0$



$i_c(0^+) = 0$ A

Solution :-

$$v_c(t) = e^{-0.8t}\left[C_1 \cos(10t) + C_2 \sin(10t)\right]$$

$$v_c(0) = C_1$$

$$-5 = C_1$$

$$i_c(0^+) = C\frac{dv_c(t)}{dt}$$

$$0 = 2*10^{-3} * \left[-0.8e^{-0.8t}\left[C_1 \cos(10t) + C_2 \sin(10t)\right] + - -\right.$$

$$C_2 = -0.4013$$

$$v_c(t) = \begin{cases} -5 & , t < 0. \\ e^{-0.8t}\left[-5\cos(10t) - 0.4013 \sin(10t)\right] & , t \geq 0 \end{cases}$$

Driven RLC circuit :

steps of solution all the same of Source Free RLC Circuit

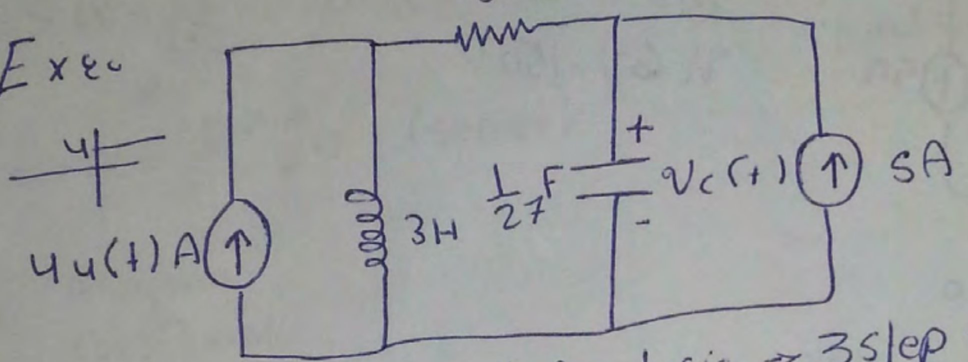
but :

over damping : $v(t) = V(\infty) + A_1 e^{s_1 t} + A_2 e^{s_1 t}$

Ciritical damping : $V(t) = v(\infty) + A_1 t e^{-\alpha t} + A_2 e^{-\alpha t}$

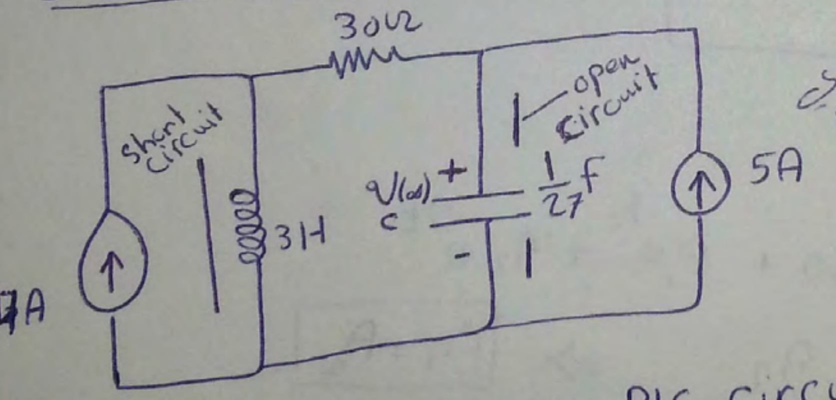
under damping : $v(t) = \underbrace{v(\infty)}_{\text{Forced response}} + \underbrace{e^{-\alpha t} [A_1 \cos(w_d t) + A_2 \sin(w_d t)]}_{\text{natural response}}$

Ex :



=> u(t) => Transient Analysis => 3 step solution

step 1 = t = ∞



(إذا بدنا نعرف توازي أو توالي نعمل kill لكل المصادر)

=> Type : Driven series RLC circuit

=> parameters : $\alpha = \frac{R}{2L} = \frac{30}{2 * 3} = 5 \text{ sec}^{-1}$

$$w_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{3 * \frac{1}{27}}} = 3 \text{ rad/sec}$$

$S_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -5+\sqrt{25-9} = -1 \sec^{-1}$

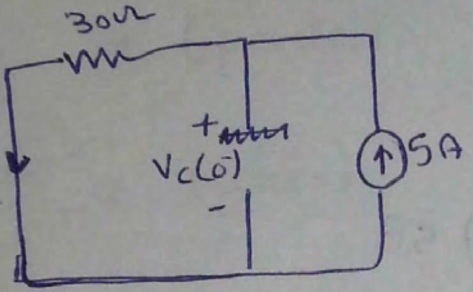
$S_2 = -\alpha - \sqrt{\alpha^2 - W_0^2} = -5-\sqrt{25-9} = -9 \sec^{-1}$

=> Solution :- $\alpha > \omega_0$ => over damped.

$v_c(t) = V_c(t) = v_c(\infty) + A_1 e^{s_1 t} + A_2 e^{s_2 t}$

$-V_c(\infty) + 5\times 30 = 0$ $\boxed{v_c(\infty) = 150\,V}$

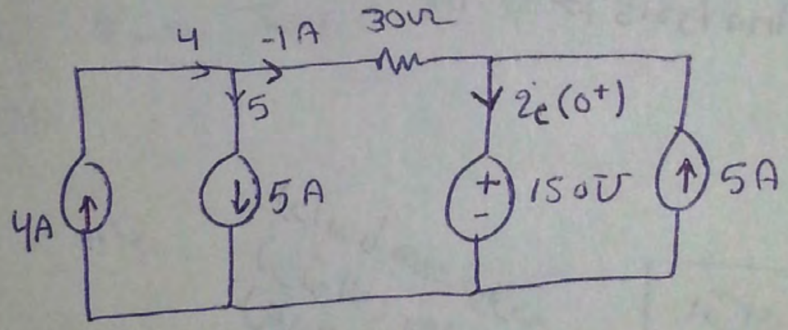
Step 2 => $t = 0^-$



$i_L(0^-) = 5A$

$v_c(0^-) = 150\,V$

step 3 => $t = 0^+ = 0$



$i_c(0^+) = 4A$

Solution :- $v_c(t) = 150 + A_1 e^{-t} + A_2 e^{-9t}$

$v_c(0) = 150 + A_1 + A_2$ => $\boxed{A_1 = -A_2}$

$i_c(t) = 0 + (-A_1 e^{-t} - 9A_2 e^{-9t})$

From ① and ②

$A_1 = 13.5$

$A_2 = -13.5$

$i_c(t) = c\frac{dv_c(t)}{dt}$

$\boxed{4 = \frac{1}{27}[-A_1 e^{-t} - 9A_2 e^{-9t}]}$

$$V_c(t) = \begin{cases} 150 & , t < 0 \\ 150 + 13.5e^{-t} - 13.5e^{-9t} & , t \geq 0 \end{cases}$$

* Lossless LC circuit :

if $R = \infty$ in the parallel RLC circuit

if $R = 0$ in the series RLC circuit

$\Rightarrow \alpha = \frac{1}{2\underset{\infty}{\cancel{R}}C} = 0$ (parallel)

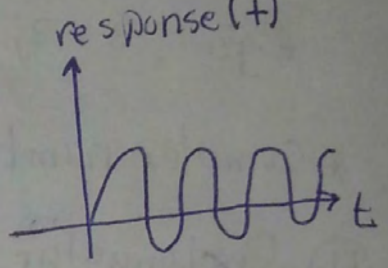
Theorm موجودة

parcticaly مش موجودة *

$\Rightarrow \alpha = \frac{\cancel{R}^{\nearrow 0}}{2L} = 0$ (series)

$\downarrow$

damping Coefficient



$\Rightarrow$ since $\alpha = 0$ There is no damping

CH 10 :- Sinsodial and steady state Analysis

Reve complex numbers

=> $x^2 = -1$ => $x = \sqrt{-1}$
$x = j$ => imaginary operator

=> $j^2 = -1$, $j^3 = -\sqrt{-1} = -j$, $j^4 = 1$, $\frac{1}{j} = -j$

=> Imaginary number = real number $\times j$

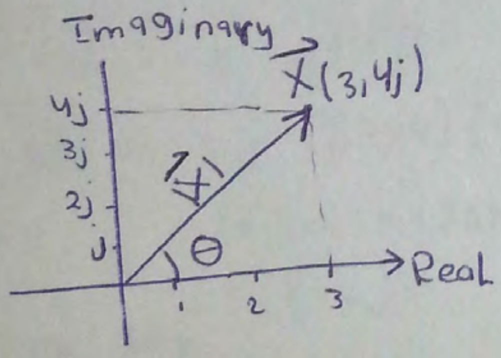
e.g :- $3\hat{j}$, $-3.1j$, $4j$

=> Complex number = real number + Imaginary number

e.g :- $3 + 3j$, $-2.1 + j2.4$, $3j$, $-5 + 1.5j$

* Complex number can be represented using two forms :-

① Rectangular Form : $\vec{X} = a + jb$



* ملاحظة

$$e^{j\theta} = \cos\theta + j\sin\theta$$

Ex: $\vec{X} = 3 + j4$

② polar form : $\boxed{\vec{X} = |\vec{X}| e^{j\angle\vec{X}}}$

Ex: $\vec{X} = 3 + j4$ (rectangular form)

$|\vec{X}| = \sqrt{3^2 + 4^2}$ (polar Form) $\qquad \vec{X} = 5e^{j\tan^{-1}\frac{4}{3}}$

$\angle\vec{X} = \tan^{-1} 4$

Ex: $\vec{X} = 3.1e^{j30^\circ}$

find $\vec{X}$ in rectangular Form?

$\vec{X} = 3.1\cos(30^\circ) + j3.1\sin(30^\circ)$

Ex: $\vec{X} = 4e^{j50}$

$\Rightarrow \vec{X} = 4\cos(50^\circ) + j4\sin(50^\circ)$

Note: $\vec{X} = \ominus 5.39\, e^{-j21.8^\circ}$

$\downarrow$ means ± 180°

$\vec{X} = 5.39\, e^{+j(-21.8^\circ - 180^\circ)} = 5.39\, e^{-j201.8^\circ}$

$\vec{X} = +5.39\, e^{j(-21.8^\circ + 180^\circ)} = 5.39\, e^{j158.2^\circ}$

* Mathematic operations:

if $\vec{X} = 3+j4$, $\vec{y} = 2-j5$

① $\vec{X} + \vec{y} = 5 - j$.

② $\vec{X} - \vec{y} = 1 + j9$.

③ $\vec{X} \times \vec{y} = (3+j4)(2-j5)$

$= 3\times2 - j3\times5 + j4\times2 + 20$

$= 26 - j7$

④ $\vec{X}^* \equiv$ Conjugate of $\vec{X}$

$\vec{X}^* = 3 \underset{\uparrow}{-} j4$

Note:

① $\vec{X} + \vec{X}^* = 2\times3 = 6$

② $\vec{X} - \vec{X}^* = j\,2\times4 = j8$

③ $\vec{X} * \vec{X}^* = (3+j4)(3-j4)$

$= 3^2 + 4^2$

$\vec{X} * \vec{X}^* =$ real numbe

⑤ $\frac{\vec{X}}{y} = \frac{3+j4}{2-j5}$

$$= \frac{(3+j4)(2+j5)}{(2-j5)(2+j5)} = \frac{6+j15+j8-20}{2^2+5^2}$$

$$= \frac{-14+j23}{29} = \frac{-14}{29} + j\frac{23}{29}$$

part ③ can be solved using polar form

$$\vec{X} = 3+j4 = 5e^{j\tan^{-1}\frac{4}{3}}$$

$$\vec{y} = 2-j5 = \sqrt{29}\, e^{j\tan^{-1}\frac{-5}{2}}$$

$$\Rightarrow \vec{X} * \vec{y} = 5\sqrt{29}\; e^{j(\tan^{-1}\frac{4}{3} + \tan^{-1}\frac{-5}{2})}$$

$$\Rightarrow \frac{\vec{X}}{\vec{y}} = \frac{5\,e^{j\tan^{-1}\frac{4}{3}}}{\sqrt{29}\; e^{j\tan^{-1}\frac{-5}{2}}} = \frac{5}{\sqrt{29}}\, e^{j(\tan^{-1}\frac{4}{3} - \tan^{-1}\frac{-5}{2})}$$

Exe. if $\vec{X} = 3e^{j60^\circ}$, $\vec{y} = 4e^{j45^\circ}$

Find $\vec{X} + \vec{y} \Rightarrow$

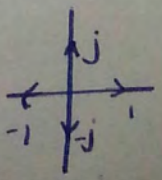
$$\vec{X} = 3\cos(60^\circ) + j\,3\sin(60^\circ)$$

$$\vec{y} = 4\cos(45^\circ) + j4\sin(45^\circ)$$

$$\vec{X} + \vec{y} = (3\cos 60^\circ + 4\cos(45^\circ) + j(3\sin 60^\circ + 4\sin 45^\circ)$$

Exe. $e^{j90^\circ} = j$ $\qquad e^{j360^\circ} = 1$

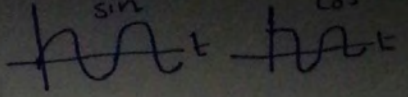
$e^{j180^\circ} = -1$

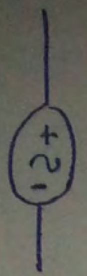


$e^{j270^\circ} = -j$

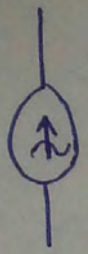
$e^{j360\times 2} = 1$

* Sinusoidal signal :





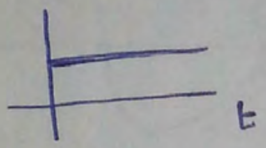
Sinusoidal voltage source (AC Voltage Source)



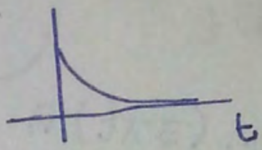
Sinusoidal current source (AC Current Source)

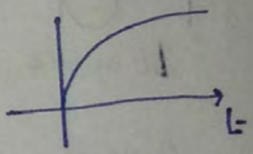
Note : Voltage or Current could be :

DC :

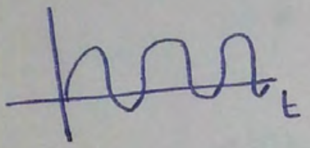


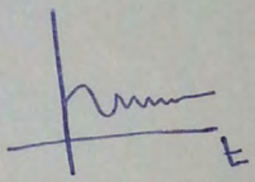
③ others





AC :





* Charactorstic of sinusoidal signal

$v(t) = v_m \sin(\underbrace{\omega t})$ V or A

v_m ↓ peak value or magnitude

ωt → angle → degree / radian

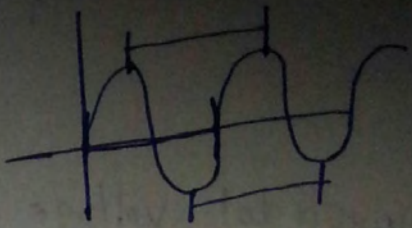
deg.	rad
360° →	2π
40° →	x

$x = \frac{2\pi * 40}{360}$

w : radian frequency (or angular frequency) [rad/sec]

$w = \frac{2\pi}{t}$

$\omega = \frac{2\pi}{T}$



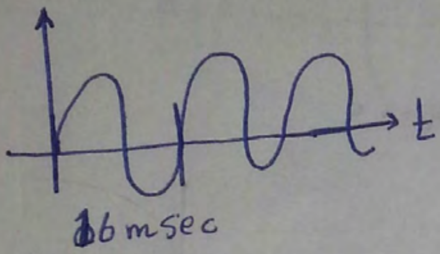
T: pericd of the signal (sec.)

$f = \frac{1}{T}$ frequancy (Hz) ↓ Hertz

Ex: $f = 60\,HZ$

$T = \frac{1}{60} = 0.016\,sec$

$= 16\,msec$



Ex: if $i(t) = 3\sin(3\pi t)\,A$

ⓐ find ω

ⓑ find T

ⓒ find f

ⓓ Draw i(t) as a function of ωt

ⓔ Draw i(t) as a function of t

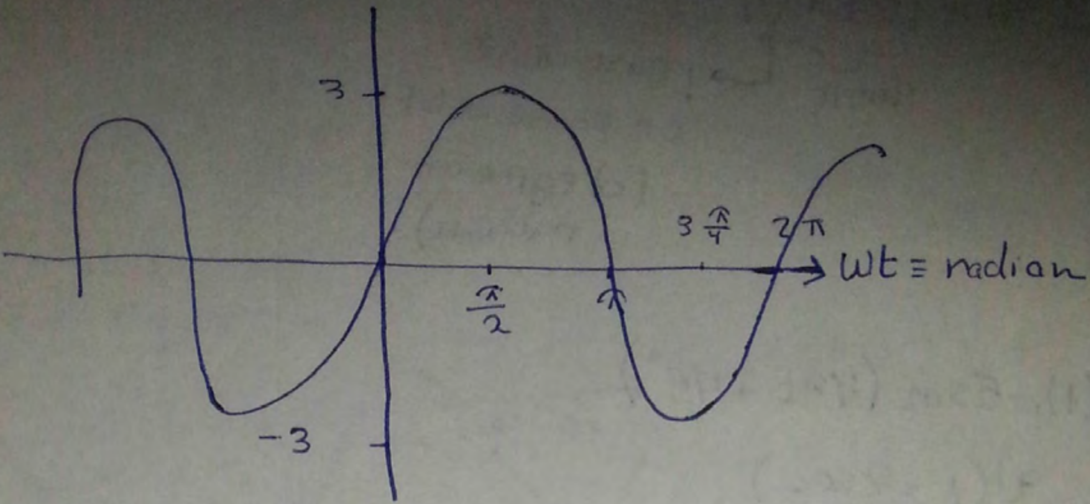
Solution:

ⓐ $\omega = 3\pi$

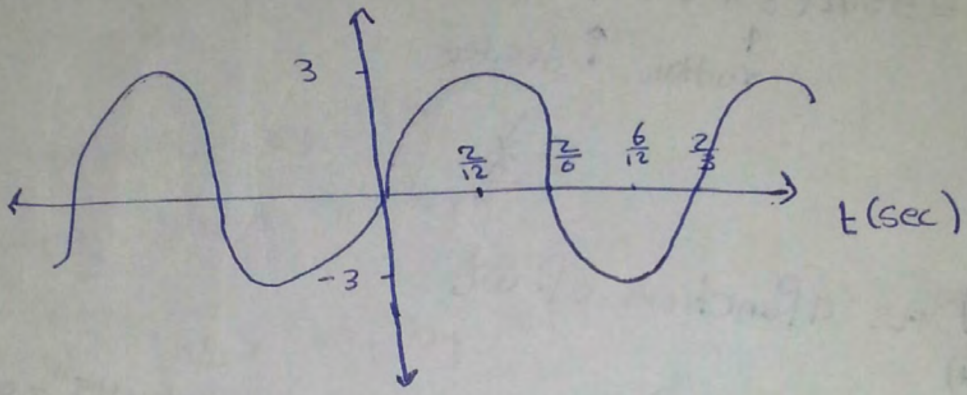
ⓑ $T = \frac{2\pi}{3\pi} = \frac{2}{3}\,sec$

ⓒ $f = \frac{1}{T} = \frac{3}{2}\,HZ$

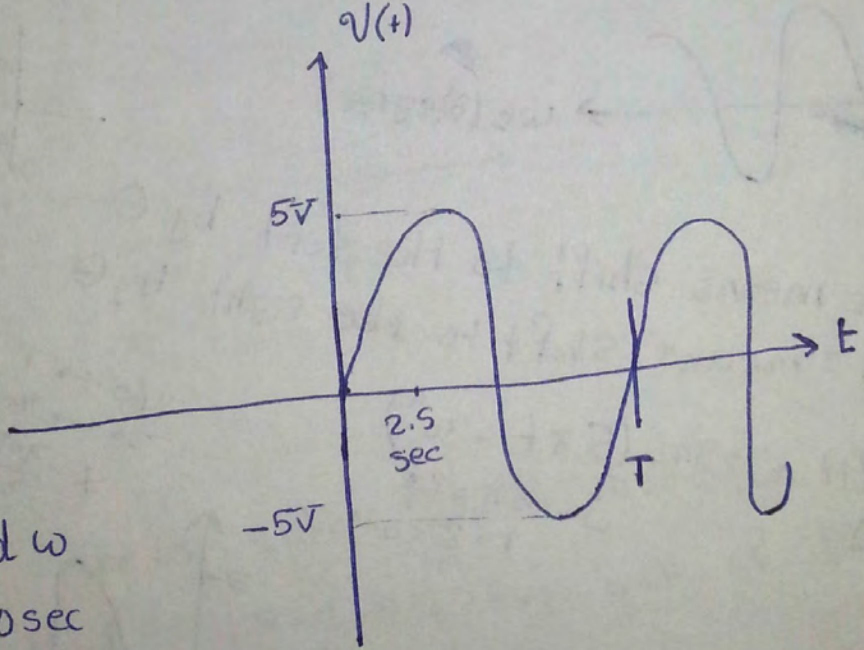
ⓓ



ⓔ



Ex :



ⓐ Find ω.

T = 10 sec

$W = \frac{2\pi}{T} = \frac{\pi}{5}$ rad/sec

ⓑ $f = \frac{1}{T} = 0.1$ HZ

ⓒ $v(t) = 5\sin\left(\frac{\pi}{2}t\right)$ V

* More general form of the sine wave :

$$v(t) = V_m \sin(\underbrace{\omega t}_{\text{angle}} + \theta)$$

θ → phase angle or phase-shift (degree or radian)

Ex : if $v(t) = 5\sin(4\pi t + 45^\circ)$

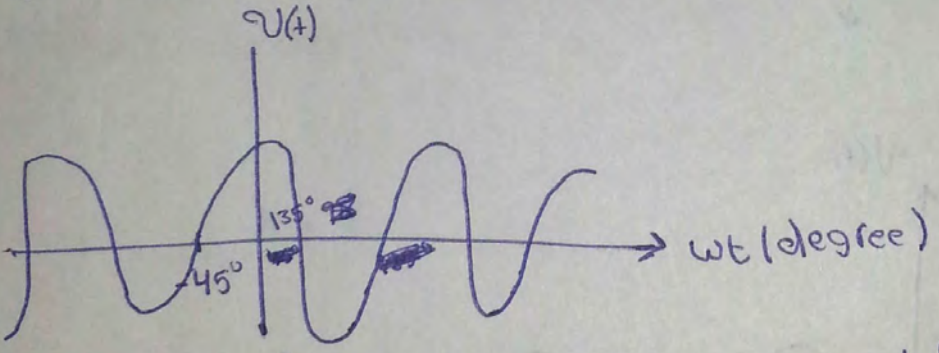
ⓐ find $v(t = 2\text{sec})$

$$v(t=2\text{sec}) = 5\sin(\underset{\uparrow \text{ radian}}{8\pi} + \underset{\uparrow \text{ degree}}{45^\circ})$$

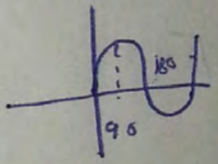
$2\pi \rightarrow 360^\circ$
$8\pi \rightarrow ?$

$\frac{\pi}{4}$

ⓑ Draw $v(t)$ as a function of ωt



$\omega t + 45^\circ = 0$
$\omega t = -45^\circ$



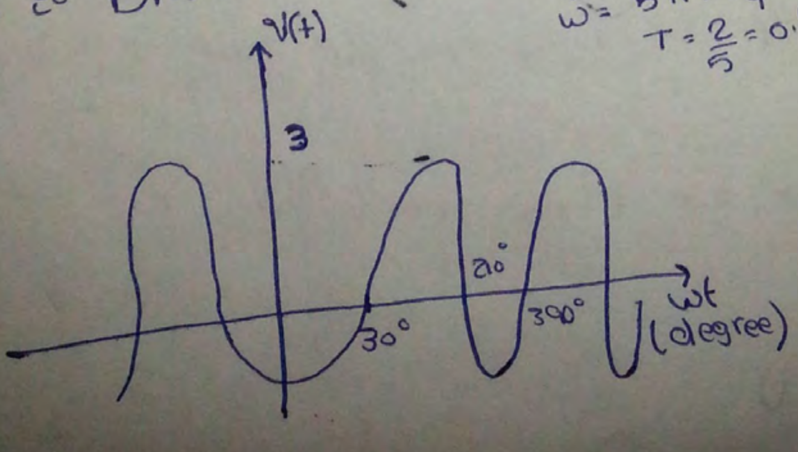
$\Rightarrow V_m \sin(\omega t + \theta) \equiv$ means shift to the left by θ

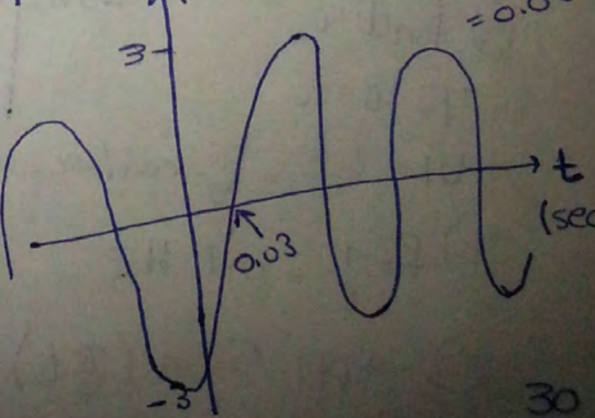
$\Rightarrow V_m \sin(\omega t - \theta) \equiv$ means shift to the right by θ

Ex : Draw $v(t) = 3\sin(5\pi t - 30^\circ)$

$\omega = 5\pi = \frac{2\pi}{T}$
$T = \frac{2}{5} = 0.4$

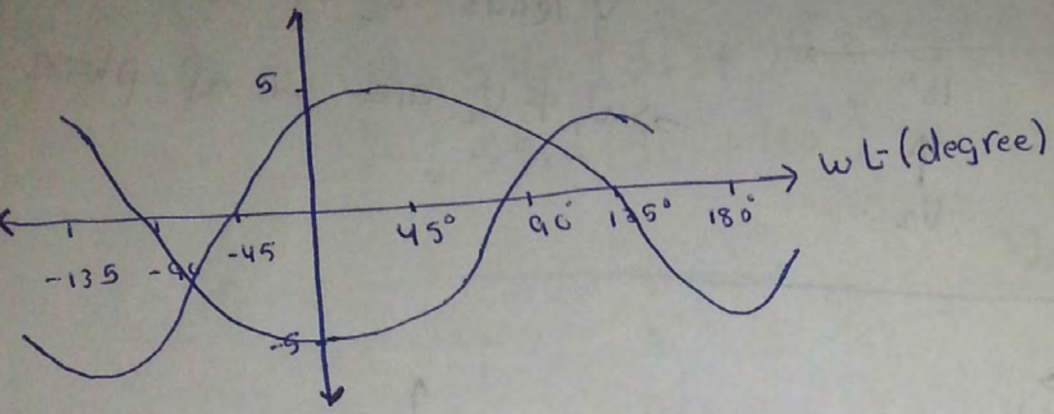
$360 \rightarrow T$
$30^\circ \rightarrow x$
$x = \frac{30T}{360} = \frac{30 \times 0.4}{360} = \frac{1.2}{36}\text{sec}$
$= 0.03$





Ex: if $v_1(t) = 5\sin(2\pi t + 45)$

$$v_2(t) = 5\sin(2\pi t - 90^\circ)$$

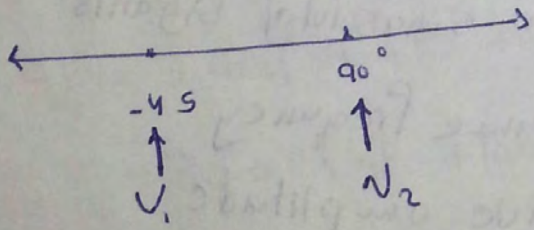


v_1 leads v_2 by 135°

or v_2 lags v_1 by 135°

or v_1 lags v_2 by -135°

or v_2 leads v_1 by -135°



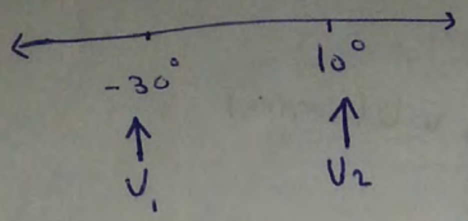
* Note

if There is aphase shift between v_1 & v_2 we Say that v_1 and v_2 are out-of-Phase.

* if there is no phase shift we say that v_1 & v_2 are in-phase.

Ex:: $v_1(t) = 3\cos(2\pi t + 30^\circ)$

$v_2(t) = 4\cos(2\pi t - 10^\circ)$



V_1 leads v_2 by 40°

$\Rightarrow V_1$ & V_2 are out-of-phase

Ex::

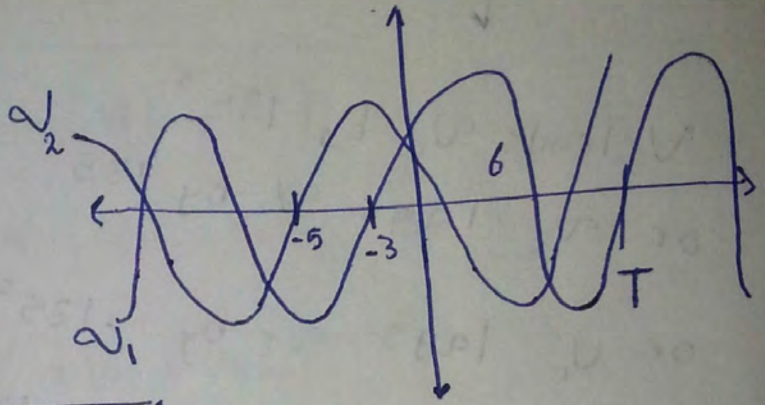
حتى نحسب (قديش سبقوا) نحن لازم بال degree نطلع ((T

$\Delta t = 2$ msec

18msec

$T \longrightarrow 360^\circ$

$2\text{msec} \longrightarrow \Delta\theta$



$$\Delta\theta = \frac{360 * 2}{18} = \boxed{\Delta\theta \Rightarrow 40^\circ}$$

---

* Note : to Compare any two sinusoidal Siganls

① They must have the same frequncy

② They must have Positive amplitude

③ They must be sine or cosine.

$-\sin(wt) = +\sin(wt \pm 180^\circ)$

$-\cos(wt) = +\cos(wt \pm 180^\circ)$

$\sin(wt) = \cos(wt - 90^\circ)$

$\cos(wt) = \sin(wt + 90^\circ)$

Ex: $v_1(t) = 5\cos(5t + 10^\circ)$

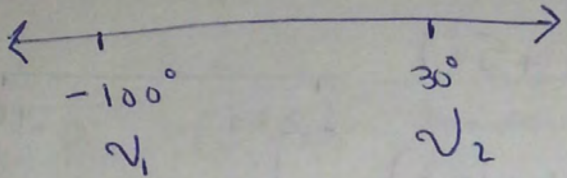
$v_2(t) = 3\sin(5t - 30^\circ)$

* لازم يطبقوا الشروط

$v_1(t) = 5\sin(5t + 10^\circ + 90^\circ)$

$= 5\sin(5t + 100^\circ)$

v_1 leads v_2 by 130°



Ex: $v_1(t) = -3\sin(4t + 10^\circ)$

$v_2(t) = -4\cos(4t - 50^\circ)$

$\Rightarrow v_1(t) = 3\sin(4t + 10^\circ + 180^\circ)$

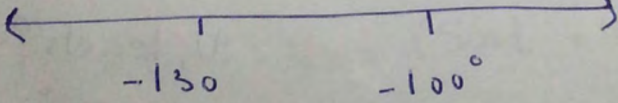
$\Rightarrow v_2(t) = 4\cos(4t + 50^\circ + 180^\circ) \Rightarrow 4\cos(4t + 130^\circ)$

$v_1(t) = 3\cos(4t + 10^\circ + 180^\circ - 90^\circ)$

$= 3\cos(4t + 100^\circ)$

$\Rightarrow v_1$ lags v_2 by 30°

$\Rightarrow v_1$ leads v_2 by -30°



Ex: $i_1(t) = 10 \sin(\underline{\underline{3t}} + 45°)$

$i_2(t) = -5 \cos(\underline{\underline{4t}} + 30°)$

We cannot compare there since different frequancy.

Ex: $v_1(t) = -3\sin(4t+10°)$

$v_2(t) = -5\sin(4t+50°)$

+180 $v_1(t) = 3\sin(4t+190°)$

$v_2(t) = 5\sin(4t+230°)$

$-230 \rightarrow v_2$ $\quad$ $-190° \rightarrow v_1$

v_2 leads v_1 by 40°

-180 $v_1(t) = 3\sin(4t-170°)$

$v_2(t) = 5\sin(4t-130°)$

$130 \rightarrow v_2$ $\quad$ $170 \rightarrow v_1$

v_2 leads v_1 by 40°

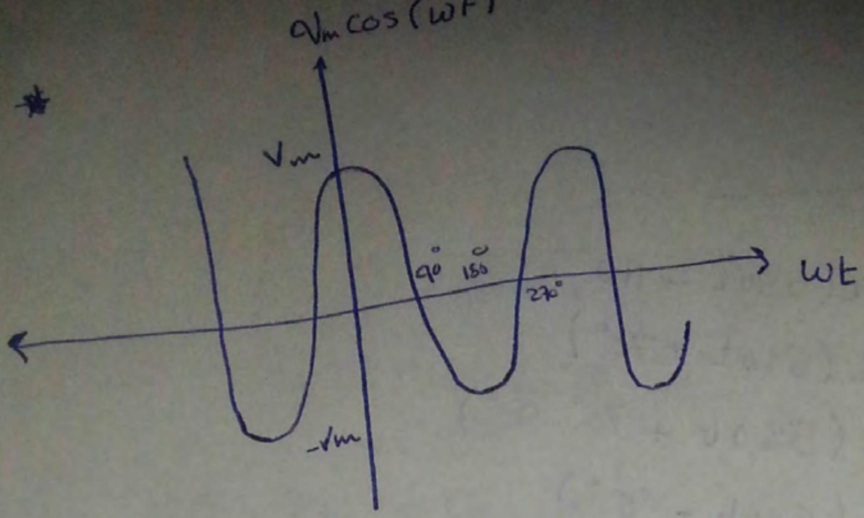
± 180 $v_1(t) = 3\sin(4t+190°)$

$v_2(t) = 5\sin(4t-130°)$

$-190° \rightarrow v_1$ $\quad$ $130° \rightarrow v_2$

v_1 leads v_2 by 320°

*



Phasor form (phasor- domain or frequancy domain) :

Ex: $v(t) = \underline{\underline{3}} \cos(200\pi t + 30°)$ ← (this signal is in the time domain)

Convert this signal from tim- domain into frequancy domain (phaser- domain)

* solution :

$\vec{V} = 3 \angle 30°$ ← phasor domain

Note: to write the phasor form of asinusoidal signals, it should be a positive cosine function.

Ex: if $i(t) = -4\cos(500t + 10°) = 4\cos(500t + 190°)$

Find i(t) in the phasor form

$\vec{I} = 4 \angle 190°$

Exe. if $v(t) = 100 \cos(400t - 30°)$

$\vec{v}' = 100 \angle -30°$

Exe. if $i(t) = -5\sin(580t - 110°)$

$= 5\sin(580t + 70°)$

$= 5\cos(580t + 70° - 90°)$

$= 5\cos(580t - 20°)$

$\vec{I} = 5 \angle -20°$

Exe. $i(t) = 8\cos(4t - 30°) + 4\sin(4t - 100°)$

Write $i(t)$ in phasor form (Frequancey - domain) (or Phasor - domain)

$i(t) = 8\cos(4t - 30°) + 4\cos(4t - 100° - 90°)$

$\bar{\bar{I}} = 8 \angle -30° + 4 \angle -190°$

$= 8e^{-j30°} + 4e^{-j190°}$

$= 8[\cos(-30°) + j\sin(-30°)] + 4[\cos(-190°) + j\sin(-190°)]$

$= 6.92 - j4 - 3.93 + j0.69$

$= 2.99 - j3.3$

$\bar{\bar{I}} = 4.45 \angle -47.8°$

Ex: if $\bar{I} = 20 + j10$ A , $\omega =$

find $i(t)$

$$\vec{I} = \sqrt{20^2 + 10^2} \angle \tan^{-1}\frac{10}{20}$$

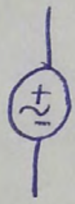
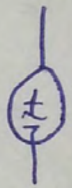
$$= 22.36 \angle 26.57^\circ$$

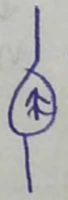
$$i(t) = 22.36 \cos(2000t + 26.57^\circ) \text{ A}$$

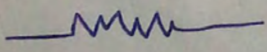
Steady state Analysis :

step 1 : Convert the circuit from the time-domain into frequancy-domain (phasor-domain)

time domain | **Frequancy domain**

Sources  $v_s(t) = V_m \cos(\omega t + \theta)$ →  $\vec{V_s} = V_m \angle \theta$

 $i_s(t) = I_m \cos(\omega t + \phi)$ →  $\bar{I_s} = I_m \angle \phi$

 R (Ω) → R (Ω)

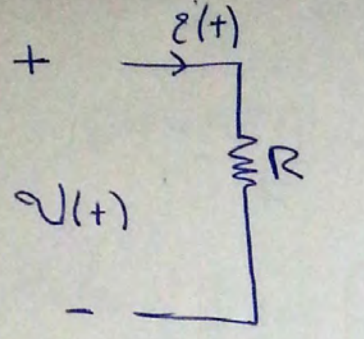
C (F) → $Z_c = \frac{1}{\boxed{j}\omega c}$ (Ω) impedence of c

complex

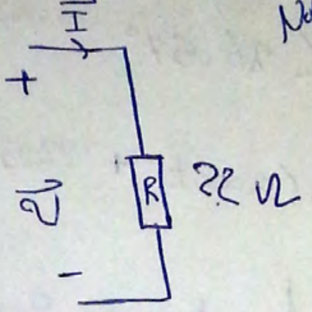
$L(H) \longrightarrow Z_L = j\omega L\ (\Omega)$

↳ impedance of L

* The Resistor in frequancy domain is R(Ω). why?!



time domain



Frequancy - domain

Note * The general form of asinsoidal signal is:

$$\underbrace{v(t)}_{\text{complex}} = \underbrace{V_m \cos(\omega t+\theta)}_{\text{Real}} + j\underbrace{V_m \sin(\omega t+}_{\text{imaganiy}}$$

$$V_m e^{j(\omega t+\theta)}$$

$$v(t) = R i(t)$$

$$V_m e^{j(\omega t+\theta)} = R i_m e^{j(\omega t+\phi)}$$

$$V_m \cancel{e^{j\omega t}} \cdot e^{j\theta} = R I_m \cancel{e^{j\omega t}} \cdot e^{j\phi}$$

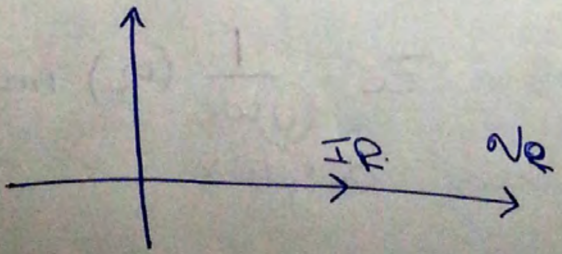
$$V_m \angle\theta = R I_m \angle\phi$$

$$\vec{V} = \underset{\Omega}{\underset{\downarrow}{R}} \vec{I}$$

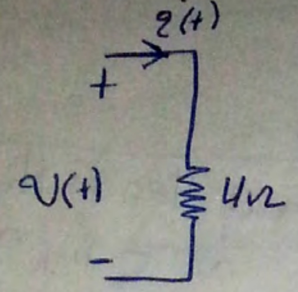
$$\theta = \phi$$

①

$$\boxed{\text{The current and voltage in R are inphase}}$$ ✓



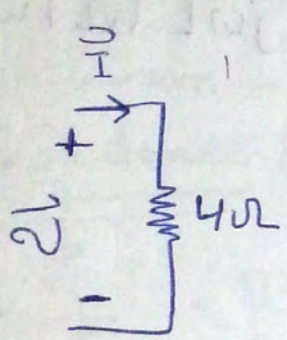
Ex: if $v(t) = 8\cos(100t - 50°)$ and $R = 4\Omega$, Find $i(t)$?!



Method 1: tim-domain

$$i(t) = \frac{v(t)}{R} = 2\cos(100t - 50°)$$

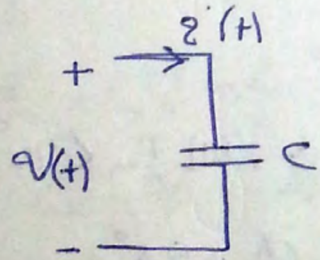
Method 2: freq-domain

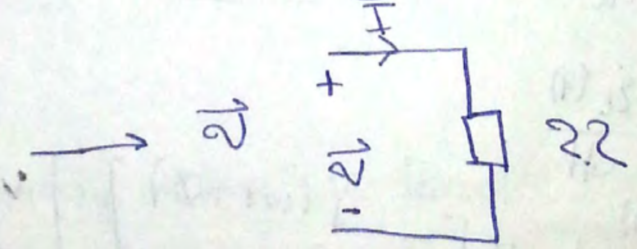


$$\Rightarrow \vec{I} = \frac{\vec{V}}{R} = \frac{8\angle -50°}{4} = 2\angle -50°$$

$$i(t) = 2\cos(100t - 50°)$$

* The capacitor in frequancy-domain is $\frac{1}{jwc}$ Why?





$$i(t) = C\frac{dV(t)}{dt}$$

$$I_m e^{j(wt+\theta)} = C\frac{d V_m e^{j(mt+\theta)}}{dt}$$

$$I_m e^{j(wt+\phi)} = C V_m j w e^{j(wt+\theta)}$$

$$I_m \cancel{e^{jwt}} \cdot e^{j\phi} = jwc\, V_m \cancel{e^{jwt}} \cdot e^{j\theta}$$

$$I_m \angle\phi = jwc V_m \angle\theta$$

$$\vec{I} = jwc\vec{V}$$

$$\vec{V} = \underbrace{\frac{1}{jwc}}_{\Omega}\vec{I}$$

$I_m \angle \phi = \omega c \angle 90^\circ * V_m \angle \theta$

$I_m \angle \phi = \omega c \, V_m \angle \theta + 90^\circ$

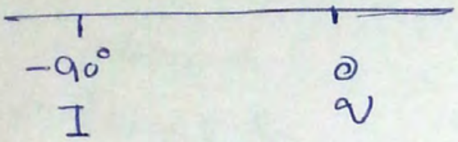
The current leads the voltage on C by 90°

$\phi = \theta + 90^\circ$

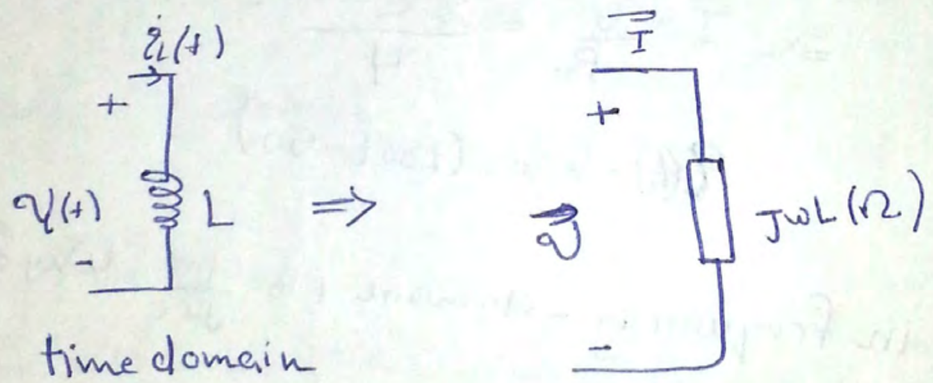
$\angle I$ $\angle V$

↓ ↓

$\cos(\omega t + 90^\circ)$ $\cos(\omega t + \theta)$



* The inductor in frequancy domain is jωL (Ω) why?



time domain

$v_L(t) = L \frac{d\, i_L(t)}{dt}$

$V_m e^{j(\omega t+\theta)} = L \frac{d}{dt}\left[I_m e^{j(\omega t+\phi)} \right]$

$V_m e^{j(\omega t+\theta)} = L\, I_m\, j\omega\, e^{j(\omega t+\phi)}$

$V_m \cancel{e^{j\omega t}} \cdot e^{j\theta} = j\omega L\, I_m \cancel{e^{j\omega t}}\, e^{j\phi}$

$V_m \angle \theta = j\omega L\, I_m \angle \phi$

$\vec{v} = j\omega L\, \vec{I}$

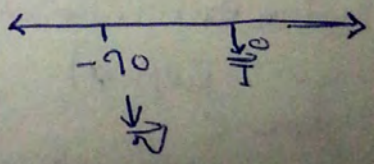
$V_m \angle \theta = \omega L \angle 90^\circ + \vec{I} \angle \phi$

$V_m \angle \theta = \omega L\, I_m \angle 90^\circ + \phi$

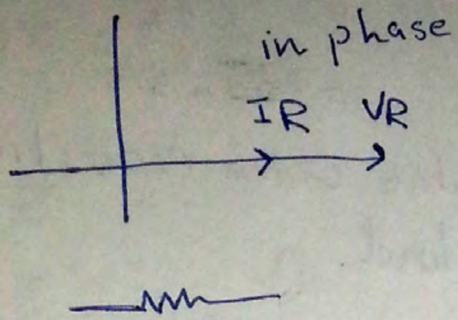
$\theta = \phi + 90^\circ$

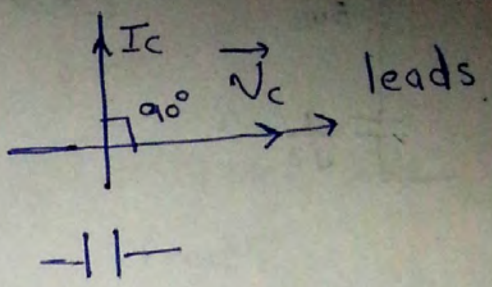
$\angle V_L = \angle I_L$

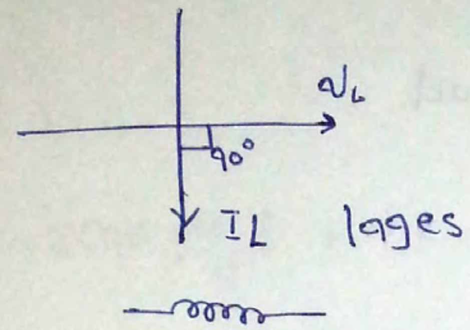
$\cos(\omega t + 90^\circ)$ $\cos(\omega t + \theta)$



The Current Lags Voltage by 90° on L







✱ Impedance Z

$\vec{Z} = \frac{\vec{V}}{\vec{I}} = R + jX$ → reactance (Ω)

↓ impedance Ω

↓ Resistance (Ω).

$\angle\vec{Z} = \angle\vec{V} - \angle\vec{I}$

if $\angle\vec{Z} = 90°$ => purely inductive load

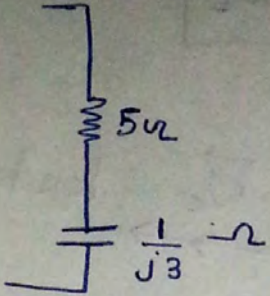
if $\angle\vec{Z} = -90°$ => Purely Capactive Load.

if $\angle\vec{Z} = 0$ => purely resistive load

if $\angle\vec{Z}$ = is positive The load is an inductive load (but not pure inductive)

if $\angle\vec{Z}$ = is negative The load is an capacitive load (but not pure capacitive)

Ex :-



$\vec{Z} = 5 - j\frac{1}{3}\ \Omega \Rightarrow \angle\vec{Z} = \tan^{-1}\frac{-\frac{1}{3}}{5}$

$= $ [illegible] -3.8°

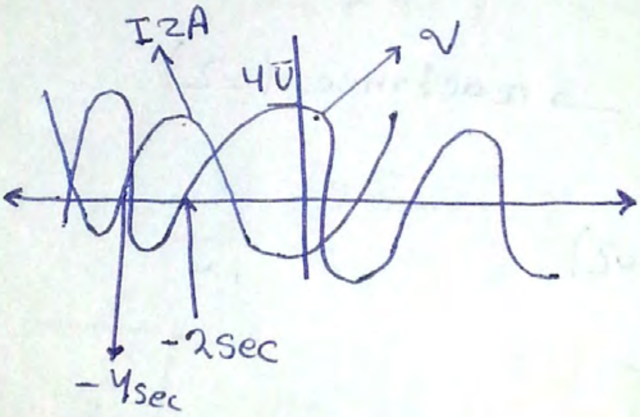
Negative value capacitive load.

Ex:- if $\vec{Z} = 3 \overset{\text{inductive load}}{\oplus} j5.5$

this loud is inductive load

if $\vec{Z} = 4 \underset{\text{capacitive load.}}{\ominus} j2$

Ex:-



ⓐ the tybe of the load ??

Capacitive load becaus $\vec{I}$ leads $\vec{V}$

ⓑ the angle of the load ??

T = 8 sec $\qquad \angle\vec{Z} = -90^\circ$

$\Delta t = 2\,sec$

$T \longrightarrow 360^\circ$

$\Delta t \longrightarrow \Delta\theta$

$\Delta\theta = \frac{360^\circ \times 2}{8} = 90^\circ$

$|\vec{Z}| = \frac{|\vec{V}|}{|\vec{I}|}$, $\angle\vec{Z} = \angle\vec{V} - \angle\vec{I}$

ⓒ reactance part of the load?

$\angle\vec{Z} = -90^\circ$

$|\vec{Z}| = \frac{4V}{2A} = 2\,\Omega$

ⓓ resistance part of the load?

$\vec{Z} = 2\angle -90^\circ$

$$Z = \underset{\overset{\uparrow}{R}}{0} - j\underset{\overset{\uparrow}{X}}{2}\ \Omega$$

$\Rightarrow v(t) = ??$

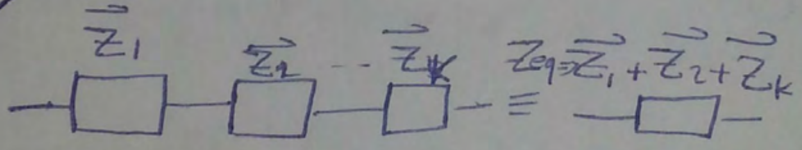
$\omega = \frac{2\pi}{T} = \frac{2\pi}{8} = \frac{\pi}{4}t$

$4\cos\left(\frac{\pi}{4}t + 0\right)$

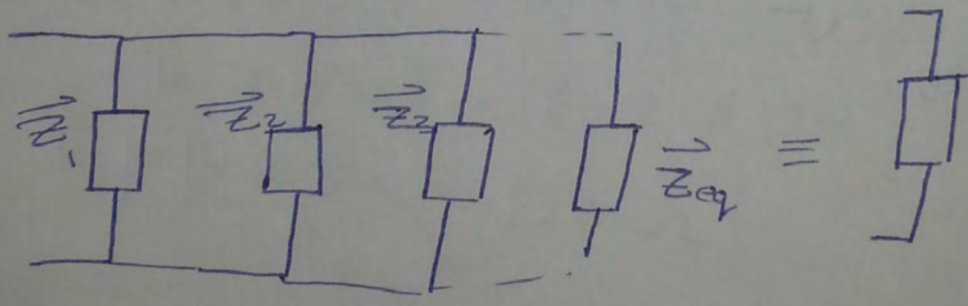
or $v(t) = 4\sin\left(\frac{\pi}{4}t + 90^\circ\right)$

$2 \rightarrow \theta$
$8 \rightarrow 360^\circ$

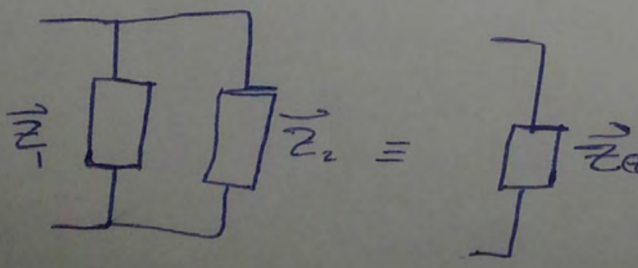
shift السالب
يعني ⊕



$Z_{eq} = \vec{Z}_1 + \vec{Z}_2 + \vec{Z}_k$



$\frac{1}{Z_{eq}} = \frac{1}{\vec{Z}_1} + \frac{1}{\vec{Z}_2} \div \cdots \frac{1}{\vec{Z}_K}$



$= \frac{\vec{Z}_1 \vec{Z}_2}{\vec{Z}_1 + \vec{Z}_2}$

$$\underset{\underset{\text{Admittance}}{\downarrow}}{\vec{Y}} = \frac{1}{\vec{Z}} = \underset{\underset{\text{Conductance}}{\downarrow}}{G} + j\,B \longrightarrow \text{susceptance}$$

(℧)

Admittance
(℧)
(S)

Conductance
(Ω)

$$\vec{Y} = \frac{1}{R+jX} * \frac{(R-jX)}{R-jX)} = \underset{\underset{G}{\downarrow}}{\frac{R}{R^2+X^2}} + j\underset{\underset{B}{\downarrow}}{\frac{-X}{R^2+X^2}}$$

Ex: if $\vec{Z} = \overset{\overset{R}{\uparrow}}{3} + j\overset{\overset{X}{\uparrow}}{5}\ \Omega$

Find B and G

$$G = \frac{R}{R^2+X^2} = \frac{3}{9+25}\ \mho$$

$$B = \frac{-X}{R^2+X^2} = \frac{-5}{9+25}\ \mho$$

$$G \neq \frac{1}{R}\ \times$$

Ex: $\vec{Z} = 4 + j2\ \Omega$

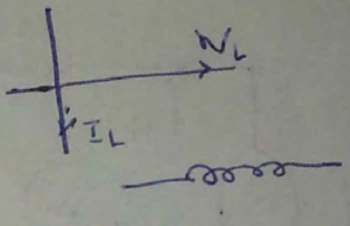
$\frac{\vec{v}}{\vec{I}} \leftarrow$

a ⇒ Find the tybe of load.

b ⇒ phase shift between the voltage and current on $\vec{Z}$

(اذا كان نقطة j
مو pure

ⓐ inductive load (But not purely)

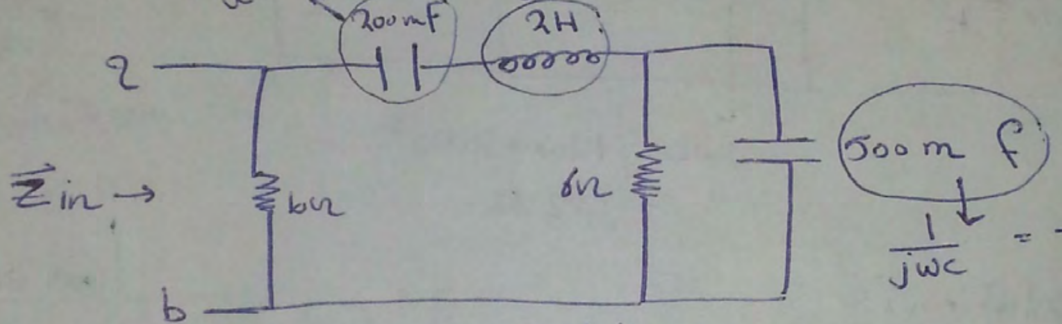


ⓑ $\vec{I}$ lags $\vec{v}$ by $\tan^{-1}\frac{2}{4}$

Ex:

$-j\frac{1}{\omega C} = -j\Omega$

$j\omega L \Rightarrow j10\Omega$



$\frac{1}{j\omega C} = -j\frac{1}{\omega C} = -j0.$

$w = 5\text{rad/sec}$

Find $\vec{Z}_{in}$

$\vec{Z}_{eq} = [6 // -j0.4 + j10 + (-j)] // 10\Omega$

$= \left[\frac{-j2.4}{6 - j0.4} + 9j\right] // 10$

$= 0.0265 + j8.602 // 10$

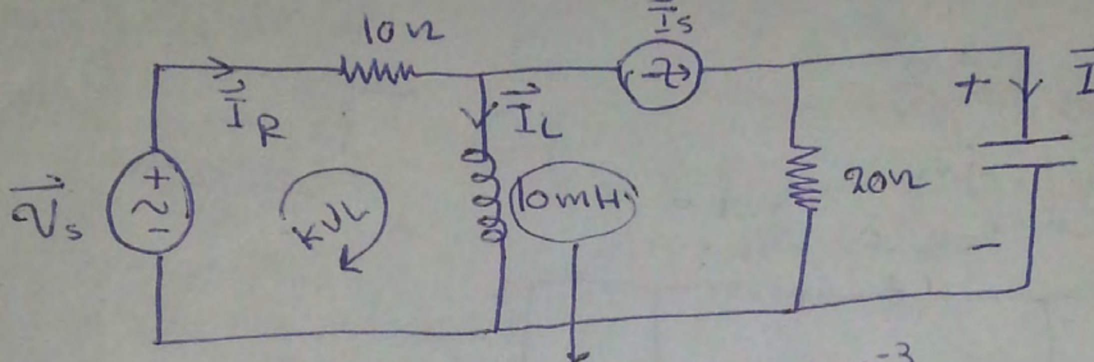
$\vec{Z}_{eq} = 4.255 + j4.929\ \Omega$

Ex: if $w = 1200$ rad/sec

$\vec{I}_c = 1.2 \angle 28°$

$\vec{I}_L = 3 \angle 53°$

* Find $\vec{I}_s$, $\vec{V}_s$, $i_R(t)$??



$\Rightarrow -j\frac{1}{wC}$

$= -j\frac{1}{1200 * 25*10}$

$= -j33.33Ω$

$jwL = j1200 * 10*10^{-3}$

$= j12Ω$

time domain.

$\boxed{i_R(t) = 3.99\cos(1200t + 17.42°) \text{ A}}$

solution:

$v_s = I_c * (-j33.33)$

$= 1.2 \angle 28° * (-j33.33)$

$\boxed{= 40 \angle -62° \text{ V}}$

$I_{20Ω} = \frac{v_c}{20} = \frac{40 \angle -62°}{20} = 2 \angle -62° \text{ A}$

$\vec{I}_s = \vec{I}_c + \vec{I}_{20} = 1.2 \angle 28° + 2 \angle -62°$

$= 2.33 \angle -31° \text{ A}$

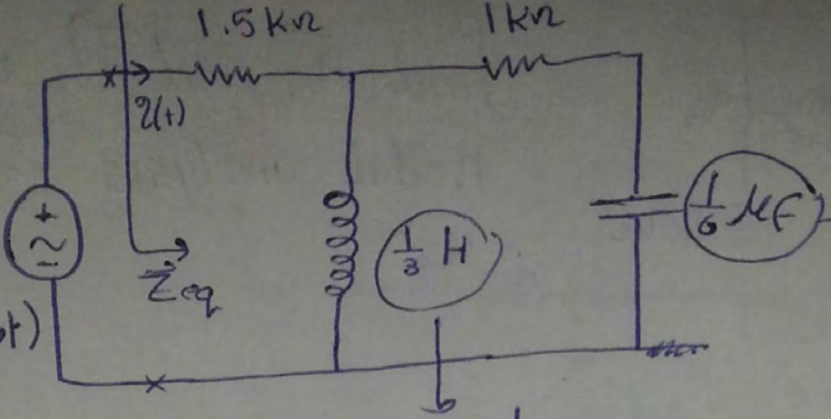
$I_R = \vec{I}_s + \vec{I}_L$

$= 2.33 \angle -31° + 3 \angle 53° = 3.99 \angle 17.42° \text{ A}$

$-V_s + 10 * I_R + j12 * (3 \angle 53°) = 0$

$V_s = 34.9 \angle 74.5°$

Ex: find $i(t)$



$v_s(t) = 40\sin(3000t)$

↓

لازم نحولها

position

cos

كان احولها

phasor.)

$-j\frac{1}{\omega C} = -j\frac{1}{3000 * \frac{1}{6} * 10^{-6}}$

$= -j2\ k\Omega$

$j\omega L$

$j * 3000 * \frac{1}{3} =$

$j1000\ \Omega = j1\ k\Omega$

$v_s(t) = 40\cos(3000t - 90^\circ)$

$\vec{v}_s = 40 \angle -90^\circ$

$\vec{Z}_{eq} = (1 - j2) // j + 1.5$

$= 2 - j1.5\ k\Omega$

$\vec{I} = \frac{\vec{v}_s}{\vec{Z}_{eq}} = \frac{40 \angle -90^\circ}{2 - j1.5} = 16 \angle -126.9^\circ\ mA$

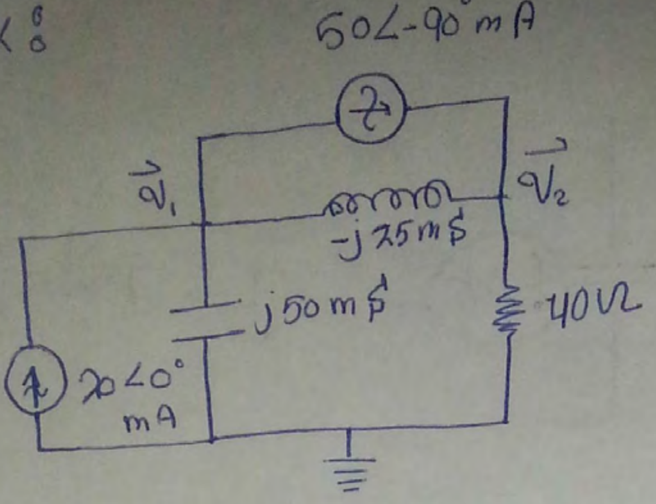
$i(t) = 16\cos(3000t - 126.9^\circ)\ mA$!!

Time domain

$-v_s(t) + i_1(t)1.5 + 3\int_0^t (i_1(t) - i_2(t))\, d\tau = 0$

$i_2 + \frac{1}{6}\frac{d\, i_2(t)}{dt} + 3\int_0^t (i_2(t) - i_1(t)) = 0$.

Ex:



find $\vec{v}_1$ and $\vec{v}_2$ using nodal analysis ?!

* At Node 1 (KCL):

$$-20\angle 0^\circ + (\vec{v}_1 - 0)j50 + (\vec{U}_1 - \vec{V}_2)(-j25) + \overset{-j50}{\overset{\uparrow}{\overbrace{50\angle -90^\circ}}} =$$

$$20 + j50 = j25\vec{V}_1 + j25\vec{V}_2 \quad \text{--} ①$$

* At Node 2 (KCL):

$$-50\angle -90^\circ + (\vec{V}_2 - \vec{v}_1)(-j25) + (\vec{v}_2 - 0)40 = 0$$

$$-j50 = j25\vec{v}_1 + (40 - j25)\vec{v}_2 \quad \text{--} ②$$

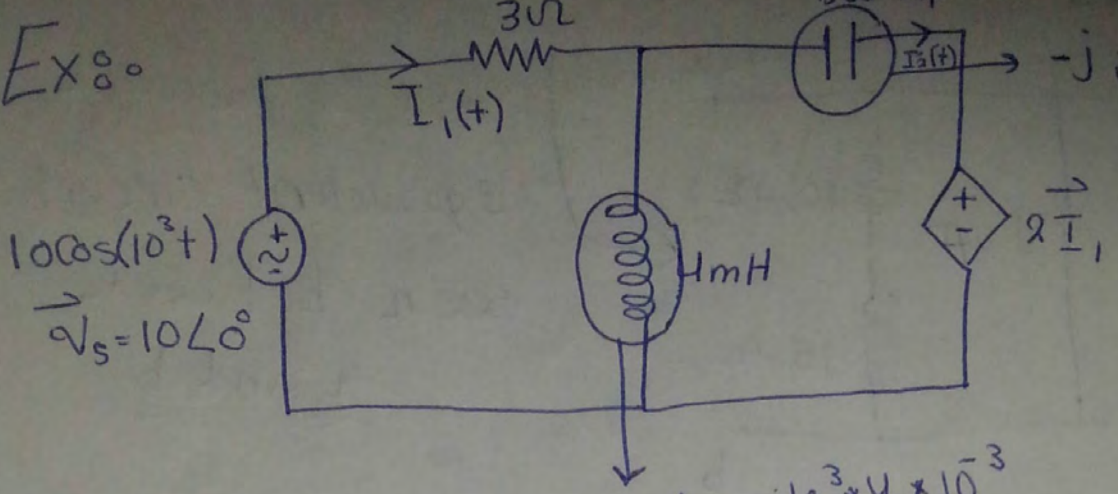
① AND ②

$$20 + j100 = (-40 + j50)\vec{v}_2$$

$$\vec{v}_2 = \frac{20 + j100}{-40 + j50} = \frac{42}{41} - j\frac{50}{41} = 1.59\angle -50^\circ$$

$$\vec{v}_1 = \frac{20 + j50 - j25(1.59\angle -50^\circ)}{j25} = 1.06\angle 23.3$$

Ex:



$$-j\frac{1}{\omega C} = -j\frac{1}{10^3 \times 500 \times 10^{-6}}$$
$$= -j2\ \Omega$$

$$j\omega L = j10^3 * 4 \times 10^{-3}$$
$$= j4\Omega$$

Q Find $i_1(t)$ and $i_2(t)$ using mesh analysis?

⇒ mesh $\underline{\underline{1}}$

$$-10 + 3\vec{I}_1 + j4(\vec{I}_1 - \vec{I}_2) = 0$$
$$-10 + (3+j4)\vec{I}_1 - j4\vec{I}_2 = 0 \quad \text{-- (1)}$$
$$I_2 = \frac{-10 + (3+j4)\vec{I}_1}{j4}$$

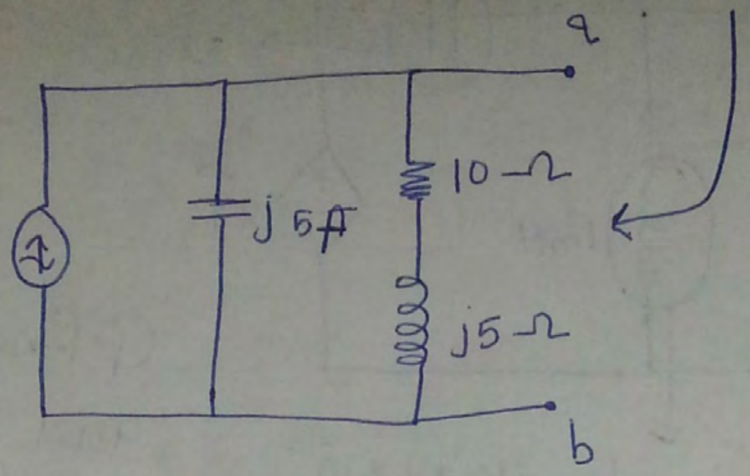
⇒ mesh $\underline{\underline{2}}$

$$j4(\vec{I}_2 - \vec{I}_1) - j2(\vec{I}_2) + 2\vec{I}_1 = 0$$
$$(2-j4)\vec{I}_1 + j2\vec{I}_2 = 0 \quad \text{--- (2)}$$

from ①

$$(2-j4)\vec{I}_1 + j2\left[\frac{10 + (3+j4)\vec{I}_1}{j4}\right] = 0$$
$$(2-j4)\vec{I}_1 - 5 + (1.5+j2)I_1 = 0$$
$$0.5 - j6 I_1 = 5 \Rightarrow \vec{I}_1 = \frac{5}{0.5-j6} = \underline{1.24\angle 29.7^\circ \text{ A}}$$
$$\underline{\vec{I}_2 = 2.77\angle 56.3^\circ \text{ A}}$$

Ex :



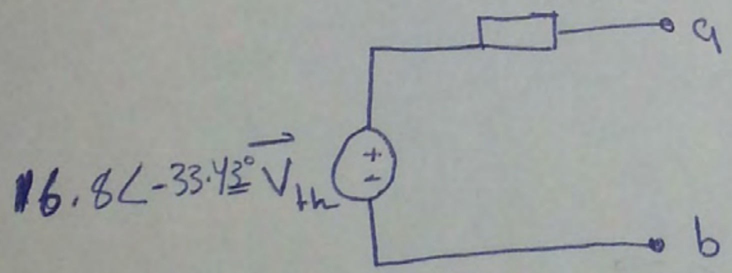
Find the thevenin equavelent circuit seen between a and b

$$\vec{Z}_{eq} = -j5 \,//\, (10 + j5) = \frac{-j5(10 + j5)}{10} = \frac{-j50 + 25}{10}$$

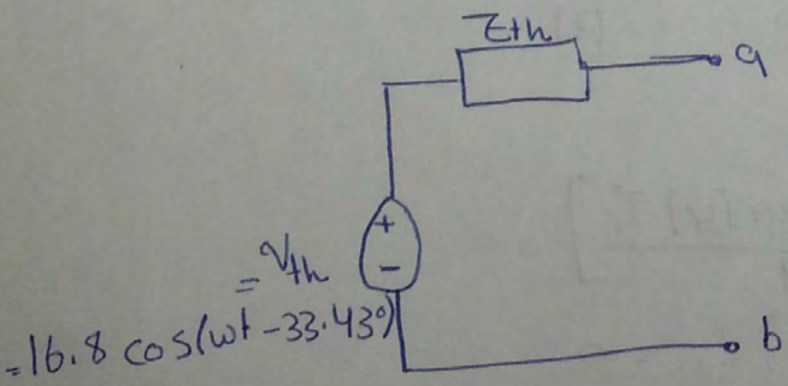
$$\Rightarrow 2.5 - j5\ \Omega$$

$$\vec{V}_{th} = 3\angle 30^\circ * \vec{Z}_{th} = 16.8\angle -33.43^\circ$$

$$Z_{th} = 2.5 - j5\ \Omega$$



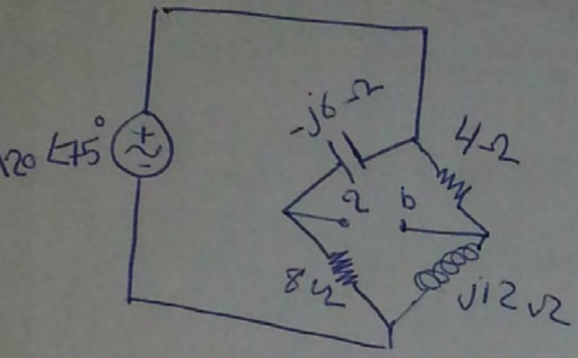
⇓ or



$I_Z = 2.77\angle 56.3^\circ$ A

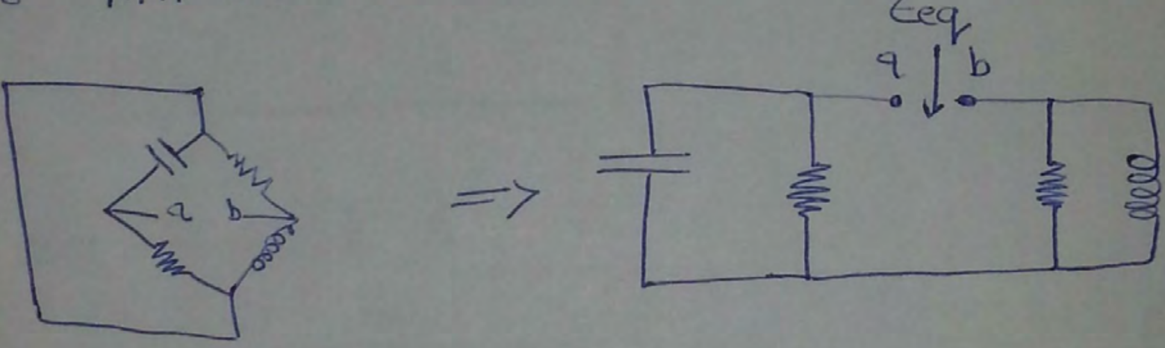
Note :- To get the maximum power transfer from a give circuit to a certain load ($\vec{Z}_L$) in the circuit $\boxed{Z_L = Z_{th}}$

Ex :-



Find the thevenin equvelent seen between a and b ??

Sol :- kill the source :-



$\vec{Z}_{eq} = (8 // -6j) + (4 // j12)$

$= \frac{8 * (-j6)}{8-j6} + \frac{4 * (j12)}{4 + j12}$

$= 2.88 - j3.84 + 3.6 + j1.2$

$= 6.48 - j2.64\ \Omega$

$V_{th} = V_{o.c} = V_a - V_b$ * using nodal analysis

$\frac{\vec{V}_a - 0}{8\Omega} + \frac{\vec{V}_a - 120\angle 75^\circ}{-j6}$ - - - (1)

$$\vec{V}_a\left(\frac{1}{8}-\frac{1}{j6}\right)=\frac{-120\angle 75^\circ}{j6} \Rightarrow \frac{\left[-120\angle 75^\circ / j6\right]}{\frac{1}{8}-\frac{1}{j6}}$$

$$\frac{\vec{V}_b-0}{j12}+\frac{\vec{V}_b-120\angle 75^\circ}{4}=\text{---}\quad(2)$$

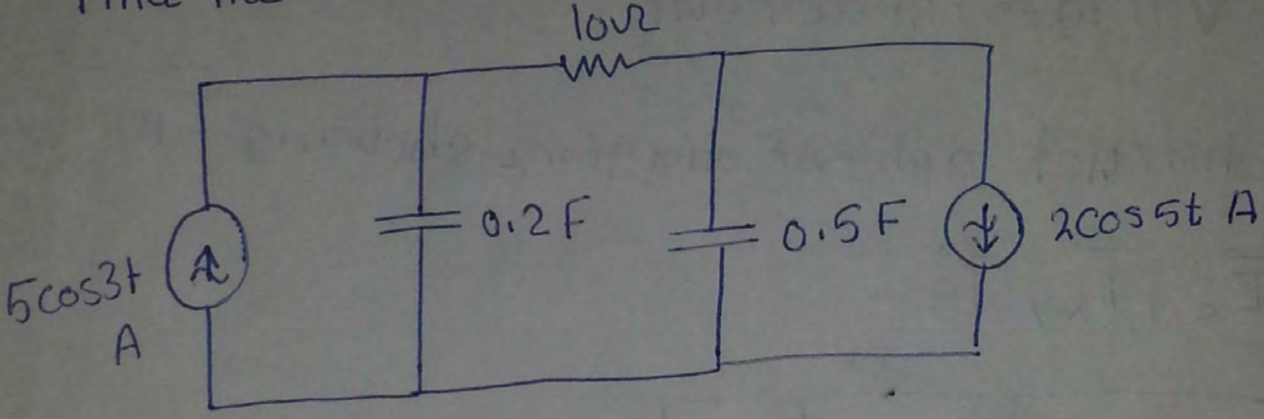
$$\vec{V}_{th}=\vec{V}_{oc}=\vec{V}_a-\vec{V}_b \Rightarrow 37.95\angle 220.31^\circ V$$

Notes:

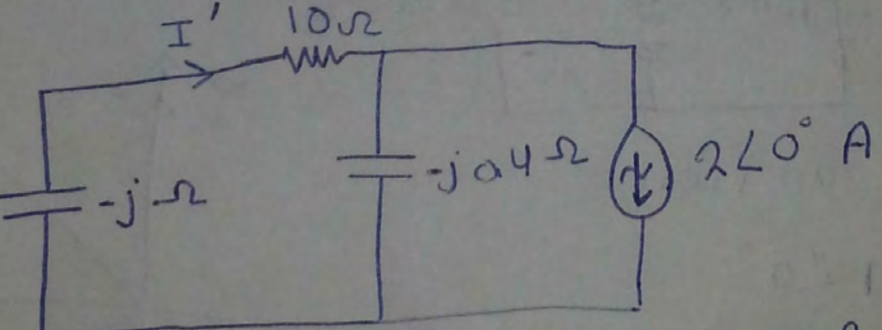
If the circuit has different Sources with different Freqances (ω) then we use

Super position

Find the Current on 10 Ω ??



due 2Cos 5t A

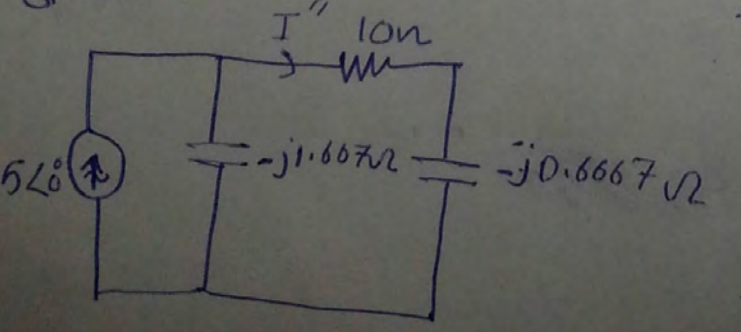


$I' = 2\angle 0^\circ \left[\frac{-j0.4}{10-j-j0.4}\right] = 79.23\angle -82.03^\circ mA \Rightarrow$

$79.23 \cos(5t-82.03^\circ) mA$

---

due 5cos 3t A



$I'' = 5\angle 0^\circ \left[\frac{-j1.667}{10-j0.6667-j1.667}\right]$

$= 811.7\angle -76.86^\circ mA$

$I'' = 811.7\cos(3t-76.86^\circ) mA$

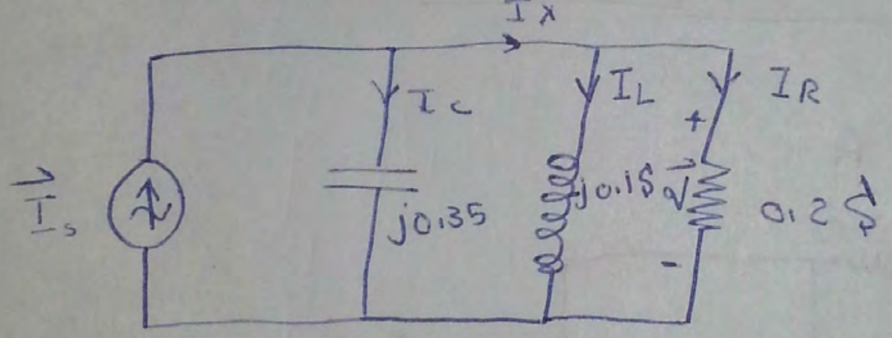
$I = I' + I''$

* phasor diagram :

it provides agraphical method for solving certain problems which may be used to check more exact analylical methodes .

=> it shows the relationships between the current and voltages in acircuit

Ex : construct aphasor diagram showing $\vec{I}_R, \vec{I}_L, \vec{I}_c, \vec{I}_X, \vec{I}_s$



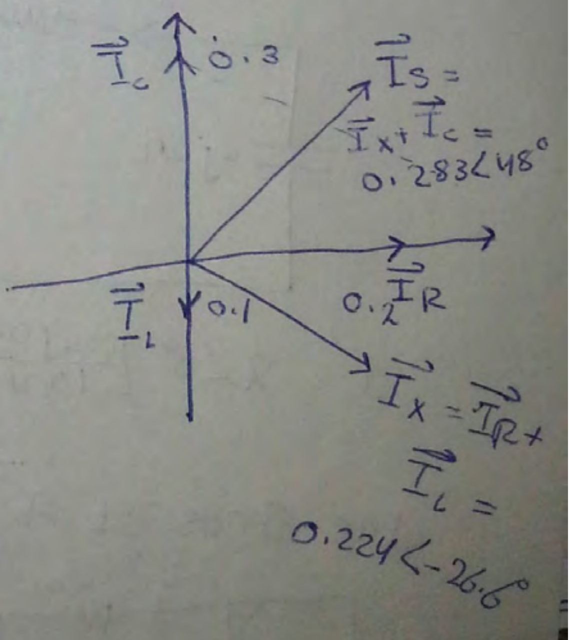
select $\vec{V}$ as arefrance

$\vec{V} : 1 \angle 0°$

$\vec{I}_R = 0.2 * 1 \angle 0° = 0.2 \angle 0°$

$\vec{I}_L = -j0.1 * 1 \angle 0° = 0.1 \angle -90°$

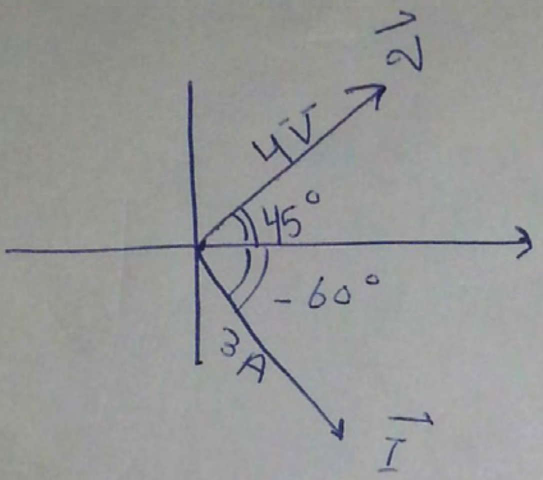
$\vec{I}_c = j0.3 * 1 \angle 0° = 0.3 \angle 90°$



[redacted]

Ex: $\vec{V}(t) = 4\cos(\omega t + 45^\circ)$ → $4 \angle 45^\circ$ phasor diagram??

$\vec{I}(t) = 3\cos(\omega t - 60^\circ)$ → $3 \angle -60^\circ$



تم بحمد الله ^_^