

CIRCUITS LAB SUMMARY

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Electric Circuit Lab

EE 219

Summary & Review

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Electrical Engineering

Note 30

- 1- This summary for final exam so you need to solve problems after studying it
- 2- Review the Summary at the End of Manual.
- 3- This is just ~~an~~ personal effort...
Good luck 😊

Experiment 1 :-

PERFORMANCE OF DC CIRCUITS

Part 1 :- identification of R and C using color coding :

① Resistors - Two Types -

* Four color code

$$AB \times 10^C \pm T$$



↑ ↑ ↑ ↓ tolerance
1st 2nd 10
A B

silver $\pm 10\%$

gold $\pm 5\%$

Examples:- green, blue, orange, gold

green 5

blue 6

orange 3

$$\text{Sol: } 356 \times 10^3 \pm 5\%$$

* Five color coding



↑ ↑ ↑ ↓ ↓ tolerance
1st 2nd 3rd 10

Brown $\pm 1\%$

red $\pm 2\%$

Example: blue gray brown red Brown

$$\text{Sol: } 681 \times 10^2 \pm 1\%$$

② capacitors :-

A	B	Z	S
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A = 1st number

B = 2nd number

Z = # of zeros

S = Symbol of Tolerance (B: $\pm 0.1\%$, C: $\pm 0.25\%$)

* Example: 104

$$10,000 \text{ PF} = 0.1 \mu\text{F}$$

* Example 103 J ($J: \pm 5\%$)

$$10,000 \text{ PF} \pm 5\%$$

$$= \underline{0.01 \mu\text{F}}$$

* Ex. 1) 274 M (M: 20%)

$$= 27,000 \text{ PF} = 0.27 \mu\text{F} \pm 20\%$$

223 J

$$22,000 \text{ PF} = 22 \text{ nF} = 0.022 \mu\text{F} \pm 5\%$$

151 K

(K: 10%)

15,100 PF

$$= 15.1 \text{ nF} \pm 10\%$$

Sometimes directly

4.7 PF

, 477 (4.7 PF), 477 (47 PF), 477 (47 nF)

$$\%e = \frac{\text{measured} - \text{nominal}}{\text{nominal}} \times 100\%$$

Part 2: Voltage & current division.

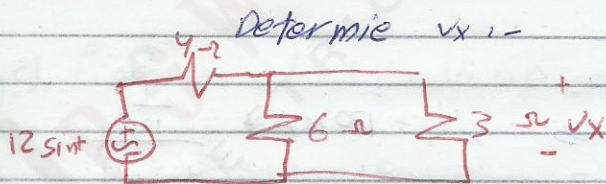
$$V_{R_1} = V_s \times \frac{R_1}{R_1 + R_2 + \dots + R_n} \quad (R_1 - R_n) \text{ Series}$$

$$I_{R_1} = I \times \frac{\frac{1}{R_1}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_n}} \quad (R_1 - R_n) \text{ Parallel}$$

Note: solve problems!

Part 3: Nodal & Mesh Analysis.

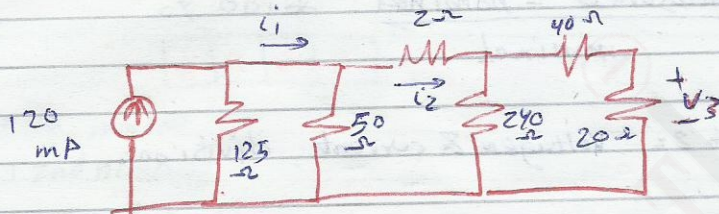
Example 3.12



$$\frac{6 \times 3}{6 + 3} = \frac{18}{9} = 2 \Omega$$

$$v_x = 12 \sin(t) \times \frac{2}{4+2} = 4 \sin t$$

practice 3.13 find i_1, i_2, V_3



$$40 + 20 = 60, \quad \frac{60 \times 240}{60 + 240} = 48, \quad 48 + 2 = 50$$

$$i_2 = 120 \text{ mA} \times \frac{1}{\frac{1}{50} + \frac{1}{50} + \frac{1}{125}} = \frac{57.45}{50} \text{ mA}$$

$$V(50) = 50 \times 50 = 2.5 \text{ V} = V(60) = i_3 \times 60 = 2.5$$

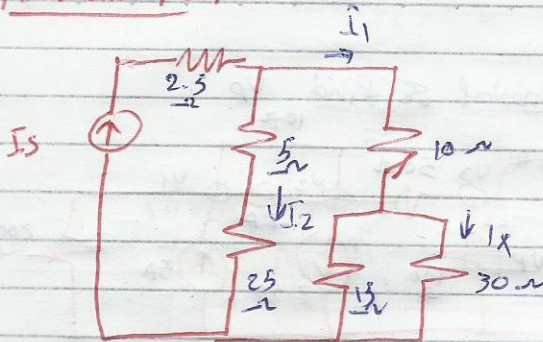
$$i_3 = 0.042 \text{ mA}$$

$$V_3 = i_3 \times 20 = 0.83 \text{ V}$$

$$\frac{56 \times 50}{100} = 28, \quad i_1 = 120 \text{ mA} \times \frac{1}{\frac{1}{125} + \frac{1}{25}} = 100 \text{ mA}$$

problem

problem 70 :-



a) find I_x if $I_1 = 12 \text{ mA}$

$$\frac{30 \times 15}{45} = 10 \quad \rightarrow \quad I_1 \times 10 = 0.12 \text{ V} = 30 \times I_x$$

$$\Rightarrow I_x = 4 \text{ mA}$$

b) find I_1 if $I_x = 12 \text{ mA}$

$$V = 30 \times I_x = 0.36 = I_1 \times 10 \Rightarrow I_1 = 36 \text{ mA}$$

c) find I_x if $I_2 = 15 \text{ mA}$

$$R_1 = \frac{30 \times 15}{30 + 15} = 10$$

$$V_1 = I_2(5 + 25) = I_1 \times (10 + R_1) \\ = 0.45 = I_1 \times 20 \Rightarrow I_1 = 22.5 \text{ mA}$$

$$V_{R_1} = I_1 \times R_1 = 0.225, \quad I_x = \frac{0.225}{30} = 7.5 \text{ mA}$$

d) I_x if $I_s = 60$

~~I_s~~

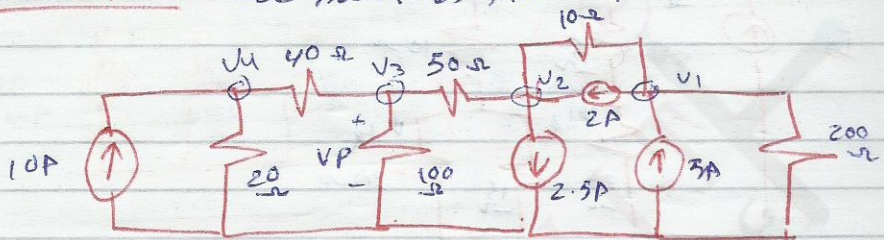
$$I_1 = 60 \times \frac{1}{20} = 36 \text{ mA}$$

$$I_x = 12 \text{ mA}$$

$$\frac{1}{20} + \frac{1}{30}$$

CH 4

Problem 3 Use nodal & find V_P



Sol.

$$\text{Node 1: } -\frac{v_1}{200} + \frac{v_1 - v_2}{10} = 3 \quad (1)$$

$$\text{Node 2: } \frac{v_2 - v_1}{10} + \frac{v_2 - v_3}{50} = -0.5 \quad (2)$$

$$\text{Node 3: } \frac{v_3 - v_2}{50} + \frac{v_3}{100} + \frac{v_3 - v_4}{40} = 0 \quad (3)$$

$$\text{Node 4: } \frac{v_4}{20} + \frac{v_4 - v_3}{40} = 10 \quad (4)$$

$$\Rightarrow 0.105 v_1 - 0.1 v_2 + 0 v_3 + 0 v_4 = 3 \quad [1]$$

$$-0.1 v_1 + 0.12 v_2 - 0.02 v_3 + 0 v_4 = -0.5 \quad [2]$$

$$\textcircled{1} v_1 - 0.02 v_2 + 0.055 v_3 - 0.025 v_4 = 0 \quad [3]$$

$$\textcircled{1} v_1 + 0 v_2 - 0.025 v_3 + 0.075 v_4 = 10 \quad [4]$$

\Rightarrow multiply (3) by 0.075 and (4) by -0.025

$$(3) - (4) = -1.5 \times 10^{-3} v_2 + 3.5 \times 10^{-3} v_3 = 0.25 \quad [5]$$

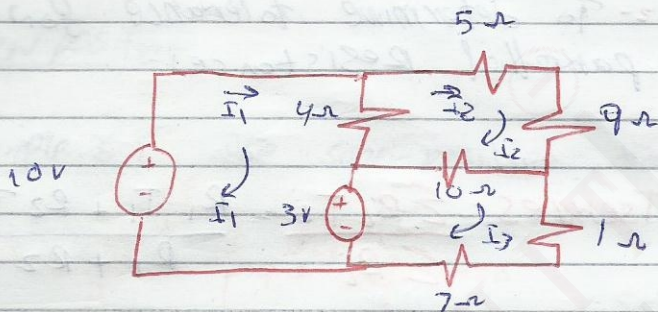
Solve (1) (2) (5)

$$v_1 = 251.26$$

$$v_2 = 273.824$$

$$v_3 = \underline{\underline{V_P}} = 171.64$$

practice 4.6 Find i_1, i_2



$$\text{mesh 1: } -10 + 4(I_1 - I_2) + 3 = 0$$

$$\text{mesh 2: } 14I_2 + 10(I_2 - I_3) + 4(I_2 - I_1) = 0$$

$$\text{mesh 3: } 8I_3 - 3 + 10(I_3 - I_2) = 0$$

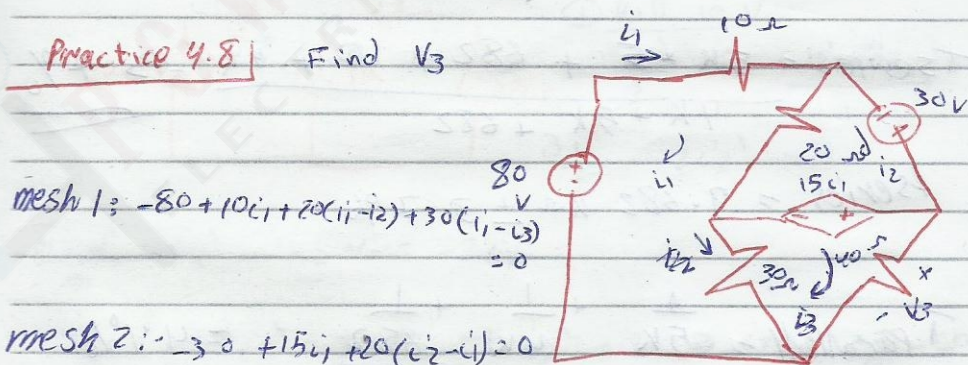
$$4I_1 - 4I_2 = 7 \quad (1)$$

$$-4I_1 + 28I_2 - 10I_3 = 0 \quad (2)$$

$$0I_1 - 10I_2 + 18I_3 = 3$$

$$I_1 = 2.22 \text{ A}, \quad i_2 = 0.47 \text{ A}, \quad i_3 = 0.43 \text{ mA}$$

Practice 4.8 Find V_3



$$\text{mesh 1: } -80 + 10i_1 + 20(i_1 - i_2) + 30(i_1 - i_3) = 0$$

$$\text{mesh 2: } -30 + 15i_1 + 20(i_2 - i_1) = 0$$

$$\text{mesh 3: } -15i_1 + 40i_3 + 30(i_3 - i_1) = 0$$

$$i_1 = 3.08, \quad i_2 = 2.27, \quad i_3 = 1.48$$

$$V_3 = 40 \times 1.48 = 79.2 \text{ V}$$

*Note:- To determine tolerance for series or parallel resistances:-

$$T_{\text{series}} = \frac{\sum R_i T_i}{\sum R_i} = \frac{R_1 T_1 + R_2 T_2 + \dots}{R_1 + R_2 + R_3 + \dots}$$

$$T_{\text{parallel}} = \frac{\sum \frac{1}{R_i} T_i}{\sum \frac{1}{R_i}} = \frac{\frac{1}{R_1} T_1 + \frac{1}{R_2} T_2 + \dots}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots}$$

Example :-

$$R_1 = 5 \text{ k}\Omega \pm 5\%$$

$$R_2 = 682 \Omega \pm 10\%$$

$$R_3 = 4 \text{ k}\Omega \pm 1\%$$

$$T_{\text{series}} = \frac{5\text{k} \times 5 + 682 \times 10 + 4\text{k}}{4\text{k} + 5\text{k} + 682} = 3.7\%$$

$$R_{\text{series}} = 9.682 \text{ k}\Omega \pm 3.7\%$$

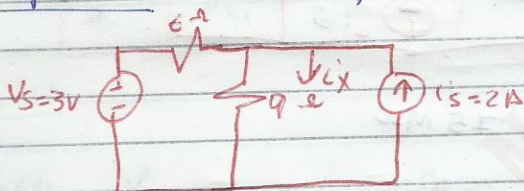
$$T_{\text{parallel}} = \frac{\frac{1}{5\text{k}} + \frac{1}{4\text{k}} + \frac{1}{682}}{\frac{1}{5\text{k} \times 5} + \frac{1}{682 \times 10} + \frac{1}{4\text{k} \times 1}} = 4.4\%$$

$$R_{\text{parallel}} = 521.85 \pm 4.4\%$$

Experiment 2: Network Theorem

① Superposition

Example 5.1 use superposition to find i_x

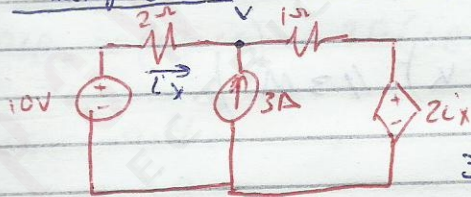


$$\text{① Kill } 3V \Rightarrow i_x' = 2 \times \frac{6}{9+6} = 0.8 \text{ A}$$

$$\text{② Kill } 2A \Rightarrow i_x'' = 3 \times \frac{9}{(9+6)9} = \frac{3}{15} = 0.2$$

$$i_x = i_x' + i_x'' = 1 \text{ A}$$

Example 5.3 find i_x



① Kill 10V

$$\frac{v'}{2} - 3 + \frac{v' - 2i_x'}{1} = 0$$

$$\frac{3v'}{2} - 2i_x' = 3$$

$$\frac{9 - v'}{2} = i_x' \Rightarrow v' = -2i_x'$$

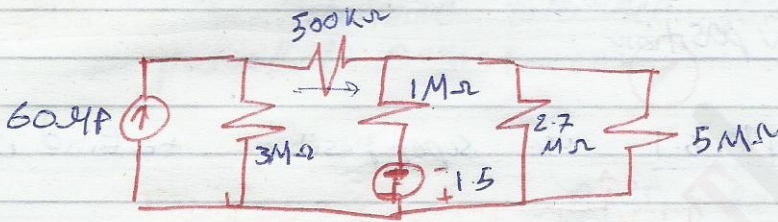
$$\frac{-6i_x'}{2} - 2i_x' = 3 \Rightarrow -5i_x' = 3 \Rightarrow i_x' = \frac{-3}{5} = -0.6 \text{ A}$$

$$\text{② Kill } 3A \Rightarrow -10 + 2i_x'' + i_x'' + 2i_x'' = 0$$

$$i_x'' = 10/5 = 2 \text{ A}$$

$$i_x = 2 - 0.6 = 1.4 \text{ A}$$

Problem 9 Find power across $500\text{ k}\Omega$



$$\textcircled{1} \frac{2.7 \times 5}{2.7 + 5} = 1.75 \text{ M}\Omega$$

$\textcircled{2}$ Kill $60\ \mu\text{A}$

$$\frac{3\text{ M}\Omega + 500\text{ k}\Omega}{A} \parallel 1.75\text{ M}\Omega = 1.17\text{ M}\Omega \quad B$$

$$V_B = -1.5 \times \frac{1.17\text{ M}\Omega}{1.17\text{ M}\Omega + 1\text{ M}\Omega} = -0.81\text{ V}$$

$$= V_A \rightarrow I_P = \frac{V_A}{A} = 0.23\ \mu\text{A}$$

$\textcircled{3}$ Kill 1.5

$$\left(\frac{1.75\text{ M}\Omega \parallel 1\text{ M}\Omega + 500\text{ k}\Omega}{A} \parallel 3\text{ M}\Omega \right)$$

$$= 0.64\text{ M}\Omega$$

$$1.136\text{ M}\Omega \quad B$$

$$= 0.824\text{ M}\Omega \quad C$$

$$I_C = 60\ \mu\text{A}$$

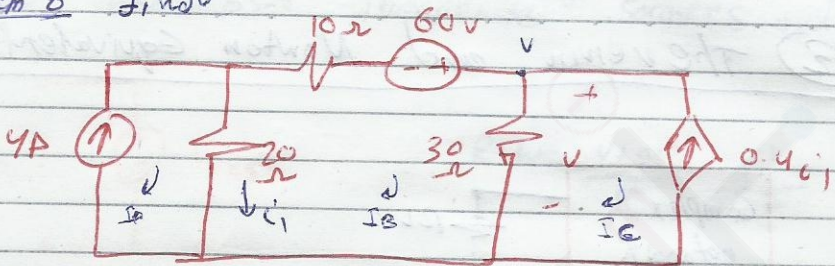
$$V_C = 49.5 = V_B \rightarrow I_B = \frac{V_B}{B} = 43.6\ \mu\text{A}$$

$$V_{500\text{ k}\Omega} = 21.78 \rightarrow I'' = 43.6\ \mu\text{A}$$

$$I = 43.83$$

$$P = 0.46\text{ mW} \quad \checkmark$$

Problem 8 find V



① Kill 4A

$$\frac{V' - 60}{30} + \frac{V'}{30} - 0.4i_1 = 0$$

$$i_1 = \frac{V' - 60}{30}$$

$$V' = 22.5 \quad i_1 = -1.25$$

② Kill 60V

$$i_A = 4$$

$$10i_B + 30(i_B - i_C) + 20(i_B - 4) = 0 \quad (1)$$

$$i_C = -0.4i_1 = -0.4(4 - i_B) = 0.4i_B - 1.6$$

$$i_1 = 4 - i_B$$

$$0.4i_B - i_C = 1.6 \quad (2)$$

$$i_B = 0.67 \quad , \quad V'' =$$

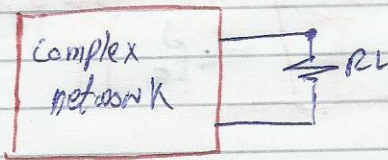
$$i_C = -1.33$$

$$i(30) = i_B - i_C = 2$$

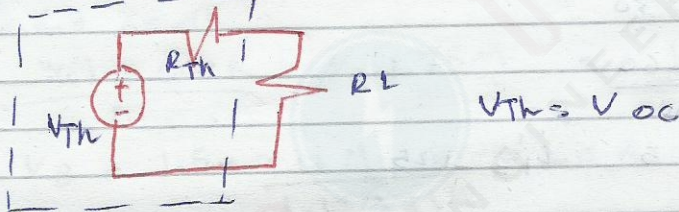
$$V'' = 2 \times 30 = 60$$

$$V = 60 + 22.5 = 82.5V$$

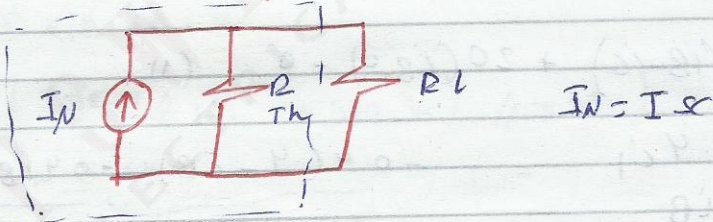
② Thevenin and Norton Equivalent circuit



① Thevenin equivalent circuit



② Norton equivalent circuit



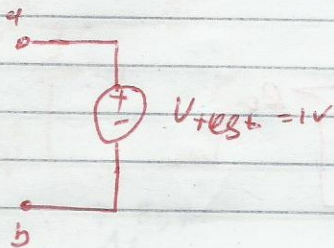
R_{Th} :- ① If there are no dependent sources.

⇒ Kill all sources, $R_{Th} = R_{eq}$.

② If there are dependent & independent sources.

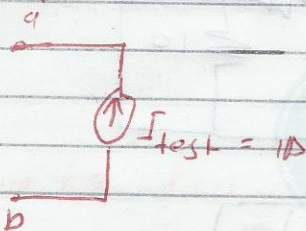
$$R_{Th} = \frac{V_{OC}}{I_{SC}}$$

③ If there are independent sources only.



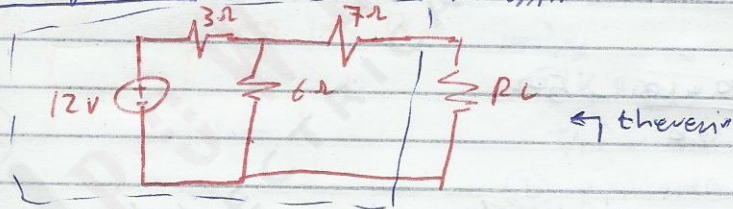
$$R_{th} = \frac{V_{test}}{I} = \frac{1}{I}$$

OR



$$R_{th} = \frac{V}{I_{test}} = V$$

Example 5.6 Determine Thevenin & Norton for left side

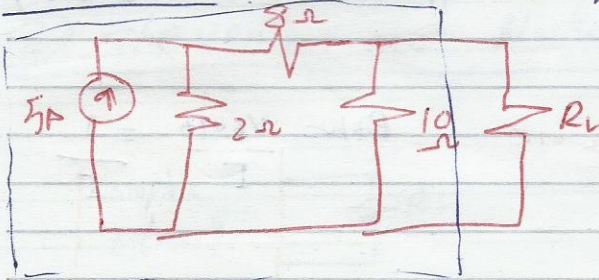


$$① V_{th} = V_{oc} = \frac{12 \times 6}{9} = 8V$$

$$② I_N = I_{sc} = \frac{12}{3} \times \frac{1}{\frac{1}{3} + \frac{1}{6} + \frac{1}{7}} = \frac{8}{9} A$$

$$R_{th} = \frac{3 \times 6}{9} + 7 = 2 + 7 = 9 \Omega$$

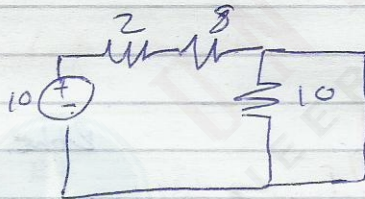
Practice 5.5 Find Norton equivalent



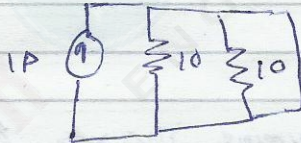
$I_{SC} = 1A$

$V_1 = 5 \times 2 = 10$

$I_1 = \frac{10}{10} = 1A$

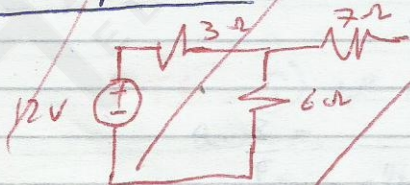


$I_{SC} = 1A$



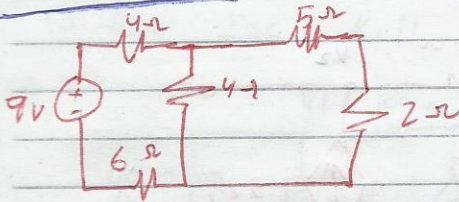
$R_{th} = \frac{10 \times 10}{20} = 5\Omega$

Example 5.7 Find thevenin equivalent

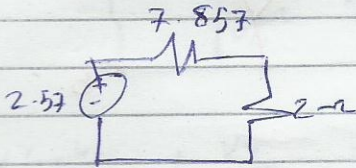


Practice 5.6

Find I through 2Ω by thevenin



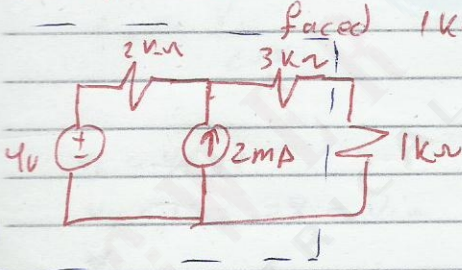
$$\textcircled{1} V_{th} = V_{oc} = 9 \times \frac{4}{4+4+6} = 2.571 \text{ V}$$



$$\textcircled{2} R_{th} = \frac{10 \times 4}{14} + 5 = 7.857$$

$$I_{2\Omega} = \frac{2.571 \times 2}{(2+7.857) \times 2} = 260.8 \text{ mA}$$

Example 5.8 Find Thevenin & Norton for the network



$$\textcircled{1} V_{th} = V_{oc}$$

kill 4V

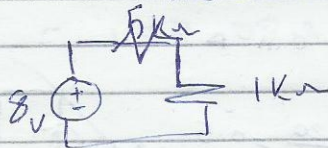
$$V_{oc}' = 4 \text{ V}$$

$$\textcircled{b} \text{ Kill } 2\text{mA}$$

$$V_{oc}'' = 4 \text{ V}$$

$$V_{th} = 4 + 4 = 8 \text{ V}$$

$$R_{th} = 5 \text{ k}\Omega$$



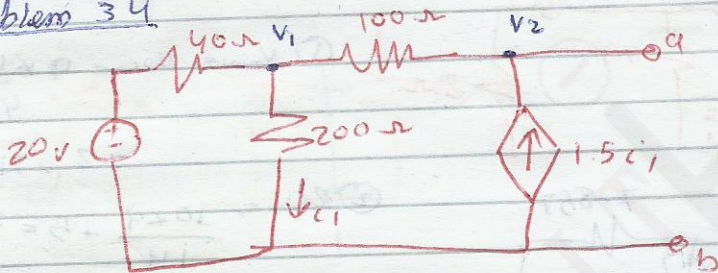
$$\textcircled{2} I_N = I_{sc} \quad \text{@ Kill } 4\text{V} \Rightarrow I_{sc}' = 2 \times \frac{2}{5} = 0.8 \text{ mA}$$

$$\textcircled{b} \text{ Kill } 2\text{mA} \quad I_{sc}'' = \frac{4}{5} = 0.8 \text{ mA}$$

$$I_{sc} = 1.6 \text{ mA}$$

Example 5.9

Problem 34



① $V_{th} = V_{oc}$

$$\frac{V_1 - 20}{40} + \frac{V_1 - V_2}{100} + \frac{V_1}{200} = 0$$

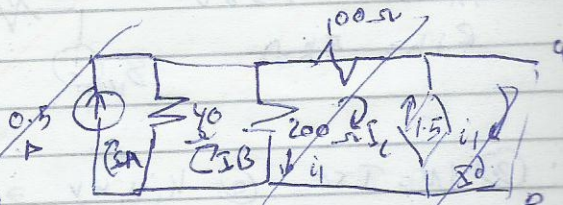
$$\frac{V_2 - V_1}{100} - 1.5 i_1 = 0$$

$$i_1 = \frac{V_1}{200}$$

$$V_1 = 22.22 \text{ V}, \quad V_2 = V_{oc} = 38.89 \text{ V}$$

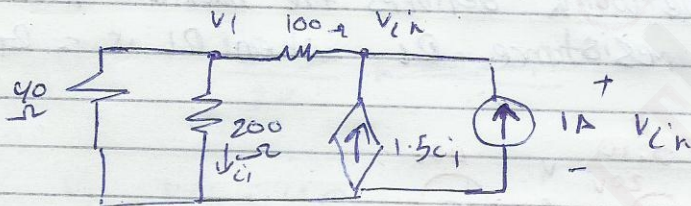
$$i_1 = 0.11 \text{ A}$$

$$I_{sc} = \frac{20}{40} = 0.5$$



$$\frac{40 \times 200}{240} = 33.33 \times 0.5 = 16.67$$

To find I_{sc} kill 20V and test + 1A



$$\frac{V_1 - V_{in}}{100} + \frac{V_1}{200} + \frac{V_1}{200} = 0$$

$$-1 - 1.5i_1 + \frac{V_{in} - V_1}{100} = 0$$

$$i_1 = \frac{V_1}{200}$$

$$V_1 = 44.44$$

$$V_2 = 177.78 = V_{in}$$

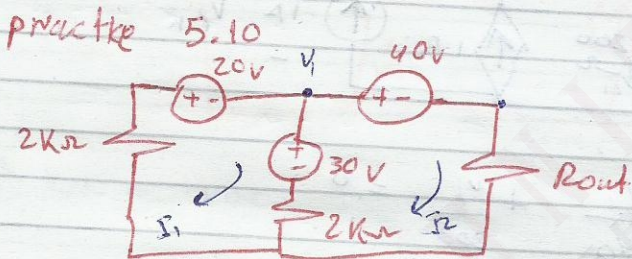
*

$$i_1 = 20.222 \text{ A}$$

$$\therefore R_{th} = \frac{V_{in}}{1A} = 177.78 \Omega$$

Maximum Power Transfer

A Network delivers the maximum power to a load resistance R_L when $R_L = R_{th}$.



a) If $R_{out} = 3k\Omega$

Find the maximum power that can deliver to any R_{out} ?

⇒ max. at $R_{out} = R_{th}$

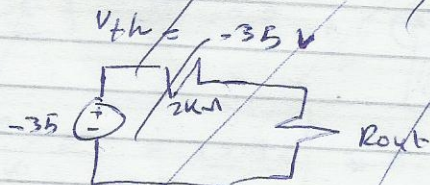
$$R_{th} = \frac{2 \times 2}{2} = \underline{2k\Omega}$$

~~$V_{th} = V_{oc}$~~

~~⊕ kill 20, 40 ⇒ $V_{oc} = 15V$~~

~~⊖ kill 20, 30 ⇒ $-V_{oc} = 40 \neq 0$, $V_{oc} = -40V$~~

~~kill 40, 30 ⇒ $V_{oc} = -20 + \frac{2}{4} = -10V$~~



~~at $R_{out} = 2k\Omega$~~

~~$V = -35 + \frac{2}{4} = -17.5$~~

$$V_{Th} = V_{oc}$$

$$\frac{V_1 + 20}{2k} + \frac{V_1 - 30}{2k} - \frac{V_1 - V_{Th}}{2k} = 0$$

$$V_1 - V_{Th} = 40$$

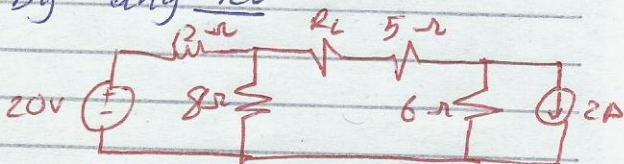
$$\Rightarrow 1 \times 10^{-3} V_1 = 5 \times 10^{-3} \Rightarrow V_1 = 5$$

$$5 - V_{Th} = 40 \Rightarrow \boxed{V_{Th} = -35 \text{ V}}$$

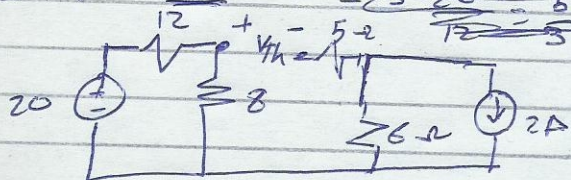
$$P_{RL} / \max = \frac{5}{4RL} V_{Th}^2 = \frac{(-35)^2}{4(2k)} = 153.13 \text{ mW}$$

Problem 47 Find Maximum power can be dissipated by any R_L

$$\textcircled{1} R_{Th} = \frac{12 \times 8}{20} + 6 + 5 = 15.8 \text{ k}\Omega$$



$$\textcircled{2} V_{Th} = V_{oc} \Rightarrow I_{5\Omega} = \frac{20}{12+8} = \frac{5}{5} \Rightarrow R = 4\Omega$$



① Kill 2A

$$V_8 = 8 \text{ V}$$

② Kill 20 V,

$$-8 + V_{Th} = 0 \Rightarrow \boxed{V_{Th} = 8 \text{ V}}$$

② Kill 20 V

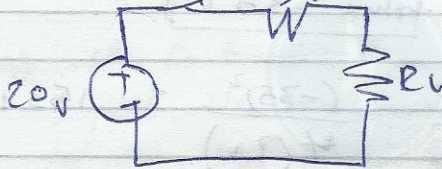
$$\frac{12 \times 8}{20} = \frac{4}{5} \cdot 8 \Omega$$

$$V_6 = -2 \times 6 = -12$$

$$\Rightarrow 12 \text{ V} + V_{th} = 0$$

$$\Rightarrow V_{th} = -12 \text{ V}$$

$$V_{th} - 12 + 8 = 20 \text{ V}$$

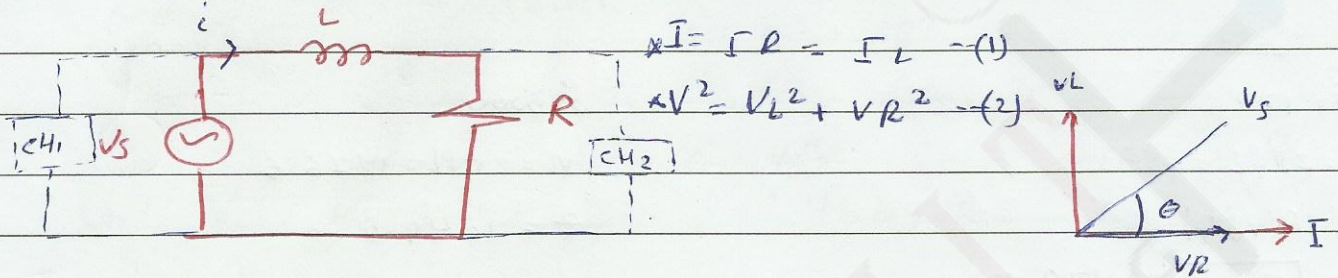


$$P_{R_L} / \max = \frac{20^2}{4 \times 15.8} = 6.33 \text{ W}$$

Experiment 3

Series RL & RC under AC Excitation

① series RL circuit



I lag V_L by 90° and lag V by θ

$$* \theta = \tan^{-1} \left(\frac{X_L}{R} \right) = \tan^{-1} \left(\frac{V_L}{V_R} \right) \dots (3)$$

(θ is -ve goes from 0 to -90)

$$* X_L = 2\pi fL \dots (4)$$

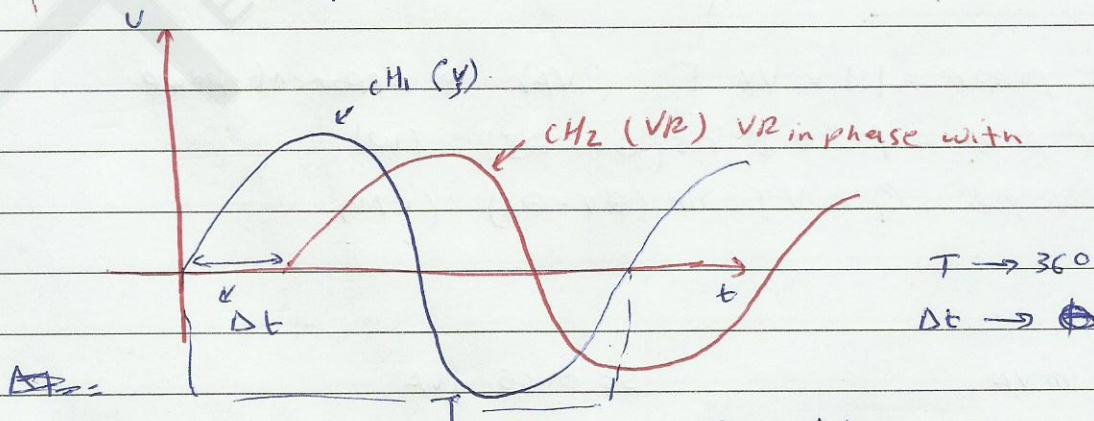
$$* I = \frac{V \angle 0}{Z \angle \theta}, Z = \frac{V_s}{I}, X_L = \frac{V_L}{I}$$

$$f \uparrow \rightarrow X_L \uparrow (2\pi fL) \rightarrow V_L \uparrow (I \times jX_L) \rightarrow V_R \downarrow (V_s \text{ const}) \rightarrow I \downarrow \left(\frac{V_R}{R} \right)$$

$$* P = V_L I \quad (\text{Power in inductor})$$

$$= L I \frac{di}{dt}$$

$$* P = I^2 R + L I \frac{di}{dt} \quad (\text{total power})$$



$$\theta = \frac{\Delta t \times 360}{T} = \Delta t \times 360 \times f \times \text{Scale}$$

Example: Let $V_s = 1 \angle 0$, $L = 47 \text{ mH}$, $R = 470 \Omega$.

determine ~~$V_s, V_L, Z, \theta, I, V_R, V_L$~~ . For $f = 100, 2000, 5000$

Sol.

$f = 100$

$X_L = 2\pi fL = 29.53$

$Z = |R + jX_L|$

$= \sqrt{R^2 + X_L^2} = 470.93 \Omega$

$\theta = \tan^{-1}\left(\frac{X_L}{R}\right) = -3.6^\circ$

$I = \frac{V_s}{Z} = \frac{1}{470.93} = 2.13 \angle -3.6 \text{ mA}$

$V_R = I * R = 2.13 \angle -3.6 * 470 * 10^{-3}$
 $= 1.001 \angle -3.6^\circ$

$V_L = I * jX_L = 2.13 \angle -3.6 * j29.53 * 10^{-3}$
 $= 0.063 \angle 86.4$

check: $\tan^{-1}\left(\frac{V_L}{V_R}\right) = \tan^{-1}\left(\frac{0.063}{1.001}\right) = -3.6 \checkmark$

$f = 5000$

$X_L = 2\pi fL = 1476.5$

$Z = 1549.5 \Omega$

$\theta = \tan^{-1}\left(\frac{X_L}{R}\right) = -72.34$

$I = 0.65 \angle -72.34 \text{ mA}$

$V_R = I * R = 0.31 \angle -72.34$

$V_L = I * jX_L = 0.96 \angle 17.66$

$\tan^{-1}\left(\frac{V_L}{V_R}\right) = \tan^{-1}\left(\frac{0.96}{0.31}\right) = -72.1 \checkmark$

Note: ① complex power $S = |S| = V * I$ (VA) \rightarrow Apparent power

② Average power = $P = V I \cos(\theta_V - \theta_I)$ (W)

③ Reactive power = $Q = V I \sin(\theta_V - \theta_I)$ (VAR)

$S = V * I = 2.13 \text{ mVA}$

$P = V * I * \cos(0 + 3.6) = 2.126 \text{ mW}$

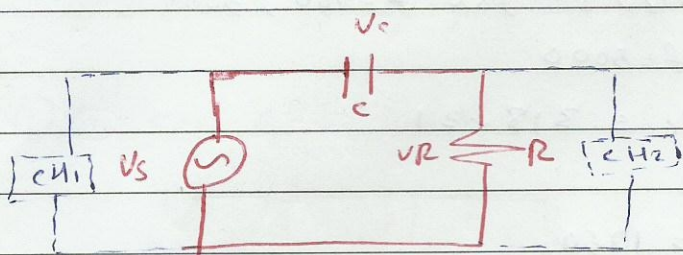
$Q = V * I * \sin(3.6) = 0.134 \text{ mVAR}$

$S = 0.65 \text{ mVA}$

$P = 0.65 \text{ m} \cos(72.3) = 0.2 \text{ mW}$

$Q = 0.65 \sin(72.3) = 0.62 \text{ mVAR}$

② Series RC circuit



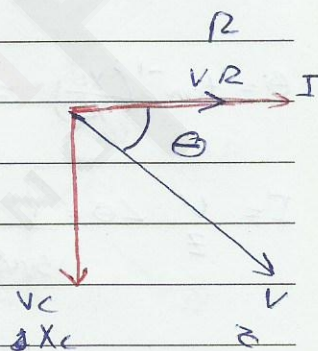
$$I = I_C = I_R$$

$$V^2 = V_C^2 + V_R^2$$

I lead V_C by 90° and lead V by θ

$$\angle I \quad \theta = +\tan^{-1}\left(\frac{X_C}{R}\right) = \tan^{-1}\left(\frac{V_C}{V_R}\right)$$

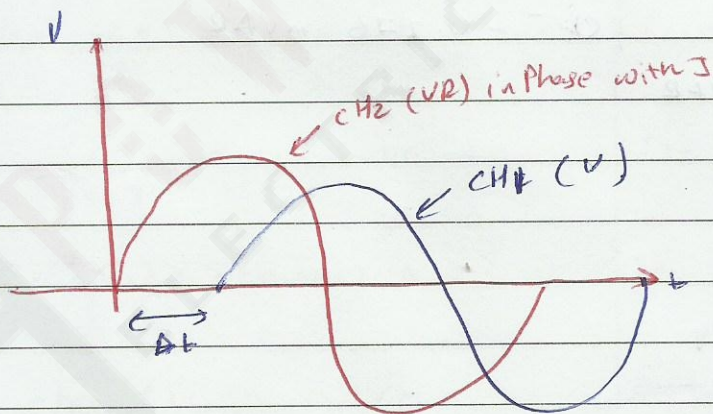
(θ goes from 90 to zero) (+ve)



$$X_C = \frac{1}{2\pi f C} \quad , \quad I = \frac{V}{Z} \angle \theta \quad , \quad Z = \frac{V_S}{I} \quad , \quad X_C = \frac{V_C}{I}$$

$$f \uparrow \rightarrow X_C \downarrow \left(\frac{1}{2\pi f C}\right) \rightarrow V_C \downarrow (I * jX_C) \rightarrow V_R \uparrow (V_{const})$$

$$\uparrow I \downarrow \left(\frac{V_R}{R}\right)$$



Example: Let $V_s = 1 \angle 0$, $C = 0.1 \mu F$, $R = 1000 \Omega$

determine, Z , θ , I , V_C , V_R , P , Q , S for $f = 100, 5000$

$f = 100 \text{ Hz}$

$$X_C = \frac{1}{2\pi f C} = 15915.5 \Omega$$

$$Z = \sqrt{X_C^2 + R^2} = 15947 \Omega$$

$$\theta = \tan^{-1}\left(\frac{X_C}{R}\right) = +86.4^\circ$$

$$I = \frac{V}{Z} \angle \theta = \frac{1}{15947} \angle 86.4 = 0.063 \text{ mA} \angle 86.4^\circ$$

$$V_C = I * -jX_C = 0.998 \angle -3.6^\circ$$

$$V_R = I * R = 0.063 \angle 86.4 \text{ mV}$$

$$|S| = V * I = 0.063 \text{ mVA}$$

$$P = VI \cos(\theta_V - \theta_I) = 3.96 \text{ m} \\ 0.00395 \text{ mW}$$

$$Q = VI \sin(\theta_V - \theta_I) = -0.063 \text{ mVAR}$$

check: $\tan^{-1}\left(\frac{V_C}{V_R}\right) = 86.4$

$$Z = \frac{V_s}{I} = \frac{1 \angle 0}{0.063 \angle 86.4} = 15873 \angle -86.4$$

$$X_C = \frac{V_C}{I} = 15841$$

$f = 5000$

$$X_C = 318.31$$

$$Z = 1050$$

$$\theta = 16.86^\circ$$

$$I = 0.952 \angle 16.86 \text{ mA}$$

$$V_C = 0.3 \angle -73.14$$

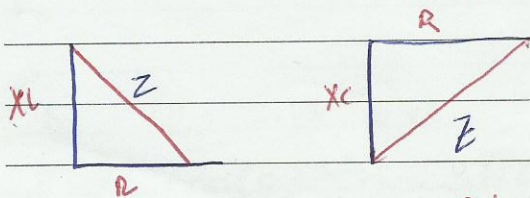
$$V_R = 0.952 \text{ V}$$

$$|S| = 0.91 \text{ mVA} \quad 0.952 \text{ mVA}$$

$$P = 0.91 \text{ mW}$$

$$Q = -0.276 \text{ mVAR}$$

* Note: Impedance triangle



$$i = \frac{V(1 - e^{-\frac{Rt}{L}})}{R}$$

Q - why the output voltage level change with f ?

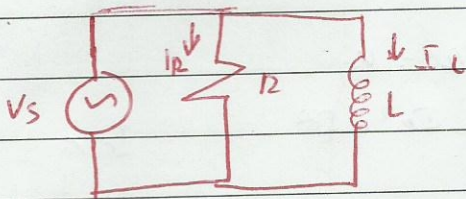
because it has internal impedance change with f .

$\frac{L}{R}$, Time constant

Experiment 4

Parallel RC and RL circuits Under AC Excitation

① Parallel RL circuit:

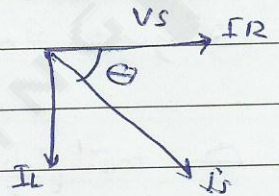


$$V_s = V_R = V_L$$

$$I_s^2 = I_L^2 + I_R^2$$

(θ goes from -90 to zero) (-ve) $\begin{cases} 0 \rightarrow \text{pure R} \\ -90 \rightarrow \text{pure L} \end{cases}$

$$\theta = -\tan^{-1}\left(\frac{I_L}{I_R}\right), \quad I_R = \frac{V_s}{R}, \quad I_L = \frac{V_s}{X_L}, \quad Y = \frac{I}{V_s}, \quad B_L = \frac{I_L}{V_s}$$



$$f \uparrow \rightarrow X_L \uparrow (2\pi fL) \rightarrow I_L \downarrow \rightarrow I_R \text{ const} \rightarrow I_s = \sqrt{I_L^2 + I_R^2} \rightarrow \text{(-ve)}$$

Example: let $V_s = 1V_0$, $L = 100 \text{ mH}$, $R = 680 \Omega$

determine I_R , I_L , I , θ ($\angle I$ with V_s), Y , B_L . For $f = 100, 5000$

$$f = 100$$

$$X_L = 2\pi fL = 62.8 \Omega$$

$$I_R = \frac{1V_0}{680} = 1.471 \text{ mA}$$

$$I_L = \frac{1V_0}{j62.8} = 0.016 \angle -90 \text{ A}$$

$$I = \sqrt{I_L^2 + I_R^2} = 0.0161 \text{ A}$$

$$\theta = -\tan^{-1}\left(\frac{I_L}{I_R}\right) = -\tan^{-1}\left(\frac{0.016}{0.00471}\right)$$

$$\approx -84.75^\circ$$

$$Y = \frac{I}{V_s} = \frac{0.0161 \angle -84.75}{1V_0}$$

$$= 0.0161 \angle -84.75 \text{ s}^{-1}$$

$$B_L = \frac{I_L}{V_s} = 0.016 \angle -90 \text{ s}^{-1}$$

$$f = 5000$$

$$X_L = 3141.6$$

$$I_R = 1.471 \text{ mA}$$

$$I_L = \frac{1V_0}{j3141.6} = 0.318 \angle -90 \text{ mA}$$

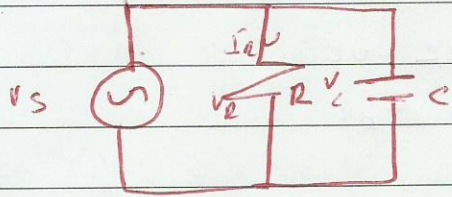
$$I = 1.505 \text{ mA}$$

$$\theta = -12.2^\circ$$

$$Y = 1.505 \angle -12.2 \text{ m s}^{-1}$$

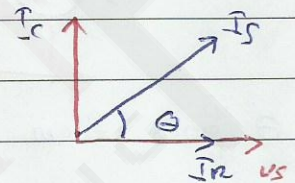
$$B_L = 0.318 \angle -90 \text{ m s}^{-1}$$

② Parallel RC circuit



$$V_s = V_R = V_C$$

$$I_s^2 = I_R^2 + I_C^2$$



(θ goes from 0 to 90) (true)

$$\theta = \tan^{-1}\left(\frac{I_C}{I_R}\right), I_R = \frac{V_s}{R}, I_C = \frac{V_s}{X_C}, Y = \frac{I}{V_s}, BC = \frac{I_C}{V_s}$$

$$f \uparrow \rightarrow X_C \downarrow \rightarrow I_C \uparrow \rightarrow I_R \text{ const} \rightarrow I_s \uparrow$$

Example: Let $R = 680 \Omega$, $C = 0.1 \mu F$, $V_s = 10$

determine I_R , I_C , I , θ , Y , BC for $f = 100, 5000$, S, P, Q

$$f = 100, X_C = \frac{1}{2\pi f C} = 15915.5 \Omega$$

$$I_R = \frac{1}{680} = 1.471 \text{ mA}$$

$$f = 5000, X_C = 318.31$$

$$I_R = 1.471 \text{ mA}$$

$$I_C = \frac{1}{-jX_C} = 62.8 \angle 90 \mu A$$

$$I_C = 3.14 \angle 90 \text{ mA}$$

$$I = 1.472 \text{ mA}$$

$$I = 3.47 \text{ mA}$$

$$\theta = \tan^{-1}\left(\frac{I_C}{I_R}\right) = 2.44^\circ$$

$$\theta = 64.9^\circ$$

$$Y = \frac{I}{V_s} = 1.472 \angle 2.44 \text{ mS}$$

$$Y = 3.47 \angle 64.9^\circ$$

$$BC = \frac{I_C}{V_s} = 62.8 \angle 90 \mu A^{-1}$$

$$BC = 3.14 \angle 90 \text{ mA}^{-1}$$

$$S = 1.472 \text{ mVA}$$

$$S = 3.47 \text{ mVA}$$

$$P = 1.47 \text{ mW}$$

$$P = 1.472$$

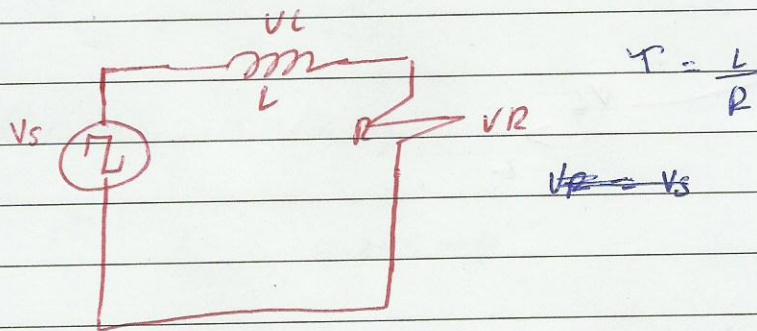
$$Q = -0.063 \text{ VAR}$$

$$Q = -3.14$$

Experiment 5:

TRANSIENT ANALYSIS

① RL circuit



* Resistor's

$$V_R = V_S (1 - e^{-t/\tau})$$

at $t=0 \rightarrow V_R = 0$

at $t = \tau \rightarrow V_R = 0.64 V_S$

at $t = 5\tau$ (large τ) $\rightarrow V_R \approx V_S$

* Inductor

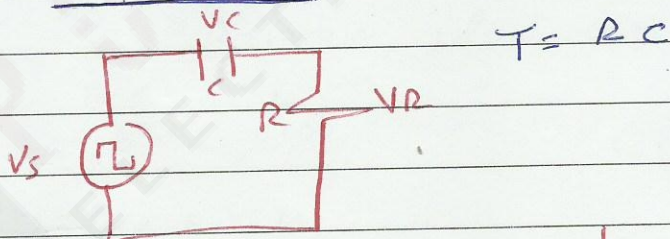
$$V_L = V_S e^{-t/\tau}$$

at $t=0 \rightarrow V_L = V_S$

at $t = \tau \rightarrow V_L = 0.36 V_S$

at $t = 5\tau \rightarrow V_L \approx 0$
(large τ)

② RC circuit



* Resistor:-

$$V_R = V_S e^{-t/\tau}$$

at $t=0 \rightarrow V_R = V_S$

at $t = \tau \rightarrow V_R = 0.36 V_S$

at $t = 5\tau \rightarrow V_R = 0$

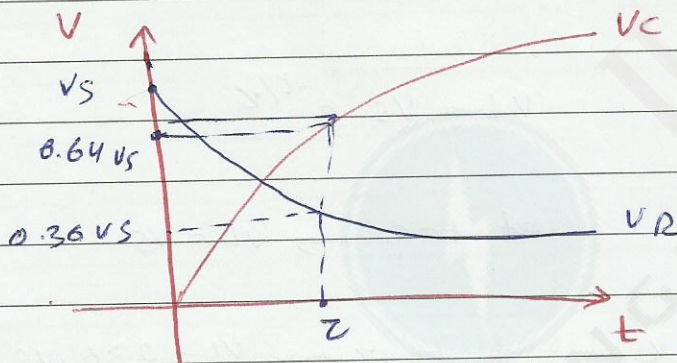
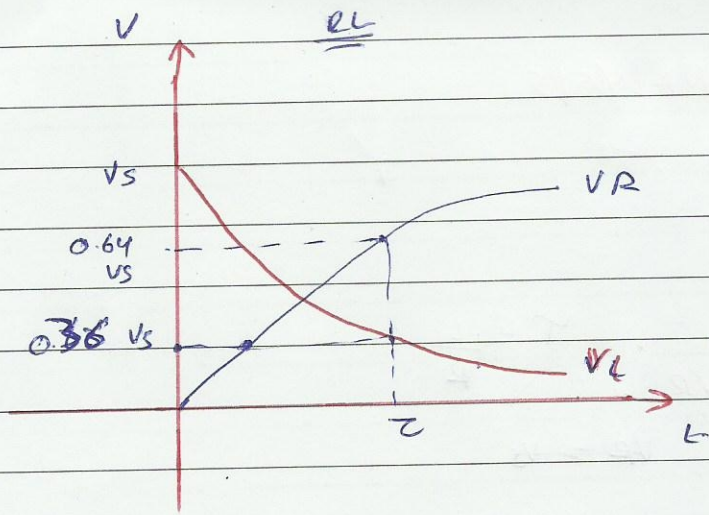
* Capacitor

$$V_C = V_S (1 - e^{-t/\tau})$$

at $t=0 \rightarrow V_C = 0$

at $t = \tau \rightarrow V_C = 0.64 V_S$

at $t = 5\tau \rightarrow V_C = V_S$



Example: in RL part, let $V_s = 4 \text{ p-p}$, $L = 100 \text{ mH}$
 $R = 470 \Omega$, find $i(t)$, $v_R(t)$, $v_L(t)$

$$\textcircled{1} T = \frac{L}{R} = \frac{1}{4700}$$

$$\textcircled{2} i_L(t) = i_f(t) + i_n(t)$$

$$i_f(t) = \frac{2}{470} = 4.255 \text{ mA}$$

$$i_n(t) = C e^{-t/\tau} \\ = C e^{-4700t}$$

$$\therefore i(t) = 4.255 \text{ mA} + C e^{-4700t}$$

$$i(0) = 4.255 \text{ mA} + C = 0$$

$$C = -4.255 \text{ mA}$$

$$\therefore i_L(t) = 4.255 - 4.255 e^{-4700t} \text{ mA} \\ = i(t)$$

$$v_R(t) = R \times i(t) \\ = 2 - 2 e^{-4700t} \text{ V}$$

$$v_L(t) = L \frac{di(t)}{dt} = 2 e^{-4700t}$$

* Now consider :-

$$\text{at } t=0, v_L = 2 = V_s$$

$$v_R = 0$$

$$\text{at } t = \frac{1}{4700} = \tau, v_L = 0.74 = 0.36 V_s$$

$$\text{at } t = 10\tau, v_L \approx 0$$

$$v_R = V_s$$

Example: in RC part, let $V_s = 4 \text{ V p-p}$, $C = 0.1 \mu\text{F}$

$R = 1000 \Omega$, find $v_C(t)$, $v_R(t)$, $i_C(t)$

$$\textcircled{1} T = RC = \frac{1}{10000}$$

$$\textcircled{2} v_C(t) = v_p(t) + v_n(t)$$

$$v_p(t) = 2 \text{ V} \\ \text{at } t=0$$

$$v_n(t) = C e^{-10000t}$$

$$v_C(t) = 2 + C e^{-10000t}$$

$$v_C(0) = 2 + C = 0, C = -2$$

$$v_C(t) = 2 - 2 e^{-10000t}$$

$$v_R(t) = V_s(t) - v_C = 2 e^{-10000t}$$

$$i_C(t) = \frac{1}{C} \int v_C(t) dt$$

Now consider :-

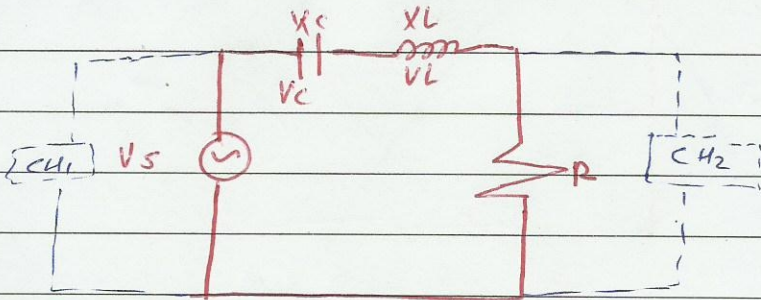
$$\text{at } t=0 \rightarrow v_C = 0$$

$$\text{at } t = \tau \rightarrow v_C = 0.64 V_s$$

Experiment 6:

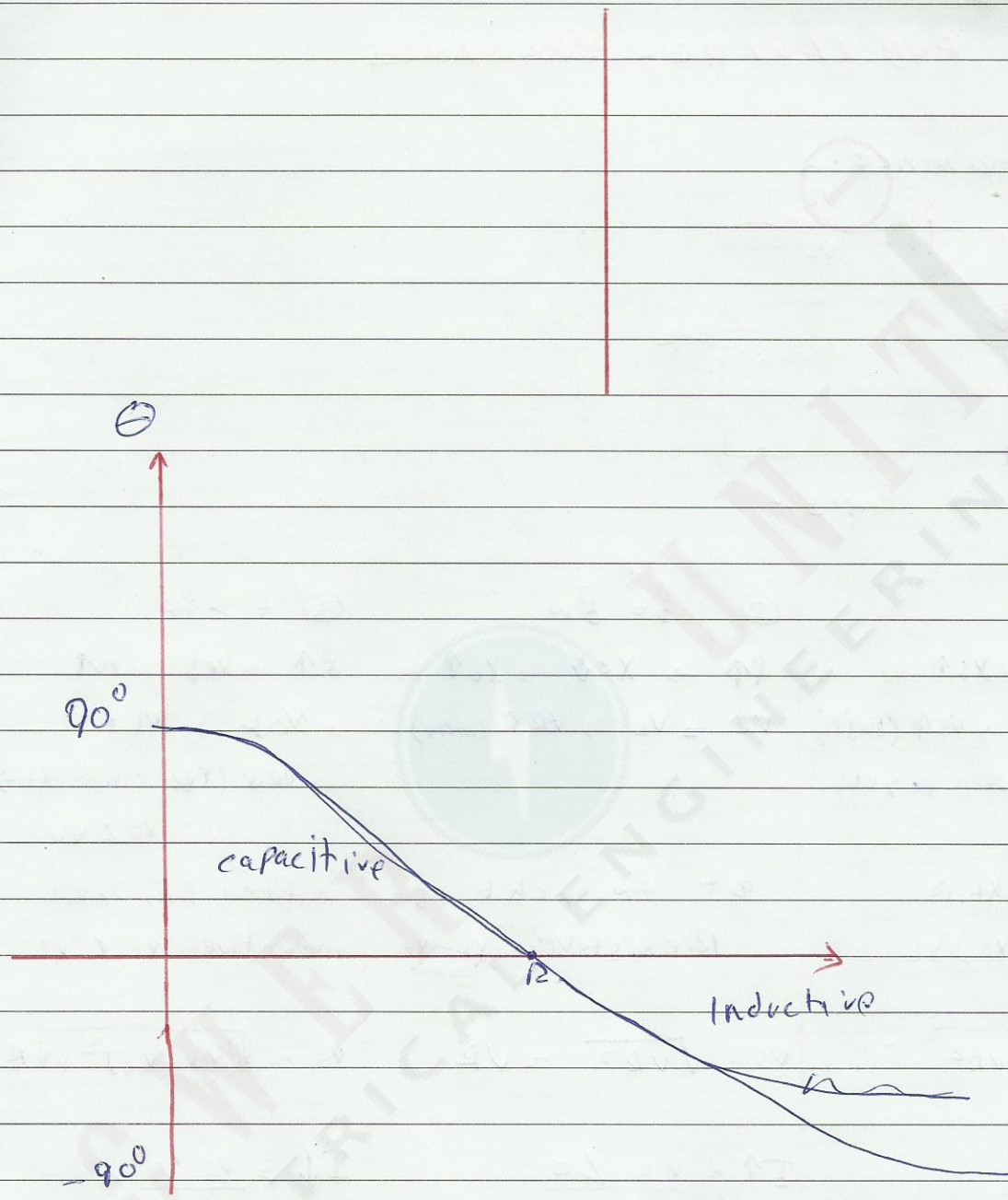
SERIES AND PARALLEL RESONANCES

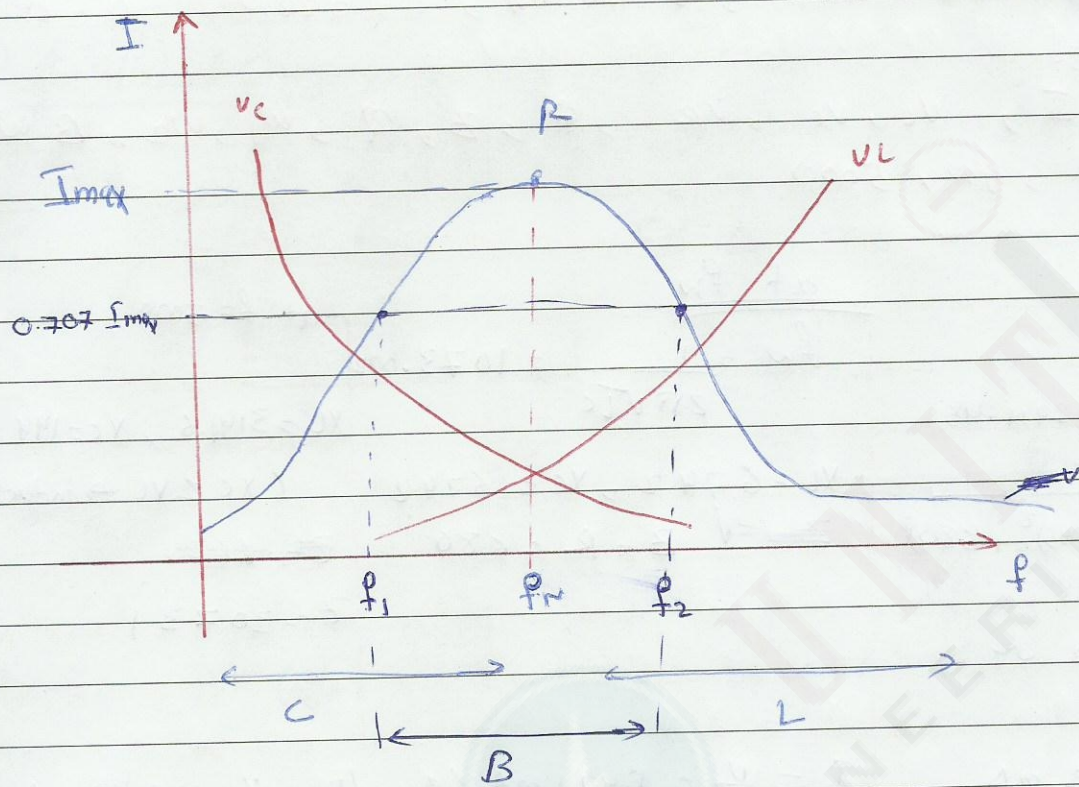
① Series Resonance :-



There are 3 cases:-

① $f < f_n$	② $f = f_n$	③ $f > f_n$
$f \uparrow \rightarrow X_c \downarrow \rightarrow X_L \uparrow$ $V_c \downarrow (I \times jX_c) \rightarrow V_L \uparrow (I \times jX_L)$ $\rightarrow V_R \uparrow (I \uparrow \text{ since } X_c > X_L)$	$f \uparrow \rightarrow X_c \downarrow \rightarrow X_L \uparrow$ $V_c = V_L \rightarrow V_R \uparrow (\text{max})$	$f \uparrow \rightarrow X_c \downarrow \rightarrow V_L \uparrow$ $\rightarrow V_c \downarrow \rightarrow V_L \uparrow$ $\rightarrow V_R \downarrow (I \downarrow \text{ since } X_L > X_c)$ $X_c < X_L$
Before f_n , ckt is capacitive $X_c > X_L$	at f_n , ckt is Resistive, $X_L = X_c$	after f_n , ckt is inductive $X_c < X_L$
$V_s = \sqrt{(X_L - X_c)^2 + R^2} I$	$V_s = \sqrt{V_R^2} = V_R$	$V_s = \sqrt{(X_L - X_c)^2 + R^2} I$
$I \uparrow = \frac{V_s}{Z} \propto \frac{1}{Z}$	$I \uparrow = \frac{V_s}{R} \propto \frac{1}{R}$	$I \downarrow = \frac{V_s}{Z} \propto \frac{1}{Z}$
$Z = \sqrt{(X_L - X_c)^2 + R^2}$	$Z = \sqrt{R^2} = R$	$Z = \sqrt{(X_L - X_c)^2 + R^2}$
$X_c > X_L$ (capacitive)	$X_c = X_L$ (resistive)	$X_c < X_L$ (inductive)
$\theta = \tan^{-1} \left(\frac{X_L - X_c}{R} \right)$ (+ve)	$\theta = \tan^{-1}(0) = 0$	$\theta = \tan^{-1} \left(\frac{X_L - X_c}{R} \right)$ (-ve)
θ goes from +90 to zero	$X_c = X_L$ $\frac{1}{2\pi f_n C} = 2\pi f_n L$ $f_n = \frac{1}{2\pi \sqrt{LC}}$	θ goes from 0 to -90





$$B = f = \frac{\omega}{2\pi}, \quad \omega = 2\pi f$$

~~$$B = f_2 - f_1 = \frac{\omega_2 - \omega_1}{2\pi}$$~~

* f_1 : lower cutoff freq (f_L)

→ At half power $= \frac{P_m}{2}$, $I = \frac{I_m}{\sqrt{2}} = 0.707 I_m$

$$\omega_1 = \omega_L = \frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

(capacitive)

* f_2 : upper cutoff freq (f_H)

$$\omega_H - \omega_L = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

* Bandwidth (B)

$$B = \frac{f_0}{Q} = f_2 - f_1$$

~~$$= \frac{R}{L} = \frac{R}{2\pi L}$$~~

* Quality factor (Q)

~~$$Q = \frac{X_L}{R} = \frac{1}{R X_C} = \frac{1}{R} \sqrt{\frac{L}{C}}$$~~

$$Q = \frac{X_L}{R} \text{ at } f_0 = \frac{\omega_0}{B}$$

$$= \frac{\omega_0}{\omega_2 - \omega_1} = \frac{f_0}{f_2 - f_1}$$

Example 3 Let $V_s = 1 \angle 0$, $L = 100 \text{ mH}$, $C = 0.22 \mu\text{F}$, $R = 680 \Omega$
determine:

f_w , I , V_L , V_C , V_R , θ , Z , Q , ω_1 , ω_2 , B.W
at $f = 300$, f_w , 5000

Sol.

$f = 300 \text{ Hz}$

$X_L = 188.5$, $X_C = 2411.45$

$I = \sqrt{(188.5 - 2411.45)^2 + (680)^2}$

$= 2324.63 \Omega$

$|I| = \frac{V}{Z} = 0.43 \text{ mA}$

$\theta = \tan^{-1} \left(\frac{X_L - X_C}{R} \right)$

$\theta = 73^\circ$

$V_L = I \times X_L = 0.0811 \text{ V}$

$V_C = I \times X_C = 1.04 \text{ V}$

$V_R = I \times R = 0.29 \text{ V}$

~~$Q = \frac{X_L}{R} = 0.277$~~

$Q = \frac{1}{R} \sqrt{\frac{L}{C}} = 0.991$

$\omega_1 = \frac{-680}{200 \times 10^{-3}} + \sqrt{\frac{680^2}{(200 \times 10^{-3})^2}}$

$\sqrt{\frac{1}{100 \times 0.22 \times 10^{-9}}}$

~~$= 2291.934532$~~

$\omega_1 = 4150.8$

$\omega_2 = 10950.8$

$f_1 = \frac{\omega_1}{2\pi} = 660.6$

~~$f_2 = \frac{\omega_2}{2\pi} = 551742.8$~~

$B = 1082.2 \text{ Hz}$

at f_w

$f_w = \frac{1}{2\pi\sqrt{LC}} = 1073.022$

$X_L = 674.2$, $X_C = 674.2$

~~$Z = \sqrt{X_L^2 + X_C^2 + R^2}$~~ $Z = R = 680$

$\theta = 0$

$V_L = 0.99$, $V_C = 0.99 \text{ V}$

$V_R = I \times R = 1 \text{ V}$

$Q = \frac{X_L}{R} = 0.991$

$B = \frac{f_w}{Q} = \frac{1073.022}{0.991}$

$= 1082.2$

at $f = 5000$

$X_L = 3141.6$, $X_C = 144.7$

($X_C < X_L \rightarrow$ inductive)

~~$Z = 666 \Omega$~~

$Z = 3073.1$

$|I| = \frac{V}{Z} = 0.33 \text{ mA}$

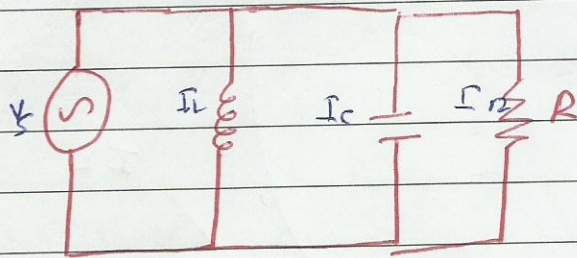
$\theta = -77.22^\circ$

$V_L = 1.04 \text{ V}$

$V_C = 0.05 \text{ V}$

$V_R = 0.2244 \text{ V}$

② Parallel Resonance



There are 3 cases:-

① $f < f_w$

$$f \uparrow \rightarrow X_L \uparrow \rightarrow X_C \downarrow \rightarrow$$

$$I_C \uparrow \left(\frac{V_S}{X_C} \right) \rightarrow I_L \downarrow \left(\frac{V_S}{X_L} \right) \rightarrow$$

$$I_R \text{ const } \left(\frac{V}{R} \right)$$

$$\downarrow I_S = \sqrt{(I_L - I_C)^2 + I_R^2}$$

$$X_C > X_L \quad (B_C < B_L)$$

Inductive parallel

$$\theta = \tan^{-1} \left(\frac{I_L - I_C}{I_R} \right) \text{ (-ve)}$$

θ goes from -90 to 0 .

② $f = f_w$

$$f_w = \frac{1}{2\pi\sqrt{LC}}$$

$$f \uparrow \rightarrow X_L \uparrow \rightarrow X_C \downarrow \rightarrow$$

$$I_C = I_L \rightarrow I_R \text{ const}$$

$$I_S = \sqrt{I_R^2} = I_R \text{ (min)}$$

$$X_C = X_L \quad (B_C = B_L)$$

Resistive

$$\theta = 0$$

③ $f > f_w$

$$f \uparrow \rightarrow X_L \uparrow \rightarrow X_C \downarrow \rightarrow$$

$$I_C \uparrow \rightarrow I_L \downarrow$$

$$I_R \text{ const}$$

$$\uparrow I_S = \sqrt{(I_L - I_C)^2 + I_R^2}$$

$$X_C < X_L \quad (B_C > B_L)$$

capacitive parallel

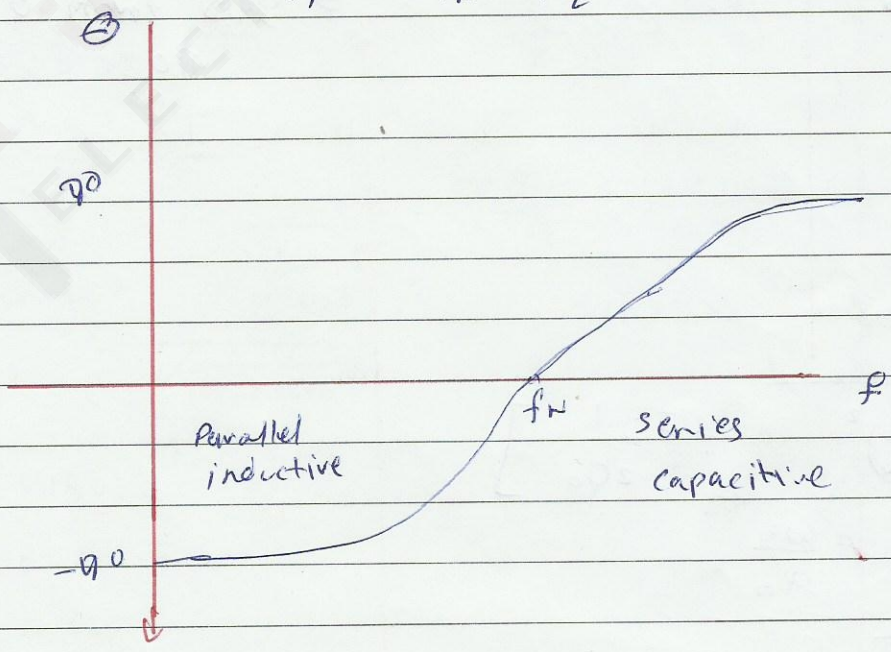
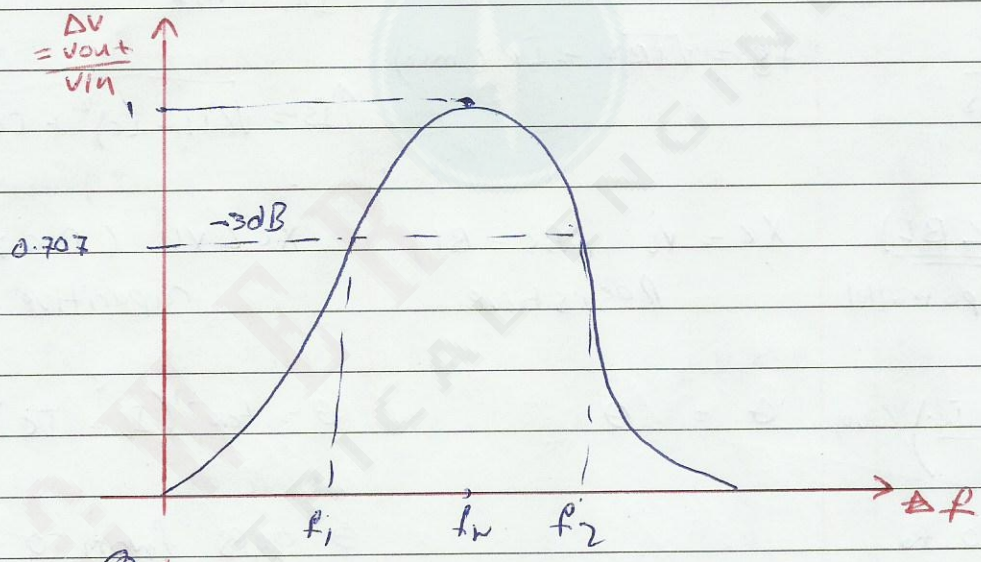
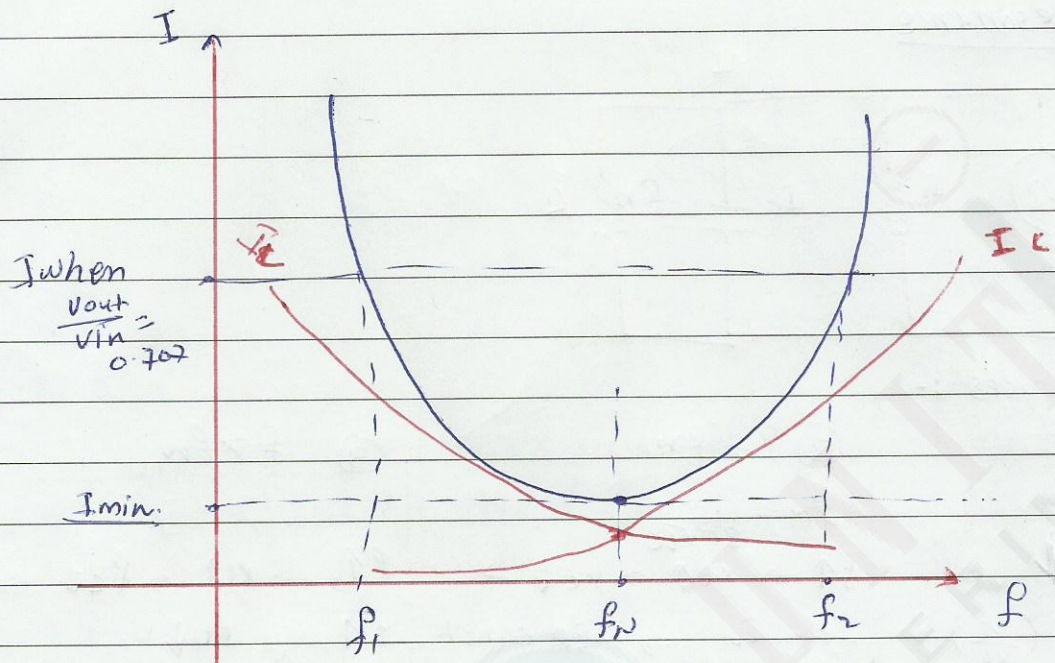
$$\theta = \tan^{-1} (I_L - I_C) \text{ (+ve)}$$

θ goes from 0 to 90 .

$$w_{1,2} = w_0 \left[\sqrt{1 + \left(\frac{1}{2Q_0} \right)^2} \right] \quad \left[\frac{1}{2Q_0} \right]$$

$$B = w_2 - w_1 = \frac{w_0}{Q_0}$$

$$Q_0 = R \sqrt{\frac{C}{L}}$$



Example :- Let $R=680$, $L=400\text{mH}$, $C=0.22\mu\text{F}$, $V_s=120$

Find f_w , I , I_L , I_C , I_R , θ , Q , ω_1 , ω_2 , BW

at $f=300$, f_w , 5000

$f=300$

$$X_L = 188.6 , X_C = 2411.45$$

$$B_L = -j5.31\text{mA}^{-1} , B_C = j0.415\text{mA}^{-1}$$

$$Y_s = -j5.31\text{mA} + j0.415\text{mA} + 1/680$$

$$= 5.11 \angle -73.3$$

$$I_R = \frac{V}{R} = 1.471\text{mA}$$

$$I_C = \frac{V}{X_C} = 0.414\text{mA}$$

$$I_L = \frac{V}{X_L} = 5.31\text{mA}$$

$$I_S = \sqrt{(5.31\text{mA} - 0.414\text{mA})^2 + 1.471\text{mA}^2}$$

$$= 5.11\text{mA}$$

$$\theta = \tan^{-1}\left(\frac{I_L - I_C}{I_R}\right) \text{ ang}$$

$$\theta = -73.3$$

$$Q = R \sqrt{\frac{C}{L}} = 1.009$$

~~ω_1~~

$f=f_w$

$$f_w = \frac{1}{2\pi\sqrt{LC}} = 1073.022$$

$$X_L = 674.2 \quad X_C = 674.2$$

$$Y = \frac{1}{R} = 1.471\text{mA}^{-1} \angle 0$$

$$I_R = 1.471$$

$$I_C = I_L = 1.48\text{mA}$$

$$\theta = 0$$

$$\omega_{1,2} = \omega_0 \left[\sqrt{1 + \left(\frac{1}{2Q_0}\right)^2} \pm \frac{1}{2Q_0} \right]$$

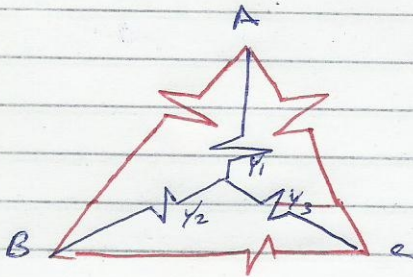
$$\omega_1 = 4185.6$$

$$\omega_2 = 10865.3$$

$$\omega_2 - \omega_1 = 6681.7 = \frac{\omega_0}{Q_0}$$

Experiment 7

Δ -Y TRANSFORMATION

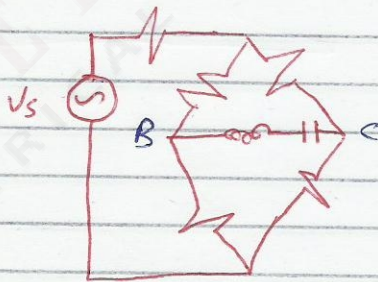


$$Y_1 = \frac{\Delta_{AC} \Delta_{AB}}{\Delta_{AC} + \Delta_{AB} + \Delta_{BC}}$$

$$\Delta_{AB} = \frac{Y_1 Y_2 + Y_2 Y_3 + Y_3 Y_1}{Y_3}$$

* Resonance

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$



① at $f < f_r \Rightarrow$
 $V_{BC} \downarrow$

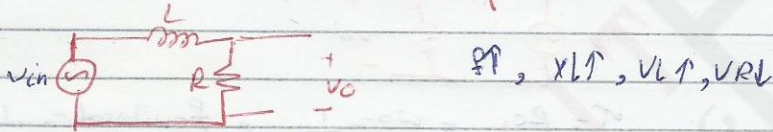
② at $f = f_r$, $X_C = X_L$, $V_{BC} = 0$

③ at $f > f_r$, $V_{BC} \uparrow$

Transfer function of a Two port Network

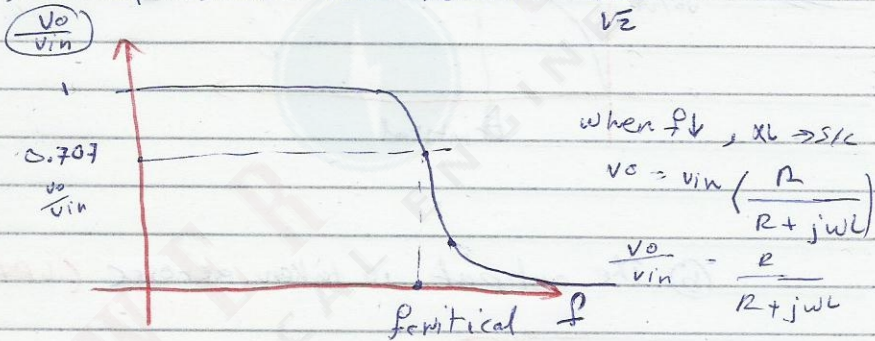
(i) RL series:

(a) The output across R (LPE)

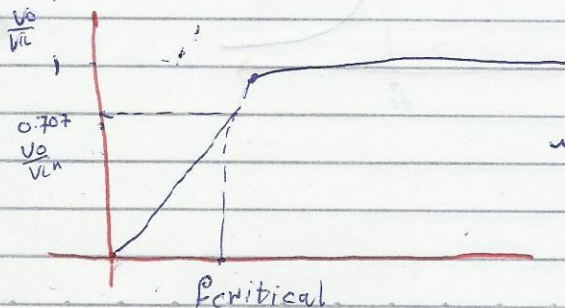
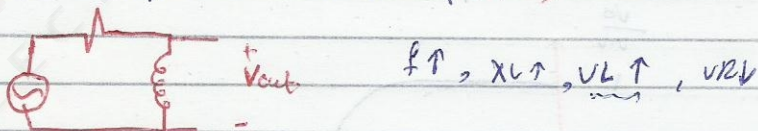


$$\tau = \frac{L}{R}, \omega_0 = \frac{1}{\tau}, f_{\text{critical}} = \frac{R}{2\pi L}$$

f_{critical} is at half power $\frac{V}{\sqrt{2}}$



(b) The output across L (HPE)



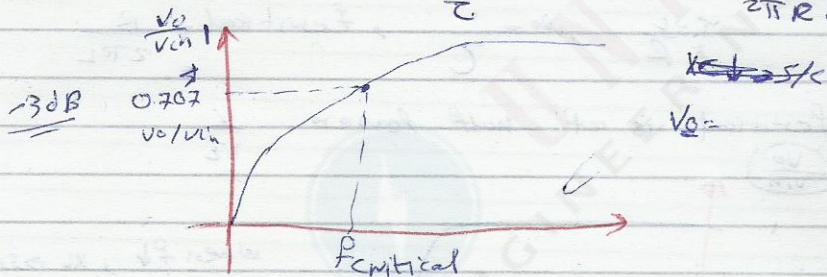
when ω is High
 $X_L \rightarrow \infty$
 $v_{in} = v_o$

② RC series, $f \uparrow$, $X_C \downarrow$, $V_R \downarrow$, $V_C \uparrow$

(a) The output is taken across R (HPF)



$$\tau = RC \Rightarrow \omega_0 = \frac{1}{\tau} \Rightarrow f_{\text{critical}} = \frac{1}{2\pi RC}$$



(b) The output is taken across C (LPF)

