CIRCUIT II NOTEBOOK

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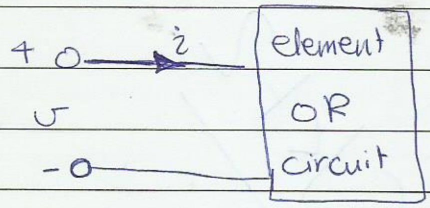
AC circuit power analysis:

1. what ??
2. why ??
3. How ??

Instantenous Power:

P(t)

Definition



by definition,

$p(t) \overset{\Delta}{=} v * i$ → current through the element according to passive (sign convention)

↑ voltage across the element or cct

power consumed by the element or cct

P of the basic passive elements:

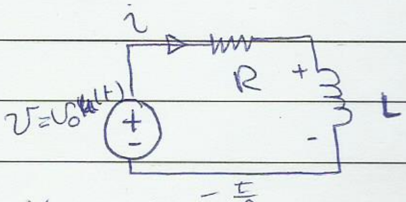
element	P
R	$P_R = v_R * i_R = v_R^2 / R = i_R^2 * R$
L	$P_L = v_L * i_L = \left(L \frac{di_L}{dt}\right) * i_L = v_L * \frac{1}{L} \int v_L \, dt$
C	$P_C = v_C * i_C = \left(C \frac{dv_C}{dt}\right) * v_C = \frac{1}{C} \int i_C \, dt * i_C$

Illustration:

find the Power P supplied to the elements in the cct

procedure:

It can be found that



$i = \frac{V_0}{R}\left(1 - e^{-\frac{R}{L}t}\right) u(t) = \underbrace{\frac{V_0}{R}}_{dc} - \underbrace{\frac{V_0}{R} e^{-\frac{t}{\tau}}}_{natural}$

$\therefore P_R = i^2 R$ ---- (2) sub (1) into (2) to find P_R

$P_L = v_L i$ but $v_L = \frac{L\,di_L}{dt}$

$= L \frac{di}{dt} * i$ -- (3) substitude (1) into (3) to find P_L

P supplied by the source, P_s

$P_s = v * i$

$= V_o\,u(t) * i$ -- (4)

substitude (1) into (4) to find P_s

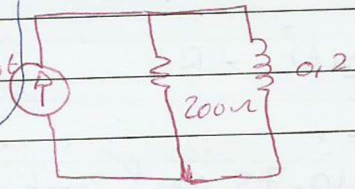
Check Power balance

$P_s \equiv P_R + P_L$

ex :>

(transient response died out)

for the given cct, $i = 12\cos 2000t$ A and assuming (steady state) conditions, find the Power being absorbed by R, L and source at $t = 1ms$



(Time domain)

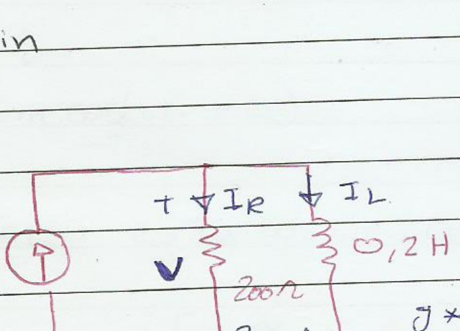
sol

objective Find $P_R(t)$, $P_L(t)$, $P_s(t)$ → then substitude $t = 1ms$

Procedure :>

Transform the circuit From Time domain → frequency Domain

$i = 12\cos 2000t$ A



$j * 2000 * 0,2$

$j400\,\Omega$

$I = 12\angle 0°$ A

frequency domain

By current divison:

$$I_L = 12\angle 0 \quad \frac{200}{200+j400} = 5.4 \angle -63.4^\circ \text{ A}$$

$\therefore$ $\boxed{i_L = 5.4\cos(2000t - 63.4^\circ)\text{ A}}$

$$I_R = 12\angle 0 - 5.4\angle -63.4^\circ$$

$$= 12\angle 0 \quad \frac{j400}{j400+200} = 10.73 \angle 26.76 \text{ A}$$

$\therefore$ $i_R = 10.73\cos(2000t + 26.76)$

$\therefore$ $P_R(t) = i_R(t)^2 * R$

$$= [10.73\cos(2000t + 26.76)]^2 * 200$$

$\therefore$ $P_R(1*10^{-3}) = \left[10.73\cos(2 + 26.76)\right]^2 * 200 = \boxed{14{,}06}\text{ kw}$

Radians

$P_L(t) = v * i_L$

$$= \left[L\frac{di_L}{dt}\right] * i_L$$

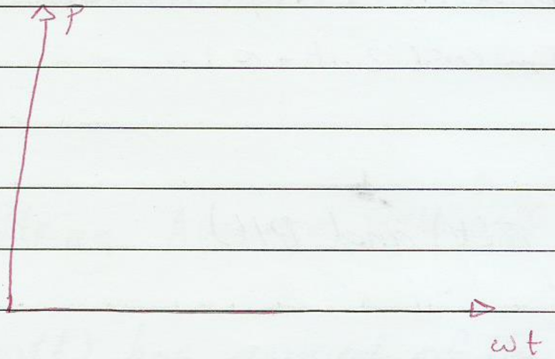
OR

$$= [200\, i_R] * i_L$$

$\therefore$ $P_L(1*10^{-3}) = \left[200 * i_R(10^{-3})\right] * i_L(10^{-3}) = \boxed{-5.69\text{ kw}}$

$$P_s = \cancel{\ } v * (-i_s) = -[P_R + P_L] = -[14.06 - 5.69]$$
$$= \boxed{-8.37}\ kw$$

Sketch P_R, P_L and P_s

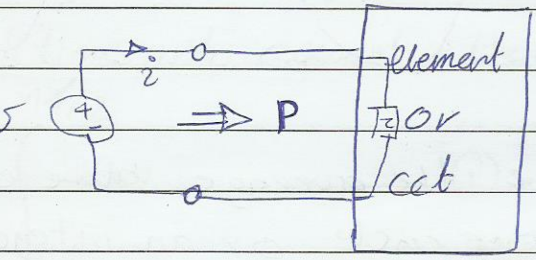


General sinusoidal Case:

* let $\underline{\underline{v(t)}} = V_m \cos\omega t$

objective

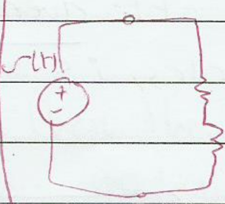
find p



<u>Procedure</u> :-

$P \triangleq v * i$

∴ let $\underline{\underline{i}} = I_m \underline{\underline{\cos}}(\omega t + \theta)$



$I_m = \frac{V_m}{\sqrt{R^2 + (\omega L)^2}}$

$\theta = -\tan^{-1}\left(\frac{\omega L}{R}\right)$

∴ $p = \underline{\underline{vi}} = V_m I_m \cos\omega t \cdot \cos(\omega t + \theta)$

$= V_m I_m \cdot \frac{1}{2}\left[\cos(2\omega t + \theta) + \cos(\theta)\right]$

$= \boxed{\frac{1}{2} V_m I_m \cos\theta + \frac{1}{2} V_m I_m \cos(2\omega t + \theta)}$ — ①

By studying (1 the followings can be deduced:

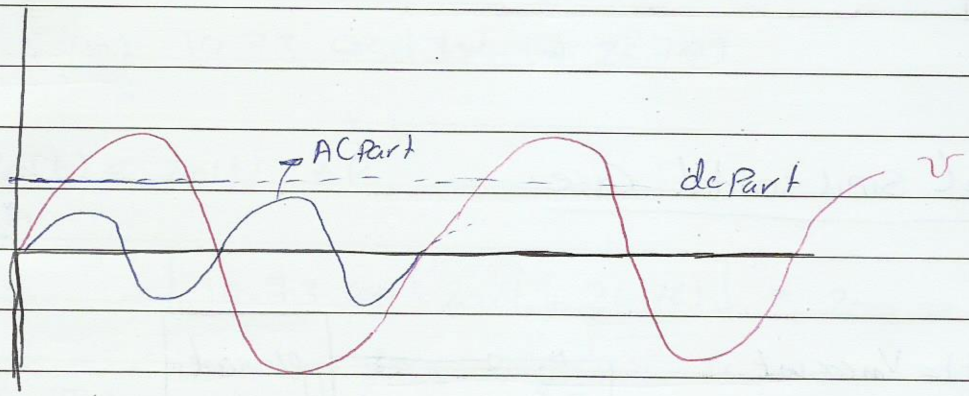
(1) P consists of 2 ~~components~~ parts :

a) a constant term = $\frac{1}{2} V_m I_m \cos\theta$

b) a sinusoidal term with a frequence twice the frequency of source = $\frac{1}{2} V_m I_m \cos(2\omega t + \theta)$

Please (2)

Sketch $v(t)$ and $P(t)$



ii) Since the average value of a pure sin wave over one cycle or an integer number of cycles = zero Then the average value P of P in (1) over a cycle or a number of cycles, is

$$\boxed{P = \frac{1}{2} V_m I_m \cos\theta}$$ (average ~~power~~ of a periodic waveform) ... (2)

Hence from (2)

a) If the element is R, then $\theta = 0°$

$\therefore P = \frac{1}{2} V_m I_m$ or $= \frac{1}{2} V_m^2 / R$ or $= \frac{1}{2} I_m^2 R$

b) If the element is L or C

then $\theta = \pm 90^\circ$

$\therefore \cos 90^\circ = 0$

$\therefore P = 0$

$\therefore$ The only element which consume Average power (P) is the Resistor

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Evaluating Average value of Periodic Instantaneous Power.

Let p(t) has a period of T Then $P \triangleq \frac{1}{T}\int_0^T p\,dt$

$$= \frac{1}{nT}\int_0^{nT} P\,dt \quad \text{where } n = 1, 2, 3 \ldots = \frac{1}{nT}\int_{-\frac{nT}{2}}^{\frac{nT}{2}} P\,dt$$

In order to take all time into account, then let $n \to \infty$

$$\therefore P = \lim_{n\to\infty} \frac{1}{nT}\int_{-nT/2}^{nT/2} \quad \text{---} \quad (1)$$

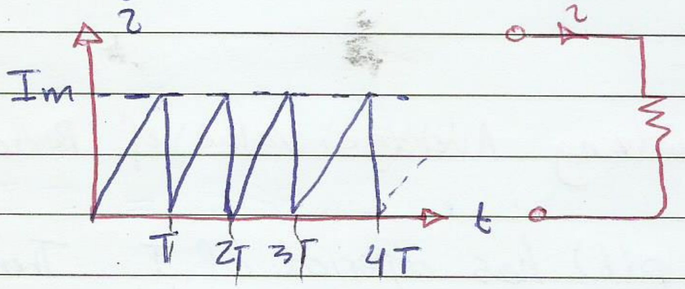
If n in (1) is changed slightly, then the integral in (1) and consequently P will change by negligible Amount

$\therefore$ nT becomes a continuous variable, say τ

Hense (1) becomes $\boxed{P = \lim_{\tau\to\infty} \frac{1}{\tau}\int_{-\tau/2}^{\tau/2} P\,dt \quad \text{---} \quad (2)}$

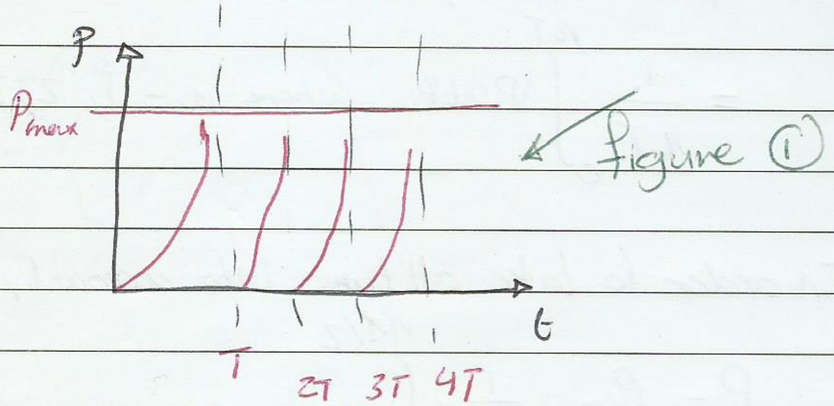
∴ Hense ② can be used to evaluate P for periodic and non periodic P by assuming that it has ∞ period.

ex: Find P supplied, to R by the following Sawtooth current
(سن المنشار)



Solution:

$$P = i^2 R$$



∴ P is periodic with a period = T as that to the current

$$\therefore P = \frac{1}{T}\int_0^T i^2 R\,dt = \frac{R}{T}\int_0^T i^2\,dt$$

∴ from figure ① , $i(t) = \frac{I_m}{T} t$ ② $0 < t < T$

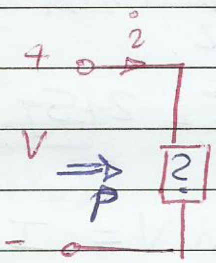
Sub ② into ①

$$\therefore P = \frac{R}{T}\int_0^T \frac{I_m^2}{T^2} t^2\,dt$$

$$= \frac{R I_m^2}{T^3}\left[\frac{1}{3}t^3\right]_0^T$$

$$\boxed{P = \frac{1}{3} R I_m^2}$$

General Sinusoidal Case:



Problem

if $v = V_m \cos(\omega t + \theta)$

and $i = I_m \cos(\omega t + \phi)$

find P

Solution:

It can be found that $T_{v\,or\,i} = \frac{2\pi}{\omega}$

Now, $p = vi = V_m I_m \cos(\omega t + \theta)\cdot\cos(\omega t + \phi)$

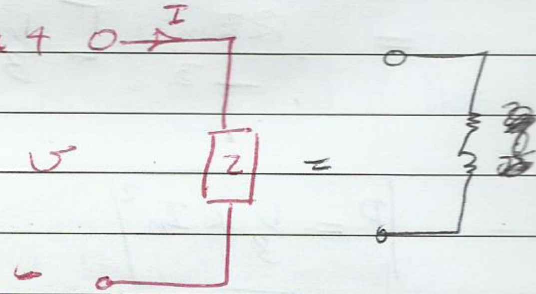
$$\therefore p = V_m I_m \frac{1}{2}\left[\cos(2\omega t + \theta + \phi) + \cos(\theta - \phi)\right]$$

$$= \frac{1}{2} V_m I_m \cos(\theta - \phi) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta + \phi) \quad \cdots (1)$$

$\therefore p$ is periodic with a period $T_p = \frac{1}{2} T_{v\,or\,i} = \frac{\pi}{\omega}$ — (2)

$$\therefore P = \frac{1}{T_p}\int_0^{T_p} p\,dt \quad \cdots (3) \equiv \boxed{\frac{1}{2} V_m I_m \cos(\theta - \phi)}$$

evaluate $\overset{\text{average}}{P}$ supplied to $Z = 6\angle 25°$ if $I = (2+j5)$ A



Sol :>

$$P = \frac{1}{2} V_m I_m \cos(\theta - \phi)$$

$$I = 2+5j = \underbrace{5.38}_{I_m} \angle \underbrace{68.2°}_{\phi}$$

$$\therefore V = IZ = 5.38\angle 68.2 * 6\angle 25 = \underbrace{32.28}_{V_m} \angle \underbrace{93.2}_{\theta}$$

$$P = 78.7 \text{ W}$$

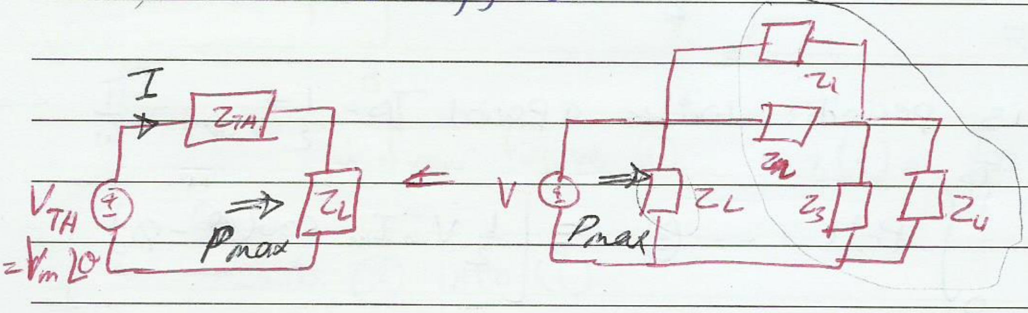
OR $P = \frac{1}{2} I_m^2 / R$

$$Z = 6\angle 25 = \underbrace{6\cos 25}_{R} + \underbrace{j6\sin 25}_{X}$$

$$\therefore P = \frac{1}{2} * (5.38)^2 * 6\cos 25 = 78.7\text{W}$$

Maximum power transfer :

Objective Find Z_L in a given cct to which max power is supplied



$$Z_{TH} = R_{TH} + jX_{TH}$$

$$Z_L = R_L + jX_L$$

Procedure: Simplify the given cct to it's thevinen equivalent

$\therefore P = \frac{1}{2} |I|^2 R_L$ ... (1)

But $I = \frac{V_{TH}}{Z_{TH} + Z_L} = \frac{V_m \angle \theta}{(R_{TH}+R_L) + j(X_{TH}+X_L)}$

$\therefore |I| = \frac{V_m}{\sqrt{(R_{TH}+R_L)^2 + (X_{TH}+X_L)^2}}$

Sub (2) into (1)

$P = \frac{1}{2} \frac{V_m^2 R_L}{(R_{TH}+R_L)^2 + (X_{TH}+X_L)^2}$ — (3)

To find Z_L to which P is max Then solve $\frac{\partial P}{\partial R_L} = 0$ — (4)

$\frac{\partial P}{\partial X_L} = 0$ ... (5)

Apply (4) and (5) to (3)

From (4) and (5) it can be found

$X_L = -X_{TH}$

$R_L = R_{TH}$

consugate

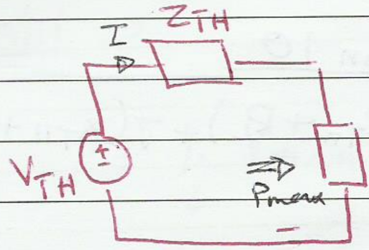
$\therefore Z_L = R_L + jX_L = R_{TH} - jX_{TH} = \boxed{Z_{TH}^*}$

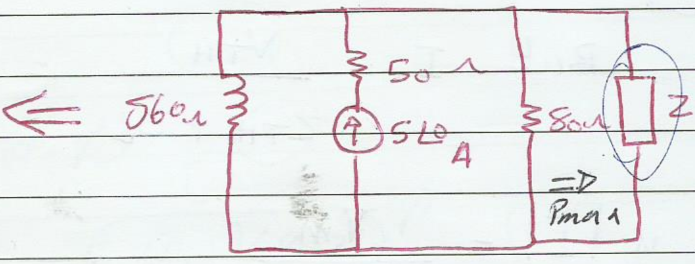
find Pmax

$P_{max} = \frac{1}{2} |I|^2 R_L$

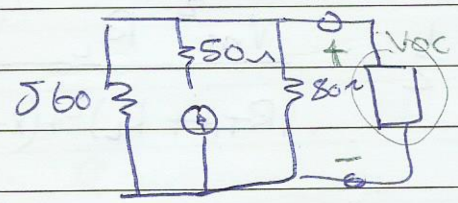
$|I| = \frac{V_m}{2 R}$

ex:2) Evaluate in the given cct. Z to which P_{max} is supplied.





* it is required to find Z_{TH}
we kill all the dependent sources



$Z_{TH} = 80 // j60$

$= \frac{80 * j60}{80 + j60} = 48 \angle 53.1$

$= 28.8 + j38.4$

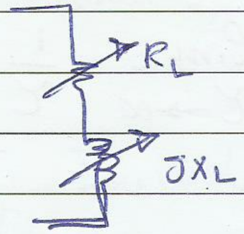
$\therefore Z = Z_{TH}^* = 48 \angle -53.1 = 28.8 - j38.4$

$V_{TH} = V_{oc} = j60 \left[5\angle 0 * \frac{80}{80 + j60} \right] = 240 \angle 53.1$

$\therefore I = \frac{V_{TH}}{Z_{TH} + Z} = \frac{240 \angle 53.1^\circ}{2 * 28.8} = 4.17 \angle 53.1$ A

$\therefore P_{max} = 1/2 * |I|^2 * 28.8 = 250.4$ W

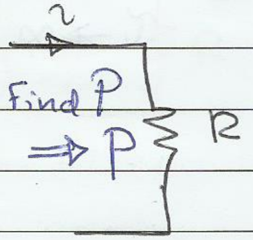
Solve from the book
find Z from P_{max} under
the following conditions:>
1) R_L fixed and $\neq R_{TH}$
2) X_L " " $\neq -X_{TH}$
3) $X_{TH} = 0$
4)
5)



⇒ Evaluating P (avg) of non periodic p (inst)

Problem:

if $i = I_{m_1} \sin w_1 t + I_{m_2} \sin w_2 t$, Find P



It's peridic if $mT_1 = nT_2$ where m and n are integers

find P

$$P = i^2 R = \int R I_{m_1}^2 \sin^2 w_1 t + \int 2 R I_{m_1} I_{m_2} \sin w_1 t \sin w_2 t + \int R I_{m_2}^2 \sin^2 w_2 t$$

∴ $P = \lim_{\tau \to \infty} \frac{1}{\tau} \int_{-\tau/2}^{\tau/2} p\, dt$ --- (2)

by substituting (1) into (2), then ther are 3 integrals to be evaluates.

① $\Rightarrow \lim_{\tau \to \infty} \frac{1}{\tau} \int_{-\tau/2}^{\tau/2} R I_{m_1}^2 \sin^2 \omega_1 t \, dt$

$= \lim_{\tau \to \infty} \frac{1}{\tau} \cdot R I_{m_1}^2 \int_{-\tau/2}^{\tau/2} \frac{1}{2} \left[1 - \cos 2\omega_1 t \right] dt = \frac{1}{2} I_{m_1}^2 R$ ③

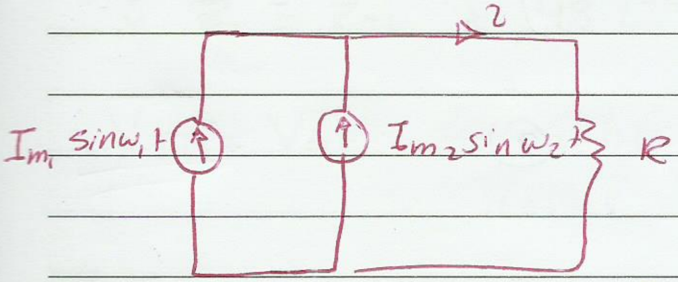
Similarly ③ $\Rightarrow \frac{1}{2} I_{m_2}^2 R$ ④

② $\lim_{\tau \to \infty} \frac{1}{\tau} \int_{-\tau/2}^{\tau/2} R I_{m_1} I_{m_2} \left[\cos(\omega_1 t - \omega_2 t) - \cos(\omega_1 t + \omega_2 t) \right] dt$

$= 0$ ⑤

i substitude ③, ④, ⑤ into ②

$P = \frac{1}{2} I_{m_1}^2 R + 0 + \frac{1}{2} I_{m_2}^2 R$



$\Rightarrow$ here we can use superposition principle

Conclusion: From ⑥ it canbe deduced that in this special case (ie current consist of components with different frequencies superposition can be applied to power calculations)

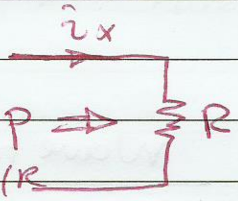
⑥ Can be extended to n components, irrespective of any phase angles:

e.x:

if $i_x = I_{m_1}\sin(\omega_1 t - \theta) - I_{m_2}\cos(\omega_2 t + \alpha) + I_{m_3}\cos(\omega_3 t - \beta) + \dots I_{m_n}\sin(\omega_n t + \phi)$

using ~~super~~ S.P:

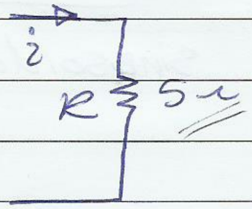
$\therefore P = \frac{1}{2} I_{m_1}^2 R + \frac{1}{2} I_{m_2}^2 R + \dots + \frac{1}{2} I_{m_2}^2 R$



OR

$= \frac{1}{2} V_{m_1}^2/R + \frac{1}{2} V_{m_2}^2/R + \dots + \frac{1}{2} V_{m_n}^2/R$

ex: if $i = (2 - 3\cos 100t)^2$ A, Find P



Solution:

$i = 4 - 12\cos 100t + 9\cos^2 100t$

$= 4 - 12\cos 100t + 9 \cdot \frac{1}{2}[1 + \cos 200t]$

$= 4 - 12\cos 100t + 4.5 + 4.5\cos 200t$

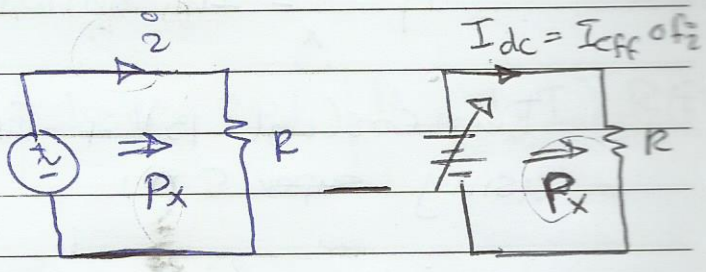
$= 8.5 - 12\cos 100t + 4.5\cos 200t$

$\therefore P = P_1 + P_2 + P_3$

$= (8.5)^2 * 5 + \frac{1}{2}(12)^2 * 5 + \frac{1}{2}(4.5)^2 * 5 = \cancel{22}\ 772$ W

Effective value of aperiodic current (i) or periodic voltage (v)

Def: The effective value of i or v (ie I_{eff} or V_{eff}) is equal to the dc current or dc voltage which supply the same average Power to a resistive R or i or v



Wave form	P_{AC}	P_{DC}	$P_{AC} = P_{DC}$
Saw tooth	$\frac{1}{3} I_m^2 R$	$I^2 R$	$\frac{1}{3} I_m^2 R = I_{eff}^2 R$ $\therefore \boxed{I_{eff} = I_m / \sqrt{3}}$
sinusoidal	$\frac{1}{2} I_m^2 R$	$I^2 R$	$\frac{1}{2} I_m^2 R = I_{eff}^2 R$ $\therefore I_{eff} = I_m / \sqrt{2}$ OR $I_m = \sqrt{2}\, I_{eff}$

Note: The domestic distribution voltage in Jordan is 220 V (effective value)

Peak value $= \sqrt{2} \times 220$

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Mathematical expression for effective value:

$$P_{AC} = P_{DC}$$

$$\frac{1}{T}\int_0^T i^2 R\,dt = I_{eff}^2 R \Rightarrow$$

$$I_{eff} = \sqrt{\frac{1}{T}\int_0^T i^2\,dt}$$

I_{eff} = square Root of the Mean of Squared current.

= RMS value

ii V_{eff} OR V_{rms}

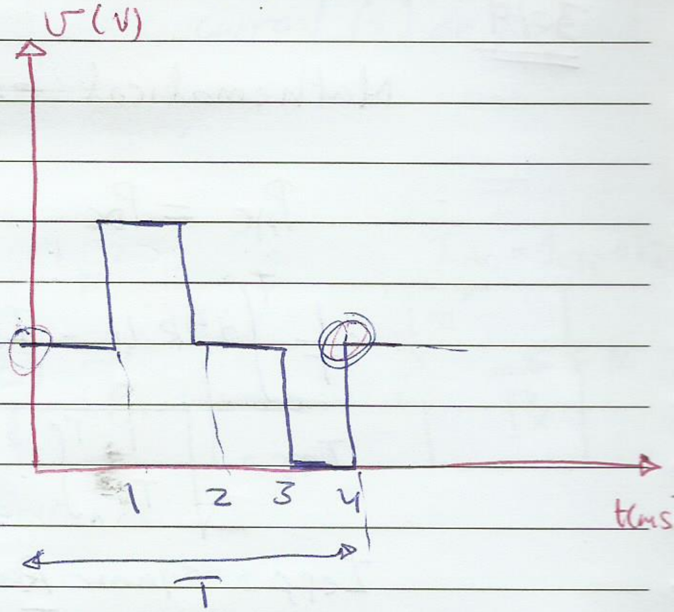
* IF $i = I_m \cos(\omega t + \theta)$ find I_{rms}!

* $$I_{rms} = \sqrt{\frac{1}{T}\int_0^T (I_m \cos(\omega t + \theta))^2\,dt}$$

where $T = \frac{2\pi}{\omega}$

it can be found $I_{RMS} = \frac{I_m}{\sqrt{2}}$

ex: Find V_{eff} of the given v

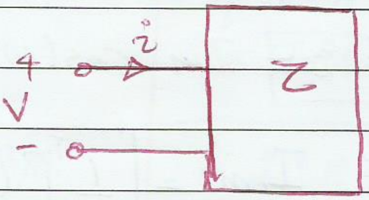


$$v_{eff} = \sqrt{\frac{1}{4\times10^{-3}}\left[\int_0^{1ms}(10)^2\,dt + \int_{1ms}^{2ms}(20)^2\,dt + \int_{2ms}^{3ms}(10)^2\,dt + 0\right]}$$

$= 12.25\ V_{rms}$

⇒ (*) Average Power in terms of effective value

It was found that:



$P = \frac{1}{2} V_m I_m \cos\theta$

$\theta = \angle V - \angle I$

since $Z = V/I$

$\therefore \angle Z = \angle V - \angle I = \theta$

$= \frac{1}{2}(\sqrt{2}\,V_{eff}) * (\sqrt{2}\,I_{eff})\cos\theta$

$$\boxed{P = V_{eff}\, I_{eff} \cos\theta}$$

※ Effective value of i or v with multiple frequencies

If $i = I_{m_1}\sin(\underset{=}{\omega_1} t - \beta) + I_{m_2}\cos(\underset{=}{\omega_2} t) - I_{m_3}\cos(\underset{=}{\omega_3} t - \beta) \ldots$

$\ldots\ I_{m_n}\sin(\underset{=}{\omega_n} t + \phi)$

Find I_{eff}

$\therefore$ As found before by superposition

$$P_{AC} = P_1 + P_2 + \dots + P_n = I_{eff_1}^2 R + I_{eff_2}^2 R + \dots + I_{eff_n}^2 R \quad \text{--- (1)}$$

But $P_{DC} = I_{eff}^2 R$ --- (2)

∴ By equating (1) and (2) ⟹

$$I_{eff}^2 \cancel{R} = I_{eff_1}^2 \cancel{R} + \dots + I_{eff}^2 \cancel{R}$$

$$\therefore I_{eff} = \sqrt{I_{eff_1}^2 + I_{eff_2}^2 + \dots + I_{eff_n}^2}$$

ex :⟹ if $i = (2 - 3\cos 100t)^2$ find I_{eff}

It was found $i = 8.5 - 12\cos 100t + 4.5\cos 200t$

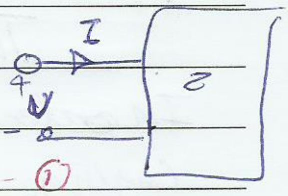
$$\therefore I_{eff} = \sqrt{I_{eff_1}^2 + I_{eff_2}^2 + I_{eff_3}^2}$$

$$= \sqrt{(8.5)^2 + \left(\frac{12}{\sqrt{2}}\right)^2 + \left(\frac{4.5}{\sqrt{2}}\right)^2} = 12.42\ A_{RMS}$$

Apparent Power and Power Factor

القدرة الظاهرية　　　　معامل القدرة

$P \triangleq V_{eff} I_{eff} \cos\theta = \frac{1}{2} V_m I_m \cos\theta$ --- (1)



Apparent Power $\triangleq V_{eff} I_{eff}$ --- (2)

Since $\cos\theta$ is Dimensionless then (1) and (2) should the same unit

But, they are different, then the Apparent Power is given the unit

since Volt - Ampar (VA)

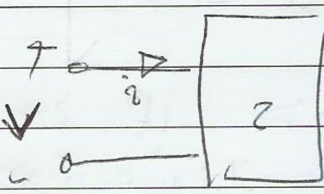
Power factor $\triangleq$ $\dfrac{\text{Average Power}}{\text{Apparent Power}}$
PF

∴ For the sinusoidal case:-

$$PF = \frac{VI\cos\theta}{VI} = \cos\theta$$

∴ θ is called PF angle.

Comments:



1) For Pure resistive Load $\theta = 0$

∴ $PF = 1$ (i.e unity PF)

$\theta = \angle V - \angle I = \angle Z$

2) For pure Inductive or capacitive Load, $\theta = \pm 90°$
∴ $PF = 0$

∴ In general $0 \leq PF \leq 1$

3) If it is given $PF = 0.5$

Then $\theta = \cancel{\pm} 60°$ (Inductive or -60 capacitor)

In order to spesify PF clearly, then the concept of Laging PF or leading PF is used

Here the reference is the current with respect to voltage.

∴ Laging PF means i lags v by $\theta = \cos^{-1}(PF)$

∘ Leading PF " " leads " " " " "

(ac current lead v) (capacitor)
(" lag ") inductor

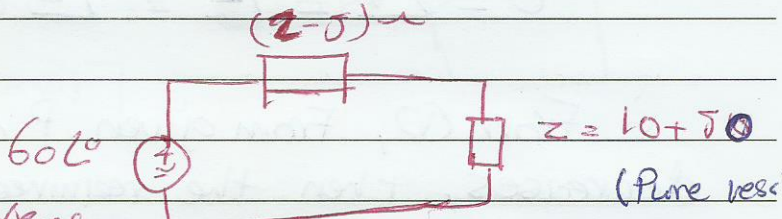
Illustration:

If $\angle V = 20^\circ$
and PF = 0.5 leading

$\therefore \angle I = 20^\circ + \cos^{-1} 0.5$
$= 20 + 60 = 80$

→ ref

find for the given cct:



1) Apparent power supplied Source
2) PF of the source

Solution:

$I_{eff} = 3.54\ A_{rms}$
Apparent power = 212.13 W
$P.F = \frac{P}{V_{eff} I_{eff}} = 0.7$

App. Power = $V_{eff} I_{eff}$

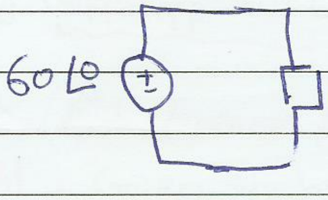
$I = \frac{60 \angle 0}{(2-j)+(10+j0)} = 4.98 \angle 4.76\ A_{rms}$

Apparent power = 60 * 4.98 = 298.8 VA

ii) PF $= \cos[\angle V - \angle I]$
$\therefore \cos[0^\circ - 4.76]$ leading
$= 0.9965$

OR: PF = $\frac{\text{Av. Power}}{\text{App. Power}}$

PF = cos ∠Z = $\cos^{-1}(\frac{1}{12})$ leading

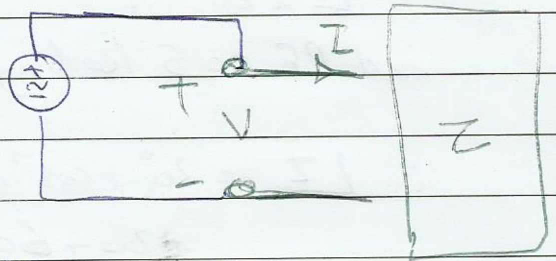


zeq = (2-j) + (10+j0)
zeq = 12 - j

capacitive

Practical Importance of PF:

since $P = V_{eff} I_{eff} \cos\theta$



$\therefore I_{eff} = \dfrac{P}{V_{eff}\cos\theta}$ —— (1)

$\theta = \angle V - \angle I = \angle Z$

$\therefore$ From (1), From given P and Veff as the PF decreases, then the required current (I_{eff}) increases

Consequently:

1) conductors or Lines with large cross sectional Area are required

$\therefore$ Higher capital cost

2) Higher losses ($I_{eff}^2 R$) in lines.

where (1) and (2) are suffered by the power Company OR distribution company

Hense $\Rightarrow$ the PF should not be below a certain ~~minimum~~ minimum acceptable level

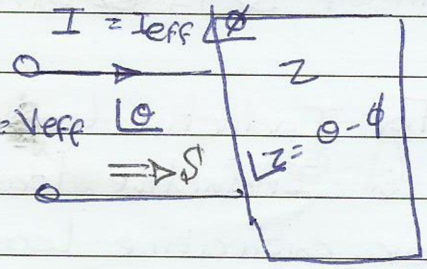
Internationally, this level is 0.85

Complex power:

Why?? This is a mathematic tool or concept used to Simplify power Calculations in AC circuits.

Definition :

$P = V_{eff} I_{eff} \cos(\theta - \phi)$ ... (1)



(1) can be written as :

$P = Re\left[V_{eff} I_{eff} e^{j(\theta - \phi)}\right]$

Re : real Part

$\therefore P = Re\left[V_{eff} e^{j\theta} * I_{eff} e^{-j\phi}\right] = Re\left[V_{eff} \angle\theta * I_{eff} \angle{-\phi}\right]$

$= Re\left[\underbrace{V * I^*}_{\text{Complex Power } S}\right]$ ... (3)

The Product $(V * I^*)$ in (3) is called complex Power, S

Comments about $S = VI^* = V_{eff}\angle\theta - I_{eff}\angle{-\phi} = V_{eff} I_{eff} \angle{\theta - \phi}$

1) The magnitude of $S = |S|^* = \underline{V_{eff} I_{eff}}$ = Apparent power (VA)

2) $\angle S = (\theta - \phi)$ = PF angle

3) $S = \underbrace{V_{eff} I_{eff} \cos(\theta - \phi)}_{Re} + j \underbrace{V_{eff} I_{eff} \sin(\theta - \phi)}_{Im}$

$\therefore Re[S] = P \equiv$ Average Power

$Im[S] = V_{eff} I_{eff} \sin(\theta - \phi)$ = Given symbole Q

$\therefore Q = V_{eff} I_{eff} \sin(\theta - \phi)$

$\therefore S = P + JQ$ Q = is called Reactive Power

Unit of Q is ~~V~~ Volt-Amper-Reactive (VAR)

$\therefore$ For Inductive Load $(\theta - \phi) > 0$ $\therefore$ Q is +ve

$\therefore$ Inductive load consumes Reactive power

For capacitive Load, $(\theta - \phi) < 0$, $\therefore$ Q is -ve

(capacitive Load consume -ve Reactive Power. ok it generates Reactive Power)

$\therefore$ In general $S = P + JQ$

* Capacitor → generators

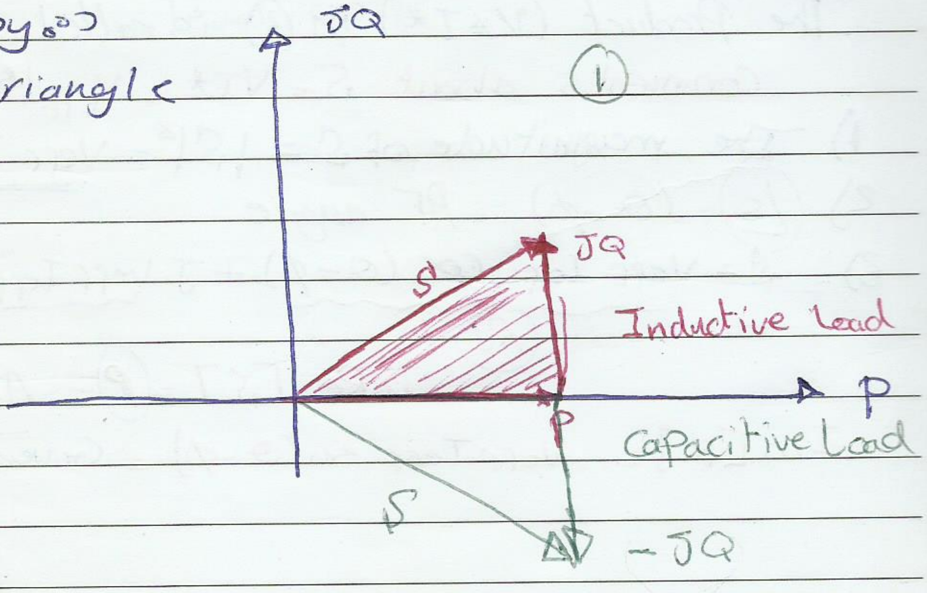
(بيزيد قيمة ال Q)

(بالتالي يقل PF)

Graphical Representation of S:

This is done by:

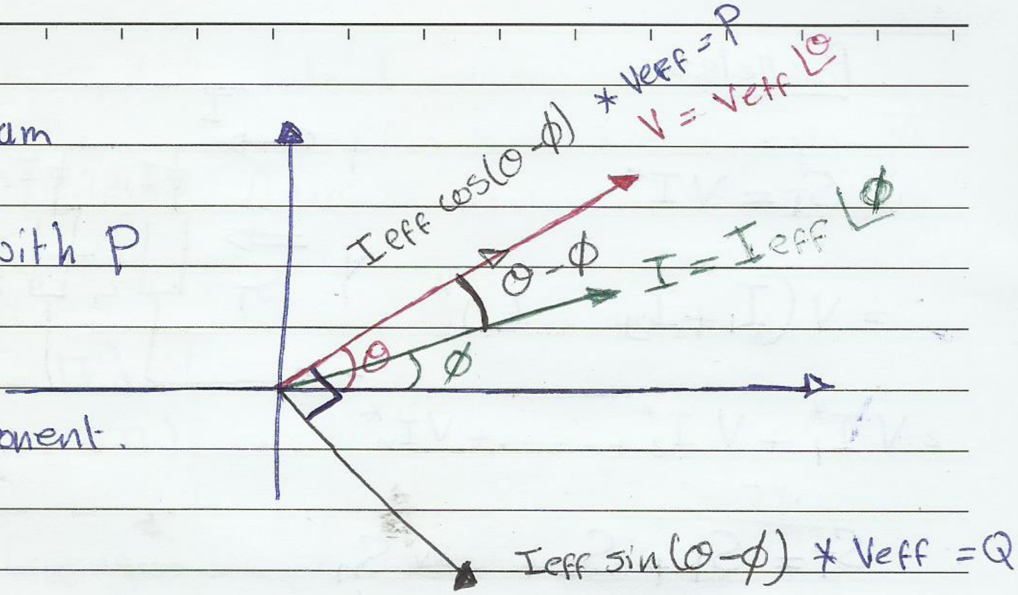
1) Power triangle



2) Phasor diagram

Q makes 90° with P

∴ Q is also called Quadrature component.



7/10:

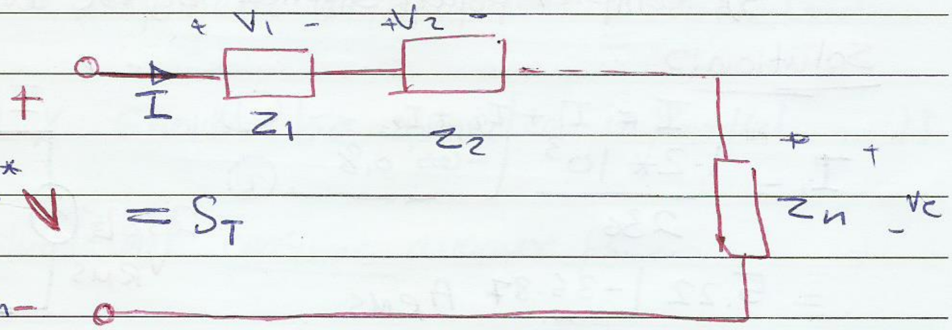
$$S = P \pm JQ = VI\cos\theta + J\,VI\sin\theta$$

$$\therefore \frac{Q}{P} = \tan\theta$$

$$\therefore Q = P\tan\theta$$

(*) Total complex power supplied to a set of loads = Σ of the individual complex power irrespective of how the loads are connected.

Series:



$S_T = V I^*$

$= (V_1 + V_2 + \dots V_n) I^*$

$= S_T$

$= V_1 I^* + V_2 I^* \dots + V_n I^*$

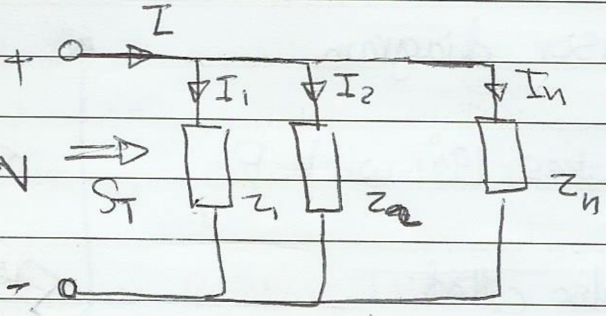
$S_T = S_1 + \dots + S_n$

Parallel:

in $S_T = VI^*$

$= V(I_1 + I_2 + \dots I_n)^*$

$= VI_1^* + VI_2^* + \dots + VI_n^*$



$\therefore S_T = S_1 + S_2 + \dots S_n$

example: An Industrial company has the following 3 loads

solve for load $I = 14.35\angle$

L_1: A pump rated at 1.2 kVA, 0.8 PF laging, 230 V_{RMS}

L_2: An Inductive motor rated at 1.6 KVA, 0.9 PF lagging, 230 V_{RMS}

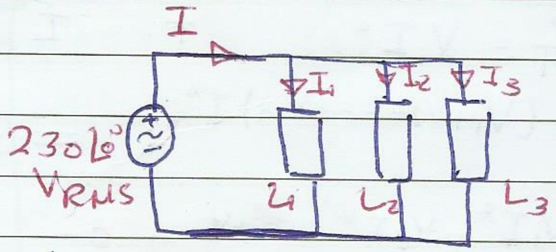
L_3: Heater rated at 900W, unity PF, 230 V_{RMS}

Find:

1) The magnitude of current supplied.
2) PF of source
3) Complex power supplied by the source

Solution:

$I = I_1 + I_2 + I_3$ — (1)



$I_1 = \frac{1.2 * 10^3}{230} \angle -\cos^{-1} 0.8$ — (2)

$= 5.22 \angle -36.87$ A_{RMS}

$I_2 = \frac{1.6 \times 10^3}{230} \angle -\cos^{-1} 0.9 = 6.96 \angle -25.84$ A_{RMS} — (3)

$I_3 = \frac{900}{230 * P.F} \angle 0 = 3.91 \angle 0$ A_{RMS} — (4)

1

$\boxed{P = VI\cos\theta \quad I = \frac{P}{V\cos\theta}}$

Substitude 2, 3, 4 into 1

$I = \underline{\underline{15.616}} \angle -23.23$ A_{RMS}

↳ magnitude

$PF = \cos(\angle V - \angle I)$

$\cos(0 - (-23.23))$

$= 0.9189$ lagging

$S = V * I^*$

$= 230 \angle 0 * 15.616 \angle 23.23$

$= 3591.68 \angle 23.23$ VA

PF Improvement or connections:

objective: To increase the power Factor of a given load in order to avoide penulty or electricity bill

ضريبة فاتورة الكهرباء

Solutions:

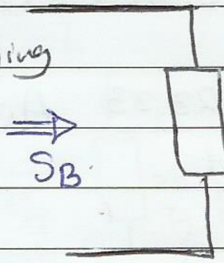
Add a component or equipment $\underline{\underline{X}}$ to the load in such away

i) X should be connected in Parallel with the load.

ii) X should not consume average Power

$\therefore P_{After} = P_{Before}$

Ex, It's required to improve the PF to 0.9 Lagging
what is the solution?



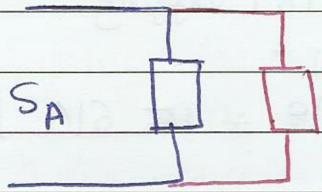
Load
1.2 KVA
0.8 PF Lagging
230 Vrms
50 Hz

Before

$S_B = 1.2 \times 10^3 \angle \cos^{-1} 0.8$

$= 1.2 \times 10^3 \angle 36.87 = (960 + J720)\,VA$

After:



$P_A = P_B = 960\,W$

since $Q = P \tan\theta$

$\therefore Q_A = P_A \tan\theta_A$

$= 960 * \tan\left[\cos^{-1} 0.9\right]$

$= 960 * \tan[25.84]$

$= 465\,VAR$

$\therefore S_A = 960 + J465 = (S_{Load} + S_x) = (960 + J720) + S_x$

$\therefore S_x = J465 - J720 = -J255\,VAR$

$\therefore$ X consumes Pure -ve Reactive Power of $\boxed{-255\,VAR}$

$\therefore$ X is a capacitor

Since $Q = VI\sin\theta$

for capacitor $\theta = -90°$

$\therefore |Q| = |V||I|$

$= |V| * |V|\,\omega C$

$= |V|^2\,\omega C$

$\therefore C = |Q| / V^2 * \omega$

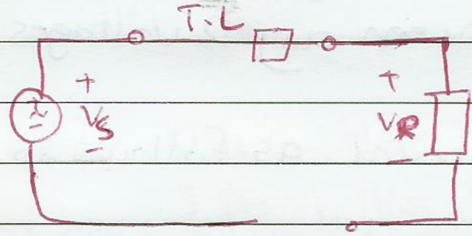
$\therefore C = 255 / (230)^2 * 2\pi(50)$

$= 15.4\ \mu F$

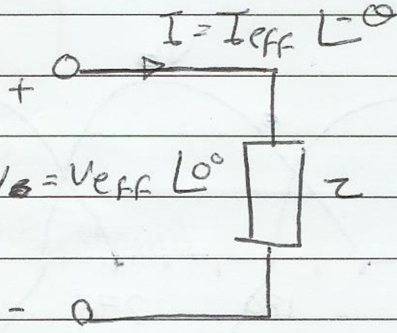
Solution: a capacitor in parallel with the load having the following specifications:>

15.4 μF, 255 VAR, 230 V_{RMS}, 50Hz

$V_R = V_S - I Z_L$



Current Lags V by θ



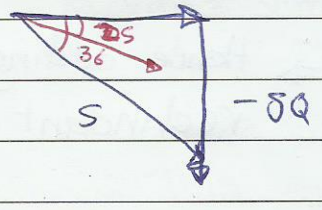
$\therefore$ Shift $= \angle V - \angle I$

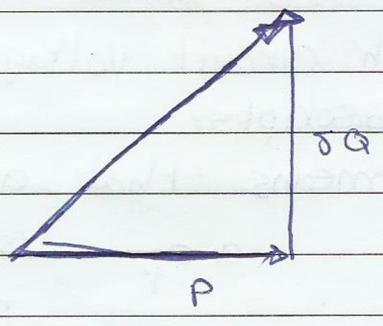
$= 0 - (-\theta) = +\theta$

$Z = \frac{V}{I} = \frac{V_{eff} \angle 0}{I_{eff} \angle -\theta} = |Z| \angle \theta$

$\therefore$ For inductive Load $\angle Z$ = +PF angle

$S = V * I^* = V_{eff} \angle 0° * I_{eff} \angle +\theta = V_{eff} I_{eff} \angle \theta$



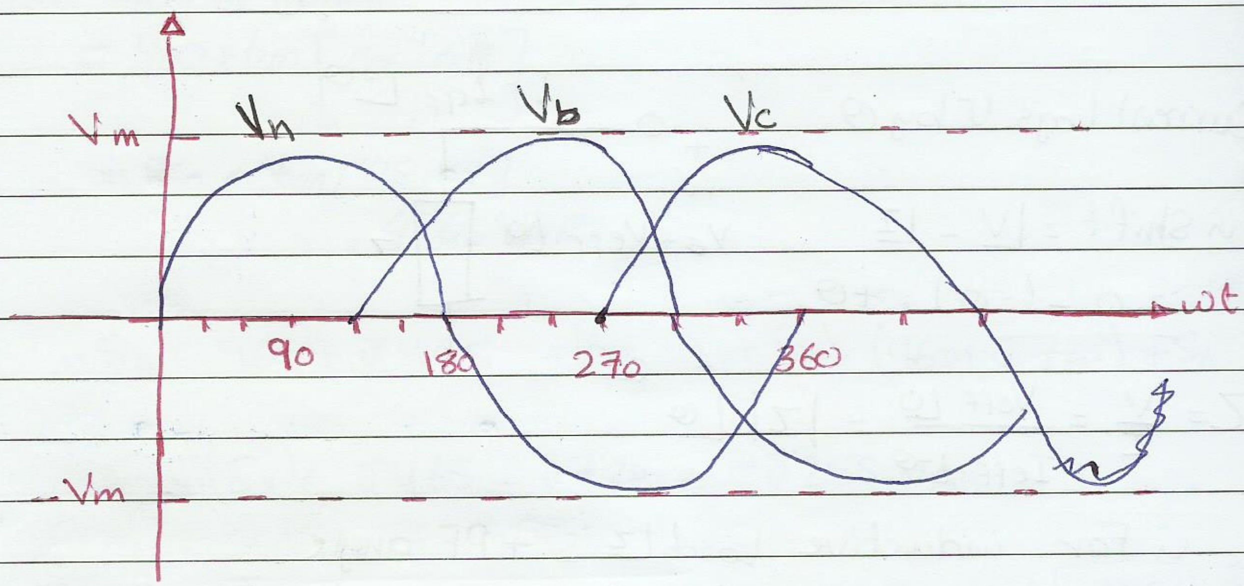


9/10:

Three phase system or circuits:

(*) Here a 3-ph source, called 3-ph ~~sy~~ synchroneous generators, generates 3 sinusoidal voltages equal in magnitude and there is a phase shift of 120° between any 2 voltages.

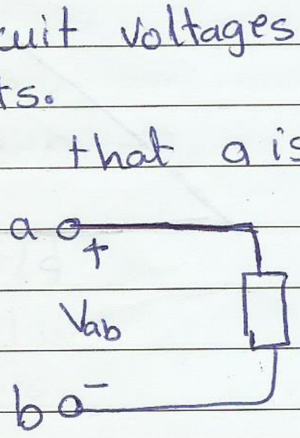
Illustrated as follows:



Double Subscript Notation:

(*) In 3-ph circuit voltages or currents are defined by double subscripts.

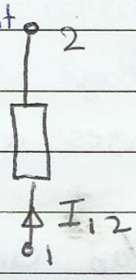
ex: V_{ab} ≡ means that a is more +ve than b

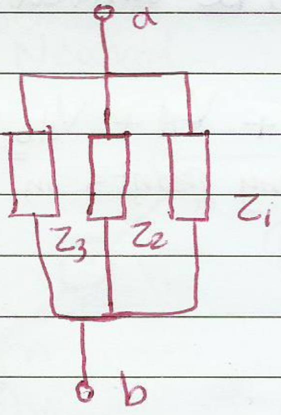


⇒ Hence adding Polarity is redundant information.

(*) A/s the current I_{12} ≡ means that current is flowing from Node 1 to Node 2

Direction of current is redundant, except when there are Parallel branches between the same nodes.





I_{ab} ≡ Here there are 3 possibilties hence one has to identify direction.

Single_phase 3wire systems:

Before analyzing the 3-Ph system, an introduction about 1-ph, 3 wires given below:

where

$V_{an} = V_{nb}$

∴ $\angle V_{an} = \angle V_{ab}$

∴ one Phase Angle

Hence the name single Phase 3 wire

by KVL:

$V_{ab} = V_{an} + V_{nb}$

∴ For $V_{an} = 120V$

∴ $V_{ab} = 240V$

∴ This system present 2 voltage levels (LV and HV)

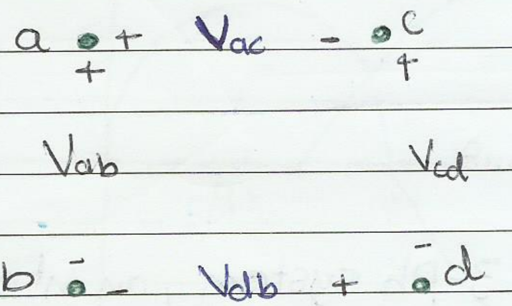
(*) a major advantage of the Double subscript Notation is that KVL can be applied directly without looking at the circuit.

Because any voltage can be written as the sum of 2 or more voltages as follows:

For e.g V_{ab} can be written as:

$$V_{ab} = V_{ac} + V_{cd} + V_{db}$$

where a, b, c and d are any points in the give ckt



$\therefore -V_{ab} + V_{ac} + V_{cd} + V_{db} = 0$

$\therefore$ KVL is satisfied

Analysis of 3-ph systems:

There are Two types of connections:

a) Y or Star connection

b) Δ Connection

(*) 3-phase source are usually Y-connected.

(*) 3-phase Loads can be Y or Δ connected

Monew

3-ph Y-connected source:

This source has:

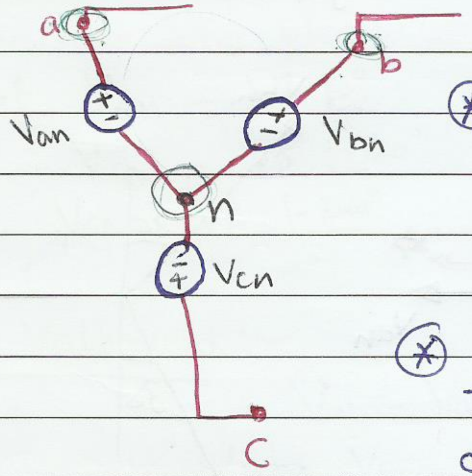
a) 3 independent terminals called Line terminals.

b) a common terminal called Neutral.

It is represented as follows:

a, b, c ≡ Line Terminals because they are connected to transmission Line.

n ≡ Neutral



Definitions:

(*) Phase voltage:
It is the voltage between a line terminal and the ~~N~~ Neutral
e.g V_{an}, V_{bn} and V_{cn}

(*) Line voltage:
It is the voltage between any pair of line terminals

(*) The domestic distribution voltage is phase voltage.

- In Jordan
~~The~~ The residential supplied voltage is:
a 50 Hz, 220 V_{RMS}, Phase voltage.

Balance 3-ph voltages: Means 3 voltages equal in magnitude and there is a phase shift = 120° between any 2 voltages.

Balanced 3-ph Voltages: To identify such voltages as Phasors, then ~~select~~ select one of them to be the Reference.

Let V_{an} to be the Ref.

$$\begin{aligned} \text{i.e. } V_{an} &= V_p \angle 0^\circ \\ V_{bn} &= V_p \angle -120 \\ V_{cn} &= V_p \angle -240 \end{aligned}$$

$V_p \equiv$ Magnitude of phase voltage usually expressed in RMS value

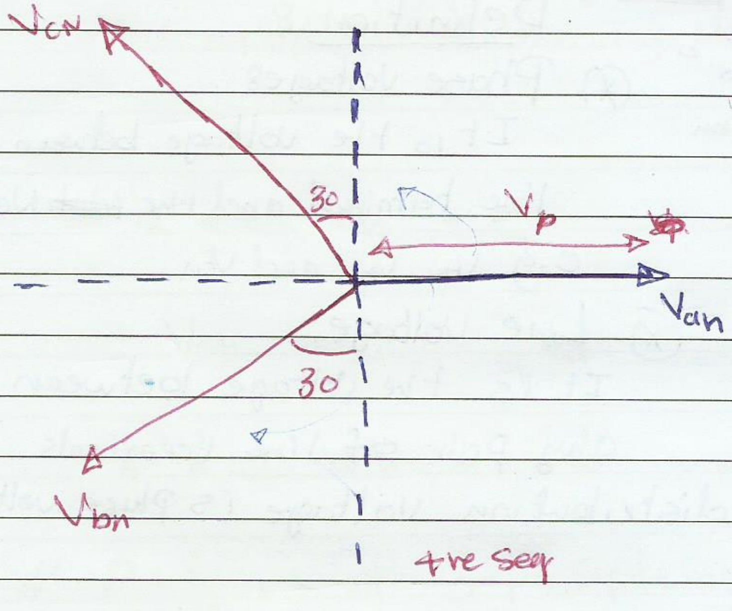
→ ~~lbs~~ +ve phase sequence OR abc sequence

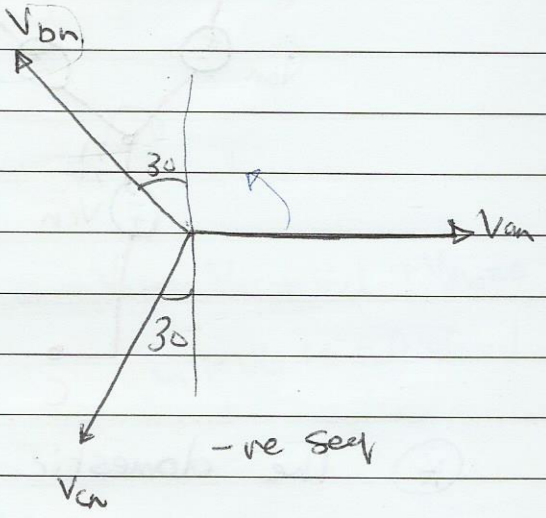
OR,

$$\begin{aligned} V_{an} &= V_p \angle 0^\circ \\ V_{bn} &= V_p \angle 120^\circ \\ V_{cn} &= V_p \angle 240^\circ \end{aligned}$$

→ -ve phase sequence or cba seq

These can be represented by phasors Diagram as follows:





Sum of Balanced Voltages OR Currents is equal to zero e.g $V_{an} + V_{bn} + V_{cn} = 0$

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Line Voltages:

These can be evaluated by:
1) analytical, or
2) graphical by using phasor diagram

i:

Assume +ve phase sequence:

Let $V_{an} = V_p \angle 0^\circ$ ---- ①

$\therefore V_{bn} = V_p \angle -120^\circ$ ---- ②

$\cdot V_{cn} = V_p \angle -240^\circ$ ---- ③

Line Voltages:

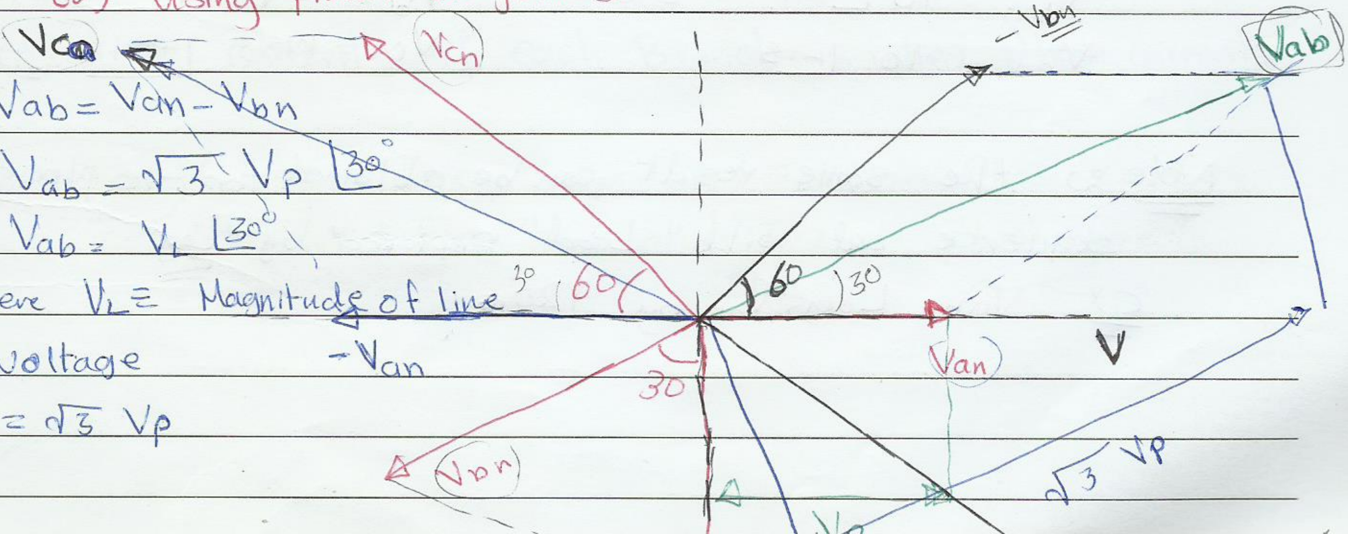
$V_{ab} = V_{an} + V_{nb} = V_{an} - V_{bn}$ ---- ④

$V_{bc} = V_{bn} - V_{cn}$ ---- ⑤

$V_{ca} = V_{cn} - V_{an}$ ---- ⑥

Substitude ①, ②, ③ into ④, ⑤ and ⑥

ii) using phasor diagram:



$V_{ab} = V_{an} - V_{bn}$

$V_{ab} = \sqrt{3}\, V_p \angle 30^\circ$

$V_{ab} = V_L \angle 30^\circ$

where $V_L \equiv$ Magnitude of line voltage

$= \sqrt{3}\, V_p$

$V_{bc} = V_{bn} - V_{cn}$ $\qquad\qquad$ $V_{bc} = V_L \angle -90^\circ$

$V_{ca} = V_{cn} - V_{an}$ $\qquad\qquad$ $V_{ca} = V_L \angle -210$

(*) It can be reduced that Line voltage Leads Phase voltage by 30°

i. V_{ab} Lead V_{an} by 30°

V_{bc} " V_{bn} " "

V_{ca} " V_{cn} " "

Conclusion :- Hence knowing any voltage (phase or line)

then the other voltages can be defuced

e.x :- If $V_{cn} = 10 \angle -15^\circ$
Find all phase and Line voltages, Assuming +ve phase sequence.

Phase voltage

$V_{an} = 10 \angle 105+120 = 225^\circ$

$V_{bn} = 10 \angle -15+120 = 105^\circ$

$V_{cn} = 10 \angle -15$

Line voltages

$V_{ab} = 10\sqrt{3} \angle 225+30 = 255^\circ$

$V_{bc} = 10\sqrt{3} \angle 105+30 = 135^\circ$

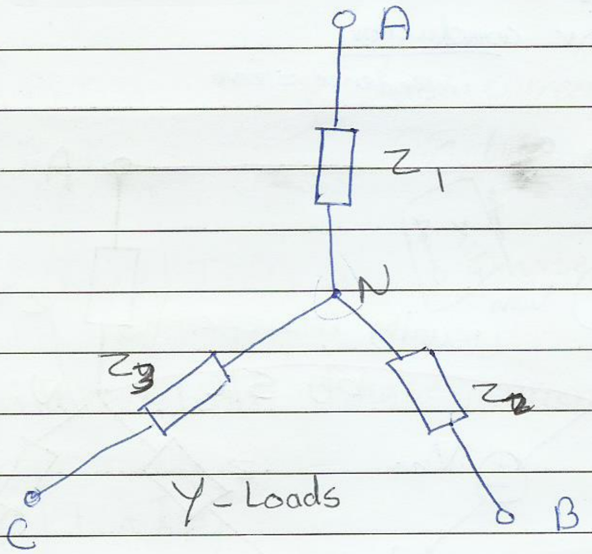
$V_{ca} = 10\sqrt{3} \angle -15+30 = 15^\circ$

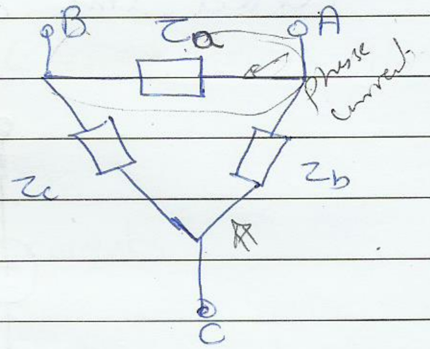
Note :- the same result can be obtained for -ve phase sequence but with "Lead" Replaced by "Lag"

ex V_{ab} Lags V_{bc} by 120°

3-Phase Loads:

This can be connected as Y or Δ as follows:





(Balanced Loads:) Means that the impedance of each Branch is the same

For Y-Load $\Rightarrow Z_1 = Z_2 = Z_3 = Z$

(Δ-Load) $\Rightarrow Z_a = Z_b = Z_c = Z$

The voltage across each Branch is called Phase voltage.

∴ For Y-Load, V_{AN}, V_{BN} and V_{CN} are phase voltages.

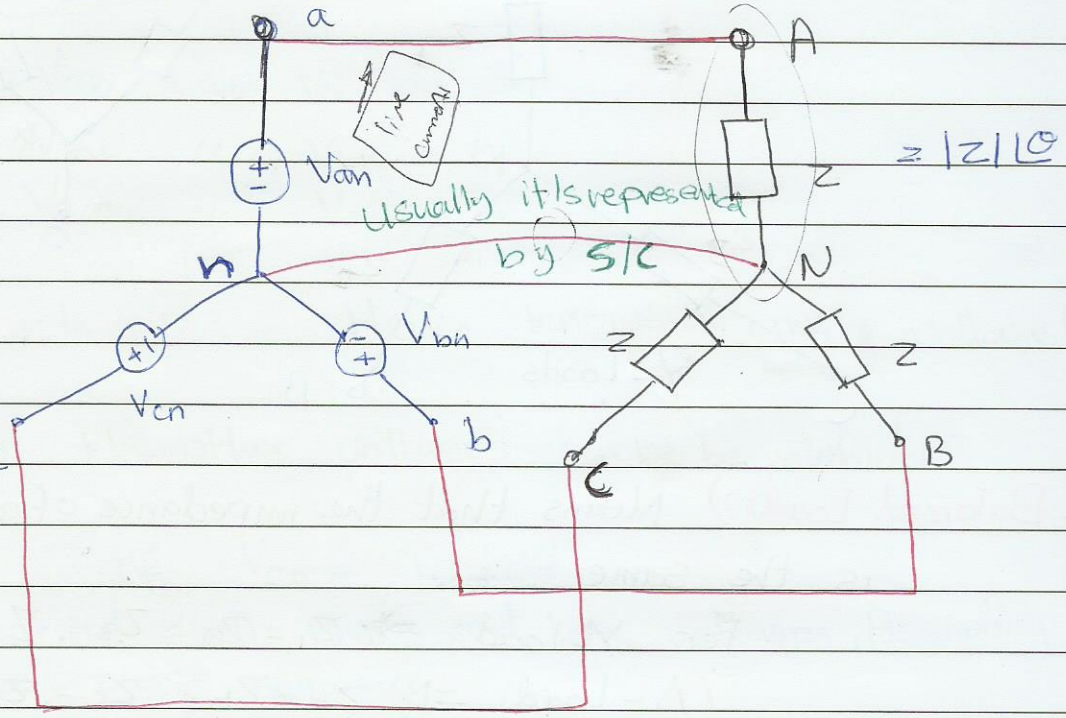
(*) The current of each Branch is called Phase current.

For Y-Load : I_{AN}, I_{BN} and I_{CN} $\Rightarrow$ Phase currents

" Δ-Load : I_{AB}, I_{BC} and I_{CA} $\Rightarrow$ Phase currents

3- phase system :-

(*) Consider a balance 3-ph Y-source supplying electrical energy to a balance 3-ph Y-load through Perfect line or conductors!
impedence = zero



objective :- Evaluate various voltages and currents and find relationships between them.

i From Fig. 1 :

$$I_{aA} = I_{AN} = V_{AN}/Z = V_{an}/Z = V_p\angle 0^\circ / |Z|\angle\theta = \frac{V_p}{|Z|}\angle -\theta = I_p\angle -\theta$$

$I_p = \frac{V_p}{|Z|}$ = Magnitude of phase current Fig 1

$$I_{bB} = I_{BN} = V_{BN}/Z = V_{bn}/Z = V_p\angle -120 / |Z|\angle\theta$$

$$= \frac{V_p}{|Z|}\angle -120 - \theta = I_p\angle -120^\circ - \theta$$

$I_{cC} = I_{CN} = V_{CN} / Z = V_{CN} / |Z| = V_P \angle -240^\circ / |Z| \angle \theta$

$= \frac{V_P}{|Z|} \angle -240 - \theta = I_P \angle -240^\circ - \theta$

I_{aA}, I_{bB}, I_{cC} Line currents Because they flow in line.

I_{AN}, I_{BN}, I_{CN} ← phase currents

∴ For Y load Phase current = Line current

$I_L = I_P$

∴ phase currents

I_{AN}, I_{BN} and I_{CN} are Balanced

∴ $I_{AN} + I_{BN} + I_{CN} = 0$

By KCL ⇒ $I_{Nn} = I_{AN} + I_{BN} + I_{CN}$

$= 0$

(*) since $I_{Nn} = 0$

Then the Line (Nn) canbe represented by any Impedence (including SC or OC) without affecting the calculation.

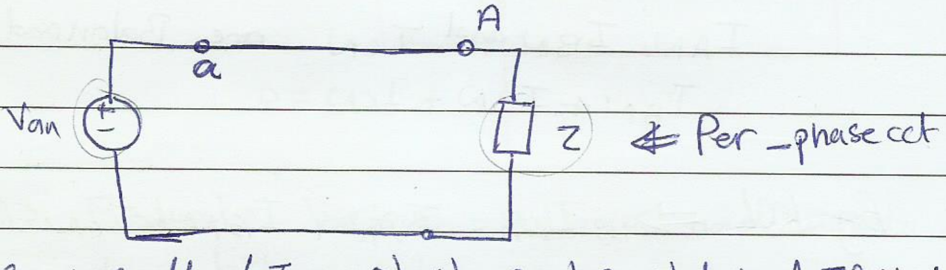
Balanced
Conclusions (Y-Y System) :->

i) Since $I_{Nn} = 0$, then it line can be represented by any impedence including S/C or O/C usually it is represented by S/C

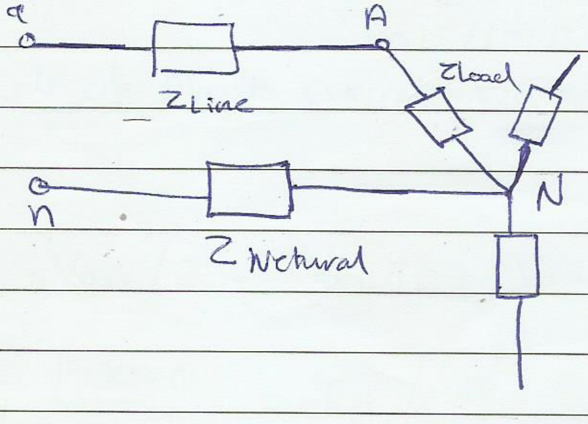
ii) $I_L = I_p$

iii) $V_L = \sqrt{3}\, V_p$

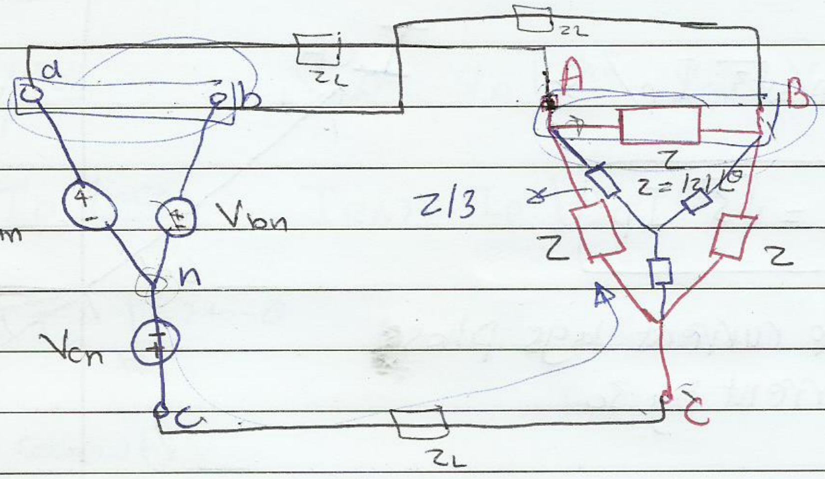
iv) since knowing the voltage and current of one phase and then the voltages and currents of other phases can be deduced, Hence the 3-ph cct. can be represented by a single phase cct called per-phase cct. as follows:



iiv) the same result ($I_{Nn} = 0$) $N_{\rightarrow}$ can be obtained If the lines are not perfect conductors, but they are balanced (i.e have equal impedances) Because Z_{Line} can be combined with Z_{Load}



$Y - \Delta$ system:



$\therefore$ For Δ-Load |phase voltage| = |Line voltage|

$$V_{AB} = V_{ab}$$

$$V_{BC} = V_{bc}$$

$$V_{CA} = V_{ca}$$

Y-Δ System (Balanced)

Phase currents

Assuming the phase sequence ①

By KCL $\Rightarrow$ $I_{aA} = I_{AB} - I_{CA} = \dfrac{V_{AB}}{Z} - \dfrac{V_{CA}}{Z} = \dfrac{V_L \angle 0}{|Z| \angle \theta} - \dfrac{V_L \angle -240}{|Z| \angle \theta}$

line current

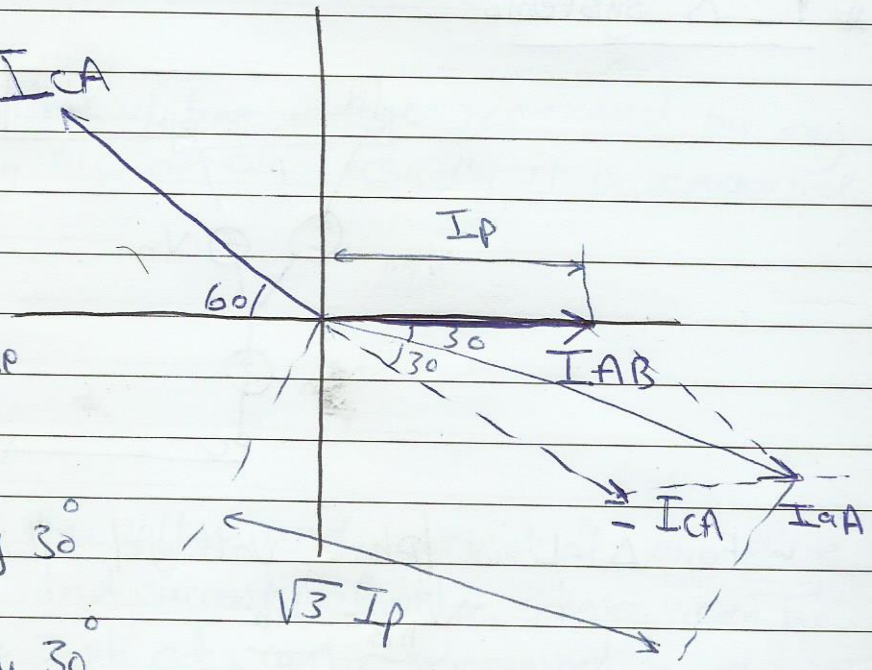
but $V_{AB} = V_L \angle 0^\circ$ $\quad \therefore I_{aA} = \dfrac{V_L}{|Z|} \angle -\theta - \dfrac{V_L}{|Z|} \angle -240-\theta$

$$= I_P \angle -\theta - I_P \angle -240-\theta$$

$I_P = \dfrac{V_L}{|Z|}$ = Magnitude of Phase current

I_{aA} can be calculated analytical or by using phase diagram

To simplify let $\theta = 0°$



$I_{aA} = \sqrt{3}\, I_p \angle -30$

$\boxed{I_L = \sqrt{3}\, I_p}$

line current lags phase current by 30

I_{aA} lags I_{AB} by $30°$

I_{bB} 〃 I_{BC} by $30°$

I_{cC} 〃 I_{CA} by $30°$

Power supplied to Balanced 3-ph loads:

Average Power:

$$P_{3\text{-}ph} = 3 * P_{1\text{-}ph} = 3 * V_p I_p \cos\theta$$

θ = phase shift between ~~(phase voltages)~~ and phase current
= $\angle Z$

$$P_{Y\text{-}Load} = 3 * \frac{V_L}{\sqrt{3}} * I_L * \cos\theta = \sqrt{3}\, V_L I_L \cos\theta$$

$$P_{\Delta\text{-}Load} = 3 * V_L * \frac{I_L}{\sqrt{3}} \cos\theta = \sqrt{3}\, V_L I_L \cos\theta$$

Instantaneous Power :-

Let $V_{AN} = V_p \angle 0^\circ$, $V_{BN} = V_p \angle -120^\circ$, $V_{CN} = V_p \angle -240^\circ$

∴ $I_{AN} = I_p \angle -\theta$ $\quad I_{BN} = I_p \angle -120-\theta$

$I_{CN} = I_p \angle -240-\theta$

∴ $v_{AN} = V_p \cos(\omega t)$

$v_{BN} = V_p \cos(\omega t - 120^\circ)$

$v_{CN} = V_p \cos(\omega t - 240^\circ)$

$i_{AN} = I_p \cos(\omega t - \theta)$

$i_{BN} = I_p \cos(\omega t - 120 - \theta)$

$i_{CN} = I_p \cos(\omega t - 240 - \theta)$

∴ $P_{AN} = v_{AN} * i_{AN} = V_p I_p \cos \omega t . \cos(\omega t - \theta) = \frac{1}{2} V_p I_p \cos\theta$
$+ \underbrace{\frac{1}{2} \cos(2\omega t - \theta)}_{V_p I_p} \quad —$ ①

E

~~$P_{BN} = v_{AN} * i_{AN} =$~~

$P_{BN} = v_{BN} * i_{BN} = I_p V_p \cos(\omega t - 120^\circ) . \cos(\omega t - \theta - 120)$

$P_{BN} = \frac{1}{2} V_p I_p \cos\theta + \underbrace{\frac{1}{2} V_p I_p \cos(2\omega t - 240 - \theta)}_{F} \quad —$ ②

$P_{CN} = v_{CN} * i_{CN} = \frac{1}{2} V_p I_p \cos\theta + \underbrace{\frac{1}{2} V_p I_p \cos(2\omega t - 480 - \theta)}_{G} \quad —$ ③

$\therefore P_{3-ph} = P_{AN} + P_{BN} + P_{CN} = ① + ② + ③ =$

$\frac{3}{2} V_p I_p \cos\theta + \underbrace{[E + F + G]}_{= 0}$

$\therefore \boxed{P = \frac{3}{2} V_p I_p \cos\theta = P}$ = constant value

This one of the adv of 3-ph circuits

$P = 3 * \frac{1}{2} VI\cos\theta = 3VI\cos\theta = \sqrt{3}\, VI\cos\theta$ W

Peak RMSphase (RMS Line)

$Q = 3 * \frac{1}{2} VI\sin\theta = 3VI\sin\theta = \sqrt{3}\, VI\sin\theta$ VAR

$S = P + JQ$ $\therefore |S| = \sqrt{P^2 + Q^2} = \underline{\underline{3VI}}$ OR $\sqrt{3}\, VI$ (VA)

ex :> If $V_{ab} = 100\angle 0$, $V_{bd} = 40\angle 180°$ and $V_{ca} = 70\angle 200°$

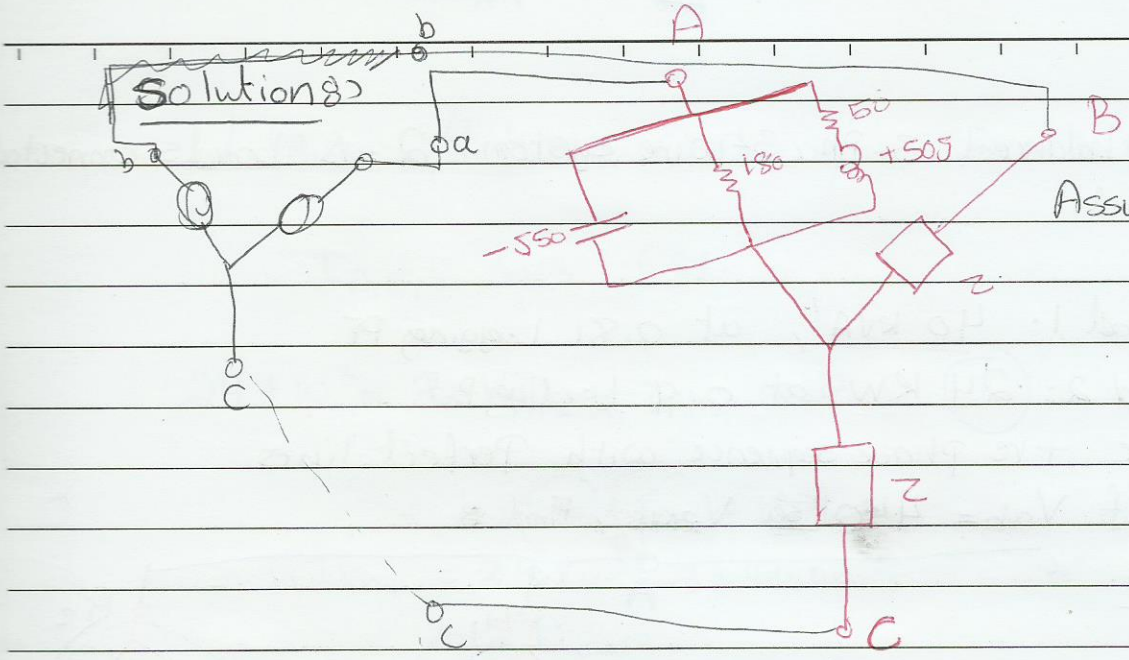
find V_{bc}

$\therefore V_{bc} = V_{ba} + V_{ac} = -V_{ab} - V_{ca} = -100\angle 0° - 70\angle 200° = 41.75\angle 145$

ex :> A balance 3-ph, 3wire system has a Y- connected Load Each phase contains 3 Loads in parallel : $-J100\,\Omega \,//\, 100\,\Omega \,//$ $(50 + J50)$

Assume +ve phase sequence with $V_{ab} = 400\angle 0$ V_{RMS}, Find the Followings.

Solution 8)



Assume perfect conductors

i) Van $\quad$ $\therefore\ V_{an} = \dfrac{400}{\sqrt{3}} \angle 0 - 30^\circ$

ii) I_{aA} $\quad$ $I_{aA} = I_{AN} = \dfrac{V_{AN}}{Z}$ ---- ①

But $V_{an} = \dfrac{400}{\sqrt{3}} \angle -30^\circ\ V_{RMS}$ ---- ②

$$\frac{1}{Z} = \frac{1}{-J100} + \frac{1}{100} + \frac{1}{50+J30} \Rightarrow Z = 50\angle 0^\circ\ \Omega \text{ ---- ③}$$

$\therefore$ by subtituting ② and ③ in to ① $\Rightarrow$

$I_{aA} = 4.62 \angle -30^\circ$

iii) Find P supplied to Load

$P = 3VI\cos\theta = \sqrt{3}\,VI\cos\theta = 3I^2_{Ph}R$

Here $\theta = 0$

$P = 3 * (4.62)^2 * 50 = ~~3201~~\ 3201.7\ W$

$$PF = \frac{\text{Average}}{\text{Apparent}}$$

ex: A balanced 3-ph, 3-wire system 2-Δ Loads connected in parallel

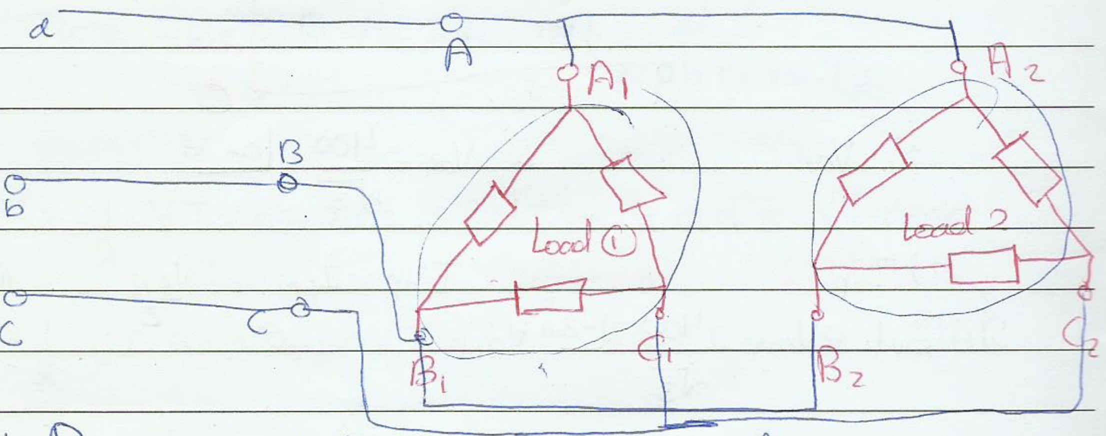
Load 1: 40 kVA, at 0.81 Lagging PF

Load 2: 24 KW, at 0.9 Leading PF

assume +ve phase sequence with Perfect lines

If $V_{ab} = 440\angle 30^\circ$ V_{RMS}, Find :

Solution:



i) Find total P supplied by source or to Loads

$$P = P_1 + P_2$$

$$40 * 0.8\ KW + 24 = 32 + 24 = 56\ KW$$

ii) Evaluate $I_{A_1B_1}$ and $I_{A_2B_2}$

$$I_{A_1B_1} = I_{ph}\angle I_{A_1B_1}$$

$$I_{A_2B_2} = I_{ph_2}\angle I_{A_2B_2}$$

$$40 * 10^3 = 3V_{ph_1} I_{ph_1} = 3 * 440 * I_{ph_1}$$

$$\therefore I_{ph_1} = 40 * 10^3 / 3 * 440 = \boxed{30.3}\ A_{RMS}$$

$\angle I_{A_1B_1} = \angle V_{A_1B_1} - \cos^{-1} 0.8 = 30 - 36.87 = -6.87$

$\therefore I_{A_1B_1} = 30.3 \angle -6.87$

$24 * 10^3 = 3 * V_{ph2} * I_{ph2} \cos\theta$

$= 3 * (440) * I_{ph2} * \underline{0.9}$

$\therefore I_{ph2} = 24 \times 10^3 / 3 * 440 * 0.9$

$= 20.2 \, A_{RMS}$

$\angle I_{A_2B_2} = \angle V_{A_2B_2} + \cos^{-1} 0.9 = 30^\circ + 25.8^\circ = \boxed{55.8^\circ}$

$\therefore I_{A_2B_2} = 20.2 \angle 55.8^\circ$ $\qquad I_{CC} \Rightarrow I_{CA}$

iii) Find $I_{aA} = \underbrace{I_{AA_1}}_{\text{line}} + I_{AA_2} \rightarrow$

$\sqrt{3} * 30.3 \angle -6.87 - 30 + \sqrt{3} * 202 \angle 55.8 - 30^\circ$

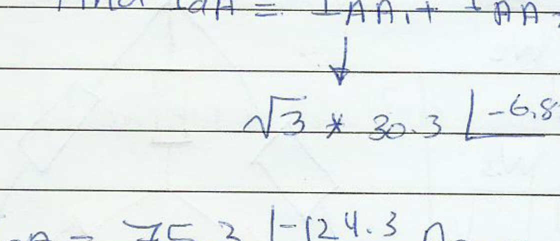
$I_{aA} = 75.3 \angle -124.3 \, A_{rms}$

Power Measurement: This is measured by Wattmeter

It consist of 2 coils:

i) Coil connected in series with Load called current coil. (CC)

ii) " " across the (Parallel) " " potential coil. (Pc)



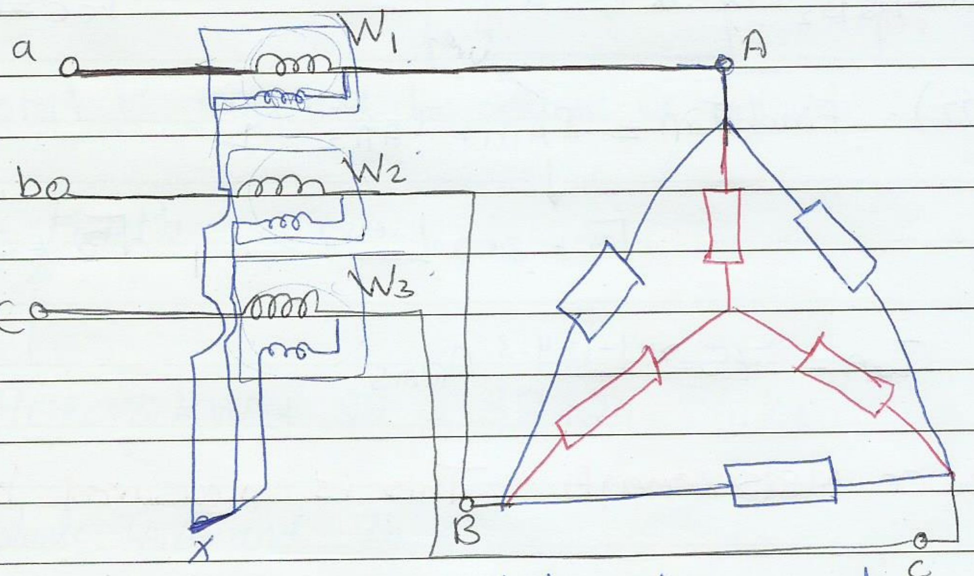
$$\text{Wattmeter Reading} = |I_{cc}| * |V_{pc}| * \cos(\angle V_{pc} - \angle I_{cc})$$

$$= |I_L| * |V_L^{\phi}| \cos(\angle V_L - \angle I_L)$$

$$= |I_L||V_L| \cos\theta \equiv P_{Load}$$

Measurment of 3-ph power:

(*) Sometimes the Neutral of the Y-Load doesnot exist, and also in the case of Δ-Load, the current coil cannot be connected in series with each phase. Hense the Wattmeters should be connected on the line terminals as follows:



(*) The potential coil is connected between each Line and a Reference point say x where x is any point in the 3-ph system

(*) Since the Wattmeter Measures the Average Power, P

$$W_1 = \frac{1}{T} \int_0^T i_{aA} V_{AX} \, dt$$

$$W_2 = \frac{1}{T}\int_0^T i_{bB}\, v_{Bx}\, dt$$

$$W_3 = \frac{1}{T}\int_0^T i_{cC}\, v_{Cx}\, dt$$

Summate the 3 Readings:

$$W = W_1 + W_2 + W_3$$

$$= \frac{1}{T}\int_0^T (i_{aA} v_{Ax} + i_{bB} v_{Bx} + i_{cC} v_{Cx})\, dt \quad \text{--- (1)}$$

By using Double Notations:

$$\left.\begin{aligned} v_{Ax} &= v_{AN} + v_{Nx} \\ v_{Bx} &= v_{BN} + v_{Nx} \\ v_{Cx} &= v_{CN} + v_{Nx} \end{aligned}\right\} (2)$$

Substitute (2) into (1) $\Rightarrow$

$$W = \frac{1}{T}\int_0^T (i_{aA} v_{AN} + i_{bB} v_{BN} + i_{cC} v_{CN})\, dt + \frac{1}{T}\int_0^T v_{Nx}(i_{aA} + i_{bB} + i_{cC})\, dt$$

Zero because Balanced currents

$$\therefore W = \frac{1}{T}\int (i_{aA} v_{AN} + i_{bB} v_{BN} + i_{cC} v_{CN})\, dt$$

= Average Power supplied to 3-ph Reading, $P_{3\phi}$

$\therefore$ Sum of Readings = $P_{3\phi}$

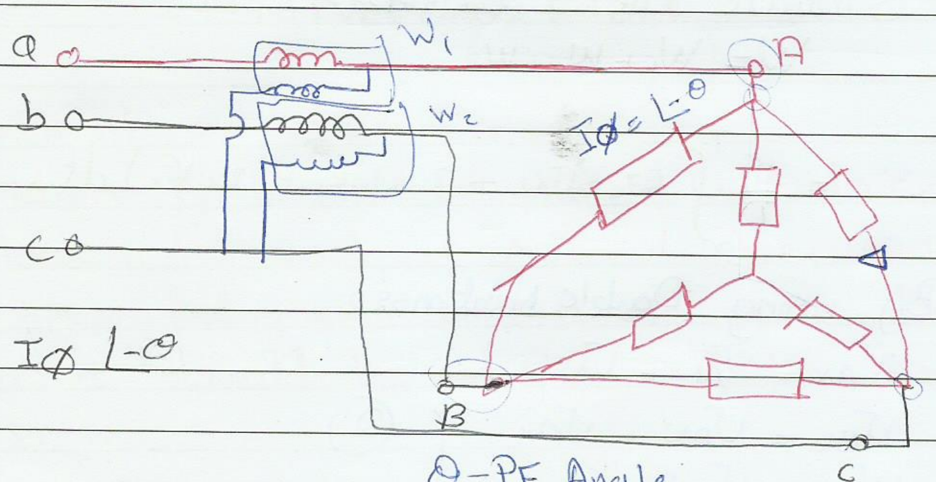
(X) If the Ref point x is taken to be one of the phases (i.e a or b or c), Then one of the Wattmeter Reading will be zero. Hense only 2 wattmeter are required. Hense the name:

Two - Wattmeter Methods:

(*) Let $V_{ab} = V\angle 0^\circ$ V_{RMS}

and assume +ve phase sequence.

(*) Let the ~~ref~~ Ref. Point phase c.



(*) Let $I_{AB} = I_\phi \angle -\theta$

θ = PF Angle

$W_1 = |I_{aA}||V_{AC}| \cos(\angle V_{AC} - \angle I_{aA})$ -- (1)

I_ϕ = phase current Magnitude

$W_2 = |I_{bB}||V_{BC}| \cos(\angle V_{BC} - \angle I_{bB})$ -- (2)

$I_{aA} = \sqrt{3}\, I_\phi \angle -\theta - 30^\circ$ -- (3)   Since $I_{AB} = I_\phi \angle -\theta$

But $V_{CA} = V \angle -240^\circ$

$V_{AC} = -V_{CA} = V \angle -60$ -- (4)

$\therefore I_{bB} = \sqrt{3}\, I_\phi \angle -\theta - 30 - 120 = -\theta - 150$ -- (5)

$V_{BC} = V \angle -120^\circ$ -- (6)

Substitude (3) and (4) into (1)

" (6) " (5) " (2) $\Rightarrow$

I (Line current) $\therefore I = \sqrt{3}\, I_\phi$

$W_1 = \sqrt{3}\, I_\phi * V * \cos(-60 - (-\theta - 30)) = \sqrt{3}\, I_\phi V \cos(\theta - 30)$ -- (7)

$W_3 = \sqrt{3}\, I_\phi * V * \cos(-120 - (-\theta - 130^\circ)) = \sqrt{3}\, I_\phi V \cos(\theta + 30)$ -- (8)

I

$\therefore W_1 + W_2 = VI\left[\cos(\theta-30) + \cos(\theta+30)\right]$

$= VI\left[2\cos\theta \cdot \cos 30 + 0\right]$

$= 2VI\cos\theta \cdot \cos 30$

$$\boxed{W_1 + W_2 = \sqrt{3}\, VI\cos\theta = P}$$

Comments and conclusions

i) $W_1 - W_2 = VI\left[\cos(\theta-30) - \cos(\theta+30)\right]$

$W_1 - W_2 = 2VI\sin\theta \cdot \sin 30 = VI\sin\theta$

$\therefore \sqrt{3}(W_1 - W_2) = \sqrt{3}\, VI\sin\theta = Q$ Reactive Power

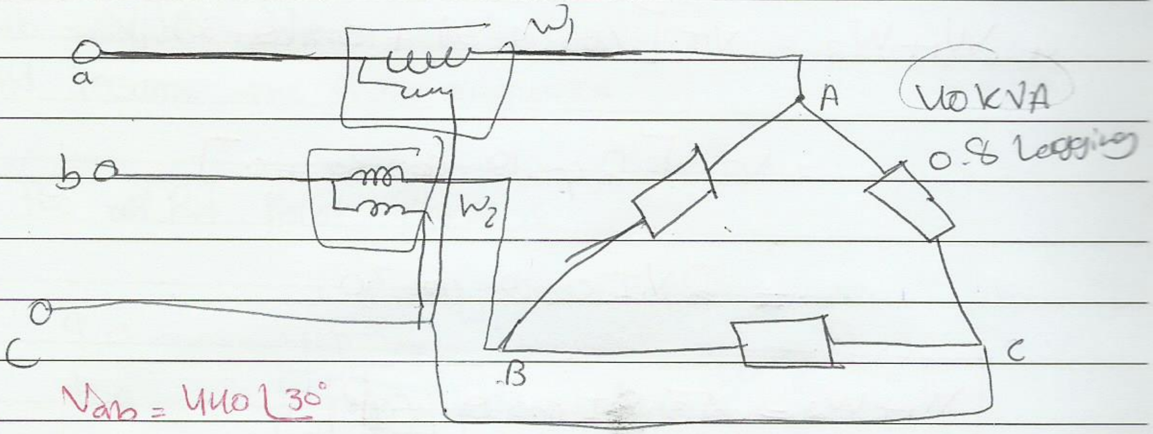
$\therefore$ Difference between Readings $\propto$ Reactive Power

ii) $\dfrac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)} = \dfrac{\sqrt{3}\, VI\sin\theta}{\sqrt{3}\, VI\cos\theta}$

$= \tan\theta$

$\therefore \theta$, pf Angle $= \tan^{-1}\left[\dfrac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}\right]$

pf $= \tan\theta$



Given $V_{ab} = 440\angle 30^\circ$

$I_{aA} = \sqrt{3} \times 30.3 \angle -36.87$

$I_{bB} = \sqrt{3} \times 30.3 \angle -36.87 - 120$

$V_{bc} = 440 \angle 30 - 120 = -90$

$W_1 = |I_{aA}||V_{ac}| \cos(\angle V_{ac} - \angle I_{aA})$

$W_2 = |I_{bB}||V_{bc}| \cos(V_{bc} - \angle I_{bB})$

$V_{ca} = 440 \angle -210$

$V_{ac} = -V_{ca} = 440 \angle -30^\circ$

By substituting

$W_1 = 23092.7$ W

$W_2 = 9070.9$ W

$W_1 + W_2 = 32.2$ kW

Magnatic couple circuit 4/11 (We can :))

<u>Definition</u> :-

These are electrical circuits which are electrically isolated but coupled (or connected) by means of magnetic field.

Principle of operation:

any current i → Generates Magnetic flux ϕ → $\phi \propto i$. If i is dc the ϕ will be dc, if i is ac then ϕ is ac → if ϕ cut a conductor or a coil, then a voltage e will be induced in this conductor or coil, where by Faraday's Law

$$e \propto \frac{d\phi}{dt}$$

$$\therefore e \propto \frac{di}{dt}$$

Induced Voltage e:

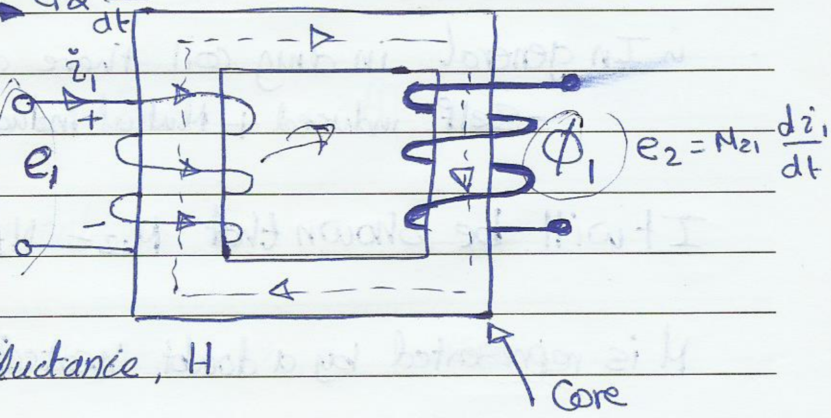
Consider the following case:

$i_1 \rightarrow \phi_1 \rightarrow e_1 \rightarrow e_1 \propto \frac{d\phi_1}{dt} \rightarrow e_1 \propto \frac{di_1}{dt}$

$$\therefore e_1 \propto \frac{di_1}{dt}$$

$$\therefore e_1 = L_1 \frac{di_1}{dt}$$

: $L_1 \equiv$ constant of proportionality called self Inductance, H



Polarity of e_1 is defined by Passive sign convention.

(*) Consider a 2nd coil, installed on the same ~~coil~~ Core.
Hence due to ϕ_1 a voltage e_2 will be induced in this 2nd coil.

$e_2 = M_{21} \frac{di_1}{dt}$

$$\therefore e_2 \propto \frac{d\phi_1}{dt} \propto \frac{di_1}{dt}$$

$$\therefore e_2 = M_{21} \frac{di_1}{dt}$$

M_{21} ≡ constant of Proportionality called Mutual Inductance

It's unit is Henry

voltage in the 2nd coil due to current in the first coil

The Polarity of the Mutually Induced voltage, e_2 can be found by Means of Dot convention which will be explained later on.

(*) If now a current i_2 flow in the 2nd coil → it will generate flux ϕ_2 → ϕ_2 will :-

1) induce a self voltage in the 2nd coil $= L_2 \frac{di_2}{dt}$ and

2) induce a mutual voltage in the 1st coil $= M_{12} \frac{di_2}{dt}$

∴ In general in any coil there are 2 induced voltages :-
Self induced + Mutual induced.

It will be shown that $M_{12} = M_{21} = M$

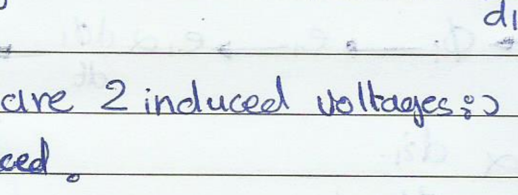
M is represented by a doubt headed arrow

Dot conventions :-

1) A dot is located at one of the terminals of each coil

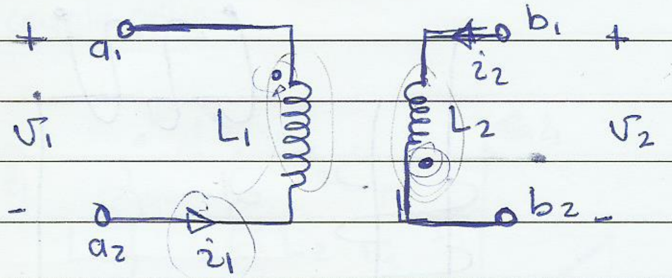
2) if the current enters a dotted terminal it will induce a voltage in the 2nd coil whose +ve Polarity is at its Dotted terminal.

" " " " " undotted " " " " " -ve " " undotted terminal



$v_1, v_2 \equiv$ Terminal Voltages

Apply KVL :-



1st Loop :-

$$-v_1 - L_1\frac{di_1}{dt} - M_{12}\frac{di_2}{dt} = 0 \quad \text{--- (1)}$$

2nd Loop :- $$-v_2 + L_2\frac{di_2}{dt} + M_{21}\frac{di_1}{dt} = 0 \quad \text{--- (2)}$$

(1), (2) are written in time domain.

Frequency domain is also applicate to $M \Rightarrow j\omega M$

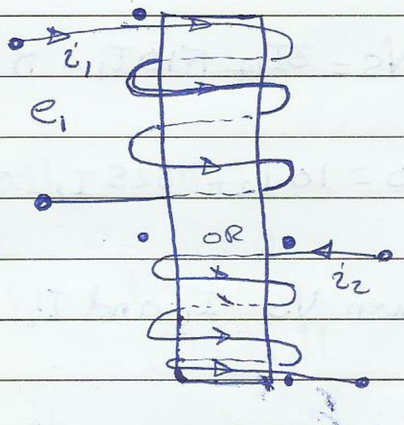
(1) $\rightarrow$ $$-V_1 - \boxed{j\omega L_1 I_1} - j\omega M I_2 = 0 \quad \text{--- (3)}$$

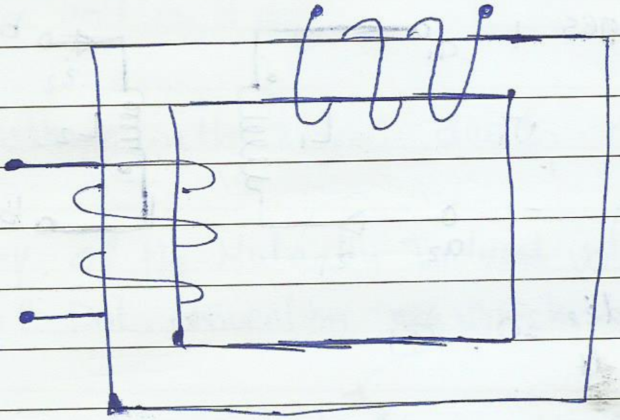
(2) $\Rightarrow$ $$-V_2 + \boxed{j\omega L_2 I_2} + j\omega M I_1 = 0 \quad \text{--- (4)}$$

Physical Meaning of Dot convention :-

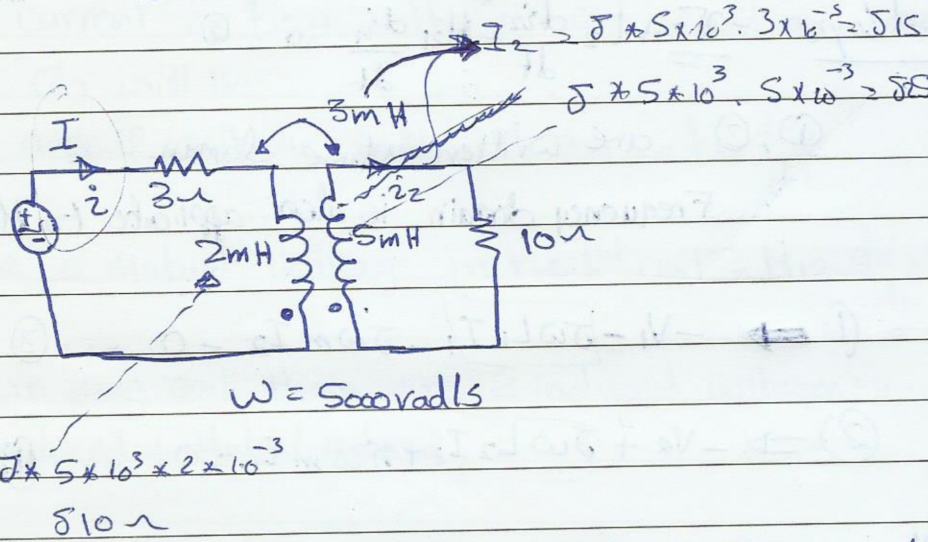
إذا كان i_1 ، i_2 نفس الاتجاه فإن ϕ يزيد

Hence : Dots are located at the ~~term~~ terminals at which entering currents produce Aiding flux (flux in the same direction)





G/11 :-



eg for the given cct. write the mesh equation in Frequency domain

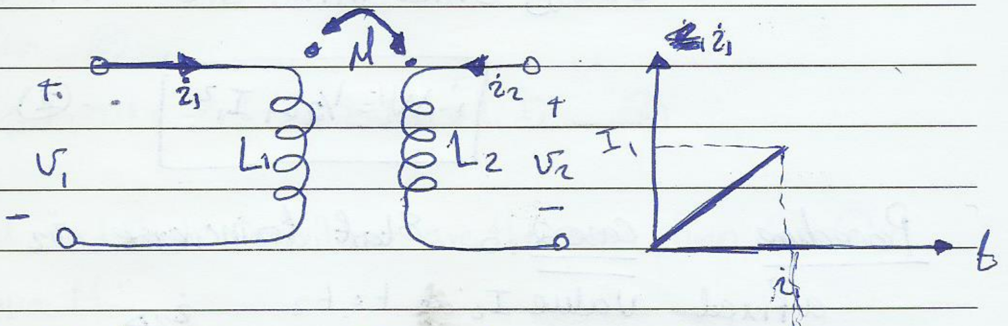
By KVL :-

$$V_S = 3I_1 + J10I_1 - J15I_2 \quad \text{--- (1)}$$

$$0 = 10I_2 + J25I_2 - J15I_1 \quad \text{--- (2)}$$

For a given Vs I_1 and I_2 can be evaluated.

* Energy stored in Mutually couple coils:



Objectives: i) To find expression For such energy. Because it has many pratical Applications

ii) To show that $M_{12} = M_{21} = M$

iii) To Find an upper Limit For M.

Procedure: case 1:

coil 2 is O/C. (i.e $i_2 = 0$)

Start to increase i_1 from zero to a certain value I_1 at $t = t_1$, and then keep it constant at this value.

Find the energy stored in this case

$$\therefore v_1 = L_1 \frac{di_1}{dt} + 0$$

$$v_2 = 0 + M \frac{di_1}{dt}$$

$\therefore$ Power supplied to coil $1 = v_1 i_1$

" " " " $2 = v_2 * 0$

$\therefore$ Energy stored in coil $1 = W_1' = \int_0^t v_1 i_1 \, dt$

$$\therefore W_1' = \int_0^t L_1 \frac{di_1}{dt} \cdot i_1 \cdot dt$$

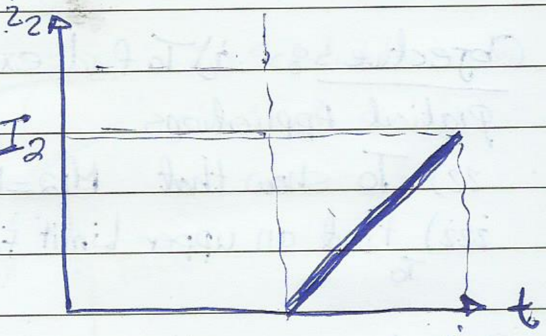
$$= \int_0^{I_1} L_1 i_1 \, di_1 = \boxed{\tfrac{1}{2} L_1 I_1^2} \text{ — (1)}$$

Energy stored in coil 2 = 0 ← W_2'

$\therefore$ Energy stored in this case = $W_1' + W_2'$

$$\boxed{\therefore W = \tfrac{1}{2} L_1 I_1^2} \quad \text{--- (2)}$$

Procedure Case 2 :- Start to increase i_2 from 0 at $t = t_1$ to a fixed value I_2 at $t = t_2$



$$v_1 = 0 + M \frac{di_2}{dt} = M_{12} \frac{di_2}{dt}$$

$$v_2 = L_2 \frac{di_2}{dt} + 0 = L_2 \frac{di_2}{dt}$$

Energy stored in 1st coil, $W_1 = \int_{t_1}^{t_2} v_1 i_1 \, dt = \int_{t_1}^{t_2} M \frac{di_2}{dt} \cdot I_1 \, dt = \int_0^{I_2} M_{12} I_1 \, di_2$

$$\boxed{W_1'' = M_{12} I_1 I_2} \quad \text{--- (3)}$$

$\therefore$ Energy in 2nd coil $W_2'' = \int_{t_1}^{t_2} v_2 i_2 \, dt$

$$\therefore W_2'' = \int_{t_1}^{t_2} L_2 \frac{di_2}{dt} \cdot i_2 \, dt = \int_0^{I_2} L_2 i_2 \, di_2$$

$$\boxed{W_2'' = \tfrac{1}{2} L_2 I_2^2} \quad \text{--- (4)}$$

$\therefore$ Total Energy ~~stored~~ stored, W = (2) + (3) + (4)

$$\boxed{W = \tfrac{1}{2} L_1 I_1^2 + \tfrac{1}{2} L_2 I_2^2 + M_{12} I_1 I_2} \quad \text{--- (5)}$$

$\cos^2\theta = 1+2\cos\theta$

Moneer

If the Process of increasing current is reversed, that then :-

Prove equation (6):

$$W = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 + M_{21} I_1 I_2 \quad \ldots (6)$$

Since the initial and Final Condition are the same, then (5) = (6)

$\therefore M_{12} = M_{21} = M$

$$\therefore \boxed{W = \frac{1}{2} I_1^2 L_1 + \frac{1}{2} L_2 I_2^2 + M I_1 I_2} \quad \ldots (7)$$

Comments and conclusions:

1) if one of the current direction is reversal, then Replace M by (-M) in (7)

ii) since t_1 and t_2 can be any values in the time domain, then W can be written by :-

$$W(t) = \frac{1}{2} L_1 i_1^2(t) + \frac{1}{2} L_2 i_2^2(t) \pm M i_1(t) i_2(t) \quad \ldots 8$$

Upper Limit of M :-

Since W can not be -ve, because one is dealing with passive elements then :-

1) if i_1 and i_2 are both +ve or -ve the Form (8), the possibility of -ve W comes from $(-M i_1 i_2)$

$$\therefore W(t) = \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 - M i_1 i_2 \quad \ldots (9)$$

(9) can be Rewritten as :-

$$W = \frac{1}{2}(\sqrt{L_1}\, i_1 - \sqrt{L_2}\, i_2)^2 + \sqrt{L_1 L_2}\, i_1 i_2 - M i_1 i_2 \quad (10)$$

∴ From (10) for +ve W ⟹ $\sqrt{L_1 L_2}\, i_1 i_2 \geq M i_1 i_2$

$$\therefore M \leq \sqrt{L_1 L_2}$$

∴ upper of Maximum value of $M = M_{max} = \sqrt{L_1 L_2}$

If i_1 and i_2 ~~have~~ have different signs

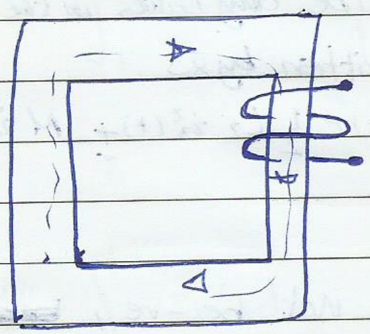
Then one can still have the same result $M \leq \sqrt{L_1 L_2}$

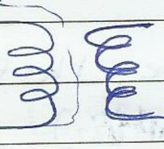
<u>Coupling Factor, k</u>

$$k \triangleq \frac{M}{M_{max}} = \frac{M}{\sqrt{L_1 L_2}}$$

$$0 \leq k \leq 1 \rightarrow \text{Tightly coupled coils}$$

loosely coupled coils



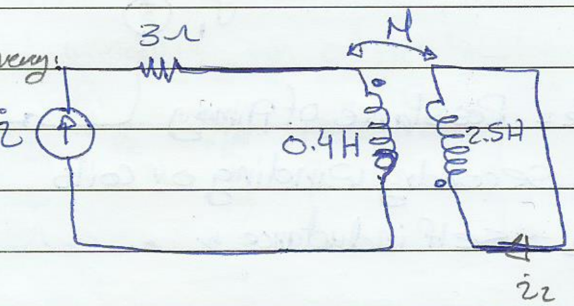


-A, A, A, B, B, A

11/11

example:

e.g For the given cct. find the energy stored at $t=0$, given



(احنا اخترنا الإتجاه)

$i = 2\cos 10t$ A

$K = 0.6$

Sol:

$\therefore w(t) = \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 + M i_1 i_2$

$L_1 = 0.4$, $L_2 = 2.5H$ | $\omega = 10$

$M = K\sqrt{L_1 L_2} = 0.6 * \sqrt{0.4 * 2.5} = 0.6$

$i_1 = 2\cos 10t$ $\therefore i_1(0) = 2$

To find i_2, Apply KVL to 2nd Mesh

$j\omega L_2 I_2 + j\omega M I_1 = 0$

$\therefore L_2 I_2 = -M I_1$

$\therefore I_2 = \frac{-M}{L_2} I_1 = \frac{0.6}{2.5} * 2\angle 0 = \frac{-1.2}{2.5}\angle 0 = -0.48$

$= 0.48 \angle 180$

$\therefore i_2 = 0.48\cos(10t + 180°)$

$i_2(0) = 0.48\cos(180) = -0.48$

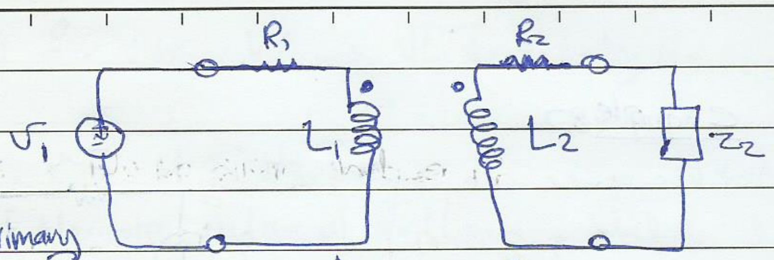
By substitude $\Rightarrow W(0) = 0.512J$

Practical Application of Mutual ~~com~~ coupling

Transformer

Construction

⇩



R_1, R_2 = Resistance of primary and secondry winding or coils

L_1, L_2 = self inductance " " " Primary winding (with the source)

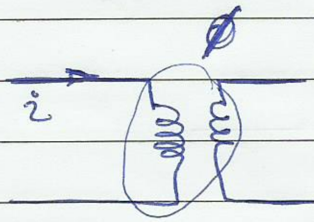
second winding (with the Load)

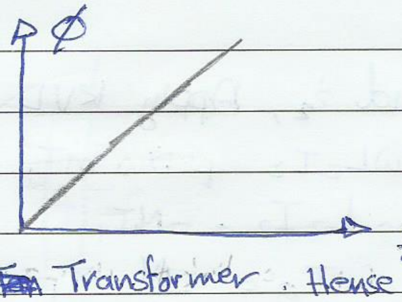
Z_L = Load Impedance

$v_1 \equiv$ Applied voltage to primary

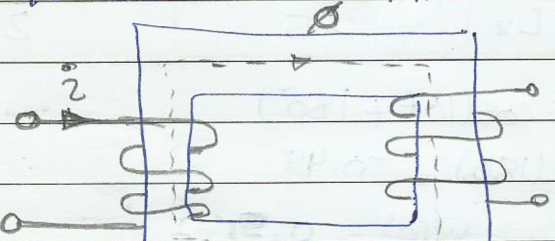
$i_1, i_2 \equiv$ Primary and secondary currents.

* Air core Transformer :)

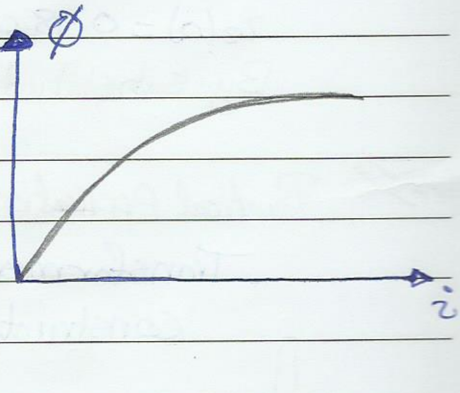




Hense it is called Linear ~~Tra~~ Transformer. Hense K is very small

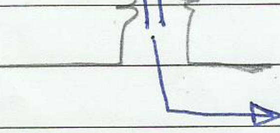


iron-core Transformer



K ~~hence~~ here is very large, Idealy = 1

Hense the Name Ideal Transformer



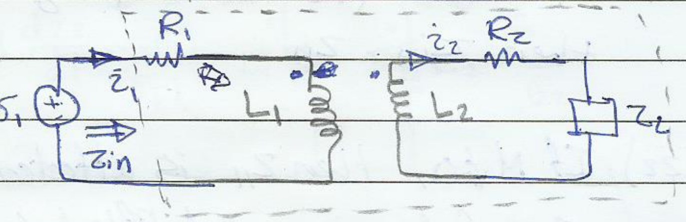
To represent Iron coil

[Medom + cct 2 + Ketabeh fanye]

Linear Transformer :-

objective :- To evaluate input impedance Z_{in} of this transformer.

Procedure :-



Is to find $Z_{in} = \frac{V_1}{I_1}$

Apply KVL to Primary cct.

$$V_1 = I_1 R_1 + j\omega L_1 I_1 - j\omega m I_2$$
$$\therefore V_1 = I_1 (R_1 + j\omega L_1) - j\omega m I_2$$
$$\therefore V_1 = I_1 Z_{11} - j\omega m I_2 \quad \ldots (1)$$

where $Z_{11} = R_1 + j\omega L_1$ and called Primary cct. Impedence

second Mesh :-

$$I_2 R_2 + I_2 Z_L + j\omega L_2 I_2 - j\omega m I_1 = 0$$
$$I_2 (R_2 + Z_L + j\omega L_2) - j\omega m I_1 = 0$$
$$I_2 Z_{22} - j\omega m I_1 = 0 \quad \ldots (2)$$

$Z_{22} = R_2 + j\omega L_2 + Z_L$, called secondary cct Impedence.

(2) $\quad I_2 = \frac{j\omega m I_1}{Z_{22}} \quad \ldots (3)$

substitude (3) into (1)

$$\therefore V_1 = I_1 Z_{11} - j\omega m \left(\frac{j\omega m I_2}{Z_{22}} \right) = I_1 \left(Z_{11} + \frac{\omega^2 m^2}{Z_{22}} \right)$$

$$\boxed{\therefore \frac{V_1}{I_1} = Z_{in} = Z_{11} + \frac{\omega^2 m^2}{Z_{22}}} \quad \ldots (4)$$

Comments and Conclusions:

i) Zin is Independent of Dot location.

ii) If $M=0$ (i.e No coupling between primary and secondary), then $Z_{in} = Z_{11}$

iii) If $M \neq 0$, then Z_{11} is affected by the factor $\frac{\omega^2 M^2}{Z_{22}}$

This factor is called Reflected Impedence

iv) if $Z_{22} = R_{22} + jX_{22}$

$$\therefore Z_{in} = Z_{11} + \frac{\omega^2 m^2}{R_{22} + jX_{22}}$$

$$\therefore Z_{in} = Z_{11} + \frac{\omega^2 m^2 (R_{22} - jX_{22})}{R_{22}^2 + X_{22}^2}$$

$$Z_{in} = Z_{11} + \frac{\omega^2 M^2 R_{22}}{R_{22}^2 + X_{22}^2} - \frac{j\omega^2 m^2 X_{22}}{R_{22}^2 + X_{22}^2}$$

Resistance from losses Point of view

$\therefore jX_{22} \longrightarrow -jX_{22}$

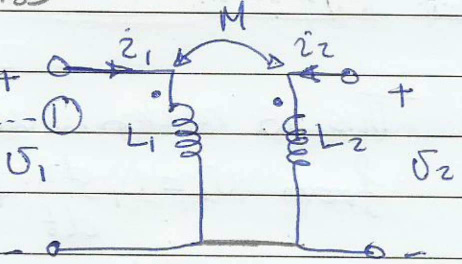
$\therefore$ Linear Transformer changes the type of load

Inductance $\rightleftarrows$ capacitive

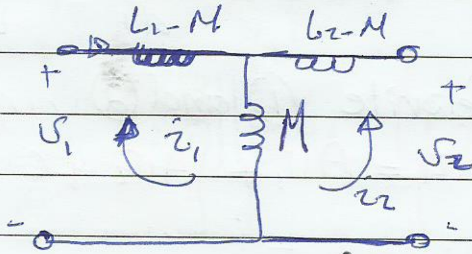
Equivalent cct of Linear Transformer

By KVL $\Rightarrow$ $v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$ ---(1)

$v_2 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$ --(2)



equivalent



figure

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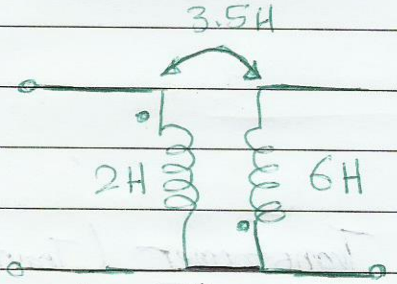
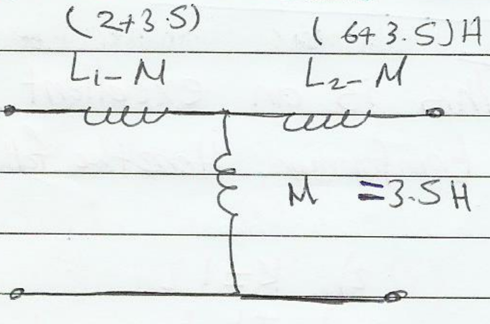
(*) If one dot Location is changed, then Replace M by -M

(*) All the elements in figure are self inductance

(*) Linear transformer can be used to synthesize or materlize Negative inductance.

ex find the equivalent T-Network





* fill in the blank Q in the exam

π - equivalent Network

This is rarely is used

from $v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$ ... (1)

$v_2 = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$ ... (2)

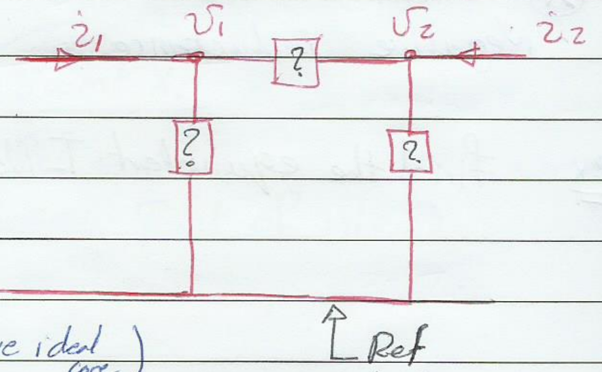
Rewrite (1) and (2), in the following forms:

$i_1 = A v_1 + A v_2 + C$ ... (3)

$i_2 = D v_2 + E v_2 + F$ ... (4)

Deduce the equivalent ckt from (3) ~~and~~ (4)

سؤال بالامتحان



π - equivalent

Ideal Transformer (Transformer use ideal core)

This is an excellent approximation for a well coupled Iron core tranformer. Here the following are assumed.

i) $K = 1$.

ii) It has no losses. Hence $R_1 = R_2 = 0$

iii) Inductive Reactance of primary and ~~sec~~ secondary windings are $\gg$ Load impedence (Z_L)

Objectives:

To evaluate:

i) Z_{in}

ii) Voltage and current relationships.

~~Procedur~~

Procedure:

The circuit is as shown in fig. 1

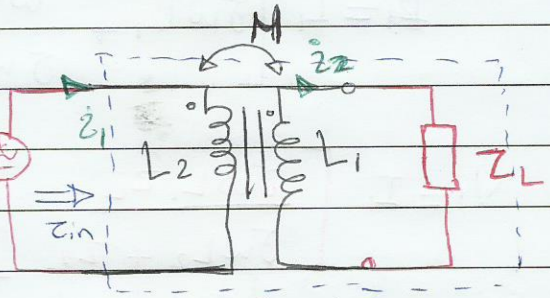


Fig. 1

* $N_1, N_2 \equiv$ Number of turns of Primary and secondary windings

$N_1 : N_2$

$1 : \frac{N_2}{N_1}$

$1 : a$

* $a = \frac{N_2}{N_1}$ is called Turns Ratio

* $L \propto N^2$

$\therefore \frac{N_2^2}{N_1^2} = \frac{L_2}{L_1} = a^2$

Since $k = \frac{M}{\sqrt{L_1 L_2}}$

$\therefore M = \sqrt{L_1 L_2}$

Apply KVL $\Rightarrow$ (Fig. 1)

Primary cct: $V_1 = j\omega L_1 I_1 - j\omega M I_2$ --- (1)

secondary cct: $j\omega L_2 I_2 + I_2 Z_L - j\omega M I_1 = 0$ --- (2)

To find Z_{in}, find Ratio of $\frac{V_1}{I_1}$

(2) $\Rightarrow I_2 = \dfrac{jwm I_1}{jwL_2 + Z_L}$ ---- (3)

substitude (3) in (1) $\Rightarrow$

$$V_1 = jwL_1 I_1 - jw\overset{M}{\cancel{m}}\left(\frac{jwm I_1}{jwL_2 + Z_L}\right) \qquad = L_1 L_2$$

$$\cancel{V_1} = I_1\left[jwL_1 + \frac{w^2 m^2}{jwL_2 + Z_L}\right] = \left[\frac{-w^2 L_1 L_2 + jwL_1 Z_L + w^2 m^2}{jwL_2 + Z_L}\right] I_1$$

so $V_1 = I_1 \left[\dfrac{jwL_1 Z_L}{jwL_2 + Z_L}\right]$

$$Z_{in} = \frac{V_1}{I_1} = \frac{jwL_1 Z_L}{jwL_2 + Z_L} = \frac{jwL_1 Z_L}{jwL_2} \qquad \text{since } wL_2 >> Z_L$$

Reflected impedence

$$\boxed{\therefore Z_{in} = \frac{L_1}{L_2} Z_L = \frac{N_1^2}{N_2^2} Z_L = \frac{Z_L}{a^2}} \quad ---- (4)$$

(it can change the ~~mag~~ magnitude of the impedence)

$\therefore$ ~~Fo~~ From (4) so

[1] I. transformer change the Magnitude of Z_L

(*) Note : Maximum Power of ~~spea~~ Amplifier (speaker

$\therefore$ This concept can be used in maximum power transfer. see the eg in textbook.

(2) To Reflect secondary impedence Z_L to primary cct. $\div$ by a^2

OR

" " primary impedence Z_{in} to secondary " $*$ by a^2

V_1

$a = 10$

(*) Current Relationship

(3) $\Rightarrow$

$$I_2 = \frac{j\omega M I_1}{j\omega M L_2} \quad \text{since } \omega L_2 \gg Z_L$$

$$\therefore \frac{I_2}{I_1} = \frac{M}{L_2} = \frac{\sqrt{L_1 L_2}}{L_2} = \sqrt{\frac{L_1}{L_2}} = \frac{N_1}{N_2} = \frac{1}{a}$$

Voltage Relationship:

Since $Z_{in} = \dfrac{Z_L}{a^2}$

$$\frac{V_1}{I_1} = \frac{1}{a^2} \cdot \frac{V_2}{I_2}$$

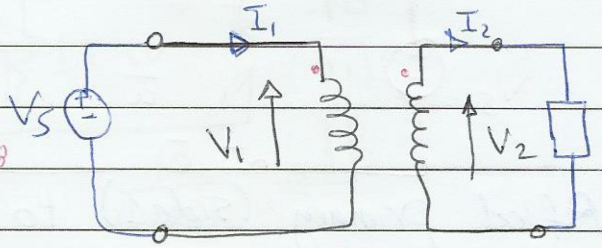
$$\therefore \frac{V_1}{V_2} = \frac{1}{a^2} \frac{I_1}{I_2} = \frac{1}{a^2} \times a = \frac{1}{a} = \frac{N_1}{N_2}$$

$$\therefore \frac{V_1}{V_2} = \boxed{\frac{I_2}{I_1}} = \frac{1}{a}$$

$\therefore V_1 I_1 = V_2 I_2$

$\therefore$ Both windings have the same (Apparent Power rating)

Conclusion:



It was found that:

$$\frac{V_2}{V_1} = \frac{I_1}{I_2} = \frac{N_2}{N_1} = a$$

$\therefore V_2 = a V_1$ ...... (1)

$I_2 = I_1 / a$ ...... (2)

$V_1 = V_2 / a$ ...... (3)

$I_1 = a I_2$ ...... (4)

$Z_{in} = Z_L / a^2$ ...... (5)

$Z_L = a^2 Z_{in}$ ...... (6)

$N_1 : N_2$

$1 : \dfrac{N_2}{N_1} = a$

side 1 ≡ Primary

side a = ~~sec~~ secondary

To Reflact Primary voltage to secondary multiply by a
″ ″ ″ current ″ ″ ÷ ″ a

OR

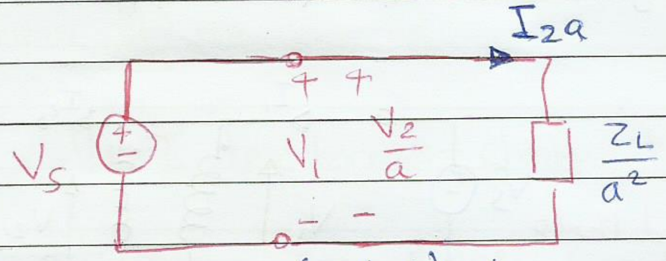
To Reflect secondary voltage to primary cct ÷ by a
″ ″ ″ current ″ ″ ″ × ″ ″

To Reflect primary impedence to secondary cct × by a^2
″ ″ secondary ″ ″ Primary ″ ÷ ″ ″

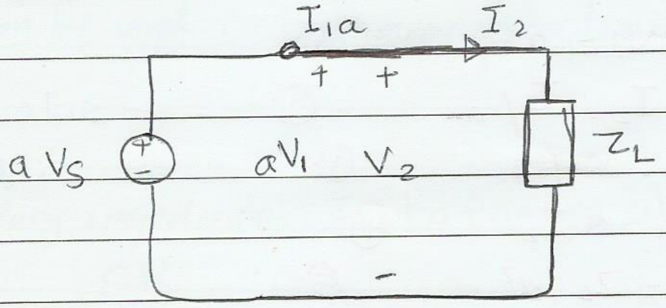
Application:

The concept of Reflection can be used to find the equivalent cct of a system which contain a transformes as follows:

Reflect secondary (side a) to primary (side 1)



Reflect primary (side 1) to secondary (side a)



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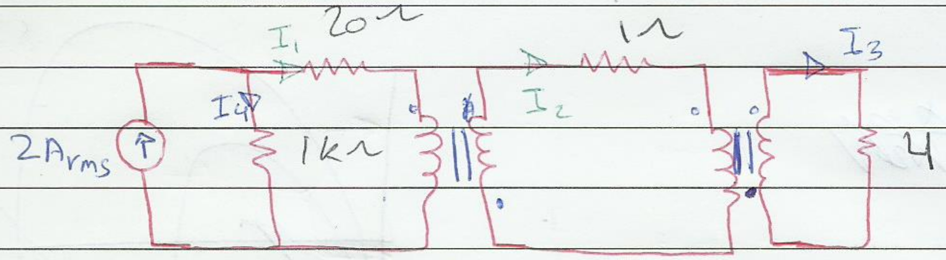
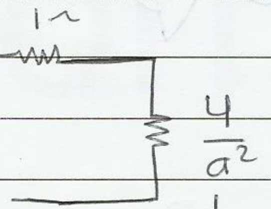


$1 : \frac{1}{5} = a_1 \qquad 1 : \frac{2}{3} = a_2$

(e.g) for a given cct evaluate the Average ~~form~~ Power Supplied to each resistance

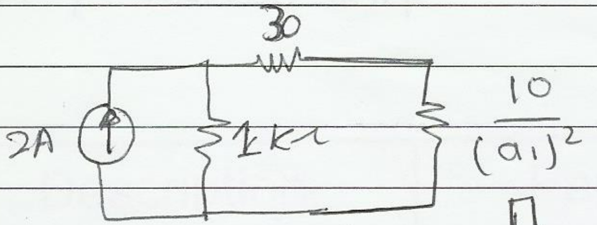
$a_1 = \frac{1}{5} \qquad a_2 = \frac{2}{3}$

Find the equivalent cct by Reflecting towards source

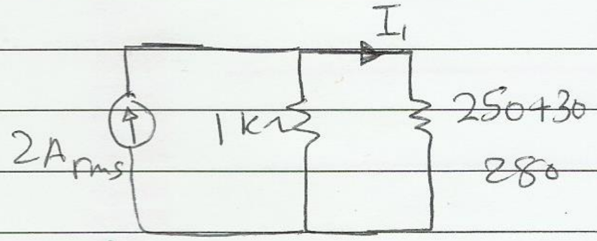


$\downarrow \frac{4}{(4/9)} = 9\Omega$

$= 9 + 1 = 10$



$\frac{10}{(1/25)} = 250\Omega$

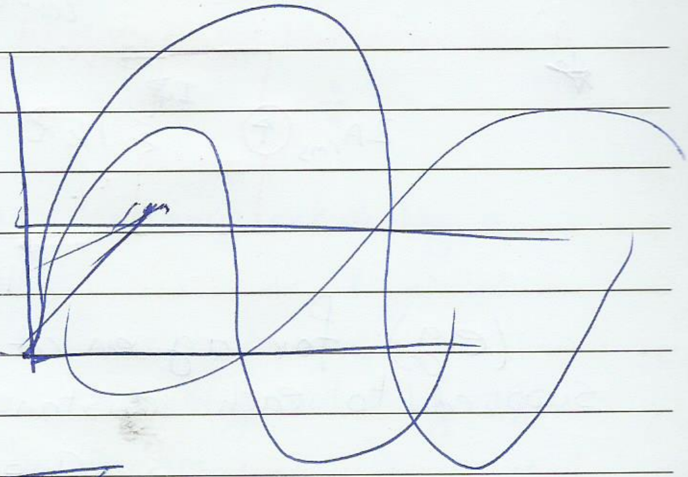


By current division:

$I_1 = 2 \times \frac{10^3}{10^3 + 20} = 1.56\,A$

$\therefore \frac{I_1}{-I_2} = \frac{1}{5} \qquad \therefore I_2 = -5\,I_1 = -5 \times 1.56 = -7.8$ Arms

$\frac{I_2}{I_3} = \frac{2}{3}$



حل السؤال

$\Rightarrow$ $I_4 = 2 - I_1 = 2 - 1.56 = 0.44$

sense $p = I^2 * R$

then evaluate P for each resistance.

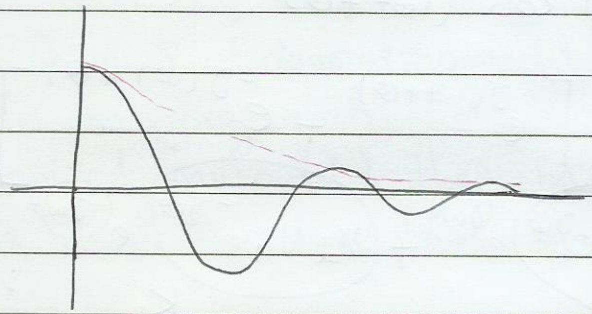
$-10 + j2i_1 + 20ji_1 - j\omega M I_2$

⋇ Complex frequency S :

→ S is introduced by using the concept of Damped Sinusoidal function and defind as follows:

$$v(t) = V_m e^{\sigma t} \cos(\omega t + \theta)$$

σ is a real number and usually it's +ve



V_m, ω and θ as defined before.

Consider the following cases:

1) let $\sigma = \omega = 0$

$\therefore v(t) = V_m \cos\theta$ = DC voltage)

2) let $\sigma = 0$

$\therefore v(t) = V_m \cos(\omega t + \theta)$ Sinusoidal)

3) let $\omega = 0$, $\therefore v(t) = V_m e^{\sigma t} \cos\theta$ = exponential

Q in exam: Define 3 in 1

$\therefore$ ① contain the 3 special cases of DC, sin, exponantial

Definition: A function f(t) is said to have a complex frequency S if it can be written as:

$f(t) = k e^{St}$ when k and S are constants Real or complex

find S of the previous special cases:

i) DC voltage

$\therefore v(t) = V_0$

$\therefore v(t) = V_0 e^{0t}$

$\therefore S = 0 \pm j0$

ii) Sinusoidal voltage

$\therefore v(t) = V_m \cos(\omega t + \theta)$

$$\therefore v(t) = V_m \frac{1}{2}\left[e^{j(\omega t+\theta)} + e^{-j(\omega t+\theta)}\right]$$

$$v(t) = \underbrace{\tfrac{1}{2} V_m e^{j\theta}}_{K_2} e^{j\omega t} + \underbrace{\tfrac{1}{2} V_m e^{-j\theta}}_{K_1} e^{-j\omega t}$$

$$\therefore v(t) = k_1 e^{S_1 t} + k_2 e^{S_2 t}$$

$$S_1 = S_2^* = j\omega$$

$S = 0 \pm j\omega$

Real $= 0$

iii) Exponential, $v(t) = V_m e^{\sigma t}$ or $A e^{\sigma t}$

$S = \sigma$

or $S = \sigma \pm j0$

20/11 :-

iv) Damped sinusoidal :-

$$v(t) = V_m e^{\sigma t} \cos(\omega t + \theta)$$

$$= V_m e^{\sigma t}\ \tfrac{1}{2}\left[e^{j(\omega t+\theta)} \cdot e^{-j(\omega t+\theta)}\right]$$

$$= \underbrace{\tfrac{1}{2} V_m e^{j\theta}}_{K_1} e^{(\sigma + j\omega)t} + \underbrace{\tfrac{1}{2} V_m e^{-j\theta}}_{K_2} e^{(\sigma - j\omega)t}$$

$$S_1, S_2 = \sigma \pm j\omega$$

∴ The S of damped sinusoidal is the general case.

∴ In solving electrical ccts. one apply the general source (i.e Damped sinusoidal), then in the final Answer substitude the value of S according to the type of source.

e.g :- Find all complex frequencies in the following functions :-

$$f(t) = \left(2e^{-100t} + e^{-200t}\right) * \sin 2000t$$

$$= \underbrace{\frac{1}{j}}_{K_1} e^{\overbrace{(-100 + j2000)}^{S_1} t} - \underbrace{\frac{1}{j}}_{K_2} e^{\overbrace{(-100 - j2000)}^{S_2} t} + \underbrace{\frac{1}{2j}}_{K_3} e^{\overbrace{(-200 + j2000)}^{S_3} t}$$

$$- \underbrace{\frac{1}{2j}}_{K_4} e^{\overbrace{(-200 - j2000)}^{S_4} t}$$

$$S_1, S_2 = -100 \pm j2000$$

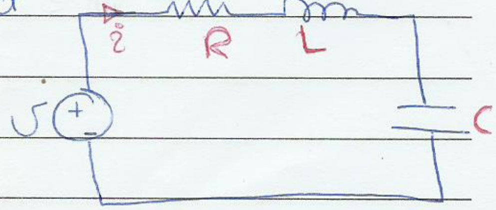
$$S_3, S_4 = -200 \pm j2000$$

Application of Damped sinusoidal function to electrical cts.

(*) Consider the following series RLC cct

If $v = V_m e^{\sigma t} \cos(\omega t + \theta)$

find i



Procedure, since the cct is linear, then

Let $i = I_m e^{\sigma t} \cos(\omega t + \phi)$ ... (2)

Transform (1) and (2) into Complex forcing function and Complex Response.

$\therefore V = Re\left[V_m e^{\sigma t} e^{j(\omega t + \theta)} \right] = Re\left[V_m e^{j\theta} e^{(\sigma + j\omega)t} \right]$

$= Re\left[V_m e^{j\theta} e^{st} \right]$ since $s = \sigma \pm j\omega$

$\therefore$ Corresponding Complex forcing function,

$v(t) = V_m e^{j\theta} e^{st}$ ... (3)

$i = Re\left[I_m e^{\sigma t} e^{j(\omega t + \theta)} \right] = Re\left[I_m e^{j\phi} e^{st} \right]$

Corresponding Complex response is

$i = I_m e^{j\phi} e^{st}$ ... (4)

By KVL:

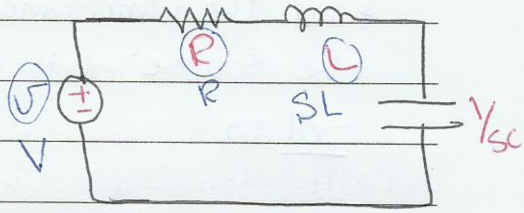
$v(t) = Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$ ... (5)

Substitude (3) and (4) into (5)

$V_m e^{j\theta} e^{st} = RI_m e^{j\phi} e^{st} + LI_m e^{j\phi} . s . e^{st} + \frac{1}{C} I_m e^{j\phi} . \frac{1}{s} e^{st}$ ... (6)

$\therefore V_m e^{j\theta} = I_m e^{j\phi} \left[R + SL + \frac{1}{SC} \right]$

$$\therefore I_m e^{j\phi} = \frac{V_m e^{j\theta}}{R + SL + 1/SC}$$



$$\therefore I_m \angle\phi = \frac{V_m \angle\theta}{R + SL + 1/SC} \quad —7$$

Comments and Conclusion 8)

i) It can be deduced that R, SL and $\frac{1}{SC}$ have the unit of Ω. There terms represent the general experssion for impedance of the Basic passive elements.

ii) Hence in solving cct. Transform it from time domain → S domain. Find the answer as a function of S.

Then Transform it to time domain

forced response ⟹ response by source

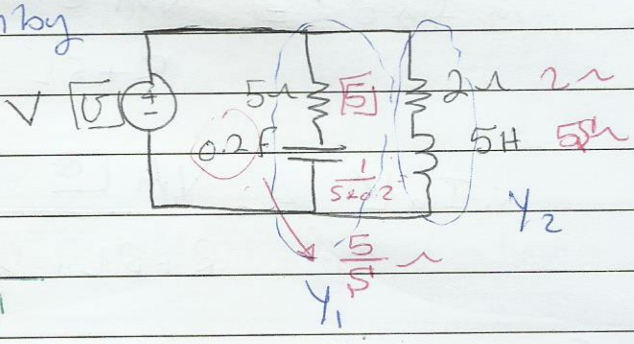
Transient response ≡ natural response = free source circuit

e.g. if $V = \underset{V_L}{12} \angle 35^\circ$ V and $S = (\underset{\sigma}{-20} + j\underset{\omega}{5})$ find $v(t)$

$\therefore V(t) = V_m e^{\sigma t} \cos(\omega t + \theta)$

Note All cct techniques, cct. laws and Impedance Combination (Series, Parallel, Δ-Y) are applicable to ccts in the S domain

Find the admittance $Y(s)$ seen by the source of the given ckt



Sol:

$$Y(s) = \frac{1}{5 + \frac{5}{s}} + \frac{1}{2 + 5s}$$

$$Y(s) = \frac{5s^2 + 7s + 5}{(5s+5)(2+5s)}$$

Transfer function:

This concept is used in solving electrical ckts, by finding Transient, forced and complex Response.

Also used in other subjects

The T.F is the Ratio of 2 functions of s and Represented by a Capital letter

e.g. E, F, G, H

For e.g $H(s) = \dfrac{F_1(s)}{F_2(s)}$

F_1 and F_2 Can be voltage or current

$\therefore H(s)$ can be:

Impedance, Z or admitance, Y

or (voltage, current) ratio

$F_1(s)$, $F_2(s)$ may be written as Polynomials

$\therefore$ In factorized forms:

$$H(s) = K \frac{(s+z_1)(s+z_2)\ldots(s+z_n)}{(s+P_1)(s+P_2)\ldots(s+P_n)}$$

Where $z_1, \ldots z_m$ and
$P_1, \ldots P_n$ constant

zeros: these are the value of S' which make $H(s) = 0$
Poles: 〃 〃 〃 〃 〃 〃 〃 〃 $= \infty$

$\therefore -z_1, -z_2 \ldots -z_m \equiv$ Zeros of $H(S)$
$-P_1, -P_2 \ldots -P_n =$ Poles 〃 〃

e.g. find the zeros and Poles of the T.F $Y(s) = \dfrac{I(S)}{V(S)}$ in the previous e.g *

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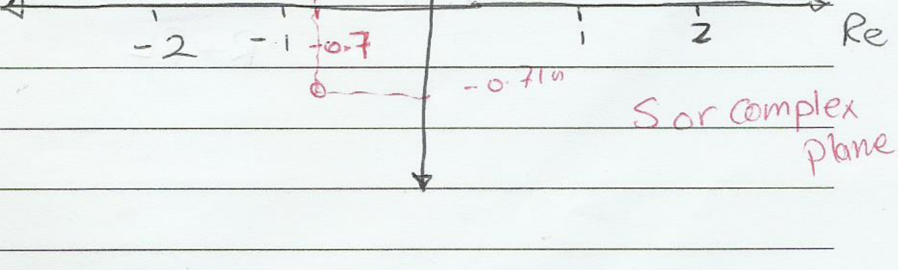
* Graphical Representation of Poles and zeros:

On the Complex plane:
zeros are represented by a small circle.
poles 〃 〃 〃 a cross 〃

e.g. In the previous e.g of $Y(S')$ it was found that:

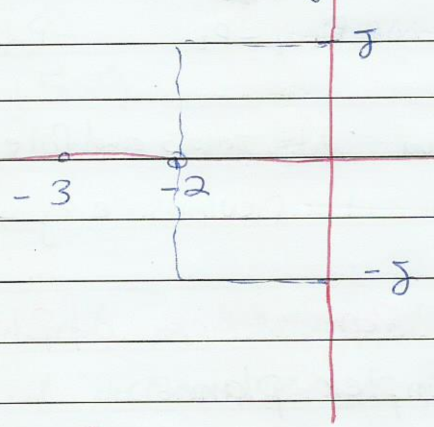
$z_1, z_2 = -0.7 \pm j\, 0.714$
$P_1 = -1$, $P_2 = -0.4$
solution: ⟹



* Hence given the pole zero diagram one can Find the corresponding T.F

(e.g) Evaluate $H(s)$ which have the following Pole - zero diagram

$$\therefore H(s) = k \frac{s(s+3)}{(s+2+j)(s+2-j)}$$



K can be evaluated If another information is given

Say, $H(1) = 4$

$$\therefore H(1) = 4 = k \frac{1 * (4)}{(3+j)(3-j)}$$

$$K = \frac{4}{10} \qquad \therefore k = 10$$

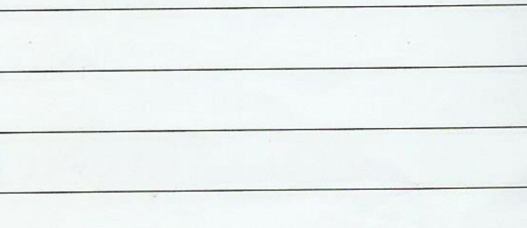
Complex Frequency Response:

↳ (voltage or current)

__Objective__ is to find the variation of Response (current or voltage) with s

Procedure: Consider the following RL cct.

Let the Response by $i(t)$ $\boxed{I(s)}$



(*) Transfer the cct to s domain

By KVL ⇒

$$V(s) = I(s)[R + SL]$$

$$\therefore I = \frac{V(s)}{R + SL}$$

(*) To sketch $|I|$ with s, consider the following cases of s

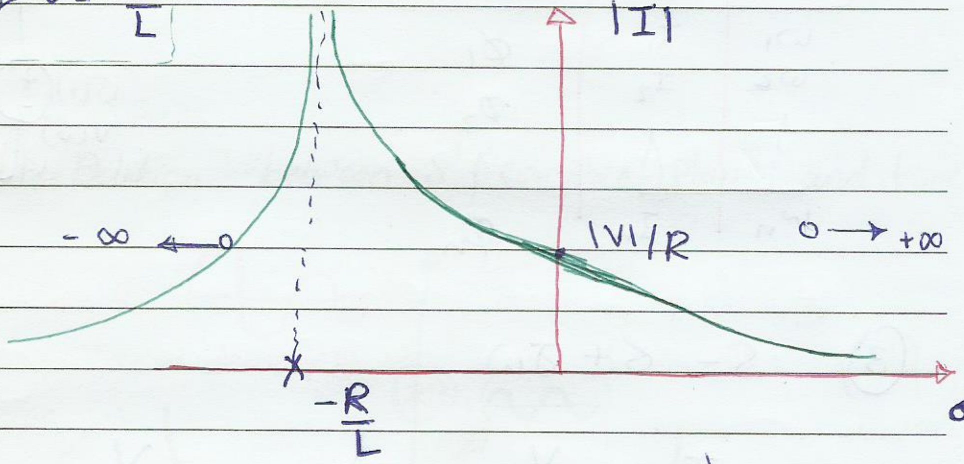
(1) $S = \sigma + j0$ (i.e exponential forcing function)

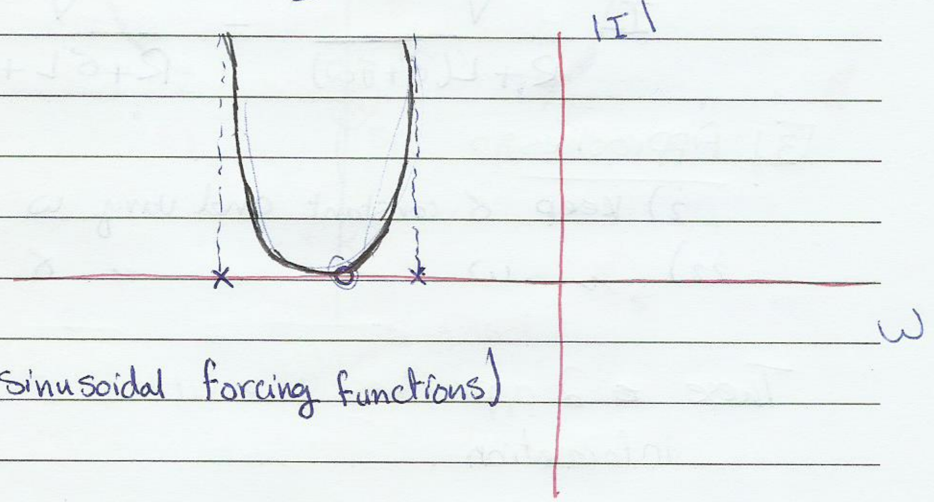
$\therefore I = \dfrac{V}{R + \sigma L}$

Pole $= -\frac{R}{L}$

Zeros $= \pm \infty$

Poles $\Rightarrow \sigma = -\frac{R}{L}$



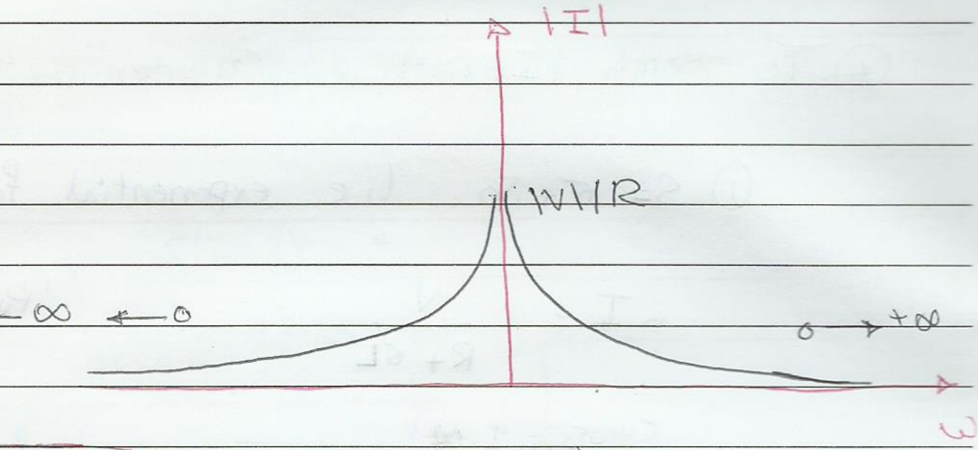


(2) $S = 0 \pm j\omega$ (i.e sinusoidal forcing functions)

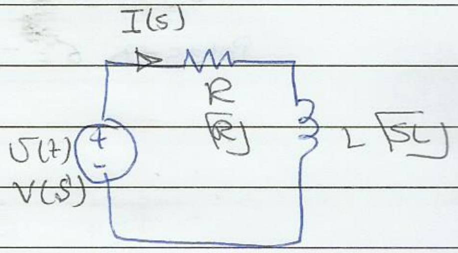
$$I = \frac{V}{R + j\omega L}$$

$\therefore |I| = |V| / \sqrt{R^2 + (\omega L)^2}$

$\angle I = \angle V - \tan^{-1} \frac{\omega L}{R}$



ω	$\|I\|$	$\angle I$
ω_1	I_1	ϕ_1
ω_2	I_2	ϕ_2
$\vdots$	$\vdots$	$\vdots$
ω_n	I_n	ϕ_n



(3) $S = \sigma \pm j\omega$

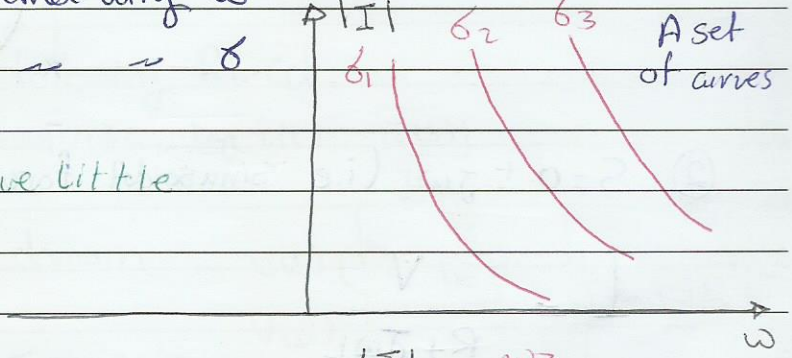
$$I = \frac{V}{R + L(\sigma + j\omega)} = \frac{V}{R + \sigma L + j\omega L}$$

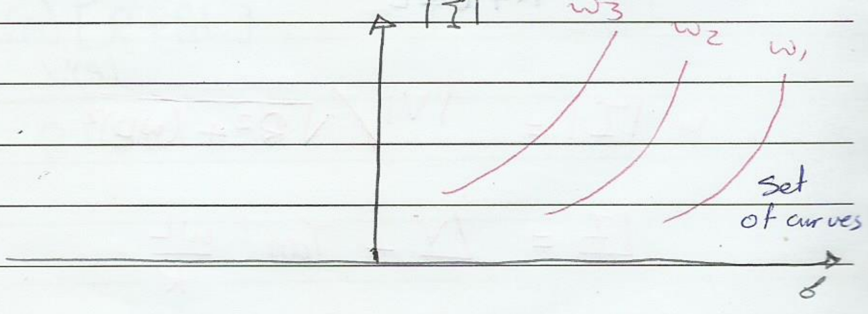
[3] Approches:

i) Keep σ constant and vary ω

ii) 〃 ω 〃 〃 〃 σ

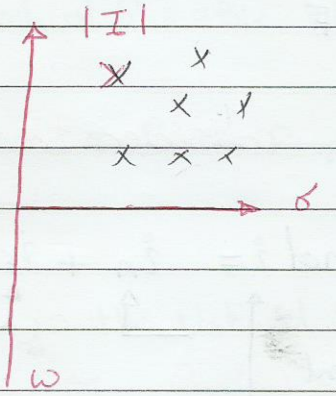
These 2 approches give little information





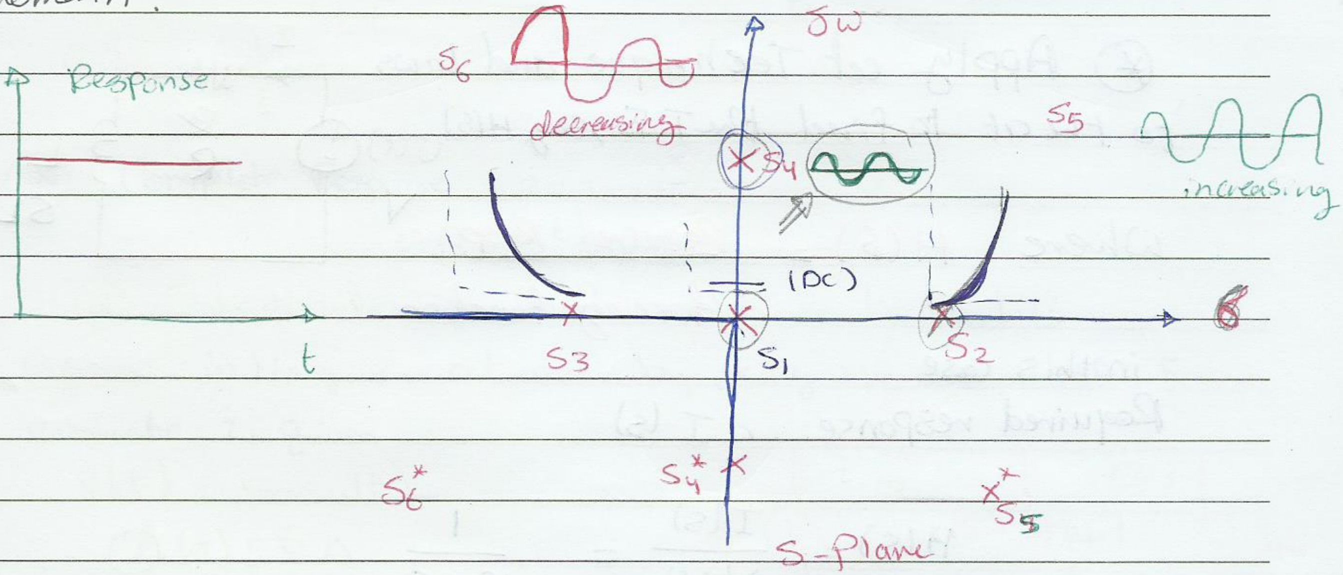
c) Three dimension sketch

A set of Points are obtained



(*) If one join the Points which have the same magnitude then the S called equipotential lines are obtained.

OR using the Relation between S (complex) plane and time domain.



Evaluating Natural, forced and complete Response by using the concept of T.F

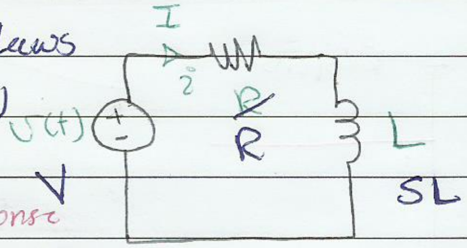
(*) To Illustrate consider the following RL cct

Objective :> find $i = i_n + i_F$

Natural or Transient — i_n; forced — i_F; Complete — i

Procedure :> Transfer the cct To S domain

(*) Apply cct Technique and laws to the cct to find the T.F, say H(S)



Where $H(S) = \dfrac{\text{Required Response}}{\text{forcing function}}$

in this case

Required response = $I(s)$

$$H(S) = \frac{I(S)}{V(S)} = \frac{1}{R+SL} \quad \text{--- (1)}$$

$$\therefore I = G(S) * V(S) \quad \text{--- (2)}$$

By knowing the forcing function (i.e $V(S)$) and it's type (i.e the value of S), then the forced response $I_F(S)$ can be evaluated from (2)

Then transform $I_f(s) \longrightarrow i_f(t)$

(*) If $V(s)$ is applied at $s' =$ a pole of $G(s)$ and in the limit when $V(s) \Rightarrow 0$
Goes to

Then a finite Response say H will be obtained at $S =$ a pole

This Response is the Natural Response

$\therefore$ In this case $I_n(s) = A$ at $S = -\frac{R}{L}$

$\therefore$ Transform $I_n(s) \longrightarrow i_n(t)$

$$\therefore i_n = A e^{-\frac{R}{L}t} \cos(0 + S) = A\cos\theta\, e^{-R/L\, t}$$

B

$$\therefore i_n = B e^{-R/L\, t}$$

(*) Complete Response $i = i_n + i_f$

$$= B e^{-\frac{R}{L}t} + i_f$$

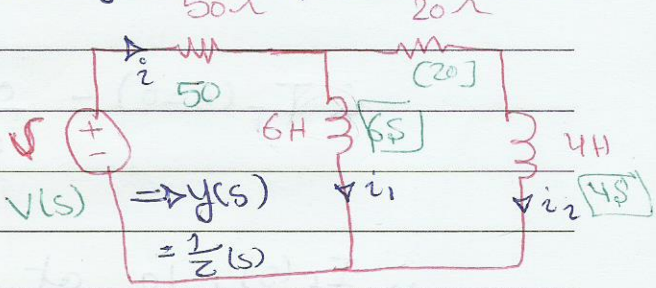
(*) Substitude initial conditions to find B

ex: in the given cct and by using the concept of T.F evaluate i, given

$V(t) = 500\, u(t)$

$i_1(0) = 5\,A$

$i_2(0) = 2\,A$



~~First~~ (*) objective

find $i = i_n + i_f$

Procedure

(*) Find $H(s) = \frac{I(s)}{V(s)} \Rightarrow Y(s)$ ... (1)

$$\therefore Z(s) = 50 + [6s \,//\, (20 + 4s)]$$

$$= 50 + \frac{6s * (20 + 4s)}{6s + 20 + 4s} \quad \text{... (2)}$$

* سؤال بالامتحان يعطيك zeros & Poles و يخليك تطلع ال $i = i_n + i_f$

* إذا السيركت كان معطيك source انت بدك تحط Currents with parallel & Voltage with series

* جرب الـ ds + حل سؤال إذا كان عندك أكثر من source نفعل الـ ... واحد

∴ By Re arrangement of (2) and substituting in (1), it can be found ∴

$$= \frac{10S + 20}{24S^2 + 620S + 1000} \quad \ldots (3)$$

finding i_n

find Poles of $H(s)$

∴ Solve $24s + 620s + 1000$

∴ $S_1 = -1.728$, $S_2 = -24.1$

$\uparrow P_1$ $\uparrow P_2$

∴ $i_n(t) = Ae^{P_1 t} + Be^{P_2 t} = Ae^{-1.728t} + Be^{-24.}$

∴ $I_F(S) = H(s)V(s)$

since $v(t) = 500u(t)$

⇓

$V(S) = 500$ and $s = 0$ since $v(t)$ dc

∴ $I_F(s=0) = \frac{0 + 20}{0 + 0 + 1005} * 500 = 10$

∴ $I_f(s) = 10$ at $s = 0$

$i_f = 10e^0 \cos(0+0)$

$= 10$

$i(t) = Ae^{-1.778t} + Be^{-24.12t} + 10$

$i = i_n + i_f$

$= Ae^{-1.728t} + Be^{-24.1t} + 10$

$i(0) =$

$A + B + 10 = 7$

$A + B = -3$

∴ Find A and B using initial condition

∴ find $i(0)$ and $\frac{di}{dt}\Big|_{t=0}$

$\frac{di(t)}{dt} = \boxed{}$

$\frac{di_1}{dt} + \frac{di_2}{dt}$

$2A - 3B = -5$

$A + B = -3$

From the given cct

By KCL $\Rightarrow i = i_1 + i_2$

$\therefore \frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$

$i(0) = i_1(0) + i_2(0) = 5 + 2 = 7$

$\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$

$= 3$

$A + B = 3$

Apply KVL to 1st Mesh

$V = 50\, i + 6 \frac{di_1}{dt}$

$V(0) = 50\, i(0) + 6 \frac{di_1}{dt}\Big|_{t=0}$

$= 500 = 50 \times 7 + 6 \frac{di_1}{dt}\Big|_{t=0}$

$\therefore \frac{di_1}{dt}\Big|_{t=0} = \frac{150}{6}$

Apply KVL to large loop

$\therefore V(0) = 50\, i(0) + 20\, i_2(0) + 4 \frac{di_2}{dt}\Big|_{t=0}$

$500 = 50 + 7 + 20 \times 2 + 4 \frac{di_2}{dt}\Big|_{t=0}$

$\frac{di_2}{dt}\Big|_{t=0} = \boxed{27.5}$

$$\frac{di_1}{dt}\bigg|_{t=0} = \frac{150}{6} + 27.5$$

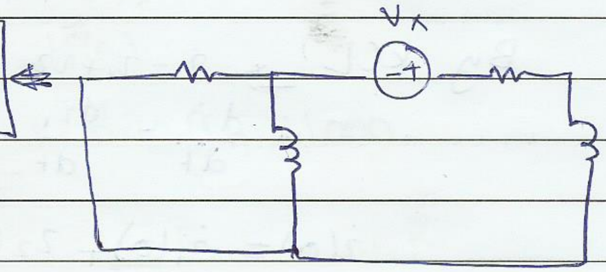
$A = 2.35$

$B = -5.35$

another ~~eq~~ question

نفس المعطيات

بعطي نفس ال poles



Frequency Responses:

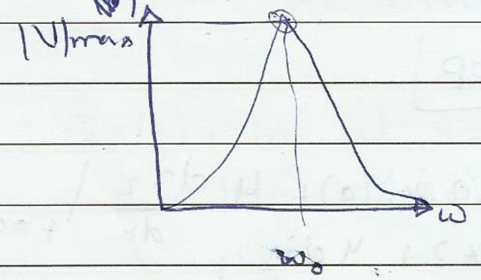
Resonance: It's the condition which occur in a system when a fixed Amplitude sinusoidal forcing function

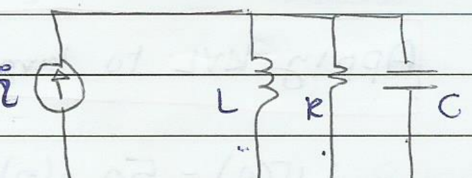
electrical ccts

Producing Maximum Response (voltage or current)

Consider the following

Parallel RLC circuit





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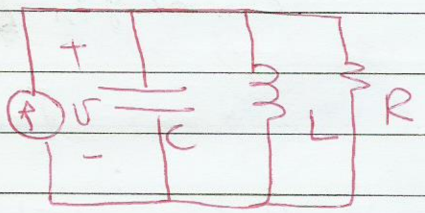
Parallel Resonance:

Objectives: Find the conditions for Resonance. Let the forcing function be i and the Resonance is v

$$V = \frac{I}{Y}$$

$$V = \frac{I}{\frac{1}{R} + j\left[\omega C - \frac{1}{\omega L}\right]} \quad \text{---} \ (1)$$



$\therefore |V|_{max}$ is when $|Y|$ is minimum

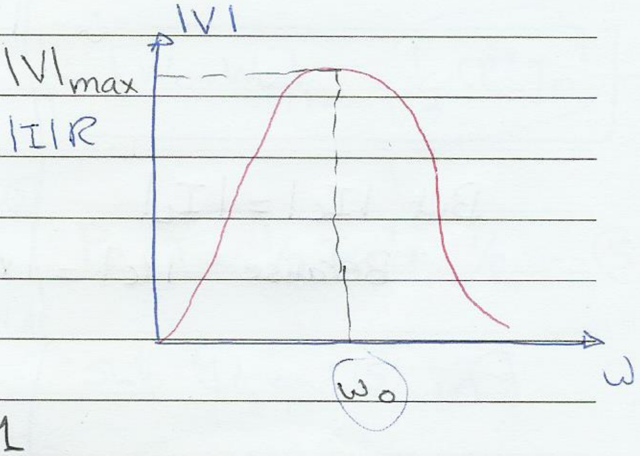
$\therefore$ For a given $|I|$, R, L and Y is minimum when

$$\left[\omega C - \frac{1}{\omega L}\right] = 0 \quad \text{---} \ (2)$$

$$\therefore \operatorname{Im}[Y] = 0$$

$$\therefore \omega C = \frac{1}{\omega L}$$

$$\therefore \omega = \frac{1}{\sqrt{LC}}$$



at Resonance $\omega = \omega_o = \dfrac{1}{\sqrt{LC}}$

(14) 18-2-20

Comments at $\omega = \omega_0$

i) $\omega L = \frac{1}{\omega C}$ $\therefore X_L = X_C$

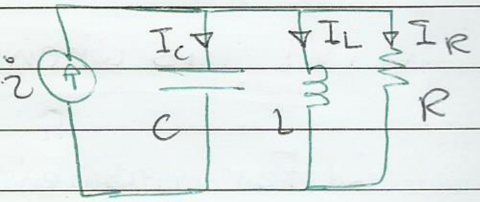
ii) $Y = 1/R$ (i.e Pure Real) $\therefore$ Input voltage (v) and current (i) are in phase.

$\therefore$ To find ω_0 for any cct, then one may find Z_{in} or Y_{in}, then solve $Im[Z_{in}] = 0$

OR

$Im[Y_{in}] = 0$

iii) $|V|_{max} = \frac{|I|}{|Y|_{max}} = \frac{|I|}{1/R}$



$\therefore |V|_{max} = |I|R$

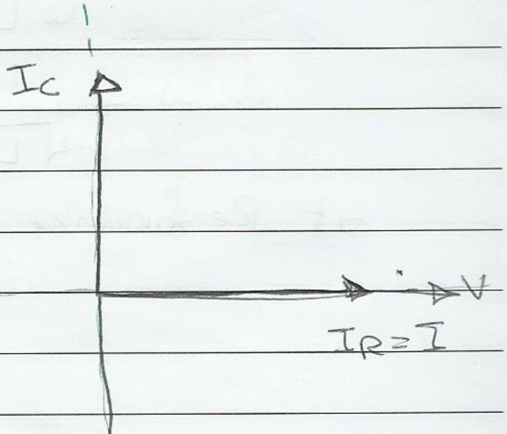
$|V|_{max} = |I|R$

$\therefore |I_R| = \frac{|V|_{max}}{R}$

$= |I|R/R = |I|$

But $|I_C| = |I_L|$

Because $|X_C| = |X_L|$



Quality Factor: Q

For any cct

$$Q \triangleq 2\pi \left[\frac{\text{maximum energy stored in the circuit}}{\text{Energy lost in one period}}\right]$$

introduced For Mathematical simplication

Objective: Evaluate Q of a Parallel RLC ckt at $\omega = \omega_0$

For $\omega = \omega_0 \Rightarrow [i.e\ Q_0]$ ~~Derv~~ Deriave

Procedure let $i = I_m \cos(\omega_0 t)$

$v = iR = R I_m \cos(\omega_0 t)$

$$Q_0 = 2\pi \frac{[w_c + w_L]_{max}}{P_R * T}$$

W_c, W_L = Energy stored in c and L Respectively

P_R = Power disipated in R

T = Periodic time

$$W_c = \frac{C}{2} V^2 = \boxed{\frac{1}{2} C R^2 I_m^2 \cos^2(\omega_0 t)} \quad \text{--(2)}$$

$$W_L = \frac{L}{2} i_L^2 \Rightarrow i_L = \frac{1}{L}\int v\,dt = \frac{1}{L}\int \cos\omega_0 t\,dt$$

$$\Rightarrow i_L = \frac{R I_m}{L \omega_0} \sin(\omega_0 t)$$

$$w_L = \frac{1}{2} L R^2 I_m^2 \frac{\sin^2(\omega_0 t)}{L^2 \omega_0^2} \quad (i.e\ \omega_0^2 = \frac{1}{LC})$$

$$\Rightarrow W_L = \frac{1}{2} L R^2 I_m^2 \frac{LC}{L^2} \sin^2(\omega t)$$

$$\boxed{\Rightarrow w_L = \frac{1}{2} R^2 I_m^2 C \sin^2(\omega_0 t)} \quad \text{--(3)}$$

~~$w_c + w_L = \frac{1}{2} C R^2 I_m^2$~~

$w_c + w_L = (2) + (3) = \frac{1}{2} C R^2 I_m^2 [\cos^2 \omega_0 t + \sin^2 \omega_0 t]$ since I_m, R and C are constants #

$$W_c + W_L = \frac{1}{2} C R^2 I_m^2$$

$$\boxed{\therefore [W_c + W_L]_{max} = \frac{1}{2} C R^2 I_m^2} \quad \text{--(4)}$$

$$P_R * T = \frac{1}{2} I_m^2 R \frac{1}{f_0} \quad \text{--(5)}$$

sub (4) and (5) in (1)

$$Q_0 = 2\pi \frac{\frac{1}{2} C R^2 I_m^2 \cdot f_0}{\frac{1}{2} I_m^2 R}$$

$$Q_0 = 2\pi f_0 C R \quad (i.e\ 2\pi f_0 = \omega_0)$$

$$\boxed{Q_0 = \omega_0 C R}$$

Comments on Q_0:

i) $Q_0 = \omega_0 CR = \frac{1}{\sqrt{LC}} CR = \sqrt{\frac{C}{L}}\, R$

ii) $Q_0 = \frac{R}{X_C} = \frac{R}{X_L}$

iii) $|I_C| = |I_L| = \omega_0 C|V| = \underbrace{(\omega_0 CR)}_{Q_0} I_m$

$\therefore |I_C| = |I_L| = Q_0 I_m$ ** Drive Q_0 ?? for series and Parallel RLC

If for e.g $I_m = 2mA$ and $Q_0 = 50$

Then $|I_C| = |I_L| = 2*50 = 100mA$

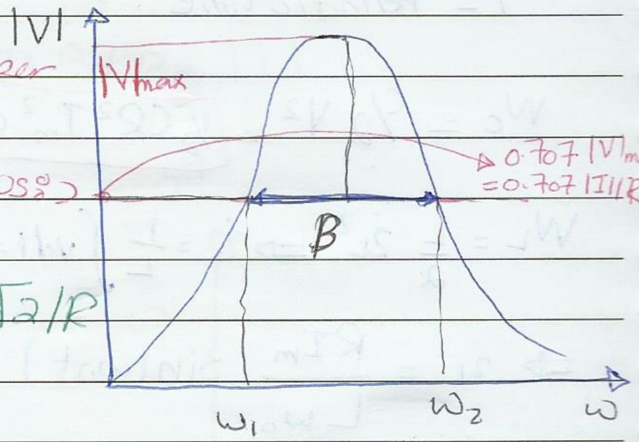
$\therefore$ Parallel RLC cct can be used as a current

Width of the Response Curve:

This is measured by the Bandwidth β

when $\beta \triangleq \omega_2 - \omega_1$

Where ω_2 and ω_1 are called upper and lower Half-power frequency respectively and defined as follows:



$|Y(\omega_1)|$ or $|Y(\omega_2)| \triangleq \sqrt{2}\,|Y|_{min} = \sqrt{2}/R$

$\therefore |V|$ at ω_1 or $\omega_2 = |I|/|Y| = I/\sqrt{2}/R = \frac{1}{\sqrt{2}} |I|R$

$= 0.707 |I|R$

$= 0.707\, |V|_{max}$

∴ P at ω_1 or ω_2 =

$$\frac{(0.707|I|R)^2}{R} = 0.5|I|^2 R$$

$= 0.5$ of P at ω_0

∴ The Name Half power frequency

Evaluation of ω_1 and ω_2 :-

$$|Y(\omega_1 \text{ or } \omega_2)| \triangleq \sqrt{2}\,|Y|_{min} = \sqrt{2}\cdot 1/R$$

Procedure $\quad Y(j\omega) = \frac{1}{R} + j\frac{1}{R}\left[\frac{\omega_0 \omega CR}{\omega_0} - \frac{R\omega_0}{\omega L \omega_0}\right]$

$$= 1/R + j\,1/R\left[\frac{Q_0\,\omega}{\omega_0} - \frac{Q_0\,\omega_0}{\omega}\right] \;\text{—}\; (1)$$

Since $\quad Q_0 = \omega_0 CR$

$$= \frac{R}{\omega_0 L}$$

Drive ω_1, ω_2

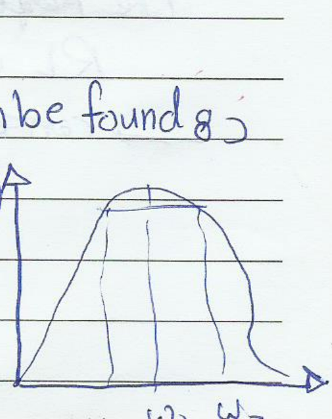
∴ From (1) at ω_1 or ω_2 $\quad [\quad\quad] = \pm 1$

$$\therefore \frac{Q_0\,\omega_2}{\omega_0} - \frac{Q_0\,\omega_0}{\omega_2} = 1 \;\text{—}\; (2)$$

$$\frac{Q_0\,\omega_1}{\omega_0} - \frac{Q_0\,\omega_0}{\omega_1} = -1 \;\text{—}\; (3)$$

By ~~substituting~~ solving (2) and (3) it can be found :-

$$\omega_2, \omega_1 = \omega_0\left[\sqrt{1+\left(\frac{1}{2Q_0}\right)^2} \pm \left(\frac{1}{2Q_0}\right)\right] \;\text{—}\; (4)$$



From (4)

i) $\beta = \omega_2 - \omega_1 = \frac{\omega_0}{Q_0}$

ii) $\omega_0 = \sqrt{\omega_1 \omega_2}$

Approximations:

i) At high Q_0, usually $Q_0 \geq 5$

$$\omega_2, \omega_1 \simeq \omega = \left[1 \pm \frac{1}{2Q_0}\right] = \omega_0 \pm \tfrac{1}{2} \omega_0 / Q_0$$

$$= \omega_0 \pm \tfrac{1}{2} \beta$$

$\therefore \omega_2$ and ω_1 are symmetrical around ω_0.

ii) at high Q_0, it can be found that

$$Y(j\omega) = \tfrac{1}{R}\left[1 + jN\right]$$

where $N = \frac{\omega - \omega_0}{\tfrac{1}{2}\beta}$

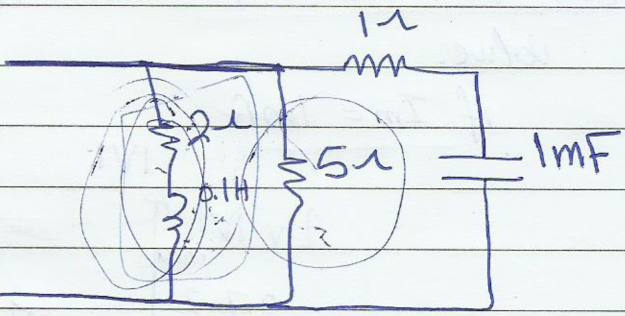
$\Rightarrow$ Number of $\tfrac{1}{2}$ Bandwidth between ω and ω_0

Conclusion:

The smaller the value of β $\rightarrow$ The sharper the Response curve $\rightarrow$ The more selective the parallel RLC ckt is $\rightarrow$ hence it is more [redacted] selective $\rightarrow$ consequently it can be used as a filter.

* لو بدك تشوف السيركت لهذا Simple Parallel أو لأ *

(e.g) for the given circuit find ω_0



Solution :-

i) find y or Z

ii) solve

$Im\,[y \text{ or } z] = 0$

imaginary part

$$\therefore Y = \frac{1}{2+0.1j\omega} + 1/5 + \frac{1}{\left(1 - \frac{j}{\omega * 0.001}\right)}$$

Find $Im[y]$.

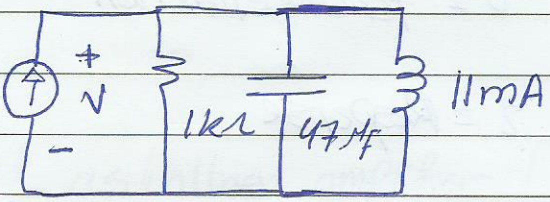
$\therefore$ It can be found $Im\,[Y] = 0$

$$0.1\omega(\omega^2+10^6) + \omega \cdot 10^3(4+0.001\omega^2) = 0$$

$$\Rightarrow \omega = 98.47 \text{ Rad/s}$$

ex 2 :-

for the given cct, $i = I_m \cos \omega t$ evaluate the following :-



i) $Q_0 \Rightarrow Q_0 = \omega_0 C R$

$$= \frac{1}{\sqrt{LC}} * C * R$$

$$= 65,37$$

* سؤال Q_0 إذا ما أعطاك يا إما Parallel

بدنا نستخدم قانون ونطلع energy

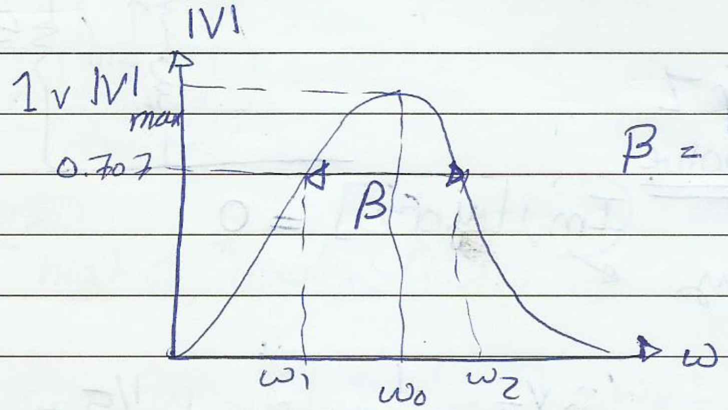
ii) find f_0

$\therefore \omega_0 = 2\pi f_0 \Rightarrow f_0 = \frac{\omega_0}{2\pi}$

$$= \frac{1}{2\pi\sqrt{LC}} = 221.3 \text{ Hz}$$

iii) sketch the response curve and show on it the main parameters value.

if $I_m = 1mA$



$\beta = \omega_2 - \omega_1$

$$|V|_{max} = I_m R = 10^{-3} \times 10^{3} = 1V$$

$$\omega_2, \omega_1 = \omega = \left[\sqrt{1+\left(\frac{1}{240}\right)^2} \pm \left(\frac{1}{240}\right)\right]$$

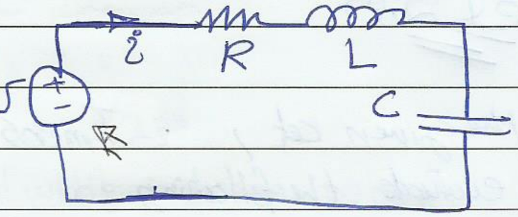
$\omega_2 = 1402$

$\omega_1 = 1380$

Series RLC Circuit

v = forcing function

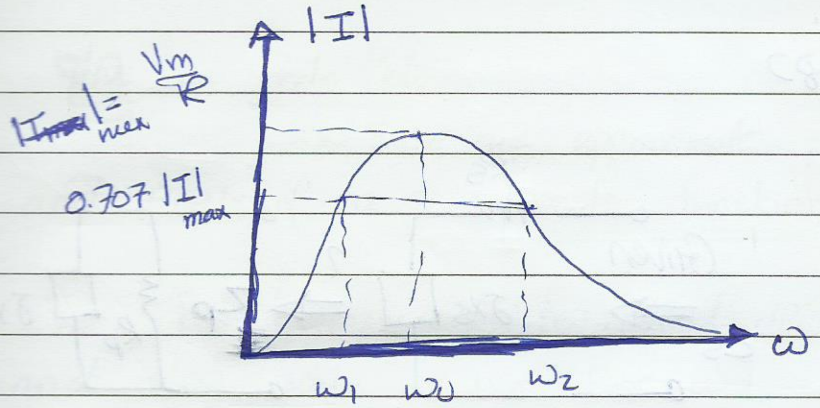
i = Response

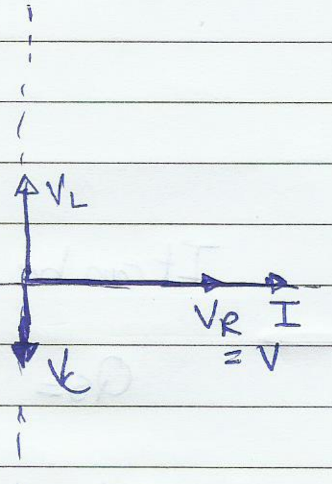


$$I = V/Z \Rightarrow \frac{V}{R + j(\omega L - 1/\omega c)}$$

$$(I)_{max} \text{ when } |Z|_{min} \Rightarrow \omega L = \frac{1}{\omega c} = 0$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$





By KVL

$$V = V_R + \underbrace{(V_C + V_L)}_{\text{صفر}}$$

$$V = V_R$$

Note the same expression of ω_1, ω_2, ω_0 and β of the Parallel RLC circuit is applicable to series RLC circuit

$$\text{But } Q_0 = \frac{\omega_0 L}{R} \dashrightarrow \text{(derive)}$$

$$\boxed{\text{OR} = \frac{1}{\omega_0 C R}}$$

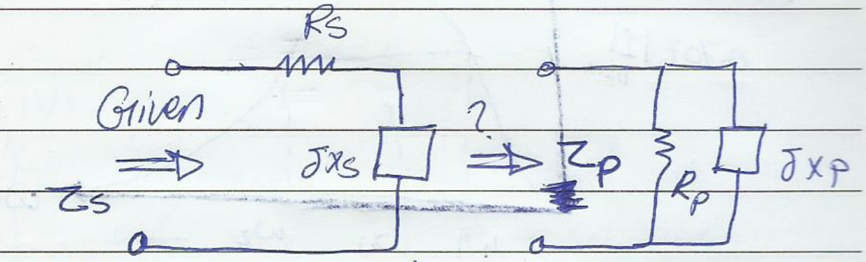
(*) Series RLC cct can be used as voltage amplifier.

Q:

(*) State the application of RLC

Other Resonant cct:

Objectives:



$S =$ series branch $P =$ Parallel branch

It can be found that

$$Q_S = \frac{X_S}{R_S} \quad \text{and} \quad Q_P = \frac{R_P}{X_P}$$

The 2 cct are equivalent if $Z_S = Z_P$

$$R_S + jX_S = \frac{R_P * jX_P}{R_P + jX_P} \quad \text{—} \; (1)$$

By equating R_e and I_m parts of (1)

$$X_P = \frac{R_S^2 + X_S^2}{X_S} = \frac{R_S^2}{X_S} + X_S = X_S\left[1 + \left(\frac{R_S}{X_S}\right)^2\right] = X_S\left[1 + \frac{1}{Q_S^2}\right]$$

$$R_P = \frac{R_S^2 + X_S}{R_S} = R_S + \frac{X_S}{R_S} = R_S\left[1 + \left(\frac{X_S}{R_S}\right)^2\right]$$

$$= R_S\left[1 + Q_S^2\right]$$

9/12 Bode Plots:

→ These are approximate plots of the magnitude and Angle of a given transfer function with frequency (ω)

But here, a log scale, called dB scale is used for the magnitude and a log scale is used for (ω)

Definition of dB scale:

Given a transfer function, say $G(s)$

Then let $s = j\omega$

$\therefore G(s) \Rightarrow G(j\omega)$

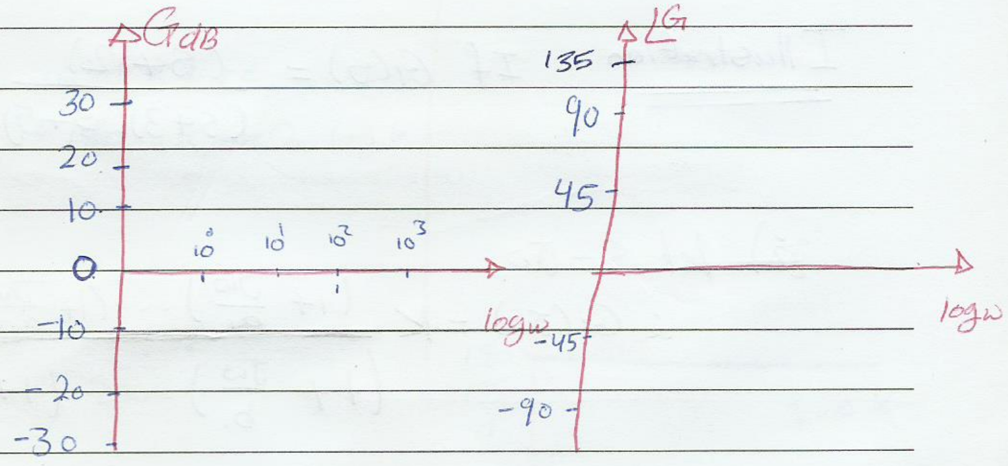
$\therefore G_{dB} \triangleq 20 \log |G(j\omega)|$

Illustration:

If a semilog graph paper is used, then the values of ω can be written on the $\log \omega$ axis.

$\lvert G(j\omega)\rvert$	G_{dB}
1	0
2	6
10	20
$10^{\pm n}$	$\pm 20n$

$\therefore$ Two required curves



Note: If the Ratio of 2 frequencies, say ω_2 and ω_1, equal to 10, then it is said that there is one Decade between ω_2 and ω_1 عشري

e.g $\dfrac{\omega_2 = 1000}{\omega_1 = 100}$

If the Ratio of 2 frequencies, say ω_2 and ω_1, equal to 2, then ~~this~~ it is said that there is one octave between ω_2 and ω_1 ثماني

e.g $\dfrac{\omega_2 = 20}{\omega_1 = 10} = 2$

Procedure:

i) Rewrite the given tranfer function in the following Factorized form:

(تحليل العوامل)

$$G(s) = K \frac{(1+\frac{s}{a_1})(1+\frac{s}{a_2})\cdots(1+\frac{s}{a_n})}{(1+\frac{s}{b_1})(1+\frac{s}{b_2})\cdots(1+\frac{s}{b_n})}$$

Where: K is a real number, $-a_1, -a_2, \ldots -a_m \equiv$ zeros of $G(s)$

$-b_1, -b_2, \ldots -b_n \equiv$ Poles of $G(s)$

Illustration If $G(s) = \dfrac{(s+2)}{(s+3)(s+5)} = \dfrac{2(1+\frac{s}{2})}{3(1+\frac{s}{3})5(1+\frac{s}{5})}$

$\therefore K = \dfrac{2}{3*5} = 2/15$

ii) let $s = j\omega$

$$\therefore G(j\omega) = K \frac{(1+\frac{j\omega}{a_1})\cdots(1+\frac{j\omega}{a_n})}{(1+\frac{j\omega}{b_1})\cdots(1+\frac{j\omega}{b_n})}$$

iii) $G_{dB} = 20\log|G(j\omega)|$

$$G_{dB} = 20\log K + 20\log\left|1+\frac{j\omega}{a_1}\right| + 20\log\left|1+\frac{j\omega}{a_2}\right| - - - +$$

$$20\log\left|1+\frac{j\omega}{a_m}\right|\Big] - \Big[20\log\left|1+\frac{j\omega}{b_1}\right| + 20\log\left|1+\frac{j\omega}{b_2}\right| - - -$$

$$+ 20\log\left[\left|1+\frac{j\omega}{b_n}\right|\right]$$

∴ to obtain (G_{dB} versus $\log\omega$) curve, plot the individual curves of K, zeros and poles and then summate them.

→ iv) $\angle G(j\omega) = \angle K + \angle 1+\frac{j\omega}{a_1} + - - - + \angle 1+\frac{j\omega}{a_m} - \Big[\angle 1+\frac{j\omega}{b_1} +$

$\angle 1+\frac{j\omega}{b_n}\Big]$

∴ Hence plot the individual curves of K, zeros, and poles and the summate then to find $\angle G(j\omega)$ viz ω

Considered the following cases of G(s) 8)

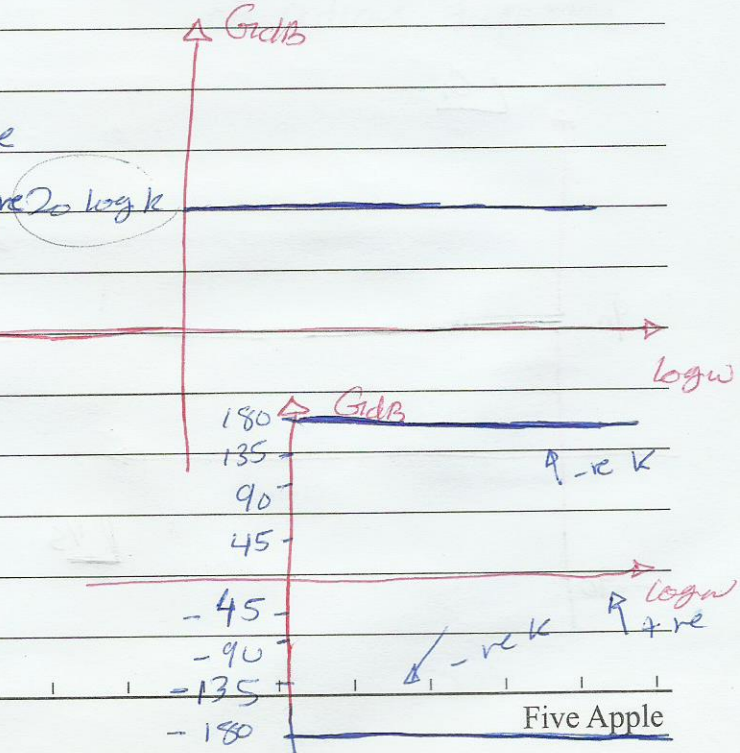
i) $G(s) = K$

$\therefore G_{dB} = 20\log|G(j\omega)| = 20\log K$

ii)

$\therefore \angle G(j\omega) = \angle K = 0°$ if K +ve

$= \pm 180$: K -ve

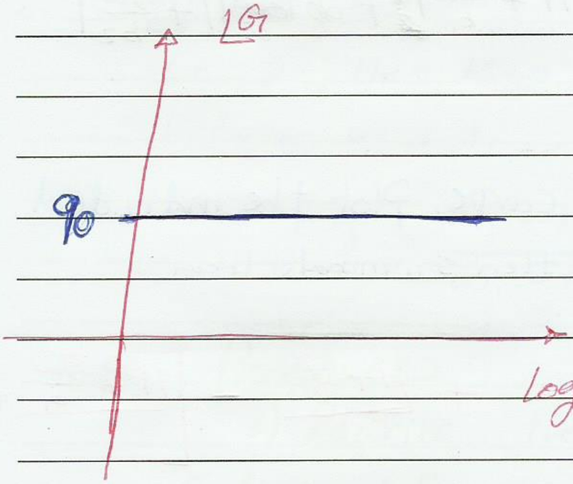


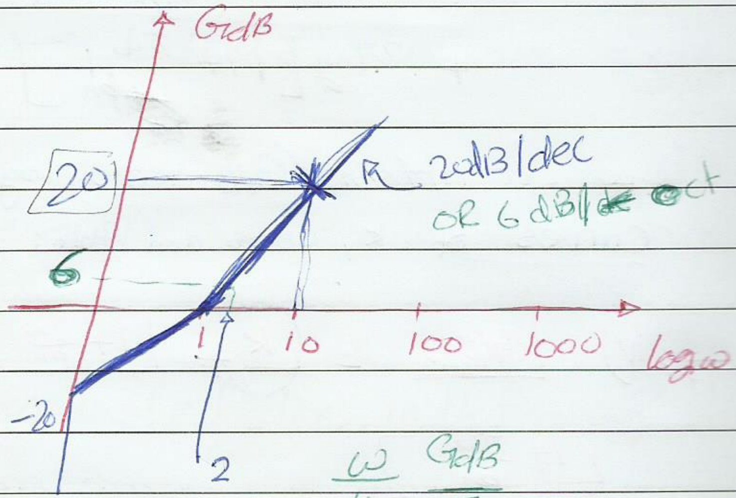
* سؤال 81 متحان - تحسيان bode plot وتكملة حلول ال 7.F

ii) $G(s) = s$

$\therefore G(j\omega) = j\omega$

$G_{dB} = 20\log|G(j\omega)| = 20\log|j\omega| = 20\log\omega$





ω	GdB
4	7
2? 8	?
	7+6=13

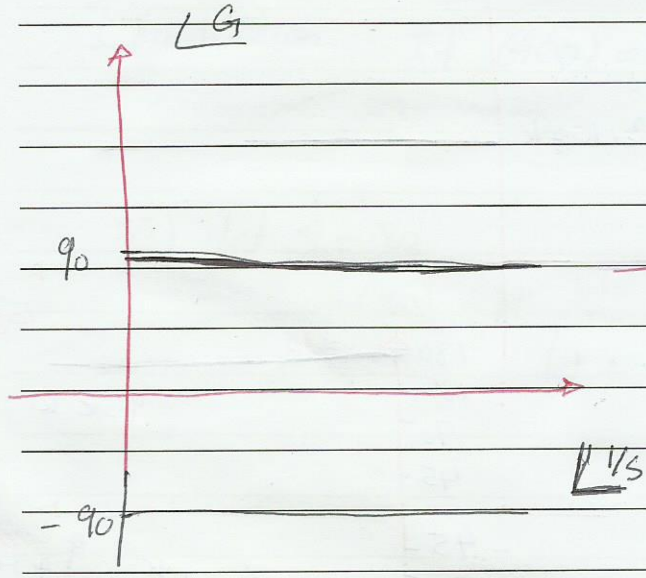
$\angle G(j\omega) = \angle j\omega = 90^\circ$

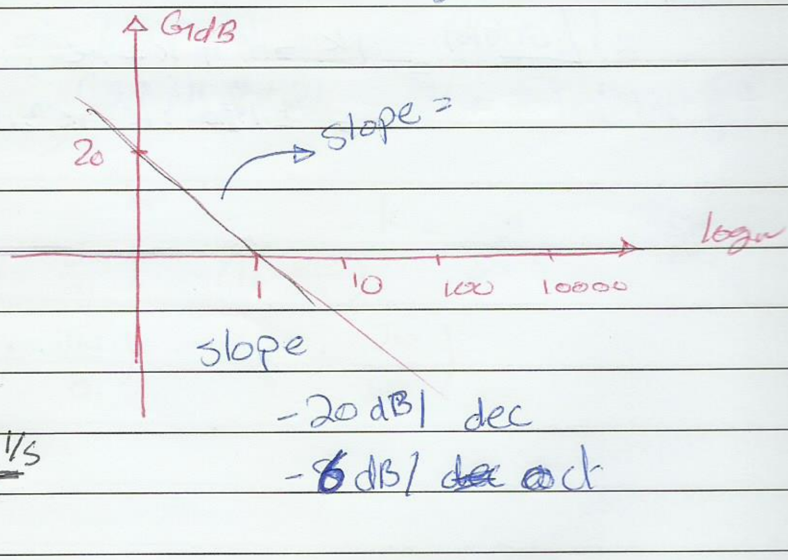
iii) $G(s) = \frac{1}{s}$

$G(j\omega) = \frac{1}{j\omega}$

$G_{dB} = 20\log|G(j\omega)| = 20\log\left|\frac{1}{j\omega}\right| = 20\log 1 - 20\log\omega$

$= -20\log\omega$





iv) $G(s) = \left(1 + \frac{s}{a}\right)$ $\therefore$ a zero at $s = -a$

$\therefore G(j\omega) = \left(1 + \frac{j\omega}{a}\right)$

$\therefore G_{dB} = 20\log|G(j\omega)| = 20\log\sqrt{1+\left(\frac{\omega}{a}\right)^2}$

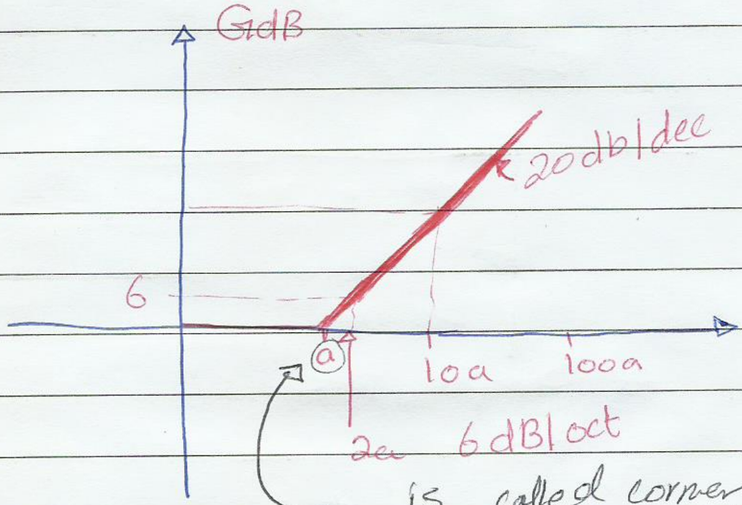
$20\log\sqrt{1+\omega^2} \leftarrow \omega \gg a$ إذا كانت

$\frac{\omega}{a} \simeq 0 \leftarrow a \gg \omega$ إذا كانت

If $\omega \ll a$, Taken when $\omega < a$, $G_{dB} = 20\log 1 = 0$

" $\omega \gg a$, " " $\omega > a$, $G_{dB} = 20\log\frac{\omega}{a}$?}

$\Rightarrow \angle G(j\omega) = \tan^{-1}\left(\frac{\omega}{a}\right)$



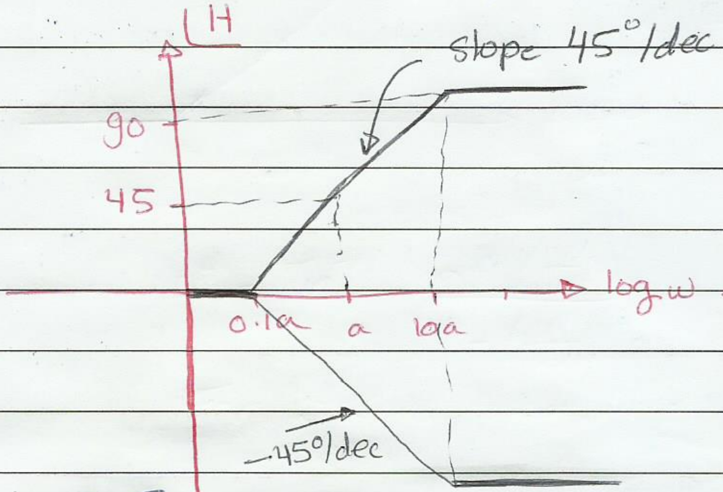
is called corner frequency OR critical frequency.

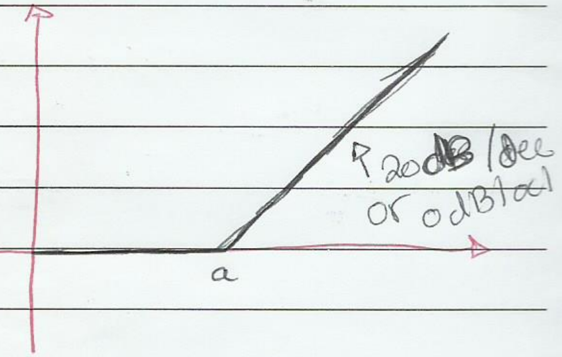
11/12

$H(s) = \left(1 + \frac{s}{a}\right)$

$\angle H(j\omega) = \tan^{-1}\left(\frac{\omega}{a}\right)$

* for $\omega \ll a$, Taken $\omega < 0.1a$, $\angle H = 0$
 " $\omega \gg a$, " $\omega > 10a$, $\angle H = 90$
 $\omega = a$, $\angle H = 45$



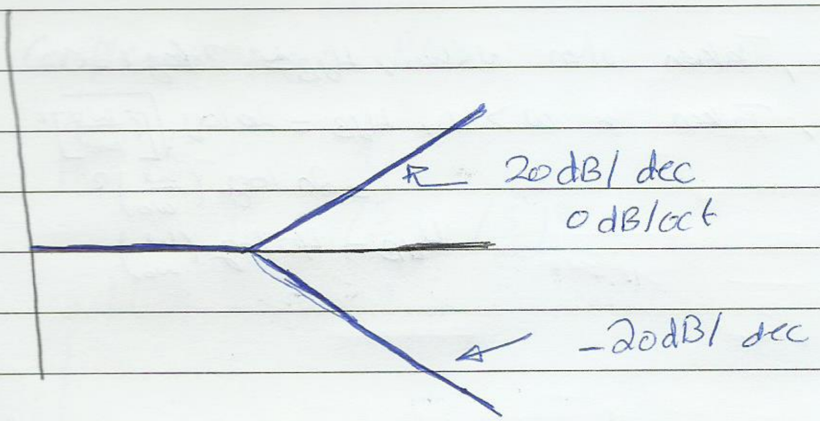


* Simple Pole.

$H(s) = \frac{1}{\left(1 + \frac{s}{a}\right)}$

$\therefore H(j\omega) = \frac{1}{1 + \left(\frac{j\omega}{a}\right)}$

$H_{dB} = 20\log 1 - 20\log\sqrt{1 + \left(\frac{\omega}{a}\right)^2} = -20\log\sqrt{1 + \left(\frac{\omega}{a}\right)^2}$



(*) Complex conjugate poles and/OR zeros

If there is a quadratic expression whos Roots are complex (i.e $b^2 < 4ac$) the rewrite it in the following form:

$$1 + 2\xi\left(\frac{s}{\omega_0}\right) + \left(\frac{s}{\omega_0}\right)^2$$

Where $0 < \xi < 1$, ω_0 = Real constant No.

Illustration:

$$H(s) = 3 + 7s + 5s^2$$

$$= 3\left[1 + \tfrac{7}{3}s + \tfrac{5}{3}s^2\right]$$

$$H(s) = 3\left[1 + \underbrace{\frac{7}{3}\cdot\frac{\sqrt{3}}{\sqrt{5}}}_{2\xi}\left(\frac{s}{\sqrt{3/5}}\right) + \left(\frac{s}{\sqrt{3/5}}\right)^2\right]$$

$\therefore \omega_0 = \sqrt{3/5}$

$$\therefore H(j\omega) = 1 + 2\xi\left(\frac{j\omega}{\omega_0}\right) + \left(\frac{j\omega}{\omega_0}\right)^2$$

$$= \left[1 - \left(\frac{\omega}{\omega_0}\right)^2\right] + 2\xi\left(\frac{j\omega}{\omega_0}\right)$$

$$\therefore H_{dB} = 20\log|H(j\omega)|$$

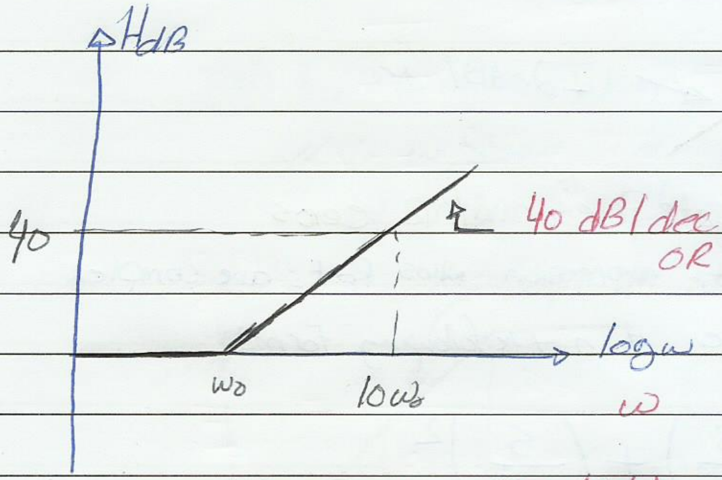
$$= 20\log\sqrt{\left[1 - \left(\frac{\omega}{\omega_0}\right)^2\right]^2 + \left[2\xi\left(\frac{\omega}{\omega_0}\right)\right]^2}$$

ii For $\omega << \omega_o$, Taken when $\omega < \omega_o$, $H_{dB} \approx 20\log 1 = 0$

$\omega >> \omega_o$, Taken " $\omega > \omega_o$, $H_{dB} = 20\log\sqrt{\left[\left(\frac{\omega}{\omega_o}\right)^2\right]^2}$

$$= 20\log\left(\frac{\omega}{\omega_o}\right)^2$$

$$H_{dB} = 40\log\left(\frac{\omega}{\omega_o}\right)$$

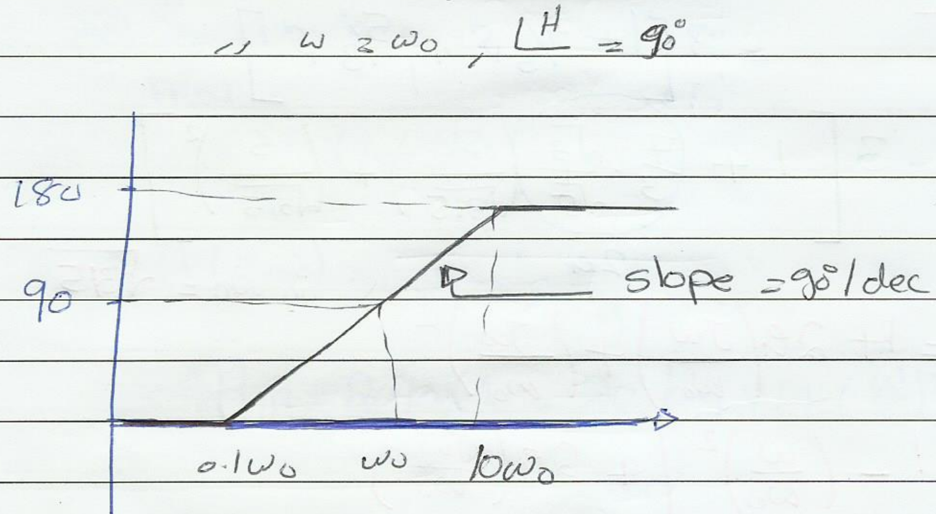


$$\angle H = \tan^{-1}\frac{2\zeta_1\left(\frac{\omega}{\omega_o}\right)}{1-\left(\frac{\omega}{\omega_o}\right)^2}$$

For $\omega << \omega_o$, taken when $\omega < 0.1\,\omega_o$, $\angle H = 0$

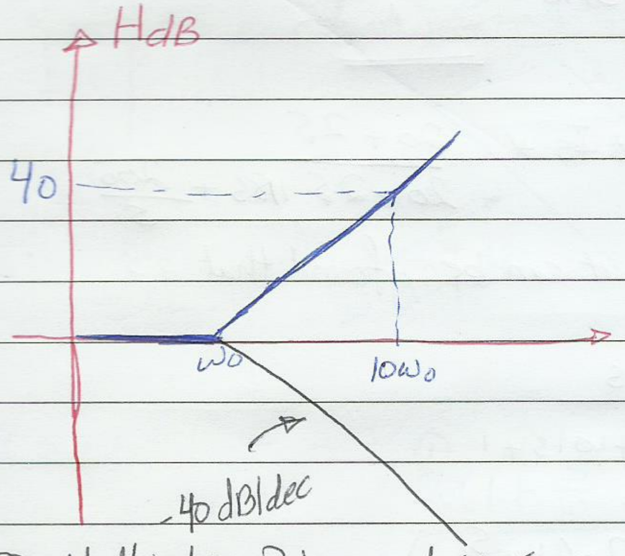
" $\omega >> \omega_o$, " " $\omega > 10\omega_o$, $\angle H = 180°$

" $\omega = \omega_o$, $\angle H = 90°$



Complex Conjugate Poles

$$H(s) = \frac{1}{1+2\zeta\left(\frac{s}{\omega_0}\right)+\left(\frac{s}{\omega_0}\right)^2}$$



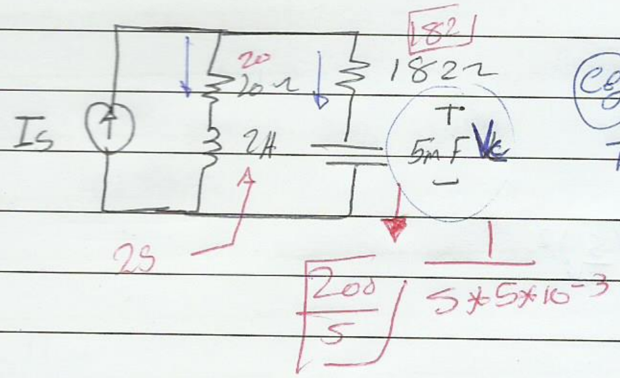
Multiple poles and zeros

$$H(s) = \left(1+ s/a\right)^{\pm m} \qquad m = 2, 3$$

In this case corner frequency $= a$

But the slopes are

1) $\pm 20m$ dB/dec
2) $\pm 45^\circ m$ /dec



Eg) Draw the Bode Plot of the T.f, $G(s) = \frac{V_c}{I_s}$

Solutions:

$$\therefore V_c = \frac{200}{S} * I_s * \frac{20+2S}{20+2S+18S+\frac{200}{S}}$$

By rearrangement it can be found that :

$$\frac{V_c}{I_s} = \frac{20+2S}{0.01S^2+1.01S+1}$$

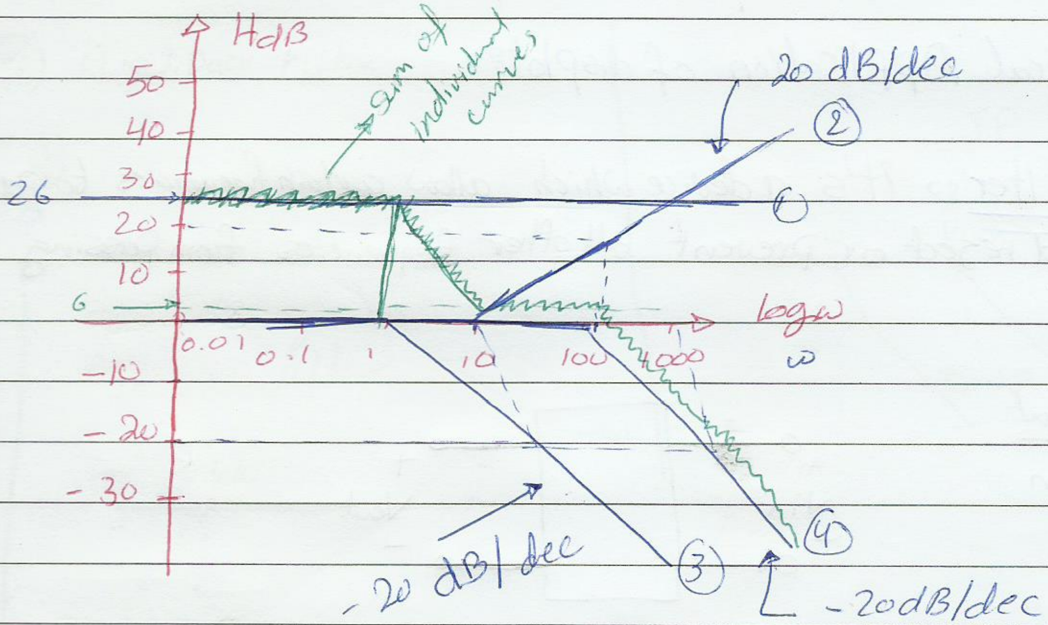
$$\therefore G(s) = \frac{V_c}{I_s} = \frac{20(1+S/10)}{0.01(S^2+101S+100)}$$

$$= \frac{2000(1+\frac{S}{10})}{(S+100)(S+1)}$$

لأنس الواحد

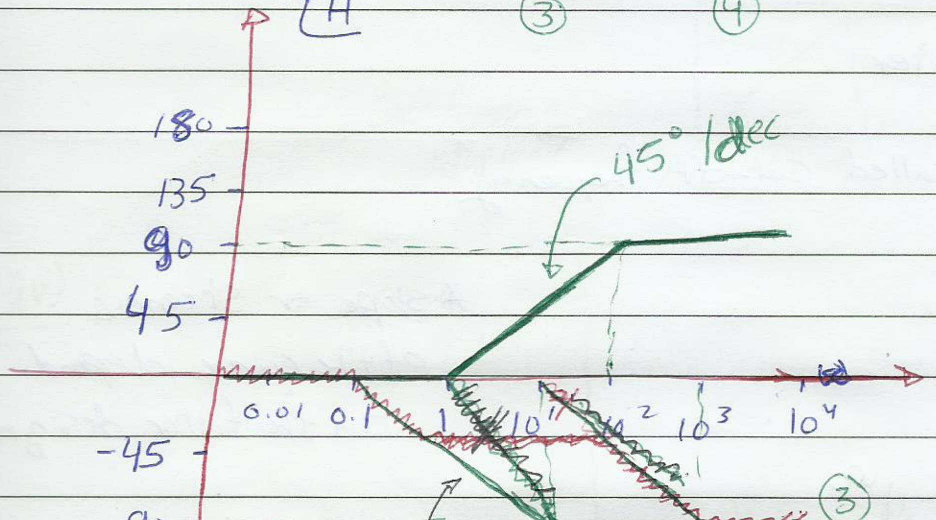
$$= \frac{200(1+\frac{S}{10})}{100(1+S)(1+\frac{S}{100})}$$

$$= \frac{\underset{①}{20}\ \underset{②}{(1+\frac{S}{10})}}{\underset{③}{(1+S)}\ \underset{④}{(1+\frac{S}{100})}}$$



23.12 :

$$H(s) = \frac{\overset{①}{20}\left(1+\overset{②}{\frac{s}{10}}\right)}{\underset{③}{\left(1+\frac{s}{100}\right)}\underset{④}{\left(1+\frac{s}{1}\right)}}$$

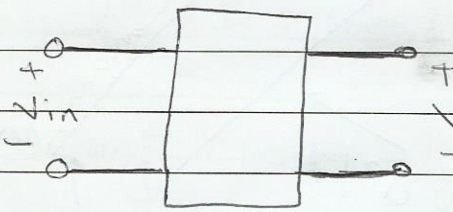


t.F وطلع dB plot بيعطيك سؤال ال *

* Practical Application of dis plots:

<u>Filter</u>: it's a device which allow certain frequencies to pass. and reject or prevent all other frequencies from passing

$H(S) = \frac{V_{out}}{V_{in}}$



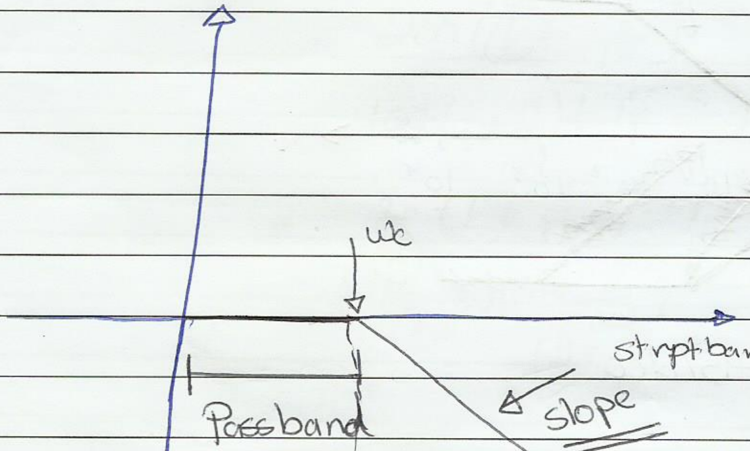
* when frequencies are allowed to pass, then $V_{out} = V_{in}$

$H(s) = 1 \qquad H_{dB} = 20 \log |H| = 20 \log 1 = 0$

* Hence by using the dB plot, filters can be classified into:

i) Low Pass Filter

$w_c \equiv$ is called cut-off frequency

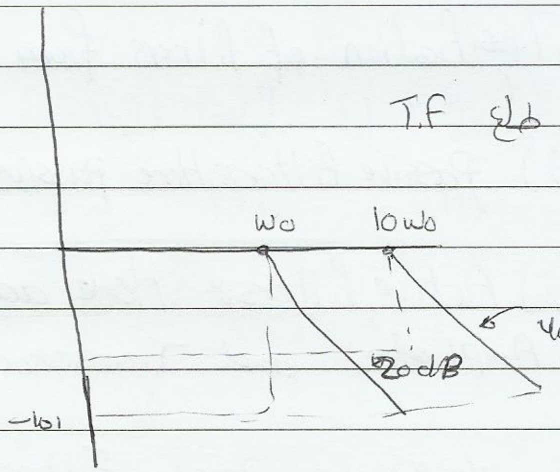


* slope or steeness of the curve depend on the filter designe

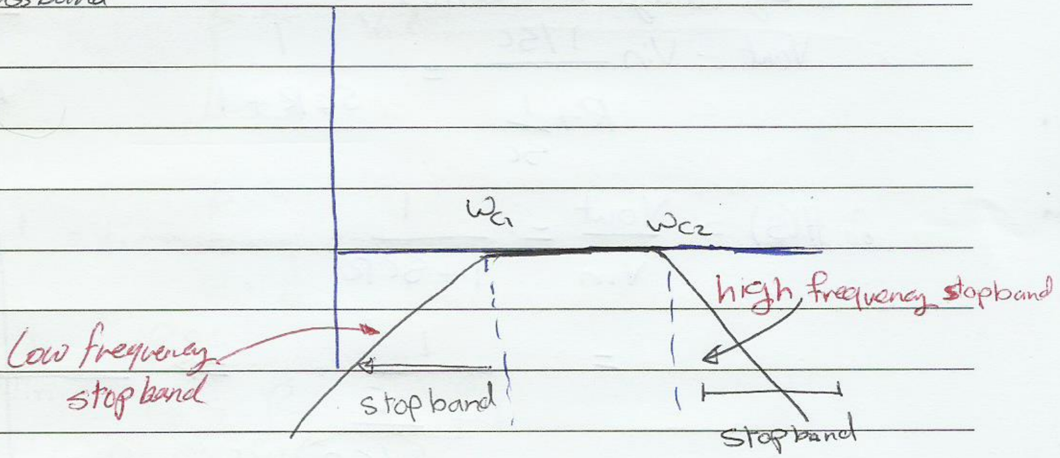
∴ signals with $w \leq w_c$ are allowed to pass

-101

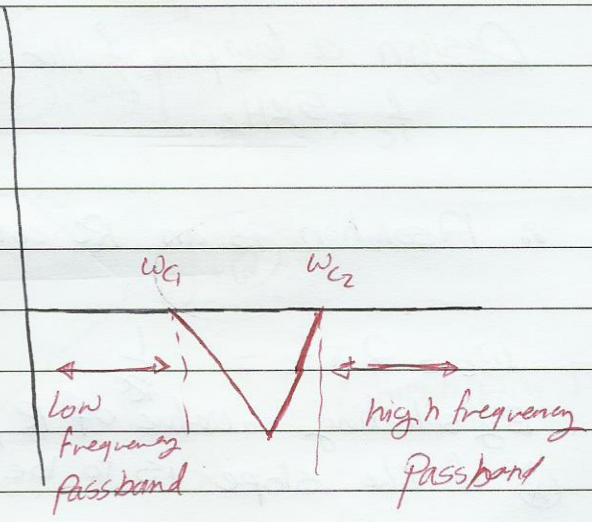
ii) Highpass Filter



iii) Band pass



iv) Band stop



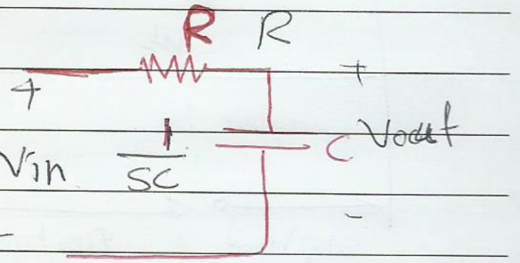
final exam:
Question about om-amp *
design passive filter in the book *

Classification of filters from construction point of view:

i) Passive filters: Here passive elements (R, L and C) are used

ii) Active filters: Here active elements such as operational Amplifier and Transistor are used.

Examples: in passive Filters:



↑ This cct is a low pass Passive filter

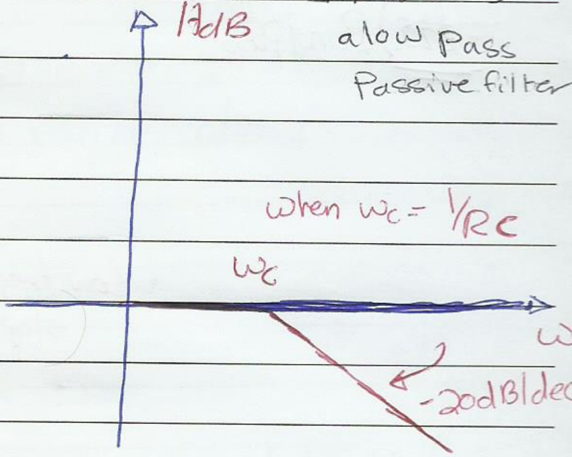
* consider the following RC ckt

∴ By voltage Division:

$$V_{out} = V_{in} \frac{1/SC}{R + \frac{1}{SC}} = \frac{1}{SCR+1}$$

$$\therefore H(S) = \frac{V_{out}}{V_{in}} = \frac{1}{1+SCR}$$

$$= \frac{1}{1 + \frac{S}{1/CR}} \quad \omega_c$$



ex: Design a low pass filter with $f_c = 5KHz$

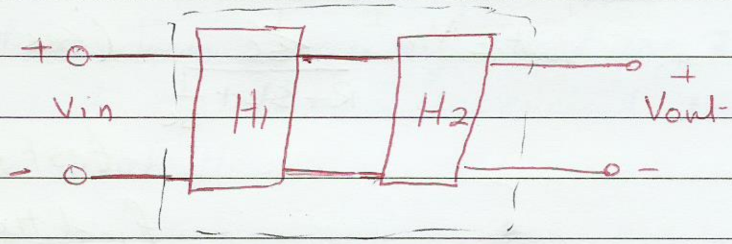
∴ A solution is an RC cct with output taken across C

$$\omega_c = 2\pi f_c = \frac{1}{RC}$$

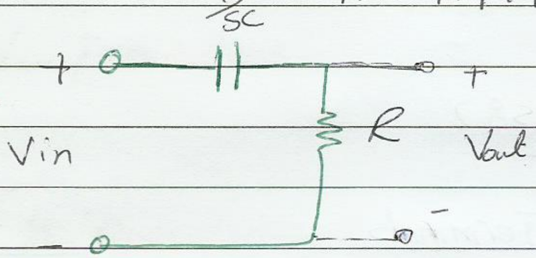
By selecting a value of R, then C can be evaluated.

* if the slope is to be for example −40dB/dec

Then $H(s) = \dfrac{1}{(\cancel{\#}1+\frac{s}{w_0})^2} = \underbrace{\dfrac{1}{(1+\frac{s}{w_0})}}_{H_1} \times \underbrace{\dfrac{1}{(1+\frac{s}{w_0})}}_{H_2}$



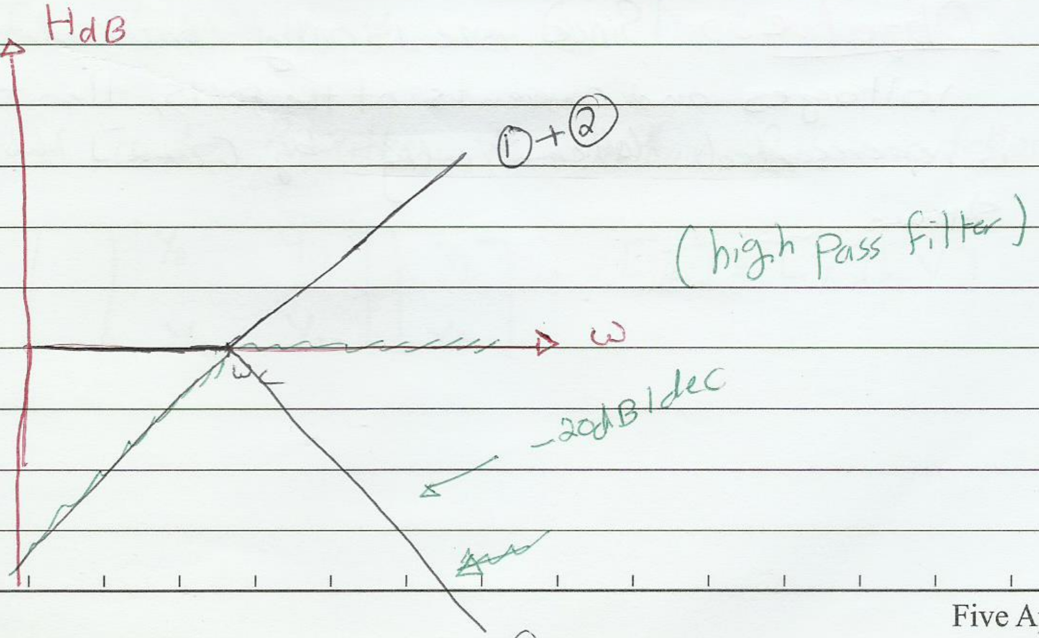
$H = H_1 \cdot H_2$

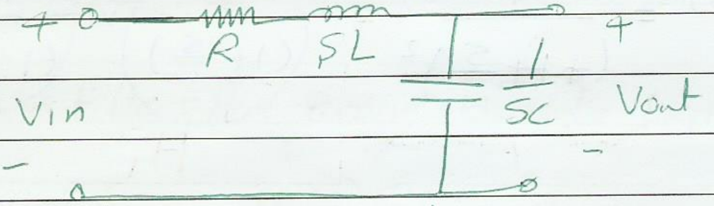


$\therefore V_{out} = V_{in} \dfrac{R}{R+\frac{1}{SC}}$

$\therefore \dfrac{V_{out}}{V_{in}} = \dfrac{RSC}{1+SCR}$

$H(s) = \dfrac{S\,Ri\;2}{\left[1+\dfrac{S}{(1/Rc)\,w_0}\right]^3}$





$$V_{out} = V_{in} \frac{\frac{1}{SC}}{R + SL + \frac{1}{SC}}$$ (see the book)

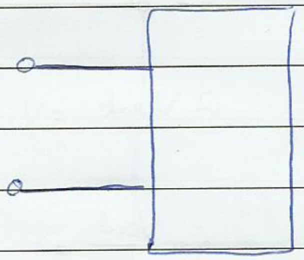
test this circuit to find the type of filter.

30/12

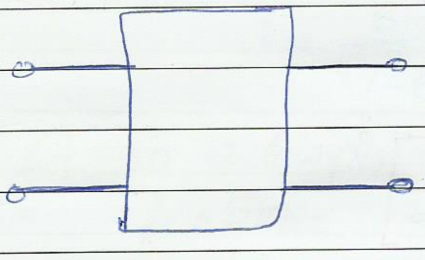
⇒ **Two Port Networks:-**

Port ≡ a Pair of Terminals

* Two-port Network :- a Network with 2 pairs of terminals.



one-port network

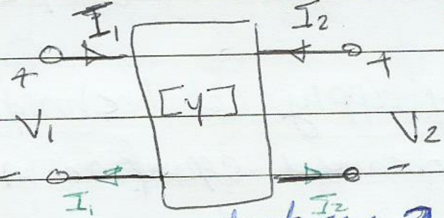


Objective :- Since one is only concerned with evaluation Voltages and currents of the ports, then the given Network is reperesented Mathematically by [2×2] Matrix as will be shown.

Y

Note network contain passive element and may contain Dependent sources but Not ~~...~~ independent sources.

Procedure:-



since there are 4 variables: V_1, V_2, I_1 and I_2. Then one may select any 2 to be independent variables, and the other 2 dependent variable are expressed as a function of them

Consequently, Parameters Can be Classification:-

i) Y - Parameters

ii) Z - 〃

iii) h - parameters

iv) t 〃

Y OR admittance parameters:-

Here I_1 and I_2 are expressed as functions of V_1 and V_2 as follow:-

$$\begin{cases} I_1 = Y_{11} V_1 + Y_{12} V_2 \quad \text{---} \quad (1) \\ I_2 = Y_{21} V_1 + Y_{22} V_2 \quad \text{---} \quad (2) \end{cases}$$

⟹ (1) and (2) can be Rewritten in a matrix ~~follow~~ as follows:-

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \Rightarrow [I] = [Y][V]$$

Methods of Evaluating [y] Parameters:

i) By apply cct techniques to the given cct. The Rearrange the obtained equation in matrix form Hence the obtained Coefficient give the [y] parameters.

ii) By applying s/c one at a time at the input and output ports, then evaluate the corresponding Parameters:

$$\therefore Y_{11} = \frac{I_1}{V_1}\bigg|_{V_2=0} \qquad Y_{12} = \frac{I_1}{V_2}\bigg|_{V_1=0}$$

$$Y_{21} = \frac{I_2}{V_1}\bigg|_{V_3=0} \qquad Y_{22} = \frac{I_2}{V_2}\bigg|_{V_1=0}$$

Y_{11} ≡ called s/c input admittance

Y_{22} ≡ " " output "

Y_{12}, Y_{21} = called s/c Transfer Admittance

If $Y_{12} = Y_{21}$ the Network is called Bilateral one

In a bilateral Networks:

If an ideal voltage source and an Ammeter are Interchanged, then the Ammeter Reading will not change

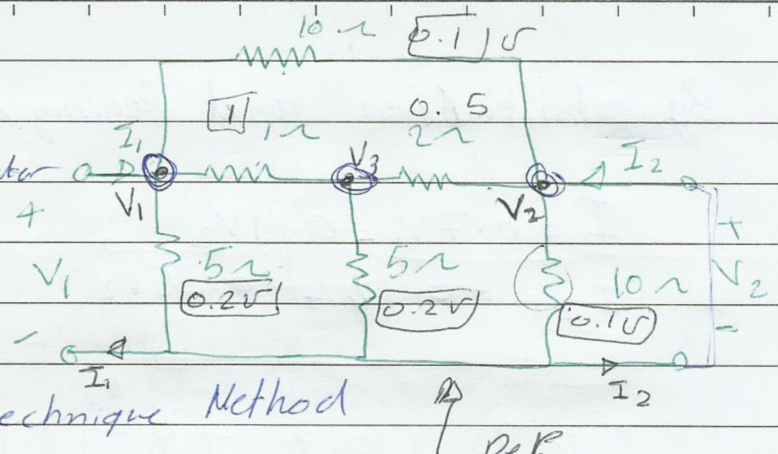
OR

" " " " current " " voltmeter " " " " "

" " voltmeter " " " " "

ex:

evaluate the [y] Parameter of the given Network



① By using cet technique Method

By using nodal technique

$$\begin{bmatrix} I_1 \\ I_2 \\ 0 \end{bmatrix} = \begin{bmatrix} 1.3 & -0.1 & -1 \\ -0.1 & 0.7 & -0.5 \\ -1 & -0.5 & 1.7 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \quad \text{--- (1)}$$

$Y_{ii} = \Sigma$ of admittance connected to i^{th} Node.

$Y_{ij} = -1 \times$ Admittance connected directly between i^{th} and j^{th} nodes.

∴ From ① ⇒

$$I_1 = 1.3V_1 - 0.1V_2 - V_3 \quad (2)$$
$$I_2 = -0.1V_1 + 0.7V_2 - 0.5V_3 \quad (3)$$
$$0 = -V_1 - 0.5V_2 + 1.7V_3 \quad (4)$$

④ $\Rightarrow V_3 = \dfrac{V_1 + 0.5V_2}{1.7}$ — ⑤

∴ substitude ⑤ into ② and ③

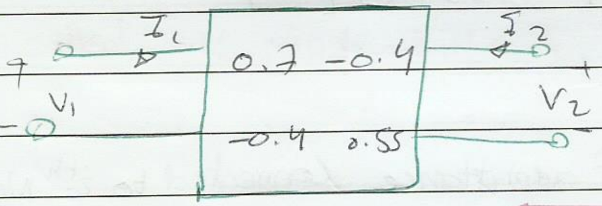
By substitution and Rearragment it can be found:

$I_1 = 0.7V_1 - 0.4V_2$

$I_2 = -0.4V_1 + 0.55V_2$

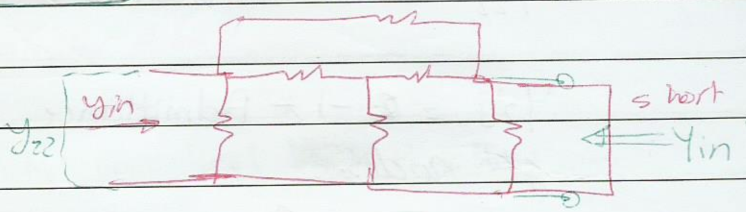
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \overset{y_{11}}{0.7} & \overset{y_{12}}{-0.4} \\ \underset{y_{21}}{-0.4} & \underset{y_{22}}{0.55} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Comment: Hence the Network is Bilateral.



* إفتكار التماثل.

S/C Technique:

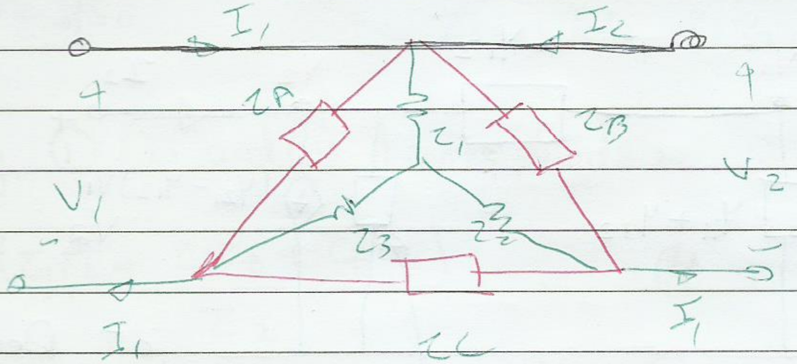


$Y_{11} = \dfrac{I_1}{V_1}\Big|_{V_2=0} = y_{in} = 0.7$

$Y_{22} = \dfrac{I_2}{V_2}\Big|_{V_1=0} = Y_{in}' = 0.55$

$Y_{12} = \dfrac{I_1}{V_2}\Big|_{V_1=0} = -0.4$

* use circut technique to find Relationship between $\dfrac{I_1}{V_2}$



Application of [Y] parameters:

Evaluating equivalent cct of some electronic ccts

Since $I_1 = Y_{11} V_1 + Y_{12} V_2$ --- (1)

$I_2 = Y_{21} V_1 + Y_{22} V_2$ --- (2)

If $Y_{12} = Y_{21}$, then the equivalent cct can be deduced directly.

If $Y_{12} \neq Y_{21}$, then equ (1) OR (2) Has to be modified in such a way the mutual element is the same.

For (e.g), Modify equ (2) as follows:

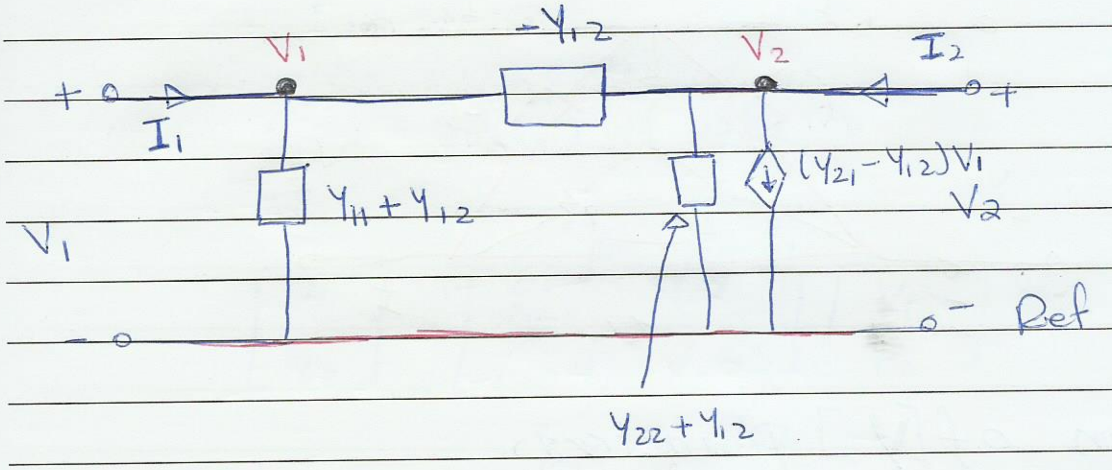
$I_2 = Y_{21} V_1 + Y_{22} V_2 + Y_{12} V_1 - Y_{12} V_1$

$= Y_{12} V_1 + Y_{22} V_2 + (Y_{21} - Y_{12}) V_1$

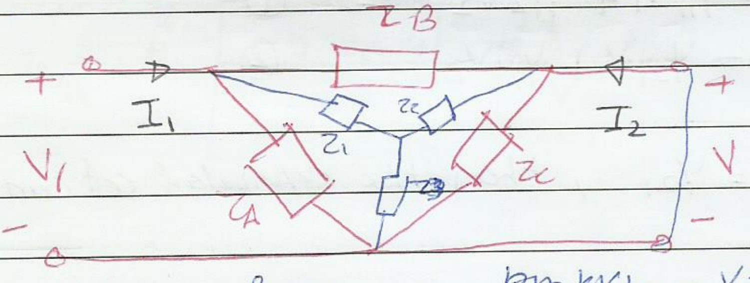
$\therefore I_2 - (Y_{21} - Y_{12}) V_1 = Y_{12} V_1 + Y_{22} V_2$ --- (3)

5 5 25 +
5 5 60
10 30 + 70
10

∴ Deduced the equivalent cct. for equation (1) and (3) as follows:



Prouf of $\Delta \rightleftarrows Y$ ~~transform~~ transformation. (7) +



for Y_{22} by KVL $-V_2 + I_1 Z_B = 0 \Rightarrow \frac{I_2}{V_2} = -\frac{1}{Z_B}$

if the Δ and Y connection are equivalent then ~~the~~ should have the same paramete.

Here Evaluate, Their [y] parameter

$$Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0} = \frac{1}{Z_A} + \frac{1}{Z_B} = \frac{1}{Z_1 + \frac{Z_2 Z_3}{Z_2+Z_3}} \quad \text{— (1)}$$

$$Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0} = \frac{1}{Z_B} + \frac{1}{Z_C} = \frac{1}{Z_2 + \frac{Z_1 Z_3}{Z_1+Z_3}} \quad \text{— (2)}$$

اعادها البست

**

$$Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0} = \frac{-1}{Z_B} = \frac{-Z_3}{Z_1 Z_2 + Z_1 Z_2 + Z_3 Z_2} \quad \ldots (3)$$

By solving (1), (2) and (3) it can be found:

$$Z_A = \frac{(Z_1 Z_2 + Z_1 Z_3 + Z_3 Z_2)}{Z_2} \quad \ldots (4)$$

$$Z_B = \frac{(\qquad)}{Z_3} \quad \ldots (5)$$

$$Z_C = \frac{(\qquad)}{Z_1} \quad \ldots (6)$$

(4), (5), and (6) can be used for $Y \rightarrow \Delta$ Transformation

OR

$$Z_1 = \frac{Z_A Z_B}{(Z_A + Z_B + Z_C)} \quad \ldots (7)$$

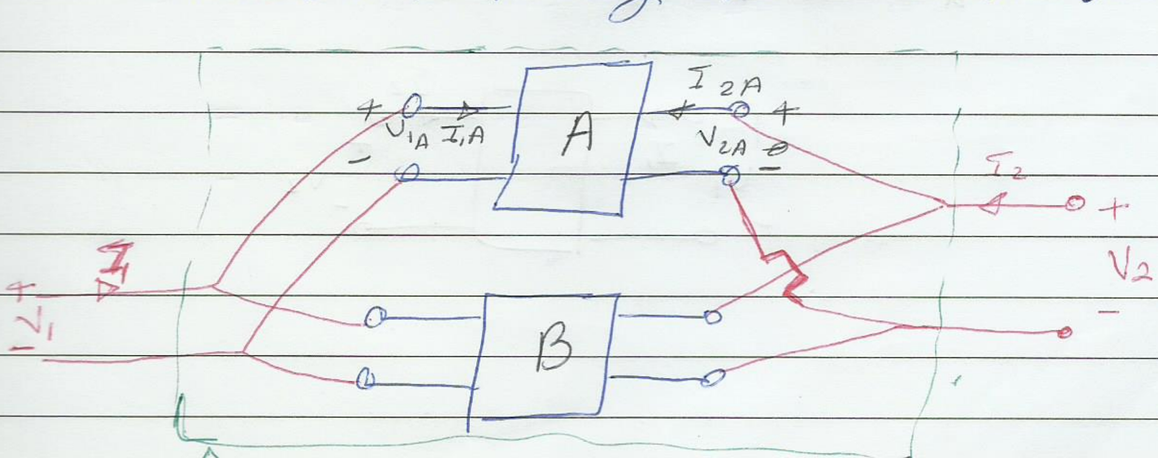
$$Z_2 = \frac{Z_B Z_C}{(Z_A + Z_B + Z_C)} \quad \ldots (8)$$

$$Z_3 = \frac{Z_A Z_C}{(Z_A + Z_B + Z_C)} \quad \ldots (9)$$

(7), (8) and (9) can be use for $\Delta \rightarrow Y$ Transformation

Y-Parameters of Parallel connected networks:

Consider the following 2 Network A and B:



$I_1 = I_{1A} + I_{1B}$

$I_2 = I_{2A} + I_{2B}$

$$\therefore \begin{bmatrix} I_{1A} \\ I_{2A} \end{bmatrix} = [Y_{12}] \begin{bmatrix} V_{1A} \\ V_{2A} \end{bmatrix} \sim ①$$

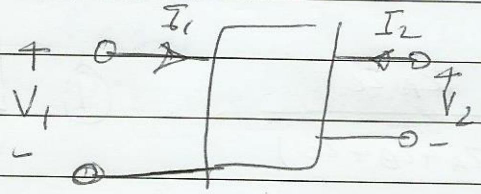
$$\begin{bmatrix} I_{1B} \\ I_{2B} \end{bmatrix} = [Y_B] \begin{bmatrix} V_{1B} \\ V_{2B} \end{bmatrix} \sim ②$$

$$\therefore ① + ② \Rightarrow \begin{bmatrix} I_{1A} + I_{1B} \\ I_{2A} + I_{2B} \end{bmatrix} = \left[[Y_A] + [Y_B] \right] \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\downarrow$$

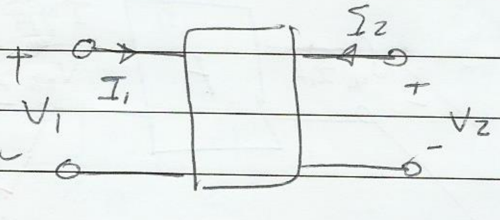
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \left[[Y_A] + [Y_B] \right] \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [Y_{eq}] \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Z_Parameters:



Defind as follows

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2 = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$



$S = \sqrt{3} \, P_p \, \sqrt{PF}$

$\cos^{-1}$

find the equivalent cct * سؤال

by Z parameter

Evaluation Techniques :-

① using cct technique as explained and illustrated in the case of [Y] Parameter.

② by performing o/c Tests at the ports as follows :-

$$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0} \qquad Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0}$$

$$Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0} \qquad Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0}$$

∴ Z parameters represent Impedances

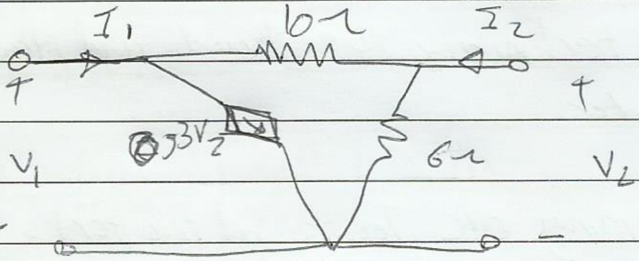
Z_{11} & Z_{22} ≡ called the o/c Input and Output Impedances.

Z_{12}, Z_{21} ≡ called o/c Transfer Impedances.

If $Z_{12} = Z_{21}$ the circuit is called bilateral.

ex 8)

evaluate the [Z] Parameters of the given network



$$Z_{11} = \frac{V_1}{I_1}\Big|_{V_2=0}$$

$$Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0}$$

By kirchoff current laws KCL

$$I_1 = 0.3V_2 + \frac{V_2}{6} = \frac{2.8}{6}V_2 \quad \text{--- (1)}$$

By KVL

$$V_1 = \frac{V_2}{6} \times 10 + V_2 = \frac{16}{6}V_2 \quad \text{--- (2)}$$

∴ Substitude (2) into (1) ⇒

$$I_1 = \frac{2.8}{6} \times \frac{6}{16}V_1$$

$$\therefore Z_{11} = \frac{V_1}{I_1} = \frac{6 \times 16}{6 \times 2.8}$$

$$= 5.71$$

$$(1) \Rightarrow \frac{V_2}{I_1} = \frac{6}{2.8} = Z_{21}$$

$$= 2.14$$

By similar Approach:

It can be found:

$Z_{12} = -4.28$

$Z_{22} = 2.14$

Equivalent cct: of [Z]

A سؤال الامتحان معادلات وطالب ال equations

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad \text{---} \quad (1)$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad \text{---} \quad (2)$$

in order to have the same Mutual element between 2 Meshes, then modify equ (1) or (2)

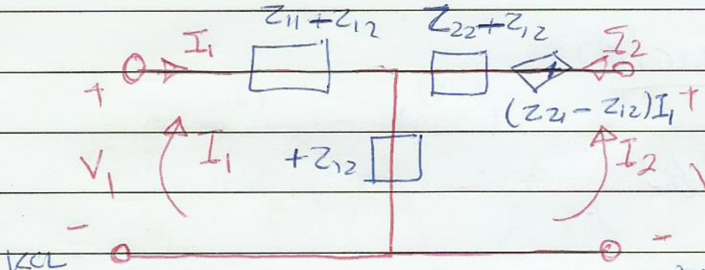
For (e.g) Modify equ. (2)

$$(2) \Rightarrow V_2 = Z_{21} I_1 + Z_{22} I_2 + Z_{12} I_1 - Z_{12} I_1$$

$$\therefore V_2 = Z_{12} I_1 + Z_{22} I_2 + (Z_{21} - Z_{12}) I_1$$

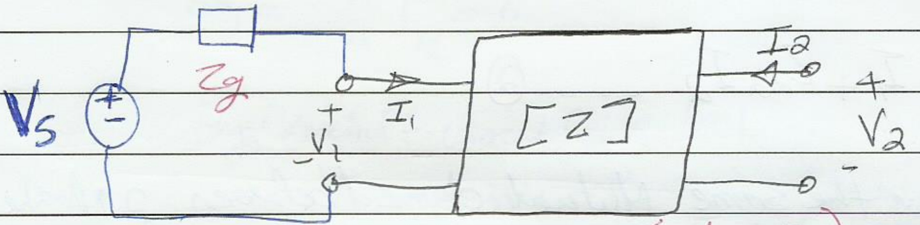
$$\therefore V_2 - (Z_{21} - Z_{12}) I_1 = Z_{12} I_1 + Z_{22} I_2 \quad \text{--} \quad (2)$$

$\therefore$ deduce the equivalent ckt by using equs (1) and (3)

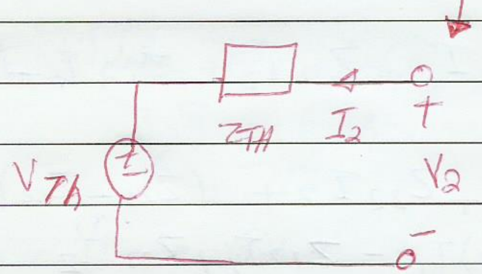


$$V_1 = I_1(Z_{11}+Z_{12}) + (I_1+I_2)(-Z_{12})$$
$$= I_1 Z_{11} + \cancel{I_1 Z_{12}} - \cancel{I_1 Z_{12}} - I_2 Z_{12}$$

Thevenen equivalent:



objectives: Find The equivalent seen from the output port.



Objectives:

find An equ:

$$V_2 = I_2 Z_{Th} + V_{TH}$$
$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad \text{--- (1)}$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad \text{--- (2)}$$
$$V_S = I_1 Z_g + V_1 \quad \text{--- (3)}$$

③ $\Rightarrow V_1 = V_S - I_1 Z_g$ -- ④

Sub ④ into ① $\Rightarrow$

$$V_S - I_1 Z_g = Z_{11} I_1 + Z_{12} I_2$$

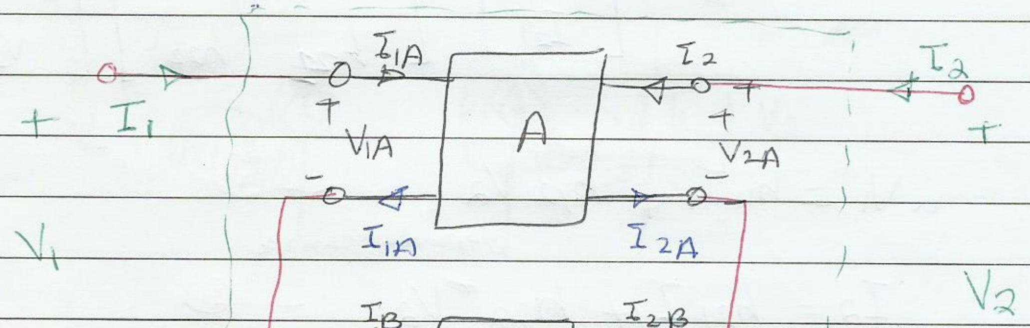
$$\therefore I_1 \times (Z_{11} + Z_g) = V_S - Z_{12} I_2$$

$$\therefore I_1 = \frac{V_S - Z_{12} I_2}{Z_{11} + Z_g} \quad \text{--- ⑤}$$

Substitude ⑤ into ② and Rearrange, it ~~an~~ can be found

$$V_2 = \underbrace{V_S \frac{Z_{21}}{Z_{11} + Z_g}}_{V_{TH}} + I_S \underbrace{\left(Z_{22} - \frac{Z_{12} Z_{21}}{Z_{11} + Z_g} \right)}_{Z_{TH}}$$

Series connection :-



Objective find [Z] of the equ network.

<u>Procedure :-</u> it can be found $I_1 = I_{1A} = I_{1B}$

$$I_2 = I_{2A} = I_{2B}$$

$$V_1 = V_{1A} + V_{1B}$$

$$V_2 = V_{2A} + V_{2B}$$

A

$$\begin{bmatrix} V_{1A} \\ V_{2A} \end{bmatrix} = \begin{bmatrix} Z_A \end{bmatrix} \begin{bmatrix} I_{1A} \\ I_{2A} \end{bmatrix} \quad \text{--- (1)}$$

B

$$\begin{bmatrix} V_{1B} \\ V_{2B} \end{bmatrix} = \begin{bmatrix} Z_B \end{bmatrix} \begin{bmatrix} I_{1B} \\ I_{2B} \end{bmatrix} \quad \text{--- (2)}$$

$\therefore$ (1) + (2) $\Rightarrow$

$$\begin{bmatrix} V_{1A} + V_{1B} = V_1 \\ V_{2A} + V_{2B} = V_2 \end{bmatrix} = \Big[\underbrace{[Z_A] + [Z_B]}_{Z_{eq}} \Big] \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Hybrid or [h] Parameters:

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$\therefore V_1 = h_{11} I_1 + \underbrace{h_{12} V_2}_{\text{dependent source}} \quad \text{--- (1)}$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \text{--- (2)}$$

Methods of evaluation:

1) As before using circuit techniques
2) by performing short circuit and open circuit ~~test~~ as follows:

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = \text{S/C input impedance}$$

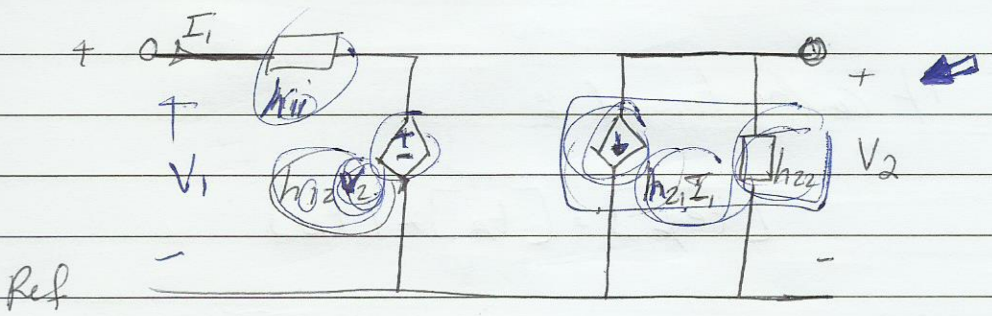
$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = \text{S/C forward current gain}$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = \text{O/C Reversed voltage gain}$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = \text{O/C output admittance.}$$

Hence the name hybrid parameter

Equivalent cct by using (1) and (2) so



is Equivalent circuit repersent of a transistor whose parameters can be measured experimentally.

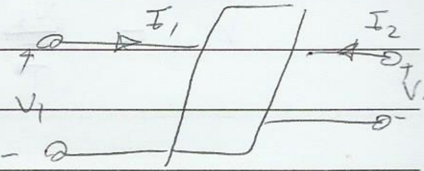
* سؤال حول Z Parameters ← Y Parameters

* إذا two Port in series Z Parameters نستخدم Z و Y two Port Parameters in parallel

Transmission Parameters [t]

* إذا cascade t Parameters

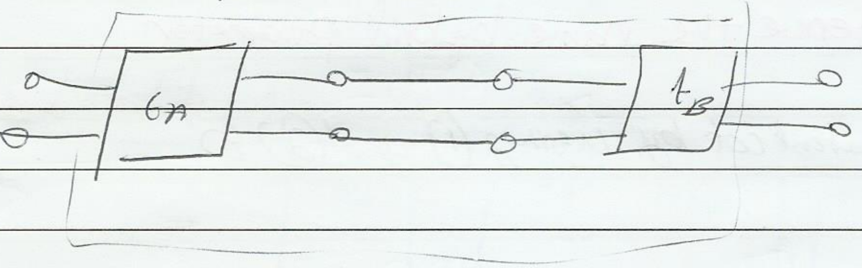
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$



The same previous method can be used to evaluate [t] parameter.

Cascade Connections:

output of the first is the input of the second



it can be found:

$$[t_{eq}] = [t_A] \times [t_B]$$