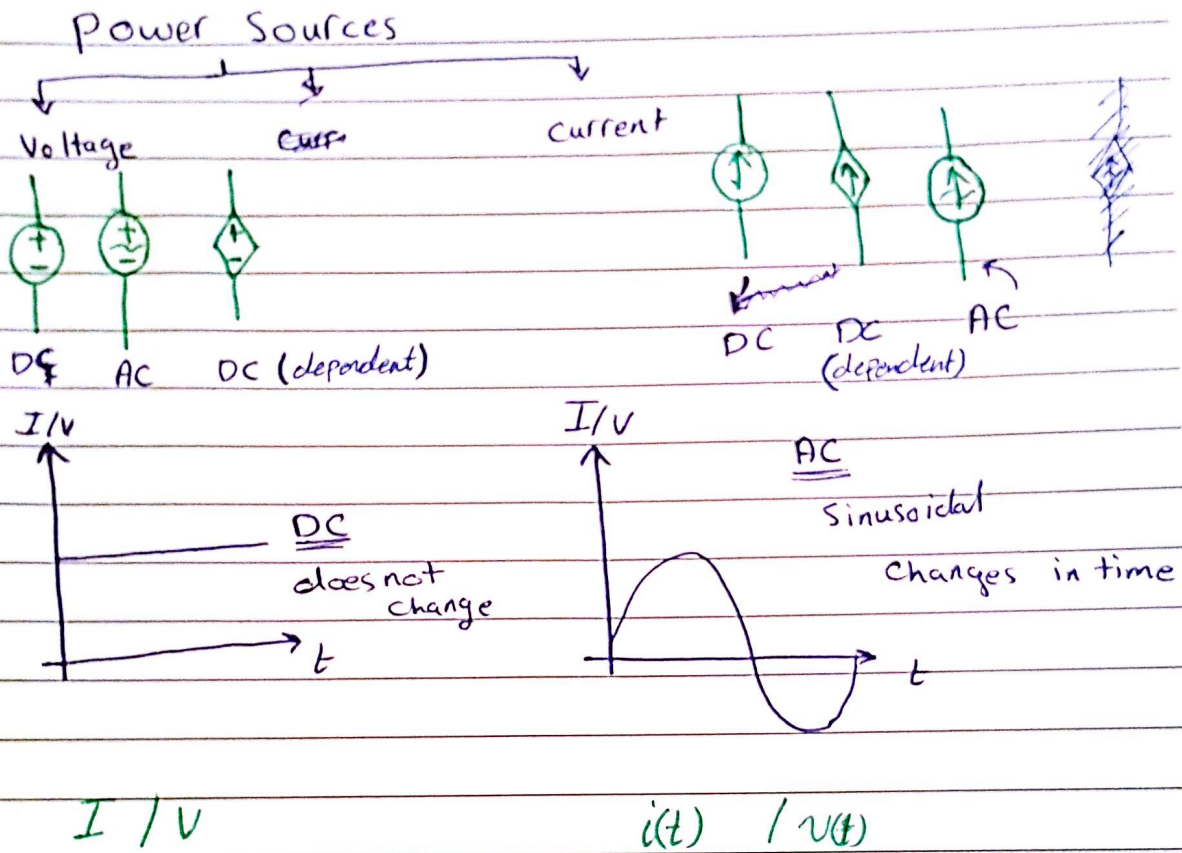
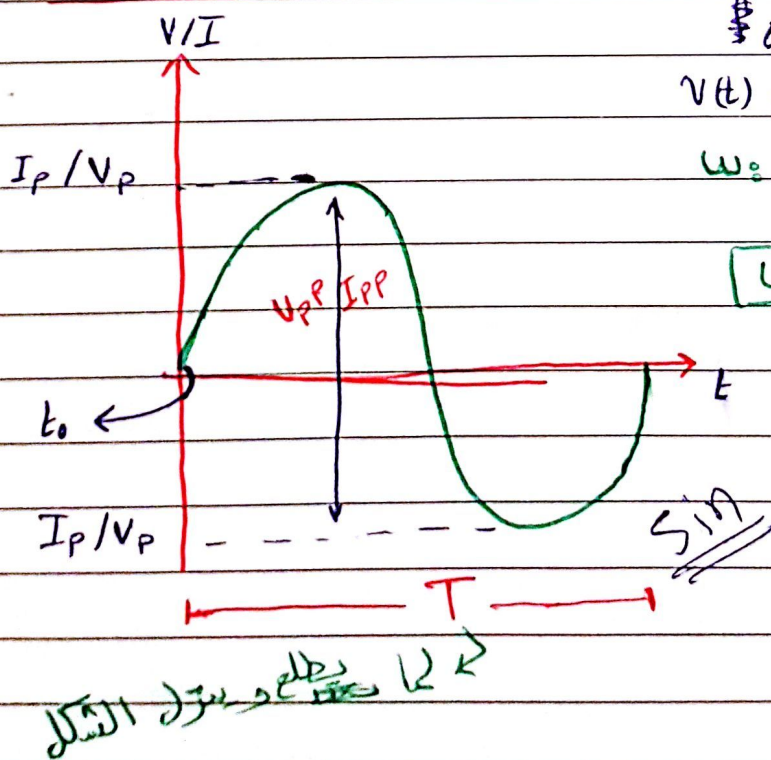


# power unit

CKTS1:



## Chapter 10: Sinusoidal



$$i(t) = i_p \sin(\omega t + \theta)$$

$$v(t) = v_p \sin(\omega t + \theta)$$

$\omega$ : angular frequency

$$\omega = 2\pi f$$

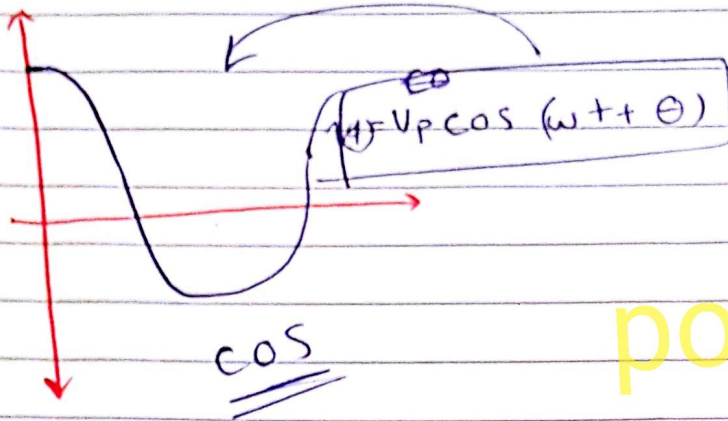
$$f = \frac{1}{T}$$

$\theta$ : phase shift

$$\theta = \frac{t_0}{T} \times 360^\circ$$

$f$ : frequency  
 $T$ : time

تغير في الجهد والقدرة



power unit

$$v(t) = v_p \cos(\omega t + \theta_1)$$

Leads or Lags ???!

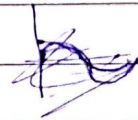
$$i(t) = i_p \cos(\omega t + \theta_2)$$

same  $\omega$

→  $\theta_1 > \theta_2$  V leads I

→  $\theta_2 > \theta_1$  I leads V

→  $\theta_1 = \theta_2$  I & V are in phase



COS → Sin

COS

~~$\cos(\theta - 90) = \sin(\theta)$~~

$\cos(\theta + 90) = \sin(\theta)$

Sin

التحويل من Sin الى Cos بزيادة 90 من phase shift

التحويل من Cos الى Sin

$$\sin 90 = \cos 0$$

$$= 1$$

$$\sin 180 = \cos 90$$

$$\sin 270 = \cos 180$$

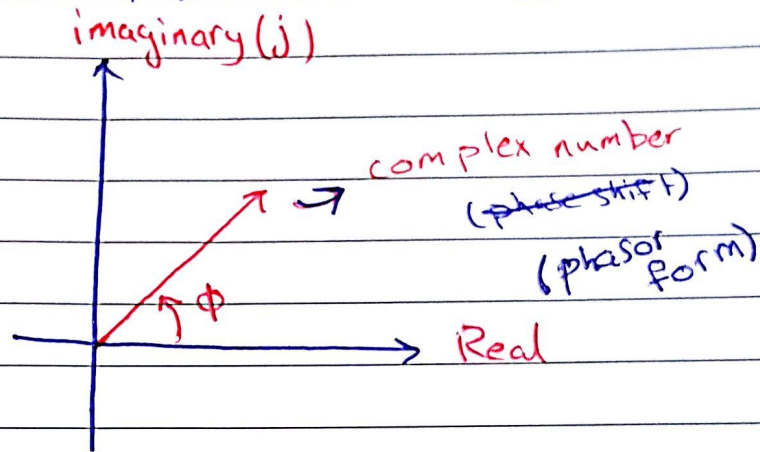
$$\sin 0 = \cos 270$$

# Complex numbers:

في نوعين من الأرقام  
real ← حقيقية  
imaginary ← خيالية

Imaginary number is  $\sqrt{-1} \Rightarrow$  ويختل  $\sqrt{-1}$  بـ  $j$

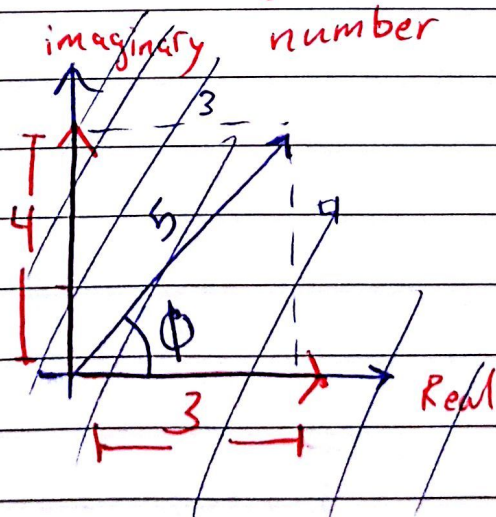
Complex numbers ← في وعند دمج الأرقام الحقيقية مع الخيالية  
imaginary Real



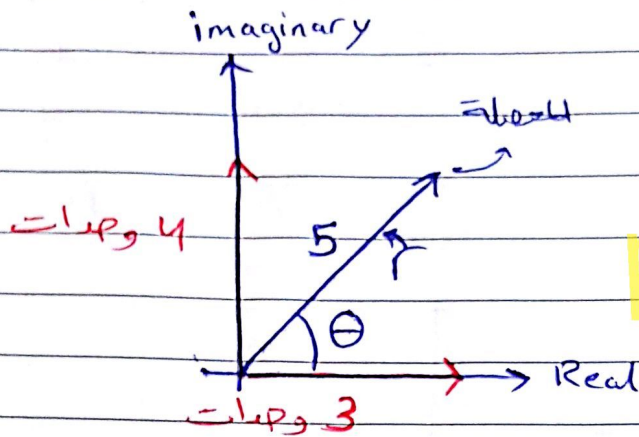
Example on complex numbers: power unit

~~$3 + 4j$~~

$3 + 4j$   
Real number ←      imaginary ←



$3 + 4j$  كما قلنا سابقا  
 الجزء الـ Real ←  $3$   
 الجزء الـ imaginary ←  $4j$



\* متجه بياننا : **power unit**

لايجاد  $\theta$

$\theta = \tan^{-1}\left(\frac{\text{imaginary}}{\text{Real}}\right) + 180^\circ$  ربع أول / ربع ← ربع ثانٍ / ثالث  
 $\theta = \tan^{-1}\left(\frac{\text{imaginary}}{\text{Real}}\right)$  ربع أول / ربع ← ربع ثانٍ / ثالث

$\theta = \tan^{-1}\left(\frac{4}{3}\right) = 53.1$

لايجاد للمتجه

$r = \sqrt{(\text{imaginary})^2 + (\text{Real})^2}$

~~r =~~

$r = \sqrt{4^2 + 3^2} = 5$

$3 + 4j$  بياننا  
 الـ phasor form  
 الـ phase shift

$3 + 4j = 5 \angle 53.1^\circ$

↓  
complex number

↓  
~~phase shift~~  
phasor form

العمليات الحسابية في ال complex numbers

$$(3+4j) + (5+6j) = 8+10j$$

جمع عادي  
ال Real مع بعض

وال imaginary مع بعض

$$(3+4j) \cdot (5+6j)$$

$$\begin{aligned} &= 15 + 20j + 18j + 6 \times 4 (j \times j) \\ &= 15 + 38j + 24(-1) \\ &= -9 + 38j \end{aligned}$$

$$\begin{aligned} j \times j &= -1 \\ (\sqrt{-1})^2 &= -1 \end{aligned}$$

\* conjugate

power unit

$$\begin{aligned} z &= x + yj \\ z^* &= x - yj \end{aligned} \quad \left. \begin{array}{l} \rightarrow \\ \leftarrow \end{array} \right\} \text{conj.}$$

$$z \cdot z^* = x^2 + y^2 \Rightarrow \text{the magnitude of the vector squared}$$

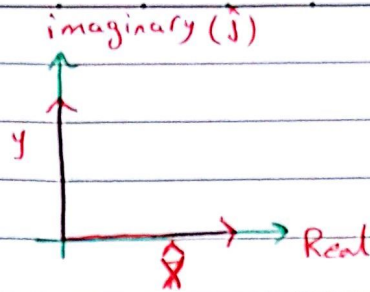
\* تمام هذه العمليات على القناة ال YouTube.com

$$\ominus = \frac{\text{Imaginary}}{\text{Real}}$$

# \* Complex numbers

$$\hat{x} + \hat{y}j$$

↓  
Real
↓  
imaginary



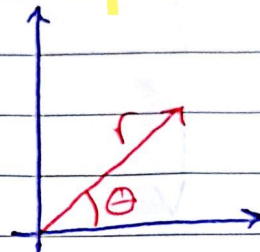
power unit

# \* phasor form

$$r \angle \theta$$

التحويل من complex إلى phasor

$$\rightarrow r = \sqrt{(\hat{x})^2 + (\hat{y})^2}$$



طول السهم (المقدار):  $r$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

ربع أول (الاج)

الزاوية بين السهم والسنين  $r$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) + 180^\circ$$

ربع ثلث (ثاني)

Sinusoidal form  $\rightarrow$  phasor form

$$v(t) = v_p \cos(\omega t + \theta)$$

↓  
 $r$

↓  
 $\theta$

$$v(t) = v_p \cos(\omega t + \theta)$$

$3+4j$

مثال:

$$r = \sqrt{3^2 + 4^2} = 5$$

$$\Rightarrow 5 \angle 53.1$$

$$\theta = \tan^{-1}\left(\frac{4}{3}\right) = 53.1$$

$$v(t) = 5 \cos(\omega t + 53.1)$$

العمليات الحسابية في ال phasor form

$$r_1 \angle \theta_1 \cdot r_2 \angle \theta_2 = r_1 r_2 \angle \theta_1 + \theta_2$$

$$\frac{r_1 \angle \theta_1}{r_2 \angle \theta_2} = \frac{r_1}{r_2} \angle \theta_1 - \theta_2$$

$$r_1 \angle \theta_1 \rightarrow r_1 \angle -\theta_1 \text{ conjugate}$$

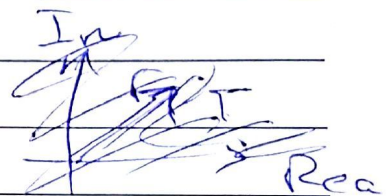
$\theta = 0 \rightarrow$  <sup>+ve</sup> Real number

$\theta = 90 \rightarrow$  +ve imaginary

$\theta = 270/-90 \rightarrow$  -ve imaginary

$\theta = 180 \rightarrow$  +ve Real  $\rightarrow$  magnitude \* ما يكتب بالـ

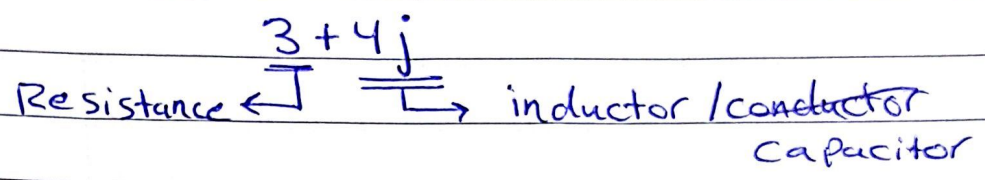
← فكر فيها  $r = \sqrt{x^2 + y^2}$   
رأسها متكون موجبة



power unit

phasor form وال phase shift  
Complex numbers

Complex numbers عبارة عن Real و Imaginary P.



Resistance الى inductor ال conductor لتحويل complex ال numbers

و كما ان مقاومته يكون خيالي imaginary

impedance ال

inductor

capacitor  
conductor

$$i(t) = \frac{1}{L} \int v(t) dt$$

$$v(t) = \frac{1}{C} \int i(t) dt$$

$$v(t) = L \frac{di(t)}{dt} = LI'$$

$$i(t) = C \frac{dv(t)}{dt} = cV'$$

$$V_L = LI'$$

$$i_c = cV'$$

$$Z = j\omega L$$

$$Z = \frac{1}{j\omega c}$$


power unit

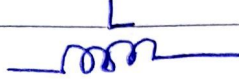
قاعدة


$$-j = \frac{1}{j} *$$



زي ما اشدنا في 3 انواع من ال devices

① Resistance  $\leftarrow$  

② Inductor  $\leftarrow$  

③ capacitor  $\leftarrow$  

دس اشدنا هم لما يكون  $v$  و  $i$  dc  
voltage current

طيب لما يكون AC؟؟!! شو نعمل

مع نعمل ال L و ال C impedance

و عشان ال L و ال C ~~م~~ ما عندنا Real Resistance

مع نستعمل ال  $Z$  (complex numbers) لتحويل ال imaginary resistance

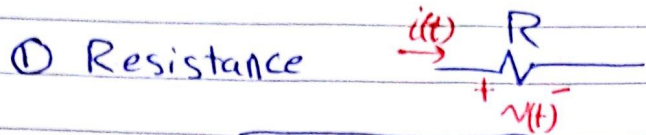
impedance means: هو ال L و ال C و ال R

في ال circuit

يكون في ال AC

circuits

power unit

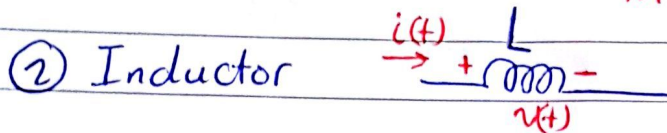


$$v(t) = i(t) \cdot R \quad / \quad v(t) \text{ \& } i(t) \text{ are in phase}$$

$$i(t) = i_m \cos(\omega t + \theta) \Rightarrow i_m \angle \theta$$

$$v(t) = R \cdot i_m \cos(\omega t + \theta) \Rightarrow R i_m \angle \theta$$

$\downarrow$   $v_m \angle \theta$



power unit

$$v(t) = L \frac{di(t)}{dt}$$

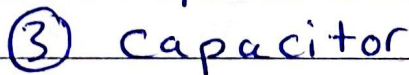
$$i(t) = i_m \cos(\omega t + \theta)$$

$$v(t) = -L \omega \sin(\omega t + \theta)$$

$$v(t) = -i_m \cdot \omega L \sin(\omega t + \theta)$$

~~AEI~~  $v_L$  leads  $i_L$  by  $90^\circ$

~~impedance of~~



$$v(t) = \frac{1}{C} \int i(t) dt$$

$$i(t) = i_m \cos(\omega t + \theta)$$

$$v(t) = \frac{1}{\omega C} \sin(\omega t + \theta)$$

$i_C$  leads  $v_C$  by  $90^\circ$

\* ELI is the ICE man

$\downarrow \downarrow \downarrow$                        $\downarrow \downarrow \downarrow$   
 $v \quad L \quad I$                        $I \quad C \quad v$

$v$  leads  $I$   
in  
Inductor

$I$  leads  $v$   
in  
capacitor

No. \_\_\_\_\_

# power unit

## \* Impedance

① Inductor  $Z = j\omega L \rightarrow$   
↓  
Angular frequency

وولت ۵۱۰  
 $V_L = I_m \omega L \sin(\omega t + \theta)$

$$V_L = (j\omega L) I$$

Capacitor  
② Conductor

$$Z = \frac{1}{j\omega C} = \frac{-j}{\omega C}$$

$$V_C = \left( \frac{-j}{\omega C} \right) I$$

Ohm's law پهنای پهنای

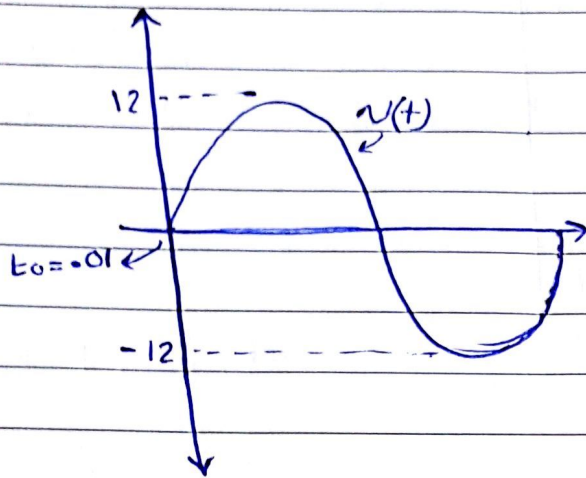
$$V = IR$$

$$\Rightarrow V = I \cdot Z$$

\* Examples & Exercises :

## power unit

II



$$\omega = 20\pi$$

find  $v(t)$ ,  $v_p$ ,  $v_{pp}$ ,  $T$ ,  $f$ ,  $\theta$  ?Sol:

$$v_{pp} = 12 - (-12) = 24 \text{ volt} \rightarrow \text{من القمة للقاع}$$

$$v_p = \frac{v_{pp}}{2} = 12 \text{ volt}$$

$$v(t) = v_p \sin(\omega t + \theta) \rightarrow \text{المترت Sin على شكل الجيب}$$

$$\theta = \frac{t_0 \times 360^\circ}{T} \quad \left| \quad T = \frac{1}{f}$$

$$= \frac{.01}{.1} \times 360 \quad \left| \quad f = \frac{\omega}{2\pi} = \frac{20\pi}{2\pi} = 10 \text{ Hz}$$

$$\theta = 36^\circ$$

$$T = \frac{1}{10} = .1 \text{ sec}$$

$$v(t) = 12 \sin(20\pi t + 36) \text{ volt} \quad \left| \quad f = 10 \text{ Hz}$$

$$\theta = 36^\circ$$

$$T = .1 \text{ Sec}$$

$$v_p = 12 \text{ volt}$$

$$v_{pp} = 24 \text{ volt}$$

No. \_\_\_\_\_

Sketch:

power unit

Sinusoid Signals

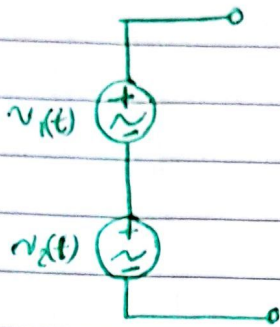
$$v(t) = v_p \cos(\omega t + \theta)$$

$$v(t) = A \cos(\omega t + \theta) : \text{Time domain form}$$

$$v(t) = A e^{j(\omega t + \theta)} = A e^{j\omega t} \cdot e^{j\theta} = A e^{j\theta} : \text{frequency domain form}$$

$$v(t) = A \cos(\omega t + \theta) \rightarrow A \angle \theta = A e^{j\theta}$$

## power unit

[2] Calculate  $V_{eq}$ ?

$$v_1(t) = 15 \cos(377t + \pi/4)$$

$$v_2(t) = 15 \cos(377t + \pi/2)$$

Sol:

$$v_1(t) = 15 \angle \pi/4$$

$$v_2(t) = 15 \angle \pi/2$$

$$v_s = v_1 + v_2 = 15 \angle 45^\circ + 15 \angle 90^\circ$$

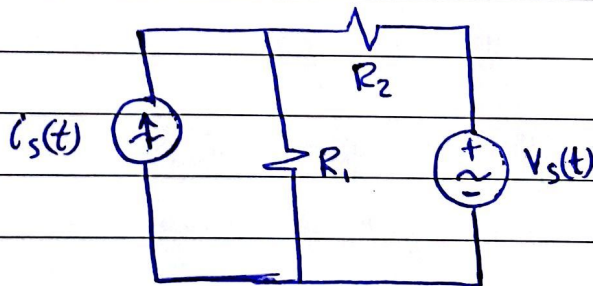
$$= 25 + 14.5j$$

$$\Rightarrow 29 \angle 30^\circ$$

$$v_s = 29 \cos(377t + 30^\circ)$$

$$v_s = 29 \cos(377t + 30^\circ)$$

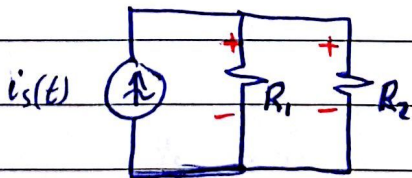
[3] compute the resistor voltages?



$$i_s(t) = 0.5 \cos(200\pi t) \text{ A}$$

$$v_s(t) = 20 \cos(2000\pi t) \text{ V}$$

$$R_1 = 150 \Omega / R_2 = 50 \Omega$$

1- Kill  $v_s$ we use superposition  $\Rightarrow$  different frequencies

$$i_{R_2}' = 0$$

$$i_{R_1}' = i_s(t) * \frac{R_2}{R_1 + R_2}$$

$$= \frac{1}{4} * \frac{1}{2} \cos(200\pi t)$$

$$i_{R_1}' = 0.125 \cos(200\pi t)$$

$$i_{R_2}' = i_s - i_{R_1}'$$

$$= (0.5 - 0.125) \cos(200\pi t)$$

$$= 0.375 \cos(200\pi t) \text{ A}$$

$$v_{R_1}' = i_{R_1}' R_1$$

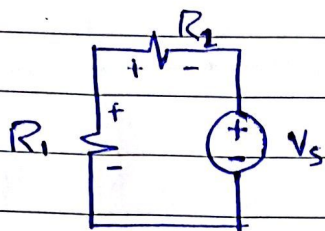
$$= 150 (0.125 \cos 200\pi t)$$

$$v_{R_2}' = i_{R_2}' R_2$$

$$= 18.75 \cos(200\pi t) \text{ Volt}$$

$$v_{R_1}' = (18.75 * \cos 200\pi t) \text{ Volt}$$

## power unit

2- Kill  $I_s$ 

$$V_{R_1}'' = V_s * \frac{R_1}{R_1 + R_2}$$

$$= 20 \cos(2000\pi t) * \frac{15\Omega}{20\Omega}$$

$$V_{R_1}'' = 15 \cos(2000\pi t) \text{ Volt}$$

$$V_{R_2}'' = V_s - V_{R_1}''$$

$$= (20 - 15) \cos(2000\pi t)$$

$$V_{R_2}'' = 5 \cos(2000\pi t) \text{ Volt}$$

$$V_{R_1}^z = V_{R_1}' + V_{R_1}''$$

$$= (18.75 \cos(200\pi t) + 15 \cos(2000\pi t)) \text{ Volt}$$

$$V_{R_2} = V_{R_2}' + V_{R_2}''$$

$$= (18.75 \cos(200\pi t)$$

$$- 5 \cos(2000\pi t)) \text{ Volt}$$

### 10.1.3 Exercise

determine the Angle by which  $v_1$  leads  $i_1$ ?

$$v_1 = 10 \cos(10t - 45)$$

a)  $i_1 = 5 \cos(10t)$

$v_1$  leads  $i_1$  by  $360 - 45$

$$= 315^\circ$$

b)  $i_1 = 5 \cos(10t - 80)$

$v_1$  leads  $i_1$  by  $35^\circ$

d)  $i_1 = 5 \cos(10t + 40)$

$v_1$  leads  $i_1$  by  $275^\circ$

e)  $i_1 = 5 \sin(10t - 19)$

$$i_1 = 5 \cos(10t - 109)$$

$v_1$  leads  $i_1$  by  $64^\circ$

$$\theta = -109 \rightarrow 251$$

$$\theta = -45 \rightarrow 315 \Rightarrow 315 - 251$$

$$-45^\circ \rightarrow 315^\circ$$

$$315 - 40 = 275^\circ$$

4] find  $i_s(t)$



$$V_s(t) = 10 \sin(\omega t + 90) \text{ volt}$$

power unit



5 Find  $i_1$  &  $i_2$

$v_s(t) = 155 \cos \omega t$

$R_s = 0.5 \Omega$

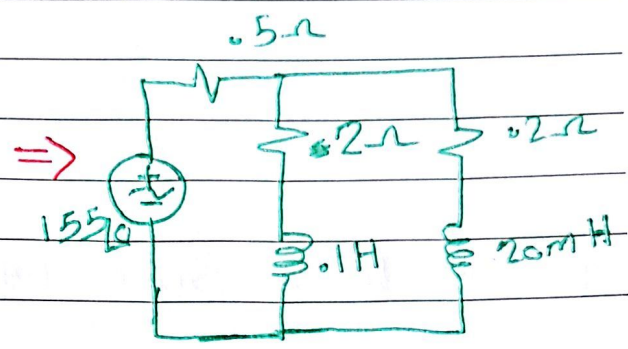
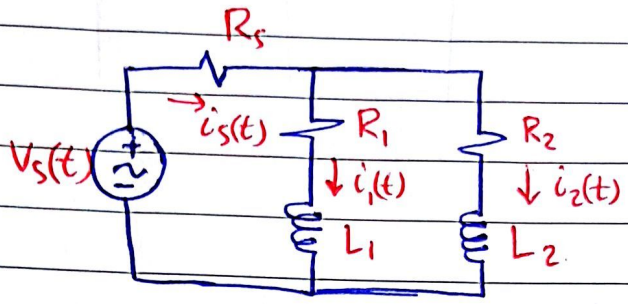
$R_2 = 0.2 \Omega$

$\omega = 377 \text{ rad/s}$

$R_1 = 2 \Omega$

$L_1 = 0.1 \text{ H}$

$L_2 = 20 \text{ mH}$



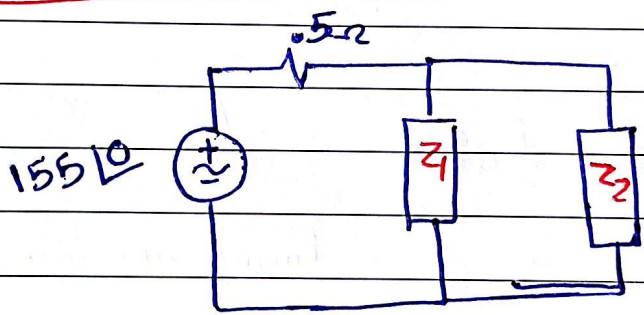
~~Imp~~ convert to impedance phasor form

$Z_L = j\omega L$

$Z_{L1} = 37.7j$

$Z_{L2} = 7.54j$

power unit



$Z_1 = 2 + 37.7j$

$Z_2 = 0.2 + 7.54j$

$Z_{eq1} = \left( \frac{1}{Z_1} + \frac{1}{Z_2} \right)^{-1}$   
 $= 0.694 + 6.28j$

$Z_{eq} = 0.694 + 6.28j$

$i_s(t) = \frac{v_s(t)}{Z_{eq}} = \frac{155 \angle 0}{0.694 + 6.28j}$

$i_1 = i_s(t) \frac{Z_2}{Z_1 + Z_2}$

$i_2 = i_s(t) \frac{Z_1}{Z_1 + Z_2}$

$i_s(t) = 24.5 \angle -83.7$

$i_1 = 4.08 \angle -82.4$

$i_1 = 4.08 \cos(377t - 82.4)$

$i_2 = 20.42 \angle -83.95$

$i_2 = 20.42 \cos(377t - 83.95)$

No. \_\_\_\_\_

[6] find  $i_a(t)$  &  $i_b(t)$

power unit

① 3 ② 60.95

$$15 \angle 0 = (0.125 - 0.122j) i_b (100 - 666.7j) + 666.7j i_b$$

$$15 \angle 0 = (575.3 \angle 96.9) i_b$$

$$i_b = 0.026 \angle -96.9$$

$$i_a = (0.125 - 0.122j) (0.026 \angle -96.9)$$

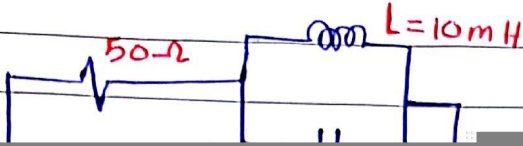
power unit

No. \_\_\_\_\_

7

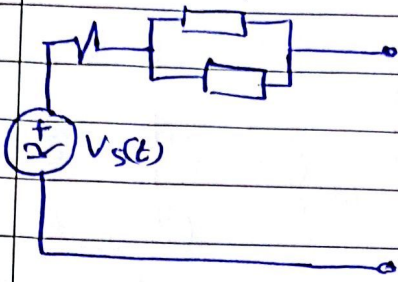
Find  $V_{th}$

~~$f = 10^3 \text{ Hz} / R_s = 50 \Omega / R_{LD} = 50 \Omega / L = 10 \text{ mH} / C = 0.1 \mu\text{F}$~~



power unit

find  $V_{th}$  /  $V_{th} = V_{oc} = V_{ld}$

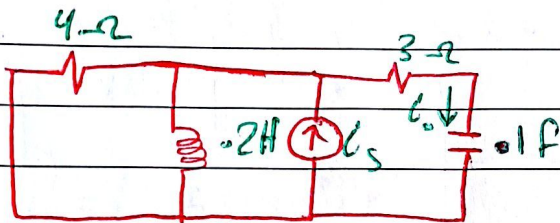


$$V_{th} = V_{oc} = V_S = V_{ld}$$

power unit

problem 2:

$$i_s = 5 \cos(10t + 40^\circ) \text{ A} \quad \text{find } i_o ??$$



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No. \_\_\_\_\_

$$\textcircled{1} \quad i_1(3-j) + (2j)i_2 = 0$$

$$\textcircled{2} \quad i_1 = 5 \angle 40^\circ + i_2$$

$$\frac{i_2 = i_3}{1-2j} \textcircled{3}$$

power unit

$$(5 \angle 40^\circ + i_2)(3-j) + (2j)i_2$$

$$((3.8 + 3.2j) + i_2)(3-j) + (2j)i_2$$

$$(3-j)i_2 + 14.6 + 5.8j + (2j)i_2 = 0$$

$$(3+j)i_2 = -14.6 - 5.8j$$

$$i_2 = (4.97 \angle -176.8^\circ) \text{ A}$$

$$i_2 = (2.22 \angle -113.4^\circ) \text{ A}$$

$$i_1 = (3.15 \angle 111.1^\circ) \text{ A}$$

$$i_0 = i_1 = 3.15 \cos(10t + 111.1^\circ)$$

No.

power unit

10.18



No. \_\_\_\_\_

$$8 \angle 45^\circ = (0.58 - .06j) V_2 - (.08 - .06j) V_1 \quad \text{--- (1)}$$

$$V_1 = (2.43 \angle -75.3^\circ) V_2 \quad \text{--- (2)}$$

power unit

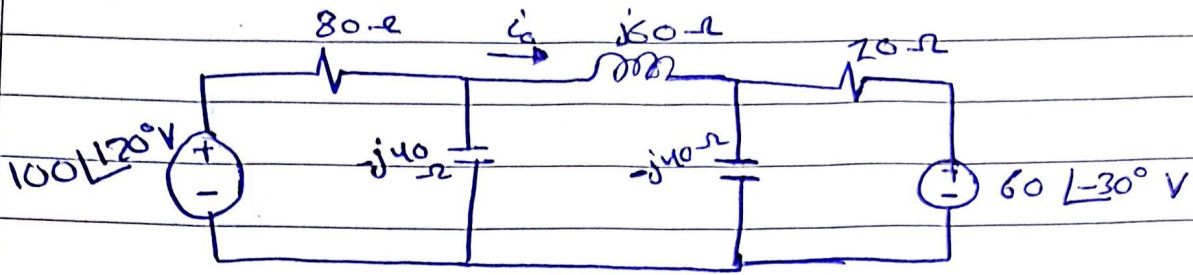
$$8 \angle 45^\circ = (0.58 - .06j) V_2 - (.08 - .06j) (2.43 \angle -75.3^\circ) V_2$$

$$8 \angle 45^\circ = (0.69 \angle 113.8^\circ) V_2$$

$$V_2 = \frac{8 \angle 45^\circ}{0.69 \angle 113.8^\circ} = 11.59 \angle -68.8^\circ$$



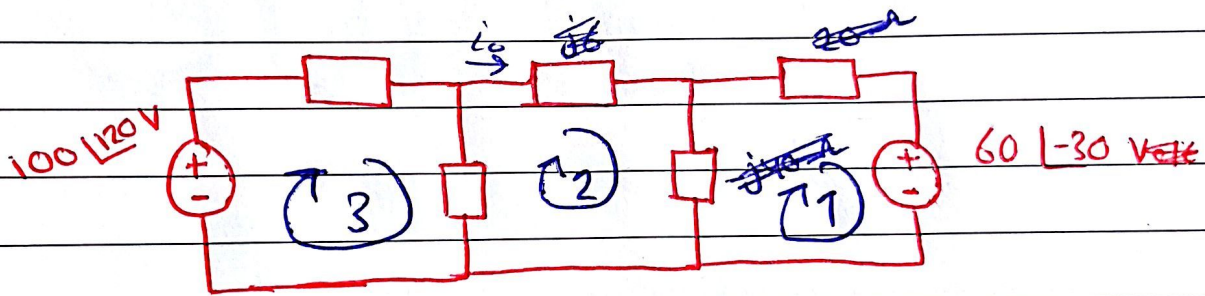
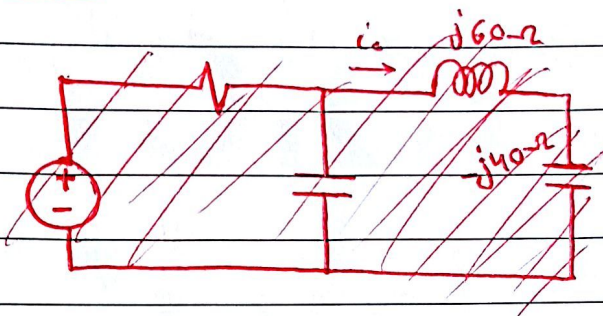
10.31



Use mesh Determine  $I_o$ :

Sol:

power unit



@ Mesh 1

$$60 \angle -30^\circ = +(20 \angle -30^\circ - j40) i_1 + (j40) i_2 = 0 \quad \text{--- (1)}$$

51.96 - 30j  
@ Mesh 2

$$i_2 (60j - 40j - 40j) + (40j) i_3 = 0 \quad \text{--- (2)}$$

$\downarrow$   
 $+(40j) i_1$

@ Mesh 3

~~100∠120~~

$$100 \angle 120^\circ = (80 - 40j) i_3 + (40j) i_2 \quad \text{--- (3)}$$

$\downarrow$   
 $-50 + 86.6j$

$$\boxed{(-20j)i_2 + (40j)i_1 + (40j)i_3 = 0} \quad \text{--- (2)}$$

$$i_2 = \frac{(-20+40j)i_1 - 51.96+30j}{40j}$$

$$\boxed{i_1 = \frac{-40j i_2 - 51.96+30j}{20-40j}}$$

$$\boxed{i_1 = (-0.8 - 0.4j)i_2 - 1.12 - 0.74j} \quad \text{--- (16)}$$

$$i_3 = \frac{-50 + 86.6j - (40j)i_2}{80 - 40j}$$

power unit

$$\boxed{i_3 = -0.93 + 0.61j + (0.2 - 0.4j)i_2} \quad \text{--- (17)}$$

$$0 = (-20j)i_2 + (40j)(\cancel{0.8} - 0.4j)i_2 - 1.12 - 0.74j + 40j(-0.93 + 0.61j + (0.2 - 0.4j)i_2)$$

$$0 = (32 + 20j)i_2 + 29.6 - 44.8j - 24.4 - 37.2j$$

$$i_2 = \frac{5.2 - 82j}{32 + 20j}$$

$$i_2 = -1.034 - 1.91j$$

$$\boxed{i_2 = 2.177 \angle -118.38^\circ \text{ Amperes} = i_0}$$