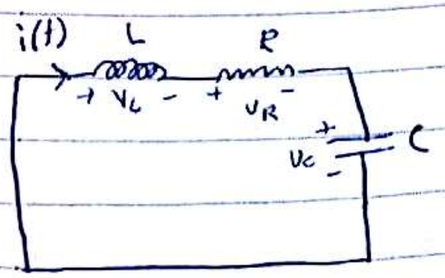
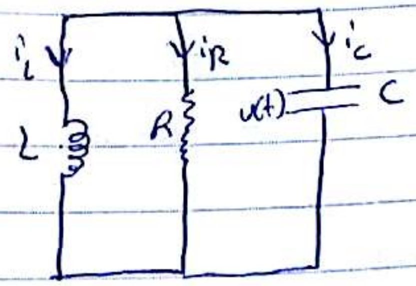
Chapter 8
RLC circuits



→ series RLC



→ parallel RLC

☒ parallel RLC :-

KCL at upper node:

$i_L + i_R + i_c = 0$

$$\frac{1}{L}\int_{t_0}^{t} v(t)\,dt + i(t_0) + \frac{v(t)}{R} + C\frac{dv(t)}{dt} = 0$$

$$C\frac{d^2 v(t)}{dt^2} + \frac{1}{R}\frac{d\,v(t)}{dt} + \frac{1}{L}v(t) = 0 \quad \text{— (1)}$$

2nd order homogeneous Diff eq.

→ to solve this eq we need 2 initial conditions $i_L(t_0)$, $v_c(t_0)$

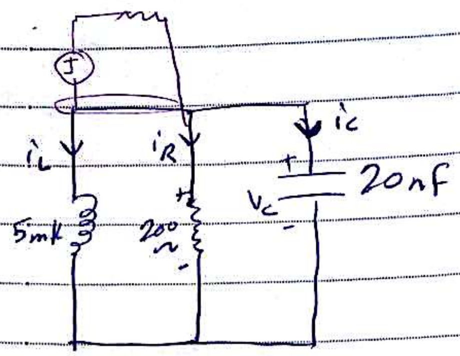
solution

$v(t) = Ae^{st} \rightarrow V = v_0\,e^{-t/\tau}$ parallel RC — (2)

sub (2) in (1)

for $t>0$

$V(0) = 60$, $i_L(0) = -0.3\,A$



$$\alpha = \frac{1}{2RC} = 125000 \; s^{-1}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 100000 \; s^{-1}$$

$\alpha > \omega_0$
over damping

$S_1 = -50000 \; s^{-1}$

$S_2 = -200000 \; s^{-1}$ $\quad |S_2| > |S_1|$

$$\boxed{v(t) = A_1 e^{-50000 t} + A_2 e^{-200000 t}} \text{ Volt}$$

$V(0) = 60$

$60 = A_1 + A_2$ — ①

$$\left.\frac{dv(t)}{dt}\right|_{t=0} = \frac{i_c(0)}{C}$$

$i_c(0) \rightarrow$ KCL

$i_C(0) + i_R(0) + i_L(0) = 0$

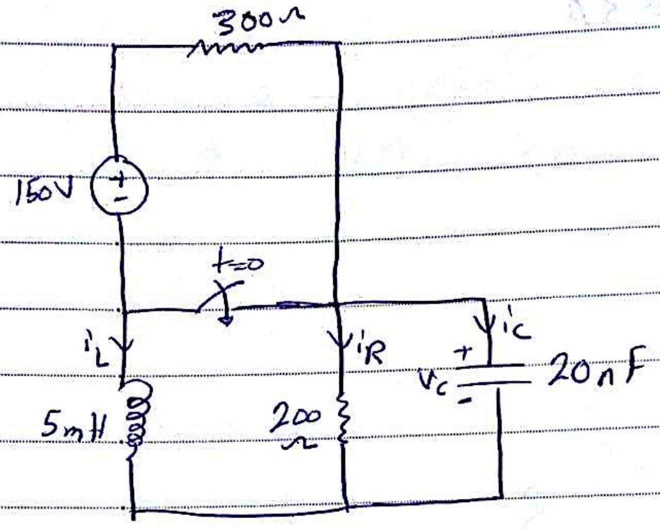
$i_c(0) = -i_R(0) - i_L(0)$

$= -\frac{60}{200} - (-0.3)$

$= -0.3 + 0.3$

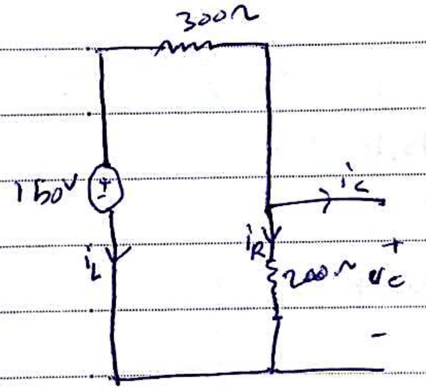
$=$ zero

* Ex: Find $v_c(t)$ for $t>0$



solution:

for $t<0$



$V_c(0^-) = V_c(0^+) = V_c(0)$

$i_L(0^-) = i_L(0^+) = i_L(0)$

$i_R(0^-) \neq i_R(0^+)$

$i_c(0^-) \neq i_c(0^+)$

$$V_c(0^-) = 150 * \frac{200}{500} = 60 \text{ volt} = v_c(0^+)$$

$$V_c(0) = 60 \text{ V}$$

$$i_R(0^-) = \frac{150}{300} = 0.3\text{ A} \neq i_R(0^+)$$

$$i_L(0^-) = -i_R(0^-) = -0.3\text{ A} = i_L(0^+)$$

$$i_c(0^-) = 0 \neq i_c(0^+)$$

* Draw $v(t)$

$v(0) = 0$

$v(\infty) = 0$

$$\frac{dv(t)}{dt} = 84(-e^{-t_m} + 6e^{-6t_m}) = 0 \qquad T_m = \text{maximum time}$$

$$6e^{-6t_m} - e^{-t_m} = 0$$

$$e^{-5t_m} = \frac{1}{6}$$

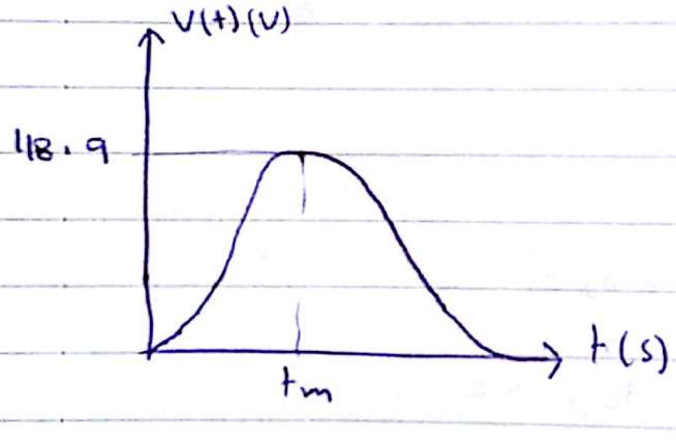
$$-5t_m = \ln\frac{1}{6}$$

$$t_m = \frac{-1}{5}\ln\left(\frac{1}{6}\right)$$

$$\boxed{t_m = 0.3583 \text{ s}}$$

$$v(t_m) = 84(e^{-t_m} - e^{-6t_m})$$

$$V_m = 48.9 \text{ volt}$$



stelling time occurs at v less 1% V_m (t_s)

$$1\% \, V_m = 0.489 \text{ volt}$$

→ to find t_s

$$v(t_s) = 84(e^{-t_s} - e^{-6t_s})$$

$$0.489 = 84e^{-t_s} - \underbrace{\cancel{e^{-6t_s}}}_{\text{decay faster}}$$

$$\boxed{t_s = 5.15 \text{ s}}$$

بعد 5 ثواني صار (L), (C) فاضيين

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -1\ s^{-1}$$
$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -6\ s^{-1}$$

$$v(t) = A_1 e^{-t} + A_2 e^{-6t} \text{ volt}$$

to find A_1, A_2

$v(0) = 0 \rightarrow A_1 + A_2 = 0$

$$\boxed{A_1 = -A_2}$$

$i(0) = 10 \rightarrow$ for inductor

* avoid the integral *

KCL: $i_C(0) = i_L(0) + i_R(0)$

$i_C(0) = 10 + \underline{zero}$ $\quad v_L(0) = v_C(0) = zero$

$i_C(0) = 10$

$i_C = C \frac{dv(t)}{dt}$

$i_C(0) = C \frac{dv}{dt}\Big|_{t=0}$

* $$\boxed{\frac{dv(t)}{dt}\Big|_{t=0} = \frac{i_C(0)}{C}}$$

$$\frac{dv(t)}{dt} = -A_1 e^{-t} - 6A_2 e^{-6t}$$

$$\frac{dv(t)}{dt}\Big|_{t=0} = -A_1 - 6A_2 = 420$$

but $A_1 = -A_2$

$A_2 - 6A_2 = 420$

$$\boxed{A_2 = -84}$$
$$\boxed{A_1 = 84}$$

$$v(t) = 84\left(e^{-t} - e^{-6t}\right) \text{ Volt}$$

* Ex:

for parallel RLC if $L = 10\,mH$, $C = 100\,\mu F$, Find R to have over damped, under damped and critical damped ?

solution:

① for over damped

$\alpha > \omega_0$

$\frac{1}{2RC} > \frac{1}{\sqrt{LC}}$

$2RC < \sqrt{LC}$

$R < \frac{\sqrt{LC}}{2C}$

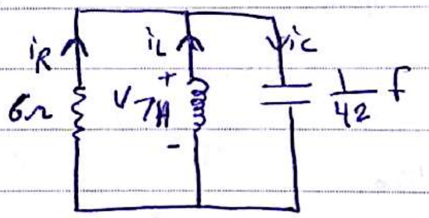
$R < 5\,\Omega$

② $r > 5\,\Omega$ under damped response

③ $R = 5\,\Omega$ critical damped

[*] The overdamped Response for parallel RLC :

Example:



Find $v(t)$ if $v(0) = 0$, $i(0) = 10\,A$?

solution:

$\alpha = 3.5\,s^{-1}$

$\omega_0 = \sqrt{6}\,s^{-1}$

$\alpha > \omega_0 \rightarrow$ over damped response

$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$

A_1, A_2, s_1, s_2 ??

$S \rightarrow \frac{1}{S}$ freq

$\alpha = \frac{1}{2RC}$ = Neper frequency

$\frac{1}{\sqrt{LC}} = w_o$ = Resonent frequency

$s_1 = -\alpha + \sqrt{\alpha^2 - w_o^2}$

$s_2 = -\alpha - \sqrt{\alpha^2 - w_o^2}$

base on α and w_o :

1) $\alpha^2 > w_o^2 \rightarrow$ real value for s_1, s_2

overdamped response

2) $\alpha^2 < w_o^2 \rightarrow$ s_1, s_2 are complex

under damped response

3) $\alpha^2 = w_o^2 \rightarrow s_1 = s_2 = -\alpha$

critical damped Response

Define :-

(zeta) $\zeta = \frac{\alpha}{w_o}$ = damping ratio.

$\zeta > 1$ over damped $\alpha > w_o$

$\zeta < 1$ under damped $\alpha < w_o$

$\zeta = 1$ critical damped $\alpha = w_o$

$$CAs^2 e^{st} + \frac{1}{R} As e^{st} + \frac{1}{L} Ae^{st} = 0$$

$$Ae^{st}\left(Cs^2 + \frac{s}{R} + \frac{1}{L}\right) = 0$$

$$Ae^{st} = 0 \begin{cases} \to A = 0 \\ \to s = -\infty \end{cases}$$

[Always zero
Trivial solution

OR

$$Cs^2 + \frac{s}{R} + \frac{1}{L} = 0$$

$$s = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$A = C, \quad B = \frac{1}{R}, \quad C = \frac{1}{L}$$

$$s = \frac{\frac{-1}{R} \pm \sqrt{(\frac{1}{R})^2 - \frac{4C}{L}}}{2C}$$

$$s = \frac{-1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \left(\frac{1}{LC}\right)}$$

$$s_1 = \frac{-1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

$$s_2 = \frac{-1}{2RC} - \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

$$v_1 = A_1 e^{s_1 t} \to \text{soln}$$

$$v_2 = A_2 e^{s_2 t} \to \text{soln}$$

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \to \text{general solution}$$

$\alpha = \omega_0 = \frac{1}{\sqrt{LC}} = 500\ s^{-1}$

$500 = \frac{1}{2RC}$

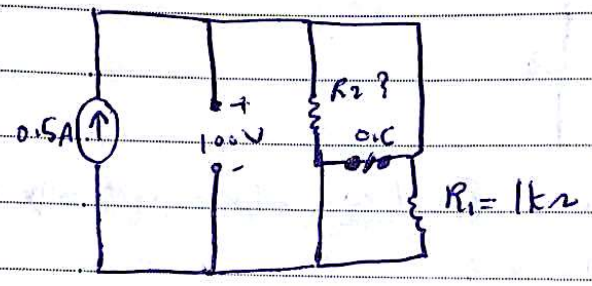
$R = \frac{1}{2C(500)} = 1k\Omega$

$\boxed{R_1 = 1k\Omega}$ * إذا طلب R over damped يكون أقل &

b) $v(0^-) = v(0^+) = V(0) = 100$ volt

if we apply t>0 , R_2 deleted

so we apply t<0



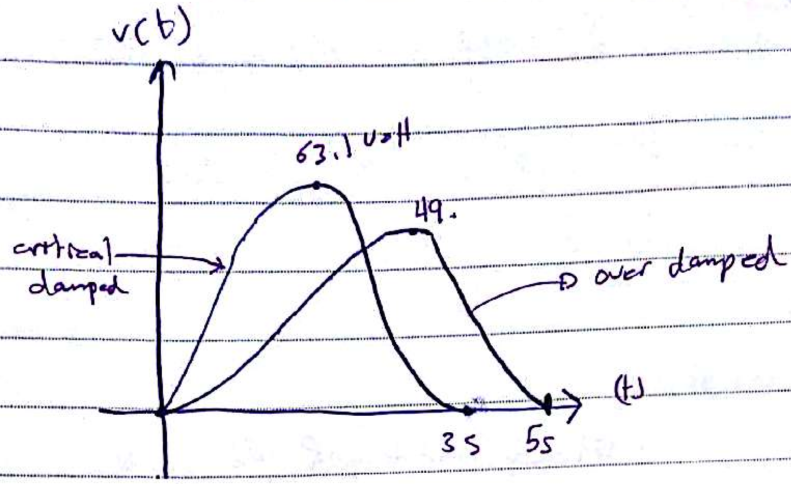
$i_{R_2} = \frac{0.5 \times 1k}{1k + R_2}$

$V_{R_2} = i_{R_2} R_2 = 100$

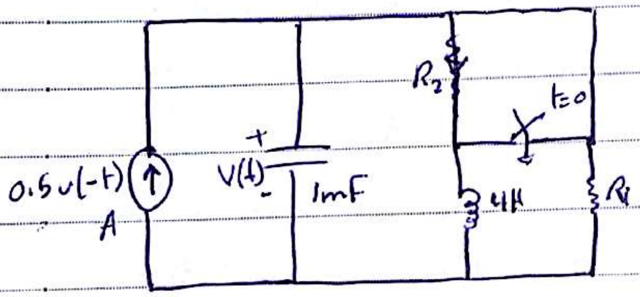
$100 = \left(\frac{0.5\,(1000)}{1000 + R_2}\right) * R_2$

$\boxed{R_2 = 250\ \Omega}$

$i_L(0^-) = i_{R_2}(0^-) = \frac{0.5 \times 1000}{1250} = 0.4A$



Example:



a) choose R_1 so that the response for $t>0$ will be critically damped

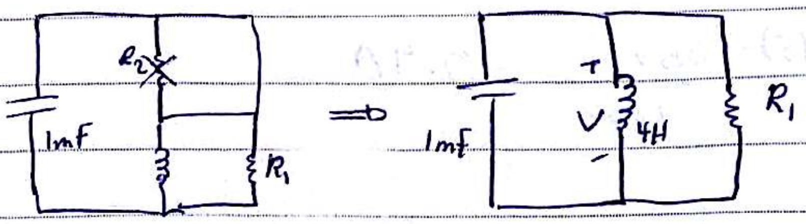
b) select R_2 to obtain $v(0) = 100V$

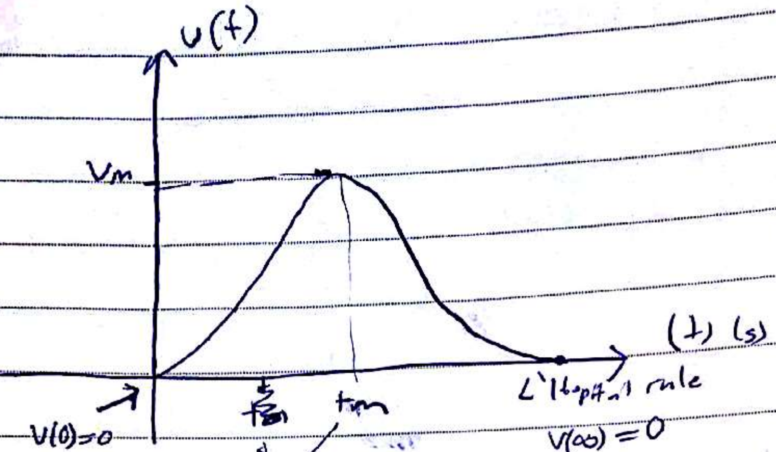
c) Find $v(t)$ at $t = 1ms$

solution:

$$0.5u(-t) = \begin{cases} 0.5, & t<0 \\ 0, & t>0 \end{cases}$$

a) for $t>0$ * For $t<0$ $v(0) = 100$ Volt





$\frac{dv(t_m)}{dt_m} = 0$

$420t\left(-\sqrt{6}e^{-\sqrt{6}t}\right) + 420e^{-\sqrt{6}t} = 0$

$+\sqrt{6} \cancel{420} t_m \cancel{e^{-\sqrt{6}t_m}} = \cancel{420} \cancel{e^{-\sqrt{6}t_m}}$

$\sqrt{6}t_m = 1$

$t_m = \frac{1}{\sqrt{6}} = 0.408 s$

$t_m|_{critical} < t_m|_{overdamped}$

$V(t_m) = 63.1$ Volt

$\boxed{*}$ $t_s \rightarrow V_m$ drop to 1 % V_m

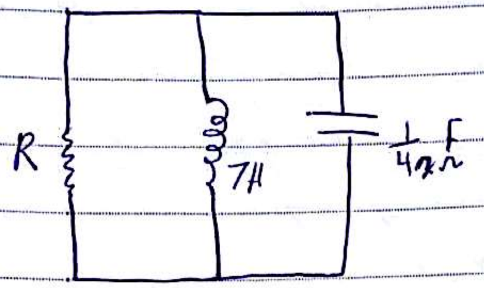
$\frac{V_m}{100} = 0.631 V$

$0.631 = 420 t_s e^{-\sqrt{6}t_s}$

$t_s = 3.12 s < t_s|_{over\ damped}$

5.15 S

Critical Damping:



$v(0) = 0$, $i(0) = 10$ A

R? to have critical damped ?!

$\alpha = \omega_0$

$\frac{1}{2RC} = \frac{1}{\sqrt{LC}} \Rightarrow 4R^2C^2 = LC$

$\boxed{L = 4R^2C}$

$R = \pm\sqrt{\frac{L}{4C}}$

$R = +\sqrt{\frac{7}{4(\frac{1}{42})}}$

$R = 8.57\,\Omega$

$R_{under} > R_{1\,critical} > R_{overdamped}$

$\alpha = \omega_0$ $\qquad \alpha > \omega_0$

$\alpha \downarrow R \uparrow$

$\alpha = \frac{1}{2RC} = \sqrt{6}\ s^{-1}$

$\omega_0 = \sqrt{6}\ s^{-1}$

$\alpha = \omega_0 = \sqrt{5}\ s^{-1}$

$S_1 = S_2 = -\alpha = -\sqrt{6}\ s^{-1}$

$S_1 = -\alpha + \sqrt{6}$

$S_2 = -\alpha - \sqrt{6}$

$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$ لو عوضنا هيك

$= A_1 e^{-\sqrt{6}t} + A_2 e^{-\sqrt{6}t} = (A_3) e^{-\sqrt{6}t}$

$A_1 + A_2$

solution:

$v(t) = A_1 t e^{-\sqrt{6}t} + A_2 e^{-\sqrt{6}t}$

*only for identical Roo

$v(0) = 0 + A_2 = 0$

$A_2 = 0$

$\boxed{v(t) = A_1 t e^{-\sqrt{6}t}}$ $t > 0$

$\left.\frac{dv(t)}{dt}\right|_{t=0} = \frac{i_c(0)}{C}$, $i_c = C\frac{dv}{dt}$

$i_c(0) = 420$ volt/second

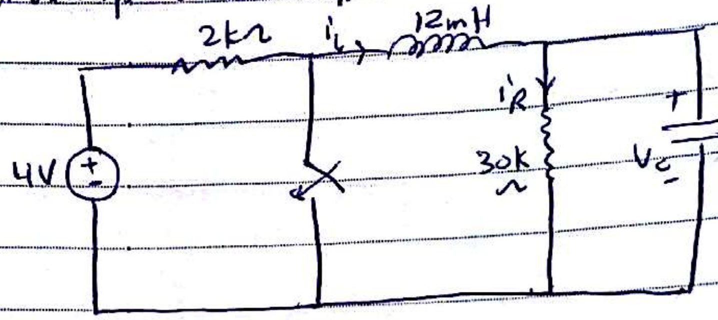
$\left. A_1 t \cdot -\sqrt{6} e^{-\sqrt{6}t} + e^{-\sqrt{6}t} A_1 = 420 \right|_{t=0}$

$e^{-\sqrt{6}(t)} A_1 = 420$

$\boxed{A_1 = 420\ V}$

$v(t) = 420\, t\, e^{-\sqrt{6}t}$ V, $t > 0$

* Example: Find $i_R(t)$ for All t:



$P = 10^{-12}$ F

solution:

$$i_R = \frac{V_c}{30000}$$

$i_R(0^-) = 125\ \mu A \neq i_R(0^+)$

$i_L(0^+) = i_L(0^-) = i_L(0) = 125\ mA$

$v_c(0^-) = v_c(0^+) = v_c(0) = 3.75$ volt

→ over damped (α > w_0)

$i_R(t) = 161.3\ e^{s_1 t} - 36.34\ e^{s_2 t}$

$s_1 = -3.063 \times 10^{+6}\ s^{-1}$

$s_2 = -13.6 \times 10^{6}\ s^{-1}$

$$\left.\frac{dv(t)}{dt}\right|_{t=0} = 0$$

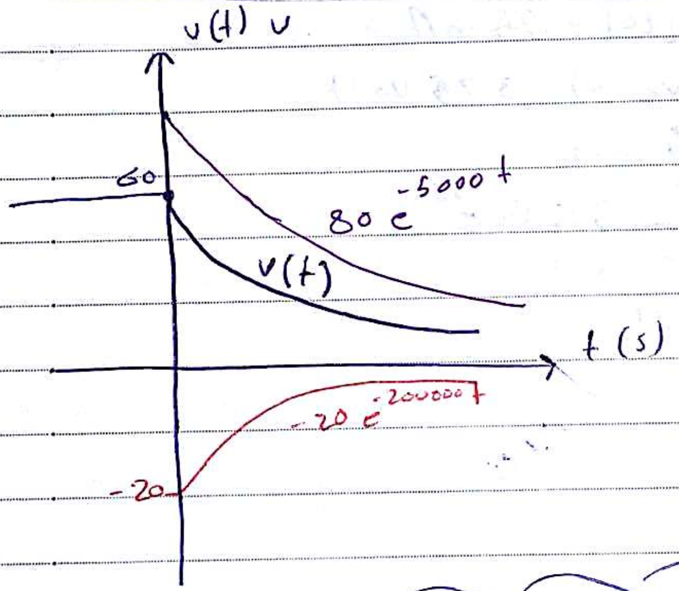
$$-50\,000\,A_1 - 200\,000\,A_2 = 0$$

$$A_1 = -4A_2 \quad (2)$$

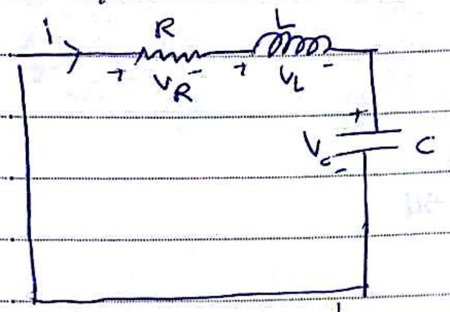
$$\boxed{A_1 = -20\ V}$$

$$\boxed{A_2 = 80\ V}$$

$$\boxed{v(t) = 80e^{-50000t} - 20e^{-200000t}\ V,\ t > 0}$$



Source free series RLC circuits :-



KVL : $iR + L\frac{di}{dt} + \frac{1}{L}\int_{t_0}^{t} i(t)\,dt + v(t_0) = 0$ * 2nd order homog differen equation

derive again :

$$\boxed{L\frac{d^2i}{dt} + R\frac{di}{dt} + \frac{1}{L}\, i(t) = 0}$$

$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$ for over damped

$i(t) = (A_1 t + A_2)\, e^{-\alpha t}$ for critical

$i(t) = (e^{-\alpha t}\,(A_1 \cos w_d t + A_2 \sin w_d t)$ for under damped.

$$\alpha = \frac{R}{2L}$$

$$w_0 = \frac{1}{\sqrt{LC}}$$

$$s_1 = -\alpha + \sqrt{\alpha^2 - w_0^2}$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - w_0^2}$$

$$w_d = \sqrt{w_0^2 - \alpha^2}$$

$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 4.75$ rad/s

$i_L(t) = e^{-1.2t}\left(B_1 \cos 4.75t + \cancel{j} B_2 \sin 4.75t\right)$

$i_L(0) = 2.027$

$\boxed{B_1 = 2.027}$ Ampere

$v(0) \Rightarrow \frac{di}{dt}$

$V = L\frac{di}{dt}$

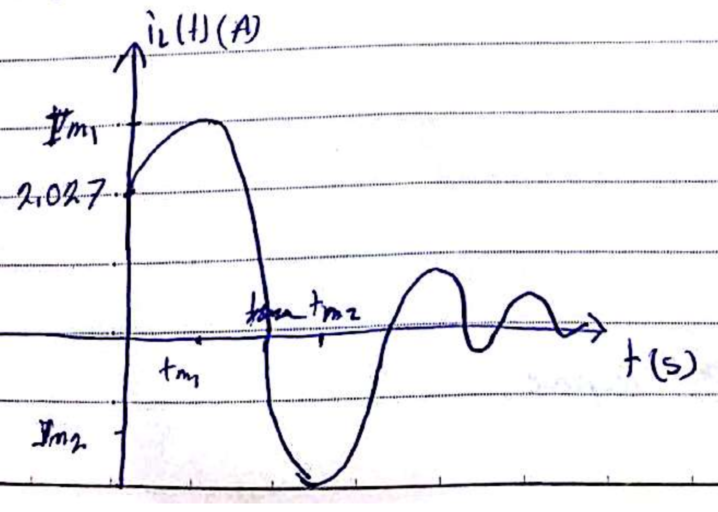
$\frac{di}{dt} = \frac{V(0)}{L} = \frac{97.3}{10} = 9.73$ A/s

$$\frac{di(t)}{dt} = e^{-1.2t}\left(-4.75B_1 \sin 4.75t + 4.75B_2 \cos 4.75t\right) + 1.2e^{-1.2t}\left(B_1 \cos 4.75t + 4.75B_2 \cos 4.75t\right)$$

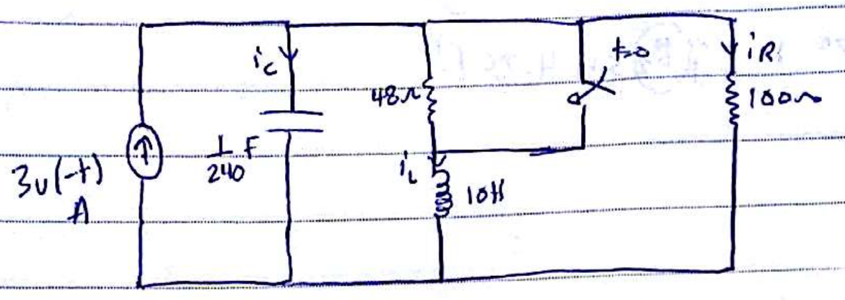
$$\left.\frac{di}{dt}\right|_{t=0} = 4.75B_2 - 1.2(B_1) = 9.73 \text{ A/s}$$

$\boxed{B_1 = 2.561 \text{ A}}$

$i_L(t) = e^{-1.2t}\left(2.027 \cos 4.75t + 2.561 \sin 4.75t\right)$ A

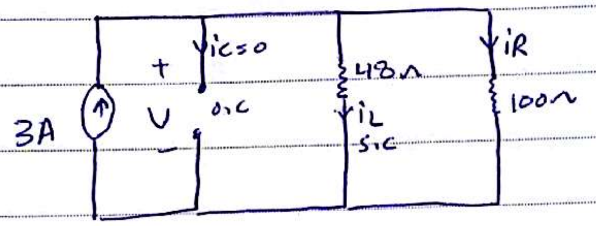


* Ex :- Find $i_L(t)$ and plot it



solutions:

for $t<0$



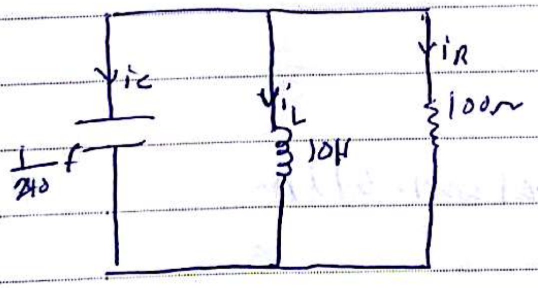
$$i_L(0^-) = \frac{3 * 100}{148} = 2.027A$$

$$V(0) = \left(\frac{300}{148}\right) * 48 = 97.3 \; V$$

$V_C(0) = 97.3$ volt

$i_L(0) = 2.027$ Ampere

for $t>0$ $\boxed{\text{parallel RLC circuits}}$



$$\alpha = \frac{1}{2RC} = 1.2 \; s^{-1}$$

$$w_0 = \frac{1}{\sqrt{LC}} = 4.899 \; s^{-1}$$

$w_0 > \alpha \rightarrow$ under damped response.

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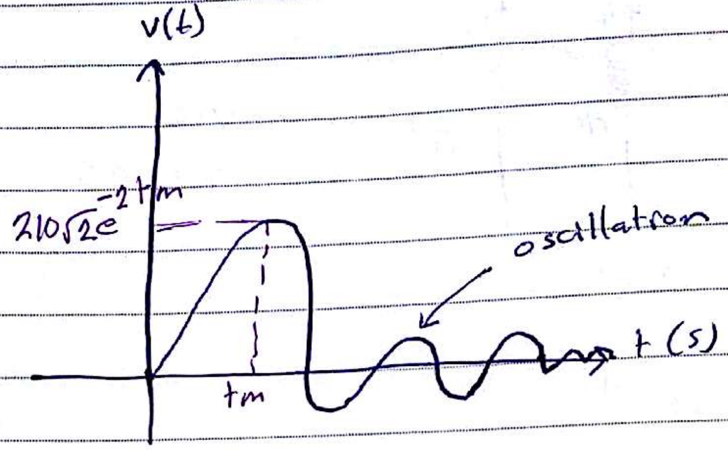
$v(t) = e^{-2t} \left(+210\sqrt{2} \sin \sqrt{2}\, t \right) V$, for $t > 0$ real

$v(0) = 0$

$v(\infty) = 0$



يضل يتذبذب
يضل لحد ما يوصل
steady state

To find maximum value:-

$$\frac{d\, v(t)}{dt} = 0$$

$V_{m_1} = 71.8\ V \rightarrow t_{m_1} = 0.435\ s$

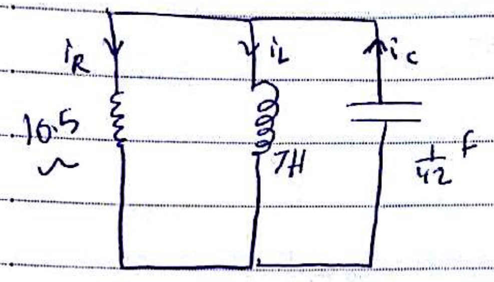
→ Next minimum

$V_{m_2} = -0.845\ V \rightarrow t_{m_2} = 2.66\ s$

settling time $\rightarrow \frac{V_m}{100} \Rightarrow 0.718$ volt

$t_s = 2.92\ s < t_s\big|_{critical} < t_s\big|_{over}$

Example:



$v(0)=0$
$i(0)=10A$

$\alpha = \frac{1}{2RC} = 2\ s^{-1}$

$w_0 = \frac{1}{\sqrt{LC}} = \sqrt{6}\ s^{-1}$

$w_d = \sqrt{w_0^2 - \alpha^2} = \sqrt{2}$ rad/s

$v(t) = e^{-2t}(B_1 \cos\sqrt{2}t + jB_2 \sin\sqrt{2}t)$

$v(0)=0$

$0 = 1 \times B_1$

$\boxed{B_1 = 0}$

$\left.\frac{dv(t)}{dt}\right|_{t=0} = \frac{i_C(0)}{C}$ $\quad i_R(0^+)=0$, $i_C(0^+)=10$

$\left.\frac{dv(t)}{dt}\right|_{t=0} = 420$ volt

$= e^{-2t}(-\sqrt{2}B_1 \cancel{\sin\sqrt{2}t} + jB_2\sqrt{2}\cos\sqrt{2}t)$ ~~420~~ $+ -2e^{-2t}(B_1 \cancel{\cos\sqrt{2}t} + jB_2 \cancel{\sin\sqrt{2}t})$

$420 = jB_2\sqrt{2}$

$B_2 = -j\frac{420}{\sqrt{2}}$

$\boxed{B_2 = -j\,210\sqrt{2} \text{ Volt}}$

[*] The Under damped Response :-

For Parallel RLC circuits

$\alpha < \omega_0$, $R\uparrow \ \alpha\downarrow$, $\zeta < 1$

$$S_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

$$S_{1,2} = -\alpha \pm j\sqrt{\omega_0^2 - \alpha^2}$$

$\omega_0 \rightarrow$ resonent Freq

$\alpha \rightarrow$ Neper freq

ω_d = Natural resonent frequency (rad/s)

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

$$\boxed{\begin{aligned} S_1 &= -\alpha + j\omega_d \\ S_2 &= -\alpha - j\omega_d \end{aligned}}$$ complex

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$ $\rightarrow$ For overdamped

$$= A_1 e^{-\alpha t} e^{j\omega_d t} + A_2 e^{-\alpha t} e^{-j\omega_d t}$$ $\alpha = \frac{1}{2RC}$ (real)

$$= e^{-\alpha t}\left[A_1(\cos\omega_d t + j\sin\omega_d t) + A_2(\cos\omega_d t - j\sin\omega_d t)\right]$$

$$= e^{-\alpha t}\left\{(A_1+A_2)\cos\omega_d t + j(A_1-A_2)\sin\omega_d t\right\}$$

$$v(t) = e^{-\alpha t}\left(B_1\cos\omega_d t + jB_2\sin\omega_d t\right)$$

$B_1 = A_1 + A_2$

$B_2 = A_1 - A_2$

c) $S_1 = S_2 = -\alpha = -500\, s^{-1}$

$v(t) = A_1 t e^{-500t} + A_2 e^{-500t}$

$v(0) = 100 \rightarrow \boxed{A_2 = 100 \text{ Volt}}$

$\left.\frac{dv(t)}{dt}\right|_{t=0} = \frac{i_c(0)}{C}$ $\quad i_C(0) = i_L(0) + i_R(0)$ for $t>0$ ~~for~~ $t \geq 0$

$= 0.4 + 0.1$

$= 50000 \text{ V/s}$ $\quad = 0.5$

$\left.\frac{dv(t)}{dt}\right|_{t=0} = A_1 t(-50000\, e^{-500t}) + A_1 e^{-500t} - 50000\, e^{-500t} = 500\,000$

$e^{-500t}(-500 A_1 t + A_1 - 50000) = 500\,000$

$e^{-500t} = 500\,000$ X

$\cancel{-500 A_1 t} + A_1 - 50000 = 500\,000$

$A_1 = 550000$ at $t=0$

$\boxed{v(t) = 550 t e^{-500t} + 0.1\, e^{-500t} \text{ kV}}$ $\quad t \geq 0$

$v(1) = --$

$i(t) = c\frac{dv(t)}{dt}$

$i(t) = 0.25\left(\frac{1}{3}(16e^{-t} - 4e^{-4t})\right)$

$= \frac{16}{3}e^{-t} - \frac{4}{3}e^{-4t}$ Ampere

→ For $R = 4\Omega$

$i(0) = 4.8A$

$v(0) = 4.8$ Volt

$\alpha = 2s^{-1}$, $w_0 = 2s^{-1}$ → critical

$s_1 = s_2 = -\alpha = -2s^{-1}$

$v_f = 24$ volt

$v(t) = 24 + (A_1t + A_2)e^{-2t}$ V

$v(t) = 24 - 19.2(t+1)e^{-2t}$ Volt

$i(t) = ((9.6t) + 4.8)e^{-2t}$ Ampere

→ For $R = 1\Omega$

$i(0) = 12A$

$v(0) = 12V$

$\alpha = 0.5s^{-1}$, $w_0 = 2s^{-1}$ → under damped

$w_d = 1.936$ rad/s

$v_f = 24$ volt

$v(t) = 24 + (A_1 \cos w_dt + A_2 \sin w_dt)e^{-\alpha t}$ volt

$A_1 = 21.694$ volt

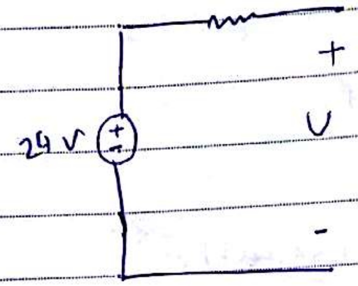
$A_2 = -12$ volt

$i(t) = (3.15 \sin 1.936t + 12\cos 1.936t)e^{-0.5t}$ Ampere

$$s_1 = -2.5 - \sqrt{(2.5)^2 - (2)^2} = -1\ s^{-1}$$

$$s_2 = -2.5 + \sqrt{(2.5)^2 - (2)^2} = -4\ s^{-1}$$

at $t = \infty$



$V_f = 24$ volt

$$v(t) = 24 + A_1 e^{-t} + A_2 e^{-4t}\ V,\ t > 0$$

from $t < 0 \rightarrow v(0) = 4 \rightarrow \boxed{A_1 + A_2 = -20}$ — 1

$$\left.\frac{dv(t)}{dt}\right|_{t=0} = \frac{i_L(0)}{C} = \frac{4}{0.25} = 16\ V/s$$

$$\frac{dv(t)}{d(t)} = -A_1 e^{-t} - 4A_2 e^{-4t}$$

$$\boxed{-A_1 - 4A_2 = 16} \text{ — 2}$$

$$A_1 = \frac{-64}{3}\ V, \quad A_2 = \frac{4}{3}\ V$$

$$v(t) = \left(24 + -\frac{64}{3} e^{-t} + \frac{4}{3} e^{-4t}\right)$$

$$v(t) = 24 + \frac{4}{3}\left(-16 e^{-t} + e^{-4t}\right) \text{ volt},\ t > 0$$

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complete response for the RLC circuits (Step Response)

$$v(t) = \underset{\text{forced}}{v_f} + v_N$$
$$= v_{ss} + v_N$$

at $t = \infty$
$L \to s.c$
$C \to o.c$

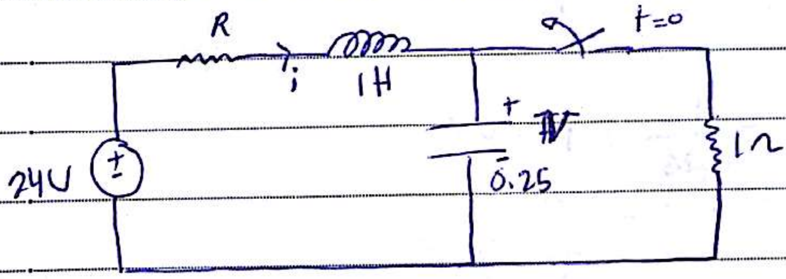
Over damped :- $v(t) = v_f + A_1 e^{s_1 t} + A_2 e^{s_2 t}$

critical :- $v(t) = v_f + (A_1 t + A_2) e^{-\alpha t}$

underdamped :- $v(t) = v_f + (A_1 \cos w_d t + A_2 \sin w_d t) e^{-\alpha t}$

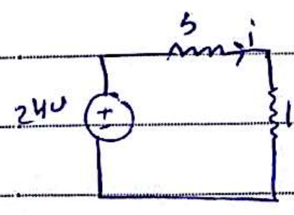
Example :-

Find $v(t)$ and $i(t)$ for $t > 0$, for $R = 5\,\Omega$, $R = 4\,\Omega$, $R = 1\,\Omega$?



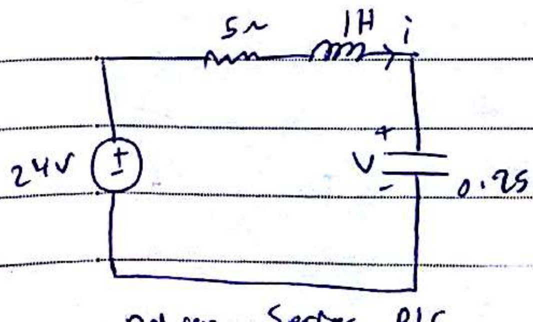
solution:-

→ for $R = 5\,\Omega$



$i_L(0^-) = \frac{24}{6} = 4$ A for $t < 0$
$v_C(0) = 4$ volt
} initial conditions

for $t > 0$



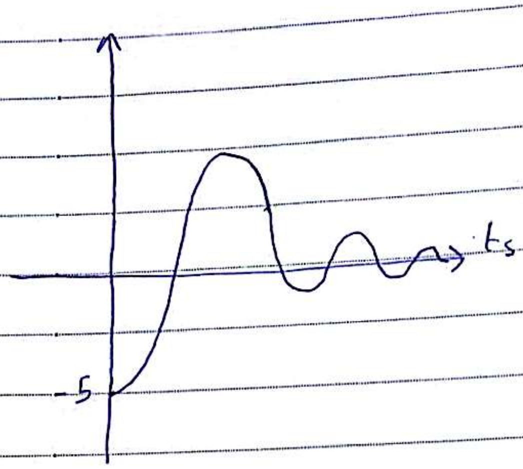
$\alpha = \frac{R}{2L} = 2.5\ s^{-1}$

$w_0 = \frac{1}{\sqrt{LC}} = 2\ s^{-1}$

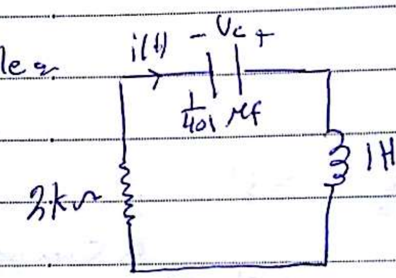
$\alpha > w_0 \rightarrow$ over damped

Driven Series RLC

$v_c(t) = e^{-0.8t}(-5\cos 9.968t + -0.4013 \sin 9.968t)\ V \quad t > 0$



Example:



Find and sketch $i(t)$?

if $i(0) = 2mA$

$v_c(0) = 2V$

solution:

series:

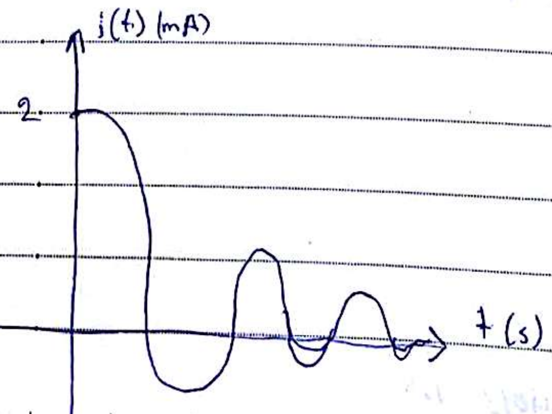
$\alpha = 1000\ s^{-1}$, $w_o = 2025\ s^{-1}$ $\longrightarrow$ under damped

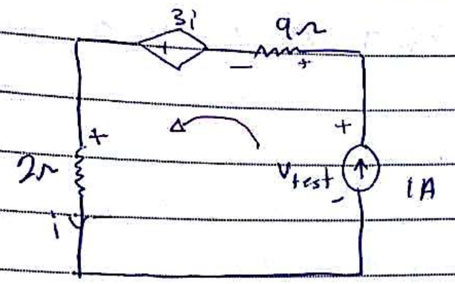
$w_d = 2000$ rad/s

$i(0) = 2mA = B_1$

$B_2 = 0$

$i(t) = 2e^{-1000t}(\cos 20000t)\ mA$





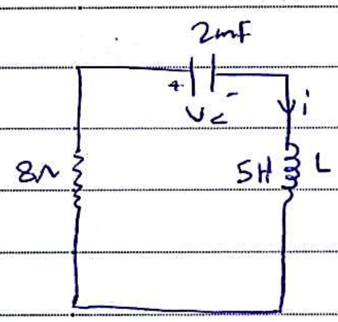
$i = 1\,A$

KVL eq:

$+2i + -3i + -9i + V_{test}$

$V_{test} = 8$ Volt

$R_{th} = \frac{V_{test}}{I_{test}} = \frac{8}{1} = 8\,\Omega$



$\alpha = \frac{R}{2L} = \frac{8}{10} = 0.8\,s^{-1}$

$\alpha < w_0 \Rightarrow$ under damped

$w_0 = \frac{1}{\sqrt{LC}} = 10$ ~~rad/s~~ s^{-1}

$w_d = \sqrt{w_0^2 - \alpha^2} = 9.968$ rad/s

$V_C(t) = e^{-0.8t}(B_1 \cos 9.968t + B_2 \sin 9.968t)$ volt

$\rightarrow V_C(0) = -5$ $\boxed{B_1 = -5 \text{ Volt}}$

$\rightarrow \left.\frac{d\,v(t)}{dt}\right|_{t=0} = \frac{i_C(0)}{C}$

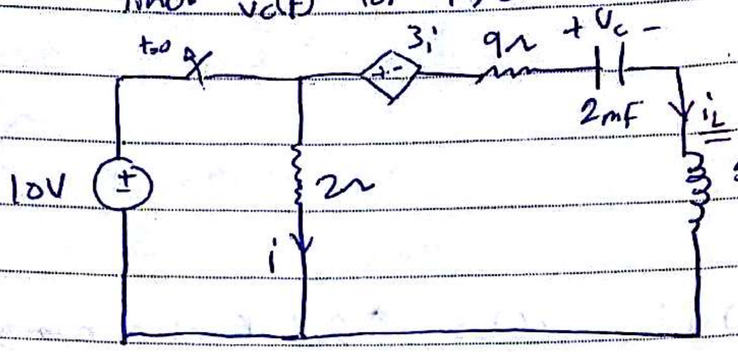
$i_C(0) = i_L(0) = 0$

$\frac{dv(t)}{dt} = -$

$\boxed{B_2 = -0.4013 \text{ Volt}}$

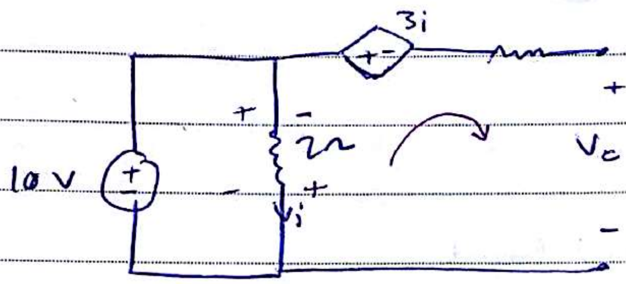
[*] Example :

find $v_c(t)$ for $t>0$



for $t<0$

switch closed , $L \to s.c$, $C \to o.c$



$i = 5\ A$

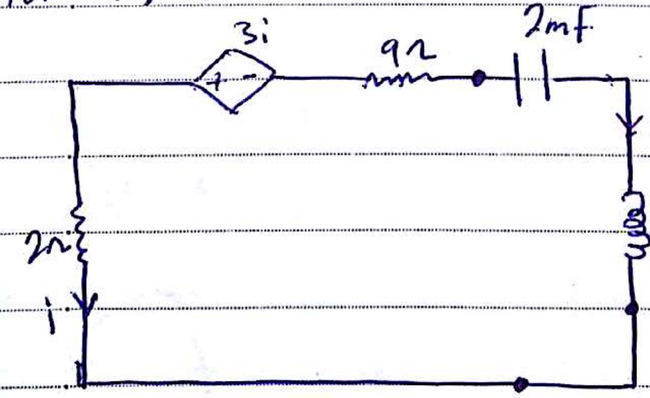
$i_L(0^-) = \text{zero} = i_L(0)$

$V_c(0^-) = 10 - 15$

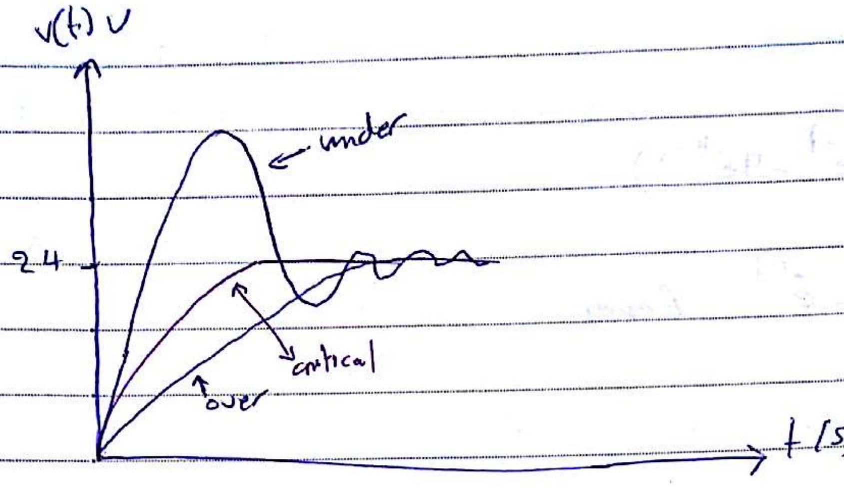
$V_c(0^-) = -5$ volt $= V_c(0)$

initial conditions

for $t>0$



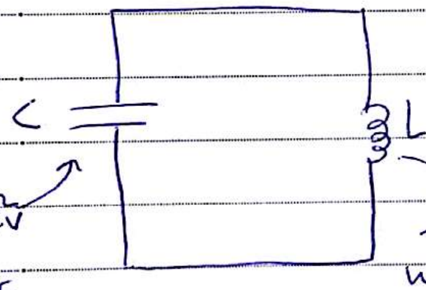
Thevenin's eq :



$\boxed{*}$ LC circuit Lossless critical

($R=0 \rightarrow$ series RLC)

($R\rightarrow\infty \rightarrow$ parallel RLC)



$\frac{1}{2}CV^2$ W_E

$\frac{1}{2}LI^2$ W_M

not practical

$$\alpha = \frac{1}{2RC} = \frac{1}{\infty} = 0$$

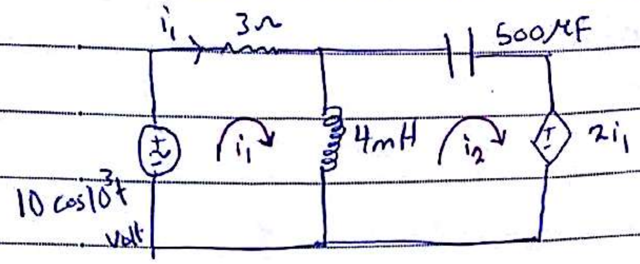
$$\alpha = \frac{R}{2L} = 0$$

There is no damping function we solve this by sine or cosine

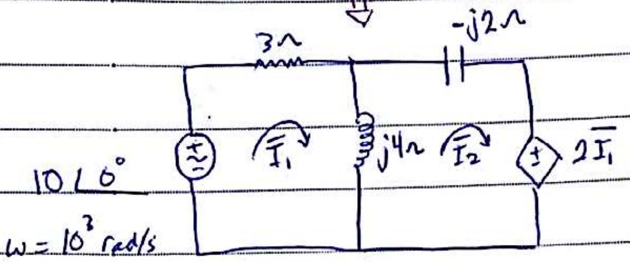
$e^{-\alpha t} = 1$

Ch. 10

* Ex: Find the time domain currents $i_1(t)$, and $i_2(t)$?



⇓



KVL at Mesh (1):

$-10\angle 0^\circ + 3\bar{I}_1 + j4(\bar{I}_1 - \bar{I}_2) = 0$

$(3 + j4)\bar{I}_1 - j4\bar{I}_2 = 10$ — ②

$10\angle 0 = 10\cos \overset{1}{0} + j10 \cancel{\sin 0}^{\text{zero}} = \underline{10}$

KVL at mesh (2):

$+j4(\bar{I}_2 - \bar{I}_1) - j2\bar{I}_2 + 2\bar{I}_1 = 0$

$(2 - j4)\bar{I}_1 + j2\bar{I}_2 = 0$ — ②

from ②:

$$\bar{I}_2 = \frac{-(2 - j4)\bar{I}_1}{j2}$$

$$\bar{I}_2 = \frac{j(2 - j4)\bar{I}_1}{2}$$

$$\bar{I}_2 = \frac{(j2 + 4)\bar{I}_1}{2}$$

$\bar{I}_2 = (j + 2)\bar{I}_1$ sub in ①

$(3+j4)\bar{I}_1 - j4(2+j)\bar{I}_1 = 10$

$(3+j4)\bar{I}_1 - j8\bar{I}_1 + 4\bar{I}_1 = 10$

$7\bar{I}_1 - j4\bar{I}_1 = 10$

$\bar{I}_1 = \frac{10}{7-j4}$

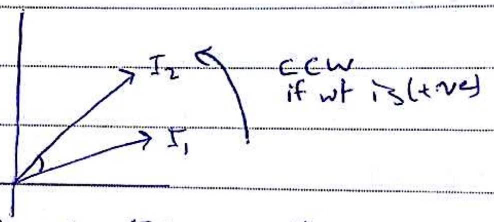
$\bar{I}_1 = \frac{10\angle 0^\circ}{\sqrt{49+16}\angle \tan^{-1}\frac{-4}{7}}$

$\bar{I}_1 = 1.24\angle 29.7^\circ$ A

$\bar{I}_2 = 2.77\angle 56.3^\circ$ A

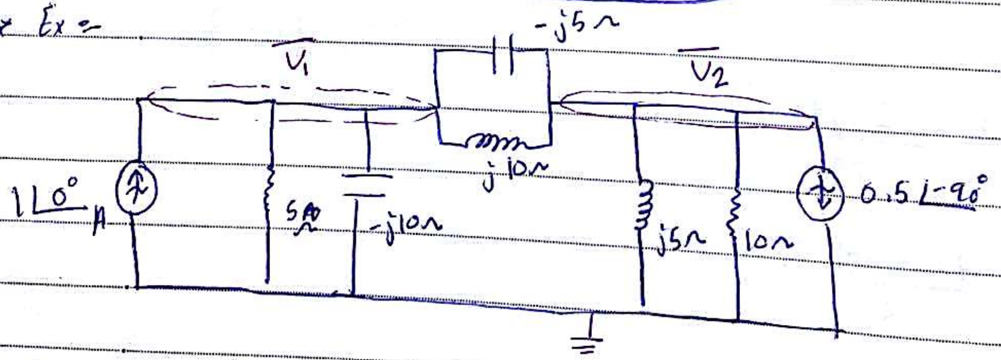
$i_1(t) = 1.24\cos(10^3 t + 29.7^\circ)$ A

$i_2(t) = 2.77\cos(10^3 t + 56.3^\circ)$ A



i_2 legs i_1 by $(56.3 - 29.7^\circ)$

* Ex -



* Nodal → (find the Nodal voltages)

3 nodes

angle $0^\circ \to 1$

$$-1 + \frac{\bar{V}_1}{5} + \frac{\bar{V}_1}{-j10} + \frac{\bar{V}_1 - \bar{V}_2}{j10} + \frac{\bar{V}_1 - \bar{V}_2}{-j5} = 0$$

$$-1 + \frac{\bar{V}_1}{5} + \frac{j\bar{V}_1}{10} + \frac{-j(\bar{V}_1 - \bar{V}_2)}{10} + \frac{j(\bar{V}_1 - \bar{V}_2)}{5} = 0$$

$$(0.2 + j0.2)\bar{V}_1 + (-j0.1)\bar{V}_2 = 1 \quad (1)$$

KCL for node(2) =

$$0.5\angle -90^\circ = 0.5\cos(-90) + j\sin(-90) = -j0.5$$

$$-j0.5 + \frac{\bar{V}_2}{10} + \frac{\bar{V}_2}{j5} + \frac{\bar{V}_2 - \bar{V}_1}{j10} + \frac{\bar{V}_2 - \bar{V}_1}{-j5} = 0 \quad (2)$$

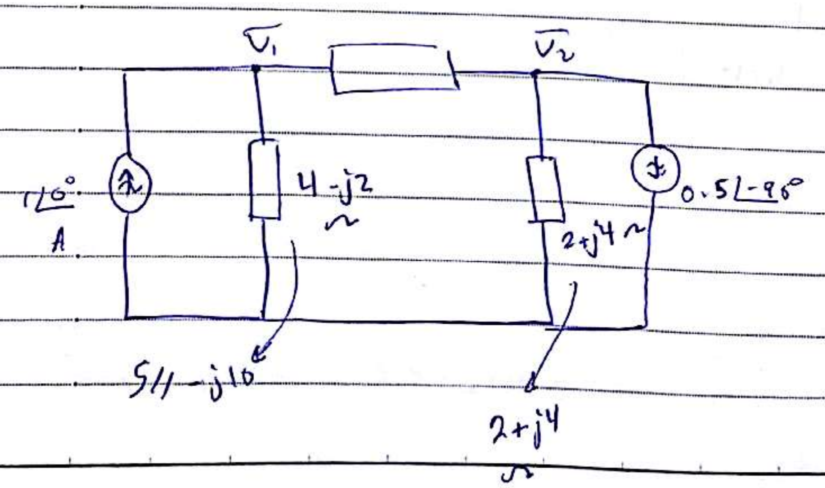
$$\bar{V}_1 = 2.24\angle -63.4^\circ \text{ V} = \boxed{1 - j2 \text{ V}}$$

$$\bar{V}_2 = 4.47\angle 116.6^\circ \text{ V} = -2 + j4 \text{ V}$$

$$v_1(t) = 2.24\cos(\omega t - 63.4^\circ) \text{ V}$$

V_2 leads by $(116.6 + 63.4)$

* Voltage leads I → Inductive
* V ~~current~~ lags I → capacitive
* V, I same angle → inphase



* دبلزم نحلها
* source Transformation.

* Find $\bar{V}_1$ using super position

linear circuit, Independent can't be killed

$\bar{V}_1$ due to $1\angle 0°$ source, $0.5\angle -90°$ → open circuit

$\bar{Z}_{eq} = (-j10 + 2+j4) \,//\, (4-j2)$

$= (2-j6) \,//\, (4-j2)$

$= 2-j2\ \Omega$

$\bar{V}_1' = 1(2-j2) = 2-j2$ Volt

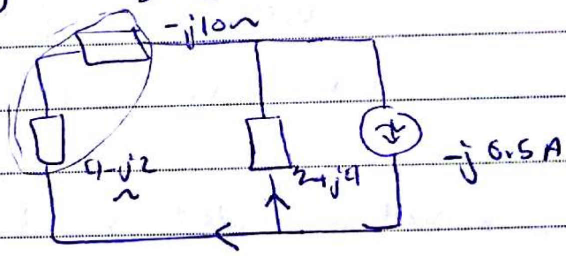
$\bar{V}_1$ due to $0.5\angle -90°$ source only

CDR:

$\bar{V}_1'' = (4-j2) * \left(-j0.5 \times \frac{2+j4}{(4-j2-j10)+(2+j4)}\right)$

$\bar{V}_1'' = (4-j2) \times (-j0.5) \left(\frac{2+j4}{6-j8}\right)$

$\bar{V}_1'' = -1$ Volt



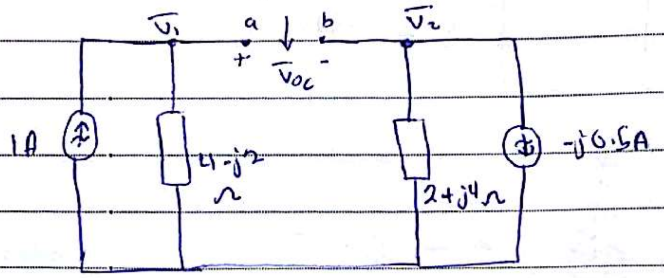
$\bar{V}_1 = \bar{V}_1' + \bar{V}_1''$

$= 2-j2-1$

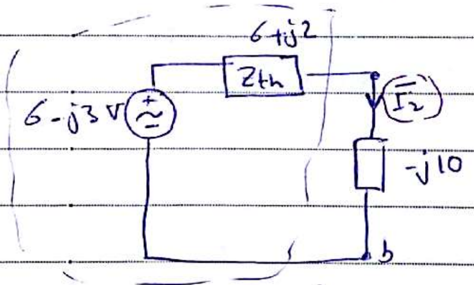
$\boxed{\bar{V}_1 = 1-j2 \text{ Volt}}$

* Find Thevenin's eq as seen by -j10Ω then find $\bar{V}_1$

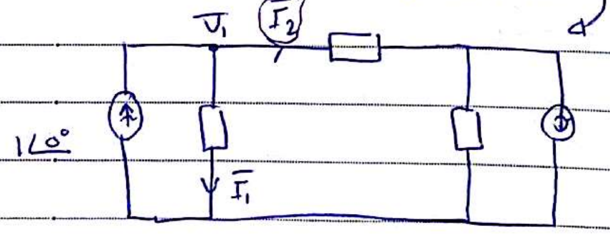
$$\bar{Z}_{th} = 4 - j2 + 2 + j4$$
$$= 6 + j2\ \Omega$$



$$\bar{V}_{oc} = \bar{V}_1 - \bar{V}_2$$
$$= 1(4-j2) - (2+j4)(+j0.5)$$
$$= 4 - j2 - j1 + 2$$
$$\bar{V}_{oc} = \bar{V}_{th} = 6 - j3 \text{ volt}$$



Find $\bar{V}_1$ in the original circuit



$$\bar{I}_2 = \frac{6 - j3}{6 + j2 - j10} = 0.06 + j0.3\ A$$

$$\bar{I}_1 = 1 - \bar{I}_2 = 0.4 - j0.3\ A$$

$$\bar{V}_1 = (0.4 - j0.3)(4 - j2)$$
$$\bar{V}_1 = 1.6 - j0.8 - j1.2 - 0.6$$
$$\boxed{\bar{V}_1 = 1 - j2 \text{ volt}}$$

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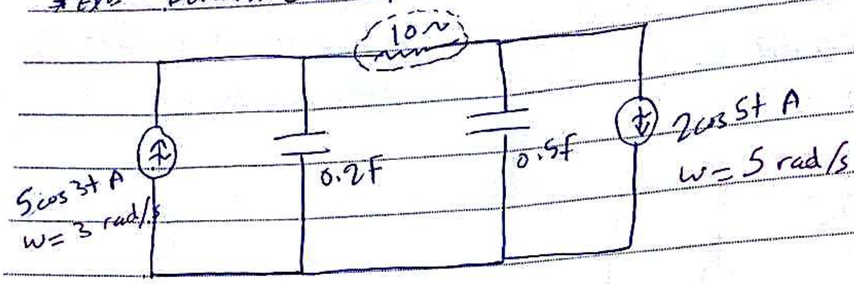
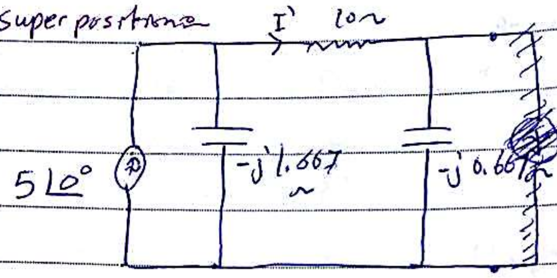
{ for two different frequensies use ~~only~~ superposition

*Ex: Determine the power dissipated by the 10Ω resistor



superposition



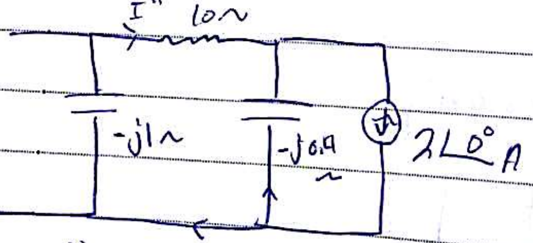
← for 5∠0°, w = 3 rad/s

CDR:

$$I' = 5\left(\frac{-j1.667}{10 - j1.667 - j0.667}\right)$$

$$I_1' = 811.7 \angle -76.86^\circ \ mA$$

for 2∠0° A only:



$$I'' = 2\left(\frac{-j0.4}{10 - j0.4 - j1}\right)$$

$$= 79.23 \angle -83.03^\circ \ mA$$

to find Power →

$i'(t) = 811.7 \cos(3t - 76.86^\circ)$ mA
$i''(t) = 79.23 \cos(5t - 82.03^\circ)$ mA

$i(t) = i'(t) + i''(t)$
$p = 10\, i^2(t)$

$\boxed{*}$ Phasor diagram

Better view for leading and lagging for all values (responces in the circuit)

i.e. $\bar{V}_1 = 6 + j8 = 10 \angle 53.1^\circ$ volt

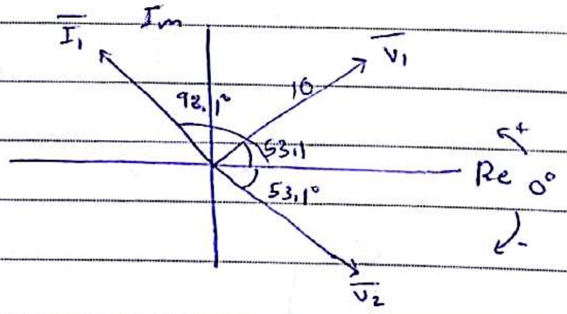
$\bar{V}_2 = 3 - j4 = 5 \angle -53.1^\circ$ V

$\bar{Y} = 1 + j1$ S $= \sqrt{2} \angle \overset{+}{45^\circ}$ → capacitance

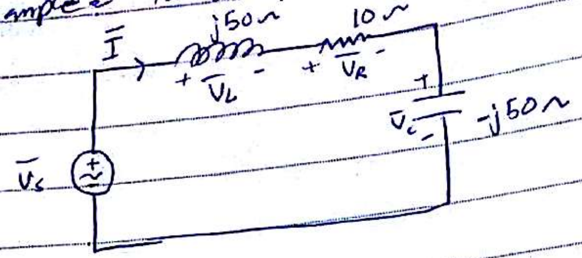
$\bar{I}_1 = \bar{V}_1 \bar{Y} = (1+j1)(6+j8) = 10\sqrt{2} \angle 98.1^\circ$ A

ωt → +ve CCW

10 V → 1 cm



* Example: for a simple RLC



reference values:

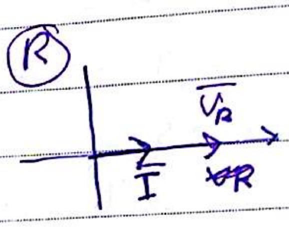
- series → $\bar{I}$ is the ref
- parallel → $\bar{V}$ is the ref

Assume:

$\omega t \to$ +ve ccw

$\bar{I} = 1\angle 0^\circ$ A as ref

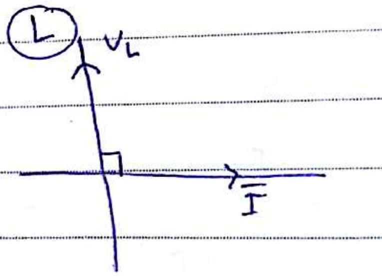
$\bar{V}_R = \cancel{100}\ \bar{I}\ R = 10\angle 0^\circ$ V



in ~~pase~~ phase

$\bar{V}_L = j\omega L \bar{I} = j50(1)$

$= 50\angle 90^\circ$ V



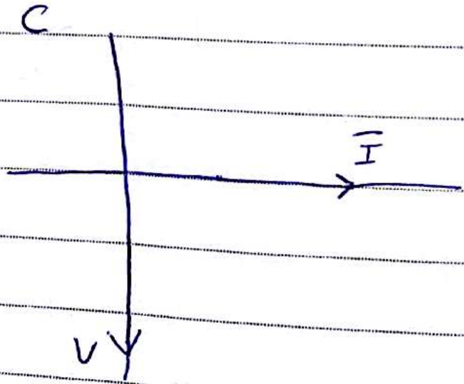
V leads I by 90°

$\bar{V}_c = +\frac{1}{j\omega c}\bar{I}$

$= -j50\bar{I}$

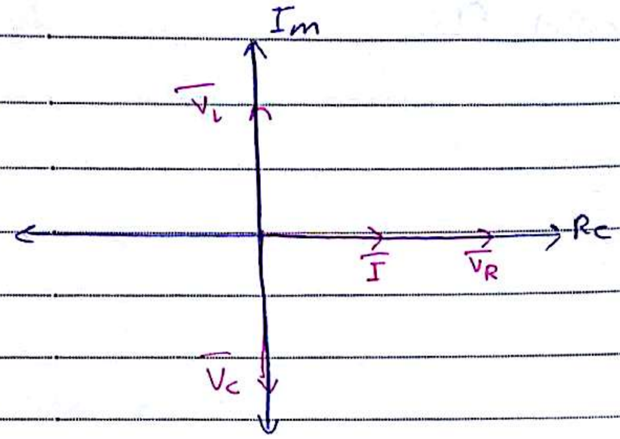
$= -j50$ Ω

$= 50\angle -90$ V



$\bar{I}$ leads $\bar{V}$ by 90°

Scanned by CamScanner



KVL :

$\bar{V}_S = \bar{V}_L + \bar{V}_R + \bar{V}_C$

$= \cancel{j50} + 10 - \cancel{j50}$

$= 10$ volt

→ $j\omega L$ cancle with $\frac{1}{j\omega C}$

circuit is at resonance (series Resonance) $\Rightarrow \bar{Z}\Big|_{\text{at resonance}} = R$

$\bar{Z} = \cancel{j\omega L} + R - \cancel{\frac{1}{j\omega C}}$

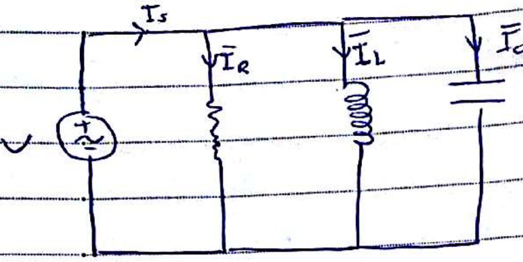
$j\omega L = \frac{-1}{j\omega C}$

$-\omega^2 LC = -1$

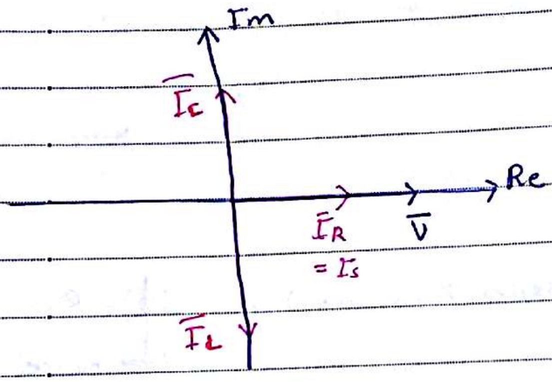
$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$$

$\boxed{*}$ Single Node-pair ckt parallel RLC

$V \rightarrow$ is ref $= 1\angle 0^\circ$



$\bar{V} = 1\angle 0^\circ$



at $\omega_0 = \frac{1}{\sqrt{LC}}$

for parallel RLC Take $I = 1\angle 0^\circ$ as a ref

$\rightarrow$ Assume $V = 1\angle 0^\circ$

$\bar{I}_R$

$\bar{I}_C$

$\bar{I}_L$

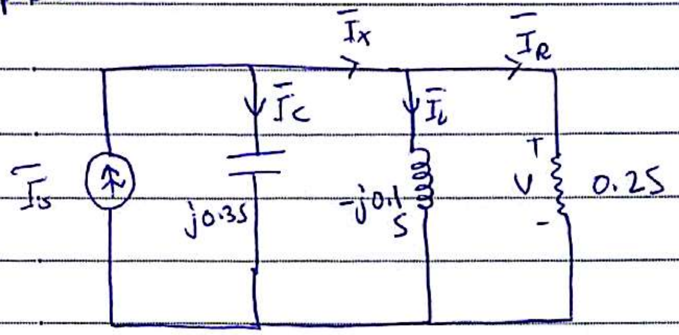
$I = I_R + I_C + I_L$

$= 0.2 + j\,0.1 \neq 1$

$\bar{V} = \frac{1\angle 0^\circ}{0.2 + j0.1}$ ← Assume

$\bar{V} = \sqrt{20}\ \angle -26.\ V$

Example



Determine how much $\bar{I}_s$ leads $\bar{I}_x$, $\bar{I}_R$, $\bar{I}_C$, $\bar{I}_L$?

Solution

Assume $\bar{V} = 1\angle 0^\circ$

$\bar{I}_R = 0.2\angle 0^\circ$

$\bar{I}_L = -j0.1(1) = 0.1\angle -90^\circ$

$\bar{I}_C = 0.3\angle 90^\circ$

$\bar{I}_x = \bar{I}_L + \bar{I}_R = 0.224\angle -26.6^\circ$ A

$\bar{I}_s = \bar{I}_x + \bar{I}_c = 0.283\angle 45^\circ$ A