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Jordan University
Mathematics Department
Calculus II, Second Exam, 18/4/2015 I

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Lecture Time: 9 - 10

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1) (3 points) Find the area of the surface obtained by rotating the curve $x = y^2 + 1$, $1 \le y \le 3$ about the x-axis.

$$\text{area} = \int 2\pi r \, ds$$

$$= \int_1^3 2\pi y \sqrt{1 + \left(\frac{dx}{dy}\right)^2} \, dy$$

$$= \int_1^3 2\pi y \sqrt{1 + (2y)^2} \, dy$$

$$= \int_1^3 2\pi y \sqrt{1 + 4y^2} \, dy \Rightarrow \text{Alter integral}$$

$$= 2\pi \cdot \frac{1}{12} (1 + 4y^2)^{3/2} \Big]_1^3 \Rightarrow \frac{\pi}{6}\left[(1+36)^{3/2} - (6)^{3/2}\right]$$

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$$\int y \sqrt{1+4y^2} \, dy$$

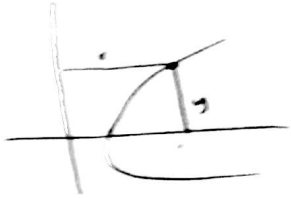
let $1 + 4y^2 = v$

$8y \, dy = dv$

$$\int \frac{dv}{8} \sqrt{v} = \frac{1}{8} \int v^{1/2} \, dv$$

$$= \frac{1}{8} v^{3/2} \cdot \frac{2}{3} = \frac{1}{12} v^{3/2}$$

$$= \frac{1}{12} (1 + 4y^2)^{3/2}$$



2) (2 points) Find the value of the sum $\sum_{n=1}^{\infty} \cos\left(\frac{\pi}{3n}\right) - \cos\left(\frac{\pi}{3(n+1)}\right)$

$$S_1 = \cos\left(\frac{\pi}{3}\right) - \cos\left(\frac{\pi}{6}\right)$$

$$S_2 = \cos\left(\frac{\pi}{3}\right) - \cancel{\cos\left(\frac{\pi}{6}\right)} + \cancel{\cos\left(\frac{\pi}{6}\right)} - \cos\left(\frac{\pi}{9}\right) = \cos\left(\frac{\pi}{3}\right) - \cos\left(\frac{\pi}{9}\right)$$

$$S_3 = \cos\left(\frac{\pi}{3}\right) - \cancel{\cos\left(\frac{\pi}{9}\right)} + \cancel{\cos\left(\frac{\pi}{9}\right)} - \cos\left(\frac{\pi}{12}\right) = \cos\left(\frac{\pi}{3}\right) - \cos\left(\frac{\pi}{12}\right)$$

$$S_n = \cos\left(\frac{\pi}{3}\right) - \cos\left(\frac{\pi}{3(n+1)}\right)$$

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} \cos\left(\frac{\pi}{3}\right) - \cos\left(\frac{\pi}{3(n+1)}\right) = \cos\left(\frac{\pi}{3}\right) - \cos(0)$$

$$= \frac{1}{2} - 1 = -\frac{1}{2}$$

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3) (3 points) Use series to write the repeated decimal $0.2\overline{7}$ as a quotient of integers (fraction).

$0\overline{27}$

$$0.\overline{27} = \frac{27}{10^2} + \frac{27}{10^4} + \frac{27}{10^6} + \frac{27}{10^8} + \cdots$$

$$\boxed{\sum_{n=1}^{\infty} \frac{27}{10^{2n}}}$$

$$\text{geometric sum} = \frac{a}{1-r} = \frac{0.27}{1 - \frac{1}{100}}$$

4) (3 points each) Test the following series for convergence:

a) $\sum_{n=1}^{\infty}\left(\frac{2+n}{3+n}\right)^n$

power unit

$\lim_{n\to\infty}\left(\frac{n+2}{n+3}\right)^n = \lim_{n\to\infty}\frac{\left(1+\frac{2}{n}\right)^n}{\left(1+\frac{3}{n}\right)^n} = \frac{e^2}{e^3} = \frac{1}{e} \neq 0$ by divergence test

the $\sum_{n=1}^{\infty}\left(\frac{2+n}{3+n}\right)^n$ is divergent

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b) $\sum_{k=1}^{\infty}\tan\left(\frac{1}{2^k}\right)$

let $b_n = \sum_{k=1}^{\infty}\frac{1}{2^k} > 0$

$\cancel{\lim_{n\to\infty}\frac{\tan(\frac{1}{2^k})}{\frac{1}{2^k}}}$ $\lim_{k\to\infty}\frac{a_n}{b_n} = \lim_{k\to\infty}\frac{\tan\left(\frac{1}{2^k}\right)}{\frac{1}{2^k}}$

let $\frac{1}{2^k} = u$

$k\to\infty \Rightarrow u\to 0$

$\lim_{n\to 0}\frac{\tan(n)}{n} = \lim_{n\to 0}\frac{\sin(n)}{n}\cdot\frac{1}{\cos(n)} = 1\cdot\frac{1}{1} = \boxed{1}$

$\lim_{k\to 0}\frac{\tan(u)}{u} = \boxed{1} \neq 0$

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but $b_n = \sum_{k=1}^{\infty}\frac{1}{2^k}$ is convergent because it is a geometric series with $-1 < r = \frac{1}{2} < 1$

so, by limit comparison test $\sum_{k=1}^{\infty}\tan\left(\frac{1}{2^k}\right)$ is convergent

c) $\sum_{n=2}^{\infty}\frac{1}{n\sqrt[3]{\ln(n)}}$

$\frac{1}{n\sqrt[3]{\ln(n)}} > 0$, $f(x) = \frac{1}{x\sqrt[3]{\ln(x)}}$, $f'(x) = \frac{-1\left(x\cdot\frac{1}{3}(\ln(x))^{-\frac{2}{3}}\cdot\frac{1}{x} + \sqrt[3]{\ln(x)}\right)}{\left(x\sqrt[3]{\ln(x)}\right)^2} = \frac{\sqrt[3]{\ln(x)} - \frac{1}{3}(\ln x)^{-\frac{2}{3}}}{\left(x\sqrt[3]{\ln(x)}\right)^2}$

$f(x)$ cont and decreasing in $[2,\infty)$

$\int_2^{\infty}\frac{1}{x\sqrt[3]{\ln(x)}} = \lim_{b\to\infty}\int_2^b\frac{1}{x\sqrt[3]{\ln(x)}} = \lim_{b\to\infty}\frac{3}{2}\left(\sqrt[3]{\ln(x)}\right)^2\Big]_2^b$

after integral

$\int\frac{1}{x\sqrt[3]{\ln x}}$ let $u = \ln x$, $du = \frac{1}{x}\,dx$

$\int\frac{du}{u^{1/3}} = \int u^{-1/3}\,du$

$\frac{3}{2}\cdot u^{2/3} + c$

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$= \infty - \frac{3}{2}\left(\sqrt[3]{\ln(2)}\right) = \infty$ divergent

So, by Integral test $\sum_{n=2}^{\infty}\frac{1}{n\sqrt[3]{\ln(n)}}$ is divergent

d) $\sum_{n=2}^{\infty}\frac{1}{n^3+\ln(n)+3}$

let $b_n = \sum_{n=2}^{\infty}\frac{1}{n^3} > 0$

$n^3+\ln(n)+3 > n^3$

$\frac{1}{n^3+\ln(n)+3} < \frac{1}{n^3}$

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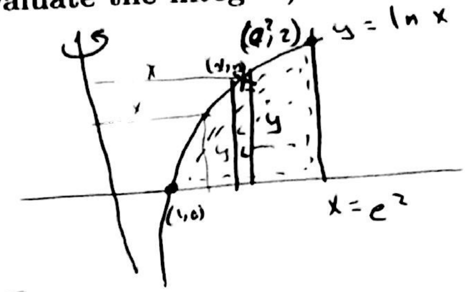
but $b_n = \sum_{n=2}^{\infty}\frac{1}{n^3}$ is convergent by a p-series $p = 3 > 1$ so, by comparison test

$\sum_{n=2}^{\infty}\frac{1}{n^3+\ln(n)+3}$ is convergent

5) Set up the integral that gives the volume obtained when the region enclosed by $y = \ln(x)$, $x = e^2$, and $y = 0$ is revolved about y-axis.

a) (3 points) Using washers method, **(Do not evaluate the integral).**

$$\int_0^2 \pi (e^2)^2 \, dy - \int_0^2 \pi (e^y)^2 \, dy$$



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b) (3 points) Using cylindrical shell method, **(Do not evaluate the integral).**

$$\int_1^{e^2} 2\pi x \, (\ln x) \, dx$$

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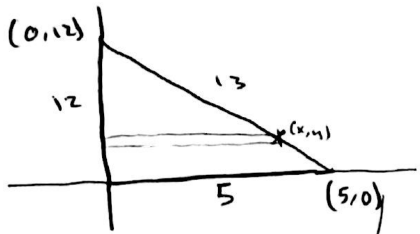
6) (4 points) Use integral to find the volume of the tetrahedron with three mutually perpendicular faces and three mutually perpendicular edges with lengths 5, 12, and 13. (Show your work).



slope $= \dfrac{0-12}{5-0} = \dfrac{-12}{5} = \boxed{-2.4}$

$y - 0 = -2.4(x-5) \Rightarrow \boxed{y = -2.4(x-5)}$

each cross section in y-axis is $\triangle$ مثلث قائم الزاوية

$\dfrac{y}{-2.4} = x - 5$

$\boxed{x = \dfrac{-y}{2.4} + 5}$

$$\int_0^{12} \frac{1}{2} (12)(x) \, dy$$

$$= \int_0^{12} 6 \left(\frac{-y}{2.4} + 5\right) dy$$

$$= 6 \int_0^{12} \left(5 - \frac{1}{2.4} y\right) dy$$

$$= 6 \left(5y - \frac{1}{2.4}\left(\frac{y^2}{2}\right)\right]_0^{12}$$

$$= 6 \left(5(12) - \frac{(12)^2}{(2.4)(2)} - 0\right)$$

$$= 6 \left(60 - \frac{144}{(2.4)(2)}\right) \text{unit}^3$$

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