الجامعة الأردنية	٠٣٠١١٠٢ تفاضل وتكامل -٢	السبت ٢٠١٥/٣/١٤
قسم الرياضيات	الامتحان الأول	

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أجب كل سؤال على صفحة منفصلة:

[1] Evaluate $\int x^5 sec^2(x^3)dx$

[2] Evaluate $\int \frac{dx}{(x-4)(x^2-16)}$

[3] Evaluate $\int \sqrt{x^2+6x+8}\ dx$

[4] Find (if exits) $\int_1^{\infty} \frac{dx}{e^x - e^{-x}}$

[5] Find the area enclosed by the curves $x = y^2 - 4y + 3$, and
$x = -y^2 + 2y + 3$

Q.1 $\int x^5 \sec^2(x^3)\, dx$

Let

$u = x^3$

$du = 3x^2 dx$

$\boxed{\frac{du}{3x^2} = dx}$

$\int \frac{x^{\cancel{5}\,3} \sec^2(u) \cdot du}{3\cancel{x^2}}$

$= \frac{1}{3} \int x^3 \sec^2(u) du$

$= \frac{1}{3} \int \underset{u}{u} \underbrace{\sec^2(u)\, du}_{dv}$

$= \frac{1}{3} \left[u \cdot \tan(u) - \int \tan(u) \cdot du \right]$

$= \frac{1}{3} \left[u \cdot \tan(u) - \ln|\sec(u)| \right] + c$

$= \frac{1}{3} x^3 \tan(x^3) - \frac{1}{3} \ln|\sec(x^3)| + c$

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$\boxed{Q.2}$

$$\int \frac{dx}{(x-4)(x^2-16)}$$

$$= \int \frac{dx}{(x-4)(x-4)(x+4)}$$

$$= \int \frac{dx}{(x-4)^2(x+4)}$$

$$\frac{1}{(x-4)^2(x+4)} = \frac{A}{x-4} + \frac{B}{(x-4)^2} + \frac{C}{x+4}$$

$$= A(x-4)(x+4) + B(x+4) + C(x-4)^2 = 1$$

$\boxed{\text{Let } x=4}$

$8B = 1 \longrightarrow \boxed{B = \frac{1}{8}}$

$\boxed{\text{Let } x=-4}$

$64C = 1 \longrightarrow \boxed{C = \frac{1}{64}}$

$\boxed{\text{let } x=0}$

$$-16A + \frac{1}{2} + \frac{16}{64} = 1$$

$$-16A + \frac{1}{2} + \frac{1}{4} = 1$$

$$-16A + \frac{3}{4} = 1 \longrightarrow 16A = \frac{3}{4} - \frac{4}{4}$$

$$16A = \frac{-1}{4}$$

$$\boxed{A = \frac{-1}{64}}$$

$$\int \frac{-\frac{1}{64}}{x-4}dx + \int \frac{\frac{1}{8}}{(x-4)^2}dx + \int \frac{\frac{1}{64}}{x+4}dx$$

$$= \frac{-1}{64}\ln|x-4| + \frac{1}{8}\left(\frac{-1}{x-4}\right) + \frac{1}{64}\ln|x+4| + c$$

$\boxed{Q.3}$

$$\int \sqrt{x^2+6x+8}\;dx \qquad \left(\frac{6}{2}\right)^2 = \boxed{9}$$

$$= \int \sqrt{x^2+6x+9-9+8}\;dx$$

$$\int \sqrt{(x+3)^2-1}\;dx \qquad \text{let } x+3 = 1\sec\theta$$

$$dx = \sec\theta\tan\theta\, d\theta$$

$$\int \sqrt{\sec^2\theta-1}\cdot\sec\theta\tan\theta\,d\theta$$

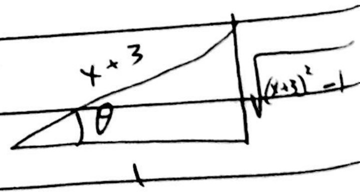
$$\int \tan^2\theta\sec\theta\,d\theta$$

$$\int (\sec^2\theta-1)\sec\theta\,d\theta$$

$$= \int \sec^3\theta\,d\theta - \int \sec\theta\,d\theta$$

$$= \frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln|\sec\theta+\tan\theta| - \ln|\sec\theta+\tan\theta| + c$$

$$= \frac{1}{2}\sec\theta\tan\theta - \frac{1}{2}\ln|\sec\theta+\tan\theta| + c$$



$$= \frac{1}{2}(x+3)\left(\sqrt{(x+3)^2-1}\right) - \frac{1}{2}\ln\left|(x+3)+\left(\sqrt{(x+3)^2-1}\right)\right| + c$$

Q.4 $\displaystyle\int_1^\infty \frac{dx}{e^x - e^{-x}}$

$$\lim_{b\to\infty} \int_1^b \frac{dx}{e^x - e^{-x}}$$

after integration $\Longrightarrow$

$$\lim_{b\to\infty} \frac{1}{2} \ln \left|\frac{e^x - 1}{e^x + 1}\right| \Bigg]_1^b$$

$$\lim_{b\to\infty} \ln\left|\frac{e^x - 1}{e^x + 1}\right|$$

$$\ln\left(\lim_{b\to\infty} \frac{e^x - 1}{e^x + 1}\right) \Rightarrow \cancel{\ln\left(\lim_{b\to\infty} \frac{e^x}{e^x}\right)}$$

$$= \ln(1) = 0$$

$$\Rightarrow \frac{1}{2}(0) - \frac{1}{2}\ln\left|\frac{e^1 - 1}{e^1 + 1}\right|$$

$$= -\frac{1}{2}\ln\left|\frac{e - 1}{e + 1}\right| \quad \text{convergeA}$$

$$\int \frac{dx}{e^x - e^{-x}}$$

$$du = e^x\,dx$$

$$\int \frac{dx}{e^x - \frac{1}{e^x}} = \int \frac{dx}{\frac{e^{2x} - 1}{e^x}}$$

$$= \int \frac{e^x\,dx}{e^{2x} - 1}$$

$$\boxed{\begin{aligned} &\text{let } u = e^x \\ &du = e^x\,dx \end{aligned}}$$

$$\int \frac{du}{u^2 - 1} = \int \frac{du}{(u-1)(u+1)}$$

$$\frac{A}{u-1} + \frac{B}{u+1} \Rightarrow A(u+1) + B(u-1) = 1$$

$\boxed{\text{let } u = 1}$

$$2A = 1 \longrightarrow \boxed{A = \frac{1}{2}}$$

$$\boxed{B = -\frac{1}{2}}$$

$$\Rightarrow \int \frac{du}{u^2 - 1} = \int \frac{\frac{1}{2}}{u - 1}\,du + \int \frac{-\frac{1}{2}}{u + 1}\,du$$

$$= \frac{1}{2}\ln|u - 1| - \frac{1}{2}\ln|u + 1|$$

$$= \frac{1}{2}\ln\left|\frac{u - 1}{u + 1}\right|$$

$$\boxed{= \frac{1}{2}\ln\left|\frac{e^x - 1}{e^x + 1}\right|}$$

Q.5

$x = y^2 - 4y + 3$

$x = y^2 - 4y + 4 - 4 + 3$

$x = (y-2)^2 - 1$

$x + 1 = (y-2)^2$

$\pm\sqrt{x+1} = y - 2$

$y = 2 \pm \sqrt{x+1}$

$x = -y^2 + 2y + 3$

$x = -(y^2 - 2y - 3)$

$x = -(y^2 - 2y + 1 - 1 - 3)$

$x = -((y-1)^2 - 4)$

$x = 4 - (y-1)^2$

$(y-1)^2 = 4 - x$

$y - 1 = \pm\sqrt{4-x}$

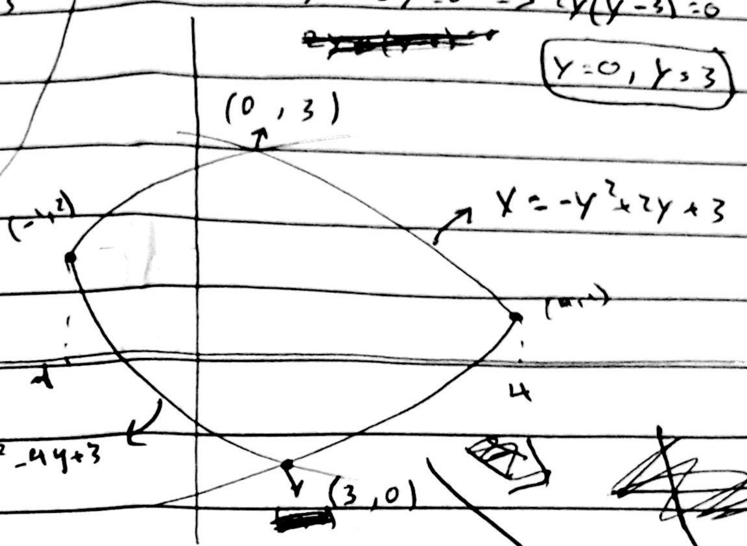
$y = \pm\sqrt{4-x} + 1$

$-y^2 + 2y + 3 = y^2 - 4y + 3$

$y^2 + y^2 - 4y - 2y + 3 - 3 = 0$

$2y^2 - 6y = 0 \Rightarrow 2y(y-3) = 0$

$y = 0, \; y = 3$



$$\text{Area} = \int_0^3 (-y^2 + 2y + 3 - y^2 + 4y - 3)\, dy$$

$$= \int_0^3 (-2y^2 + 6y)\, dy$$

$$= \left(\frac{-2}{3} y^3 + 3y^2\right)\Big]_0^3$$

$$= \left(\frac{(-2)(27)}{3} + 27\right) - 0$$

$$= -18 + 27$$

$$= 9 \text{ unit}^2$$