THE UNIVERSITY OF JORDAN

0301101 Calculus I — Second test — Nov. 29, 2014

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Instructor's name: Class days and time:

Questions 1 to 11: fill in the blanks with the answers only. Each question is worth 2 marks.

___Q1) The vertical asymptote(s) of $f(x) = \frac{\tan x}{x^3-4x}$ is (are) $x=2,\ x=-2$

___Q2) $\frac{d^{21}}{dx^{21}}(2^x) =$ $2^x(\ln 2)^{21}$

___Q3) If $f(x) = x^4$, then $\lim\limits_{x \to 1} \frac{f(x)-1}{x-1} =$ 4

___Q4) The function $f(x) = \frac{x-1}{|x|-1}$ has removable discontinuity at the point(s) $(1,0)$

___Q5) $\lim\limits_{x \to 1} \frac{2\sin(x^2-1)}{x-1} =$ 4

___Q6) If $f(x) = \sin^{-1}(\log_2 x)$, then $f'(x) =$ $\frac{1}{\sqrt{1-(\log_2 x)^2}} \cdot \frac{1}{x \ln 2}$

___Q7) If $f'(x) = \frac{x}{x^2+1}$, $g(x) = \sqrt{x}$ and $h(x) = f(g(x))$, then $h'(2) =$ $\frac{1}{6}$

___Q8) Using linear approximation (linearization), the best estimate of $(0.99)^3$ is $= 1 - 3(0.01)$

___Q9) The horizontal asymptote(s) of $f(x) = \frac{\sqrt[3]{2x^3+1}}{x-3}$ is (are) $y = \sqrt[3]{2}$

___Q10) If $f(x) = x^2 - 6x + 21, x \leq 3$, then $(f^{-1})'(16) =$ $-\frac{1}{4}$

Questions 11 and 12: Each part is worth 4 marks. Write all solution steps.

___ Q11) Let $y = (\sin 3x)^{2x}$. Find $\frac{dy}{dx}$.

$\ln y = \ln(\sin 3x)^{2x}$

$\ln y = 2x \ln(\sin 3x)$

$\frac{\frac{dy}{dx}}{y} = 2x \cdot \frac{3\cos 3x}{\sin 3x} + \ln(\sin 3x) \cdot 2$

$\frac{dy}{dx} = \left(6x \cot 3x + 2\ln(\sin 3x)\right) \cdot y \Rightarrow \frac{dy}{dx} = \left(6x \cot 3x + 2\ln(\sin 3x)\right) \cdot (\sin 3x)^{2x}$

(4)

___ Q12) Given the curve $x^2 + 2y^2 = 1$.

a) Find the point(s) on this curve where the tangent line has the slope 1.

$x^2 + 2y^2 = 1$

$2x + 4y\frac{dy}{dx} = 0$

$4y\frac{dy}{dx} = -2x$

$\frac{dy}{dx} = \frac{-2x}{4y}$

$\boxed{\frac{dy}{dx} = \frac{-x}{2y}}$

$\frac{dy}{dx} = \text{slope} = 1$

$\frac{dy}{dx} = \frac{-x}{2y}$

$1 = \frac{-x}{2y}$

$2y = -x$

$\boxed{y = \frac{-x}{2}} \to (1)$

$x^2 + 2y^2 = 1$

$x^2 + 2\left(\frac{-x}{2}\right)^2 = 1$

$x^2 + 2\frac{x^2}{4} = 1$

$x^2 + \frac{1}{2}x^2 = 1$

$\frac{3}{2}x^2 = 1$

$\boxed{x = \pm\sqrt{\frac{2}{3}}}$

$x = +\sqrt{\frac{2}{3}} \Rightarrow y = \frac{1}{2}\left(-\sqrt{\frac{2}{3}}\right)$

point (1) $\left(\sqrt{\frac{2}{3}}, -\frac{1}{2}\sqrt{\frac{2}{3}}\right)$

$x = -\sqrt{\frac{2}{3}} \Rightarrow y = \frac{1}{2}\left(--\sqrt{\frac{2}{3}}\right)$

point (2) $\left(-\sqrt{\frac{2}{3}}, \frac{1}{2}\sqrt{\frac{2}{3}}\right)$

(4)

b) Find $\frac{d^2y}{dx^2}$.

$x^2 + 2y^2 = 1$

$2x + 4y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-2x}{4y}$

$\frac{dy}{dx} = \frac{-x}{2y}$

$\frac{d^2y}{dx^2} = \frac{2y(-1) - (-x) \cdot 2\frac{dy}{dx}}{(2y)^2}$

$= \frac{-2y - \left(-x \cdot 2 \cdot \frac{-x}{2y}\right)}{(2y)^2} \Rightarrow \frac{d^2y}{dx^2} = \frac{-2y - \frac{x^2}{y}}{(2y)^2}$

(4)