The University of Jordan
Mathematics Department

Second Exam

23

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Calculus III (0301201)

Name: نعيم حسن الحطيش

Student Number: 0127430

Section: 11:00 - 9:30

Lecturer: د. حسن النحال

(Q₁) (6 points): Given the curve

$$C:\vec{r}(t)=(\tan^{-1}t)\hat{i}+(t-\tan^{-1}t)\hat{j}+\left(\frac{1}{\sqrt{2}}\ln(t^2+1)\right)\hat{k},\quad -2\le t\le 6.$$

Find the following:

(a) Find the arc length of C.

$x=\tan^{-1}t \to x'=\frac{1}{1+t^2}$ $\quad y=t-\tan^{-1}t \to y'=1-\frac{1}{1+t^2}$

$z=\frac{1}{\sqrt{2}}\ln(t^2+1) \to \frac{1}{\sqrt{2}}\frac{2t}{t^2+1}$

$\text{Arc}=\int_{-2}^{6}\sqrt{\left(\frac{1}{1+t^2}\right)^2+1+\left(\frac{1}{1+t^2}\right)^2-\frac{2}{1+t^2}+\frac{1}{2}\frac{4t^2}{(t^2+1)^2}} = \int_{-2}^{6}\sqrt{\frac{2t^2+2}{(1+t^2)^2}-\frac{2}{1+t^2}+1}$

1

(b) Find the parameteric equation of the tangent line to C at the point (0, 0, 0).

$\vec{r}'(t)=\left\langle \frac{1}{1+t^2}\hat{i}+1-\frac{1}{1+t^2}\hat{j}+\frac{1}{\sqrt{2}}\frac{2t}{t^2+1}\hat{k}\right\rangle$

$x=t$

$y=0$

$z=0$

2

(c) Find the curvature of C at (0, 0, 0).

$r'=\langle 1,0,0\rangle$ $\quad r''=\left\langle\frac{-2t}{(1+t^2)^2},\frac{-2t}{(1+t^2)^2},\frac{1}{\sqrt{2}}\frac{2(t^2+1)-4t^2}{(t^2+1)^2}\right\rangle$

$r''=\langle 0,0,2\rangle$

$r'\times r''=\begin{vmatrix}1&0&0\\0&0&\sqrt{2}\end{vmatrix}=\langle 0,-2,0\rangle$

$|r'\times r''|=2$

$|r'|=1$

1.5

$\text{curvature}=\frac{|r'\times r''|}{|r'|^3}=\frac{2}{1}=2$

$\frac{1}{2\sqrt{2}}$

(Q_2) (6 points): Evaluate the following if it exists:

(a) $\lim\limits_{(x,y)\to(0,0)} \dfrac{e^{-\frac{1}{\sqrt{x^2+y^2}}}}{\sqrt{x^2+y^2}}$

$r = x^2 + y^2$

$= \lim\limits_{r\to 0} \dfrac{e^{-\frac{1}{\sqrt{r}}}}{\sqrt{r}} \to \lim\limits_{r\to 0} \dfrac{2\sqrt{2}\, e^{-\frac{1}{\sqrt{r}}}}{\frac{1}{2\sqrt{2}}} = \cancel{8e}$

$= \lim\limits_{r\to 0} 8\, e^{-\frac{1}{\sqrt[3]{r}}} = \infty$ doesn't exist

2

(b) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^{3/2}\,y}{x^3+y^2}$

1) x ~~axis~~ axis $\lim\limits_{x\to 0} \dfrac{x^{3/2}\times 0}{x^3+0} = 0$

$x = y^{\frac{2}{3}}$ $\to \lim\limits_{y\to 0} \dfrac{(y^{\frac{2}{3}})^{\frac{3}{2}} \times y}{(y^{\frac{2}{3}})^3 + y^2} = \lim\limits_{y\to 0} \dfrac{y^2}{y^2+y^2}$

$= \frac{1}{2}$

$\frac{1}{2} \neq 0$

lim doesn't exist

3

$f \to u \to x, y, z$

(Q₃) (6 points): Let $F = f\left(\underbrace{x^2-y^2}_{u}, \underbrace{y^2-z^2}_{w}, \underbrace{z^2-x^2}_{m}\right)$ show that:

$$yz\frac{\partial F}{\partial x} + xz\frac{\partial F}{\partial y} + xy\frac{\partial F}{\partial z} = 0.$$

$u = (x^2-y^2,\ y^2-z^2,\ z^2-x^2)$

$\cancel{\frac{df}{dx} = \frac{df}{du}\cdot\frac{du}{dx} = \frac{df}{du}\,2x}$

$\frac{df}{dx} = \frac{df}{du}\cdot\frac{du}{dx} + \frac{df}{dm}\cdot\frac{dm}{dx} = \frac{df}{du}\cdot 2x + \frac{df}{dm}\cdot -2x$

$\frac{df}{dy} = \frac{df}{dw}\cdot\frac{dw}{dy} + \frac{df}{du}\cdot\frac{du}{dy} = \frac{df}{dw}\cdot 2y + \frac{df}{du}\cdot -2y$

$\frac{df}{dz} = \frac{df}{dm}\cdot\frac{dm}{dz} + \frac{df}{dw}\cdot\frac{dw}{dz} = \frac{df}{dm}\,2z + \frac{df}{dw}\cdot -2z$

$yz\left(\frac{df}{du}\cdot 2x + \frac{df}{dm}\cdot -2x\right) + xz\left(\frac{df}{dw}\,2y + \frac{df}{du}\cdot -2y\right) + yx\left(\frac{df}{dm}\,2z + \frac{df}{dw}\cdot -2z\right)$

$= 0$

(Q₄) (2 points): Let $f(x, y) = \frac{y}{x+y}$. Find a unit vector $\hat{u}$ for which $D_{\hat{u}} f(2, 3) = 0$.

$\cancel{D_u(2,3)}\ D_{\hat{u}} = \nabla f \cdot \hat{u}$

$f_x = \frac{-y}{(x+y)^2} \to \frac{-3}{25}$

$\nabla f = \left\langle \frac{-3}{25}, \frac{2}{25} \right\rangle$

$f_y = \frac{(x+y)-y}{(x+y)^2} \to \frac{2}{25}$

$\hat{u} = \langle 0, 0 \rangle$

(Q5) (4 points): Find all points on the surface $x^2+y^2-z^2=1$ at which the normal line is parallel to the line through $p(1,-2,1)$ and $Q(4,0,-1)$.

$\overrightarrow{PQ} = \langle 3, +2, -2 \rangle$

1

$f = x^2 + y^2 - z^2 - 1$

$\nabla f = \langle 2x, 2y, -2z \rangle \rightarrow$ normal ~~of~~

$\nabla f \times pq = 2x \quad 2y \quad -2z$

1

(Q_6) (6 points): Find and classify all critical points as local maximum, minimum or saddle point for the function $f(x, y) = 4xy - x^4 - y^4$.

$f_x = 4y - 4x^3 \cancel{4} = 0 \quad \to y = x^3$

$f_y = 4x - 4y^3 = 0$

$\cancel{4}x - \cancel{4}x^9 = 0$

$x(1 - x^8) = 0 \quad \to x = 0, 1, -1$

$\cancel{x(1-x^4)}$

$(0,0) \quad (1,1) \quad (-1,-1)$

$f_{xx} = -12x^2$

$f_{yx} = -12y^2 \qquad f_{xy} = 4$

6

$D = f_{x4} \cdot f_{yy} - f_{x4}$

$(0,0) \to 0 \cdot 0 - 4 = -4$ (saddle.

$(1,1) \to -12 \times -12 - (4)^2 = 144 - \cancel{4}\,36 = \cancel{140}\,+$ (Local max

$(-1,-1) \to -12 \times -12 - (4)^2 = 144 - \cancel{4}\,36 = \cancel{140}\,+$ (Local max

$f_{xx} = -12 < 0$