Calculus 3 second

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قسم الرياضيات   تفاضل وتكامل 3   الاثنين 22 / 8 /2005

الجامعة الأردنية   الامتحان الثاني b   6:40 - 7:40

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Q1: Let $D_{(2i+j)}f(1,1) = 2\sqrt{5}, D_{-3i}f(1,1) = -3$ then $D_{(i-j)}f(1,1) =$

(a) $\sqrt{2}$ (b) $-\sqrt{2}$ (c) $\frac{\sqrt{2}}{2}$ (d) $\frac{-\sqrt{2}}{2}$ (e) 0

Q2: Let $f(x,y) = ye^{-x} + \cos x$, then $\frac{\partial^3 f}{\partial y \partial x^2} =$

(a) ye^{-x} (b) e^{-x} (c) $\cos y$ (d) $\sin y$ (e) 0

Q3: Parametric equations of the tangent line to the curve of intersection of the surfaces : $3z = \sqrt{x^2+y^2}, 2x + y - 3z = 5$ at the point $\left(3,4,\frac{5}{3}\right)$ are :

(a) $x = 3 - 3t, y = 4 + 21t, z = \frac{5}{3} - 5t$ (b) $x = 3 - 3t, y = 4 - 21t, z = \frac{5}{3} + 5t$

(c) $x = 3 - 3t, y = 4 + 21t, z = \frac{5}{3} + 5t$ (d) $x = 3 + 3t, y = 4 + 21t, z = \frac{5}{3} + 5t$

(e) none

Q4: Let $f(x,y) = \frac{x}{y}, z = x + f(x,y)$ then $y\frac{\partial z}{\partial y} + x\frac{\partial z}{\partial x} =$

(a) y (b) xy (c) x (d) $-x$ (e) 0

Q5: A point on the surface $x(y+z)+2x(1-y)+z^2=1$ at which the tangent plane is parallel to xz-plane is :

(a) $(-2-,3,1)$ (b) $(2,-3,1)$ (c) $(2,3,-1)$ (d) $(2,3,1)$ (e) $(-2,3,1)$

Q6: If the maximum value of the directional derivative of a function f at a point $P(1,1,-1)$ is equal to 12 and this value occurs in the direction of $\vec{a}=3i-4j+\sqrt{11}k$, then the gradient of f at $P(1,1,-1)$ is equal to :

(a) $(6,-8,2\sqrt{11})$ (b) $(6,8,2\sqrt{11})$ (c) $(6,-8,\sqrt{11})$ (d) $(-6,-8,2\sqrt{11})$ (e) $(6,-8,-2\sqrt{11})$

Q7 : Let $f(x,y)=\dfrac{y\sqrt{x}}{3y^2+x}$, then one of the following is true

(a) domain of f is $\Re\times\Re-\{(o,o)\}$

(b) along the curve $y-\sqrt{x}=0$, we have $\lim_{(x,y)\to(0^+,0)} f(x,y)=\frac{1}{4}$

(c) f is continuous at $(-1,1)$

(d) along the curve $y-x=0$, we have $\lim_{(x,y)\to(0^+,0)} f(x,y)=1$

(e) f has no critical points .

Q8: : Let $z=f(t), t=x^2g(y), g(1)=2, g'(1)=3, f'(2)=\frac{1}{2}, f''(2)=1$

then $\dfrac{\partial^2 z}{\partial x\partial y}$ at the point $(x,y)=(1,1)$ us equal to :

(a) 3 (b) $\frac{3}{2}$ (c) $\frac{9}{2}$ (d) 9 (e) 0

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Q9 : The critical points of the function

$$f(x,y) = 3x^2 - 3y^2 - x^3 + 6xy,$$

are classified as follows :

(a) $f(0,0)$ is a saddle point and $f(4,4)$ is local maximum

(b) $f(0,0)$ is a saddle point and $f(4,4)$ is local minimum

(c) Both $f(0,0)$ and $f(4,4)$ are local maximum

(d) Both $f(0,0)$ and $f(4,4)$ are local minimum

(e) $f(0,0)$ is a local minimum and $f(4,4)$ is a saddle point

Q10: Parametric equations of the normal line to the surface

$x^2 + e^x \cos y + ye^z = 2$ at the point $\left(0, \frac{\pi}{2}, 0\right)$ is :

(a) $x = 0, y = \frac{\pi}{2}, z = \frac{\pi}{2}t$ (b) $x = 0, y = \frac{\pi}{2}t, z = \frac{\pi}{2}$

(c) $x = 0, y = \frac{\pi}{2}, z = \frac{\pi}{2}$ (d) $x = 0, y = \frac{\pi}{2}, z = 0$ (e) none

Q11 : The directional derivative of $f(x,y) = x^2y - 2y^2x + z^2$ at $P(1,1,-1)$ is equal to 0 in the direction of the vector :

(a) $(2,2,3)$ (b) $(2,1,-3)$ (c) $(0,-2,-3)$ (d) $(2,-2,3)$ (e) $(-2,-2,3)$

Q12 : The double integral $\int_1^3 \int_0^{\ln x} f(x,y)\,dydx$ with order of integration reversed will be :

(a) $\int_1^3 \int_0^{\ln y} f(x,y)\,dxdy$ (b) $\int_0^{\ln 3} \int_{e^y}^3 f(x,y)\,dxdy$ (c) $\int_0^{\ln 3} \int_{e^x}^3 f(x,y)\,dxdy$

(d) $\int_0^{\ln 3} \int_{e^x}^3 f(x,y)\,dydx$ (e) $\int_0^{\ln 3} \int_{e^y}^3 f(x,y)\,dydx$

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Dept. of Math. Math. 0301201b Calculus 3 Exam.I
Univ. Of Jordan Sun.31-7-2005 6:40-7:40 P.M.

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Section : 1,50 - 2,50

CIRCLE the Correct answer or write the final answer .

* Consider the vectors : $\vec{a}=2i-j+k, \vec{b}=3i+2j-2k$, and answer questions : 1,2,and 3 .

1- $Proj_{\vec{a}}\vec{b}$ is the vector :

(a) $\frac{1}{3}(3i+2j-2k)$ (b) $\frac{2}{\sqrt{17}}(3i+2j-2k)$

(c) $\frac{1}{3}(2i-j+k)$ (d) $\frac{2}{\sqrt{17}}(2i-j+k)$

2- $\|\overrightarrow{2a}\times\vec{b}\| = \boxed{14\hat{j}+14\hat{k}}$

3-A vector orthogonal to both $\vec{a},\vec{b}$ and of norm 3 is : $\boxed{\frac{3}{\sqrt{98}}(7\hat{j}+7\hat{k})}$

* Consider the the vectors $\vec{a},\vec{b}$ such that $\|\vec{a}\|=5, \|\vec{b}\|=3, \|\vec{a}-\vec{b}\|=2$ then answer questions : 4 and 5 .

4- $\|2\vec{a}+\vec{b}\| =$

5- $\overrightarrow{3b}\bullet(2\vec{a}\times\vec{b}) = \boxed{0}$

$\|a\|^2 = 9+4+4$
17

$6+-2-2 = \frac{2}{6} \quad \frac{1}{3}$

$\sqrt{98} * x = 3$

$x = \frac{3}{\sqrt{98}}$

$n = \langle 1, 2, -1 \rangle$

6- Parametric equations of the line passing through the point $P(2,3,1)$ and perpendicular to the plane $x+2y-z=10$ are :

(a) $x=3-t, y=-2+2t, z=-4+t$ (b) $x=2+t, y=3+2t, z=1-t$
(c) $x=1+2t, y=2+3t, z=-1+1t$ (d) $x=1+3t, y=2-2t, z=1-4t$

* Consider the points : $P(2,1,0), Q(4,0,1), R(5,3,-2), S(2,3,1)$, and answer questions : 7,8, 9 and 10 .

$\langle 2, -1, 1 \rangle$

7- Parametric equations of the line passing through the points P, Q are :
(a) $x=2+3t, y=1+2t, z=1-2t$ (b) $x=2+2t, y=1-t, z=+t$
(c) $x=2+3t, y=1-2t, z=-2t$ (d) $x=2+3t, y=1+2t, z=-2t$

PQ

8- An equations of the plane determined by the points P, Q, S is :
(a) $3x+2y-4z=8$ (b) $3x-4z=1$
(c) $y+z=1$ (d) $3x+2y=8$

P X Q
$PQ = \langle 2, -1, 1 \rangle$
$PS = \langle 0, 2, 1 \rangle$

$\langle 3, 2, -4 \rangle$ $\sqrt{9+4+16}$

9-The area of the triangle with vertices P, Q, S is : $\boxed{\frac{\sqrt{29}}{2}}$

10 - Determine whether the points P, Q, S are collinear (justify your answer)

$\boxed{\text{not Collinear}}$ $\boxed{PQ \times PS \neq 0}$

$-\hat{i} + 2\hat{j} + 2\hat{k}$

$$\begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 0 & 2 & 1 \end{bmatrix} \begin{matrix} \hat{i} & \hat{j} \\ 2 & -1 \\ 0 & 2 \end{matrix}$$

$-\hat{i} + 4\hat{k}$

$-3\hat{i} + 4\hat{k} - 2\hat{j}$

$\langle -3, -2, 4 \rangle$

$-6 \quad -2 \quad 4$

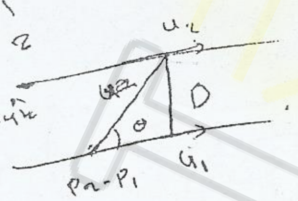
$\overrightarrow{P_1P_2} = u = \langle 1, 1, -2 \rangle$

$P_1 \langle 1, 0, 1 \rangle$

$P_2 \langle 2, 1, -1 \rangle$

sin θ = ‖D‖ / ‖u‖

D = ‖u‖



‖u × u_1‖ / ‖u_1‖

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-5, 3, 3, -5, 1 parllell

• **Consider the lines :**

$L_1 : x = 1+t, y = 2t, z = 1-t; L_2 : x = 2-2t, y = 1-4t, z = -1+2t$,

and answer questions : 1`1,and12 .

$u_1 \langle 1, 2, -1 \rangle$, $u_2 = \langle -2, -4, 2 \rangle$

11-The distance between the two lines L_1, L_2 is equal to :

$\boxed{\frac{\sqrt{35}}{6}}$

12- An equations of the plane determined by the lines L_1, L_2 is :

(a) $8x - y + 5z = 21$ (b) $3x + y + z = 4$

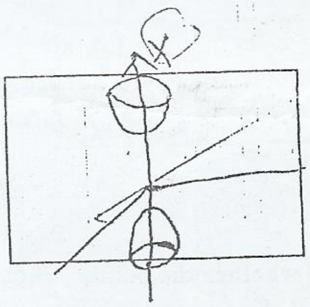
(c) $3x - y + z = 0$ (d) $3x - y + z = 4$

In questions 13,14 Identify and sketch .

13- (a) $z^2 + y^2 = x^2 - 4$

$x^2 - z^2 - y^2 = 4$

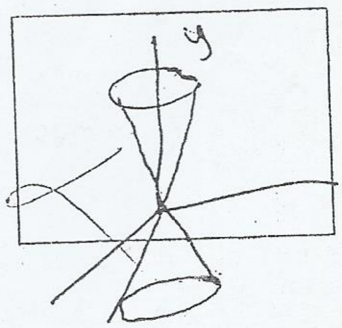
Hyp. of two sheet



$y = \frac{x^2}{2} + \frac{z^2}{2}$

(b) $2y = x^2 + z^2$

Circular cone



±y

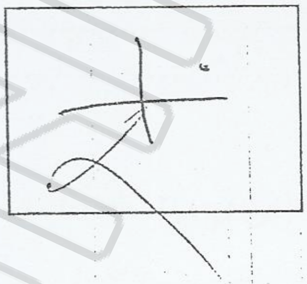
$x = \rho \quad , y = 0 \quad , z = \rho \cos \frac{\pi}{6}$

$\cos \phi = \frac{\sqrt{3}}{2}$

$\rho = 0 \quad , \theta = 0 \quad \phi = \frac{\pi}{6}$ $\quad \frac{2}{\sqrt{3}}$

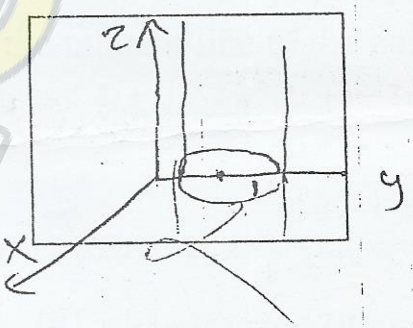
14- (a) $\varphi = \frac{\pi}{6}$

~~plane~~
point



(b) $r = 2\sin\theta$

cylindrical surface



$r = 2\sin\theta$

$x = r\cos\theta \quad , \quad y = \sin\theta$

$r^2 = 2r\sin\theta$

$\boxed{r^2} = 2y$

$x = 2y$

$r^2 =$

$r = \sqrt{x^2 + y^2}$

$x^2 + y^2 - 2y = 0$

$x^2 + (y - 1)^2 = 1$