The University of Jordan
Mathematics Department

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First Exam

14/11/2012
Calculus III (0301201)

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Section: 11:00 — 9:30 **Lecturer:** [illegible]

(Q_1) (12 points): Given the four points

$$P_1(1, 0, -1),\ P_2(0, 2, 3),\ P_3(-2, 1, 1),\ P_4(3, 3, 3).$$

(a) Find parametric equations of the line passing P_1 and parallel to the line P_3P_4.

$\overrightarrow{P_3P_4} = \langle 5, 2, 2 \rangle$

$x = 1 + 5t$

$y = 2t$

$z = -1 + 2t$

(b) Find the direction cosines of $\overrightarrow{P_3P_4}$.

$\overrightarrow{P_3P_4} = \langle 5, 2, 2 \rangle$

$|\overrightarrow{P_3P_4}| = \sqrt{25 + 4 + 4} = \sqrt{33}$

$\cos\alpha = \frac{5}{\sqrt{33}}$

$\cos\beta = \frac{2}{\sqrt{33}}$

$\cos\gamma = \frac{2}{\sqrt{33}}$

(c) Find $\text{Proj}_{\overrightarrow{P_1P_4}} \overrightarrow{P_2P_3}$.

$\overrightarrow{P_2P_3} = \langle -2, -1, -2 \rangle$ $\overrightarrow{P_1P_4} = \langle 2, 3, 4 \rangle$

$\text{proj}_{\overrightarrow{P_1P_4}} \overrightarrow{P_2P_3} = \frac{\overrightarrow{P_2P_3} \cdot \overrightarrow{P_1P_4}}{|\overrightarrow{P_1P_4}|^2} P_1P_4 = \frac{-4 + -3 - 8}{(\sqrt{4 + 9 + 16})^2} \langle 2, 3, 4 \rangle$

$= \frac{-15}{29} \langle 2, 3, 4 \rangle = \langle \frac{-30}{29}, \frac{-45}{29}, \frac{-60}{29} \rangle$

$P_1\langle 1,0,-1\rangle$ $P_2\langle 0,2,3\rangle$ $P_3(-2,1,1)$

$P_4(3,3,3)$

(d) Find the area of the triangle $P_1P_2P_3$.

$\vec{P}_{12} = \langle -1, 2, 4\rangle \Rightarrow |\vec{P}_{12}| = \sqrt{1+4+4} = 3$

$\vec{P}_{23} = \langle -2,-1,-2\rangle \Rightarrow |\vec{P}_{23}| = \sqrt{4+1+4} = 3$

$\vec{P}_{31} = \langle +3,+1,-2\rangle \Rightarrow |\vec{P}_{31}| = \sqrt{9+1+4} = \sqrt{14}$

~~area = 1/2~~ area $= \frac{1}{2}|P_{12}\times P_{23}| = \frac{\sqrt{125}}{2}$

$P_{12}\times P_{23} = \begin{vmatrix} -1 & 2 & 4 \\ -2 & -1 & -2 \end{vmatrix} = \langle 0,-10,5\rangle$

$|P_{12}\times P_{23}| = \sqrt{100+25} = \sqrt{125}$

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(e) Find the distance between the point P_4 and the plane $P_1P_2P_3$.

$|P_{12}\times P_{23}| = \langle 0,-10,5\rangle = n$

plan $P_1P_2P_3$ ~~$-10(y+10)+5(z$~~

$-10(y-2)+5(z-4)=0 \rightarrow -10y-20+5z-20=0$

$-10y+5z-40=0$

$\frac{|ax_0+by_0+cz_0+d|}{\sqrt{a^2+b^2+c^2}} = \frac{|-10\times 3+5\times 3-40|}{\sqrt{0+100+25}} = \frac{-30+15-40}{\sqrt{125}}$

$= \frac{-55}{\sqrt{125}}$

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(f) Find the cosine of the angle between the two planes $P_1P_2P_3$, $P_1P_2P_4$.

$P_1P_4 = \langle 2,3,4\rangle$ ~~$P_1P_3 = \langle -3,+1,+2\rangle$~~ $P_{12} = \langle -1,2,4\rangle$

$P_1P_4\times P_{12} = \begin{vmatrix} 2 & 3 & 4 \\ -1 & 2 & 4 \end{vmatrix} = \langle 4,-12,7\rangle$

$n_1 = \langle 0,-10,5\rangle$

$n_2 = \langle 4,-12,7\rangle$

$|n_1| = \sqrt{125}$

$|n_2| = \sqrt{16+144+49} = \sqrt{209}$

plane $P_1P_2P_4 \Rightarrow 4(x-3)-12(y-3)+7(z-3)=0$

$4x-12-12y+36+7z$ ~~-36~~ $-21=0$

$4x-12y+7z+3=0$

$\cos\theta = \frac{n_1\cdot n_2}{|n_1||n_2|} = \frac{0+120+35}{\sqrt{209}\sqrt{125}} = \frac{155}{\sqrt{209}\sqrt{125}}$

(Q$_2$) (4 points): Find the equation of the plane that contains the line $x = 3t$, $y = 1 + t$, $z = 2t$ and is parallel to the line of intersection of the planes: $x - 4y + 2z = 0$ and $2x - y + z = 0$.

P_1 P_2

at $x=0$: $-4y + 2z = 0$

$2y + 2z = 0$

$6y + 4z = 0$ $y=0$ $z=0$

$(0,0,0)$ point of intersection

$n_1 = \langle 1, -4, 2 \rangle$ $n_2 = \langle 2, -1, 1 \rangle$

$n_1 \times n_2 = \begin{vmatrix} 1 & -4 & 2 \\ 2 & -1 & 1 \end{vmatrix} = \langle 2, 3, +7 \rangle = n_5$

n_3 of Line $= \langle 3, 1, 2 \rangle$

n_4 (for plane) $= n_3 \times n_5 = \begin{vmatrix} 3 & 1 & 2 \\ 2 & 3 & 7 \end{vmatrix} = \langle 1, -25, 11 \rangle$

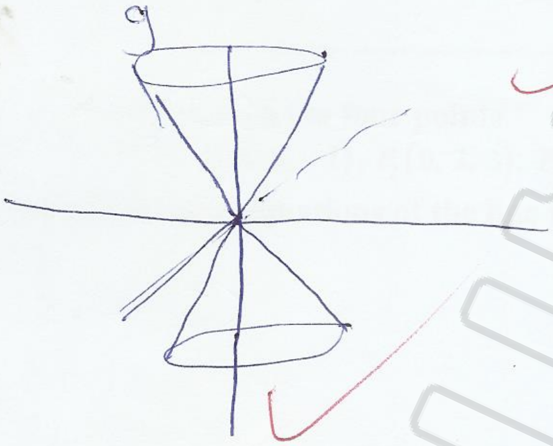
at $t=0$ $(0, 1, 0)$

equation of plane $\Rightarrow$ $x - 25(y-1) + 11z = 0$

(Q$_3$) **(4 points): Identify and sketch the following surfaces.**

(a) $12z^2 - 3x^2 = 4y^2$.

hyperbolic Cone along y axis



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(b) $z + 2x^2 = 4$.

$z = 4 - 2x^2$

parabolic Culinder along y axis