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University of Jordan
Department of Mathematics

First Exam | Math.: 0301201 | 28/06/2012

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* Given $\overrightarrow{P_1P_2} = \langle 2, 3, -4\rangle$, $\overrightarrow{P_1P_3} = \langle 1, 0, 1\rangle$, answer questions 1, 2, 3.

3/3 **Q1) Proj** $\overrightarrow{P_2P_3}$ over $-2\overrightarrow{P_1P_2}$

$A = -2\overrightarrow{P_1P_2} = \langle -4, -6, 8\rangle$

$\overrightarrow{P_2P_3} = \overrightarrow{P_1P_3} - \overrightarrow{P_1P_2} = \langle -1, -3, 5\rangle$

$\frac{\overrightarrow{P_2P_3} \cdot A}{|A|^2} \cdot A = \frac{4+12+40}{(\sqrt{16+36+64})^2} * (-4\hat{i} - 6\hat{j} + 8\hat{k})$

$\text{Proj}_{-2\overrightarrow{P_1P_2}} \overrightarrow{P_2P_3} = -\frac{248}{116}\hat{i} - \frac{372}{116}\hat{j} + \frac{496}{116}\hat{k}$

2/2 **Q2) Comp** $\overrightarrow{P_1P_2}$ over $\overrightarrow{P_1P_2} \times \overrightarrow{P_2P_3}$

$A = \overrightarrow{P_1P_2} \times \overrightarrow{P_2P_3} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -4 \\ -1 & -3 & 5 \end{vmatrix}$

$A = 3\hat{i} - 6\hat{j} - 3\hat{k}$

$\text{Comp}_A \overrightarrow{P_1P_2} = \frac{\overrightarrow{P_1P_2} \cdot \vec{A}}{|\vec{A}|} = \frac{6 - 12 + 12}{|\vec{A}|} = \text{Zero}$

Q3) find a vector of length 3 orthogonal to both $\overrightarrow{P_1P_2}$ & $\overrightarrow{P_1P_3}$.

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$\overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3}$ ⊥ both

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -4 \\ 1 & 0 & 1 \end{vmatrix} = 3\hat{i} - 6\hat{j} - 3\hat{k}$

$3\vec{u} = 3\frac{\overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3}}{|\overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3}|} = \frac{3\hat{i} - 6\hat{j} - 3\hat{k}}{\sqrt{9+36+9}}$

$3\vec{u} = \frac{9\hat{i}}{\sqrt{54}} - \frac{18\hat{j}}{\sqrt{54}} - \frac{9\hat{k}}{\sqrt{54}}$

3/3 **Q4) Identify the following surfaces:**

(i) $x^2 = z - y^2 - 1$

$z = x^2 + y^2 + 1$ — I hyperbolic paraboloid

(ii) $x^2 = y^2 + z^2 + 1$

II hyperboloid of 2-sheets

(iii) $x^2 = z - 1$

III Cylinder

* Given the point $Q(1, 2, -3)$, the line $l: x=t,\ y=2-t,\ z=3+2t$, and the two planes $P_1: x+y+z=1$, $P_2: x-2y+3z=5$. Answer questions 5, 6, 7, 8.

Q5) Find parametric equations for the line through the point $Q(1, 2, -3)$, that is parallel to both planes P_1 & P_2.

$n_1 = \langle 1,1,1\rangle$

$n_2 = \langle 1,-2,3\rangle$

$n_1 \times n_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & -2 & 3 \end{vmatrix}$

$= 5\hat{i} - 2\hat{j} - 3\hat{k} \parallel P_1 \,\&\, P_2 \Rightarrow d_{l_1} = \langle 5,-2,-3\rangle$

l_1: $x = 1+5t$, $y = 2-2t$, $z = -3-3t$

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Q6) Find an equation of the plane containing the line l and perpendicular to the plane P_1.

$n_1 = \langle 1,1,1\rangle$

$t=0 \Rightarrow a(0,2,3) \in l \,\&\, P$

Plane: P, normal: $\vec{n}$

$P \perp P_1 \quad n_1 \perp n \Rightarrow \vec{n}_1 \cdot \vec{n} = 0$

$a_1 + a_2 + a_3 = 0$

$\vec{n} = \langle 2,-1,-1\rangle$

$P: 2x - 1(y-2) - 1(z-3) = 1$

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Q7) Find the angle between the planes P_1 & P_2.

$n_1 = \langle 1,1,1\rangle$

$n_2 = \langle 1,-2,3\rangle$

θ between P_1 & P_2 = θ between n_1 & n_2

$\cos\theta = \dfrac{n_1 \cdot n_2}{|n_1||n_2|} = \dfrac{1-2+3}{\sqrt{3}\sqrt{1+4+9}} = \dfrac{2}{\sqrt{3\times 14}}$

$\theta = \cos^{-1}\left(\dfrac{2}{\sqrt{3\times 14}}\right)$

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Q8) Find the distance from the point $Q(1, 2, -3)$ to the line l.

$t=0 \Rightarrow P(0,2,3) \in l$

$d_l = \langle 1,-1,2\rangle$

$\overrightarrow{PQ} = \langle 1,0,-6\rangle$

$\overrightarrow{QP} \times d_l = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & -6 \\ 1 & -1 & 2 \end{vmatrix} = -6\hat{i} - 8\hat{j} - \hat{k}$

$|\overrightarrow{QP} \times d_l| = \sqrt{101}$

$h = \dfrac{|\overrightarrow{QP} \times d_l|}{|d_l|} = \dfrac{\sqrt{101}}{\sqrt{6}} = h$

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