THE UNIVERSITY OF JORDAN

0301101 Calculus I   First test   Oct. 25, 2014

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Questions 1 to 8: fill in the blanks with the answers only. Each question is worth 1.5 marks.

___ Q1) If the domain and range of $f(x)$ are $[-1,3]$ and $[0,4]$, respectively, then the

range of $g(x) = 1 - 2f^{-1}(3x)$ is $[-5,3]$

___ Q2) If the domain of $f(x)$ is $[-2,1)$ and $g(x) = 2x + 1$, then the domain of

$(f \circ g)(x)$ is $[-\frac{3}{2}, 0)$

___ Q3) If $Log_2(5x - 1) - Log_2 x = 1$, then $x = \frac{1}{3}$

___ Q4) $\cos^{-1}\left(\cos\left(\frac{9\pi}{8}\right)\right) = \frac{7\pi}{8}$

___ Q5) $\cos\left(2\tan^{-1}\left(\frac{2}{5}\right)\right) = \frac{21}{29}$

___ Q6) If $f(x) = \frac{3x^2+1}{\ln(5-x)}$, then the domain of $f(x)$ is $(-\infty, 5) - \{4\}$

___ Q7) The graph of $y = x^2 \tan x$ is symmetric about origin

___ Q8) If $4^x - 2^x - 6 = 0$, then $x = Log_2 3$

power unit

Questions 9 and 10: Each question is worth 4 marks.

___ **Q9)** Find the function g that is obtained from $f(x) = |2x + 3|$ by the following steps. Write the answer of each step.

a) Reflecting the graph of $y = f(x)$ about the y-axis $|-2x+3|$ ..... then

b) shifting 2 units down $|-2x+3| - 2$ then

c) stretching vertically by a factor of 2 units $2\left(|-2x+3| - 2\right)$ then

d) reflecting about the x-axis $-2\left(|-2x+3| - 2\right) \Rightarrow -2|-2x+3| + 4$

(4)

___ **Q10)** *Solve and show your work.* Let $f(x) = \ln(2x + 1) - \ln(1 - 2x)$

a) Determine whether the function f is even, odd or neither.

$f(x) = \ln \dfrac{2x+1}{1-2x}$

$f(-x) = \ln \dfrac{-2x+1}{1+2x}$

$f(x) \neq f(-x) \leftarrow$ not even

$\begin{cases} -f(x) = -\ln \dfrac{2x+1}{1-2x} \\ = \ln \left(\dfrac{2x+1}{1-2x}\right)^{-1} \\ = \ln \dfrac{1-2x}{2x+1} \end{cases}$

(1)

$f(-x) = -f(x)$ ---- so odd function

b) Find $f^{-1}(x)$.

$f(x) = \ln(2x+1) - \ln(1-2x)$

$y = \ln \dfrac{2x+1}{1-2x}$

$e^{y} = e^{\ln \frac{2x+1}{1-2x}}$

$e^{y} = \dfrac{2x+1}{1-2x} \Rightarrow 2x+1 = e^{y} - 2xe^{y}$

$2x + 2xe^{y} = e^{y} - 1$

$x(2 + 2e^{y}) = e^{y} - 1$

$x = \dfrac{e^{y} + 1}{2 + 2e^{y}}$

$f^{-1}(x) = \dfrac{e^{x} - 1}{2 + 2e^{x}}$

$f(x) = \ln \dfrac{2x+1}{1-2x}$

$x = \ln \dfrac{2y+1}{1-2y}$

$e^{x} = e^{\ln \frac{2y+1}{1-2y}}$

$e^{x} = \dfrac{2y+1}{1-2y}$

$2y + 1 = e^{x} - 2e^{x}y$

$2y + 2e^{x}y = e^{x} - 1$

$y(2 + 2e^{x}) = e^{x} - 1$

$y = \dfrac{e^{x} - 1}{2 + 2e^{x}}$

(3)

$f^{-1}(x) = \dfrac{e^{x} - 1}{2 + 2e^{x}}$

power unit