

[Handwritten scribbles]

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الثلاثاء ٢٠١٧/٤/١١	التفاضل و التكامل ١: الامتحان الثاني	الجامعة الأردنية
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In Q1-Q8, write the final answer in the provided blank, 2 points each.

Q1: $y = x \tanh^{-1} 3x + \ln \sqrt{9 - x^2}$, then $y' = \dots$

[Handwritten solution for Q1]

$$y' = \frac{x}{4x^2 + 1} + \tanh^{-1}(3x) \cdot 3 + \frac{1}{2} \cdot \frac{-2x}{\sqrt{9-x^2}}$$

[Crossed out part: $2\sqrt{4-x^2}$]

[Handwritten solution for Q1]

$$y' = x \cdot \frac{1}{3x^2+1} + \tanh^{-1}(3x) \cdot 3 + \frac{1}{2} \cdot \frac{-2x}{\sqrt{9-x^2}}$$

[Other scribbles: 6 , $(4-x^2)^{1/2}$, $\frac{1}{2}$, $\frac{-2x}{\sqrt{4-x^2}}$, $\frac{1}{\sqrt{4-x^2}}$, $\frac{1}{\sqrt{9-x^2}}$]

Q2: $\tanh x = \frac{12}{13}$, then $\cosh x = \dots$

[Handwritten solution for Q2]

$$\cosh(x) = \dots$$

[Handwritten scribbles for Q2]

Q3: $x^y = y^x$, then $y' = \dots$

[Handwritten scribbles for Q3]

Q4: $T(z) = 2^z \log_2 z$, then $T'(z) = \dots$

[Handwritten solution for Q4]

$$T'(z) = 2^z \cdot \frac{1}{z \ln 2} + \log_2 z \cdot 2^z \cdot \ln 2$$

[Handwritten scribbles for Q4]

$$2^z \cdot \frac{1}{z \ln 2} + \log_2 z \cdot 2^z \cdot \ln 2$$

$$h'(x) = \frac{1}{2} (4 + 3f(x))^{-\frac{1}{2}} \cdot 3f'(x)$$

Q5: $h(x) = \sqrt{4 + 3f(x)}$, $f(1) = 7$, $f'(1) = 4$ then $h'(1) = \dots$

$$h'(x) = \frac{1}{2} (4 + 3f(x))^{-\frac{1}{2}} \cdot 3f'(x)$$
$$h'(1) = \frac{\frac{1}{2} \cdot 3f'(1)}{(4 + 3f(1))^{-\frac{1}{2}}} = \frac{\frac{1}{2} \cdot 12}{\sqrt{4 + 3 \cdot 7}} = \frac{6}{\sqrt{4 + 21}} = \frac{6}{\sqrt{25}} = \frac{6}{5}$$

$h'(1) = \frac{6}{5}$

Q6: $f(x) = \sqrt{3 + x^2}$ then its linear approximation near $x = 1$...

~~$f(x) = \sqrt{3 + x^2}$~~
 ~~$f'(x) = \frac{x}{\sqrt{3 + x^2}}$~~
 ~~$f'(1) = \frac{1}{2}$~~
 ~~$L(x) = f(1) + f'(1)(x - 1)$~~
 ~~$L(x) = 2 + \frac{1}{2}(x - 1)$~~
 ~~$L(x) = \frac{1}{2}x + \frac{3}{2}$~~
 $f(1) = 2$

Q7: $\lim_{\theta \rightarrow \frac{\pi}{6}} \frac{\sin \theta - \frac{1}{2}}{\theta - \frac{\pi}{6}} = \dots$



Solve Q9-Q11 completely, 5 points each.

Q9: Find the equation of the tangent line to the curve

$$x^2 + y^2 = (2x^2 + 2y^2 - x)^2 \text{ at the point } \left(0, \frac{1}{2}\right).$$

~~$2x + 2y \frac{dy}{dx} = 2(2x^2 + 2y^2 - x) \cdot (4x + 4y \frac{dy}{dx} - 1)$~~

$$\ln(x^2 + y^2) = 2 \ln(2x^2 + 2y^2 - x) \rightarrow \frac{2x + 2y \frac{dy}{dx}}{x^2 + y^2} = \frac{4x + 4y \frac{dy}{dx} - 1}{2x^2 + 2y^2 - x}$$

$$\frac{x + y \frac{dy}{dx}}{x^2 + y^2} = \frac{4x + 4y \frac{dy}{dx} - 1}{2x^2 + 2y^2 - x} \rightarrow (x + y \frac{dy}{dx}) \cdot (2x^2 + 2y^2 - x) = (x^2 + y^2) \cdot (4x + 4y \frac{dy}{dx} - 1)$$

Q10: Find all points on which the curve $y = 2\sin x + \sin^2 x$ a horizontal tangent.

$$y' = 2\cos x + 2\sin x \cos x = 0$$

$$2\cos x + \cos^2 x = 0$$

$$m=0 = f'(x) = g'$$

Q11: Find all asymptotes of the function $f(x) = \frac{\sqrt{4x^2+1}}{3x-5}$.

V. e H

V.A $3x-5=0 \rightarrow 3x=5 \rightarrow x = \frac{5}{3}$

H.A

$$\lim_{x \rightarrow \frac{5}{3}} \frac{\sqrt{4x^2+1}}{3x-5}$$

$$= \frac{\sqrt{4\left(\frac{5}{3}\right)^2+1}}{3 \cdot \frac{5}{3} - 5} = \frac{\sqrt{4\left(\frac{5}{3}\right)^2+1}}{0}$$

$$= \infty$$

$x = \frac{5}{3}$ is V.A

$$= \lim_{x \rightarrow \infty} \frac{\sqrt{4x^2+1}}{3x-5}$$

$$= \lim_{x \rightarrow \infty} \frac{\sqrt{4 + \frac{1}{x^2}}}{x \left(3 - \frac{5}{x}\right)}$$

$$\lim_{x \rightarrow \infty} = \frac{\sqrt{4 + \frac{1}{\infty}}}{3 - \frac{5}{\infty}} = \frac{\sqrt{4 + 0}}{3}$$

$$= \frac{2}{3}$$