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The University of Jordan

DEPARTMENT OF MATHEMATICS

Calculus I

EXAM 2C / 1st SEMESTER 2016-2017

Date: 03/12/2016

29 / 30

الاسم : .... الرقم الجامعي: (..........) وقت المحاضرة: (..........)

16.5

Instructions: *The test one two-sided page; make sure you do both sides. You CANNOT use a calculator on any part of this exam. The point value of each problem is indicated in brackets. Finally, before you start to work a problem, be sure that you understand what is being asked.*

For questions 1 to 12, fill in the blank with the correct answer. **Only correct answers count. [1.5 pts each]**

1. If $4x^2+2xy+y^2=12$, then $\dfrac{dy}{dx}$ at the point (1,2) is equal to ....-2....

2. $\displaystyle\lim_{h\to 0}\frac{9(1+h)^9-9}{h}=$ ....81....

3. The equation of the tangent line to the graph of $y=\sin^{-1}\left(\dfrac{x}{4}\right)$ at the origin ....$y=\dfrac{1}{4}x$....

4. $\dfrac{d}{dx}\left[\log_7\left(x^2+e^2\right)\right]=$ ....$\dfrac{2x}{(x^2+e^2)\ln 7}$....

5. $f(x)=\dfrac{\sin(x)}{\sqrt{16-x^2}}$, is continuous on ....$R-\{-4,4\}$....

6. $\displaystyle\lim_{x\to\frac{\pi}{2}^+} e^{\sec(x)}=$ ....0 (zero)....

7. If $g(0)=2$, $g'(0)=3$ and $f(x)=\dfrac{4-3e^{2x}}{x+g(x)}$, then $f'(0)=$ ....-4....

8. $\displaystyle\lim_{x\to 0}\frac{1-\cos^2(5x)}{x^2}=$ ....25....

9. If $f(x)=\sec^3(6^x)$, then $f'(x)=$ ....$\cancel{3\sec^2(6^x)\cdot\tan(6^x)\cdot 6^x\ln 6}$ $3\sec^3(6^x)\cdot\tan(6^x)\cdot 6^x\ln 6$

10. If $f(x)=\ln(x+3)$, then $\displaystyle\lim_{x\to 2}\frac{f(x)-f(2)}{x-2}=$ ....$\dfrac{1}{5}$....

11. The vertical asymptote of $f(x)=\dfrac{x-2}{x^2-5x+6}$ is $x=$ ....3....

12. $\displaystyle\lim_{x\to -4}\frac{2x+8}{|x|+4}=$ ....0 (Zero)....

For question 13, 14, and 15, sufficient work must be shown to receive credit.

13. [4 pts] Find $\lim_{x\to-\infty} \sqrt{x^2+3x}+x$.

$\lim_{x\to-\infty} -\infty + \infty x$

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$\lim_{x\to-\infty} \frac{\sqrt{x^2+3x}+x}{1} \times \frac{\sqrt{x^2+3x}-x}{\sqrt{x^2+3x}-x}$

$= \lim_{x\to-\infty} \frac{\cancel{x^2}+3x-\cancel{x^2}}{\sqrt{x^2+3x}-x}$

$\lim_{x\to-\infty} \frac{3x}{|x|\sqrt{1+\frac{3}{x}} - x}$

$\lim_{x\to-\infty} \frac{3x}{-x\left(\sqrt{1+\frac{3}{x}}+1\right)}$

$\lim = \frac{3}{-(1+1)}$

$\boxed{= -\frac{3}{2}}$

14. [4 pts] Use the linear approximation to estimate $\sqrt[3]{1001}$.

4

$f(x) = \sqrt[3]{x}$

$b = 1001$

$a = 1000$

$f(x) = x^{\frac{1}{3}}$

$f'(x) = \frac{1}{3} x^{-\frac{2}{3}}$

$f'(x) = \frac{1}{3\sqrt[3]{x^2}}$

$f(b) = f(a) + f'(a)(b-a)$

$f(b) = 10 + \frac{1}{300}(1001-1000)$

$f(b) = 10 + \frac{1}{300} \longrightarrow f(\sqrt[3]{1001}) = \frac{3001}{300}$

15. [4 pts] Differentiate $f(x) = \dfrac{\sqrt[4]{x}\,\sin^7(x)}{(x-1)^9 e^{x^2}}$.

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$y = \frac{\sqrt[4]{x}\,\sin^7(x)}{(x-1)^9 e^{x^2}}$

$\ln y = \ln(\sqrt[4]{x}) + \ln(\sin^7(x)) - \left(\left(\ln (x-1)^9 + \ln e^{x^2}\right)\right)$

$\ln y = \frac{1}{4}\ln x + 7\ln\sin x - 9\ln x-1 - x^2$

$\frac{y'}{y} = \frac{1}{4x} + \frac{7\cos x}{\sin x} - \frac{9}{x-1} - 2x$

$y' = \frac{\sqrt[4]{x}\,\sin^7 x}{(x-1)^9 e^{x^2}}\left(\frac{1}{4x} + 7\cot x - \frac{9}{x-1} - 2x\right)$

Good Luck