The University of Jordan
Department of Mathematics
Calculus I, First Exam

Student's Name: _____ Student's Number: _____

Instructor's Name: _____ Lecture's time: _____

Q1) (10.5 marks) Fill the blank with the correct answer

1) $\text{Cos}^{-1}(\text{Cos}\frac{10\pi}{9}) =$ $\frac{8\pi}{9}$

2) If the range of $f(x)$ is $[-2, 5]$, and $h(x) = 3 - 2f(3x - 1)$, then the range of $h(x)$ is $[7, -7]$

3) The domain of $h(x) = \sqrt{4 - x^2}\,\text{Sin}^{-1}(2x - 4)$ is $[\frac{3}{2}, \frac{5}{2}]$

4) If $\text{Log}_x(4x - 3) = 2$, then $x =$ $\{1, 3\}$

5) $\text{Sec}(\text{Sin}^{-1}(\frac{3}{5}) + \text{Cos}^{-1}(\frac{4}{5})) =$ $\frac{7}{5}$

6) Let $f(x) = x^2 + 2x + 4$, if $h(x)$ is obtained from $f(x)$ by shifting $f(x)$ one unit right and two units down, then $h(x) = (x-1)^2 + 2(x-1) + 4 - 2$
$= (x-1)^2 + 2(x-1) + 2 = x^2 - 2x + 1 + 2x - 2 + 2 = x^2 + 1$

7) If $h(x) = 5^x$ and the domain of $f(x)$ is $[3, 8]$, then the domain of $(f \circ h)(x)$ is

Q2) (3 marks) Let $f(x) = \frac{e^x}{2+e^x}$. Find $f^{-1}(x)$.

$y = \frac{e^x}{2+e^x}$

$x = \frac{e^y}{2+e^y}$

$e^y = x(2+e^y)$

$e^y = 2x + e^y x$

$e^y - e^y x = 2x$

$e^y(1-x) = 2x$

$e^y = \frac{2x}{1-x}$

take ln

(2)

Q3) (3 marks) Show that the function $f(x) = \frac{x^3 \cos x}{x^2+3} + \ln\left(\frac{1-x}{1+x}\right)$ is an odd function.

$f(-x) = -f(x)$

$f(-x) = \frac{(-x)^3 \cos(-x)}{(-x)^2+3} + \ln\left(\frac{1-(-x)}{1+(-x)}\right)$

$f(-x) = \frac{-x^3 \cos x}{x^2+3} - \ln\left(\frac{1-x}{1+x}\right)$

$f - f = -\left(\frac{x^3 \cos x}{x^2+3} + \ln\left(\frac{1-x}{1+x}\right)\right)$

(2)

Q2) (4 marks) If $\frac{4x^2-8x}{x^3-8} \le 5f(x)+1 \le \frac{\sqrt{x+7}-3}{\sqrt{x+2}-2}$ on open interval near $x = 2$. Find $\lim_{x\to 2} f(x)$.

by sandwich thm

$\lim_{x\to 2} \frac{4x^2-8x}{x^3-8} \le \lim_{x\to 2} 5f(x)+1 \le \lim_{x\to 2} \frac{\sqrt{x+7}-3}{\sqrt{x+2}-2}$

(2)

$\lim_{x\to 2} \frac{4x(x-2)}{(x-2)(x^2+2x+4)} \le \lim_{x\to 2} 5f(x)+1 \le \lim_{x\to 2} \frac{\sqrt{x+7}-3}{\sqrt{x+2}-2} \times \frac{\sqrt{x+7}+3}{\sqrt{x+7}+3} \times \frac{\sqrt{x+2}+2}{\sqrt{x+2}+2}$

$\le$ $\le$