$\frac{14}{30}$

University of Jordan
Department of Mathematics

Second Exam **Math.: 0301201** **19/07/2012**

د. خالد ... [illegible]

Name: [illegible] **Number:** 0119203 **Section:** ....

Q₁) Reparametrize the curve $\vec{r}(t)=\left(\frac{2}{t^2+1}-1\right)\hat{i}+\frac{2t}{t^2+1}\hat{j}$ with respect to arc length parameter s, measured from the point $(1, 0)$ in the direction of increasing t.

$\frac{2}{t^2+1}-1=\alpha$

$1=\frac{2}{t^2+1}-1$

$t^2=0$

$\boxed{t=0}$

$S = \int_{t=0}^{t} \vec{r}(u)\,du = \int_0^t \frac{2}{u^2+1}-1+\frac{2u}{u^2+1}\,du$

$u^2+1=z$

$dz=2u$

$\int\frac{dz}{z}=\ln z=\ln u^2+1\Big|_0^t$

$\tan^{-1}u-u\Big|_0^t+\ln u^2+1\Big|_0^t$

$(\tan^{-1}t-t)-0+\ln t^2+1 \;\cancel{-\ln 1}$

$S=\tan^{-1}t-t+\ln t^2+1$

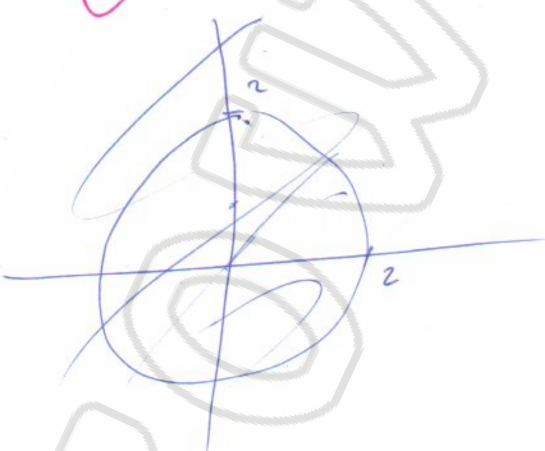
zero

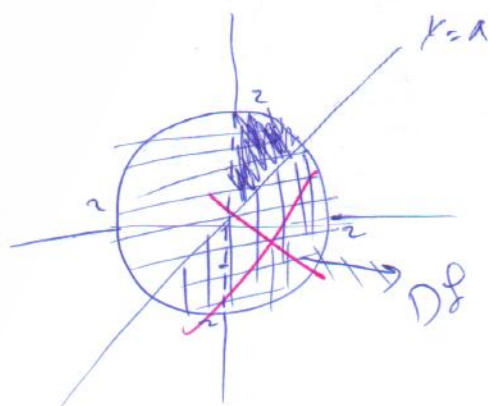
Q₂) Sketch the domain of this function $f(x, y)=\sqrt{4-x^2-y^2}+\frac{\ln x}{\sqrt{y-x}}$.

(1)

$4-x^2-y^2\geq 0$ & $x>0$ &

$4\geq x^2+y^2$ $y-x>0$





Q3) Write an equation of the tangent plane to the surface $z = x - \tan^{-1} yz$ at the point $(0, 0, 0)$.

$\vec{\nabla} f = f_x \hat{i} + f_y \hat{j} + f_z \hat{k}$

$= 1\hat{i} + \frac{1 \cdot z}{1+(yz)^2}\hat{j} + \frac{y}{1+(yz)^2}\hat{k}$ $\quad \left(1+\frac{y}{1+(yz)^2}\right)\hat{k}$

$\vec{\nabla} f(0,0,0)$

$= \hat{i} + 0 + 0 \quad \therefore \vec{n} = \hat{i}$

$T_p: \; 1(x-0) = 0$

$= \boxed{x = 0}$

$\frac{1}{3}$

Q4) If $z = xy + xf\left(\frac{y}{x}\right)$, show that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = xy + z$.

$\cancel{\frac{\partial z}{\partial x} = \frac{-f_x}{f_z}}$: $\quad xy + xf(u) - z = 0 \qquad u = \frac{y}{x}$

zero

$\frac{\partial z}{\partial x} = \frac{-f_x}{f_z} = +\frac{y + x f'(u) + f(u)}{+1} \qquad \frac{\partial u}{\partial x} = \frac{-y}{x^2}$

$x\frac{\partial z}{\partial x} = x\,\frac{y + x\left(\frac{-y}{x^2}\right) + f\left(\frac{y}{x}\right)}{1}$

$\frac{\partial z}{\partial y} = \frac{-f_y}{f_z} =$

$y\,\frac{-f_y}{f_z} = y\left(x + x f'\left(\frac{y}{x}\right)\cdot\frac{1}{x}\right)$

$\boxed{x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = xy + z}$

Q5) If the maximum $Df(2, 1, -1) = 4$, and it occurs in the direction of $2\hat{i} - \hat{j} - 2\hat{k}$. Find $Df(2, 1, -1)$ in the direction towards the origin.

$\frac{2}{5}$

$\vec{\nabla} f$ in the same Direction with $2\hat{i} - \hat{j} - 2\hat{k}$

$\frac{2\hat{i} - \hat{j} - 2\hat{k}}{\sqrt{4+1+4}} = 4\left(\frac{2}{3}\hat{i} - \frac{1}{3}\hat{j} - \frac{2}{3}\hat{k}\right) = \vec{\nabla} f$

$\vec{a} = -2\hat{i} - \hat{j} + \hat{k}$

$\vec{u}_a = \frac{-2}{\sqrt{6}}\hat{i} - \frac{1}{\sqrt{6}}\hat{j} + \frac{1}{\sqrt{6}}\hat{k}$

$D_{\vec{a}} f = \vec{\nabla} f \cdot \vec{u}_a$

$= \frac{-4}{3\sqrt{6}} + \frac{1}{3\sqrt{6}} - \frac{2}{3\sqrt{6}}$

$\boxed{= \frac{-5}{3\sqrt{6}}}$

Q₃) Write an equation of the tangent plane to the surface $z = x - \tan^{-1} yz$ at the point $(0, 0, 0)$.

$\vec{\nabla} f = f_x \hat{i} + f_y \hat{j} + f_z \hat{k}$

$= 1\hat{i} + \frac{1 \cdot z}{1+(yz)^2}\hat{j} + \frac{y}{1+(yz)^2}\hat{k}$

$\left(1 + \frac{y}{1+(yz)^2}\right)\hat{k}$

$\vec{\nabla} f(0,0,0)$

$= \hat{i} + 0 + 0 \quad \therefore \vec{n} = \hat{i}$

$T_p: \quad 1(x-0) = 0$

$= \boxed{x = 0}$

$\frac{1}{3}$

Q₄) If $z = xy + xf\left(\frac{y}{x}\right)$, show that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = xy + z$.

$xy + x f(u) - z = 0 \qquad u = \frac{y}{x}$

$\frac{\partial u}{\partial x} = \frac{-y}{x^2}$

$\frac{\partial z}{\partial x} = \frac{-f_x}{f_z} = + \frac{y + x f'(u) + f(u)}{+1}$

zero

$x\frac{\partial z}{\partial x} = x\frac{y + x\left(\frac{-y}{x^2}\right) + f\left(\frac{y}{x}\right)}{1}$

$\frac{\partial z}{\partial y} = \frac{-f_y}{f_z} =$

$y \frac{-h_y}{f_z} = \left(x + x f'\left(\frac{y}{x}\right) \cdot \frac{1}{x}\right)$

$\boxed{x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = xy + z}$

Q₅) If the maximum $Df(2, 1, -1) = 4$, and it occurs in the direction of $2\hat{i} - \hat{j} - 2\hat{k}$. Find $Df(2, 1, -1)$ in the direction towards the origin.

$\frac{2}{5}$

$\vec{\nabla} f$ in the same Direction with $2\hat{i} - \hat{j} - 2\hat{k}$

$\frac{2\hat{i} - \hat{j} - 2\hat{k}}{\sqrt{4+1+4}} = 4\left(\frac{2}{3}\hat{i} - \frac{1}{3}\hat{j} - \frac{2}{3}\hat{k}\right) = \vec{\nabla} f$

$\vec{a} = -2\hat{i} - \hat{j} + \hat{k}$

$\vec{u}_a = \frac{-2}{\sqrt{6}}\hat{i} - \frac{1}{\sqrt{6}}\hat{j} + \frac{1}{\sqrt{6}}\hat{k}$

$D_a f = \vec{\nabla} f \cdot \vec{u}_a$

$= \frac{-4}{3\sqrt{6}} + \frac{1}{3\sqrt{6}} - \frac{2}{3\sqrt{6}}$

$\boxed{= \frac{-5}{3\sqrt{6}}}$

Q$_6$) $\lim_{(x,y)\to(0,0)} \dfrac{\sin xy}{xy^2+2xy}$.

$= \lim_{(x,y)\to(0,0)} \dfrac{\sin xy}{xy(y+2)} = 1 \cdot \lim_{y\to 0} \dfrac{1}{y+2}$

$= \dfrac{1}{2}$

3

3

use $\lim_{x\to a} f(x)\,g(x) = \lim_{x\to a} f(x) \lim_{x\to a} g(x)$

Q$_7$) Find and classify the criticals for local maximum, minimum or saddle for the function $f(x, y) = 3x - x^3 - 2y^2 + y^4 + 5$.

$f_x = 3 - 3x^2$

$f_y = -4y + 4y^3$

$f_x = 0 \Rightarrow 3 - 3x^2 = 0$

$3x^2 = 3$

$x^2 = 1$

$x = \mp 1$

$\boxed{x=1}$ $\boxed{x=-1}$

$f_y = 0 \Rightarrow 4y(-1+y^2) = 0$

$y = 0$ $y^2 = 1$

$y = \mp 1$

$\boxed{y=0}$ $\boxed{y=1}$ $\boxed{y=-1}$

7

$f_{xx} = -6x$ $f_{yy} = -4 + 12y^2$ $f_{xy} = 0$

~~$D = f_{xx} \cdot f_{yy} - (f_{xy})^2 = -6x(-4+12y^2)$~~

pts ① $(1,0)$ ④ $(-1,0)$

② $(1,1)$ ⑤ $(-1,1)$

③ $(1,-1)$ ⑥ $(-1,-1)$

$D = f_{xx} \cdot f_{yy} - (f_{xy})^2$

① $= -6 \cdot -4 = 24$

$D > 0$ $f_{xx} < 0$

local max on pt $(1,0)$

② $-6 \cdot 8 = -48$

$D < 0$ saddel

③ $-6 \cdot 8 = -48$

$D < 0$ saddel

④

$6 \cdot -4 = -24$

$D < 0$ saddel

⑤ $6 \cdot 8 = 48$ pt $(-1,1)$

$D > 0$ $f_{xx} > 0$ Local min

⑥ $6 \cdot 8 = 48$ $(-1,-1)$ Local min