الجامعة الأردنية	تفاضل وتكامل 2: الاختبار الثاني	الثلاثاء: 2012/4/24
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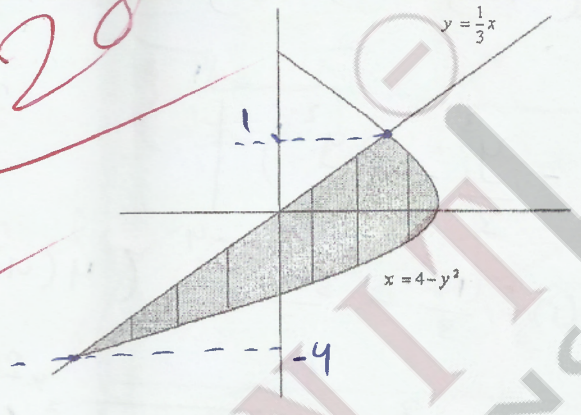
[1] (a) Set up the integral that gives the area of the shaded region



$y = \frac{1}{3}x$

$3y = x$

$4 - y^2 = 3y$

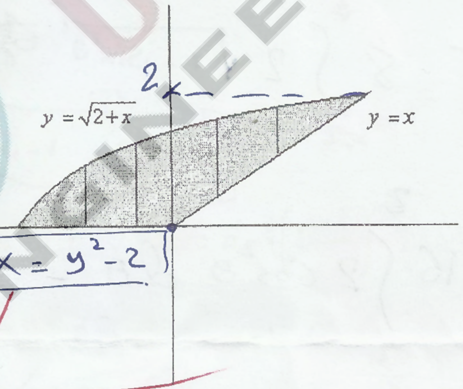
$y^2 + 3y - 4 = 0$

$(y+4)(y-1) = 0$

$y = -4 \; , \; y = 1$

خلف الورقة ⟹

(b) Set up the integral that gives the volume of the solid whose base is the shaded region and each cross section perpendicular to the x-axis is equilateral triangle.



$\boxed{y = x}$, $y = \sqrt{x+2}$

$y^2 = x + 2 \Rightarrow \boxed{x = y^2 - 2}$

$y = y^2 - 2$

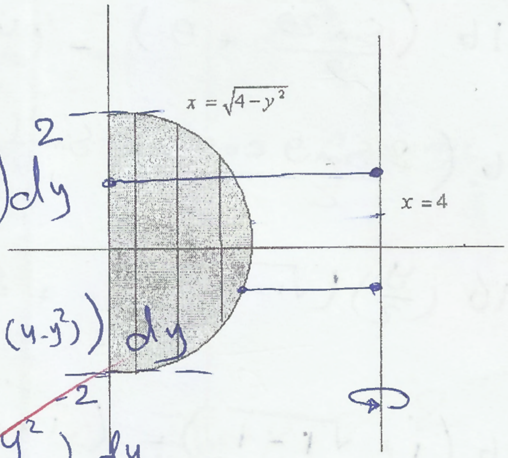
$y^2 - y - 2 = 0$

$(y-2)(y+1) = 0$

$y = 2 \; , \; y = -1$

Set up: $A = \int_0^2 (y - (y^2 - 2))\,dy$

[2] Find the volume of the solid generated by rotating the shaded region about the line $x = 4$.



$V = \pi \int_{-2}^{2} \left((4-0)^2 - (4 - \sqrt{4-y^2})^2 \right) dy$

$V = \pi \int_{-2}^{2} 16 - \cancel{16+} \left(16 - 8\sqrt{4-y^2} + (4-y^2)\right) dy$

$V = \pi \int_{-2}^{2} (\cancel{16} - \cancel{16} + 8\sqrt{4-y^2} - 4 + y^2)\, dy$

$V = \pi \left(8\int_{-2}^{2} \sqrt{4-y^2}\,dy - \int_{-2}^{2} 4 + y^2\,dy \right).$

$x = \sqrt{4-y^2}$

$x^2 = 4 - y^2 \rightarrow x^2 + y^2 = 4$

$\hookrightarrow \boxed{r = 2}$

$y = 2 \sin\theta$

$dy = 2\cos\theta\, d\theta$

خلف الورقة ⟹

$Z = \sqrt{1+\frac{1}{x}}$

$dZ = \frac{\frac{-1}{x^2}}{2\sqrt{Z}}$

$Z = 1 + \frac{1}{x}$

$dZ = \frac{-1}{x^2}$

$\frac{1}{x} = Z - 1$

$x = \frac{1}{Z-1}$

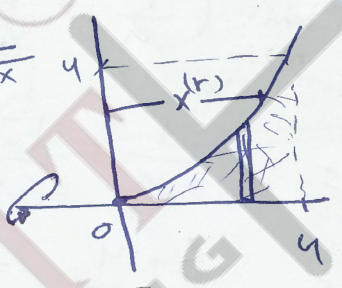
$\frac{\sqrt{Z}}{(Z-1)}$

As

[3] Find the surface area generated by rotating $y = 2\sqrt{x}$, $0 \le x \le 4$ about the ~~y-axis~~. x-axis

$$A_s = 2\pi \int r\, ds$$

$$y = 2\sqrt{x}, \quad \frac{dy}{dx} = \cancel{2}\frac{1}{\cancel{2}\sqrt{x}}$$



$$A_s = 2\pi \int_0^4 x\sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$A_s = 2\pi \int_0^4 x\sqrt{1+\left(\frac{1}{\sqrt{x}}\right)^2}\, dx$$

$$A_s = 2\pi \int_0^4 \cancel{x}\sqrt{1+\frac{1}{x}}\, dx$$

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$Z = \sqrt{1+\frac{1}{x}}$

$1 + \frac{1}{x}$, $\frac{-1}{x^2}$

[4] (a) Find the limit of the sequence $\{a_n\}_{n=1}^{\infty}$, where $a_n = \left(1-\frac{1}{n}\right)^{n^2}$.

~~$\lim_{n\to\infty}\left(1-\frac{1}{n}\right)^{n^2}$~~ ... (by root . test) $\rightarrow \left(1+\frac{-1}{n}\right)^n$

$$\lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty}\left(\left(1-\frac{1}{n}\right)^{n^2}\right)^{\frac{1}{n}} = \lim_{n\to\infty}\left(1-\frac{1}{n}\right)^n$$

$$\hookrightarrow \boxed{\lim_{x\to\infty}\left(1+\frac{a}{x}\right)^x = e^a}$$

$$= e^{-1} = \frac{1}{e} < 1$$

Converge by Root test

(b) Find the sum $\sum_{n=2}^{\infty} \frac{1}{(n+1)(n+3)}$

$$\frac{1}{(n+1)(n+3)} = \frac{A}{(n+1)} + \frac{B}{(n+3)}$$

$$\boxed{A(n+3) + B(n+1) = 1}$$

$n = -3 \rightarrow -2B = 1 \Rightarrow \boxed{B = -\frac{1}{2}}$

$n = -1 \Rightarrow 2A = 1 \Rightarrow \boxed{A = \frac{1}{2}}$

Jump is "2"

خلي بالك ⟹

$$\sum_{n=1}^{\infty}\left(\frac{1/2}{(n+1)} - \frac{1/2}{(n+3)}\right)$$

$$\sum = \left(\frac{1/2}{2} - \cancel{\frac{1/2}{4}}\right) + \left(\frac{1/2}{3} - \cancel{\frac{1/2}{5}}\right) + \left(\cancel{\frac{1/2}{4}} - \frac{1/2}{6}\right)$$

$$+ \cdots + \left(\cancel{\frac{1/2}{n}} - \frac{1/2}{(n+2)}\right) + \left(\cancel{\frac{1/2}{(n+1)}} - \frac{1/2}{n+3}\right)$$

[5] Test for convergence

(a) $\sum_{n=1}^{\infty} \frac{\cos^2(n)}{n^{\frac{3}{2}}+5n}$

$0 \leq \cos^2(n) \leq 1$

$\frac{\cos^2(n)}{n^{3/2}+5n} \leq \frac{1}{n^{3/2}}$ (*) but $\frac{1}{n^{3/2}}$ is Converge by P-Series $P=3/2>1$

$\downarrow$ Converge by C.T

$\downarrow$ Converge

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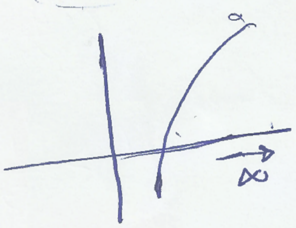
$\therefore \frac{\cos^2(n)}{n^{3/2}+5n}$ is Converge

(b) $\sum_{n=2}^{\infty} \frac{(-1)^n}{n\ \ln(n)}$

$\sum_{n=2}^{\infty} (-1)^n \frac{1}{n\ln(n)} = \sum (-1)^n a_n$

$\boxed{\left(\frac{1}{n\ln(n)}\right)' = \frac{-1\ (\cancel{x}\cdot\frac{1}{\cancel{x}}+\ln(n)\cdot 1)}{n^2\ln^2(n)} \quad a_n \text{ is decresing}}$

$\therefore a_n$ is decresing ...... (1)



$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1 * \frac{1}{n}}{\frac{n\ln(n)}{n}}$

$= \lim_{n\to\infty} \frac{\frac{1}{n}}{\ln(n)} = \frac{0}{\infty}$

2.5

$= \lim_{n\to\infty} \frac{\frac{-1}{n^{\cancel{2}}}}{\frac{1}{\cancel{x}}} = \lim_{n\to\infty} \frac{-1}{n} = 0$

$\therefore \lim_{n\to\infty} a_n = 0$ ...... (2)

$\therefore \sum_{n=2}^{\infty} \frac{(-1)^n}{n\ln(n)}$ is Converge by Alternating Series