الجامعة الأردنية	الاختبار الأول: تفاضل وتكامل 2	الثلاثاء: 2012/3/13
اسم الطالب: يزن عدلي رشيد الحمدان		مدرس المادة: د. حسين النجار
الرقم الجامعي: 0110494		وقت المحاضرة: 9 - 10

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Evaluate the following integrals:

[1] $\int \frac{x}{\cos^2 x}\,dx$

$= \int x \sec^2 x \, dx$ ... by Parts

$u = x$ $dv = \sec^2 x\,dx$

$du = dx$ $v = \tan x$

$\int x \sec^2 x\,dx = x\tan x - \int \tan x\,dx$

$= x\tan x - \int \frac{\sin x}{\cos x}\,dx$

$z = \cos x$

$dz = -\sin x\,dx$

$dx = \frac{dz}{-\sin x}$

$\Rightarrow = x\tan x + \int \frac{\cancel{\sin x}}{z}\frac{dz}{\cancel{\sin x}}$

$= x\tan x + \ln|z| + c$

$\int \frac{x}{\cos^2 x}\,dx = x\tan x + \ln|\cos x| + c$

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[2] $\int \frac{dx}{2\sin x - \tan x}$

$= \int \frac{dx}{2\sin x - \frac{\sin x}{\cos x}}$

$= \int \frac{dx}{\frac{2\sin x\cos x - \sin x}{\cos x}}$

$= \int \frac{\cos x\,dx}{\sin x(2\cos x - 1)}$

$z = \cos x$

$dz = -\sin x$

$dx = \frac{dz}{-\sin x}$

$= \int \frac{-z}{\sin^2 x(2z-1)}\,dz$

$\boxed{\sin^2 x = 1 - \cos^2 x} \boxed{= 1 - z^2}$

$= \int \frac{-z}{(1-z^2)(2z-1)}$

$= -\int \frac{z\,dz}{(1-z)(1+z)(2z-1)}$ P.F

$$\boxed{\frac{z}{(1-z)(1+z)(2z-1)} = \frac{A}{1-z} + \frac{B}{1+z} + \frac{C}{2z-1}}$$

4.5

تكملة [2] يتبع الحل خلف الورقة

Continue

[2] $\frac{z}{(1-z)(1+z)(2z-1)} = \frac{A}{1-z} + \frac{B}{1+z} + \frac{C}{2z-1}$

$$= \frac{A(1+z)(2z-1) + B(1-z)(2z-1) + C(1+z)(1-z)}{(1-z)(1+z)(2z-1)}$$

$$\boxed{A(1+z)(2z-1) + B(1-z)(2z-1) + C(1+z)(1-z) = z}$$

$z=1 \Rightarrow A(3)(1) = 1 \Rightarrow \boxed{A = \frac{1}{3}}$

$z=\frac{1}{2} \Rightarrow C(\frac{3}{2})(\frac{1}{2}) = \frac{1}{2} \Rightarrow \frac{3}{4}C = \frac{1}{2} \Rightarrow \boxed{C = \frac{2}{3}}$

$z=-1 \Rightarrow B(-2)(-3) = -1 \Rightarrow -6B = -1 \Rightarrow \boxed{B = \frac{1}{6}}$

$\therefore \int \frac{z\,dz}{(1-z^2)(2z-1)} = -\frac{1}{3}\int \frac{dz}{1-z} + -\frac{1}{6}\int \frac{dz}{1+z} + -\frac{2}{3}\int \frac{dz}{2z-1}$

$$= -\left(\frac{1}{3}\ln|1-z| + \frac{1}{6}\ln|1+z| + \frac{2}{3}\ln|2z-1|\right) + C$$

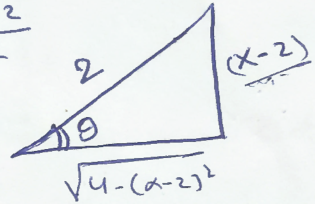
$\int \frac{d\alpha}{2\sin\alpha - \tan x} = -\frac{1}{3}\ln|1-\cos x| \pm \frac{1}{6}\ln|1+\cos x| - \frac{2}{3}\ln|2\cos\alpha - 1| + C$

[3] $\int \sqrt{4x - x^2}\, dx$

$$4x - x^2 = -(x^2 - 4x + 4) + 4$$

$$= -(x-2)^2 + 4 = 4 - (x-2)^2$$

$$\therefore \int \sqrt{4 - (x-2)^2}\, dx \qquad \begin{cases} x - 2 = 2\sin\theta \Rightarrow \sin\theta = \dfrac{x-2}{2} \\ dx = 2\cos\theta\, d\theta \end{cases}$$



$$= \int \sqrt{4 - 4\sin^2\theta} \cdot 2\cos\theta\, d\theta$$

$$= \int 2\sqrt{\cos^2\theta} \cdot 2\cos\theta\, d\theta$$

$$= \int 4 \cos\theta \cos\theta\, d\theta$$

$$= 4 \int \cos^2\theta\, d\theta$$

$$\boxed{\cos^2\theta = \tfrac{1}{2}(1 + \cos 2\theta)}$$

4

$$\Rightarrow \sin^{-1}(\sin\theta) = \sin^{-1}\left(\frac{x-2}{2}\right)$$

$$\Rightarrow \theta = \sin^{-1}\left(\frac{x-2}{2}\right)$$

$$\Rightarrow \cos\theta = \frac{\sqrt{4-(x-2)^2}}{2}$$

$$= 4 \int \tfrac{1}{2}(1 + \cos 2\theta)\, d\theta \Rightarrow \tfrac{4}{2}\int d\theta + \tfrac{4}{2}\int \cos 2\theta\, d\theta$$

$$\tfrac{4}{2}\theta + \tfrac{4}{2}\,\frac{\sin 2\theta}{2} + C$$

$$\tfrac{4}{2}\sin^{-1}\left(\frac{x-2}{2}\right) + \tfrac{4}{2}\,(2\sin\theta\cos\theta) + C$$

$$= 2\sin^{-1}\left(\frac{x-2}{2}\right) + 4\left(\frac{x-2}{2}\right)\left(\frac{\sqrt{4-(x-2)^2}}{2}\right) + C$$

[4] $\int \frac{x}{(x+2)(x^2+4)}\, dx$

$$\lim_{b\to\infty}\int_0^b e^{-\sqrt{x}}$$

e^{-z}

[5] Find (if exists) $\int_0^\infty e^{-\sqrt{x}}\,dx$

$$\left\{\begin{array}{l} z = -\sqrt{x} \\ dz = \dfrac{-1}{2\sqrt{x}}\,dx \\ dx = -2\sqrt{x}\,dz = -2z\,dz \end{array}\right.$$

$\lim \int_{\square}^{\square} -2z\,e^{z}\,dz$ by Parts

$u = -2z$ ① $dv = e^{z}\,dz$

$du = -2\,dz$ ② $v = e^{z}$

$$\int_{\square}^{\square} -2ze^{z}dz = -2ze^{z}\Big|_{\square}^{\square} + \int_{\square}^{\square} 2e^{z}\,dz$$

$$= -2ze^{z}\Big|_{\square}^{\square} + 2e^{z}\Big|_{\square}^{\square}$$

4.5

$$= +2\sqrt{x}\,e^{-\sqrt{x}}\Big|_0^\infty + 2e^{-\sqrt{x}}\Big|_0^\infty$$

$$= \lim_{b\to\infty} 2\sqrt{x}\,e^{-\sqrt{x}}\Big|_0^b + \lim_{b\to\infty} 2e^{-\sqrt{x}}\Big|_0^b$$

$\boxed{e^0 = 1}$

$2e^{-\sqrt{b}} - 2(1)$