$\frac{20.5}{30}$

University of Jordan **Dept. of Math.**

Math. 0301101 **Second Exam** **8-12-2011**

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Q_1) The following question contains ten multiple choice problems, each is 1.5 mark. Write (**x**) on the correct answer.

1) $\lim\limits_{t\to 0}\frac{\sqrt{t^2+9}-3}{t^2}=$

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{5}$ (d) $\frac{1}{6}$ (e) $\frac{1}{8}$

2) $\lim\limits_{x\to -\infty}\frac{4x+\sqrt{x^2+4}}{4x+1}=$

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{3}{4}$ (d) 1 (e) $\frac{5}{8}$

3) Let f be any function such that $\boxed{f(2)=2}$ and $\boxed{f'(2)=3}$, then $\lim\limits_{x\to 2}\frac{xf(x)-4}{x-2}=$

(a) 2 (b) 3 (c) 4 (d) 8 (e) 10

4) If $f(x)=\sin 2x$, then $f^{(41)}(x)=$

(a) $2^{41}\cos 2x$ (b) $-2^{41}\sin 2x$ (c) $-2\sin 2x$ (d) $-2\cos 2x$ (e) *none*

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5) If $f(x)=x^{78}+2x^{65}+x^{50}-x^{40}+3x^{35}+4x^{10}-2x^3+7$, then $f^{(50)}(0)=$

(a) $2(65!)$ (b) $(50!)$ (c) $(65!)$ (d) $(78!)+2(65!)$ (e) *none*

6) If $f(x)=e^{4x}$, then the local linear approximation of f at $x=0$ is,

(a) $1+x$ (b) $1+2x$ (c) $1+3x$ (d) $1+4x$ (e) $1+5x$

7) If $\frac{d}{dx}f(6x)=x^3$, then $f'(6)=$

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{5}$ (e) $\frac{1}{6}$

8) If $y=2\cos\left(\sin^{-1}x\right)$, then $\frac{dy}{dx}=$

(a) $\frac{1}{\sqrt{1-x^2}}$ (b) $-\frac{\cos x}{\sqrt{1-x^2}}$ (c) $\frac{-2x}{\sqrt{1-x^2}}$ (d) $\frac{-3x}{\sqrt{1-x^2}}$ (e) $\frac{-4x}{\sqrt{1-x^2}}$

9) If $f(x)=\cosh(2\ln x)$, then $f'(x)=$,

(a) $2x-\frac{2}{x^3}$ (b) $3x-\frac{3}{x^4}$ (c) $4x-\frac{4}{x^5}$ (d) $5x-\frac{5}{x^6}$ (e) *none*

10) If $f(x)=3\tanh^{-1}(\cos x)$, $x\neq n\pi$, then t $f'(x)=-\csc x$,

(a) $-2\csc x$ (b) $-\csc 2x$ (c) $-3\csc x$ (d) $-\csc 3x$ (e) *none*

Removable

In the following questions **show** your work in details.

Q2) ($6+1$ *marks*) Let $f(x) = \frac{x^2-2x}{2|x|-x^2}$, Find,

a) The point(s) at which f has a vertical asymptotes, or a hole.

b) The Horizontal asymptotes of f.

$$f(x) = \frac{x^2-2x}{2|x|-x^2}$$

a) $2|x| - x^2 = 0$

$2x - x^2 = 0$ OR $2x + x^2 = 0$

$x(2-x) = 0$

$x(2+x) = 0$

$\boxed{x=0}$; $\boxed{x=2}$

$\boxed{x=0}$; $\boxed{x=-2}$

$-2 \quad 0 \quad 2$

$\lim_{x\to -2} \frac{x^2-2x}{2|x|-x^2} = \frac{x^2-2x}{-2x-x^2} = \frac{4-(-4)}{0} = \infty$

$\lim_{x\to 0} \frac{x^2-2x}{2|x|-x^2} = \frac{0}{0}$?!

$\lim_{x\to 2} \frac{x^2-2x}{2x-x^2} = \frac{0}{0}$

1½

∴ Vertical asymptotes is $\boxed{x=-2}$

b) $\lim_{x\to\infty} \frac{x^2-2x}{2x-x^2} = \frac{x^2(1-\frac{2}{x})}{x^2(\frac{2}{x}-1)} = -1$

$\lim_{x\to -\infty} \frac{x^2-2x}{-2x-x^2} = \frac{x^2(1-\frac{2}{x})}{-x^2(\frac{2}{x}+1)} = \frac{1-0}{0+1} = \frac{1}{1} = 1$

½

∴ H.A is

$y = -1$

$y = 1$

Q3) ($2+2+4$ *marks*). If $y^2 - x^2y + x^3 = 13$. Find,

a) y', y''

b) The equation of the tangent(s) line at $x = 2$.

y':

a) $2yy' - (x^2y' + y\,2x) + 3x^2 = 0$

$2yy' - x^2y' = -2xy - 3x^2$

$\frac{y'(2y-x^2)}{(2y-x^2)} = \frac{-(2xy+3x^2)}{(2y-x^2)}$

1½

$y'' = \frac{(2y-x^2)\left[-(2xy'+y(2)+6x)\right] + (2xy+3x^2)(2y'-x^2)}{(2y-x^2)^2}$

b) at $x = 2$

$y^2 - (2)^2y + (2)^3 = 13$

$y^2 - 4y + 8 = 13$

$y^2 - 4y - 5 = 0$

$(y-5)(y+1) = 0$

$y = 5$; $y = -1$

$(2,5)$

$2(5)y' - (4y' + 5(2)(2)) + 3(4) = 0$

$10y' - 4y' + 20 + 12 = 0$

$6y' + 32 = 0 \Rightarrow \boxed{y' = \frac{-32}{6}}$

$(y - y_1) = \frac{-32}{6}(x - x_1)$

$y - 5 = -\frac{32}{6}(x-2)$

$y = -\frac{32}{6}x + \frac{32}{3} + 5$ ---- (1)

$(2,-1) \Rightarrow 2(-1)y' - (4y' + (-1)(2)(2)) + 3(4) = 0$

$-2y' - 4y' - 4 + 12 = 0$

$-6y' + 8 = 0 \Rightarrow \boxed{y' = \frac{8}{6}}$

$y + 1 = \frac{8}{6}(x-2)$

$y = \frac{8}{6}x - \frac{8}{3} - \frac{3\times1}{3\times1} = \frac{8}{6}x - \frac{11}{3}$

(2) ----