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University of Jordan

Dept. of Math.

Math. 0301101

Second Exam

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Instructor:

Q1) The following question contains ten multiple choice problems, each is 1.5 mark. Write (x) on the correct answer.

1) $\lim_{t \rightarrow 0} \frac{\sqrt{t^2+9}-3}{t^2} =$
 (a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{5}$ (d) $\frac{1}{6}$ (e) $\frac{1}{8}$

2) $\lim_{x \rightarrow -\infty} \frac{4x + \sqrt{x^2+4}}{4x+1} =$
 (a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{3}{4}$ (d) 1 (e) $\frac{5}{8}$

3) Let f be any function such that $f(2) = 2$ and $f'(2) = 3$, then $\lim_{x \rightarrow 2} \frac{xf(x)-4}{x-2} =$
 (a) 2 (b) 3 (c) 4 (d) 8 (e) 10

4) If $f(x) = \sin 2x$, then $f^{(41)}(x) =$
 (a) $2^{41} \cos 2x$ (b) $-2^{41} \sin 2x$ (c) $-2 \sin 2x$ (d) $-2 \cos 2x$ (e) none

5) If $f(x) = x^{78} + 2x^{65} + x^{50} - x^{40} + 3x^{35} + 4x^{10} - 2x^3 + 7$, then $f^{(50)}(0) =$
 (a) $2(65!)$ (b) $(50!)$ (c) $(65!)$ (d) $(78!) + 2(65!)$ (e) none

6) If $f(x) = e^{4x}$, then the local linear approximation of f at $x = 0$ is,
 (a) $1+x$ (b) $1+2x$ (c) $1+3x$ (d) $1+4x$ (e) $1+5x$

7) If $\frac{d}{dx} f(6x) = x^3$, then $f'(6) =$
 (a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{5}$ (e) $\frac{1}{6}$

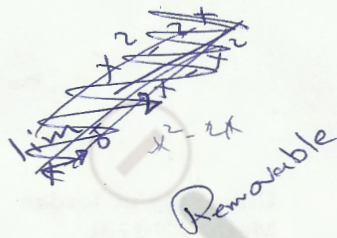
8) If $y = 2 \cos(\sin^{-1} x)$, then $\frac{dy}{dx} =$
 (a) $\frac{1}{\sqrt{1-x^2}}$ (b) $-\frac{\cos x}{\sqrt{1-x^2}}$ (c) $-\frac{2x}{\sqrt{1-x^2}}$ (d) $\frac{-3x}{\sqrt{1-x^2}}$ (e) $\frac{-4x}{\sqrt{1-x^2}}$

9) If $f(x) = \cosh(2 \ln x)$, then $f'(x) =$
 (a) $2x - \frac{2}{x^3}$ (b) $3x - \frac{3}{x^4}$ (c) $4x - \frac{4}{x^5}$ (d) $5x - \frac{5}{x^6}$ (e) none

10) If $f(x) = 3 \tanh^{-1}(\cos x)$, $x \neq n\pi$, then $f'(x) = -\csc x$,
 (a) $-2 \csc x$ (b) $-\csc 2x$ (c) $-3 \csc x$ (d) $-\csc 3x$ (e) none

$\frac{1}{2} \frac{2t}{t^2+9} = \frac{2t}{t^2+9}$
 $\frac{2t}{t^2+9} = \frac{2}{t+\frac{9}{t}}$
 $\frac{2}{t+\frac{9}{t}}$

$\sin 2x = 2 \cos x \sin x$
 $\cos 2x = \cos^2 x - \sin^2 x$
 $\sin^2 2x = 4 \sin^2 x \cos^2 x$
 $\cos^2 2x = \cos^4 x - \sin^4 x$



In the following questions show your work in details.

Q2) (6+1 marks) Let $f(x) = \frac{x^2 - 2x}{2|x| - x^2}$, Find,

- The point(s) at which f has a vertical asymptotes, or a hole.
- The Horizontal asymptotes of f .

$$f(x) = \frac{x^2 - 2x}{2|x| - x^2}$$

\therefore Vertical asymptotes is $x = -2$

$$b) \lim_{x \rightarrow \infty} \frac{x^2 - 2x}{2x - x^2} = \frac{x^2(1 - \frac{2}{x})}{x^2(\frac{2}{x} - 1)} = -1$$

$$\lim_{x \rightarrow -\infty} \frac{x^2 - 2x}{2x - x^2} = \frac{x^2(1 - \frac{2}{x})}{-x^2(\frac{2}{x} - 1)} = \frac{1-0}{0+1} = 1$$

\therefore H.A is $y = -1$

$$y = -1$$

$$y = 1$$

$$a) 2|x| - x^2 = 0$$

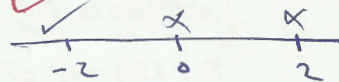
$$2x - x^2 = 0$$

$$\text{OR } 2x + x^2 = 0$$

$$x(2-x) = 0$$

$$x(2+x) = 0$$

$$x = 0; x = -2 \quad x = 0; x = 2$$



$$\lim_{x \rightarrow -2} \frac{x^2 - 2x}{2|x| - x^2} = \frac{x^2 - 2x}{-2x - x^2} = \frac{4 - (-4)}{0} = \infty$$

$$\lim_{x \rightarrow 0} \frac{x^2 - 2x}{2|x| - x^2} = \frac{0}{0} \text{ ?!}$$

Q3) (2+2+4 marks). If $y^2 - x^2y + x^3 = 13$. Find,

- y', y''
- The equation of the tangent(s) line at $x = 2$.

y' :

$$a) 2yy' - (x^2y' + y2x) + 3x^2 = 0$$

$$2yy' - x^2y' = -2xy - 3x^2$$

$$y'(2y - x^2) = \frac{(2xy + 3x^2)}{(2y - x^2)}$$

$$y' = \frac{2xy + 3x^2}{2y - x^2}$$

$$y'' = \frac{(2y - x^2)[-(2xy' + y(2) + 6x)] + (2xy + 3x^2)(2y' - x^2)}{(2y - x^2)^2}$$

$$y + 1 = \frac{8}{6}(x - 2)$$

$$y = \frac{8}{6}x - \frac{8}{3} = \frac{4}{3}x - \frac{8}{3}$$

$$(2) \dots$$

b) at $x = 2$

$$y^2 - (2^2)y + (2^3) = 13$$

$$y^2 - 4y + 8 = 13$$

$$y^2 - 4y - 5 = 0$$

$$(y - 5)(y + 1) = 0$$

$$y = 5 \quad y = -1$$

(2, 5)

$$2(5)y' - (4y' + 5(2)(2) + 3(4)) = 0$$

$$10y' - 4y' + 20 + 12 = 0$$

$$6y' + 32 = 0 \Rightarrow y' = -\frac{32}{6}$$

$$(y - y_1) = \frac{-32}{6}(x - x_1)$$

$$y - 5 = \frac{-32}{6}(x - 2)$$

$$y = \frac{-32}{6}x + \frac{32}{3} + 5 \dots (1)$$

$$(2, -1) \Rightarrow 2(-1)y' - (4y' + (-1)(2)(2) + 3(4)) = 0$$

$$-2y' - 4y' - 4 + 12 = 0$$

$$-6y' + 8 = 0 \Rightarrow y' = \frac{8}{6}$$