$\frac{17}{20}$

University of Jordan Dept. of Math. Math. 0301101 First Exam 27-10-2011

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الرقم الجامعي : ٠١١٠٤٩٤

المدرس:

الرقم المتسلسل :

وقت المحاضره:

Q1) The following question contains seven multiple choice problems, each is 1.5 mark. Write (x) on the correct answer.

1) If $f(x) = \frac{x+1}{x^2+1}$, $g(x) = \sqrt{x^2+3x-1}$, then $(f\circ g)(2) =$

(a) $\frac{1}{2}$ (b) $\frac{5}{2}$ (c) $\frac{2}{5}$ (d) $\frac{3}{5}$ (e) $\frac{4}{5}$

2) If $f(x) = x^2$, $g(x) = \sqrt{x+1}$, then Domain $f\circ g =$

(a) $\mathbb{R}$ (b) $\mathbb{R}\backslash\{-1\}$ (c) $(-\infty,-1)$ (d) $(-\infty,1]$ (e) $[-1,\infty)$

7.5

3) If $\log_x(5x-4) = 2$, then $x =$

(a) $\{1,4\}$ (b) $\{1\}$ (c) $\{4\}$ (d) $\{1,3\}$ (e) $\{3\}$

4) If $f(x) = e^x + 3e^{-x} - 1$, then $f^{-1}(3) =$

(a) $\ln(1)$ (b) $\ln(\frac{3}{4})$ (c) $\ln(\frac{3}{5})$ (d) $\ln(\frac{4}{5})$ (e) $\ln(\frac{3}{2})$

5) If $x = \frac{1}{2}\ln 9 - \ln 2$, then $e^{2x} =$

(a) e^3 (b) 3 (c) $\frac{3}{2}$ (d) $\frac{9}{4}$ (e) $\frac{9}{2}$

6) If $f(x) = \frac{3}{4+2\cos x}$, then range $f=$

(a) $[\frac{1}{2},\frac{3}{4}]$ (b) $[\frac{1}{2},\frac{3}{2}]$ (c) $[\frac{1}{4},\frac{3}{2}]$ (d) $[\frac{1}{4},\frac{3}{4}]$ (e) *none*

7) Let $f(x) = x^2+2x$, If f is shifted 2units right, 3units up then reflected about the Y-axis, we obtained,

(a) $g(x) = x^2+2x+3$ (b) $g(x) = x^2-2x+3$ (c) $g(x) = x^2+2x-3$
(d) $g(x) = x^2-2x-3$ (e) $g(x) = x^2-6x+3$

$(x^2+2x+1)-1$

$(x+2)^2$

x^2+2x+3

x^2+2

$(x^2+1)^2-1$

$(-x^2-1)^2+2$

$(x^2+2x+1)-1$

$(x+1)^2-1$

$(x-1)^2+2$

$x^2+2x+1+2$

x^2+2x+3

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B

In the following questions **show** your work in details.

Q₂) $(3+2\ marks)$ Let $f(x) = 3 - 2\sin^{-1}(2x-1)$ Find,

a) The domain and range of f.

① domain:

$-1 \le 2x-1 \le 1$

$+1 \quad +1 \quad +1$

$\frac{0}{2} \le \frac{2x}{2} \le \frac{2}{2}$

$0 \le x \le 1$

Domain $f(x) = [0, 1]$

② Range:

~~Range f(x) = R~~

$-\frac{\pi}{2} \le \sin^{-1}(2x-1) \le \frac{\pi}{2}$

$\pi \ge -2\sin^{-1}(2x-1) \ge -\pi$

$3+\pi \ge 3-2\sin^{-1}(2x-1) \ge 3-\pi$

Rang $f(x) = [3-\pi, 3+\pi]$

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b) $f\left(\frac{1}{2}+\frac{1}{2}\sin\frac{5\pi}{4}\right)$

$f\left(\frac{1}{2}+\frac{1}{2}\sin\frac{5\pi}{4}\right) = 3 - 2\sin^{-1}\left(2\left(\frac{1}{2}+\frac{1}{2}\sin\frac{5\pi}{4}\right)-1\right)$

$f\left(\frac{1}{2}+\frac{1}{2}\sin\frac{5\pi}{4}\right) = 3 - 2\sin^{-1}\left(\cancel{2}\,\frac{1}{\cancel{2}}\left(1+\sin\frac{5\pi}{4}\right)-1\right)$

but $\sin\frac{5\pi}{4} = -\sin\left(\frac{5\pi}{4}-\frac{4\pi}{4}\right) = -\sin\frac{\pi}{4}$

$\therefore f\left(\frac{1}{2}+\frac{1}{2}\sin\frac{5\pi}{4}\right) = 3 - 2\sin^{-1}\left(\left(\cancel{1}-\sin\frac{\pi}{4}\right)-\cancel{1}\right)$

$f\left(\frac{1}{2}+\frac{1}{2}\sin\frac{5\pi}{4}\right) = 3 - 2\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

$f\left(\frac{1}{2}+\frac{1}{2}\sin\frac{5\pi}{4}\right) = 3 - 2 * -\frac{\pi}{4}$

$f\left(\frac{1}{2}+\frac{1}{2}\sin\frac{5\pi}{4}\right) = 3 - \frac{\pi}{2}$

$1\frac{1}{2}$

Q₃) $(3+2\ marks)$ Let $f(x) = \frac{e^{2x}-1}{e^{2x}+1}$. Find,

a) $f^{-1}(x)$

$y = \frac{e^{2x}-1}{e^{2x}+1}$

$x = \frac{e^{2y}-1}{e^{2y}+1}$

$x(e^{2y}+1) = e^{2y}-1$

$xe^{2y} + x = e^{2y} - 1$

$x + 1 = e^{2y} - xe^{2y}$

$x + 1 = e^{2y}(1-x)$

$e^{2y} = \frac{x+1}{1-x}$

$\ln(e^{2y}) = \ln\left(\frac{x+1}{1-x}\right)$

$2y = \ln\left(\frac{x+1}{1-x}\right) \Rightarrow y = \frac{1}{2}\ln\frac{(x+1)}{(1-x)} = f^{-1}(x)$

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b) Classify $f^{-1}(x)$ as even, odd, or neither.

$f^{-1}(x) = \frac{1}{2}\ln\frac{(x+1)}{(1-x)}$

$f^{-1}(x) = \frac{1}{2}(\ln(x+1) - \ln(1-x))$

$f^{-1}(-x) = \frac{1}{2}(\ln(-x+1) - \ln(1-(-x)))$

$f^{-1}(-x) = \frac{1}{2}(\ln(1-x) - \ln(x+1))$

$f^{-1}(-x) = -\frac{1}{2}(\ln(x+1) - \ln(1-x))$

$\therefore f^{-1}(-x) = -f^{-1}(x)$

$f^{-1}(x)$ is odd function

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