+ Convolution :-

- → integral
- → uses:

$h(t)$ = impulse response

$$x(t) \longrightarrow \boxed{\text{LTI sys}} \longrightarrow y(t)$$

$$y(t) = x(t) * h(t)$$

- we use the convolution to find the output of (LTI) sys.

$$\longrightarrow \boxed{\text{LTI sys.}} \longrightarrow y(t)$$

1- $x(t)$ — $y(t)$
2- $x(t) = \delta(t)$ — $y(t) = h(t)$
3- $x(t)$ $u(t)$ unit step — $y(t) = S(t)$ unit step response

$$\delta(t) \longrightarrow \boxed{\text{LTI}} \longrightarrow h(t)$$

$$x(t) \longrightarrow \underset{h(t)}{\boxed{\text{LTI}}} \longrightarrow y(t) = x(t) * h(t)$$

- we solve the conv. by:-
① math.
② graphical procedure.
③ Table.

$$y(t) = x(t) * h(t) = \int_{-\infty}^{+\infty} h(\tau)\, x(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{+\infty} x(\tau)\, h(t-\tau)\, d\tau$$

* $x(t) * h(t) = y(t)$
* $x(t-a) * h(t-b) = y(t-a-b)$
* $x(t) * \delta(t) = x(t)$
* $x(t-a) * \delta(t) = x(t-a)$
* $\delta(t) * \delta(t) = \delta(t)$
* $e^{at} u(t) * u(t) = \dfrac{e^{at} u(t) - u(t) e^{0}}{a-0} = \dfrac{e^{at}-1}{a} u(t)$
* $t^{m} u(t) * t^{n} u(t) = \dfrac{m!\, n!}{(m+n+1)!} t^{(n+m+1)} u(t)$
* $h(t-a) * h(t-b) = y(t-b-a)$

Q :- $x(t) = u(t-1) + u(t-2)$

$h(t) = u(t+1)$

Find $y(t)$?

Sol :- $y(t) = x(t) * h(t) = h(t) * x(t)$

$= u(t+1) * (u(t-1) + u(t-2))$

$= u(t+1) * u(t-1) + u(t+1) * u(t-2)$

$= (t-0)\, u(t+1-1) + u(t+1-2)(t+1-2)$

$= t\, u(t) + (t-1)\, u(t-1)$

يفضل أن يبقى معاملها (1)

Q :-

$x(t) = \delta(2t-2) = \dfrac{1}{|2|} \delta(t-1) = \dfrac{1}{2} \delta(t-1)$

$h(t) = u(t+1) + u(t-2)$

Find , $y(t)$

$y(t) = \frac{1}{2}(u(t) + u(t-3))$

Q:-
$$\delta[2-n] * \delta[n+6] = \delta[n+4]$$

note:- $\delta[an] = \delta[n]$

$$\delta[2-n] = \delta[-1(n-2)] = \delta[n-2]$$

Q:-
$x(t) = rect(t)$

$h(t) = \delta(t - \frac{1}{2}) + \delta(t + \frac{1}{2})$

Find $y(t)$:-

$y(t) = rect(t - \frac{1}{2}) + rect(t + \frac{1}{2})$

$\Longrightarrow y(t) = rect(\frac{t}{2})$

Q:- $S(t) = (1 - e^{-t})\, u(t)$

Find $h(t)$?!

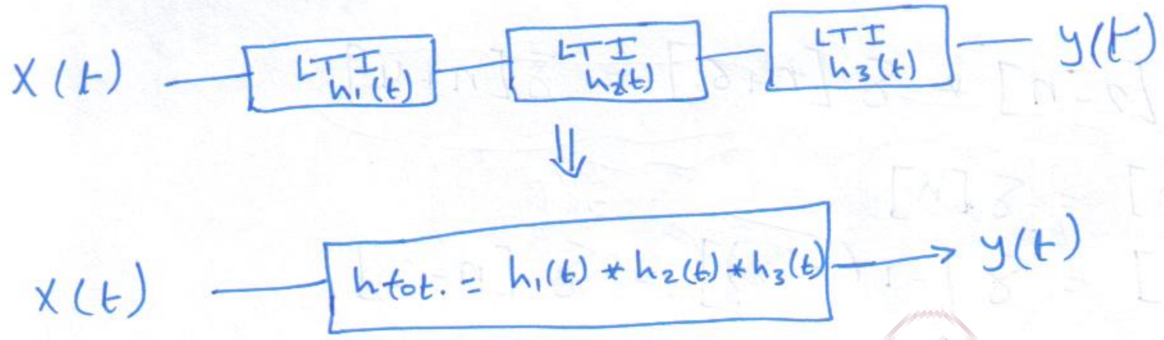
$\boxed{* \delta(t) = \frac{du(t)}{dt}}$

$\boxed{\delta(t) \overset{\text{ضرب}}{*} x(t) = x(0)}$

$\boxed{* h(t) = \frac{ds(t)}{dt}}$

$h(t) = \frac{ds(t)}{dt} = e^{-t}\, u(t) + \cancel{\delta(t)(1 - e^{-t})}$ 0.

$\Longrightarrow h(t) = e^{-t}\, u(t)$



Q8-

~~$x_1[n] = \delta$~~

$h_1[n] = 2\delta[n] + \delta[n-2]$

$h_2[n] = -\delta[n] + \delta[n-1]$

Connected serially, Find overall response?

$h_{total} = -2\delta[n] + 2\delta[n-1] - \delta[n-2] + \delta[n-3]$

Q8-

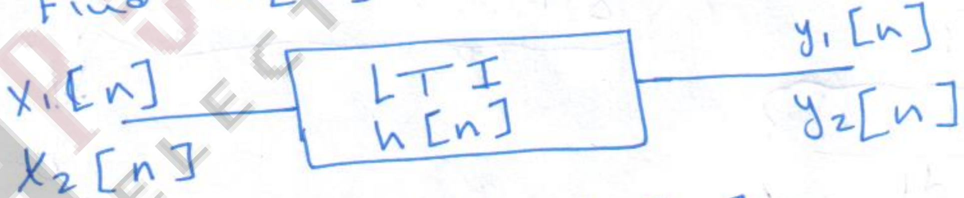
$x_1[n] = \delta[n] + \delta[n-1] + \delta[n-2]$

$y_1[n] = \cos(n)$

$x_2[n] = \delta[n-1] + \delta[n-2]$

$y_2[n] = \sin[n]$

Find $h[n]$



$y_1[n] = x_1[n] * h[n]$

$\cos(n) = h[n] + h[n-1] + h[n-2]$ ⋯ ①

$y_2[n] = x_2[n] * h[n]$

$\sin(n) = h[n-1] + h[n-2]$ ⋯ ②

$\cos(n) - \sin(n) = h[n]$

$$x(t) \longrightarrow \boxed{h_1(t)} \longrightarrow \boxed{h_2(t)} \longrightarrow y(t)$$

if

1) $h_1(t) = \delta(t-1)$, find h_{total}.

$h_2(t) = \delta(t-8)$

$$h_{tot.} = h_1 * h_2 = \delta(t-9)$$

2) $h_1 = u(t-2) - u(t)$

$h_2 = u(t-1) + u(t+2)$

find h_{tot}.

<u>Sol</u> :-

حل السؤال لحالك يا مهندس :p

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* System :-

→ Stable → if $\int_{-\infty}^{\infty} |h(t)| dt$ = رقم , other wise ⟹ <u>unstable</u>

→ casual if :-

$$y(t) = x(t) * h(t) = \int_{-\infty}^{+\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{t} x(\tau)\, h(t-\tau)\, d\tau$$

Causal

تحدد حدود التكامل من "unit step"

Q :- $h(t) = e^{-t} u(t-1)$

<u>- Stable or unstable :-</u>

$$\int_{-\infty}^{\infty} e^{-t} u(t-1)\, dt = \int_{1}^{\infty} e^{-t}\, dt$$

$$= -e^{-t} \Big]_{1}^{\infty} = (0 - -e^{-1})$$

$$= e^{-1} \text{ " رقم "}$$

⟹ <u>Stable</u> #

<u>Causal or non causal :-</u>

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, e^{-(t-\tau)}\, u(t-\tau-1)\, d\tau$$

$$= \int_{-\infty}^{t-1} x(t)\, e^{(t-\tau)}$$

Causal

<u>Note</u> :- $u(t-\tau-1) = u(-\tau + (t+1))$

Q:-

$$y(t) = \int_{-\infty}^{t} e^{-2(t-\tau)}\, x(\tau - 1)\, d\tau$$

① Find $h(t)$.
② Causal or non causal.
③ Stable or non stable.
④ repeat (1, 2, 3) if

$$y(t) = \int_{-\infty}^{+\infty} e^{-2(t-\tau)}\, x(\tau-1)\, dt$$

Sol:-

$$x(t) \longrightarrow \boxed{LTI} \longrightarrow y(t)$$

$x(t) = \delta(t)$ $\qquad$ $y(t) = h(t)$

Note:-

$$\delta(t-1) \overset{\text{ضرب}}{X} x(t) = x(1)\, \delta(t-1)$$

$$\delta(t-b)\, X\, x(t) = x(b)\, \delta(t-b)$$

$$\int x(t)\, \delta(t-b) = x(b)$$

$$h(t) = \int_{-\infty}^{t} \underbrace{e^{-2(t-\tau)}}_{x(t)}\ \underbrace{\delta(t-1)}_{\delta(t-1)}\, d\tau$$

$$= \int_{-\infty}^{t} \underbrace{e^{-2(t-1)}}_{\text{constant}}\, \delta(t-1)\, d\tau$$

$$\Longrightarrow = e^{-2(t-1)} \int_{-\infty}^{t} \delta(t-1)\, d\tau = e^{-2(t-1)}\, u(t-1)$$

لو كانت حدود التكامل $(-\infty \leftarrow \infty)$

← جواب التكامل $=(1)$ وليس unit step

Q8-

$$x[n] = \delta[n-2]$$

$$y[n] = u[n-3]$$

Find $h[n]$

$$y[n] = h[n] * x[n]$$

$$u[n-3] = h[n] * \delta[n-2]$$

$$\Downarrow$$

$$u[n-1]$$

Q8- $\displaystyle\int_{-\infty}^{\infty} u(\tau-1)\, u(t-\tau)\, d\tau$?

$$\int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

مقارنة مع قانون .Conv $\Longrightarrow$ $h(t-\tau) = u(t-\tau)$

$$x(\tau) = u(\tau-1)$$

$$\Longrightarrow y(t) = x(t) * h(t)$$

$$= u(t-1) * u(t)$$

$$= (t-1)\, u(t-1)$$

---

* important Rule :

① $x(t)\,\delta(t-b) = x(b)\,\delta(t-b)$

② $x(t) * \delta(t-b) = x(t-b)$

③ $\displaystyle\int_{-\infty}^{\infty} x(t)\,\delta(t-b) = x(b)$

④ $\displaystyle\int_{-\infty}^{\infty} \delta(t-b)\,dt = u(t-b)$

⑤ $\displaystyle\int_{-\infty}^{\infty} \delta(t-b)\,dt = 1$

\#

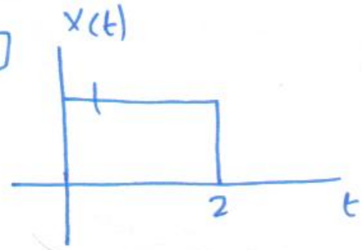
* graphical procedure :-

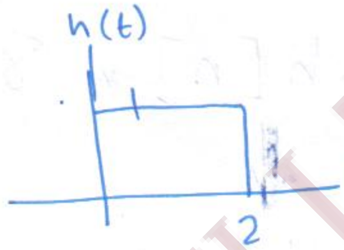
$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

if $x(t) = h(t) = u(t) - u(t-2)$, Find $y(t)$?

Sol :-

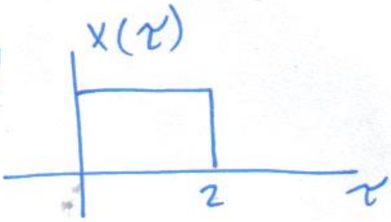
$\boxed{1}$

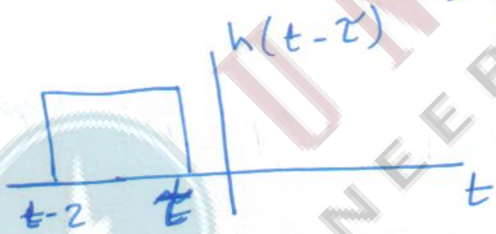




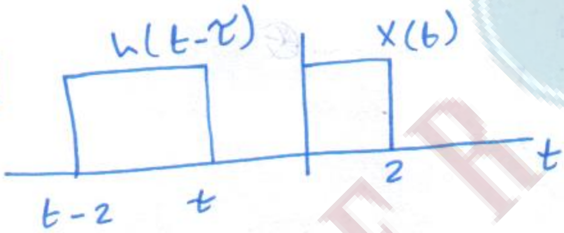
mean reflect then shift by $(-\tau)$

$\boxed{2}$

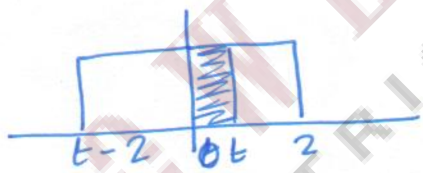




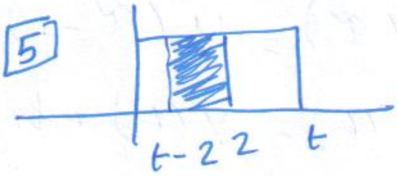
$\boxed{3}$



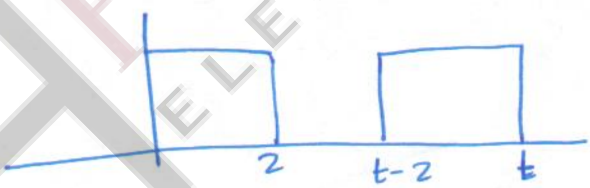
$\boxed{4}$



$\boxed{5}$



$\boxed{6}$



$$y(t) = \begin{cases} 0 & , t < 0 \\ t & , 0 < t < 2 \\ 4-t & , 0 < t < 4 \\ 0 & , 4 < t \end{cases}$$

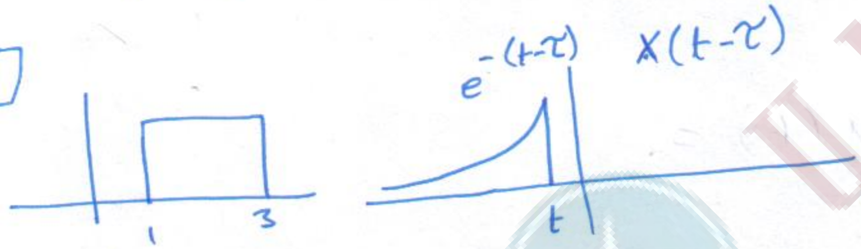
$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$x(t) = e^{-t}\, u(t)$$

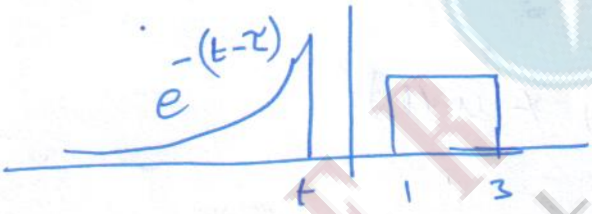
$$h(t) = u(t-1) - u(t-3)$$

1 $h(t) =$ ~~[illegible]~~ $x(t)$

2



1

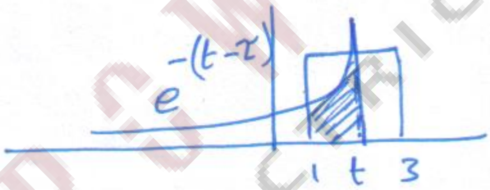


① $y(t) =$ ———

$= 0 \quad , t < 1$

No overlap

2



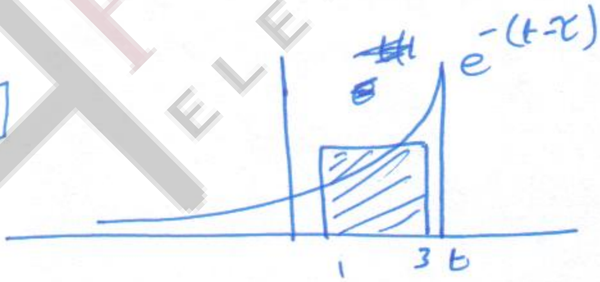
② $y(t) = -\int_1^t (1)\, e^{-(t-\tau)}\, d\tau$

$= e^{-t} \int_1^t e^{\tau}\, d\tau$

$= e^{-t}\left(e^{\tau}\Big]_1^t\right) = e^{-t}(e^{t} - e^{1})$

$= (1 - e^{1-t}), \quad 1 < t < 3$

3



③ $= e^{-(t-3)} - e^{-(t-1)} \qquad 3 < t$

$$* \quad t^n u(t) * t^m u(t) = \frac{n!\, m!}{(n+m+1)!}\, t^{(n+m+1)!}\, u(t)$$

Q8-

$$t\, u(t) * u(t) = t^{1} u(t) * t^{0} u(t)$$

$$= \frac{1!\, 0!}{(0+1+1)!}\, t^{(0+1+1)}\, u(t)$$

$$= \frac{1}{2} t^2 u(t)$$

Q8-

$$(t\, u(t-1)) * u(t) =$$

نجمع ونطرح لل (t)

علما انه لا يوجد فيها shift

$$(t-1+1)\, u(t-1) * u(t)$$

$$\big((t-1)\, u(t-1) + u(t-1) \big) * u(t)$$

$$= \frac{1}{2} (t-1)^2 u(t-1) + (t-1)\, u(t-1)$$

Convolution

+ إعداد المهندس رضا الحسيري

Power_Unit +

No.	$X_1(t)$	$X_2(t)$	$X_1(t) * X_2(t)$
-1-	$X(t)$	$\delta(t-a)$	$X(t-a)$
-2-	$e^{at}\,u(t)$	$u(t)$ $\quad \frac{e^{at}u(t)-u(t)}{a-0}$	$\frac{1-e^{at}}{-a}\,u(t)$
-3-	$u(t)$	$u(t)$	$t\,u(t)$
-4-	$e^{at}\,u(t)$	$e^{bt}\,u(t)$	$\left(\frac{e^{at}-e^{bt}}{a-b}\right)u(t)$
-5-	$e^{at}\,u(t)$	$e^{bt}\,u(-t)$	$\frac{e^{at}u(t)+e^{bt}u(-t)}{b-a}$
-6-	$t^{m}\,u(t)$	$t^{n}\,u(t)$	$\frac{m!\,n!}{(n+m+1)!}\,t^{n+m+1}\,u(t)$
-7-	$\delta(t)$	$\delta(t)$	$\delta(t)$
-8-	$\delta(t-a)$	$\delta(t-b)$	$\delta(t-a-b)$

* <u>Note</u>:- (a) & (b) constant.