



Electrical Circuits (1) (EE211)
Second Exam
3rd Term, 2016-2017
July 6th, 2017. ⌚ 14:00 – 15:30

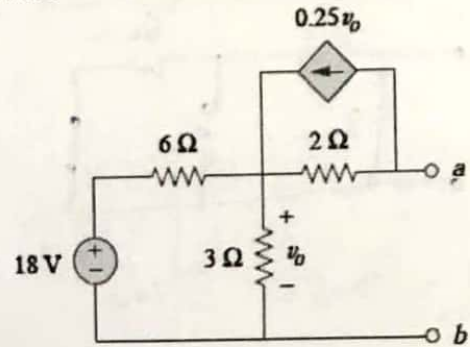
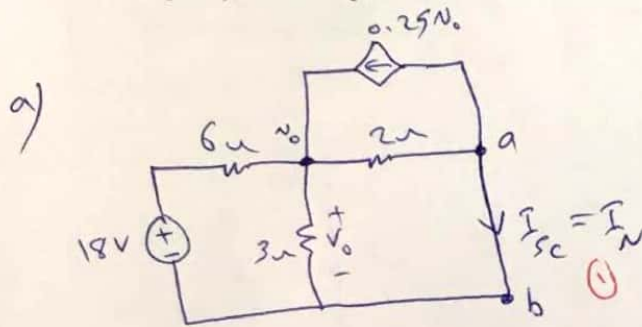
Key

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Reg. No.:

Problem 1: (5 points)

Consider the given circuit and answer the following questions:

- (2 points) find the equivalent Norton current seen between a and b.
- (2 points) find the equivalent Thevenin voltage seen between a and b.
- (1 point) find the equivalent Norton resistor seen between a and b.



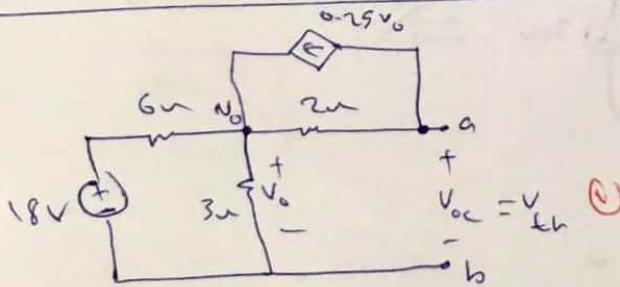
Nodal:

$$\frac{V_o - 18}{6} + \frac{V_o}{3} + \frac{V_o}{2} - \frac{V_o}{4} = 0 \Rightarrow \boxed{V_o = 4V}$$

KCL at (a):

$$I_{sc} = \frac{V_o}{2} - \frac{V_o}{4} = 2 - 1 \Rightarrow \boxed{I_N = 1A}$$

b)



Nodal:

$$\frac{V_o - 18}{6} + \frac{V_o}{3} + \frac{V_o - V_{oc}}{2} - \frac{V_o}{4} = 0 \quad \text{--- (1)}$$

$$\frac{V_{oc} - V_o}{2} + 0.25V_o = 0 \quad \text{--- (2)}$$

(2) in (1)

$$\frac{V_o}{2} = 3 \Rightarrow \boxed{V_o = 6V} \quad , \quad V_{oc} =$$

KVL:

$$* \frac{V_{oc}}{2} = \frac{V_o}{4} + \frac{V_o}{2} = \frac{V_o}{4}$$

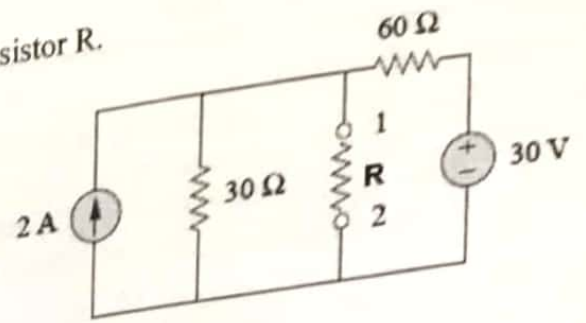
$$V_{oc} = \frac{V_o}{2} = \boxed{3V}$$

$$c) \quad \boxed{R_{th} = \frac{V_{th}}{I_N} = 3\Omega}$$

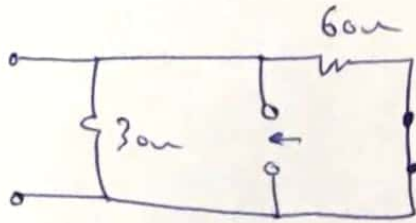
Problem 2: (5 points)

Consider the given circuit and answer the following questions:

- (2 points) Find the value of the resistor R connected between 1 and 2 to get the maximum power transfer to this resistor.
- (3 points) Find the maximum power transfer to the resistor R .



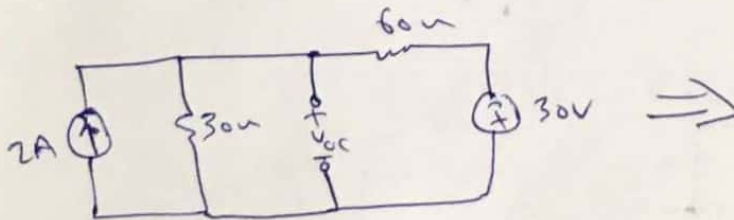
a) $R = R_{th}$ (1)



$$R_{th} = 30 \parallel 60 = 20\Omega$$

$R = 20\Omega$ (1)

b) $P_{max} = \frac{V_{th}^2}{4R_{th}}$ (1)



nodal:

$$\frac{V_{oc}}{30} - 2 + \frac{V_{oc} + 30}{60} = 0$$

$$\frac{V_{oc}}{20} = 2 + \frac{1}{2} \Rightarrow V_{oc} = 50V$$
 (1)

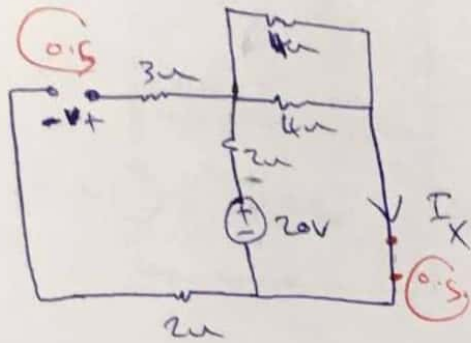
$$P_{max} = \frac{(50)^2}{80} = 31.25W$$
 (0.5)

Problem 3: (5 points)

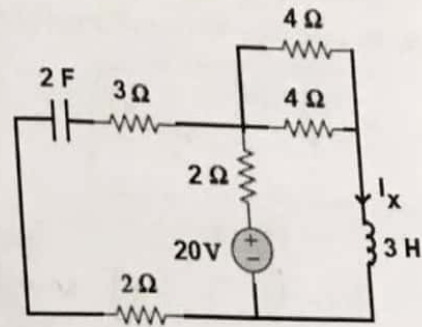
Consider the given circuit and answer the following questions:

- a) (2.5 points) find the current I_x .
- b) (2.5 points) find the stored energy in the capacitor.

a)



$$I_x = \frac{20}{4} = 5A$$



$$b) W_c = \frac{1}{2} C V^2 \quad (1)$$

$$\text{KVL: } -V + 2(-I_x) + 20 = 0$$

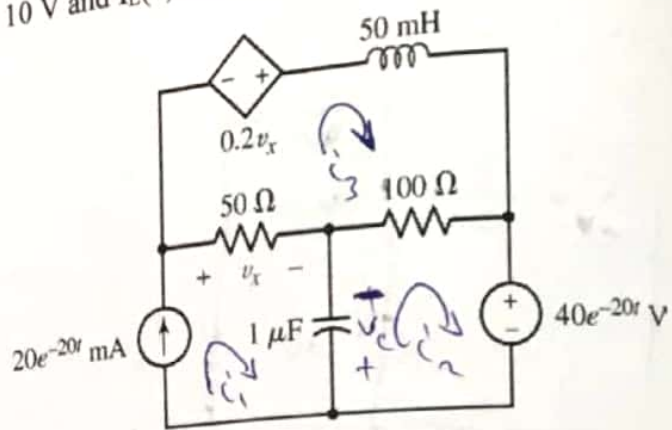
$$V = -10V \quad (1)$$

$$W_c = 100 \text{ J} \quad (0.5)$$

Problem 4: (5 points)
 Write a complete set of mesh equations if $v_c(0) = 10 \text{ V}$ and $i_L(0) = 5 \text{ A}$. (don't solve the equations)

$$i_1 = 20e^{-20t} \text{ mA} \quad (0.5)$$

$$v_x = 50(i_1 - i_3) \text{ V} \quad (0.5)$$



KVL at mesh (2)

$$\frac{1}{C} \int_0^t (i_2(t) - i_1(t)) dt + 100(i_2 - i_3) + 40e^{-20t} = 0 \quad (1)$$

$$10^6 \int_0^t i_2 dt + 100i_2 - 100i_3 = -40e^{-20t} - 10 + 10^6 \int_0^t 20e^{-20t} dt \quad (2)$$

KVL at mesh (3):

$$-0.2v_x + 50 \times 10^{-3} \frac{di_3}{dt} + 100(i_3 - i_2) + 50(i_3 - i_1) = 0 \quad (3)$$

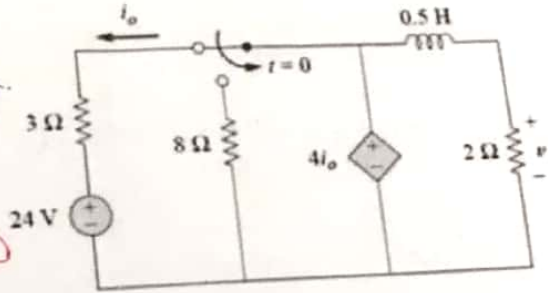
$$-100i_2 + 160i_3 + 0.05 \frac{di_3}{dt} = 50(0.02e^{-20t}) + 10e^{-20t}$$

$$-100i_2 + 160i_3 + 0.05 \frac{di_3}{dt} = 20e^{-20t} \quad (4)$$

Problem 5: (5 points)

Consider the given circuit and answer the following questions:

- Find the type of this circuit for $t \geq 0$.
- Find the time constant.
- Find $v(0^-)$ and $i_o(0^-)$.
- Find $v(0^+)$ and $i_o(0^+)$.
- Find $v(t)$ for all t .



a) source free series RL circuit. (1)

b) $\tau = \frac{L}{R} = \frac{0.5}{2} = 0.25 \text{ s}$ (1)

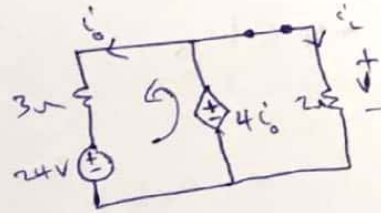
c) KVL:

$$-4i_o + 3i_o + 24 = 0$$

$$i_o(0^-) = 24 \text{ A}, \quad t < 0 \quad (0.5)$$

$$v = 4i_o \Rightarrow v(0^-) = 96 \text{ V} \quad (0.5)$$

$\Rightarrow t < 0$:



d) $i_o(0^+) = 0$ \rightarrow open circuit. (0.5)

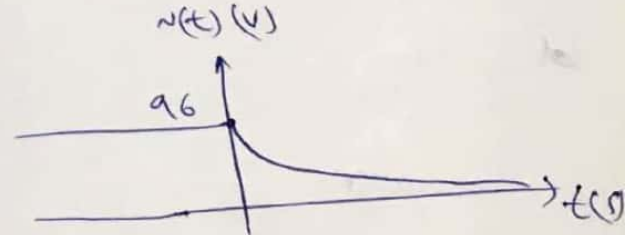
$$v_o(0^+) = 2 i_o(0^+) = 2 i_o(0^-)$$

$$= 2 \frac{96}{2} = 96 \text{ V} \quad (0.5)$$

$\Rightarrow t > 0$:



e)
$$v(t) = \begin{cases} 96, & t < 0 \\ 96 e^{-4t}, & t > 0 \end{cases} \quad (1)$$

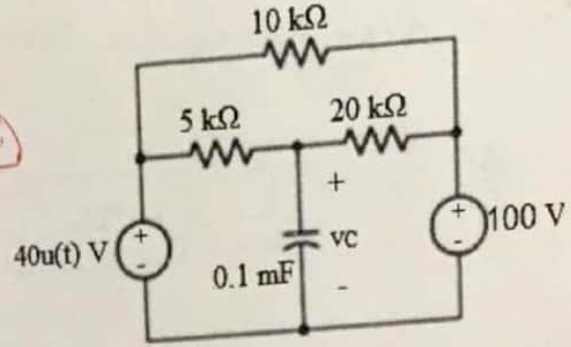


$$v(t) = 48(1 + 48 e^{-4t})$$

Problem 6: (5 points)

- Find the type of this circuit for $t \geq 0$.
- Find the time constant.
- Find $v_c(0)$.
- Find $v_c(t)$ for all t .
- Sketch $v_c(t)$ for all t .

a) Driven RC parallel circuit. (1)



b) $\tau = R_{th} C$

$R_{th} = 5k \parallel 20k = 4k\Omega$ (0.5)

$\tau = 0.4s$ (0.5)

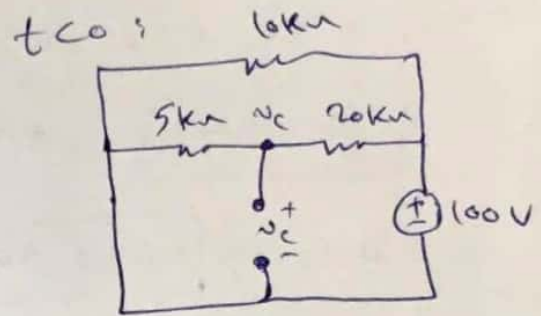
c) $v_c(0^-) = v_c(0^+) = v_c(0^+)$

$v_c(0) = ?$

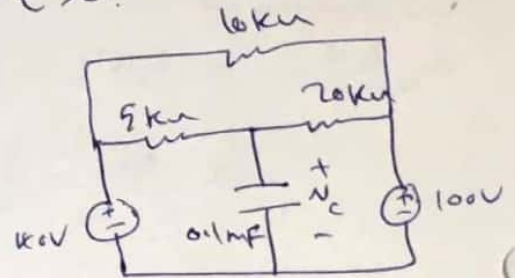
Nodal:

$$\frac{v_c}{5000} + \frac{v_c - 100}{20000} = 0$$

$$\frac{5 v_c}{20000} = \frac{1}{200} \Rightarrow v_c(0) = 20V$$
 (1)



$t > 0^+$:



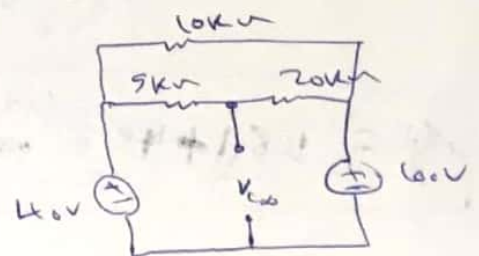
d) $v_{c\infty} =$

Nodal:

$$\frac{v_{c\infty} - 40}{5000} + \frac{v_{c\infty} - 100}{20000} = 0$$

$$\frac{v_{c\infty}}{4} = 8 + 5 \Rightarrow v_{c\infty} = 52V$$

at $t = \infty$



$$v_c(t) = 52 - (52 - 20)e^{-t/0.4}$$

$$v_c(t) = 52 - 32e^{-t/0.4}$$
 (1)

