Probability & Random Variables

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Notebook

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First Exam
Solutions

Q1 IF PDF is given by $f_X(x) = \dfrac{q}{q^2+x^2}$, where q is a (+ve) constant number, find CDF?

Given That: $\displaystyle\int \frac{1}{a^2+b^2u^2}\,du = \frac{1}{ab}\tan^{-1}\left(\frac{bx}{a}\right)$

Solution: CDF is given by $F_X(x) = \displaystyle\int_{-\infty}^{x} f_X(\acute{x})\,d\acute{x}$

$$\Rightarrow F_X(x) = \int_{-\infty}^{x} \frac{q}{q^2+u^2}\,du = \cancel{q}\left(\frac{1}{\cancel{q}}\tan^{-1}\left(\frac{x}{q}\right)\right)\Big|_{-\infty}^{x}$$

$$= \tan^{-1}\left(\frac{x}{q}\right) - \tan^{-1}(-\infty) = \boxed{\frac{\pi}{2} + \tan^{-1}\left(\frac{x}{q}\right)} \ \#.$$

Q2 Given a Box with 10 cards Numbered from 1 → 10 mixed up, if card is drawn & at least it is with number 5 what is the probability of a card to be drawn with number 10 to appear?

Solution:

10 cards

since at least with number 5 ⇒ Assume Like a 6 cards in the box.

$$\Rightarrow P\{\#10\} = \frac{1}{6} = \boxed{0.167} \ \#$$

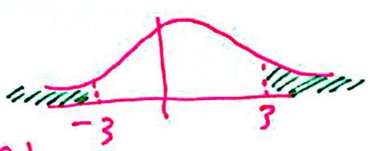
Q3 Given a Gaussian Distribution with the following PDF:

$$f_X(x) = \frac{1}{\sqrt{8\pi}}\, e^{\frac{-(x-1)}{8}}, \quad \text{find } P\{|X|>3\}?$$

Solution:

from $f_X(x)$ you can find that $\sigma_x = 2$ & $a_x = 1$.

$P\{|X|>3\} = 1 - P\{|X|<3\} = 1 - P\{-3<X<3\}$



$= 1 - \left(P\{X<3\} - P\{X<-3\}\right)$

$F_X(3) = F\left(\frac{3-1}{2}\right) = F(1)$

$F_X(-3) = F\left(\frac{-3-1}{2}\right) = F(-2)$

$\hookrightarrow = 1 - F(1) + F(-2)$

$= 1 - 0.8413 + 1 - 0.9773 = \boxed{0.1814} \ \#$

Q4| a Box1 with 4 Red balls & 3 green balls & Box2 with 6 green balls & 8 red balls

if R.V Defined as:

$$X = \begin{cases} 0 & , \text{green ball from box1.} \\ 1 & , \text{red ball from box2.} \\ 2 & , \text{otherwise.} \end{cases}$$

Find $P\{X=2\}$?

Solution: $P\{X=2\} = P\{\text{red ball from B1 OR green ball from B2}\}$

$= P\{\text{red} \cap B1\} + P\{\text{green} \cap B2\}$

$= P(\text{red}/B_1)\,P(B_1) + P(\text{green}/B_2)\,P(B_2)$

$= \frac{4}{7} * \frac{1}{2} + \frac{6}{14} * \frac{1}{2} = \boxed{0.5}$ #

Q5| Unfair Die rolled to appear {2} with probability 40% if it is Rolled 8 times what is the probability of showing the number 2 @ least 7 times?

Solution:

$N=8$, $K \geq 7$ $\Rightarrow P(K \geq 7) = P\{K=7\} + P\{K=8\}$

$= \binom{8}{7} 0.4^7\, 0.6^1 + \binom{8}{8} 0.4^8\, 0.6^0 = \boxed{85.197 * 10^{-4}}$ #

Q6| For a certain experement it found that $P(A) = 0.7$, $P(B/A) = 0.3$, $P(A/B) = 0.4$, find $P(B)$?

Solution:

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B/A)\,P(A)}{P(B)}$$

$$\Rightarrow P(B) = \frac{0.3 * 0.7}{0.4} = \boxed{0.525} \ \#$$

Q7| A box contain 5 red balls & 8 green balls, if a two ball drawn without replacement. What is the probability of drawing the first ball red given that the second is red?

Solution:

Let: 1^{st} red $\equiv A$, 2^{nd} red $\equiv B$

first was red $\equiv A_1$, first was green $\equiv A_2$

red	green
5	8

Total = 13

$$P(A|B) = \frac{P(B/A)\,P(A)}{P(B)} \quad ; \; P(B/A) = \frac{4}{12} \quad ; \; P(A) = \frac{5}{13}$$

$$P(B) = P(B|A_1)P(A_1) + P(B|A_2)P(A_2) = \frac{4}{12}\cdot\frac{5}{13} + \frac{5}{12}\cdot\frac{8}{13} = \frac{5}{13}$$

$$P(A|B) = \frac{\frac{4}{12}\cdot\frac{5}{13}}{\frac{5}{13}} = \frac{4}{12} = \boxed{0.333} \; \#$$

Q8| Given $f_X(x) = 0.4\,\delta(x-2) + a\,\delta(x-4) + b\,\delta(x-6) + 0.1\,\delta(x-8)$

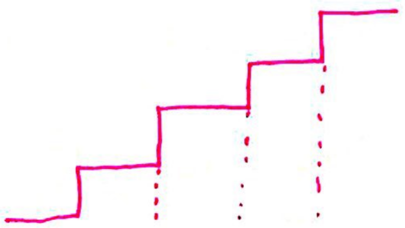
The CDF was:

$$F_X(x) = \begin{cases} c, & x < 2 \\ d, & 2 \leq x < 4 \\ e, & 4 \leq x < 6 \\ f, & 6 \leq x < 8 \\ g, & x > 8 \end{cases}$$

Find the value of (e) given $a = 0.2$?

solution:

$g = 1$, $c = 0$, $d = 0.4$, $f = 0.9$

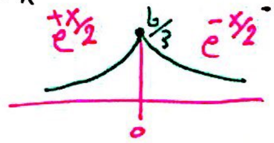


$\Rightarrow \; 0.4 + a = e$

$e = 0.2 + 0.4$

$\Rightarrow \boxed{e = 0.6} \; \#$

Q9| Given $f_X(x) = \frac{b}{3} e^{-|x|/2}$, find b such that $f_X(x)$ is PDF?



solution:

$$\int_{-\infty}^{\infty} f_X(x)\,dx = 1 = 2\int_0^{\infty} \frac{b}{3} e^{-x/2}\,dx$$

$$\frac{2b}{3} \left. \frac{e^{-x/2}}{\frac{-1}{2}} \right|_0^{\infty} = \frac{-4b}{3}(0-1) = 1 \Rightarrow \boxed{b = \frac{3}{4}} \; \#$$

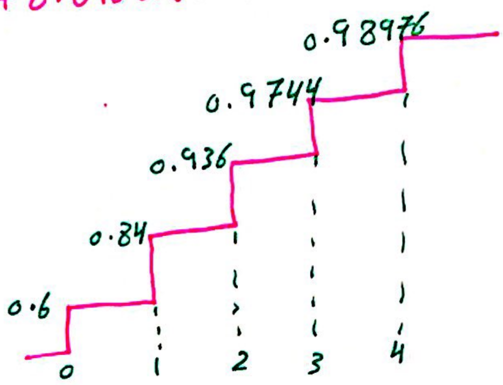
Q10 Given $f_X(x) = \sum_{K=0}^{\infty} (0.6)(0.4)^K \delta(x-K)$

Find $P\{1 \le X \le 4 / X < 3\}$?

Solution:

$$F_X(x) = \sum_{K=0}^{\infty} (0.6)(0.4)^K u(x-K)$$

$$= 0.6\,u(x-0) + 0.24\,u(x-1) + 0.096\,u(x-2) + 0.0384\,u(x-3) + 0.01536\,u(x-4)$$



$$P\{1 \le X \le 4 / X<3\} = \frac{P\{1 \le X \le 4 \cap X<3\}}{P\{X<3\}}$$

$$= \frac{P\{1 \le X < 3\}}{P\{X<3\}}$$

$$= \frac{F_X(3) - F_X(1) + P\{X=1\}}{P\{X<3\}} = \frac{0.936 - 0.84 + 0.24}{0.936} = \boxed{0.359}$$

#

Q11 Given An expirement with exponential R.V with $X \sim exp(1,3)$ Then find $P\{0 < X < 5\}$?

Solution: $f_X(x) = \begin{cases} \frac{1}{b} e^{-\frac{(x-a)}{b}} & , x \ge a \\ 0 & , x < a \end{cases} = \begin{cases} \frac{1}{3} e^{-\frac{(x-1)}{3}} & , x \ge 1 \\ 0 & , x < 1 \end{cases}$

$$P\{0 < X < 5\} = \int_1^5 \frac{1}{3} e^{-\frac{(x-1)}{3}} dx = \cancel{\tfrac{1}{3}} e^{\frac{1}{3}} \left. \frac{e^{-x/3}}{\cancel{-\frac{1}{3}}} \right|_1^5 = e^{\frac{1}{3}}\left(e^{-\frac{1}{3}} - e^{-5/3}\right)$$

$$= \boxed{0.7364}$$ #

Q12 Given a sample space $S = \{1,2,3,4,5,6\}$ if An event $A = \{1,3,4\}$, which event of the following is independent with A :

a) $\{4\}$ b) $\{1,2\}$ c) $\{1,2,4,5\}$ . . . .

Solution: for part c :

it must achieve this $P\{event\} * P\{A\} = P\{event \cap A\}$

$P\{\{1,2,4,5\}\} = \frac{4}{6}$

$P\{A\} = \frac{3}{6}$

$P\{even \cap A\} = P\{1,4\} = \frac{2}{6} = \frac{1}{3}$ ✓

$P\{event \cap A\} = \frac{3}{6} * \frac{4}{6} = \frac{1}{3}$

Answer $\boxed{c}$ #

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