PROBABILITY

Ch.1 Summary

POWER UNIT

ELECTRICAL ENGINEERING

Chapter (1)

(*) prob → exp. → نتائج التجربة → The outcomes of the experiement (sample space «S»)

- → Event
- → Prob. of the event $\{P(A)\}$

Notes: 1) $P(A) \equiv$ Prob. of the event (A)

2) event : any outcome of the experiement

3) Prob. of the event : a non (−ve) number depends on the definition of the event.

(*) We have (3) facts of probability :

1) $0 < P(A) < 1$

2) $P(S) = 1$

3) $\sum_{i=1}^{N} P(A_i) = 1$; Mutually exclusive events

→ * إذا كان نسبة النجاح (0.7) فإنّ نسبة الرسوب هي (0.3)

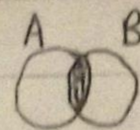
(*) Operations on the (set) : collection of objects

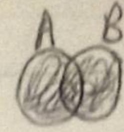
let (S) ...

1) $A \cup \bar{A} = S$

2) $A \cap \bar{A} = \Phi$ «empty set»

- $\cap \rightarrow$ intersection (AND)
- $\cup \rightarrow$ union (OR)
- $\bar{A} \rightarrow$ complement of A

$A \cap B \longrightarrow$  $\rightarrow$ Intersection

$A \cup B \longrightarrow$  $\rightarrow$ Union

⊛ Joint Probability:

1) $\text{Prob}(A \cup B) = P(A) + P(B) - P(A \cap B)$

2) $\text{Prob}(A \cap B) = P(A) + P(B) - P(A \cup B)$

3) If (A) & (B) are **mutually exclusive** then:

$$P(A \cap B) = 0$$

4) If (A) & (B) are **IND** events then:

$$P(A \cap B) = P(A) \odot P(B)$$ → ضرب

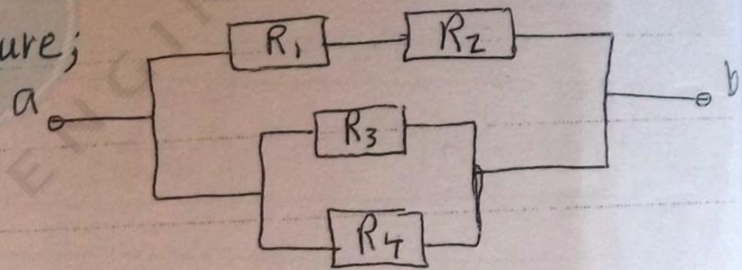
Note:
1) One of them → OR → $\cup$ → توالي
2) both of them → AND → $\cap$ → توازي

Ex: Group of students, if P(pass in math) = 0.6 and P(pass in chemistry) = 0.5, and P(pass in both of them) = 0.4. If we choose a student redondly, find the prob. that he pass <u>one of them</u> ??

solution $P(M \cup C) = \underbrace{P(M)}_{\text{Math}} + \underbrace{P(C)}_{\text{Chemistry}} - \underbrace{P(M \cap C)}_{\text{both}}$

$\rightarrow P(M \cup C) = 0.6 + 0.5 - 0.4 \Rightarrow \therefore \boxed{P(M \cup C) = 0.7}$

Ex: Given the shown figure;



if $P(R_1 \text{ fails}) = 0.01$

$P(R_2 \text{ fails}) = 0.03$

$P(R_3 \text{ fails}) = 0.02$

$P(R_4 \text{ fails}) = 0.01$

if all repeates fail <u>independently</u>

then, find the probability that the signal **will not arrive point (b)** ??

solution will not arrive point (b) means ;

$\rightarrow$ path (1) fail <u>and</u> path (2) fail

$\rightarrow$ (R_1 fail <u>or</u> R_2) <u>and</u> (R_3 <u>and</u> R_4)

$P(R_1 \cup R_2) \cap P(R_3 \cap R_4)$

but... since they are IND events then:

$$P(R_1 \cup R_2) \cap P(R_3 \cap R_4) =$$

$$\left[P(R_1) + P(R_2) - P(R_1 \cap R_2)\right] \cdot \left[P(R_3) \cdot P(R_4)\right]$$

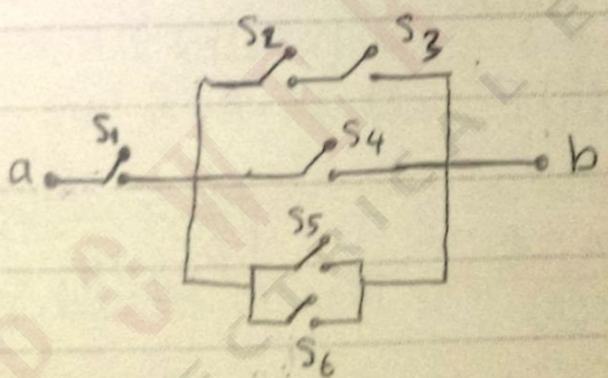
$$= \left[P(R_1) + P(R_2) - \{P(R_1) \cdot P(R_2)\}\right] \cdot \left[P(R_3) \cdot P(R_4)\right]$$

$$= \{0.01 + 0.03 - (0.01)(0.03)\} \cdot \{(0.02)(0.01)\}$$

→ $\boxed{P(\text{not arrive}) = K}$ ※

→ $\boxed{P(\text{arrive}) = 1 - K}$ ※

<u>Ex:</u>



given $P(\underset{open}{S_i}) = 0.01$

where $i = 1, 2, \ldots, 6$

if all switches operate independently, find prob (not arrive point b) ??

※ Conditional Probability:

$$\boxed{P(A|B) = \frac{P(A \cap B)}{P(B)} \; ; \; P(B) \neq 0}$$

Note: $P(A|B)$ —تقرأ→ probability of (A) given (B) (علماً بأن)

↳ (B) certanity event {حدث مؤكد}

Ex:

	B_1	B_2	B_3
R	2	4	6
G	3	5	7

1) $P(R|B_1) = \frac{2}{5}$ ⟹ احتمال سحب كرة حمراء من Box (1)

2) $P(G|B_3) = \frac{7}{13}$ ⟹ احتمال سحب كرة خضراء من Box (3)

Ex (1.4.10): If $P(A)_{failed} = 0.03$, $P(B)_{failed} = 0.01$

$P(B|A)_{failed} = 0.06$, then find:

1) $P(A \cap B)_{failed}$ 2) $P(A|B)$ 3) Is A, B are IND ??

Solution

1) $P(B|A) = \frac{P(A \cap B)}{P(A)} \Longrightarrow \therefore \boxed{P(A \cap B) = 18 \times 10^{-4}}$

2) $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{18 \times 10^{-4}}{1 \times 10^{-2}} \Longrightarrow \therefore \boxed{P(A|B) = 18 \times 10^{-2}}$

3) to find if A & B are IND then:

$P(A) \cdot P(B) \overset{??}{=} P(A \cap B)$

$3 \times 10^{-4} \neq 18 \times 10^{-4}$

$\Longrightarrow \therefore$ **Not IND**

(*) Bay's Theorem:

	B_1	B_2	B_3
R	2	4	6
G	3	5	7

1) $P(R \mid B_2) = \frac{4}{9}$ → conditional
(حدث مؤكد → B_2)

2) $$P(B_2 \mid R) = \frac{P(R \mid B_2) \cdot P(B_2)}{P(R)}$$ → Bay's
(حدث مؤكد → R) ; $P(R)$ → Total Probability

** Note: to find total probability $P(R)$ then:

→ $P(R) = P(R \mid B_1)\,P(B_1) + P(R \mid B_2)\,P(B_2) + P(R \mid B_3)\,P(B_3)$

→ $P(R) = \frac{2}{5} * \frac{1}{3} + \frac{4}{9} * \frac{1}{3} + \frac{6}{13} * \frac{1}{3} \implies P(R) = K$

$\implies \therefore P(B_2 \mid R) = \dfrac{\frac{4}{9} * \frac{1}{3}}{K}$

ملاحظة: ١) إذا لم يؤثر الحدث المؤكد على باقي الأحداث ← conditional

٢) إذا أثر الحدث المؤكد على باقي الأحداث ← Bay's

٣) إذا ذكر في السؤال {likely hood أو equally likely أو same prob.}

فإن: احتمال B_1 = احتمال B_2 = احتمال B_3

→ $P(B_1) = P(B_2) = P(B_3) = \frac{1}{3}$ ← لأنهم (٣) صناديق

٤) إذا لم يذكر في السؤال أحد الكلمات السابقة. يجب أن تعطى في السؤال.

٥) إذا كان الحدث المؤكد هو:
- المرسل ← conditional
- المستقبل ← Bay's

3) $P(B_2|G) \longrightarrow$ Bay's

4) $P(R|B_3) \longrightarrow$ Conditional

} $\Longrightarrow$ Tolerance : Gold = ∓5% ; Silver = ∓10%

5) $P(G) = ??$ $\longrightarrow$ $\boxed{P(G) = 1 - P(R)}$ OR <u>another way</u>:

$$P(G) = P(G|B_1)\,P(B_1) + P(G|B_2)\,P(B_2) + P(G|B_3)\,P(B_3)$$

Ex: Given:

	5%	10%	Total
10 Ω	2	4	6
20 Ω	3	5	8
30 Ω	1	6	7

R = 10
R = 20
R = 30

If they are equally likely, find: → $P(5\%|20\Omega)$ → $P(30\Omega|10\%)$

(solution) 1) $P(5\%|20\Omega) = \frac{3}{8}$ → conditional

2) $P(30\Omega|10\%)$ → Bay's

$$\Longrightarrow P(30\Omega|10\%) = \frac{P(10\%|30\Omega) \cdot P(30)}{P(10\%)}$$

but ... $P(30) = \frac{1}{3}$ ← equally likely

now, $P(10\%) = P(10\%|10\Omega)\,P(10) + P(10\%|20)\,P(20) + P(10\%|30\Omega)\,P(30\Omega)$

$$\Rightarrow P(10\%) = \frac{4}{6} * \frac{1}{3} + \frac{5}{8} * \frac{1}{3} + \frac{6}{7} * \frac{1}{3} = K$$

$$\Longrightarrow \therefore \boxed{P(30\Omega|10\%) = \frac{\frac{6}{7} * \frac{1}{3}}{K}}$$

Ex (1.4.5):

	B_1	B_2	B_3	Total
0.01	20	95	25	140
0.1	55	35	75	165
1	70	80	145	295
Total	145	210	245	600

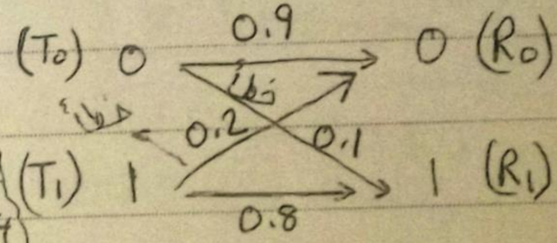
If $\{B_1, B_2, B_3\}$ equally likely find → $P(0.01 | B_1)$, $P(B_2 | 0.1)$??

solution 1) $P(0.01 | B_1) = \frac{20}{145}$ → conditional

Bay's → 2) $P(B_2 | 0.1) = \frac{P(0.1 | B_2) \cdot P(B_2)}{P(0.1)} = \frac{\frac{35}{210} * \frac{1}{3}}{K}$

where; $P(0.1) = K = \left\{ \frac{55}{145} * \frac{1}{3} + \frac{35}{210} * \frac{1}{3} + \frac{75}{245} * \frac{1}{3} \right\}$

Ex: Binary Channel ⟹



if $P(T_0) = 0.6$ & $P(T_1) = 0.4$

1) $P(R_0) = 0.9 * 0.6 + 0.2 * 0.4 \implies \boxed{P(R_0) = 0.62}$

2) $P(R_1) = 1 - 0.62 \implies \boxed{P(R_1) = 0.38}$

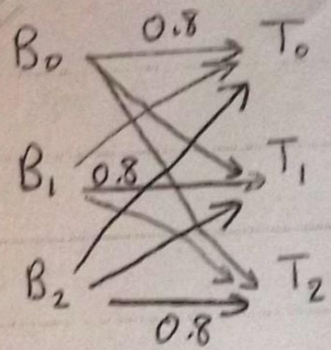
3) $P(R_0 | T_1) = 0.2$ → conditional

4) $P(R_1|T_0) = 0.1$ → conditional

5) $P(T_0|R_1) = \dfrac{P(R_1|T_0) \cdot P(T_0)}{P(R_1)} = \dfrac{0.1 * 0.6}{0.38}$

Ex (1.4.11): {Terrnary Channel} → يرسل (٣)
ويستقبل (٣)

If $P(A_i|B_j) = \begin{cases} 0.8 , & i=j \\ 0.1 , & i \neq j \end{cases}$ and $\begin{cases} P(B_0) = 0.5 \\ P(B_1) = 0.3 \\ P(B_2) = 0.2 \end{cases}$



then find:

1) $P(T_0) = 0.8*0.5 + 0.1*0.3 + 0.1*0.2$

→ $\boxed{P(T_0) = 0.45}$

2) $P(T_1) = 0.8*0.3 + 0.1*0.5 + 0.1*0.2$ → $\boxed{P(T_1) = 0.31}$

3) $P(T_2) = 0.8*0.2 + 0.1*0.3 + 0.1*0.5$ ⟹ $\boxed{P(T_2) = 0.24}$

{أو حل آخر} → $P(T_2) = 1 - (0.45 + 0.31) = 0.24$ #

4) $\boxed{P(T_1|B_1) = 0.8}$ #

5) $P(B_0|T_2) = \dfrac{P(T_2|B_0) \cdot P(B_0)}{P(T_2)} = \dfrac{0.1 * 0.5}{0.24}$ #

④ Bernoulli Trials : (تجربة لها احتمالين فقط)

we use it when we have an experiement with (2) outcomes

$$\Rightarrow \boxed{P(K) = \binom{N}{K} P^K (1-P)^{N-K}}$$ where :

N ≡ number of trials , K ≡

⇒ for large (N) " N > 70 " & small (P) then :

$$\Rightarrow \boxed{P(K) = \binom{N}{K} P^K (1-P)^{N-K} \approx \frac{(NP)^K e^{-NP}}{K!}}$$

Note} $\binom{N}{K} = \frac{N!}{K!(N-K)!}$ → In Calculater use $\boxed{nCR}$ مفتاح

Ex: A basketball player will shoot ⑩ (→ N) balls, given that P(hit the ring) = (0.7) (→ P), find the probability that he will hit the ring ⑤ (→ K) times ??

$$P(K=5) = \binom{10}{5} (0.7)^5 (0.3)^5 = 0.1029$$ ※

Note} P(he hit the ring (2) times at the most) = {0, 1, 2}

P(hit the ring (7) times at least) = {7, 8, 9, 10}

Ex (1.7.4): If $N=5$, $P(\text{arrive late})=0.4$ find:

a) for $K=3,4,5 \rightarrow \binom{5}{3}(0.4)^3(0.6)^2 + \binom{5}{4}(0.4)^4(0.6)^1$

$\dots \binom{5}{5}(0.4)^5(0.6)^0 = \boxed{0.184}$

b) $K=0 \Rightarrow \binom{5}{0}(0.4)^0(0.6)^5 \longrightarrow (0)$ غاب

$\binom{5}{5}(0.6)^5(0.4)^0 \longrightarrow$ (5) حضر أو حل آخر

Ex (1.7.12): If $N=250$, $P(\text{not-return})=0.01$

& not more (3) $\equiv \{K=0,1,2,3\}$

$$\Longrightarrow e^{-250(0.01)}\left[\frac{250(0.01)^0}{0!}+\frac{250(0.01)^1}{1!}+\frac{250(0.01)^2}{2!}+\frac{250(0.01)^3}{3!}\right]$$