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The University of Jordan

School of Engineering

Department of Electrical Engineering

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Probability and Random Variables, EE321, Second Exam

شعبة ②

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Q1. a) Find $F_Y(y)$ and $f_Y(y)$ for $Y = -4X + 3$ and $f_X(x) = 2e^{-2x}u(x)$.

b) What is $\bar{X}$, and $\bar{Y}$?

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$$E[x] = \bar{x} = \int_{-\infty}^{\infty} x f_x(x) dx$$

$$= \int_{-\infty}^{\infty} x \, 2e^{-2x} u(x) dx$$

$$= \int_{0}^{\infty} x \, 2e^{-2x} dx$$

$$= 2 \int_{0}^{\infty} x e^{-2x} dx$$

$$\rightarrow u = x \quad dv = e^{-2x} dx$$

$$du = dx \quad v = \frac{e^{-2x}}{-2}$$

$$uv - \int v du$$

$$= 2\left(x \frac{e^{-2x}}{-2} \Big|_0^\infty + \int_0^\infty \frac{e^{-2x}}{+2} dx \right)$$

$$= 2\left(\frac{-1}{2}(0-0) + \frac{e^{-2x}}{-4} \Big|_0^\infty \right)$$

$$2\left(\frac{1}{4} - \frac{1}{-4} \right) = 2 \times \frac{1}{4} = 0.5$$

$$\bar{x} = 0.5$$

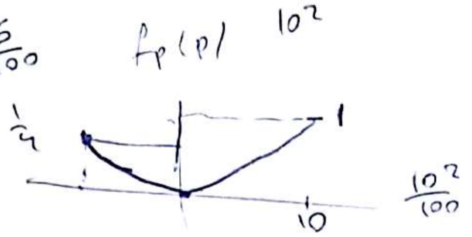
$$\bar{Y} = E[-4x+3]$$

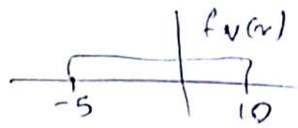
$$= -4E[x] + 3$$

$$= -4 \times 0.5 + 3$$

$$= -2 + 3 = 1$$

$\frac{(-5)^2}{100} = \frac{25}{100}$





Q2. The voltage across a resistor R= 100 Ω is a uniform random variable **V** between -5 and 10 volts. Determine the density function of the power $f_P(p)$ and find its average value $\bar{P}$.

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$$\bar{P} = \frac{V^2}{R} = \frac{V^2}{100}$$

$$f_v(v) = \begin{cases} \frac{1}{15}, & -5 < v < 10 \\ 0, & \text{elsewhere} \end{cases}$$

$$f_p(p) = \int_{-\infty}^{\infty} p(v)\, f_v(v)\, dv$$

???

$$= \int_{-5}^{10} \frac{v^2}{100} \cdot \frac{1}{15}\, dv$$

$$= \frac{1}{1500}\left(\frac{v^3}{3}\Big|_{-5}^{10} \right)$$

$$= \frac{1}{1500}\left(\frac{1000}{3} - \frac{-125}{3} \right)$$

$$= \frac{1}{4}$$

$$f_p(p) = \begin{cases} \frac{1}{4}, & 0 < p < 1 \\ 0, & \text{elsewhere} \end{cases}$$

$$\bar{P} = \int_{-\infty}^{\infty} p\, f_p(p)\, dp$$

$$= \int_0^1 p\, \frac{1}{4}\, dp$$

$$\frac{1}{4}\, \frac{p^2}{2}\Big|_0^1 = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$$

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$$w\left(ja - \frac{wb}{4}\right)$$

$$e^{\cancel{w^2}\left(\cancel{\frac{ja}{w}}\right.}$$

Q3. A random variable X, has the following characteristic function:

$$\emptyset_X(\omega) = \exp(j\omega a - \frac{\omega^2 b}{4})$$

$$e^{\left(jwa - \frac{w^2 b}{4}\right)}$$

Find the mean and the variance of the random variable.

$$e^{jwa} \cdot e^{-\frac{w^2 b}{4}}$$

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$$m_1 = (-j) \left.\frac{d\phi_x(w)}{dw}\right|_{w=0}$$

$$\left.\frac{d\phi_x(w)}{dw}\right|_{w=0} = ja\, e^{jwa} \cdot e^{-\frac{w^2 b}{4}} + e^{jwa}\left(-\frac{2wb}{4}\, e^{-w^2 b/4}\right)\Bigg|_{w=0}$$

zero

$$= ja \times -j = a$$

$$(-j)^2 = -1$$

$$\cancel{\boxed{mean = m_1 = a}}$$

$$(-j)^2 \left.\frac{d^2\phi_x(w)}{dw^2}\right|_{w=0} = (ja)^2 e^{jwa} \cdot e^{-w^2 b/4} + ja\, e^{jwa}\left(\cancel{-\frac{2wb}{4}}\right) e^{-\frac{w^2 b}{4}}$$

zero

$$+ \cancel{ja}\, e^{jwa}\left(-\frac{2b}{4}\, e^{-w^2 b/4} + \left(\cancel{-\frac{2wb}{4}}\right)^2 e^{-w^2 b/4}\right)$$

zero

$$+ ja\, e^{jwa}\left(\cancel{-2wb}\, e^{-w^2 b/4}\right)\Bigg|_{w=0}$$

zero

$$= \left((ja)^2 - \frac{2b}{4}\right) \times (-j)^2$$

$$= \left(j^2 a^2 - \frac{2b}{4}\right) = \left(-a^2 - \frac{2b}{4}\right)^{x-1} = \cancel{-a}$$

$$m_2 = a^2 + \frac{2b}{4}$$

$$\sigma_x^2 = m_2 - m_1^2 = \cancel{+a^2} + \frac{2b}{4} \cancel{-a^2} = \cancel{-2a^2} \cancel{-\frac{2b}{4}}$$

$$\cancel{\sigma_x^2 = a^2 + \frac{2b}{4} - a^2 = \frac{2b}{4}}$$

$$e^{\left(jwa - \frac{w^2 b}{4}\right)} = \frac{e^{\left(jwa - \frac{w^2 b}{4}\right)}}{\cancel{\left(ja - \frac{2wb}{4}\right)}}$$

$$\left(ja\cancel{z} - \frac{2wb}{4}\right) e^{jwa - \frac{w^2 b}{4}}$$

$$\left(ja - \frac{2wb}{4}\right) e^{\left(jwa - \frac{w^2 b}{4}\right)}$$

$$ja\left(e^{jwa - \frac{w^2 b}{4}}\right) \quad -2wb\, e^{\left(jwa - \frac{w^2 b}{4}\right)}$$

$$\cancel{(ja)}$$

$$ja\left(ja - \frac{2wb}{4}\right) \quad -\frac{2b}{4}\, e^{jwa - \frac{w^2 b}{4}}$$

$$(ja)^2$$

$F_x(1.77) = 0.9616$

$F_x(0.67) = 0.7486$ $\quad$ $F_x(1.79) = 0.9633$

$\sigma_x = 0.89 \simeq 0.9$

Q4. A Gaussian random variable, for which $\bar{X} = 0.6$ and $\sigma_X^2 = 0.8$, is transformed to a new random variable by the transformation :

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$$Y = T(X) = \begin{cases} 4 & 1.0 \le X < \infty \\ 2 & 0 \le X < 1.0 \\ -2 & -1 \le X < 0 \\ -4 & -\infty < X < -1 \end{cases}$$

a) Find the density function of Y.

b) Find the mean and variance of Y.

① $P\{Y=-4\} = P\{-\infty < x < -1\} = F_x(-1) = F_x\left(\frac{-1-0.6}{0.89}\right) = F_x(-1.79)$

$= 1 - F_x(1.79)$

$= 1 - 0.9633 = \cancel{0.0384}\ 0.0367$

② $P\{Y=-2\} = P\{-1 < x < 0\} = F_x(0) - F_x(-1) = F_x\left(\frac{0-0.6}{0.89}\right) - F_x\left(\frac{-1-0.6}{0.89}\right)$ $\quad -0.67$ $\quad \cancel{-0.666}$

$= (1 - F_x(0.67)) - 0.0367$

$= 1 - 0.7486 - 0.0367 = \underbrace{0.2514}_{F_x(0)} - 0.0367 = 0.2147$

③ $P\{Y=2\} = F_x(1) - F_x(0) = F_x\left(\frac{1-0.6}{0.89}\right) - F_x\left(\frac{0-0.6}{0.89}\right)$ $\quad 0.44$

$= 0.67 - 0.2514 = 0.4186$

④ $P\{Y=4\} = 1 - F_x(1) = 1 - 0.67 = 0.33$

$f_Y(y) = 0.0367\,\delta(y+4) + 0.2147\,\delta(y+2) + 0.4186\,\delta(y-2)$
$+ 0.33\,\delta(y-4)$

$\bar{Y} = 0.0367(-4) + 0.2147 \times (-2) + 0.4186(2) + 0.33(4)$
$= 1.581$

$\overline{Y^2} = 0.0367(-4)^2 + 0.2147(-2)^2 + 0.4186(2)^2 + 0.33(4)^2$
$= 8.4$

$\sigma_y^2 = \overline{Y^2} - \bar{Y}^2 = 8.4 - (1.581)^2 = 5.9$

Q5. Two random variables X and Y have the joint probability density function

$$f_{XY}(x,y) = \begin{cases} Cx^2y & 0 < y < x < 2 \\ 0 & elsewhere \end{cases}$$

$c\int_0^x \int_y^2 x^2y \, dx\,dy$

a) Find the constant C so that this is a valid joint density function.

b) Find the marginal density functions of X and Y, that is $f_X(x)$ and $f_Y(y)$.

a) let $c=1$ Not Accepted

c can be represented by variable y → $\frac{x^3}{3}\Big|_y^2$

$c = \dfrac{1}{\frac{32}{10} - \frac{y^5}{10}}$ but it's a constant $\propto$

$\boxed{\frac{10}{32} < C < \infty}$ depending on y

b) $f_X(x) = \int_0^x x^2y \, dy$

$f_X(x) = x^2\left(\frac{y^2}{2}\Big|_0^x\right) = x^2 \frac{x^2}{2} = \frac{x^4}{2}$, $0 < x < 2$

$f_Y(y) = \int_y^2 x^2y \, dx$

$= y\left(\frac{x^3}{3}\Big|_y^2\right) = y\left(\frac{8}{3} - \frac{y^3}{3}\right)$

$f_Y(y) = \frac{8}{3}y - \frac{y^4}{3}$, $0 < y < 2$

other page →

$$c \int_{\cancel{y}}^{\cancel{2}} \int_{0}^{x} x^2 y \, dy \, dx$$

$$0$$

$$x^2 \left. \frac{y^2}{2} \right|_{0}^{x}$$

C = constant

$$c \int_{y}^{2} x^2 \frac{x^2}{2} \, dx$$

$$\int_{y}^{2} \frac{x^4}{2} \, dx$$

$$c \left. \frac{x^5}{10} \right|_{y}^{2}$$

$$\left(\frac{32}{10} - \frac{y^5}{10} \right) = 1$$

$$\frac{1}{\frac{32}{10} - \frac{y^5}{10}} \quad \alpha$$

Table B-1 Values of $F(x)$ for $0 \le x \le 3.89$ in steps of 0.01

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998	.9998
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998
3.6	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
3.7	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
3.8	.9999	.9999	.9999	.9999	.9999	.9999	.9999	1.0000	1.0000	1.0000