

$$\frac{32}{40}$$

$$\frac{16}{20}$$

The University of Jordan

School of Engineering

Department of Electrical Engineering

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Probability and Random Variables, EE321

Name: عمر عبدالرحمن أحمد عبد ربه



شعبة ٥

Student Number: 0144364

Q1. The sample space for an experiment is  $S = \{-3 \leq s \leq 4\}$ . List all possible values of the following random variables:

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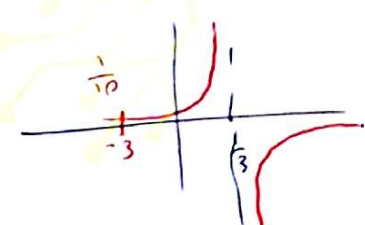
- a)  $X = 2s$
- b)  $X = 2s^2 - 1$
- c)  $X = \cos(\pi s)$
- d)  $X = (1 - 3s)^{-1}$

$a) X_1 = \{-6 \leq X \leq 8\}$   
 $b) X_2 = \{-1 \leq X \leq 31\}$   
 $c) X_3 = \{1 \leq X \leq 13\}$   
 $d) X_4 = \{-\frac{1}{11} \leq X \leq 1\} \cup \{\frac{1}{10} \leq X < \infty\}$

$$\begin{aligned} -3 \times 2 &= -6 \\ 4 \times 2 &= 8 \\ 0 &\Rightarrow 2(0)^2 - 1 = -1 \\ 4^2 - 1 &= 15 \\ 8^2 - 1 &= 63 \\ 4^2 &= 16 \times 2 = 32 \end{aligned}$$

solution

d)  $X = \frac{1}{1-3s}$



$$S_X = \{-\infty < X \leq -\frac{1}{11}\} \cup \{\frac{1}{10} \leq X < \infty\}$$

$(1-0)^{-1} = 1$   
 $(1-12)^{-1} = -\frac{1}{11}$   
 $(1+9)^{-1} = \frac{1}{10}$

$$\sigma_x^2 = 400$$

Q2. If  $X$  is  $N(1000, 400)$ ; find,

a) 1)  $P\{X > 1024\}$       2)  $P\{X < 1024 / (X > 961)\}$       3)  $P\{31 < \sqrt{X} < 32\}$

b)  $f_x(x) / \{35 < x < 1025\}$

$$\mu_x = 1000 \quad \sigma_x = 20$$

$$\left\{ \begin{array}{l} F(1.2) = 0.8849 \\ F_x(1.95) = 0.9744 \end{array} \right.$$

$$\begin{aligned} a) 1) P\{X > 1024\} &= 1 - F_x\left(\frac{1024 - 1000}{20}\right) \\ &= 1 - F(1.2) = 1 - 0.8849 = 0.1151 \end{aligned}$$

$$2) P\{X < 1024 / (X > 961)\} = P\{X < 1024 \cap X > 961\}$$

$$= \frac{P\{X < 1024\} - P\{X < 961\}}{P\{X > 961\}}$$

$$= \frac{F_x\left(\frac{1024 - 1000}{20}\right) - F_x\left(\frac{961 - 1000}{20}\right)}{1 - F_x\left(\frac{961 - 1000}{20}\right)} = \frac{F_x(1.2) - F_x(-1.95)}{1 - F_x(-1.95)}$$

$$= \frac{0.8849 - 1 + 0.9744}{1 - 1 + 0.9744} = 0.8818$$

$$3) P\{31 < \sqrt{X} < 32\} = P\{961 < X < 1024\}$$

$$= F_x\left(\frac{1024 - 1000}{20}\right) - F_x\left(\frac{961 - 1000}{20}\right) = F_x(1.2) - (1 - F_x(1.95))$$

$$= 0.8849 - 1 + 0.9744 = 0.8593$$

b  
→

$$b) f_x(x | \{35 < x < 1025\}) = \begin{cases} f_x(x) & , a < x < b \\ \int_a^b f_x(x) = F_x(b) - F_x(a) & \\ 0 & , \text{otherwise} \end{cases}$$

$$f_x(x) = \frac{1}{\sqrt{2\pi} \sigma x^2} e^{-\frac{(x-\mu)}{2\sigma^2}}$$

$$= \frac{1}{\sqrt{2\pi} (400)} e^{-\frac{(x-1000)}{2(400)}} \quad \mu = 1000 \quad \sigma = 20$$

$$= \frac{1}{\sqrt{800\pi}} \cdot e^{-\frac{1000}{800}} \cdot e^{-\frac{x}{400}}$$

$$\int_{35}^{1025} f_x(x) dx = \int_{35}^{1025} \frac{1}{\sqrt{800\pi}} e^{-\frac{1000}{800}} e^{-\frac{x}{400}}$$

$$= \frac{1}{\sqrt{800\pi}} e^{-\frac{5}{4}} \int_{35}^{1025} e^{-x/400}$$

$$= 0.069 \left( \frac{e^{-x/400}}{-\frac{1}{400}} \right)_{35}^{1025} = 27.6 \left( e^{-35/400} - e^{-1025/400} \right)$$

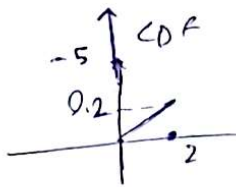
$$= 27.6 (0.839) = 23.1$$

$$f_x(x | 35 < x < 1025) = \begin{cases} \frac{0.07 e^{-\frac{(x-1000)}{800}}}{0.8944} & , 35 < x < 1025 \\ 0 & , \text{otherwise} \end{cases}$$

$$= F_x\left(\frac{1025-1000}{20}\right) - F_x\left(\frac{35-1000}{20}\right)$$

$$= F_x(1.25) - F_x(-48.25)$$

$$= 0.8944 \quad 1 - F_x(48.25)$$



3

0.2

Q3. A random variable X has the following distribution function:

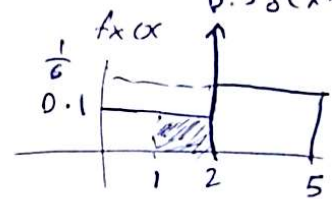
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$$F_X(x) = \begin{cases} 0 & x < 0 \\ 0.1x & 0 \leq x < 2 \\ \frac{1}{6}(x+1) & 2 \leq x < 5 \\ 1 & x \geq 5 \end{cases}$$

0.2

- a) Find  $f_X(x)$ .  
 b) Evaluate  $P\{1 < x < 2\}$  and  $P\{0 < x \leq 2\}$ .

كل وحدة لجال  $\frac{1}{6} \times 3 = \frac{3}{6} = 0.5$   
 $0.3 \delta(x-2)$



a)  $f_X(x) = \frac{dF_X(x)}{dx} = \begin{cases} 0 & x < 0 \\ 0.1 & 0 \leq x < 2 \\ \frac{1}{6} & 2 \leq x < 5 \\ 0 & x \geq 5 \end{cases}$

b)  $P\{1 < x < 2\} = \int_1^2 f_X(x) dx = (2-1)(0.1) = 0.1$

$P\{0 < x \leq 2\} = \int_0^2 f_X(x) dx = (2-0)(0.1) = 0.2$

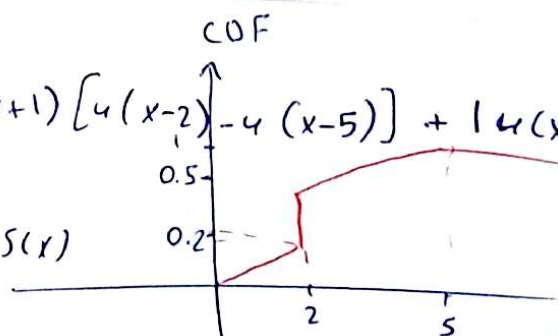
$\downarrow$   $F_X(2) - F_X(0) = \frac{1}{6}(2+1) = 0.5$

$F_X(x) = 0.1 \times [\mu(x) - \mu(x-2)] + \frac{1}{6}(x+1) [\mu(x-2) - \mu(x-5)] + 1 \mu(x-5)$

$F_X(x) = 0.1 [\mu(x) - \mu(x-2)] + 0.1(0) \mu(x)$

$-0.1(2) \delta(x-2) + \frac{1}{6} [\mu(x-2) - \mu(x-5)] + \frac{1}{6}(3) \delta(x-2)$

$-\frac{6}{6} \delta(x-5) + \delta(x-5)$



Q4. We have four boxes. Box 1 contains 2000 components of which 5% are defective. Box 2 contains 500 components of which 40% are defective. Boxes 3 and 4 contain 1000 each with 10% defective. We select at random one of the boxes, and we remove at random a single component.

- a) What is the probability that the selected component is defective?  
 b) If the selected component is defective, what is the probability that it came from Box 2?

	5% Box <sub>1</sub>	40% Box <sub>2</sub>	10% Box <sub>3</sub>	10% Box <sub>4</sub>
G	1900	300	900	900
D	100	200	100	100
total	2000	500	1000	1000

G: Good  
D: Defective

$$a) P(D) = P(D/Box_1)P(Box_1) \oplus P(D/Box_2)P(Box_2) \oplus P(D/Box_3)P(Box_3) \oplus P(D/Box_4)P(Box_4)$$

$$P(Box_1) = P(Box_2) = P(Box_3) = P(Box_4) = 0.25 = \frac{1}{4}$$

$$= \left(\frac{100}{2000}\right)\left(\frac{1}{4}\right) \oplus \left(\frac{200}{500}\right)\left(\frac{1}{4}\right) \oplus \left(\frac{100}{1000}\right)\left(\frac{1}{4}\right) \oplus \left(\frac{100}{1000}\right)\left(\frac{1}{4}\right)$$

$$= \frac{1}{80} + \frac{1}{10} + \frac{1}{40} + \frac{1}{40} = \frac{13}{80}$$

$$b) P(Box_2/D) = \frac{P(D/Box_2)P(Box_2)}{P(D)} = \frac{\left(\frac{200}{500}\right)\left(\frac{1}{4}\right)}{\frac{13}{80}} = \frac{2}{13}$$

Table B-1 Values of  $F(x)$  for  $0 \leq x \leq 3.89$  in steps of 0.01

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998	.9998
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998
3.6	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
3.7	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
3.8	.9999	.9999	.9999	.9999	.9999	.9999	.9999	1.0000	1.0000	1.0000