CH 1 1/10

1.1 Set of Definitions

Aset is acollections of objects (capital letter)
An element is an object in the set (Small letter)

rolling adice $A = \{1, 2, 3, 4, 5, 6\}$

Rull method

$a \in A$ if the element abelogs A
$a \notin A$ if " " adoesn't belong to set A

⇒ To describe aset Tabular Method

element are anumerical explicity

Ex: the set of all integer between 5 and 10

$A = \{6, 7, 8, 9\}$

$A = \{5 < a < 10\}$ → A is uncontable set / in Finit set

Finite set — ends up to alimited number
in Finit set

$A = \{1, 2, 3, 4, 5, 6\}$

* Null set denoted by ϕ has no element $A = \{\ \}$

⇒ if set A has all its elements in set B with no elements of B is in not in A
$A \subseteq B$
$B \subseteq A$
$A = B$
we call A is a subset of B

⇒ if at lest one element of B is not in A
$A \subset B$
we call set A is proper subset of B

e.g ⸬ $A = \{1, 2, 3\}$
$B = \{1, 2, 3, 4\}$
A is proper subset of B

e.g ⸬ $A = \{1, 2, 3\}$
$B = \{1, 2, 3\}$
A is subset of B

⇒ Two sets are disjoint (mutual exclusive) if there is no element common among them

e.g $A = \{2, 4, 6\}$ A and B are disjoint

$B = \{1, 3, 5\}$

Ex 1.1.1 / P.4

$A = \{1, 3, 5, 7\}$, $B = \{1, 2, 3, \dots\}$, $C = \{0.5 < c \le 8.5\}$

$D = \{0.0\}$, $E = \{2, 4, 6, 8, 10, 12, 14\}$, $F = \{-5.0 < f \le 12.5\}$

A) Con, Tabular, Finit
B) inFinit, con, Tabular
C) non con, inFinit, Rule
D) Con, Finit, Tabular
E) Finit, Con, Tabular
F) noncon, inFinit, Rule

$A \subset B$
$D \subset F$
A; D, E are mutually exclusive
D, B are mutually exclusive

⇒ universal set S include every Chois.
$S = A = \{1, 2, 3, 4, 5, 6\}$ Rolling adice experiment

$A = \{2, 4, 6\}$ out come even
$B = \{1, 3, 5\}$ out come odd
$C = \{1, 2\}$ out come is less than 3

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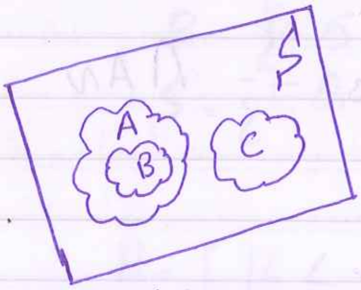
The total # of subset created form $S = 2^n$

Bonus: list all 64 possibilies at describe the experiment to get them?

=> Set operations

* Universal set represented by rectangular.
* all subsets are represented by an area.



* A and C are mutual exclusive
* $B \subset A$

=> The different of two set A and B (A - B)
All element is in A not in B

e.g.: $A = \{0.6 < a \leq 1.6\}$

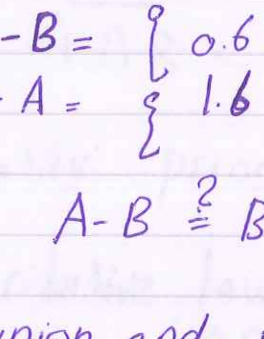
$B = \{1.0 \leq b < 2.5\}$

$A - B = \{0.6 < c < 1.0\}$

$B - A = \{1.6 < c \leq 2.5\}$

$A - B \stackrel{?}{=} B - A$ $\quad A - B \neq B - A$

=> union and intersection



* The union of two sets is given as $A \cup B$, all elements in A and B without repetition

⇒ The intersection of two sets $A \cap B$ represents all common elements

* two sets that are disjoint have their intersetion equals to ϕ.

⇒ For N sets $A, A_2, ----, A_N$

$$\bigcup_{n=1}^{N} A_n = A_1 \cup A_2 \cup A_3 --- \cup A_N$$

$$\bigcap_{n=1}^{N} A_n = A_1 \cap A_2 \cap A_3 --- \cap A_N$$

⇒ Complement of aset A

$\bar{A} = S - A$ $\quad A \cup \bar{A} = S$

$\bar{S} = \phi$ $\quad A \cap \bar{A} = \phi$

$\bar{\phi} = S$

* Algebric laws :

1) Commutative law

$A \cap B = B \cap A$

$A \cup B = B \cup A$

2) distributive law

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

3) Associative law

$A \cup (B \cup C) = (A \cup B) \cup C = A \cup B \cup C$

$A \cap (B \cap C) = (A \cap B) \cap C = A \cap B \cap C$

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4/10

4) De Morgan's Law :-

$$\overline{A \cup B} = \bar{A} \cap \bar{B}$$
$$\overline{A \cap B} = \bar{A} \cup \bar{B}$$

Ex 1.2.2 / P8

$$\overline{A \cap B} = \bar{A} \cup \bar{B}$$

$A = \{2 < a \leq 16\}$
$B = \{5 < b \leq 22\}$
$S = \{2 < s \leq 24\}$

$\bar{A} = \{16 < a \leq 24\}$

$\bar{B} = \{2 < b \leq 5 , 22 < b \leq 24\}$

$\bar{A} \cup \bar{B} = \{2 < c \leq 5 , 16 < c \leq 24\}$ ✓

$A \cap B = C = \{5 < c \leq 16\}$

$\overline{A \cap B} = D = \{2 < d \leq 5 , 16 < d \leq 24\}$ ✓

5) Duality principle :- $\cap$ $\cup$ S ϕ

distributive law

$A \cap (B \cup C)$

Dual $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

4/10

1.3 probability introduced through sets and Relative Frequnancy &c

Freq: 1 2 3 4 5 6 7 8 9 10
H H H T T T H T H H

$\frac{n_H}{N} = \frac{6}{10}$ The probability

عدد المرات

$A = \{1, 2, 3\}$ Trial is the process of conducting an experment.

out come is the out put of the experment

possible outcomes it'm give sample space S

* Fair = unbiased all out comes have same chance to occur.

rolling adie

$\boxed{\text{sub sets}}$

$A = \{\text{evennumber}\}$ $B = \{\text{odd number}\}$

$= \{2, 4, 6\}$ $= \{1, 3, 5\}$

set of probability

$\{\frac{3}{6}, \frac{3}{6}\}$

There would be two sets one set is the possible out comes and other set is the likelihood of the out comes.

→ sample space is the universe set which has all possible out comes

$S = \{ 0 < s \leq 12 \}$

let A = { odd numbe result when spinning the wheel }

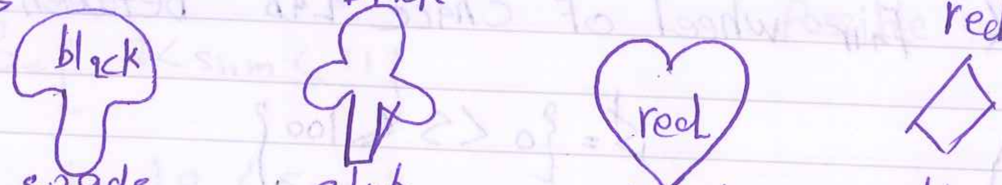
$A = \{ 1, 3, 5, 7, 9, 11 \}$

B = { numbers between 3 and 5 }

$B = \{ 3 < b < 5 \}$

* Cards Deck

S black



red spade

$\frac{1}{4}$ $\frac{13}{52}$

spade club heart diamond

bonus

how many possible events I can create !

N --- 52 $2^N = 4.5 \times 10^{15}$

Certain event 100% $p(S) = 1$

immpossible event 0% $p(\phi) = \phi$

The probability of an event

denoted & $\boxed{p(A)} \geq 0$ +ve $p(\phi) = 0$

$\boxed{P[A]}$

$P\{\cdots\}$

$P[S] = 1$

For N events 4/10

$A_1, A_2, \ldots, A_n$ $n = 1, 2, \ldots, N$

iff $A_i \cap A_j = \text{zero} = \phi$ $i \neq j = 1, 2, \ldots, N$

$$P\left[\bigcup_{n=1}^{N} A_n\right] = \sum_{n=1}^{N} P[A_n]$$

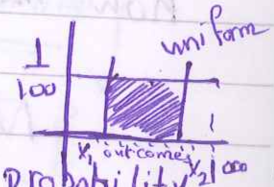
Ex $\frac{1.3.1}{\text{P.11}}$ 6/10/2015

X fair wheel of chace Lab between 0 and 100

$S = \{0 < S \leq 100\}$

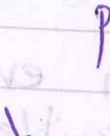
Q probability of an event between $X_1 < X \leq X_2$

$$p(X) = \frac{1}{100}(X_2 - X_1)$$



probability density Function

$$X_n = \frac{n}{N} 100$$



$N \to \infty$

$P(A_n) = 0$

$A_n = \{X_{n-1} < X \leq X_{n+1}\}$

$P\{\text{const}\} = 0$

in a continuous sample space

6/10/2015
Ex 1.3.2
page 12

Mathmatical Model of Experiments

1) Assign sample space.

2) define the event of intersel

S =

(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 5)
(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

A B C

6 6 two trials (exp)
Possible outcomes = $6^2 = 36$

$A = \{ sum = 7 \}$

$B = \{ 8 < sum \leq 11 \}$

$C = \{ 10 < sum \}$

3) make probability assignment?

$P(A) = \frac{6}{36} = \frac{1}{6}$

$P(C) = \frac{3}{36} = \frac{1}{12}$

$P(B) = \frac{9}{36} = \frac{3}{12}$

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$$P\{B \cap C\} = \frac{2}{36} = \frac{1}{18}$$

$$P\{B \cup C\} = \frac{10}{36} = \frac{5}{18}$$

Probability as a Relative Frequency

n number of trials

n_s number of succese in the trials

$$P\{A\} = \frac{n_s}{n}$$

When n is very large probability as relative freqency will reach the Probability using set theory

=> flipping a coin 1000 times

$n_H = 490$

$n_T = 510$

$\overline{1000}$

$$P(H) = \frac{490}{1000} = 0.49$$

$$P(T) = \frac{510}{1000} = 0.5$$

$S = \{H, T\}$ $\quad A = \{H\}$ $\quad P\{A\} = \frac{1}{2}$

Ex $\frac{1.3.3}{P14}$

40 resistors

same size
same shape

relative Freq.
cans we make it For ever.

without return

18 resistor are 10Ω
12 // are 22Ω
33 // are 27Ω
17 // are 47Ω
$\overline{40}$

$$P(\text{drowing } 10\Omega) = \frac{18}{80}$$

$$P(\text{drowing } 22\Omega) = \frac{12}{40}$$

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$p\{\text{drowing } 10\Omega / 22\Omega\} = \frac{18}{79}$

1.4 Joint and conditional probability

$P(A \cap B) \neq 0$ events A and B are Joint

$P(A \cap B) = P[A] + P[B] - P(A \cup B)$

$P(A \cup B) = P[A] + P[B] - P[A \cap B]$

$P[A \cup B] \leq P[A] + P[B]$

$P[A \cup B] = P[A] + P[B]$ when $A \cap B = \phi$

* Condional probabilits

we need to Find the propability of an event A given prior Knowledge of the occurance of another event B which affects the even A

Rol adie

$B = \{2, 4, 6\}$

$A = \{4\}$

$P[A/B]$

* Conditional probability &c 11/10/2015

⇒ event A occrance depends on the occurance of event B

$P\{B\}$ priori probability.

Rolling a die $B = \{2, 4, 6\}$

$A = \{4\}$

$P\{A\} = \frac{1}{6}$ $P(A \backslash B) = \frac{P\{A \cap B\}}{P\{B\}} = \frac{\frac{1}{6}}{\frac{3}{6}} = \frac{1}{3}$

$P\{B\} \neq 0$

$P\{B\} > 0$

$P\{A/B\} \geq 0$ - if event A and B are mutually exclosive the $P\{A \cap B\} = 0 \rightarrow P\{A \backslash B\} = 0$

- $P\{A/B\} > 0$ because $P\{B\} > 0$ and $P\{A \cap B\} > 0$

- $P\{S \backslash B\} = 1 = \frac{P\{S \cap B\}}{P\{B\}} = \frac{P\{B\}}{P\{B\}} = 1$

$$P\left\{\bigcup_{n=1}^{N} A_n\right\} = \sum_{n=1}^{N} P\{A_n\}$$

For even A and C which mutually exclosive

$P(A \cap C) = 0$

$P(A \cup C \backslash B) = P\{A \backslash B\} + P\{C \backslash D\}$

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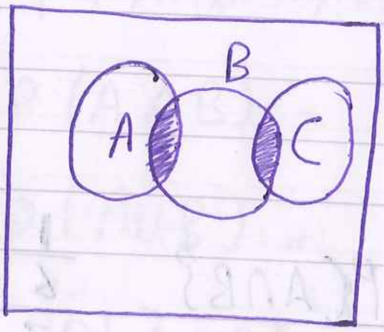
$$P\{A \cup C \backslash B\} = \frac{P\{(A \cup C) \cap B\}}{P\{B\}}$$

$$= \frac{P\{(A \cap B) \cup (C \cap B)\}}{P\{B\}}$$

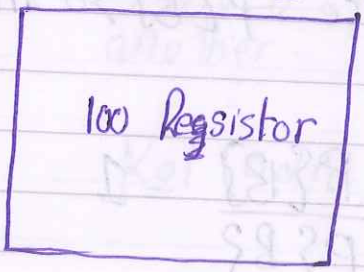


$\ast$ $P\{A \cup B\} = P\{A\} + P\{B\} - P\{A \cap B\}$

$$= \frac{P\{(A \cap B)\}}{P\{B\}} + \frac{P\{C \cap B\}}{P\{B\}}$$

$$= P\{A \backslash B\} + P\{C \backslash B\}$$

Ex(1.4.1) P16



$R(\Omega)$	5%	10%	total
22	10	14	24
47	28	16	44
100	24	8	32
	62	38	100

A = { drawing a 47Ω resistor}

B = { drawing a resistor with 5% tolerance }

C = { drawing a 100Ω resistor}

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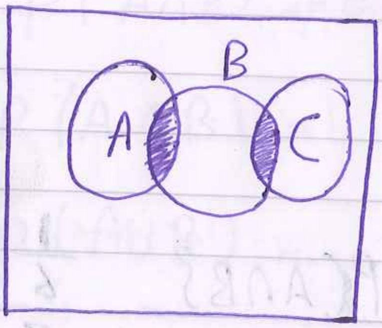
$$P\{A\cup C \backslash B\} = \frac{P\{(A\cup C)\cap B\}}{P\{B\}}$$

$$= \frac{P\{(A\cap B)\cup(C\cap B)\}}{P\{B\}}$$



$*$ $P\{A\cup B\} = P\{A\} + P\{B\} - P\{A\cap B\}$

$$= \frac{P\{(A\cap B)\}}{P\{B\}} + \frac{P\{C\cap B\}}{P\{B\}}$$

$$= P\{A\backslash B\} + P\{C\backslash B\}$$

Ex(1.4.1) P16

100 Regsistor

R (Ω)	5%	10%	total
22	10	14	24
47	28	16	44
100	24	8	32
	62	38	100

A = { drawing a 47-Ω resistor }

B = { drawing a resistor with 5% tolerance }

C = { drawing a 100 Ω resistor }

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$$P\{A\} = P\{A \cap B_1\} + P\{A \cap B_2\} + P\{A \cap B_3\} --- + P(A \cap B_N)$$

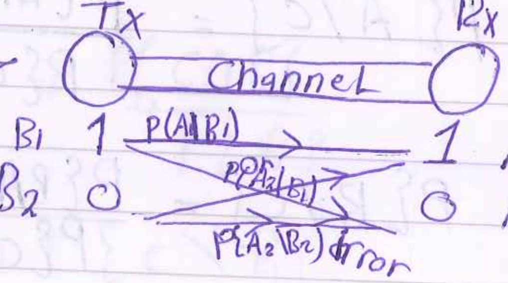
$$P\{A \setminus B_n\} = \frac{P(A \cap B_n)}{P(B_N)}$$

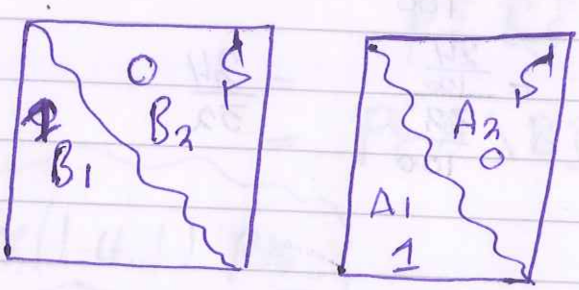
$$P\{A\} = \sum_{n=1}^{N} P\{A \setminus B_n\} P\{B_n\}$$

Ex. 1.4.2 / P18

Binary Comm. System. (two level system 0,1)

simplest comm. system $\Rightarrow$





$\Rightarrow$ Symetric channle.

* assume $P\{B_1\} = 0.6$
$P\{B_2\} = 0.4$

$P\{A_1 \setminus B_1\} = 0.9$

$P\{A_2 \setminus B_2\} = 0.9$

$P\{A_1 \setminus B_2\} = 0.1$

$P\{A_2 \setminus B_1\} = 0.1$

① $P\{A_1\} = P\{A_1 \setminus B_1\} P\{B_1\} + P\{A_1 \setminus B_2\} P\{B_2\}$
$= (0.9)(0.6) + (0.1)(0.4)$
$= 0.58$

② $P\{A_2\} = P\{A_2 \setminus B_1\} P\{B_1\} + P\{A_2 \setminus B_2\} P\{B_2\}$
$= (0.1)(0.6) + (0.9)(0.4)$
$= 0.42$

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③ $P\{B_1 \setminus A_1\}$ ≡ posteriori probability

$$p(B_1 \setminus A_1) = \frac{P(A_1 \cap B_1)}{P(A_1)} = \frac{P(A_1 \setminus B_1)\, P(B)}{P(A_1)} = \frac{0.9 * 0.6}{0.58}$$

$$= 0.931$$

$$* \quad p(A_1 \setminus B_1) = \frac{P(A_1 \cap B_1)}{P(B_1)} \Rightarrow$$

④ $$p(B_2 \setminus A_2) = \frac{P(A_2 \setminus B_2)\, P(B_2)}{P(A_2)}$$

$$= \frac{0.9 * 0.4}{0.42} = 0.857$$

$$* \quad p(B_1 \setminus A_2) = \frac{P(A_2 \setminus B_1)\, P(B_1)}{P(A_2)}$$

$p(B_2 \setminus A_1)$ do it!

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1.5 independent events

The occurance of event A doesnot depend in the occurace of event B.

$$P\{A \cap B\} = P\{A\}\ P\{B\}$$

=> condition probability

$$P\{A \backslash B\} = P\{A\}$$

$$P\{A \backslash B\} = \frac{P\{A \cap B\}}{\underset{\neq 0}{P\{B\}}} = \frac{P\{A\} \cdot \cancel{P\{B\}}}{\cancel{P\{B\}}} = P\{A\}$$

$$\underbrace{P\{A \cap B\}}_{\text{A and B are disjoint}} = \underset{\neq 0}{P\{A\}} \cdot \underset{\neq 0}{P\{B\}}$$

=> Two events can't be disjoint and independent at the same time.

=> if the events are independent they can't be disjoint

Ex 1.5.1 P 31

52 Cards

event A = { selecting a King }

event B = { selecting a Jack or a queen }

event C = { selecting a heart }

1) $p(A) = \frac{4}{52}$

2) $p(B) = \frac{4}{52} + \frac{4}{52} - 0 = \frac{8}{52}$

3) $P(c) = \frac{13}{52}$

4) $P(A \cap B) = 0$

5) $P(A \cap C) = \frac{1}{52}$

6) $P(B \cap C) = \frac{2}{52}$

$P(B) + P(c) - p(B \cup C)$

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if A and C are independent ??

check $p(A \cap C) \overset{?}{=} p(A) \cdot P(C)$

$\frac{1}{52} \overset{?}{=} \frac{4}{52} \cdot \frac{13}{52}$ ✓

$P(A \cap B) \overset{?}{=} P(A) \cdot P(B)$

$0 \neq \frac{4}{52} \cdot \frac{8}{52}$

$P(B \cap C) \overset{?}{=} P(B) \cdot P(C)$

$\frac{2}{52} = \frac{8}{52} \cdot \frac{13}{52} = \frac{2}{52}$ ← statically independent

For Multi random Variables they are statistically independent if they satisfy set of equations

=> # of equation $2^N - N - 1$

=> if $A_1, A_2, \ldots, A_n, \ldots, A_N$

A, B, C

$2^3 - 3 - 1 = 4$

$P(A \cap B) \overset{?}{=} P(A) \cdot P(B)$

$P(A \cap C) \overset{?}{=} P(A) \cdot P(C)$

$P(A \cap C) \overset{?}{=} P(B) \cdot P(C)$

$P(A \cap B \cap C) \overset{?}{=} P(A)\, P(C) \cdot P(B)$ => This condition is not sufficient for independence three .t.v

1.5.2 / p 23

52 - card deck

drawing 4 cards

with replacement 13/10/2015

A_1, A_2, A_3, A_4 drawing all in First time, 2nd, 3rd, 4th

$$p(A_1 \cap A_2 \cap A_3 \cap A_4) = p(A_1) \cdot p(A_2)\, p(A_3)\, p(A_4)$$
$$= \frac{4}{52} \cdot \frac{4}{52} \cdot \frac{4}{52} \cdot \frac{4}{52}$$

=> with out replacement

$$p(A_1 \cap A_2 \cap A_3 \cap A_4) = p(A_1) \cdot p(A_2 \backslash A_1) \cdot p(A_3 \backslash A_1 \cap A_2) \cdot p(A_4 \backslash A_1 \cap A_2 \cap A_3)$$
$$= \frac{4}{52} \cdot \frac{3}{51} \cdot \frac{2}{50} \cdot \frac{1}{49}$$

* properties for statistically independent event.

=> Assume event

$A_1, A_2, ----, A_n, ---, A_N$

are statistically independent

$A_i \cup A_j$ $\quad i \neq j$

$A_i \cap A_j$

$\overline{A_i}$

then any event A_n will be independent from {union, intersection, compliment} of other event.

=> if there is two events A and B A independent of $\bar{B}$

$$p(A \cap \bar{B}) = p(A) \cdot p(\bar{B})$$

=> if there is three events A, B and C

A independent of $(B \cup C)$
$(B \cap C)$

$$p(A \cap (B \cup C)) = p(A) \cdot p(B \cup C) = p(A)[p(B) + p(C)] - p(A)p(C)$$

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$$P(A \cap (B \cap C)) = P(A) \cdot P(B \cap C)$$
$$= P(A) \cdot P(B) \cdot P(C)$$

1.6 combinal Experiments.

e.g $\underbrace{\text{Fliping a coin}}_{\text{subexperiment.}}$ and role die in the same time

=> Subexperments are conducted simulatansously.
=> Repeat experiment many times.
=> Combined event sample space $= S'$.

$$S' = S'_1 \times S'_2 \cdots \times S'_N$$

(S'_1, S'_2)

Sample space

(H, 1)	(T, 1)
(H, 2)	(T, 2)
(H, 3)	(T, 3)
(H, 4)	(T, 4)
(H, 5)	(T, 5)
(H, 6)	(T, 6)

$S'_1 \in S'_1$ $\qquad$ $S'_2 \in S'_2$

* flip coin 2 trial

(H, H)
(H, T)
(T, H)
(T, T)

Probability Notebook

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BY : MARAH ALOMARI

18/10/2015

$N \; \{P\} \; , N-K \; , K$

Two approximation

1) if $N, N-K, K$ are large and P is relatively large

De-Moivre-laplace approximation

$$\binom{N}{K} P^{K} (1-P)^{N-K} \simeq \frac{1}{\sqrt{2\pi NP(1-P)}} \cdot e^{-\frac{(K-NP)^2}{2NP(1-P)}}$$

P is small

$$\binom{N}{K} P^{K} (1-P)^{N-K} \simeq \frac{(NP)^{K} e^{-NP}}{K!}$$

Poisson approximation

EX 1.7.3 / P31

Firing bullets for 3 sec
2400 bullets / min

K number of success

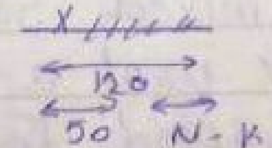
$$\frac{2400}{60} \times 3 = 120 \text{ bullets}$$

$p(h) = 0.4$

$p(m) = 0.6$

$N = 120$

$K = 50$



$$0.0659 = \binom{120}{50} 0.4^{50} \, 0.6^{120-50}$$ ← defcult to calculation

$$0.0693 = \frac{1}{\sqrt{2\pi(120)(0.4)(0.6)}} \cdot e^{-\frac{(50-120\times 0.4)^2}{2\times 120\times 0.4\times 0.6}}$$ ← we do the approx.

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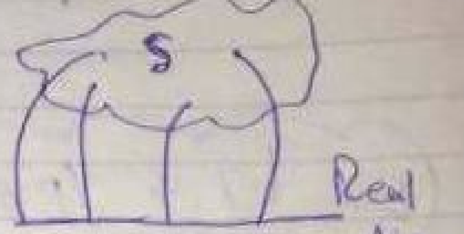
problem solve it 18/10/2015

① 1.1.8 ② 1.1.12 ③ 1.2.8 ④ 1.2.15 ⑤ 1.3.6
⑥ 1.3.11 ⑦ 1.4.3 ⑧ 1.4.5 ⑨ 1.4.12 ⑩ 1.5.5
⑪ 1.5.6 ⑫ 1.6.3 ⑬ 1.7.2 ⑭ 1.7.10.

CH.2

X is arandom variable that maps the element of the Sample Space of an experiment to real line using aspecific function.

maping can
1) point to-point
2) multi point-to-point
3) point-to-multipoint



random variable can be
1) real
2) complex

⇒ The element of sample space S_i while the elements of X are x_i

X: capital letter
x_i: small letter.

⇒ It is possible to maping continuous sample space or discrete or mixed sample spaces in to discrete or continuous R.V with a condition that continuous R.V has to be mapped from continuous S.

Cont. → cont.
discrete → discrete
cont → discrete

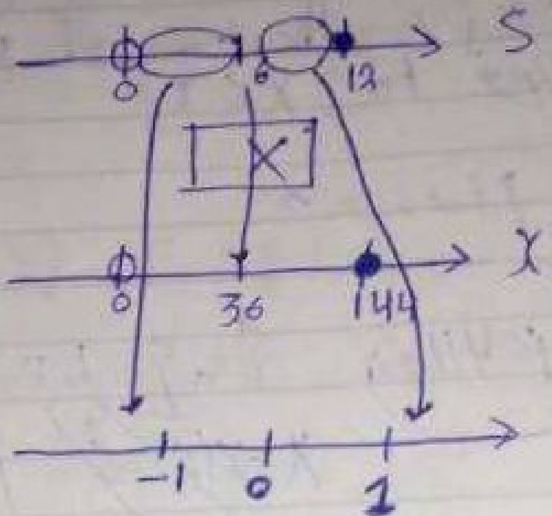
18/10/2015

Ex 2.1.2 / p. 42

possible outcomes 0 to 12

$S = \{0 < S \leq 12\}$

define R.V. $X = x(s) = s^2$



Define X are any
$0 \leq s < 6$ is -1
$6 \leq s \leq 12$ is 1

mapping

⇒ Conditions for R.V.s

1) map each point in S to a single position (point) on real line

2) even $\{X \leq x\}$ shall be an event for any real number less than or equal ~~to~~ x

⇒ The probability of event $\{X \leq x\}$ is the sum of probabilities of all events $\{X \leq x\}$ is given by $P\{X \leq x\}$

$P\{X = \infty\} = P\{X = -\infty\} = 0$

conditions for R.V:

1) point-to-point mapping

2) set $\{X \le x\}$ includes all real numbers $\le X$ on real line

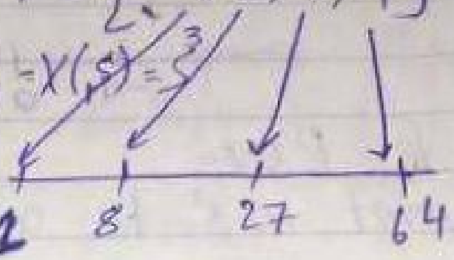
3) $P\{X \le x\}$ is the sum of probabilities of all events corresponding to $\{X \le x\}$

4) $P(-\infty) = P(\infty) = 0$

Ex 2.1.3 / P44:

$S = \{1, 2, 3, 4\}$

$X = X(s) = s^3$



$P(4) = \frac{10}{24}$

$P(X=1) = \frac{4}{24}$

$P(X=\cancel{27}) = \frac{7}{24}$

$P\{X \le 3\} = \frac{4}{24} + \frac{3}{24} + \frac{7}{24} = \frac{14}{24}$

$P(1) = \frac{4}{24}$,

$P(2) = \frac{3}{24}$,

$P(3) = \frac{7}{24}$

2.2 Distribution Functions

CDF
Cumulative distribution Function
⇓
distribution Function ⇒ $F_X(x) = P\{X \le x\}$
$G_X(x)$

PDF
probability density Function
⇓
Density Function

20/10/2015

① $F(-\infty) = 0$ Not $p(X=\infty) = 0$

② $F_X(\infty) = 1$

③ $0 \le F_X(x) \le 1$

④ $x_1 \le x_2$

$F_X(x_1) \le F_X(x_2)$

$F_X(x)$ is an increasing Function

⑤ $p\{x_1 < X \le x_2\} =$

$F_X(x_2) - F_X(x_1)$

⑥ $F_X(x^+) = F_X(x) + F_X(x)$ continuous at the right

Check conditions ①, ②, ④ and ⑥ to verify if CDF is a valid Fun

⇒ Set $\{X \le x_1\}$

⇒ set $\{x_1 < X \le x_2\}$

are mutually exclusive

$p\{X \le x_2\} =$

$p\{X \le x_1 \cup x_1 < X \le x_2\} =$

$p\{X \le x_1\} + p\{x_1 < X \le x_2\}$

$F_X(x_1)$

$p\{x < X \le x_2\} = F_X(x_1) - F_X(x_2)$

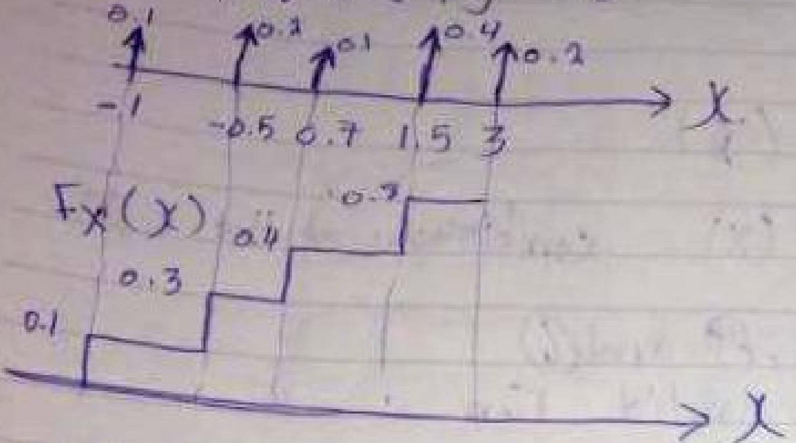
20/10/2015

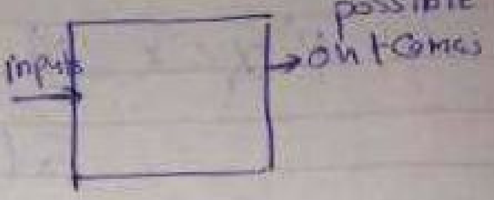
Ex 2.2.1 / p46

X has value in the set $\{-1, -0.5, 0.7, 1.5, 3\}$.

The corresponding probabilities are $\{0.1, 0.2, 0.1, 0.4, 0.2\}$

$P\{X < -1\} = 0$





$\Rightarrow F_X(4) = P\{X \leq 4\} = 1$

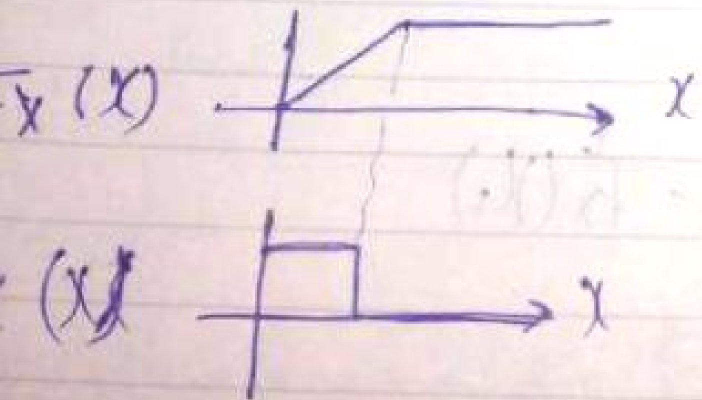
$\Rightarrow P\{X = 4\} = 0$

$$F_X(x) = p(x = 0.1)\, u(x+1) + p(x = 0.2)\, u(x+0.5) + p(x = 0.7)\, u(x - 0.7) + \dots$$

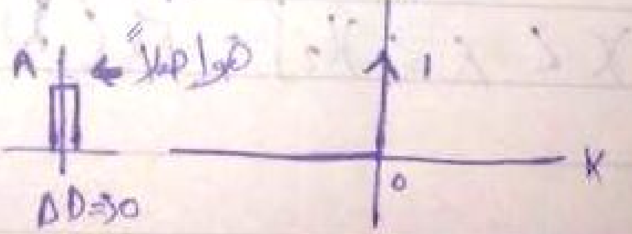
$$= \sum_{i=1}^{N} p(x_i)\, u(x - x_i).$$

2.3 Density Function (pdf) $\{ f_X(x) = \frac{d}{dx} F_X(x) \}$, $f_X(x)$

$\frac{d}{dx} F_X(x)$, $F_X'(x)$ derivative event



$$\frac{d}{dx} u(x) = \delta(x)$$



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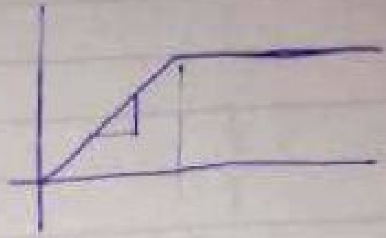
CDF $F_X(x) = P\{X \le x\}$

$$F_X(x) = \sum_{i=1}^{N} p(x_i)\, u(x - x_i)$$

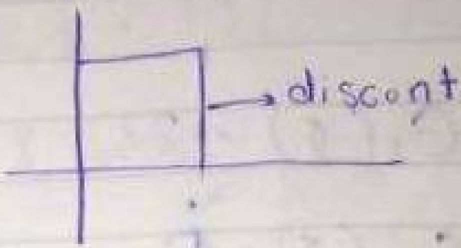
pdf $f_X(x) = \frac{d}{dx} F_X(x)$

probability of pdf



① $f_X(x) \geq 0$

② $\int_{-\infty}^{\infty} f_X(x)\, dx = 1$



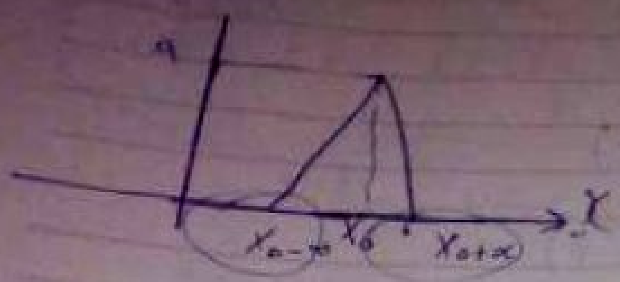
$$f_X(x) = \sum_{i=1}^{N} p(x_i)\, \delta(x - x_i)$$

③ $F_X(x) = \int_{-\infty}^{x} f_X(y)\, dy$

④ $P\{x_1 < x \le x_2\} = \int_{x_1}^{x_2} f_X(x)\, dx$

Ex 2.3.1 / p49 80

22/10/2015



check the validity of PDF

$g_x(x) \geq 0$

$$\int_{-\infty}^{\infty} g_x(x)\,dx \overset{?}{=} 1$$

$$\frac{1}{2}\, 2\alpha \times a = 1$$

$$\alpha a = 1$$

$$a = \frac{1}{\alpha}$$

$G_x(x) = ??$

$$G_x(x) = \int_{-\infty}^{x} g_x(y)\,dy$$

$y = mx + b$

$$y = \frac{a}{\alpha} x + b \qquad y = \frac{1}{\alpha^2}\left[x - x_0 + \alpha\right]$$

$$G_x(x) = \begin{cases} 0 & x < x_0 - \alpha \\ \frac{1}{2\alpha^2}(x - x_0 + \alpha)^2 & x_0 - \alpha < x < x_0 \\ \cancel{} & x_0 \leq x < x_0 + \alpha \\ 1 & x \geq x_0 + \alpha \end{cases}$$

$$\frac{1}{2} + \frac{1}{\alpha}(x - x_0) - \frac{1}{2\alpha^2}(x - x_0)^2$$

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Ex 2.3.2 / P50

$X_0 = 8$
$\alpha = 5$
$a = \frac{1}{5} = 0.2$

$* \ f_X(x) = \begin{cases} 0 & x < 3 \\ \frac{x-3}{25} & 3 \leq x < 8 \\ 0.2 - \frac{x-8}{25} & 8 \leq x < 13 \\ 0 & x \geq 13 \end{cases}$

$* \ G_X(x) = \begin{cases} \frac{1}{50}(x-3)^2 \\ \frac{1}{2} + 0.2(x-8) - \frac{1}{50}(x-8)^2 \end{cases}$

$P\{4.5 < x \leq 6.7\} =$

$$\int_{4.5}^{6.7} \frac{x-3}{25}\, dx = 0.2288$$

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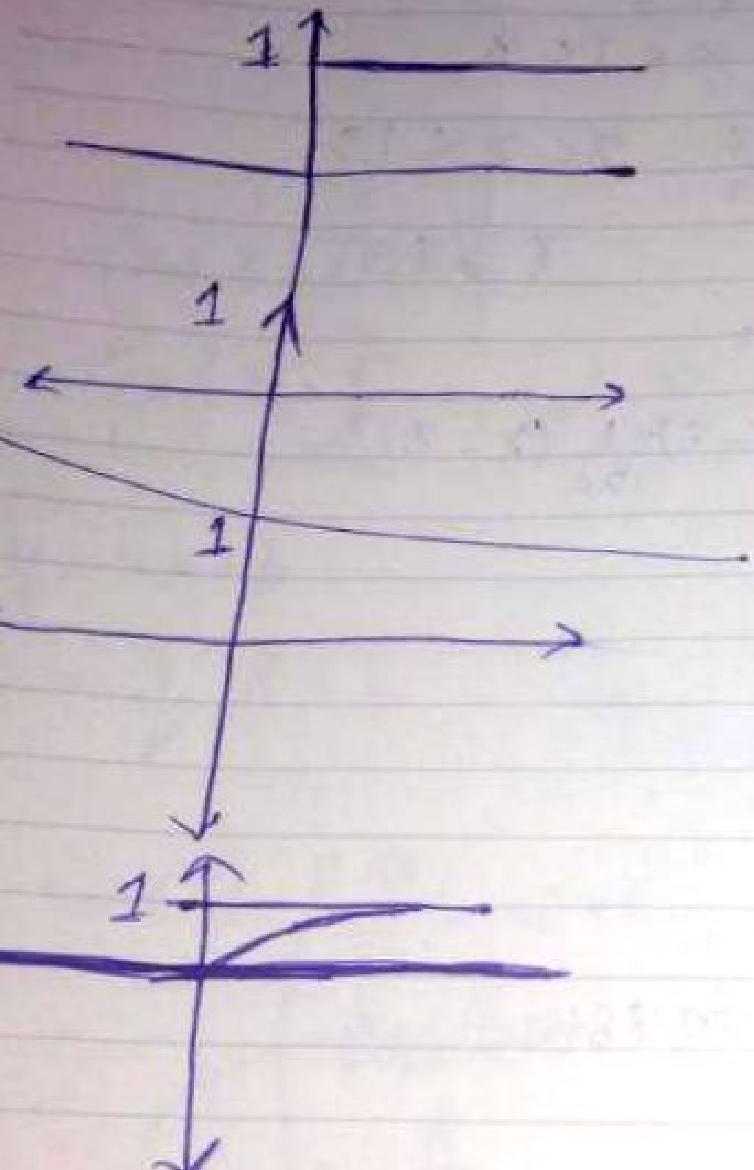
Ex 2.33 / p.50

$F_X(x) = u(x)\left[1 - e^{-\frac{x^2}{b}}\right] \quad b > 0$

CDF function



Find $f_X(x) = \frac{d}{dx} F_X(x)$

$$f_X(x) = u(x)\frac{d}{dx}\left(1 - e^{-\frac{x^2}{b}}\right) + \left(1 - e^{-\frac{x^2}{b}}\right)\frac{du(x)}{dx}$$

$$= u(x)\left(\frac{2x}{b} e^{-\frac{x^2}{b}}\right) + \cancel{\left(1 - e^{-\frac{x^2}{b}}\right)\delta(x)}$$

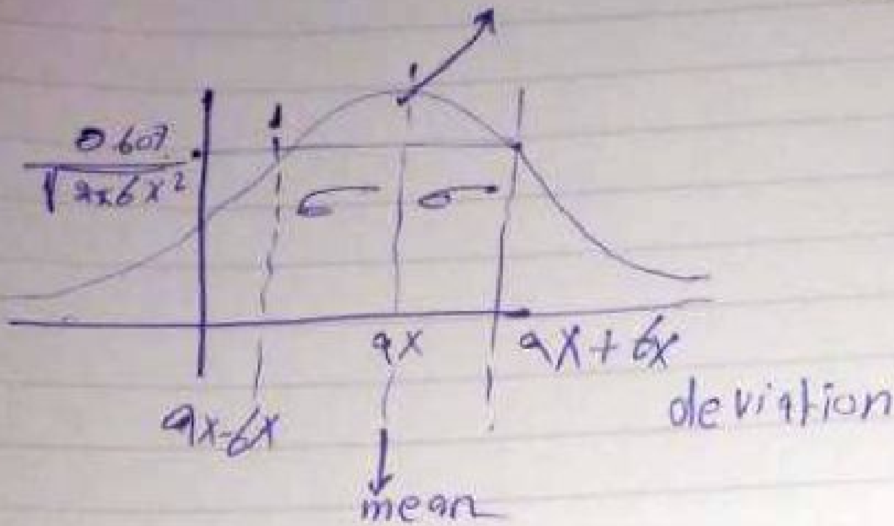
zero

22/10/2015

2.4 Gaussan R.V

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma_X^2}} \cdot e^{-\frac{(x-a_X)^2}{(2\sigma)^2}}$$

$$-\infty < x < \infty$$



$p(-\infty) = 0$

$p(\infty) = 5$

CDF

$$F_X(x) = \frac{1}{\sqrt{2\pi\sigma_X^2}} \int_{-\infty}^{x} e^{-\frac{(y-a_X)^2}{2\sigma_X^2}}$$

Probability Notebook

POWER UNIT

DR . AHMAD ATYA

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25/10/2015

2.4 The Gaussian Distributions

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma_x^2}} \cdot e^{-\frac{(x-a_x)^2}{2\sigma_x^2}} \qquad -\infty < x < \infty$$

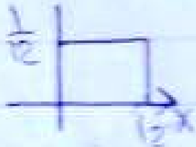
⇒ Example of Gaussian distribution
- production
- noise → thermal noise
  → shut noise

$X + Y = Z \Rightarrow$ Gaussian distribution

(X: random, Y: random)

wheel of Fortion

Bouns: creat R.V from 0-1, counter of 10^3 By Matlab

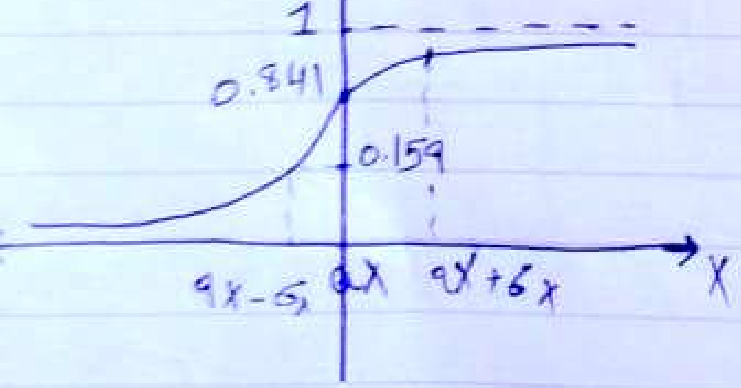


$$F_X(x) = \int_{-\infty}^{x} f_X(y)\,dy \quad \leftarrow \text{CDF}$$

$$= \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi\sigma_x^2}} e^{-\frac{(y-a_x)^2}{2\sigma_x^2}}\,dy$$

$$P\{X \le x_i\} = F_X(x_i)$$

$$P[X > x_i] = 1 - P[X \le x_i] = 1 - F_X(x)$$



1

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$$u = \frac{y - 4x}{6x}$$

$$du = \frac{1}{6x}\, dy \longrightarrow dy = 6x\, du$$

$$y = -\infty \Rightarrow u = -\infty$$
$$y = X \Rightarrow u = \frac{X - 4x}{6x}$$

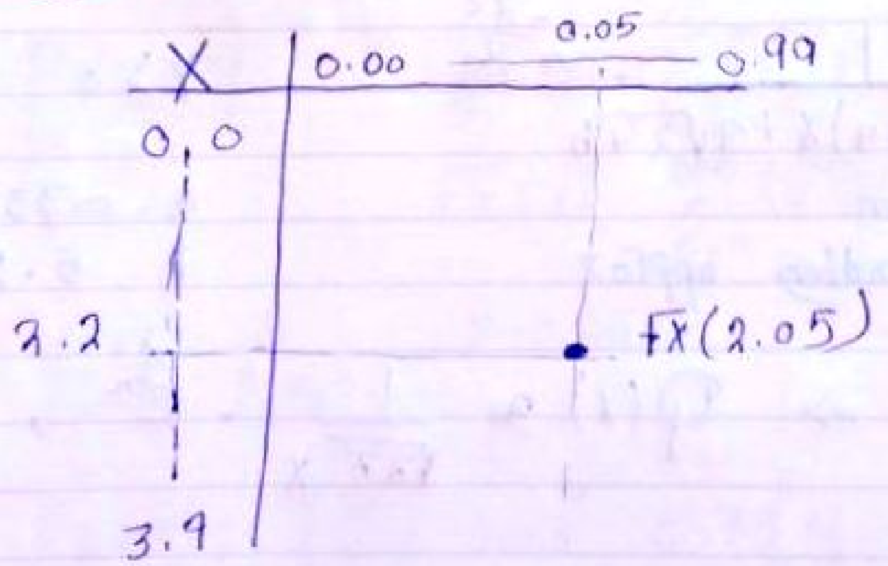
$$Fx(x) = \int_{-\infty}^{\frac{X-4x}{6x}} \frac{1}{\sqrt{2\pi}\, \cancel{6x}}\, e^{-\frac{u^2}{2}}\, \cancel{6x}\, du$$

$$= Fx\left(\frac{X-4x}{6x}\right)$$

$$Fx(2) = Fx\left(\frac{2-3}{3}\right) \qquad \begin{array}{l} x \\ 4x = 3 \\ 6x = 5 \end{array}$$

use the Table

Appendix B / P.418



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Ex 2.4.1/P53

$P\{X < 5.5\}$ $\quad a_X = 3$, $\sigma_X = 2$

$$F_X(5.5) = F_X\left(\frac{5.5-3}{2}\right)$$

$$F_X(1.25) = 0.8944$$

Q- Function :-

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-y^2/2}\, dy$$

$$\Rightarrow \bar{F}_X(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-y^2/2}\, dy, \quad a_X = 0, \ \sigma_X = 1$$

Q used digital Comm. system.

$$Q(x) = 1 - F_X(x)$$

$$Q(x) \approx \frac{1}{(1-a)x + a\sqrt{x^2+b}} \cdot e^{-\frac{x^2}{2}} \qquad x \geq 0$$

Borjession and Sundbery approx

$a = 0.339$
$b = 5.510$

Abramuvitz Approx $\Rightarrow$ $Q(x) \approx \frac{1}{\sqrt{2\pi}\, x} \cdot e^{-\frac{x^2}{2}}, \quad x > 3$

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Ex 2.4.3 / page 54 32

$a_x = 7$

$\sigma_x = 0.5$

we can't use the Table
we use the approx.

$P = \{X \leq 7.3\}$

$F_X(x) = F_X(7.3)$

$= F_X\left(\frac{X - a_x}{\sigma_x}\right)$

$= F_X\left(\frac{7.3 - 7}{0.5}\right)$

$= F_X\left(\frac{0.3}{0.5}\right)$

$= F_X(0.6) \leftarrow$ Table (Actual)

$= 1 - \Phi(0.6) \leftarrow$ approx 1

$\% = \left| \frac{\text{Actual} - \text{approx}}{\text{Actual}} \right|$

2.5 other Distribution and densits Func &c

<u>Binomial</u>

$$f_X(x) = \sum_{k=0}^{N} \binom{N}{k} p^k (1-p)^{N-k} \delta(x-k)$$

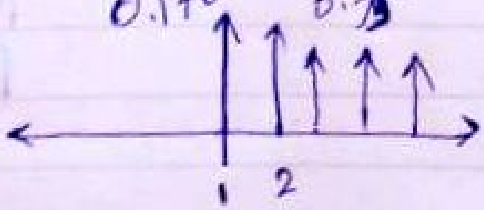
$$F_X(x) = \sum_{k=0}^{N} \binom{N}{k} p^k (1-p)^{N-k} u(x-k)$$

Ex: $N = 6$, $p = 0.25$

Find $f_X(x)$ &c

$$F_X(x) = \binom{6}{0} 0.25^0\, 0.75^6 \delta(x) + \binom{6}{1} 0.25^3\, 0.75^{0.5} \delta(x-1) + \dots$$

$= 0.178 \qquad + 0.75$



$\rightarrow$

4

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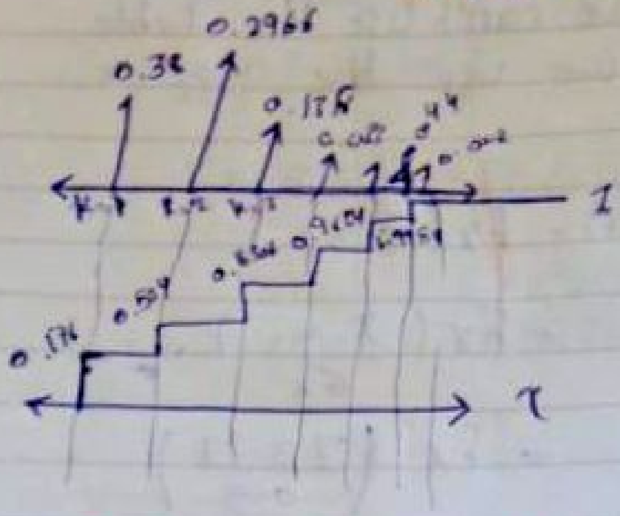
2.5 Binomial Distribution ⇒

⇒ Bernoli Trial



radar application

2.5 poisson ⇒

$$\Rightarrow f_X(x) = e^{-b} \sum_{K=0}^{\infty} \frac{b^K}{K!} \delta(x-K)$$

$b > 0$

$b = \lambda T$

$$\Rightarrow F_X(x) = e^{-b} \sum_{K=0}^{\infty} \frac{b^K}{K!} u(x-K)$$

⇒ in bionmial pdf

$N \rightarrow$ large

$P \rightarrow$ small

$b = NP$

binomial distribution will converge to poisson distribution

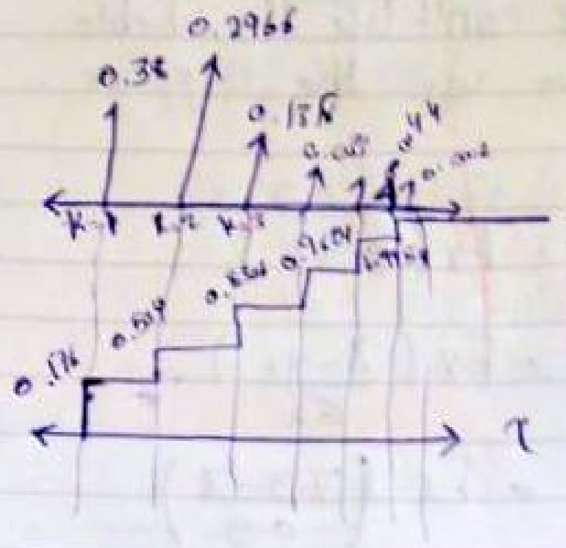
$N = 1000$

$P = 0.1$

27/10/2015

2.5 Binomial Distribution :-

⇒ Bernoli Trial



radar application

2.5 poisson :-

$$\Rightarrow f_X(x) = e^{-b} \sum_{K=0}^{\infty} \frac{b^K}{K!} \delta(x-K)$$

$b > 0$

$b = \lambda T$

$$\Rightarrow F_X(x) = e^{-b} \sum_{K=0}^{\infty} \frac{b^K}{K!} u(x-K)$$

⇒ in bionmial pdf

$N \rightarrow$ large

$P \rightarrow$ small

$b = NP$

binomial distribution will convarge to poisson distribution

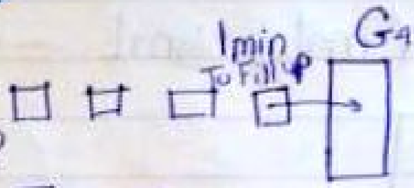
$N = 1000$

$P = 0.1$

Ex 2.5.1 | p. 56

27/10/2015

What is the Prob. that waiting line occurs?



$\lambda = 50/h$

$= P\{X > 1\}$

$= P\{X \geq 2\}$

$$b = \lambda T = \frac{50}{60\,min} \times 1\,min = \frac{5}{6}$$

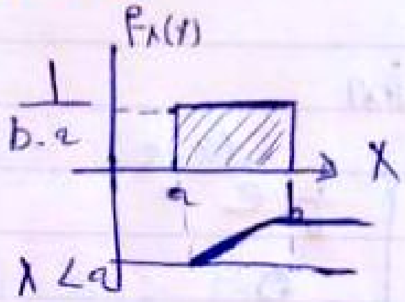
$$P\{X > 1\} = 1 - P\{X < 1\}$$

$$F_X(1) = \sum_{k=0}^{1} \frac{e^{-b} b^k}{k!} u(x-k)$$

$$= e^{-5/6}\left[\frac{\left(\frac{5}{6}\right)^0}{0!} + \frac{(5/6)^1}{1!}\right]$$

$$P\{X > 1\} = 1 - e^{-5/6}\{1 + 5/6\}$$

Uniform distribution



$-\infty < a < \infty$

$-\infty < b < \infty$

$a < b$

$$f_X(x) = \begin{cases} 0 & x < a \\ \frac{1}{b-a} & a \leq x < b \\ 0 & x \geq b \end{cases}$$

$$F_X(x) \begin{cases} 0 & x < a \\ \frac{1}{b-a}(x-a) & a \leq x < b \\ 1 & x \geq b \end{cases}$$

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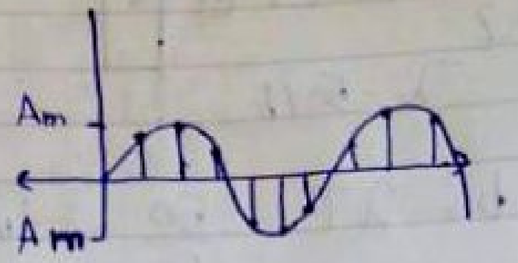
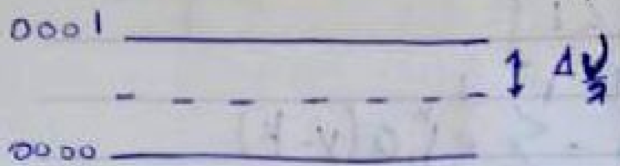
⇒ quantization nois $\overset{\text{use}}{\Rightarrow}$ digital Comm.

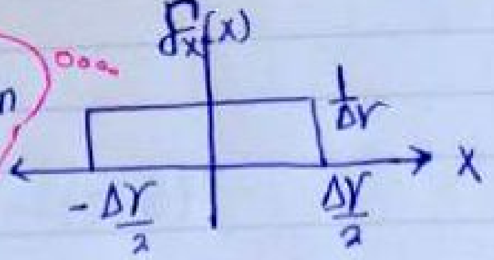
Analog signal — digital signal
A D C



$-\ \Delta V = \frac{2A}{L}$ $\quad L = 2^n$



quantization Error

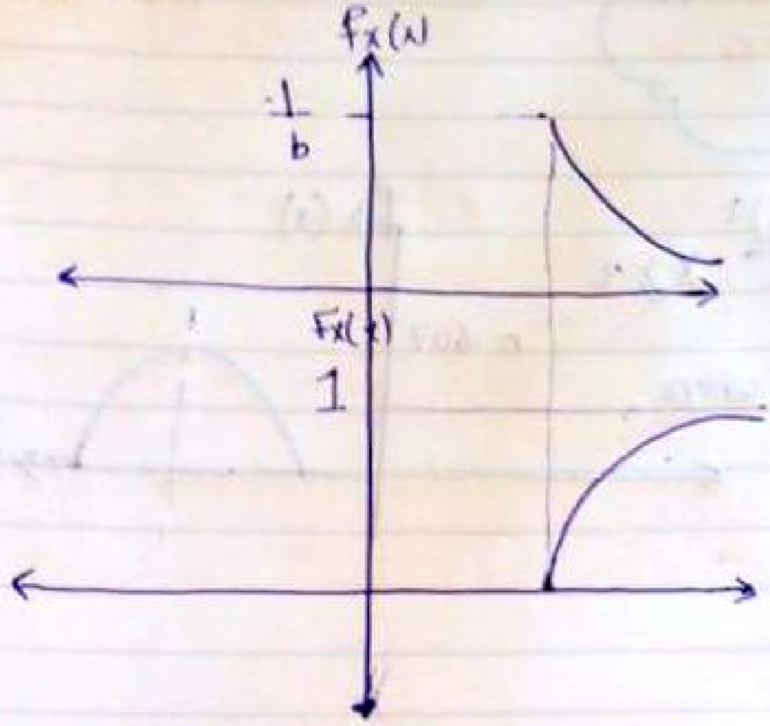


Exponential distribution

Radar application

$$f_X(x) = \begin{cases} \frac{1}{b} e^{-\left(\frac{x-a}{b}\right)} & x \geq a \\ 0 & x < a \end{cases}$$

$$F_X(x) = \begin{cases} 0 & x < a \\ 1 - e^{\left(\frac{-x+a}{b}\right)} & x \geq a \end{cases}$$



Ex 2.5.2 / p58 g_0

$$f_P(p) = \begin{cases} \frac{1}{P_0} e^{-P/P_0} & p > 0 \\ 0 & p < 0 \end{cases}$$

P_0 is the average amount of power receivid

$$P\{\text{received power} > P_0\} = 1 - \underbrace{P\{P \le P_0\}}_{F_P(P_0)}$$

$$P\{P > P_0\} = 1 - \left[1 - e^{-P/P_0}\right]_{P = P_0}$$

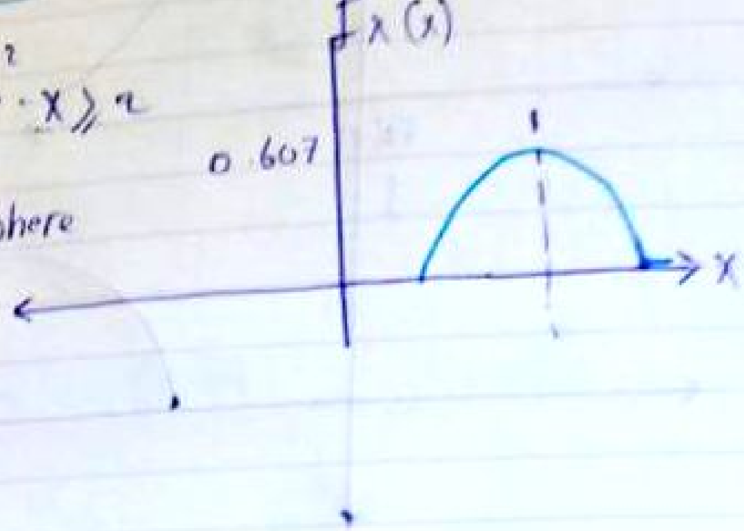
$$= 1 - \{1 - e^{-1}\}$$

$$= 0.368$$

27/10/2015

RayLeigh Distribution :-

$$f_X(x) = \begin{cases} \frac{2}{b}(x-a)\, e^{\frac{-(x-a)^2}{b}} & x \geq a \\ 0 & \text{else where} \end{cases}$$



Probability Notebook

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1/11/2015

CH.3 operasion on R.V Expectation

→ expectation is related averaging process of R.V

Can be called :-

- the mathematical expectation of x
- the expected value of x
- The mean of x
- The statistical averge of x

Denoted by $E[x]$

Exe. 3.1.1 / P78

4 people $1\$ = 100¢$

25 ¢
10 ¢
5 ¢
1 ¢

coins < 1\$ recored the amount (0.75 \$)

Coins > 1\$ (2.75 \$) 0.75

18¢,	45¢,	64¢,	72¢,	77¢,	95¢
8	12	28	22	15	5

Fraction average

$$\bar{x} = 18\frac{8}{90} + 45\frac{12}{90} + 64\frac{28}{90} + 72\frac{22}{90} + 77\frac{15}{90} + 95\frac{5}{90}$$

1/11/2013

$$\bar{X} = \sum_{i=1}^{N} x_i \, p(x_i) = E[X]$$ ← amount Fraction average.

→ X R.V continous

$$E[X] = \int_{-\infty}^{\infty} x \, f_X(x) \, dx$$

$$f_X(x) = \sum_{i=1}^{N} p(x_i) \, \delta(x - x_i)$$

$$E[X] = \int_{-\infty}^{\infty} x_i \sum_{i=1}^{N} p(x_i) \, \delta(x - x_i)$$

$$= \sum_{i=1}^{N} p(x_i) \sum_{-\infty}^{\infty} x_i \, \delta(x - x_i)$$

$$= \sum_{i=1}^{N} p(x_i) \, x_i$$

$* \; f(x)\delta(x-a) = f(a)$

Ex 3.1.2 / p.79

$\bar{X}$ of X with expontantial distribution

$$f_X(x) = \begin{cases} \frac{1}{b} e^{-\frac{(x-a)}{b}} & x > a \\ 0 & x < a \end{cases}$$

$$E[X] = \int_{a}^{\infty} x \frac{1}{b} e^{-\frac{(x-a)}{b}} \, dx$$

⇒ Appendix / p. 419

$$E[X] = \frac{e^{a/b}}{b} \left[e^{-a/b} (ab + b^2) \right] = a + b$$

1/11/2015

→ For symmetric R.V the $E[X]$ will be the mean center point

$$f_X(x+a) = f_X(-x+a)$$

Expected Value of the Function of R.V

assume $g(x)$ is Function of R.V X

$$E[g(x)] = \int_{-\infty}^{\infty} g(x)\, f_X(x)\, dx$$

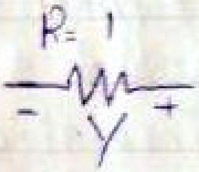
$$E[aX + bX^2 + cX^3 + \cdots] = aE[X] + bE[X^2] + cE[X^3]$$

Ex 3.1.3 p.79 ec

R.V V described using a Rayleigh distribution

$$f_V(v) = \begin{cases} \frac{2}{b}(v-a)\, e^{-\frac{(v-a)^2}{b}} & v \geq a \\ 0 & v < a \end{cases}$$

$a = 0$, $b =$



$$Y = g(V) = V^2$$

Find average power across 1Ω resistor

$$E[Y] = E[V^2]$$

$$= \int_{-\infty}^{\infty} v^2 f_V(v)\, dv$$

$u = v^2$

$du = 2v\, dv$

$v\, dv = \frac{1}{2} du$

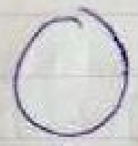
$$= \int_{0}^{\infty} v^2 \frac{2}{5} v\, e^{-\frac{v^2}{5}}\, dv$$

1/11/2015

$$\int_0^\infty u \frac{2}{5} \frac{1}{2} du \; e^{\frac{-u}{20}}$$ $v = 0$ $u = 0$, $v = \infty$ $v = \infty$

$E[V^2] = 5$

Ex 3.1.4 / p 80 sc



L distict
symbols x_i
$i = 1, 2, \ldots, L$

$p(x_i$, is aprobability of symbol $x = x_i$

- Facts of information Theorey 20

are statisfical for logarithim function

1) information $\propto \dfrac{1}{P(x_i)}$
2) information of two indepenclent source is should add
3) information should be positive
4) information should be zero of certain event $p(x) = 1$
5) L mini is 2 (0, 1)

$Ix = -\log P(x_i)$

(17)

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3.2 Moment (about origin)

$$m_n = E(x^n) = \int_{-\infty}^{\infty} x^n f_x(x)\,dx$$

$$m_0 = \int_{-\infty}^{\infty} x^0 f_x(x)\,dx = 1$$

$$m_1 = E(x) = \int_{-\infty}^{\infty} x\, f_x(x)\,dx$$

⟹ Central moment (Reference about the mean)

$$n_{th} \text{ central moment} = M_n = \int_{-\infty}^{\infty} (x-\bar{x})^n f_x(x)\,dx$$

$$= E[(x-\bar{x})^n]$$

$$M_0 = 1$$

$$M_1 = \int_{-\infty}^{\infty} (x-\bar{x})\, f_x(x)\,dx = E[(x-\bar{x})] = 0$$

$$M_2 = \int_{-\infty}^{\infty} (x-\bar{x})^2 f_x(x)\,dx = \underline{E[(x-\bar{x})^2]}$$

variance σ_x^2

$$= E(x^2 - 2x\bar{x} + \bar{x}^2)$$

$$= E[x^2] - E[2x\bar{x}] + E[\bar{x}^2]$$

$$= m_2 - 2\bar{x}\,E[x] + m_1^2$$

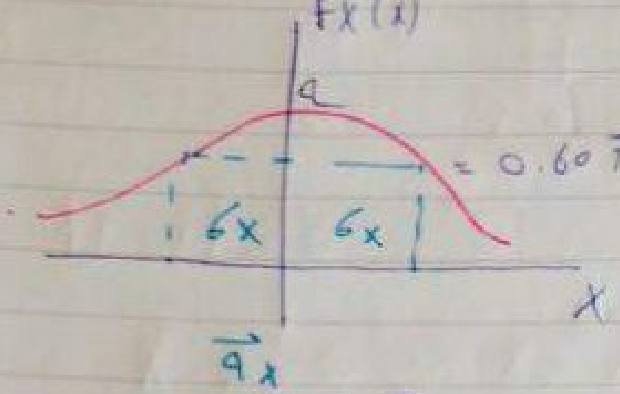
$$= m_2 - 2m_1^2 + m_1^2$$

$$M_2 = m_2 - m_1^2$$

10/11/2015

variance $= \sigma_x^2$ $\quad \sigma_x^2 = m_2 - m_1^2$

$\pm\sqrt{\sigma_x^2}$ $\quad +\sqrt{\sigma_x^2}$ represent standard deviation



$m_1 = \bar{X}$

Example 3.2.1/82 $\quad X$ with exp function density

$$f_X(x) = \begin{cases} \frac{1}{b} e^{-\frac{(x-a)}{b}} & x > a \\ 0 & x < a \end{cases}$$

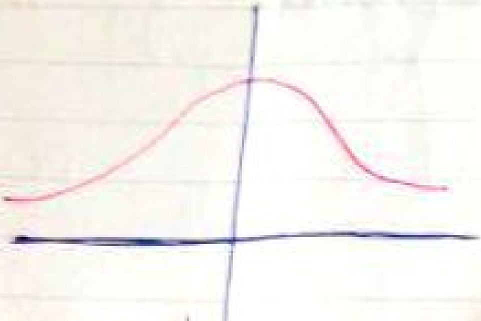
$$\sigma_x^2 = E[(x-\bar{x})^2] = m_2 - m_1^2$$

$$m_2 = \int_a^\infty x^2 \frac{1}{b} e^{-\frac{(x-a)}{b}} dx$$

$$m_1 = \int_a^\infty x \frac{1}{b} e^{-\frac{(x-a)}{b}} dx$$

$$\int x^2 e^{ax} dx = e^{ax}\left[\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3}\right]$$

$$\sigma_x^2 = E[(x-\bar{x})^2] = \int_a^{\infty} (x-\bar{x}) \frac{1}{b} e^{-\frac{(x-a)}{b}} dx$$ 10/11/2015



symetic

M_3 = defines the measure of asymetry of density function

$$M_3 = E[(x-\bar{x})^3]$$
$$= E[(x-\bar{x})(x-\bar{x})^2] = m_3 - 3m_1 m_2 + 2m_1^3$$
$$= \int_{-\infty}^{\infty} (x-\bar{x})^3 f_x(x)\, dx$$

$M_3 \Rightarrow$ Skew

skewness or skew Coefficient $= \frac{M_3}{\sigma_x^3}$

Ex 3.2.2/82

$$f_x(x) = \begin{cases} \frac{1}{b} e^{-\frac{(x-a)}{b}} & x \geq a \\ 0 & x < a \end{cases}$$

$$\sigma_x^2 = M_2 = m_2 - m_1$$

Calculate m_1, m_2, m_3

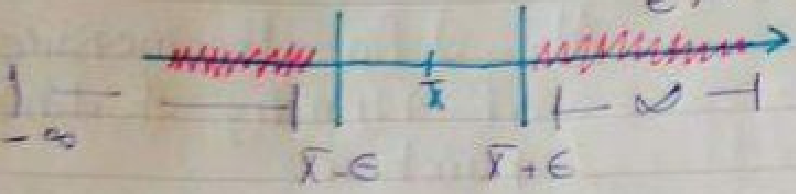
$M_3 = 2b^3$ $\quad \frac{M_3}{\sigma_x^3} = 2 \leftarrow$ not symetric

Cheby Cheves inequality

10/11/2015

$$P\{|X-\bar{X}| \geq \epsilon\} \leq \frac{\sigma_X^2}{\epsilon^2}$$

$\epsilon > 0 \Rightarrow$ very small

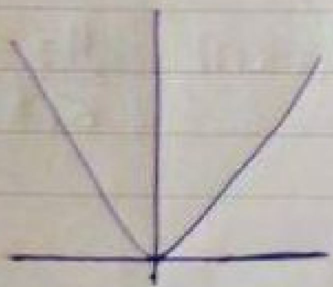


$$|X-\bar{X}| \geq \epsilon$$

$$X-\bar{X} \geq \epsilon \Rightarrow X \geq \bar{X}+\epsilon$$

$$-(X-\bar{X}) \geq \epsilon$$

$$X-\bar{X} \leq -\epsilon \Rightarrow X \leq \bar{X}-\epsilon$$



$$P\{|X-\bar{X}| \geq \epsilon\} = \int_{-\infty}^{\bar{X}-\epsilon} f_X(x)\,dx + \int_{\bar{X}+\epsilon}^{\infty} f_X(x)\,dx$$

$$\sigma_X^2 = \int_{-\infty}^{\infty} (X-\bar{X})^2 f_X(x)\,dx = \int_{-\infty}^{\bar{X}-\epsilon} (X-\bar{X})^2 f_X(x)\,dx$$

$$+ \int_{\bar{X}-\epsilon}^{\bar{X}+\epsilon} (X-\bar{X})^2 f_X(x)\,dx + \int_{\bar{X}+\epsilon}^{\infty} (X-\bar{X})^2 f_X(x)\,dx$$

10/11/2015

$$\sigma_x^2 \geq \int_{-\infty}^{\bar{x}-\epsilon} (x-\bar{x})^2 f_x(x)\,dx + \int_{\bar{x}+\epsilon}^{\infty} (x-\bar{x})^2 f_x(x)\,dx$$

$\epsilon = 0$ they are equal

$$\sigma_x^2 \geq \int_{-\infty}^{\bar{x}-\epsilon} \epsilon^2 f_x(x)\,dx + \int_{\bar{x}+\epsilon}^{\infty} \epsilon^2 f_x(x)\,dx$$

$|x - \bar{x}| \geq \epsilon$

$|x - \bar{x}|^2 \geq \epsilon^2$

$$\frac{\sigma_x^2}{\epsilon^2} \geq \int_{-\infty}^{\bar{x}-\epsilon} f_x(x)\,dx + \int_{\bar{x}+\epsilon}^{\infty} f_x(x)\,dx$$

$$P\{|x-\bar{x}| < \epsilon\} \geq 1 - \frac{\sigma_x^2}{\epsilon^2}$$

$\epsilon \to 0 \quad x \to \bar{x}$

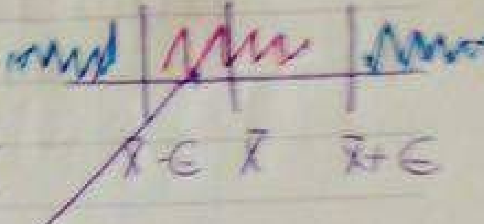
$$P\{|x-\bar{x}| < \epsilon\} = 1$$

$\sigma_x^2 \to 0$

12/11/2015

Ex 3.2.3/p 83

$$p\{|X-\bar{X}| \geq \epsilon\} \leq \frac{\sigma_x^2}{\epsilon^2}$$



$$p\{|X-\bar{X}| < \epsilon\} \geq 1 - \frac{\sigma_x^2}{\epsilon^2}$$

$p\{X < \bar{X} - 3\sigma_x\}$ or $\{X > \bar{X} + 3\sigma_x\}$ then the $p\{|X-\bar{X}| \geq \epsilon\} \leq \frac{\sigma_x^2}{\epsilon^2}$

$\epsilon = 3\sigma_x$ $= \frac{\sigma_x^2}{9\sigma_x^2}$

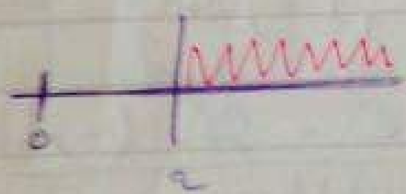
$= \frac{1}{9}$

⇒ Markov's inequality :- ← proof 1 Bonn's

For non negative R.V X

$$p\{X \geq a\} \leq \frac{1}{a} E[X]$$

$a > 0$



3.3 Functions that gives moments &c.

1) Characteristic fun – $\Phi_x(\omega)$

2) Moment generating fun – $M_x(u)$

$s = j\omega$

$$\Phi_x(\omega) = E\{e^{j\omega x}\}$$

$$= \int_{-\infty}^{\infty} f(x)\, e^{j\omega x}\, dx$$

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$$f_x(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \Phi_x(\omega)\, e^{-j\omega x}\, d\omega$$

$$m_n = (-j)^n \frac{d\,\Phi_x(\omega)}{d\omega}\Bigg|_{\omega=0}$$

Ex: 3.3.1 / P85 ٨٥ First moment means the mean.

$$f_x(x) = \begin{cases} \frac{1}{b} e^{-\frac{(x-a)}{b}} & x \geq a \\ 0 & x < a \end{cases}$$

$$\Phi_x(\omega) = \int_{-\infty}^{\infty} f_x(x)\, e^{j\omega x}\, dx$$

$$= \int_{a}^{\infty} \frac{1}{b}\, e^{-\frac{(x-a)}{b}} \cdot e^{j\omega x}\, dx$$

$$\frac{-(\frac{1}{b}-j\omega)}{-(\frac{1}{b}-j\omega)} * \frac{e^{a/b}}{b} \int_{a}^{\infty} e^{-x(\frac{1}{b}-j\omega)}\, dx$$

$$= \frac{e^{-a/b}}{b(\frac{1}{b}-j\omega)}\, e^{-x(\frac{1}{b}-j\omega)} \Bigg|_{a}^{\infty} = \frac{e^{j\omega a}}{1-j\omega b}$$

$$\frac{d\,\Phi_x(\omega)}{d\omega}\Bigg|_{\omega=0} = \frac{(1-j\cancel{\omega}b)(ja)\,\cancel{e^{j\omega a}}^{\,1} - \cancel{e^{j\omega a}}^{\,1}(-jb)}{(1-j\omega b)^2}$$

$$= \frac{ja+jb}{1}$$

$$m_1 = (-j)^1 (ja+jb)$$

$$= a+b$$

$$M_x(\nu) = \int_{-\infty}^{\infty} f_x(x)\, e^{\nu x}\, dx$$

$$m_n = \frac{d\,M_x(\nu)}{d\nu}\Bigg|_{\nu=0}$$

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Chernoff's Inequality :

$$P\{X \geq a\} \leq M_x(\nu)\, e^{-a\nu}$$

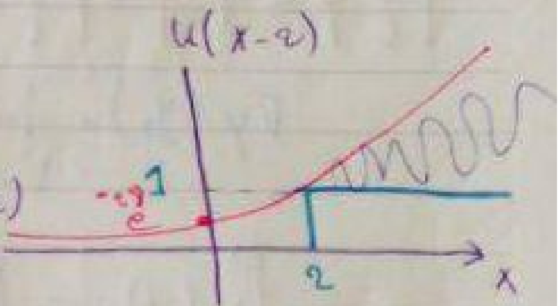
$-\infty < a < \infty$

Markov's $\leq \frac{1}{a} E(x)$

$\longrightarrow u(x-a) \leq e^{\nu(x-a)}$

$\longrightarrow f_X(x)\, u(x-a) \leq f_X(x)\, e^{\nu(x-a)}$

$$\int_{-\infty}^{\infty} f_X(x)\, u(x-a)\, dx \leq \int_{-\infty}^{\infty} f_X(x)\, e^{\nu(x-a)}\, dx$$



$$P\{X \geq a\} \to \int_{a}^{\infty} f_X(x)\, dx \leq e^{-a\nu} \underbrace{\int_{-\infty}^{\infty} f_X(x)\, e^{\nu x}\, dx}_{M_x(\nu)}$$

3.4 Transformation of R.V :

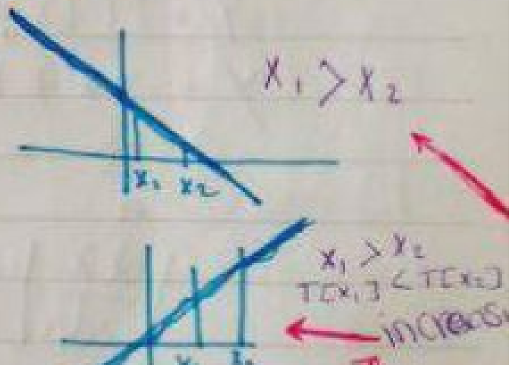
3 Cases.

input $\longrightarrow$ T_x $\longrightarrow$ Y $\quad f_Y(y)$

X

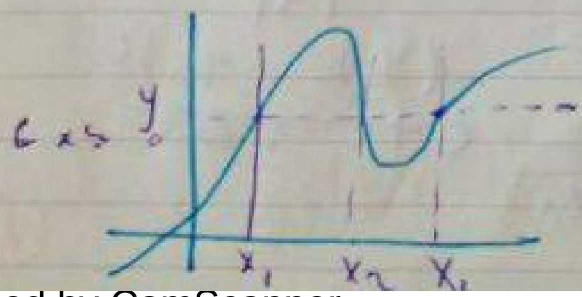
✓ $f_X(x)$ diode

✓ $F_X(x)$ o-p-amp



1) X continous + T(x) continous ⟶ monotonic / nonmonotonic

2) X discontinuous + T[x] is continous



$F_X(x) = P\{X \leq x\}$

$F_Y(y) = P\{Y \leq y\}$

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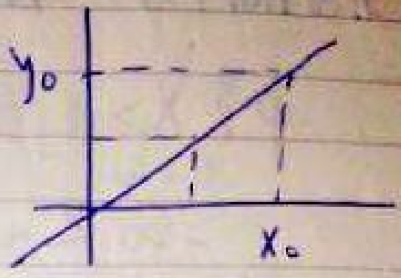
monotonicly increasing ↓

$P\{Y \le y_0\} = P\{X \le x_0\}$ ⇒

$F_Y(y_0) = F_X(x_0)$

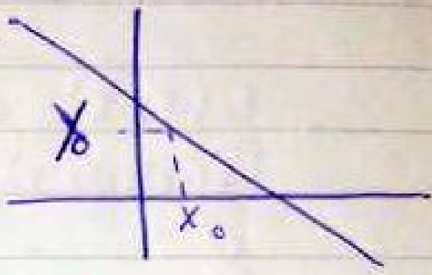


monotonicly deacrising ↓

$P\{Y \le y_0\} = P\{X \ge x_0\}$

$F_Y(y_0) = 1 - F_X(x_0)$

$1 - P\{X \le x_0\}$



$$f_Y(y) = f_X(x)\left|\frac{dx}{dy}\right|$$ ← inverse of the slop

⇐ $X \rightarrow \boxed{T[x]} \rightarrow Y$

Ex 3.4.1 / P 89 ec

$Y = T[X]$

$Y = aX + b$

$\frac{Y-b}{a} = X$

$\left|\frac{dx}{dy}\right| = \frac{1}{|a|}$

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma_x^2}} e^{-\frac{(x-\eta_x)^2}{2\sigma_x^2}}$$

a, b are constant real ←

$$f_Y(y) = f_X(x)\left|\frac{dx}{dy}\right|$$

$$= \frac{1}{\sqrt{2\pi\sigma_x^2}} e^{-\frac{(x-\eta_x)^2}{2\sigma_x}} \cdot \frac{1}{|a|}$$

$$= \frac{1}{|a|}\frac{1}{\sqrt{2\pi\sigma_x^2}} e^{-\frac{\left(\frac{y-b}{a} - \eta_x\right)^2}{2\sigma_x^2}}$$

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Sat
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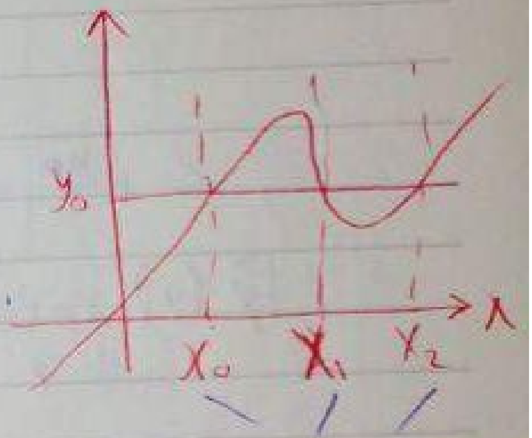
$$= \frac{1}{\sqrt{2\pi a^2 \sigma_x^2}} \, e^{-\frac{\left(\frac{y-b-a a_x}{a}\right)^2}{2\sigma_x^2}}$$

$$= \frac{1}{\sqrt{2\pi \sigma_y^2}} \, e^{-\frac{(y-(b+a a_x))^2}{2\sigma_y^2}}$$

$$= \frac{1}{\sqrt{2\pi \sigma_y^2}} \, e^{-\frac{(y-a_y)^2}{2\sigma_y^2}}$$

$\sigma_y^2 = a^2 \sigma_x^2 \leftarrow$ $\qquad a_y = b + a x \leftarrow$

⇒ non monotonic



roots of T(x)
→ not cross zero
→ has same y

$$P\{Y \ge y_0\} = P\{x \mid Y \le y_0\}$$

$$F_Y(y_0) = \int_{x \mid y \le y_0} f_X(x)\, dx \leftarrow$$

derive with respect y_0

$$f_Y(y_0) = \frac{d}{dy_0} \int_{x \mid y < y_0} f_X(x)\, dx$$

single point

$$f_Y(y) = \frac{\sum_n f_X(x_n)\big|_{x_n}}{\left|\frac{dy}{dx}\right|_{x = x_n}}$$

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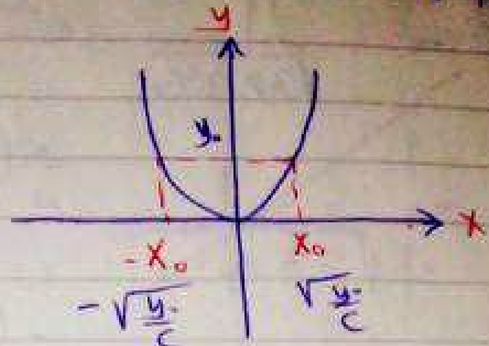
Ex 3.4.2 / p91



solution 1: $Y = T(X) = cX^2$

$$X = \pm\sqrt{\frac{y_0}{c}}$$

$$\frac{dy}{dx} = 2cx$$

$$f_Y(y) = \frac{f_X(x_0)}{\left|\frac{dy}{dx}\right|_{x=x_0}} + \frac{f_X(-x_0)}{\left|\frac{dy}{dx}\right|_{x=-x_0}}$$

$$= \frac{f_X\left(\sqrt{\frac{y_0}{c}}\right)}{\left|2c\sqrt{\frac{y_0}{c}}\right|} + \frac{f_X\left(-\sqrt{\frac{y_0}{c}}\right)}{\left|2c-\sqrt{\frac{y_0}{c}}\right|}$$

$$c\sqrt{\frac{y_0}{c}} = \sqrt{\frac{c^2 y_0}{c}} = \sqrt{cy_0} \Leftarrow$$

$$f_Y(y) = \frac{f_X\left(\sqrt{\frac{y_0}{c}}\right) + f_X\left(-\sqrt{\frac{y_0}{c}}\right)}{2\sqrt{cy_0}}$$

To generalized $y_0 = y \Leftarrow$

solution 2:

$$P\{Y \le y\} = P\{X \mid Y < y\}$$

$$F_Y(y) = \int_{-\sqrt{\frac{y}{c}}}^{\sqrt{\frac{y}{c}}} f_X(x)\,dx$$

$\Rightarrow$ Leibniz's Rule:

$$f_Y(y) = f_X\left(\sqrt{\frac{y}{c}}\right)\frac{d\left(\sqrt{\frac{y}{c}}\right)}{dy} - f_X\left(-\sqrt{\frac{y}{c}}\right)\frac{d\left(-\sqrt{\frac{y}{c}}\right)}{dy}$$

$$\frac{d\left(\frac{y}{c}\right)^{\frac{1}{2}}}{dy} = \frac{1}{2}\left(\frac{y}{c}\right)^{\frac{-1}{2}} \cdot \frac{1}{c}$$

$$= \frac{1}{2}\,\frac{1}{\sqrt{\frac{y}{c}}} \cdot \frac{1}{c} = \frac{1}{2\sqrt{\cancel{c}^2\frac{y}{\cancel{c}}}} = \frac{1}{2\sqrt{cy}}$$

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3) Transformation of discrete R.V Ec

$$Y = T(X)$$

X is discrete

→ Same values of Y for many value of X

→ single value of Y for single value of X

* $F_X(x) = \sum_n p(x_n)\, u(x - x_n)$

* $f_X(x) = \sum_n p(x_n)\, \delta(x - x_n)$

point-to-point tranformation

$Y = T[X]$

$X = x_n$

$y_n = T[x_n]$

* $f_Y(y) = \sum_n p(y_n)\, \delta(y - y_n)$

* $F_Y(y) = \sum_n p(y_n)\, u(y - y_n)$

For more than on x_n give same y_n

$$P(y_n) = \sum_n p(x_n)$$

Ex 3.4.3 / p.92

$$Y = 2 - X^2 + \frac{X^3}{3}$$

X	$P(x_n)$	Y
-1	0.1	2/3
0	0.3	2
1	0.4	4/3
2	0.2	2/3

$$F_Y(y) = 0.3\, u(y - \tfrac{2}{3}) + 0.3\, u(y-2) + 0.4\, u(y - 4/3)$$

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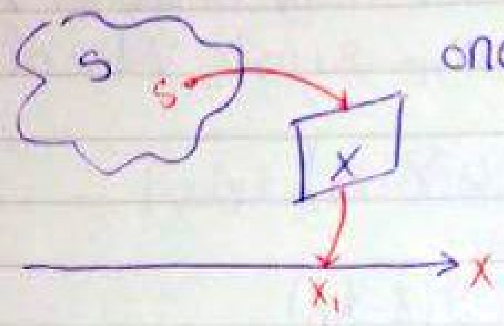
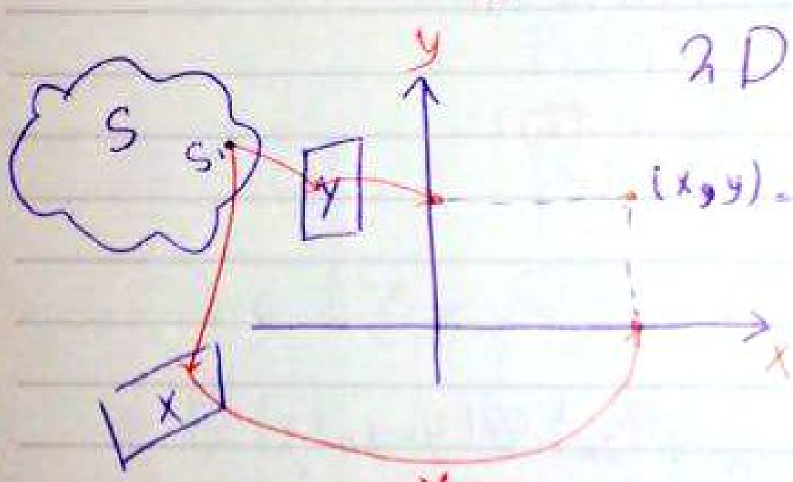
3.1.1, 3.1.5, 3.1.11, 3.2.9, 3.2.16, 3.2.22, 3.2.33, 3.2.36, 3.4.1, 3.4.7, 3.4.10, 3.4.14.

CH 4: Multiple R.Vs



one dimension

$\{X \le x_0\}$

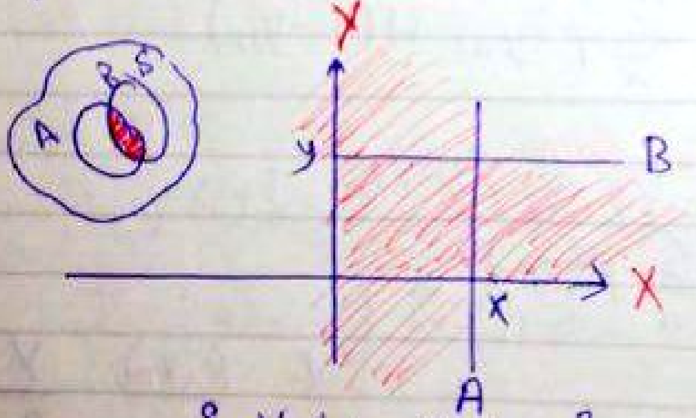


2D

$(x, y) = (X(s_1), Y(s_1))$

point in vector R.V

The Joint Sample Space S_J



$A = \{X \le x\}$

$B = \{Y \le y\}$

$P\{A\} = F_X(x)$

$P\{B\} = F_Y(y)$

$\{X \le x, Y \le y\} = F_{X,Y}(x, y)$ joint distribution function $= A \cap B$

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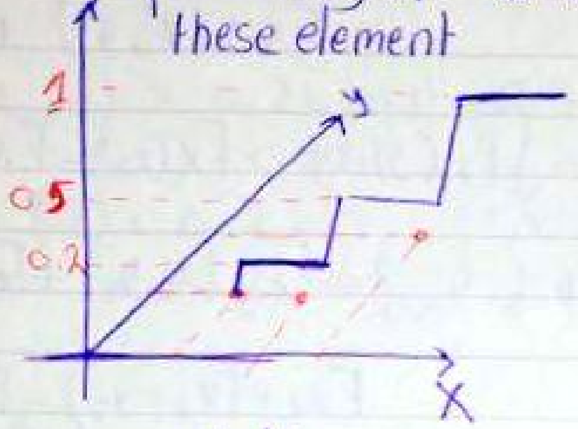
Ex: 4.2.1 / p110

discrets random variabl X and Y

S_J has elements $(1,1), (2,1), (3,3)$

Y probability of these element $0.2, 0.3, 0.5$



$F_{X,Y}(x,y)$??

$$\ast \; F_{X,Y}(x,y) = \sum_{n=1}^{N}\sum_{m=1}^{M} P(x_n, y_n)\, u(x-x_n)\, u(y-y_n)$$

$$= \overset{0.2}{\cancel{P(1,1)}}\, u(x-1)\, u(y-1) + \overset{0.3}{\cancel{P(2,1)}}\, u(x-2)\, u(y-1) + \overset{0.5}{\cancel{P(3,3)}}\, u(x-3)\, u(y-3)$$

$$P\{X \le x, Y \le y\} = F_{X,Y}(x,y)$$

→ if we have N R.Vs

$$X_1, X_2, X_3, \ldots, X_n, \ldots, X_N$$

$$F_{X_1, X_2, \ldots, X_n, \ldots, X_N}(x_1, x_2, \ldots, x_n, \ldots, x_N) =$$

$$P\{X_1 \le x_1, X_2 \le x_2, \ldots, X_n \le x_n, \ldots, X_N \le x_N\}$$

$\ast$ propablies of Distribution func:

(1) $F_X(-\infty) =$ $\quad F_{X,Y}(-\infty,-\infty) = 0$

$F_{X,Y}(-\infty,y) = 0 \quad F_{X,Y}(x,-\infty) = 0.$

$F_X(\infty) = 1$ ② $F_{X,Y}(\infty,\infty) = 1$

$0 \le F_X(x) \le 1$ $\quad$ $0 \le F_{X,Y}(x,y) \le 1$

$F_X(x^+)$ $\quad$ $F_{X,Y}(x^+,y^+)$ non decreasing function

$$P\{x_1 < X \le x_2\} = \int_{x_1}^{x_2} f_X(y)\,dy \Rightarrow F_X(x_2) - F_X(x_1)$$

⑤ $$P\{x_1 \le X \le x_2,\ y_1 < Y \le y_2\} = F_{X,Y}(x_2,y_2) + F_{X,Y}(x_1,y_1) - F_{X,Y}(x_1,y_2) - F_{X,Y}(x_2,y_1)$$

$F_{X,Y}(x,\infty) = F_X(x)$

$F_{X,Y}(\infty,y) = F_Y(y)$

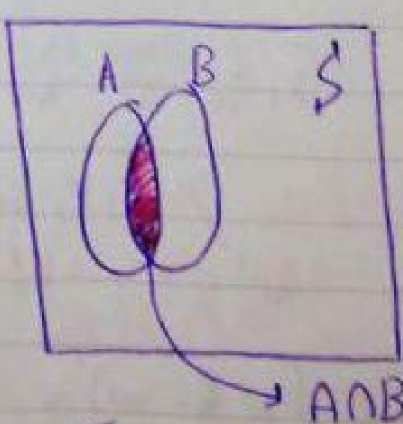
marginal distribution Function

To verify validity of $F_{X,Y}(x,y)$ we take ①, ②, ⑤

* Marginal Distribution functions:

$F_X(x) = F_{X,Y}(x,\infty)$

$F_{X,Y}(x,y) = P\{X \le x, Y \le y\}$



$y \to \infty$

$B \to S$

$A \cap B = A$

$P\{X \le x, Y \le y\} = P\{X \le x\} = F_X(x)$

Ex 4.2.2 / P 18 80

$$F_{X,Y}(x,y) = 0.2\,u(x-1)\,u(y-1) + 0.3\,u(x-2)\,u(y-1) + 0.5\,u(x-3)\,u(y-3)$$

$$F_X(x) = 0.2\,u(x-1) + 0.3\,u(x-2) + 0.5\,u(x-3)$$

$$F_Y(y) = 0.2\,u(y-1) + 0.3\,u(y-1) + 0.5(y-3)$$
$$= 0.5\,u(y-1) + 0.5\,u(y-3)$$

$\Rightarrow$ N R.Vs $X_1, X_2, \dots, X_n, \dots, X_N$

$N-K$

To get K dimensional marginal CDF you put $N-K$ R.Vs to infinity

4 R.Vs $\left\{ \begin{array}{l} N=3 \\ K=2 \end{array} \right.$ $\left. \begin{array}{l} N=3 \\ K=1 \end{array} \right\}$ 2 R.V

4.3 Joint Density and it's properties :-

1. $F_X(x)$, $F_{X,Y}(x,y)$

2. $f_X(x) = \frac{d}{dx} F_X(x)$, $f_{X,Y}(x,y) = \frac{d^2 F_{X,Y}(x,y)}{dx\,dy}$

3) $\sum_n p(x_i)\, u(x-x_i)$, $\sum_i \sum_j p(x_i, y_j)\, u(x-x_i)\, u(y-y_j)$

* discrete x, y

$$f_{X,Y}(x,y) = \sum_i \sum_j p(x_i, y_j)\, \delta(x-x_i)\, \delta(y-y_j)$$

$$f_{X_1, X_2 \dots X_N}(x_1, x_2, \dots x_N) = \frac{d^N F(x_1, x_2 \dots x_N)}{dx_1\, dx_2 \dots dx_n}$$

properties of densits func :-

validity

① $f_{X,Y}(x,y) \geq 0$

② $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{X,Y}(x,y)\, dx\, dy = 1$ (area under pdf)

③ $F_{X,Y}(x,y) = \int_{-\infty}^{x}\int_{-\infty}^{y} f_{X,Y}(\xi_1, \xi_2)\, d\xi_2\, d\xi_1$

④ $F_X(x) = \int_{-\infty}^{x}\int_{-\infty}^{\infty} f_{X,Y}(\xi_1, \xi_2)\, d\xi_2\, d\xi_1$

marginal distribution function

$$F_Y(y) = \int_{-\infty}^{\infty}\int_{-\infty}^{y} f_{X,Y}(\xi_1, \xi_2)\, d\xi_1\, d\xi_2$$

$f_X(x) =$

marginal density function

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⑤ $P\{x_1 < X \le x_2,\ y_1 < Y \le y_2\} = \int_{x_1}^{x_2}\int_{y_1}^{y_2} f_{X,Y}(x,y)\,dy\,dx$

⑥ $f_X(x) = \int_{-\infty}^{\infty} f_{XY}(x,y)\,dy$

$= \frac{d}{dx} F_X(x)$

$f_Y(y) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dx$

Ex 4.3.1 /P115

$$g_{X,Y}(x,y) = \begin{cases} b\,e^{-x}\cos y & 0 \le x \le 2,\quad 0 \le y \le \pi/2 \\ 0 & \text{other} \end{cases}$$

b constant

$g_{X,Y}(x,y) > 0$

$\int_0^{\pi/2}\int_0^{2} b\,e^{-x}\cos y\,dx\,dy =$

$b\int_0^{\pi/2}\cos y\,dy\int_0^{2} e^{-x}\,dx =$

$b\,[\sin y]_0^{\pi/2}\,[-e^{-x}]_0^{2} = b(1-e^{-2}) = 1$

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Ex 4.3.2 / p/15

$f_X(x)$ For $f_{X,Y}(x,y) = u(x)u(y)x\,e^{-x(y+1)}$

$f_Y(y)$

$$f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dy$$

$$= \int_{\cancel{-\infty}\ 0}^{\infty} u(x)\cancel{u(y)}\,x\,e^{-x(y+1)}\,dy$$

$$= \int_0^{\infty} u(x)\,x\,e^{-x(y+1)}\,dy$$

$$= u(x)e^{-x}\int_0^{\infty} x\,e^{-xy}\,dy$$

$$f_X(x) = u(x)(-1)e^{-x}e^{-xy}\Big|_0^{\infty}$$

$$= u(x)\,e^{-x}$$

$$f_Y(y) = \int_{\cancel{-\infty}\ 0}^{\infty} \cancel{u(x)}\,u(y)\,x\,e^{-x(y+1)}\,dx$$

$$= \int_0^{\infty} u(y)\,x\,e^{-x(y+1)}\,dx \qquad a = -(y+1)$$

$$\int x e^{ax}dx = e^{ax}\left[\frac{x}{a} - \frac{1}{a^2}\right]\Big|_0^{\infty}$$

$$f_Y(y) = \frac{u(y)}{(1+y)^2}$$

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$\rightarrow$ N R.Vs

$$X_1, X_2, \ldots, X_n, \ldots, X_N$$

K marginal pdf
$K < N$ cdf

N
K
N - K integration

$$f_{X_1, X_2, \ldots, X_K}(x_1, x_2, \ldots, x_K) = \underbrace{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f_{X_1, X_2, \ldots, X_N}(x_1, x_2, \ldots, x_N)}_{N-K \text{ integration}}$$

Note:
$N = 3$
$K = 2$
X_1, X_2, X_3

$$f_{X_1, X_2}(x_1, x_2) = \int_{-\infty}^{\infty} f_{X_1, X_2, X_3}(x_1, x_2, x_3)\, dx_3$$

4.4 Conditional Distribution and density

$$F_X(x \mid B) = P\{X \le x \mid B\}$$

$$F_X(x \mid X \le b) = \begin{cases} \dfrac{F_X(x)}{F_X(b)} & x < b \\ 1 & x \ge b \end{cases}$$

$$f_X(x \mid X \le b) = \begin{cases} \dfrac{f_X(x)}{\int_{-\infty}^{b} f_X(x)\, dx} & x < b \\ 0 & x \ge b \end{cases}$$

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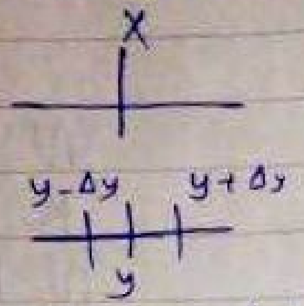
$P\{X \leq x \mid B)$



$\Delta y \sim 0$

$B = \{Y \leq y\}$

$B = \{y - \Delta y < Y \leq y + \Delta y\}$

point conditioning : finding distribution of a R.v conditional on occurance of the other R.v with a specific point

$$F_X(x \mid B) = F_X(x \mid y-\Delta y < Y \leq y+\Delta y)$$

$$= \int_{-\infty}^{x} \int_{y-\Delta y}^{y+dy} f_{x,y}(\xi_1, \xi_2)\, d\xi_2\, d\xi_1$$

$$\int_{y-\Delta y}^{y+\Delta y} f_y(\xi)\, d\xi$$

→ Conditional Distribution and Density func ⇐

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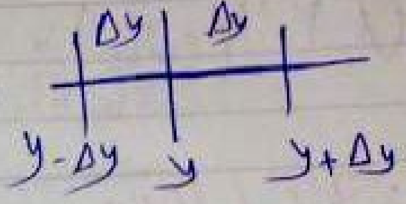
$$F_X(x|B) = \frac{P\{X \le x | B\}}{P\{B\}}$$

→ point conditioning



$\Delta y \to 0$

$$P\{X \le x | Y = y_k\}$$

$$B = \{y - \Delta y \le Y \le y + \Delta y\}$$

$$A = \{X \le x\}$$

$$P\{A|B\} = P\{X \le x, y - \Delta y \le Y \le y + \Delta y\}$$

$$F_X(x|B) = \frac{P\{X \le x, y-\Delta y \le Y \le y+\Delta y\}}{P\{y-\Delta y \le Y \le y+\Delta y\}}$$

Conditional distribution

$$= \frac{\int_{-\infty}^{x} \int_{y-\Delta y}^{y+\Delta y} f_{X,Y}(\zeta_1, \zeta_2)\, d\zeta_1\, d\zeta_2}{\int_{y-\Delta y}^{y+\Delta y} f_Y(\zeta)\, d\zeta}$$

descret
$$\begin{cases} X & x_i \quad i = 1, \dots, N \\ Y & y_j \quad j = 1, \dots, M \end{cases}$$

$$f_{X,Y}(x,y) = \sum_{i=1}^{N} \sum_{j=1}^{M} P(x_i, y_j)\, \delta(x - x_i)\, \delta(y - y_j)$$

$$F_X(x \mid y = y_k) = \sum_{i=1}^{N} \frac{p(x_i, y_k)}{p(y_k)} u(x - x_i)$$

$$f_X(x \mid Y = y_k) = \sum_{i=1}^{N} \frac{p(x_i, y_k)}{p(y_k)} \delta(x - x_i)$$

Ex: 4.4.1/P117

$p(x_1, y_1) = \frac{2}{15}$, $p(x_2, y_1) = \frac{3}{15}$

$p(x_1, y_3) = \frac{4}{15}$, $p(x_2, y_2) = \frac{1}{15}$

$p(x_2, y_3) = \frac{5}{15}$ $\quad f_X(x \mid Y = y_3) = ??$

$$f_{X,Y}(x, y) = \frac{2}{15}\delta(x - x_1)\delta(y - y_2) + \frac{3}{15}\delta(x - x_2)\delta(y - y_1) + \frac{4}{15}\delta(x - x_1)\delta(y - y_3) + \frac{1}{15}\delta(x - x_2)\delta(y - y_2) + \frac{5}{15}\delta(x - x_2)\delta(y - y_2)$$

$$p(y_3) = p(x_1, y_3) + p(x_2, y_3) = \frac{4}{15} + \frac{5}{15} = \frac{9}{15}$$

$$(f_X \mid y_3) = \frac{p(x_1, y_3)}{p(y_3)}\delta(x - x_1) + \frac{p(x_2, y_3)}{p(y_3)}\delta(x - x_2)$$

$$= \frac{\frac{4}{15}}{\frac{9}{15}}\delta(x - x_1) + \frac{\frac{5}{15}}{\frac{9}{15}}\delta(x - x_2)$$

$$= \frac{4}{9}\delta(x - x_1) + \frac{5}{9}\delta(x - x_2)$$

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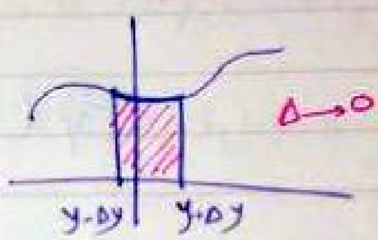
$$F_X(x \mid y-\Delta y \leq Y \leq y+\Delta y) = \frac{\int_{-\infty}^{x}\int_{y-\Delta y}^{y+\Delta y} f_{X,Y}(z_1,z_2)\,dz_2\,dz_1}{\int_{y-\Delta y}^{y+\Delta y} f_Y(z)\,dz \neq 0}$$

$$= \lim_{\Delta y \to 0} \frac{\int_{-\infty}^{x}\int_{y-\Delta y}^{y+\Delta y} f_{X,Y}(z_1,z_2)\,dz_2\,dz_1}{\underbrace{\int_{y-\Delta y}^{y+\Delta y} f_Y(z)\,dz}_{2\Delta y\, f_Y(y)}}$$



$$= \lim_{\Delta y \to 0} \frac{\int_{-\infty}^{x} f_{X,Y}(z_1,y)\,dz_1\, \cancel{2\Delta y}}{\cancel{2\Delta y}\, f_Y(y) \neq 0}$$

> 0

$$F_X(x \mid y) \Rightarrow \frac{\int_{-\infty}^{x} f_{X,Y}(z_1,y)\,dz_1}{f_Y(y)}$$

$$F_Y(y \mid x) = \frac{\int_{-\infty}^{y} f_{X,Y}(x,z)\,dz}{f_X(x)} \longrightarrow \text{when we derive this is constant}$$

$\Downarrow$

$P\{Y \leq y \mid X = x\}$

$$f_X(x \mid y) = \frac{d\, F_X(x \mid y)}{dx} = \frac{f_{X,Y}(x,y)}{f_Y(y)}$$

Set theorem $\Rightarrow$ $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

$$f_Y(y \mid x) = \frac{f_{X,Y}(x,y)}{f_X(x)}$$

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Ex 4.4.2 /119 Q.

o) $f_Y(y|x)$?

$f_{X,Y}(x,y) = u(x)u(y)\, x\, e^{-x(y+1)}$

Ex 4.3.2 / P115

$$f_X(x|y) = \frac{f_{X,Y}(x,y)}{f_Y(y)} > 0$$

$$f_Y(y|x) = \frac{f_{X,Y}(x,y)}{f_X(x)}$$

$f_X(x) = u(x)\, e^{-x}$

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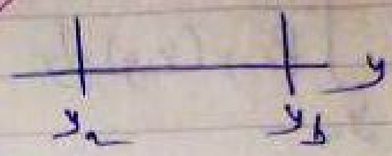
$$F_X(x \mid y_a < Y \le y_b) = \frac{\int_{y_a}^{y_b}\int_{-\infty}^{x} f_{X,Y}(\xi, y)\, d\xi\, dy}{\int_{y_a}^{y_b} f_Y(y)\, dy \ne 0}$$

$$\int_{y_a}^{y_b} f_Y(y)\,dy = F_Y(y_b) - F_Y(y_a)$$

$A = \{X \le x\} \to X$

$B = \{y_a < Y \le y_2\} \to Y$

Set Theorem $P\{A \mid B\} = \dfrac{P(A \cap B)}{P(B)}$



$B = \{y_a < Y \le y_b\}$

$P\{X \le x \mid y_a < Y \le y_b\}$

$F_X(x \mid y_a \le y \le b)$

$$f_X(x) = \frac{d}{dx} F_X(x)$$

$$F_X(x) = \int_{-\infty}^{\infty} f_X(x)$$

$$= \frac{F_{X,Y}(x, y_b) - F_{X,Y}(x, y_a)}{F_Y(y_b) - F_Y(y_a)}$$

Ex 4.4.3 / p/206 Find $f_X(x \mid Y \le y)$?? $\quad -\infty < Y \le y$

$$\text{for} \Rightarrow f_{X,Y}(x,y) = u(x)\,u(y)\,x\,e^{-x(y+1)}$$

$$f_X(x \mid y_a < Y \le y_b) = \frac{\int_{y_a}^{y_b} f_{X,Y}(x,y)\,dy}{\int_{y_a}^{y_b} f_Y(y)\,dy}$$

$$= \int_{-\infty}^{y} f_Y(y)\,dy :$$

$$\Rightarrow f_Y(y) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dx = \frac{u(y)}{(1+y)^2}$$

Ex 4.3.2 / P115

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$$\int_{y_a}^{y_b} f_{x,y}(x,y)\,dy = \int_{-\infty}^{y} u(x)\,u(y)\,x e^{-x(y+1)}\,dy =$$

$$= u(x)\int_{0}^{y} x e^{-x(y+1)} = u(x)\,e^{-x}(1-e^{-xy})$$

$$\Rightarrow \int_{y_a}^{y_b} f_Y(y)\,dy = \int_{-\infty}^{y} \frac{u(y)}{(1+y)^2}\,dy$$

$$= \frac{y}{1+y} \qquad y>0,\; x>0$$

=> interval conditioning

$$f_X(x \mid Y \leq y) = \frac{u(x)\,e^{-x}(1-e^{-xy})}{\frac{y}{y+1}}$$

$$= \frac{y+1}{y}\,u(x)\,u(y)\,e^{-x}(1-e^{-xy})$$

=> point conditioning

$$f_X(x \mid y) = (y+1)^2 x\, e^{-x(y+1)}\, u(x)\, u(y)$$

4.5 statistically independant R.V of

Set Theorem

event A   event B $P(A \cap B) = P(A) \cdot P(B)$

$A \rightarrow X$

$B \rightarrow Y$

$$P\{X \leq x, Y \leq y\} = P\{X \leq x\} \cdot P\{Y \leq y\}$$

$$F_{X,Y}(x,y) = F_X(x) \cdot F_Y(y)$$

$$f_{X,Y}(x,y) = f_X(x) \cdot f_Y(y)$$

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$P(A \backslash B) = P(A)$
$P(B \backslash A) = P(B)$ } set theorem

$F_X(x \backslash Y \leq y) = F_X(x)$

$F_Y(y \backslash X \leq x) = F_Y(y)$

→ For N R.Vs

$X_1, X_2, \ldots, X_n, \ldots X_N$

$A_i = \{X \leq x_i\}$

the random Variable are Statistically independent if any group of R.Vs is independent From any other groups.

4.6 Distribution and Density of a Sum of R.V.

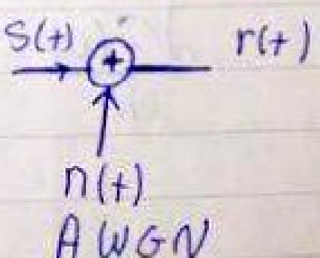
=> (Statistically independent R.V)

X , X & Y are indep.

Y

$W = X + Y$

Com System



$F_W(w) = P\{W \leq w\}$
$= P\{X + Y \leq w\}$

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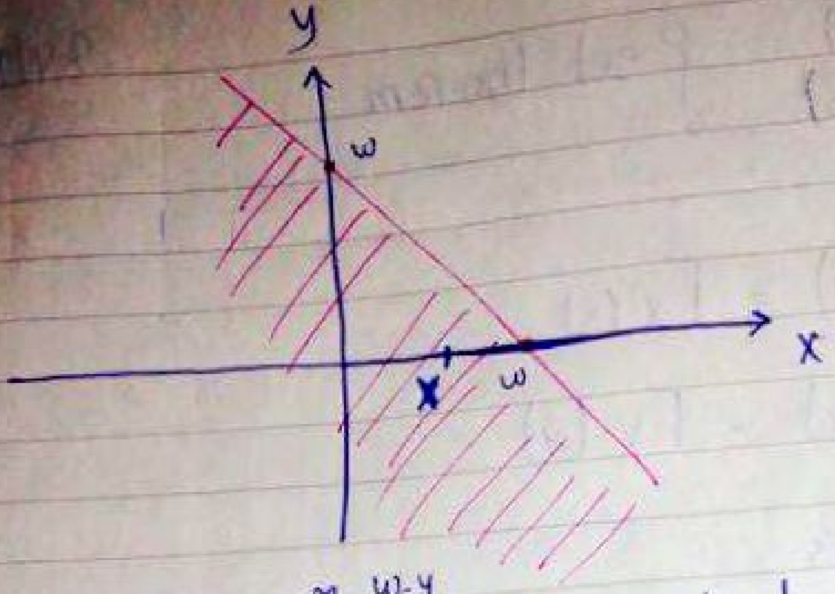
$y = w - x$



$$F_W(w) = \int_{-\infty}^{\infty}\int_{-\infty}^{w-y} \underbrace{f_{X,Y}(x,y)}_{f_X(x)\cdot f_Y(y)}\,dx\,dy$$

$$= \int_{-\infty}^{\infty} f_Y(y) \cdot \int_{-\infty}^{w-y} f_X(x)\,dx\,dy$$

$\Rightarrow$ Leibniz's Rule

Appendix (G)

$$G(u) = \int_{\alpha(u)}^{\beta(u)} H(x,u)\,dx$$

$$f_W(w) = \frac{d}{dw}\int_{-\infty}^{\infty} f_Y(y)\int_{-\infty}^{w-y} f_X(x)\,dx\,dy$$

$$\frac{dG(u)}{du} = H(\beta(u),u)\frac{d\beta(u)}{du} - H(\alpha(u),u)\frac{d\alpha(u)}{du} + \int_{\alpha(u)}^{\beta(u)} \frac{d\,H(x,u)}{du}\,dx$$

$$f_W(w) = \int_{-\infty}^{\infty} f_Y(y)\, f_X(w-y)\,dy$$

$$f_W(w) = f_X(x) * f_Y(y)$$

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Ex 4.6.1 /p 123

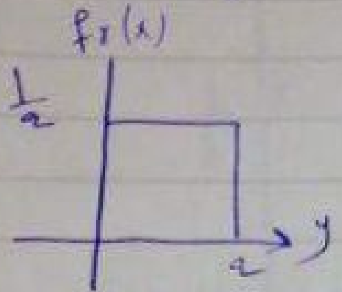
$W = X + Y$

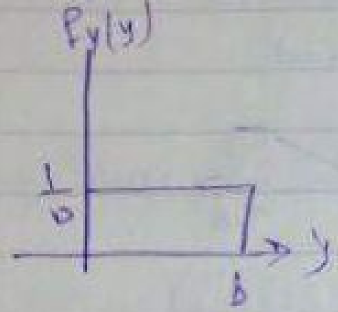
$f_X(x) = \frac{1}{a}\left(u(x) - u(x-a)\right)$ $\qquad 0 < a < b$

$X > 0$

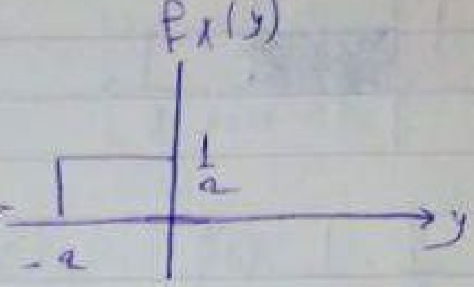
$f_Y(y) = \frac{1}{b}\left(u(y) - u(y-b)\right)$ $\qquad Y > 0$

$W > 0$





$\Rightarrow$



$W = X + Y$

$f_W(w) = f_X(x) * f_Y(y)$

$$= \int_{-\infty}^{\infty} f_Y(y)\, f_X(w-y)\, dy$$

① $f_X(-y)$

② w is a shift for the y

$w > 0$

Value of $f_X(y)$

③

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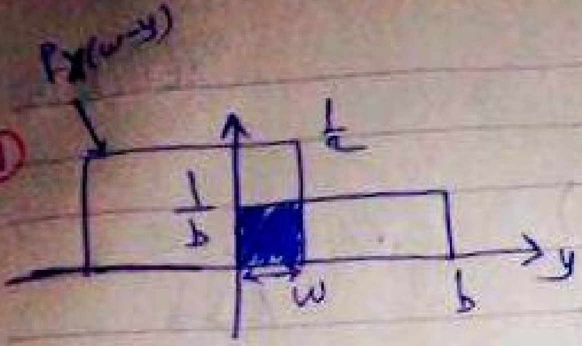
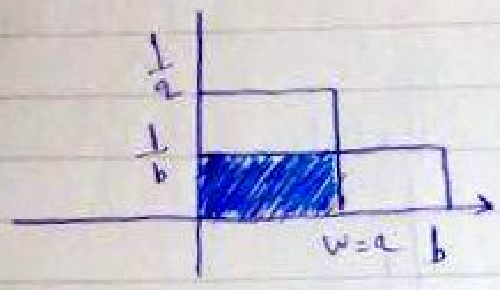
1

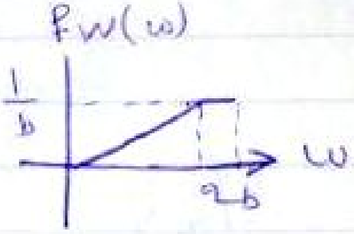


$\frac{1}{b} \cdot \frac{1}{a} \cdot w$

2



$\frac{1}{b} \cdot \frac{1}{2} \cdot w \Rightarrow$



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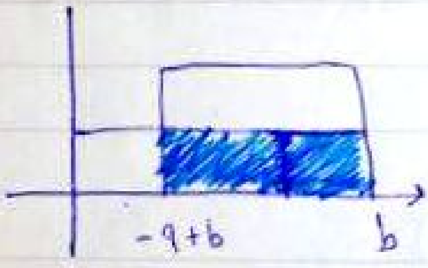
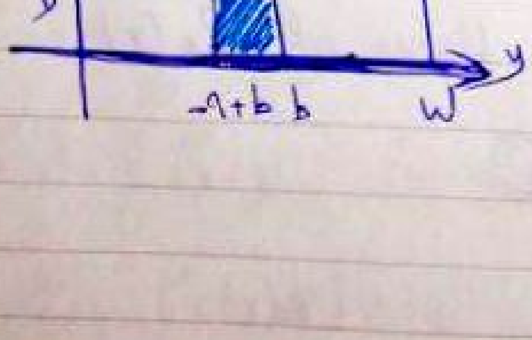


$w = b$

$\frac{1}{b}$

4



$\frac{1}{2b}(b+a-w)$

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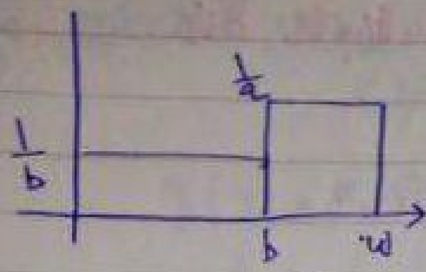
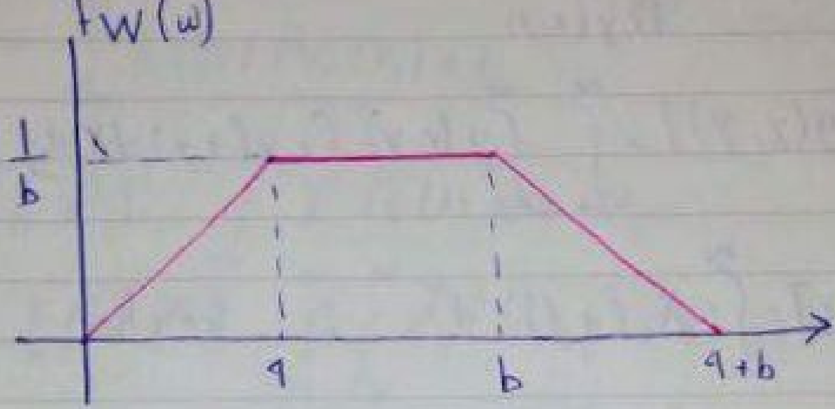


$w = b + a$



→ Sum of several indep RVs

$X_1, X_2, \ldots, X_n, \ldots, X_N$

$Y = X_1 + X_2 + \ldots + X_N$

$$f_Y(y) = \underbrace{f_{X_1}(x_1) * f_{X_2}(x_2)}_{f_{Y_1}(y_1)} * \ldots * f_{X_N}(x_N)$$

$$\int_{-\infty}^{\infty} f_{X_2}(x_2)\, f_{X_1}(w - x_2)\, dx_2$$

4.1.2, 4.2.1, 4.2.4, 4.2.8, 4.2.11, 4.3.2, 4.3.10, 4.3.15, 4.3.19, 4.4.1, 4.4.8, 4.5.3, 4.5.5, 4.5.10, 4.6.2, 4.6.4, 4.6.10, 4.6.13

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Ch. 5 operations on Multiple R.Vs

$$g(X, Y)$$

$$\downarrow \quad \downarrow$$

$$x \quad y$$

$$\downarrow \quad \downarrow$$

$$f_{X,Y}(x,y)$$

$Y = 5X - x$

$$\bar{g} = E[g(X, Y)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x, y)\, f_{X,Y}(x,y)\, dx\, dy$$

$$E[X] = \int_{-\infty}^{\infty} x\, f_X(x)\, dx \longleftarrow \text{remeber .}$$

→ For N $X_1, X_2, ---, X_N$

$g(X_1, X_2 -- X_N)$ $f_{X_1, -- X_N}(x_1, x_2, -- x_N)$

$$\bar{g} = E[g(--)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} -- \int_{-\infty}^{\infty} g(x_1, -- x_N)\, dx_1 -- dx_N$$

Ex 5.1.1 / p142

weighted R.Vs

$$g(X_1, X_2, --, X_N) = \sum_{i=1}^{N} \alpha_i X_i$$

$$= E[\sum_{i=1}^{N} \alpha_i X_i]$$

$$= \sum_{i=1}^{N} E[\alpha_i X_i]$$

$$= \sum_{i=1}^{N} \alpha_i E[X_i]$$

$$= \sum_{i=1}^{N} \alpha_i \bar{X_i}$$

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— The expected value of sum of R.V.s is the sum of expected value of ind. R.V

$$g(x_1, x_2, \ldots x_N) = g(x_i)$$

$$\bar{g} = E[g(x_1)]$$

$$= \int_{-\infty}^{\infty} g(x_1) f_{x_1}(x_1)\, dx_1$$

$$m_n = E[x^n] = \int_{-\infty}^{\infty} x^n f_x(x)\, dx \Leftarrow$$

$$m_{n,k} = E[x^n y^k] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^n y^k f_{x,y}(x,y)\, dx\, dy$$

$$m_{n0} = E[x^n]$$

$$m_{0k} = E[y^k]$$

order of the joint moment is $n+k$

order is 2 $\rightarrow$ $\left.\begin{matrix} m_{02} \\ m_{11} \\ m_{20} \end{matrix}\right\}$ 2nd moment of joint R.V.s

m_{01} called center of gravity of joint fun $f_{x,y}(x,y)$

$m_{11} = E[XY]$ correlation R.V X and Y

$$R_{xy} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x\, y\, f_{x,y}(x,y)\, dx\, dy$$

X and Y are statistically independent

$$R_{x,y} = m_{10}\, m_{01} = E[X]\, E[Y]$$

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Probability Notebook

POWER UNIT

DR . AHMAD ATYA

BY : MARAH ALOMARI

29/11/2015

$$m_{02} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^{0} y^{2} f_{X,Y}(x,y)\,dx\,dy$$

تصحيح
للمحاضرة
الماضية

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^{2} f_{X,Y}(x,y)\,dx\,dy$$

$$= \int_{-\infty}^{\infty} y^{2}\,dy \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dx$$

$$= \int_{-\infty}^{\infty} y^{2} f_{Y}(y)\,dy$$

if R.Vs X and Y statically independent, they are un correlated but if they are un correlated, they are not neccessary independent.

→ The R.Vs X and Y are orthogonal if

$$R_{XY} = 0$$

Ex 5.1.2 /P142

$\bar{X} = 3$ $\quad \underbrace{\sigma_X^2 = 2}_{X}$

2nd moment of X

$$E[X^2] = \sigma_X^2 + \bar{X}^2 = 2 + 3^2 = 11$$

$$Y = -6X + 22 \leftarrow \text{dependent on } X$$

$$\bar{Y} = E[-6X + 22]$$

$$= -6\bar{X} + 22 = 4$$

→

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$R_{xy} = E[XY]$

$= E[X(-6X+22)]$

$= E[-6X^2 + 22X]$

$= -6E[X^2] + 22E[X]$

$= -6(11) + 22(3)$

$= 0$

X and Y orthogonal

$R_{xy} = E[X]\,E[Y]$

$= 3 \cdot 4$

$0 \neq 12$

Corolated
they are dependent
$R_{xy} = E[X] \cdot E[Y]$

independent $\longrightarrow$ uncorrolated

uncorrolated $\longrightarrow$ dependent
$\longrightarrow$ independent

N R.Vs $\quad X_1, X_2, ---, X_n, ---, X_N$

$n_1, n_2, ---, n_n, ---, n_N$

$n_1 + n_2 + --- + n_N$ order of the moment

$$m_{n_1 n_2 --- n_N} = E[X_1^{n_1} X_2^{n_2} --- X_N^{n_N}]$$

$$= \underbrace{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} --- \int_{-\infty}^{\infty}}_{N\text{- integral}} x_1^{n_1} x_2^{n_2} --- x_N^{n_N} f_{X_1, X_2, ---X_N}(x_1, ---x_2 --- x_N)\, dx_1\, dx_2 --- dx_N$$

$\left.\begin{matrix} m_{01} \\ m_{10} \end{matrix}\right\}$ Central Gravity

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Joint Central Moment μ_{nk}

$n+k$ order

$$\mu_{nk} = E[(X-\bar{X})^n (Y-\bar{Y})^k]$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x-\bar{X})^n (y-\bar{Y})^k f_{X,Y}(x,y)\, dx\, dy$$

Variance of X $\sigma_X^2 = \mu_{20} = E[(X-\bar{X})^2]$

$$= \int_{-\infty}^{\infty} (x-\bar{X})^2 f_X(x)\, dx$$

$$\sigma_Y^2 = \mu_{02} = E[(Y-\bar{Y})^2] = \int_{-\infty}^{\infty} (y-\bar{Y})^2 f_Y(y)\, dy$$

2nd central joint moment

→ Covariance $= \mu_{11}$
$= C_{XY}$

Note
$E[g(x)]$
$E[ax^2] + E[bx] + E[c]$

$$C_{XY} = E[(X-\bar{X})(Y-\bar{Y})]$$

$$= E[XY - X\bar{Y} - \bar{X}Y + \bar{X}\bar{Y}]$$

$$= E[XY] - E[X]\bar{Y} - \bar{X}E[Y] + \bar{X}\bar{Y}$$

$$= R_{XY} - E(X)E(Y)$$

if X and Y are not correlated

$$C_{XY} = E[X]E[Y] - E[X]E[Y] = 0$$

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$C_{xy} = R_{xy} - E[X]E[Y]$

$R_{xy} = E[XY]$

normalized 2nd order moment :- (ρ)

$$(\rho) = \frac{\mu_{11}}{\sqrt{\mu_{20}\mu_{02}}} = \frac{C_{xy}}{\sigma_x \sigma_y}$$

$$= E\left[\left(\frac{X-\bar{X}}{\sigma_x}\right)\left(\frac{Y-\bar{Y}}{\sigma_y}\right)\right]$$

$$0 \leq \rho \leq 1$$

→ For N R.Vs $X_1, X_2, \ldots X_N$

$$\mu_{n_1, n_2, n_3 \ldots, n_N} = E[(X_1-\bar{X})^{n_1} \ldots (X_N-\bar{X}_N)^{n_N}]$$
$$n_1+n_2+\ldots+n_N$$

$$= \int_{-\infty}^{\infty}\!\!\int_{-\infty}^{\infty} (x_1-\bar{x}_1)^{n_1} \ldots (x_N-\bar{x}_N)^{N_N} f_{X_1,X_2 \ldots X_N}(x_1, x_2 \ldots x_N)\, dx_1\, dx_2 \ldots dx_N$$

Summary :-

1) Correlation $R_{xy} = E[XY]$
2) Covariance $C_{xy} = R_{xy} - E[X]E[Y]$
3) orthogonal $R_{xy} = 0$, $C_{xy} = -E[X]E[Y]$
4) uncorrelated $R_{xy} = E[X]E[Y]$

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Ex 5.1.3 / p145 :- Find the variance of X

$\sigma_X^2 = E[(X-\bar{X})^2] = E[\sum_{i=1}^{N} \alpha_i (X_i - \bar{X}_i) \sum_{j=1}^{N} \alpha_j (X_j - \bar{X}_j)]$

$= E[X^2] - \bar{X}^2$

$= \sum_{i=1}^{N} \sum_{j=1}^{N} \alpha_i \alpha_j \underbrace{E[(X_i - \bar{X}_i)(X_j - \bar{X}_j)]}_{?}$

1) assume X_i are uncorrolated
2) if $i \neq j \Rightarrow$ equal zero $\quad C_{X_i X_j}$

$R_{X_i X_j} = E[X_i] E[X_j]$

if $i = j \Rightarrow$

$E[(X_i - \bar{X}_i)^2] \Rightarrow$ the veriance

$$C_{X_i, X_j} = \begin{cases} 0 & i \neq j \\ \sigma_{X_i}^2 & i = i \end{cases}$$

$\sigma_X^2 = \sum_{i=1}^{N} \alpha_i \sigma_{X_i}^2$

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$C_{xy} = R_{xy} - E[X]E[Y]$

1) $R_{xy} = E[X]E[Y]$

Y and X same varianse

$C_{xy} = 0$

2) if X & Y are orthogonal and either N both R.Vs mean are zero

$C_{xy} = -E[x]E[Y]$

$$\Phi_X(\omega) = E[e^{j\omega}]$$

$$= \int_{-\infty}^{\infty} f_X(x)e^{j\omega x}\,dx$$

$$f_X(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \Phi_X(\omega)e^{-j\omega x}\,d\omega$$

5.2 Joint Characteristic Func:

X and Y

$$\Phi_{X,Y}(\omega_1,\omega_2) = E[e^{j\omega_1 x} e^{j\omega_2 y}]$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{X,Y}(x,y)\, e^{j\omega_1 x} e^{j\omega_2 y}\,dx\,dy$$

$$f_{X,Y}(x,y) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \Phi_{X,Y}(\omega_1,\omega_2)e^{-j\omega_1 x} e^{-j\omega_2 y}\,d\omega_1 d\omega_2$$

$$\phi_X(\omega_1) = \Phi_{X,Y}(\omega_1, 0)$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{X,Y}(x,y)\, e^{j\omega_1 x} \cancel{e^{j\omega_2 y}}^{\,1}\,dx\,dy$$

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$$= \int_{-\infty}^{\infty} e^{j\omega_1 x} \underbrace{\int_{-\infty}^{\infty} f_{x,y}(x,y)\, dy}_{f_x(x)}\, dx$$

$$= \int_{-\infty}^{\infty} f_x(x) e^{j\omega_1 x}\, dx$$

$$m_{nk} = (-j)^{n+k} \left. \frac{d^{n+k}\, \Phi_{x,y}(\omega_1,\omega_2)}{d\omega_1^n\, d\omega_2^k} \right|_{\omega_1=\omega_2=0}$$

$n+k$

Ex 5.2.1 / p147 80

$$\Phi_{x,y}(\omega_1,\omega_2) = e^{-\omega_1^2 - 8\omega_2^2}$$

Show that R.Vs x and y have zero mean and uncorrelated

$$\bar{X} = m_{10}$$

$$\bar{Y} = m_{01}$$

$$R_{XY} = E[XY] = \mu_{11}$$

$$\bar{X} = m_{10} = E[X]$$

$$= -j \left. \frac{d\, \Phi_x(\omega_1,\omega_2)}{d\omega_1} \right|_{\omega_1=\omega_2=0}$$

$$= -j \left[-2\omega_1 e^{-\omega_1^2 - 2\omega_2^2} \right]_{\omega_1=\omega_2=0}$$

$$= 0$$

$$\bar{Y} = m_{01} = E[Y]$$

$$= -j \frac{d}{d\omega_2} \left[\Phi_{X,Y}(\omega_1,\omega_2) \right]_{\omega_1=\omega_2=0}$$

$$= 0$$

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$R_{x,y} = E[XY] = m_{11}$

$= (-j)^{1+1} \cdot (-2\omega_1)(-16\omega_2) \cdot e^{-2(\omega_1^2 + 3\omega_2^2)} \Big|_{\omega_1 = \omega_2 = 0}$

$= 0$

→ N R.Vs $X_1, X_2, \ldots X_n, \ldots N_N$

$\Phi_{X_1, X_2, \ldots, X_n}(\omega_1, \omega_2, \ldots, \omega_N) = E[e^{j\omega_1 x_1} e^{j\omega_2 x_2} \ldots e^{j\omega_N x_N}$

$= E[e^{\sum_{i=1}^{N} \omega_i x_i}$

$m_{n_1 n_2 \ldots n_N} = (-j)^R \dfrac{\partial^R \Phi_{X_1, X_2, \ldots, X_N}(\omega_1, \omega_2, \ldots, \omega_N)}{\partial \omega_1^{n_1} \partial \omega_2^{n_2} \ldots \partial \omega_n^{n_N}} \Bigg|_{\omega_i = 0 \quad i = 1, \ldots, N}$

$R = n_1 + n_2 + \ldots + n_N$

Ex 5.2.2 /p148 let $Y = X_1 + X_2 + \ldots + X_N$ (N statistically indep. R.Vs)

$f_{X_1, X_2, X_3 \ldots X_N}(x_1, x_2, \ldots x_N) = \prod_{i=1}^{N} f_{X_i}(x_i)$

$\Phi_Y(\omega) = E[e^{j\omega Y}]$

$= E[e^{j\sum_{i=1}^{N} \omega_i x_i}]$

$= E[e^{j\omega_1 x_1} e^{j\omega_2 x_2} \ldots e^{j\omega_N x_N}]$

$= \Phi_{X_1, X_2, \ldots, X_N}(\omega_1, \omega_2, \ldots \omega_N)$