20/12

No.

حل امتحان السكند:

Q1: Random Variable X

$Y = g(x)$

a) Find $f_Y(y)$ if $g(x) = 2F_X(x) + 4$

Sol

$g(x)$ non decreasing & ~~$4 < y < 6$~~

$4 < g(x) < 6$

because continuos ⇒ monotonically increasing

$$f_Y(y) = \frac{f_X(x_1)}{|g'(x_1)|} = \frac{\cancel{f_X(x_1)}}{2\cancel{f_X(x_1)}} = \frac{1}{2}, \quad 4<x<6$$

$$f_Y(y) = \begin{cases} \frac{1}{2}, & 4 < x < 6 \\ 0, & o.w \end{cases}$$

b) Find $y = g(x)$ such that y is uniform between $(6, 10)$, $6 < Y < 10$

$$Y = g(x) = 4F_X(x) + 6$$

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Q2 Poisson density function

$$f_x(x) = e^{-b} \sum_{k=0}^{\infty} \frac{b^k}{k!} \delta(x-k)$$

a) Find $\Phi_x(\omega)$

$$\Phi_x(\omega) = E[e^{j\omega x}] = \int_{-\infty}^{\infty} \left(e^{-b} \sum_{k=0}^{\infty} \frac{b^k}{k!} \delta(x-k)\right) e^{j\omega x} dx$$

$$\cancel{= \sum_{k=0}^{\infty} \frac{b^k}{k!} \delta(x-k) \int_{-\infty}^{\infty}}$$

$$= e^{-b} \sum_{k=0}^{\infty} \frac{b^k}{k!} \int_{-\infty}^{\infty} e^{j\omega x} \delta(x-k) \, dx$$

$$= e^{-b} \sum_{k=0}^{\infty} \frac{b^k}{k!} e^{j\omega k}$$

$$= e^{-b} \sum_{k=0}^{\infty} \frac{(b e^{j\omega})^k}{k!} \rightarrow \text{Taylor Series}$$

~~Taylor~~

$$\Phi_x(\omega) = e^{-b(1-e^{j\omega})}$$

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b) Find mean & variance

$$m_1 = (-j)\frac{d\Phi_x(\omega)}{d\omega}\Big|_{\omega=0} = b$$

mean

$$m_2 = (-j)^2\frac{d^2\Phi_x(\omega)}{d\omega^2}\Big|_{\omega=0} = b + b^2$$

$$\sigma_x^2 = m_2 - m_1^2 = b + b^2 - b^2 = b$$

variance

coordinate rotation

Q3 $\theta = \pi/6$, X & Y R.Vs

$C_{xy} = 0.8$, $\sigma_x = 1.9$

$$Y_1 = X\cos\theta + Y\sin\theta$$

$$Y_2 = -X\sin\theta + Y\cos\theta$$

a) find Y_1 & Y_2 in terms of X & Y

$$Y_1 = \frac{\sqrt{3}}{2}X + \frac{1}{2}Y$$

$$Y_2 = -\frac{1}{2}X + \frac{\sqrt{3}}{2}Y$$

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b) Find σ_y^2

$$\theta = \frac{1}{2}\tan^{-1}\left(\frac{2C_{xy}}{\sigma_x^2 - \sigma_y^2}\right)$$

$$\frac{\pi}{6} = \frac{1}{2}\tan^{-1}\left(\frac{2(0.8)}{(1.9)^2 - \sigma_y^2}\right)$$

$$\boxed{\sigma_y^2 = 2.686}$$

Q4 ~~$f_{xy}(x,y) = 0.4\,\delta(x+\alpha)$~~ $\delta(y-2)$

$$f_{xy}(x,y) = 0.4\,\delta(x+\alpha)\,\delta(y-2) + 0.3\,\delta(x-\alpha)\,\delta(y-2) + 0.1\,\delta(x-\alpha)\,\delta(y-\alpha) + 0.2\,\delta(x-1)\,\delta(y-1)$$

a) X & Y are uncorrelated

$C_{xy} = 0$, $C_{xy} = R_{xy} - \bar{X}\bar{Y}$, $R_{xy} = \bar{X}\bar{Y}$

$$E[XY] = E[X]\cdot E[Y]$$

$$E[X] = 0.4(-\alpha) + 0.3(\alpha) + 0.1(\alpha) + 0.2(1)$$
$$= 0.2$$

$$E[Y] = 0.4(2) + 0.3(2) + 0.1(\alpha) + 0.2(1)$$
$$= 0.1\alpha + 1.6$$

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$$E[XY] = 0.4(-\alpha)(2) + 0.3(\alpha)(2) + 0.1(\alpha)(\alpha) + 0.2(1)(1)$$

$$= -0.8\alpha + 0.6\alpha + 0.1\alpha^2 + 0.2$$

$$= 0.1\alpha^2 - 0.2\alpha + 0.2$$

$$E[XY] - \bar{X}\bar{Y} = 0.1\alpha^2 - 0.2\alpha + 0.2 - (0.2)(0.1\alpha + 1.6)$$

$$\boxed{\alpha_1 = 2.65} \quad \boxed{\alpha_2 = -0.45}$$

b) X & Y are orthogonal

$R_{xy} = 0$, $E[XY] = 0$

$$0.1\alpha^2 - 0.2\alpha + 0.2 = 0$$

$$\boxed{\alpha_1 = 1 + j}$$

$$\boxed{\alpha_2 = 1 - j}$$

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Q5 X_1, X_2 and X_3 are statistically independent

$\overline{X}_1 = -1$, $\sigma_{x_1}^2 = 2.0$

$\overline{X}_2 = 0.6$, $\sigma_{x_2}^2 = 1.5$

$\overline{X}_3 = 1.8$, $\sigma_{x_3}^2 = 0.8$

$Y = X_1 + X_2 + X_3$

$\overline{Y} = \overline{X}_1 + \overline{X}_2 + \overline{X}_3 = 1.4$

$\sigma_y^2 = \sigma_{x_1}^2 + \sigma_{x_2}^2 + \sigma_{x_3}^2 = 4.3$

$$f_y(y) = \frac{1}{\sqrt{2\pi(4.3)}} e^{-(y-1.4)^2/2(4.3)} \qquad -\infty < y < \infty$$

$$Y \sim N(1.4, 4.3)$$

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