$\frac{11}{30}$

University of Jordan
Faculty of Engineering & Technology
Department of Electrical Engineering
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EE321, Second Exam

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Q1A Gaussian random variable X~N(0.8,0.25) is transformed to a new random variable Y by the following transformation:

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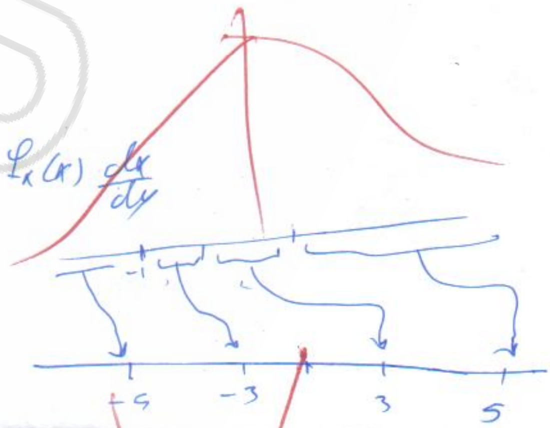
$$Y = T(X) = \begin{cases} -5 & -\infty < X < -1 \\ -3 & -1 \le X < 0 \\ 3 & 0 \le X < 1.0 \\ 5 & 1.0 \le X < \infty \end{cases}$$

a) Find the density and distribution functions of Y.
b) Find the mean and variance of Y.

$f_Y(y) = f_X(x)\,\frac{dx}{dy}$



$$f_Y(y) = \frac{1}{\sqrt{2\pi\sigma_Y^2}}\, e^{-\frac{(y-a_Y)^2}{2\sigma_Y^2}}$$

$$f_Y(y) = \frac{1}{\sqrt{2\pi\sigma_Y^2}} \int_{-\infty}^{\infty} e^{-\frac{(y-a_Y)^2}{2\sigma_Y^2}}$$

$P\{X<0\} = \frac{2}{4} = 0.5$

$F_Y(0) = 0.5$

$\frac{1}{\sqrt{2\pi\sigma_Y^2}} = 0.5$

$\sigma_Y = 0.79$

Q2. Two random variables X and Y have a joint probability density function:

$$f_{XY}(x,y)=\begin{cases} kx^2y & 0<y<x<2 \\ 0 & elsewhere \end{cases}$$

a) Find the constant k such that $f_{XY}(x,y)$ is a valid joint density function.

b) Find the marginal densities of X and Y.

c) Are X and Y statistically independent?

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$f_{XY}(x,y) \geq 0 \Rightarrow k \geq 0$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{XY}(x,y)\,dx\,dy = 1 = \int_0^2\int_0^x kx^2y\,dy\,dx$$

$$\int_0^2 \frac{kx^4}{2}\,dx = \frac{kx^5}{10}\Big|_0^2 = \frac{k\,2^5}{10} - \frac{kx^5}{10} = 1$$

let $K = 1$

b) $f_X(x) = \int_{-\infty}^{\infty} x^2y\,dy$

$$\int_0^x \frac{x^2y^2}{2} = \boxed{\frac{x^4}{2} = f_X(x)}$$

$f_Y(y) = \int_{-\infty}^{\infty} x^2y\,dx$

$$= x^3y\Big|_y^2$$

$$\boxed{8y - y^4 = f_Y(y)}$$

c) $f_{XY}(x,y) \overset{?}{=} f_X(x)f_Y(y)$

$$x^2y \overset{?}{=} \frac{x^4}{2}(8y - y^4)$$

$$4x^4y - \frac{x^4y^4}{2} \neq x^2y$$

they are ~~not~~ independent

Q3. Statistically independent random variables X and Y have respective means $\overline{X} = 1.0$, $\overline{Y} = -0.5$. Their second moments $E[X^2] = 4$, $E[Y^2] = 2.75$.
A new random variable is defined as

$$W = 5X^2 + 3Y + 1$$

Find: R_{XY}, C_{XY}, $\overline{W}$, R_{WY} and C_{WY}.

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$\boxed{R_{XY} = m_{11} = \bar{X}\bar{Y} = -0.5}$

$\boxed{C_{XY} = R_{XY} - \bar{X}\bar{Y} = \text{Zero}}$ because they are independent

$E[W] = 5E[X^2] + 3E[Y] + 1$

$= 4 * 5 + 3(-0.5) + 1$

$= 20 - 1.5 + 1$

$\boxed{\overline{W} = 19.5}$

$R_{WX} = \bar{W}\bar{Y} = -0.5 * 19.5 = \boxed{-9.75 = R_{WY}}$

$C_{WY} = R_{WY} - \bar{W}\bar{Y} = -9.75 - 19.5 * (-0.5) = \text{Zero}$

Q4. Given the random process

$$X(t) = A\cos(w_0 t + \theta)$$

Where A is Gaussian random variable N(1,4), w_0 , and θ are constants. Find:

1) The expected value of X(t).
2) The autocorrelation function of X(t).
3) Is X(t) wide sense stationay?

$\cancel{E[X(t)] = \int A\cos(w_0 t+\theta)\, f_X(x)\, dt}$

$E[X(t)] = \int_{-\infty}^{\infty} A\cos(w_0 t+\theta)\, f_X(x)\, dx$

$R_{XX} = E[X(t) \cdot X(t+\tau)]$

$= (A\cos(w_0 t+\theta))(A\cos(w_0 t + w_0\tau + \theta))$

$A = \frac{1}{\sqrt{8\pi}} e^{-\frac{(x-1)^2}{8}}$

WSS: $R_{XX}(t,t+\tau) = E[X(t) \cdot X(t+\tau)] = R_{XX}(\tau)$

$E[X(t)] = \text{constant}$