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Q1- Suppose three indistinguishable boxes has balls as follows,

| Color | Box1 | Box 2 | Box3 |
|-------|------|-------|------|
| Red | 2 | 4 | 3 |
| White | 3 | 1 | 4 |
| Blue | 5 | 3 | 3 |



A box is selected at random and one ball is selected at random at its observed to be red, what is the probability that box three is selected?

1

$$P(\text{Red from box } i) = \frac{P(\text{Box } i | \text{Red}) \cdot P(\text{Red})}{\sum_{i=1}^3 P(\text{Box } i | \text{Red}) \cdot P(\text{Red})}$$

event A: {red in Box 3}

~~$P(A) = \frac{3}{10}$~~

Event A : Box 3

Event B : Red

$$P(A) = \frac{1}{3}$$

$$P(B) = \frac{9}{28}$$

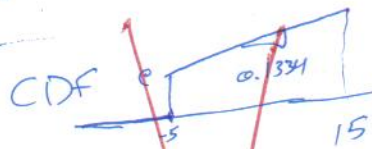
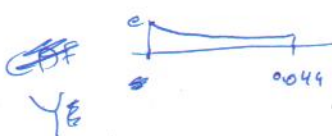
$$P(A|B) = P(A) \cdot P(B) = \frac{3}{28}$$

PROG

Q2- If X is uniform Random Variable on the interval $(-5, 15)$. Define $Y = e^{\frac{-X}{5}}$.

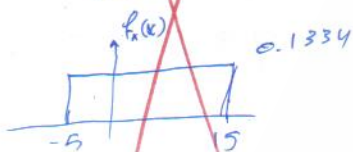
Find:

- 1- The pdf of Y .
- 2- The expected value of Y .
- 3- The variance of Y .



≈ 0.1334

~~pdf~~ $f_{Y(y)} = \frac{d}{dx} F(x) =$



$-\ln y = \frac{x}{5}$

$f_Y(y) = f_X(x) \frac{dx}{dy}$

POWER

Q3- If X is a normal random variable with a mean $\bar{X} = 80$, and a standard deviation $\sigma_x = 10$. Compute the following probabilities

1- $P\{X \leq 80\}$

2- $P\{65 \leq X \leq 100\}$

3- $P\{X > 70\}$

4- $P\{85 \leq X \leq 95\}$

5- $P\{|X - 80| \leq 80\}$

2

~~$F_x\left(\frac{X-80}{10}\right)$~~



① $P\{X \leq 80\} = F_x(0) = 0.5$

② $P\{65 \leq X \leq 100\}$

~~$P\{65 \leq X \leq 100\}$~~

~~$= F_x(100) - F_x(65)$~~

~~$= F_x\left(\frac{100-80}{10}\right) - F_x\left(\frac{65-80}{10}\right) = F_x(2) - F_x(-1.5)$~~

$P\{X \leq 100\} = F_x\left(\frac{100-80}{10}\right) = F_x(2) = 0.9773$

$P\{X \geq 65\} = 1 - F_x\left(\frac{65-80}{10}\right) = 1 - F_x(-1.5)$

$= F_x\left(\frac{95-80}{10}\right) = F_x(1.5) = 0.9332$

$P\{65 \leq X \leq 100\} = 0.9773 - 0.9332 = 0.0441$

3) $P\{X > 70\} = 1 - F_x\left(\frac{70-80}{10}\right)$



~~$= 1 - F_x(-1) = 1 - 0.2420 = 0.7580$~~

④ $P\{X \leq 95\} = F_x\left(\frac{95-80}{10}\right) = F_x(1.5) = 0.9332$

$P\{X \geq 85\} = 1 - F_x\left(\frac{85-80}{10}\right) = 1 - F_x(0.5) = 1 - 0.6915 = 0.3085$

$P\{85 \leq X \leq 95\} = 0.9332 - 0.3085 = 0.6247$

⑤ $P\{10 \leq X \leq 160\}$

Q4- A Random Variable X has a pdf of the form

$$f_X(x) = \begin{cases} ax^2 & 0 < x \leq 2 \\ ax & 2 < x \leq 3 \end{cases}$$

Find:

- 1- The value of a .
- 2- $P\{2 < X \leq 3\}$
- 3- The CDF $F_X(x)$

① ~~$f_X(x) \geq f_X(x)$~~
 ~~$a > 0$~~
 ~~$f_X(x_2) \geq f_X(x_1)$~~
 ~~$x_2 > x_1$~~
 ~~$f_X(x) \geq 0$~~
 ~~$a > 0$~~

1

$$\int_{-\infty}^{\infty} f_X(x) dx = 1 = \int_0^2 ax^2 dx + \int_2^3 ax dx = 1$$

$$\frac{ax^3}{3} \Big|_0^2 + \frac{ax^2}{2} \Big|_2^3 = 1$$

$$= \frac{8}{3}a + \frac{9}{2}a - a = 1$$

$$a \left(\frac{8}{3} + \frac{9}{2} - 1 \right) = 1$$

$$a = -2.167$$

② $P\{2 < X \leq 3\} = \int_2^3 a dx =$

POWER

TABLE B-1
Values of $F(x)$ for $0 \leq x \leq 3.89$ in steps of 0.01

| x | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|-----|-------|-------|-------|-------|-------|-------|-------|--------|--------|--------|
| 0.0 | .5000 | .5040 | .5080 | .5120 | .5160 | .5199 | .5239 | .5279 | .5319 | .5359 |
| 0.1 | .5398 | .5438 | .5478 | .5517 | .5557 | .5596 | .5636 | .5675 | .5714 | .5753 |
| 0.2 | .5793 | .5832 | .5871 | .5910 | .5948 | .5987 | .6026 | .6064 | .6103 | .6141 |
| 0.3 | .6179 | .6217 | .6255 | .6293 | .6331 | .6368 | .6406 | .6443 | .6480 | .6517 |
| 0.4 | .6554 | .6591 | .6628 | .6664 | .6700 | .6736 | .6772 | .6808 | .6844 | .6879 |
| 0.5 | .6915 | .6950 | .6985 | .7019 | .7054 | .7088 | .7123 | .7157 | .7190 | .7224 |
| 0.6 | .7257 | .7291 | .7324 | .7357 | .7389 | .7422 | .7454 | .7486 | .7517 | .7549 |
| 0.7 | .7580 | .7611 | .7642 | .7673 | .7704 | .7734 | .7764 | .7794 | .7823 | .7852 |
| 0.8 | .7881 | .7910 | .7939 | .7967 | .7995 | .8023 | .8051 | .8078 | .8106 | .8133 |
| 0.9 | .8159 | .8186 | .8212 | .8238 | .8264 | .8289 | .8315 | .8340 | .8365 | .8389 |
| 1.0 | .8413 | .8438 | .8461 | .8485 | .8508 | .8531 | .8554 | .8577 | .8599 | .8621 |
| 1.1 | .8643 | .8665 | .8686 | .8708 | .8729 | .8749 | .8770 | .8790 | .8810 | .8830 |
| 1.2 | .8849 | .8869 | .8888 | .8907 | .8925 | .8944 | .8962 | .8980 | .8997 | .9015 |
| 1.3 | .9032 | .9049 | .9066 | .9082 | .9099 | .9115 | .9131 | .9147 | .9162 | .9177 |
| 1.4 | .9192 | .9207 | .9222 | .9236 | .9251 | .9265 | .9279 | .9292 | .9306 | .9319 |
| 1.5 | .9332 | .9345 | .9357 | .9370 | .9382 | .9394 | .9406 | .9418 | .9429 | .9441 |
| 1.6 | .9452 | .9463 | .9474 | .9484 | .9495 | .9505 | .9515 | .9525 | .9535 | .9545 |
| 1.7 | .9554 | .9564 | .9573 | .9582 | .9591 | .9599 | .9608 | .9616 | .9625 | .9633 |
| 1.8 | .9641 | .9649 | .9656 | .9664 | .9671 | .9678 | .9686 | .9693 | .9699 | .9706 |
| 1.9 | .9713 | .9719 | .9726 | .9732 | .9738 | .9744 | .9750 | .9756 | .9761 | .9767 |
| 2.0 | .9773 | .9778 | .9783 | .9788 | .9793 | .9798 | .9803 | .9808 | .9812 | .9817 |
| 2.1 | .9821 | .9826 | .9830 | .9834 | .9838 | .9842 | .9846 | .9850 | .9854 | .9857 |
| 2.2 | .9861 | .9864 | .9868 | .9871 | .9875 | .9878 | .9881 | .9884 | .9887 | .9890 |
| 2.3 | .9893 | .9896 | .9898 | .9901 | .9904 | .9906 | .9909 | .9911 | .9913 | .9916 |
| 2.4 | .9918 | .9920 | .9922 | .9925 | .9927 | .9929 | .9931 | .9932 | .9934 | .9936 |
| 2.5 | .9938 | .9940 | .9941 | .9943 | .9945 | .9946 | .9948 | .9949 | .9951 | .9952 |
| 2.6 | .9953 | .9955 | .9956 | .9957 | .9959 | .9960 | .9961 | .9962 | .9963 | .9964 |
| 2.7 | .9965 | .9966 | .9967 | .9968 | .9969 | .9970 | .9971 | .9972 | .9973 | .9974 |
| 2.8 | .9974 | .9975 | .9976 | .9977 | .9977 | .9978 | .9979 | .9979 | .9980 | .9981 |
| 2.9 | .9981 | .9982 | .9982 | .9983 | .9984 | .9984 | .9985 | .9985 | .9986 | .9986 |
| 3.0 | .9987 | .9987 | .9987 | .9988 | .9988 | .9989 | .9989 | .9989 | .9990 | .9990 |
| 3.1 | .9990 | .9991 | .9991 | .9991 | .9992 | .9992 | .9992 | .9992 | .9993 | .9993 |
| 3.2 | .9993 | .9993 | .9994 | .9994 | .9994 | .9994 | .9994 | .9995 | .9995 | .9995 |
| 3.3 | .9995 | .9995 | .9996 | .9996 | .9996 | .9996 | .9996 | .9996 | .9996 | .9997 |
| 3.4 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9998 | .9998 |
| 3.5 | .9998 | .9998 | .9998 | .9998 | .9998 | .9998 | .9998 | .9998 | .9998 | .9998 |
| 3.6 | .9998 | .9999 | .9999 | .9999 | .9999 | .9999 | .9999 | .9999 | .9999 | .9999 |
| 3.7 | .9999 | .9999 | .9999 | .9999 | .9999 | .9999 | .9999 | .9999 | .9999 | .9999 |
| 3.8 | .9999 | .9999 | .9999 | .9999 | .9999 | .9999 | .9999 | 1.0000 | 1.0000 | 1.0000 |

Although a closed-form solution for $Q(x)$ is not known, an excellent approximation is

$$Q(x) \approx \left[\frac{1}{0.661x + 0.339\sqrt{x^2 + 5.51}} \right] \frac{e^{-x^2/2}}{\sqrt{2\pi}} \quad x \geq 0 \quad (\text{B-8})$$

which is due to Börjesson and Sundberg, 1979. The maximum absolute relative error in the approximation for $Q(x)$ is given as 0.27 percent for any $x \geq 0$. By using the approximation (B-8) for $Q(x)$ in (B-7), an excellent approximation for $F(x)$ is realized.