University of Jordan

Faculty of Engineering & Technology

Department of Electrical Engineering

Course Title: Probability & Random Variables	Exam: 1^{st} Exam	Date: Nov/05/2015
Course No.: 0903321	Semester: 1^{st} Term 2015-2016	Time Period: 1:00 Hr.
Instructor: Dr. Ahmad Atieh & Prof Mohammed Khasawneh		

Q.1	Q.2	Q.3	Total /20
5	7	4	16/22

Student Name: [redacted]

Student Number: [redacted]

Section: [redacted]

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Q1)

~~P(transmit 0) = P(T1)~~

A digital communication system transmits "on the average" equal number of 0 and 1 bits over a noisy channel. Assume the probability of error at the receiver when detecting a 0 bit is 5×10^{-9}, and the probability of error when detecting a 1 bit is 1×10^{-9}. **Show your work**

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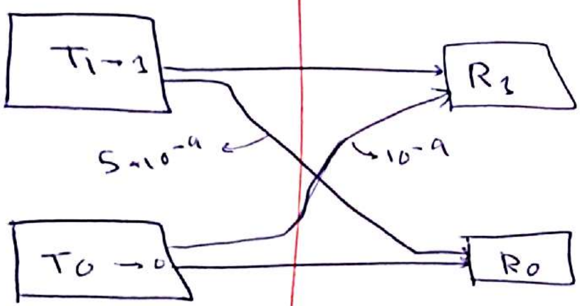
a) Sketch a probabilistic model for such communication system?

b) Calculate the probability of error for the communication system?

$P(T_1) = P(T_0) = \frac{1}{2}$

p(error detecting 0) = 5×10^{-9}

p(error " 1) = 10^{-9}



(a) P(error detection)

$\equiv P\{0 \text{ error} \mid 1\}$

$= 5 \times 10^{-9}$

p(error detection)

$\equiv P\{1 \text{ error} \mid 0\}$

$= 10^{-9}$

~~$P(R_0 \mid T_0) =$~~

Look @ page #5

P(error detecting 0)

$= P(\text{recieving } 0 \mid \text{transmitting } 1)$

$= 5 \times 10^{-9}$

P(error detecting 1)

$= P(\text{recieving } 1 \mid \text{transmitting } 0)$

$= 10^{-9}$

(a)

$P(R_0) = P(R_0 \mid T_0)\, P(T_0) + P(R_0 \mid T_1)\, P(T_1)$

$P(R_0 \mid T_0) = 5 \times 10^{-9} \cdot \frac{1}{2} + (1 - 5 \times 10^{-9}) \cdot \frac{1}{2}$

T_0 $= .5$

$P(R_1) = .5$

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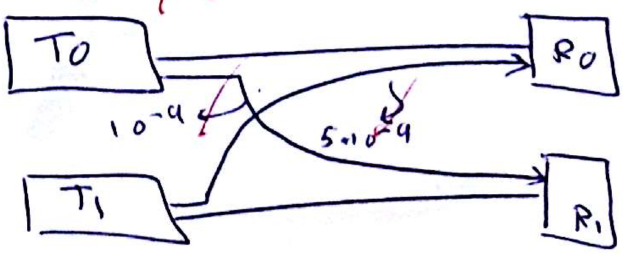

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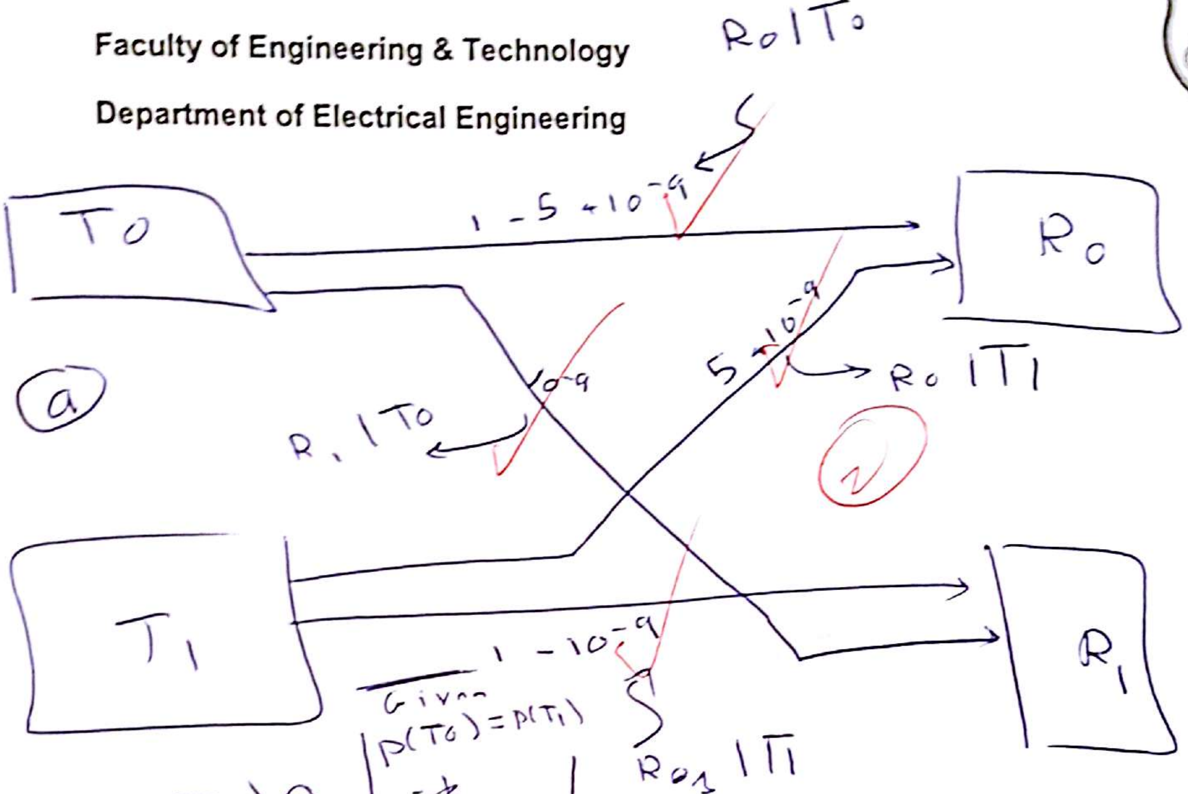

Given
$p(T_0)=p(T_1)$
$=\frac{1}{2}$
$p(R_0|T_1)$
$=5\times10^{-9}$
$p(R_1|T_0)$ 10^{-9}

$\cancel{p(R_0|T_0)\Rightarrow}$

$\cancel{p(?)\,R_0}$

$\cancel{p(R_0)=}$

$p(R_0|T_0)+p(R_0|T_1)=1$

$p(R_0|T_0)=1-5\times10^{-9}$

$p(T_0)=p(T_1)=1/2$

$p(R_0|T_1)=5\times10^{-9}$

$p(R_1|T_0)=10^{-9}$

$p(R_1|T_1)=1-p(R_1|T_0)$

$=1-10^{-9}$

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(b) $p(\text{error})=p(R_0|T_1)\,p(T_1)+p(R_1|T_0)\,p(T_0)$

$=5\times10^{-9}\cdot\frac{1}{2}+10^{-9}\cdot\frac{1}{2}$

$p(\text{error})=3\times10^{-9}$

5

5' 9"

6 5 Feet 9 inch

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+7

Q2)

An assembly line technician checked out electrical resistors in a batch size of 1000. Suppose that the probability of any electrical resistor coming out defective is 0.002. **Show your work** $P(def) = .002$

a. Find the probability that no more than 3 components come out defective (3, 2, 1, 0) def.

b. Find the probability of exactly 5 components are defective given that 3 resistors have been observed to be defective already

c. What is the probability that only 3 components are defective

\# of resistors = 1000

$p(def) = .002$ $\quad p(undef) = .998$

ⓐ $P\{X$

P{3 or less def}

$\cancel{= \binom{3}{0}(.002)^0(1-.002)^3}$

{0 def or 1 def or 2 def or 3 def}

P{3 or less out of 1000 is def}

$P\left\{\binom{1000}{0} \text{ 0 def.} + \binom{1000}{1} \text{ 1st def.} + \binom{1000}{2} \text{ 2nd def} + \binom{1000}{3} \text{ 3rd def}\right\}$

$P(A) = \left[\frac{1000!}{(1000-0)! \cdot 0!}\right] \cdot (.002)^0 \cdot .998^{1000}$

$+ \left[\frac{1000!}{999! \cdot 1!}\right] \cdot (.002)^1 \cdot .998^{999}$

$+ \left(\frac{1000!}{998! \cdot 2!}\right) \cdot (.002)^2 \cdot (.998)^{998}$

$+ \left(\frac{1000!}{997! \cdot 3!}\right)(.002)^3 \cdot (.998)^{997}$

$\Rightarrow .8573 = P\{3 \text{ or less def}\}$

ⓑ P{def $A = 3$ def.

$P(A) = \binom{1000}{3} \cdot (.002)^3 \cdot .998^{997}$

$= .1806$

$P(2 \text{ def.} \mid A)$

$P(2 \text{ def} \mid A) = P\left(\binom{997}{2}(.002)^2 \cdot .998^{995}\right)$

$P(B) \Rightarrow$ .27094 ~~.26986~~ $= P(2 \text{ def} \mid 3 \text{ def}) = P(B)$

ⓒ $P\left(\binom{1000}{3} \cdot (.002)^3 \cdot .998^{997}\right)$

$P(C) = .1806$

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Q3)

The joint probabilities of students' weights (W) and heights (H) in a class are listed in the following table. Find the following and **show your work**:

a) The probability of student's height is in the range 5′ 8″ – 6′?
b) The probability of student's weight is in the range 160 – 190 lbs given that his height is 5′ 8″ – 6′?
c) The probability of students' weight is in the range 190 – 220 lbs given that his ~~weight~~ h is more than 5′ 8″?

	W_1 100–130	W_2 130–160	W_3 160–190	W_4 190–220	W_5 220–250	$P[H_i]$
H_1 5′ – 5′4″	0.08	0.04	0.02	0	0	0.14
H_2 5′4″ – 5′8″	0.06	0.12	0.06	0.02	0	0.26
H_3 5′8″ – 6′	0	0.06	0.14	0.06	0	0.26
H_4 6′ – 6′4″	0	0.02	0.06	0.10	0.04	0.22
H_5 6′4″ – 6′8″	0	0	0	0.08	0.04	0.12
	.14	.24	.28	.26	.08	1

(a) $P(5'8'' - 6') = P(H_3)$ (For All weights)

$= .26$

2

(b) $P(W_3 \mid H_3) = \dfrac{P(W_3 \cap H_3)}{P(H_3)}$

$= \dfrac{.14}{.26} = .5385$

(c) ~~P(W4~~

$P(C) = p(W_4 \mid H_3)\, p(H_3)$

$+ p(W_4 \mid H_4)\, p(H_4)$

$+ p(W_4 \mid H_5)\, p(H_5)$

(c) $p(C) = \cancel{\dfrac{.06}{.26}}\ .26$

$= p(W_4 \cap H_3) + p(W_4 \cap H_4) + p(W_4 \cap H_5)$

$= .06 + .1 + .08$

$= .24$

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