University of Jordan
Faculty of Engineering & Technology
Department of Electrical Engineering
1st Semester – A.Y. 2015/2016

Course: Power System Analysis (2) – 0903482 (3 Cr.)

Instructor: Dr. Eyad A. Feilat

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Course Website: **http://eacademic.ju.edu.jo/E.Feilat/Material/Forms/AllItems.aspx**

Catalog Data: Power system protection: layout of substations, requirements and elements of protection systems, relays. Directional and non-directional over current and earth fault feeder protection. Differential protection as applied to feeders. Principles of distance protection. Economic operation of power systems, classical economic dispatch, the transmission loss equation, automatic generation control. Power system stability: rotor dynamics and the swing equation, the power angle equation, synchronizing power coefficient, equal-area criterion of stability, introduction to multi-machine stability studies.

Prerequisites by Course: EE 0903481 – Power System Analysis (1)

Prerequisites By Topic: Student should have a background of the following topics:

- Basic principles of power system components and its representation
- Calculation of shortcircuit currents.
- principles of synchronous machines.

Textbook: **Power System Analysis J.J. Grainger & W. D. Stenvson (1994) Mc-Graw Hill.**

References:

- copmputer – Adided power system analysis G. L. Kusic prentic hall .
- power system protections volumes 1, 2 & 3 edited by electricity council , mcdonald.
- Power Generation, Operation & ControlBy A. J. Wood & B. F. WollenbergJohn Wiley.
- Electrical power system protection 2013 by A.wright C.Christopoulos.

Schedule & Duration: 16 Weeks, 45 lectures (50 minutes each) plus exams.

Minimum Student Material: Textbook, class handouts, scientific calculator, and an access to personal computer**.**

Minimum College Facilities: Classroom with blackboard and projection display facilities, library, and computational facilities.

Course Objectives: The overall objective of this course is to provide the student with basic knowledge and proficiency in the basic principles of protecting the different components of the power system during abnormal conditions with emphasis on feeder protection. It also aims to acquaint the student with techniques used for operating power generation systems in an ecomomic manner, and methods used to investigate the stability of synchronous machines running in parallel.

Course Outcomes and Relation to ABET Student Outcomes:

Upon successful completion of this course, a student shouldbe able to :

1.	Understand the basic principles of power systems protection, identify the protection system components and be familiar with the principle of operation of earth fault, overcurrent directional and nondirectional relays, differential and distance relays	[a,c,e, k,f]
2.	be familiar with classical enconomic operation and automatic control of power stations	[a,e, k]
3.	Study the dynamics of the power system during abnormal conditions	[a, e,]

Course Topics:

	Topic Description	Hrs
1.	**Power Ssytem Protection:** Layout of electrical substations, requirements of a successful protection system, current and voltage transformers, electromechanical and static relays. Directional and non directional over current and earth fault protection schemes and relay setting. Voltage and current balance differential protection schemes for feeders, pilot wire protection, summation circuits. Distance protection: principle of operation, distance- time schemes, methods of distance measurement, setting, acceleration schemes, practical considerations.	**21**
2.	**Economic Operation of Power Systems:** Distribution of load between units within a plant and between plants. Classical economic dispatch, automatic generation control, examples.	**9**
3.	**Power System Stability:** The stability problem, rotor dynamics and the swing equation, the power angle equation, synchronizing power coefficients, equal area criterion of stability. Classical multi machine stability studies. Step-by-step solution of the swing curve. Factors affecting stability.	**15**

Ground Rules:

Attendance is required and highly encouraged. To that end, attendance will be taken every lecture. All exams (including the final exam) should be considered **cumulative**. Exams are closed book. No scratch paper is allowed. You will be held responsible for all reading material assigned, even if it is not explicitly covered in lecture notes.

Assessments:

Exams and Quizzes

Grading policy:

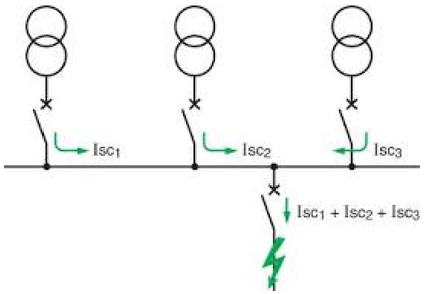
First Exam		**20 %**
Midterm Exam		**30 %**
Final Exam		**50 %**
	Total	**100%**

Last Updated:

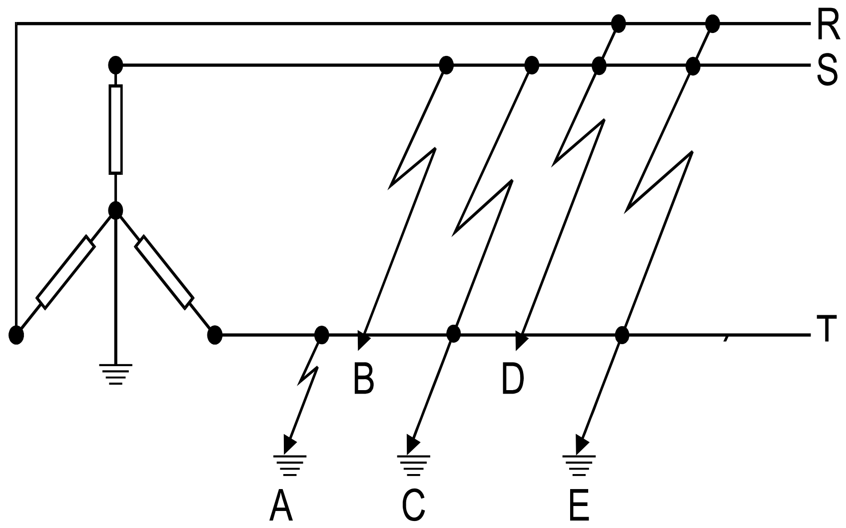
October r 2014

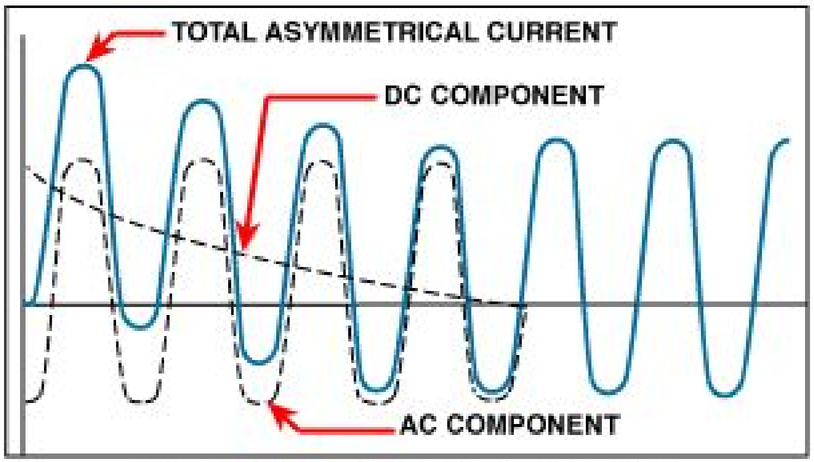
EE482-Power System Analysis II

Dr. E. A. Feilat
Electrical Engineering Department
School of Engineering
University of Jordan



Topic 1-1: Fault Types and Calculations



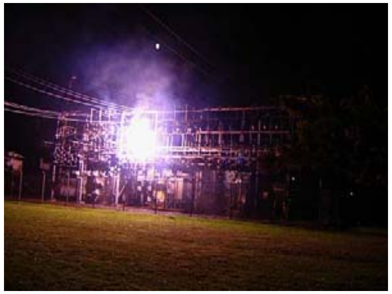


SHORT CIRCUIT CURRENT COMPONENTS

What is a Power System Fault?

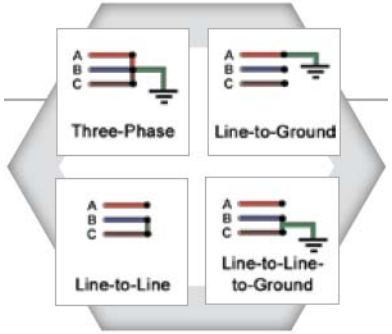
- **A power system fault is the breakdown of insulation (between conductors, or between a phase conductor and ground) which results in excess current flow.**

Possible Faults



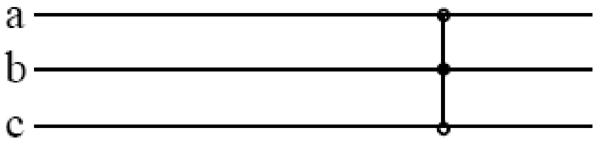
- **Cable Faults**
- **Transformer faults**
- **Busbar Faults**

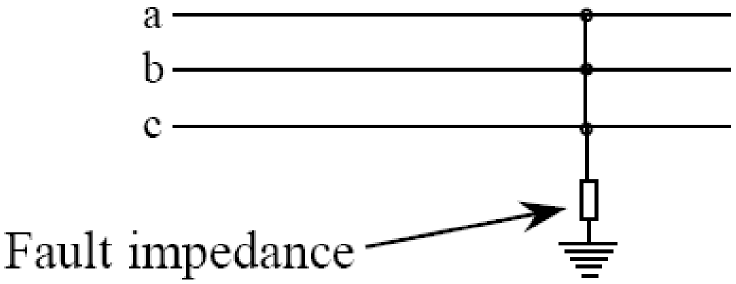
Types of Faults



- **Balanced Faults (Symmetrical Faults)**

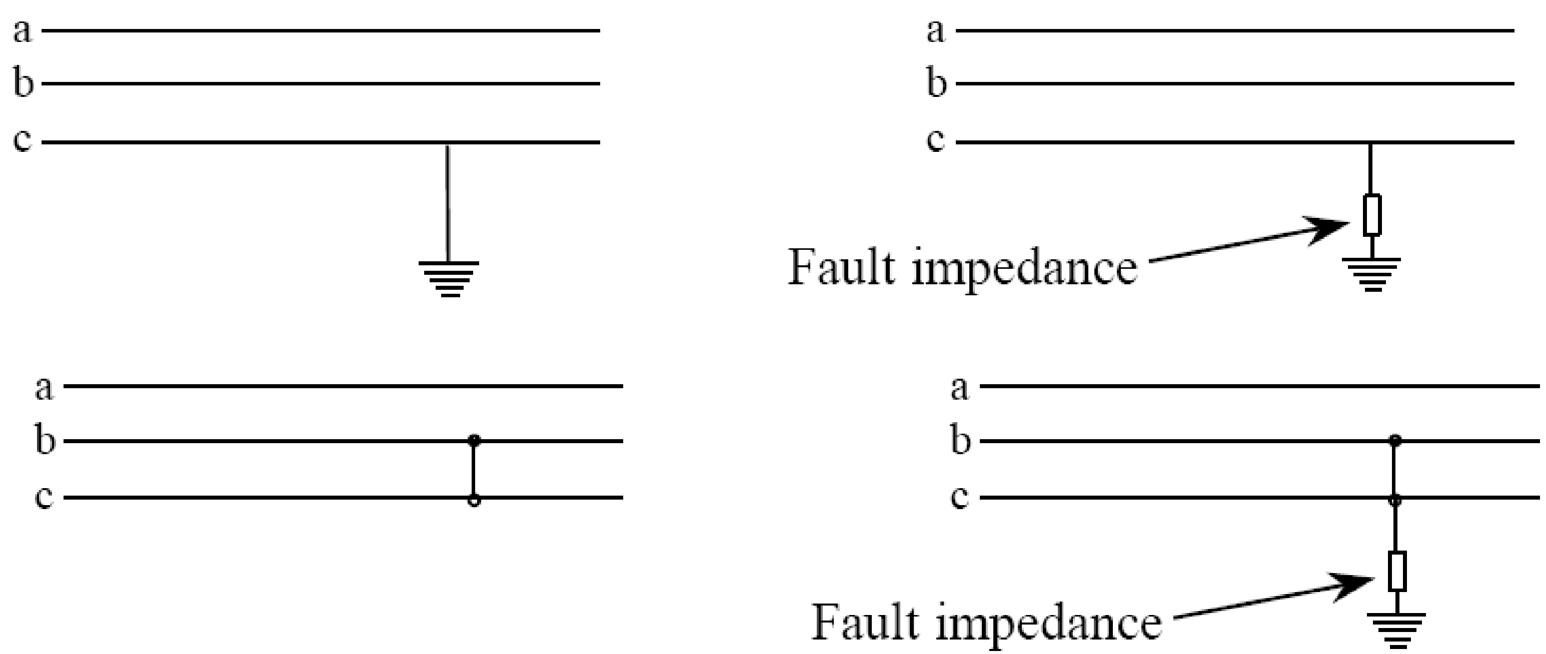
➲ **3-Phase Fault (with or without ground) ➔ (5%)**





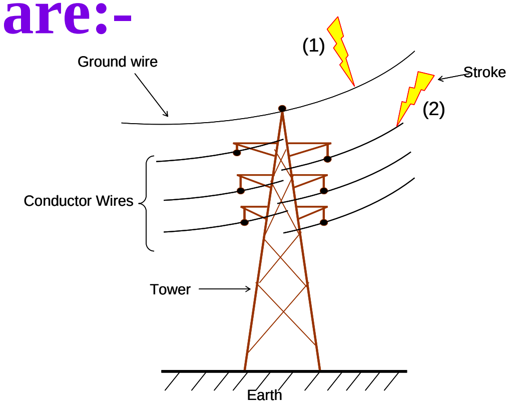
Types of Faults

- **Unbalanced Faults (Unsymmetrical Faults)**
  - **Single phase (Phase-Ground) → (70%)**
  - **Two phase to ground (Phase-Phase-Ground) → (15%)**
  - **Two phase (Phase-Phase) → (10%)**

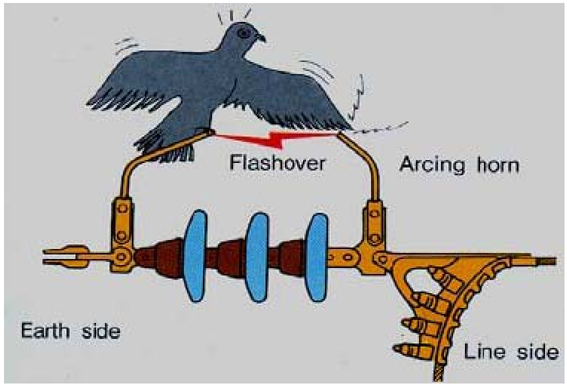


Causes of Faults on Power System

➔ **The most common causes of faults on OHL are:-**



- **Lightning**
- **Contaminated Insulators**
- **Punctured or broken insulators**
- **Birds and animals**
- **Cars hitting lines and structures**
- **Ice and snow loading**
- **Wind**

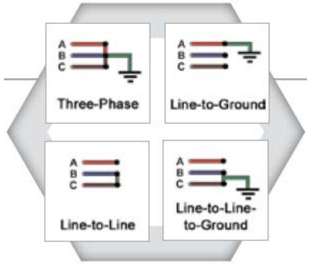


Causes of Faults on Power System

➔ **In electrical machines, cables and transformers, faults are caused by:**

- **Failure of insulation because of moisture**
- **Mechanical damage**
- **Flashover caused by overvoltage or abnormal loading.**

Simple Calculation of Short- Circuit Currents



- Calculation Methods
  - Ohmic Method - Where all impedances are expressed in Ohms
  - Per Unit Method - Where all impedances are expressed in pu values. **Used to simplify calculations on systems with more than 2 voltages.**

Standardized I_{sc} calculations

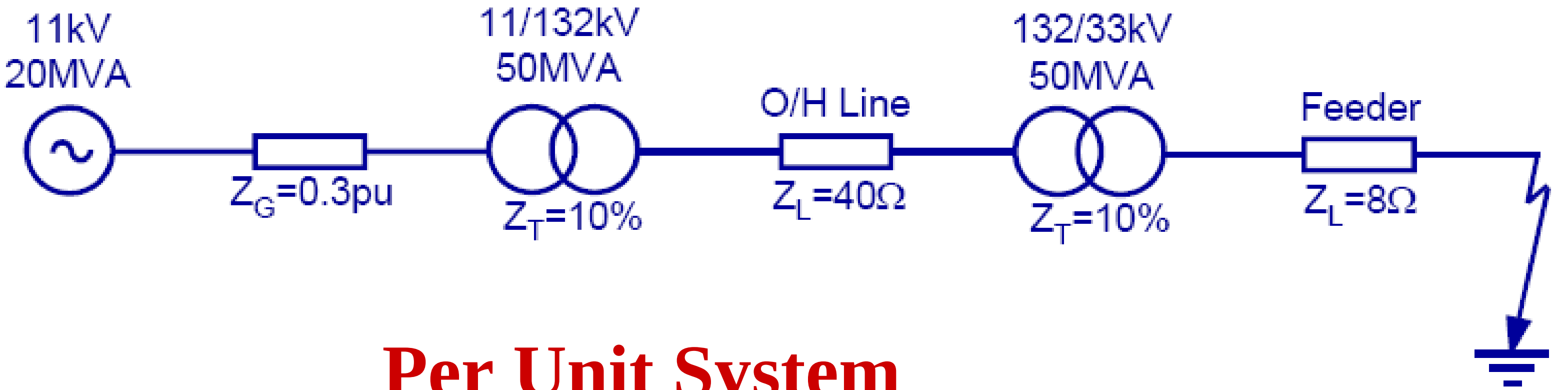
The Impedance Method-Classical

- ➔ Used to calculate fault currents at any point in a network with a high degree of accuracy.
- ➔ The impedance method, reserved primarily for LV networks.
- ➔ This method involves adding the various resistances and reactances of the fault loop separately, from the source to the given point, and then calculating the corresponding impedance.
- ➔ The I_{sc} value is obtained by applying Ohm's law:

$$I_{sc} = \frac{V_{LL}}{\sqrt{3}\sum Z}$$

Different Voltages

How do we Analyze?

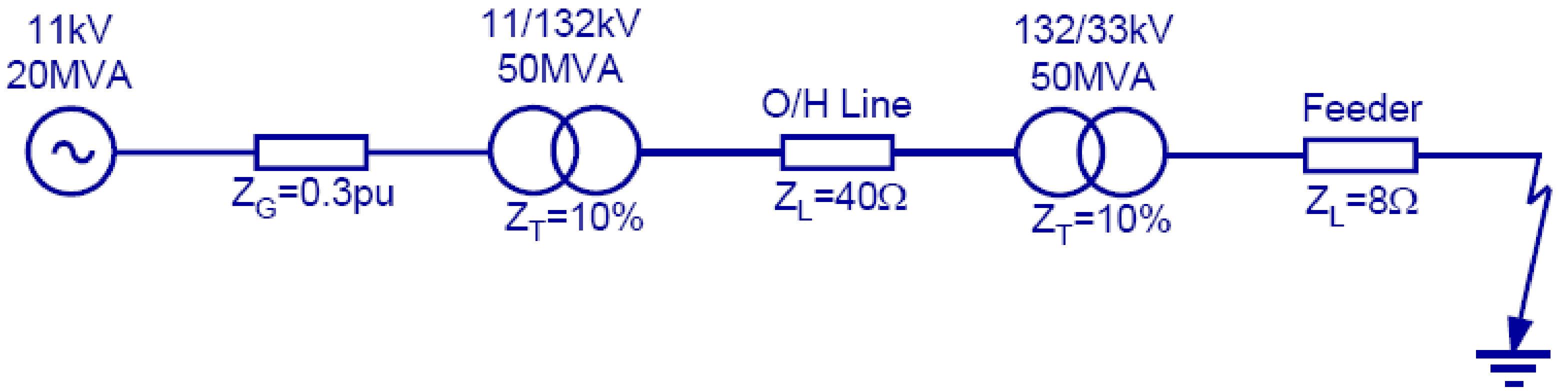


Per Unit System

Definition:

$$\text{Per Unit Value of a Quamtity} = \frac{\text{Actual Value}}{\text{Base value of the Same units}}$$

Base Quantities and Per Unit Values



- Particularly useful when analyzing large systems with several voltage levels
- All system parameters referred to common base quantities
- Base quantities fixed in one part of system
- Base quantities at other parts at different voltage levels depend on ratio of intervening transformers

Base Quantities and Per Unit Values

Base quantities normally used :-

- **Base MVA**

$\mathbf{MVA_{BASE} = MVA_b = MVA_{3Ph}}$

- **Constant at all voltage levels**
- **Value ~ MVA rating of largest item of plant or 100MVA**

- **BASE VOLTAGE**

$\mathbf{kVA_{BASE} = kV_b = LL\ Voltage\ in\ kV}$

- **Fixed in one part of system**
- **This value is referred through transformers to obtain base voltages on other parts of system.**
- **Base voltages on each side of transformer are in same ratio as voltage ratio.**

Base Quantities and Per Unit Values

Other Base quantities:-

BASE Impedance = $$Z_b = \frac{(kV_b)^2}{MVA_b}$$ **in Ohms**

BASE Current = $$I_b = \frac{MVA_b}{\sqrt{3} \times kV_b}$$ **in kA**

Base Quantities and Per Unit Values

$$\text{Per Unit Value} = \frac{\text{Actual Value}}{\text{Base value of the Same units}}$$

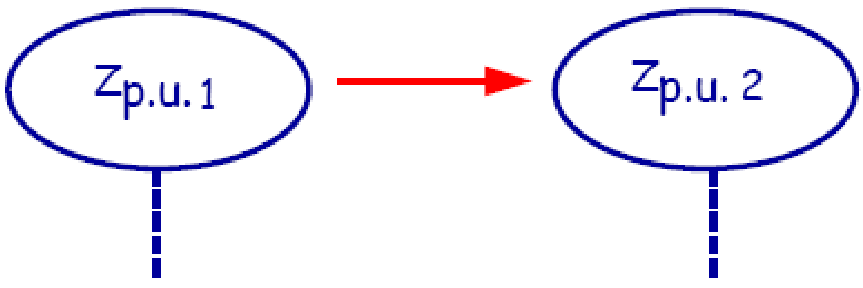
$$\text{Per Unit MVA} = MVA_{p.u.} = \frac{MVA_a}{MVA_b}$$

$$\text{Per Unit Voltage} = kV_{p.u.} = \frac{KV_a}{KV_b}$$

$$\text{Per Unit Impedance} = Z_{p.u.} = \frac{Z_a}{Z_b} = Z_a \cdot \frac{MVA_b}{(kV_b)^2}$$

$$\text{Per Unit Current} = I_{p.u.} = \frac{I_a}{I_b}$$

Conversion of Per Unit Values from One Set of Quantities to Another



Z_{b1} Z_{b2}

MVA_{b1} MVA_{b2}

kV_{b1} kV_{b2}

Actual $Z = Z_a$

$$Z_{p.u.1} = \frac{Z_a}{Z_{b1}}$$

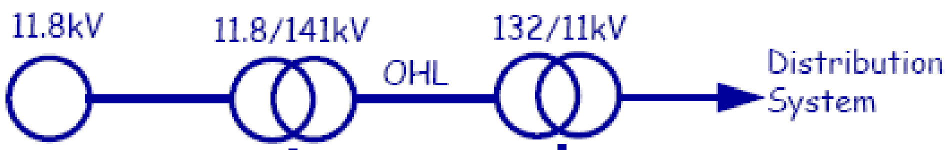
$$Z_{p.u.2} = \frac{Z_a}{Z_{b2}} = Z_{p.u.1} \times \frac{Z_{b1}}{Z_{b2}}$$

$$= Z_{p.u.1} \times \frac{(kV_{b1})^2}{MVA_{b1}} \times \frac{MVA_{b2}}{(kV_{b2})^2}$$

$$= Z_{p.u.1} \times \frac{MVA_{b2}}{MVA_{b1}} \times \frac{(kV_{b1})^2}{(kV_{b2})^2}$$

Transformer – Base Voltage Selection

Base voltage on each side of a transformer must be in the same ratio as voltage ratio of transformer.



Incorrect selection of kV_b	11.8kV	132kV	11kV
Correct selection of kV_b	$\frac{132 \times 11.8}{141}$ = 11.05kV	132kV	11kV
Alternative correct selection of kV_b	11.8kV	141kV	$\frac{141 \times 11}{132}$ = 11.75kV

Procedure For Calculating Maximum Fault Current

1. **Draw a single-line diagram of the power system.**
2. **Collect detailed impedance data for all of the components of the power system. i.e *Resistance R* and *Reactance X***
3. **Although fault current can be calculated using the *Ohmic* method, it is usually simpler to use the *Per-Unit Method* where all of the impedances are referred to an arbitrarily chosen common *BASE MVA*.**
4. **Convert all of the various impedances to *per-unit* values with a common base MVA.**
5. **Find the total *Resistance R*, and *Reactance X*, from the source to the fault.**
6. **Calculate the total Impedance:**

$$Z = \sqrt{R^2 + X^2}$$

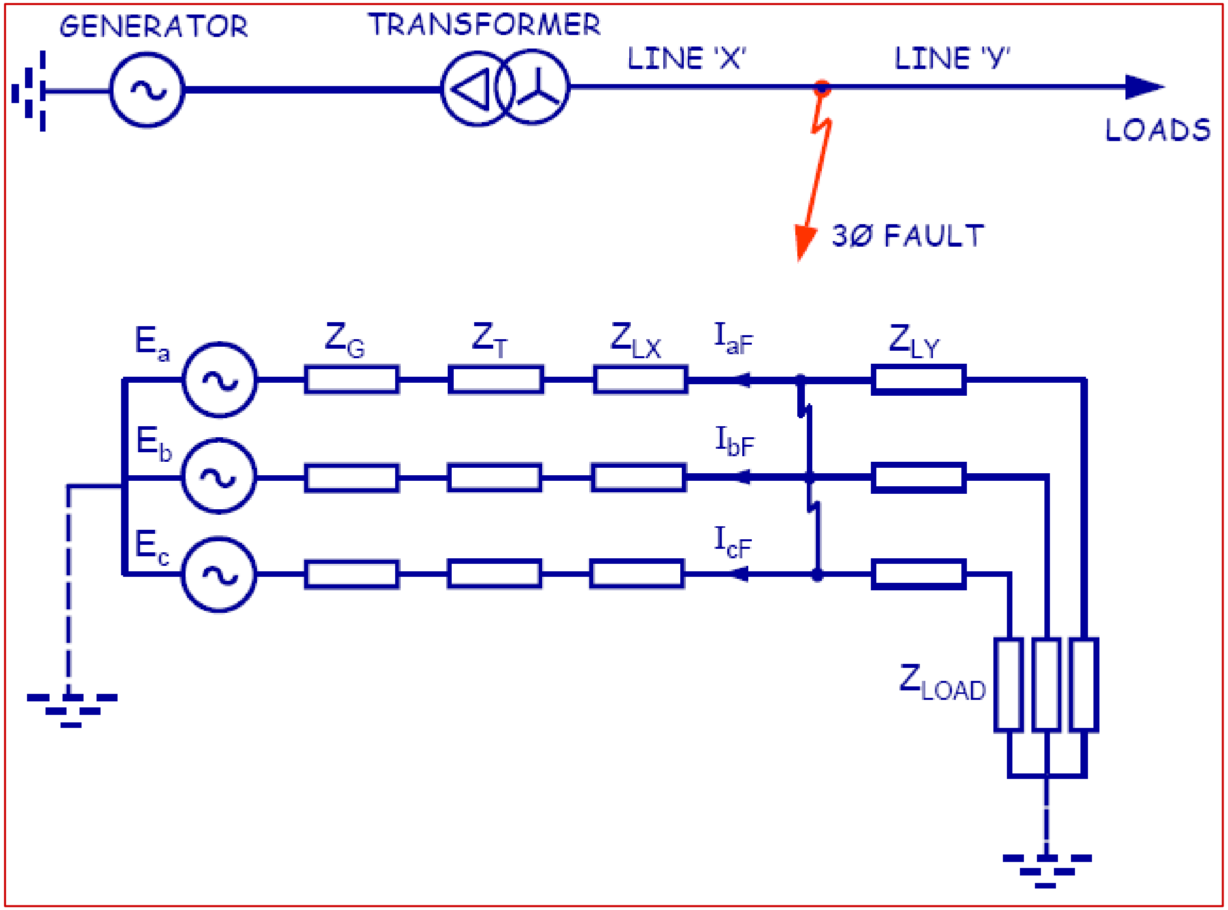
Power System Fault Analysis

Balanced 3-Phase Faults

- ➔ **RARE:- Majority of faults are unbalanced**
- ➔ **CAUSES:-**
  - ➢ **System energization with maintenance earthing clamps still connected.**
  - ➢ **1-Phase faults developing into 3-Phase faults**
- ➔ **3-Phase faults may be represented by 1-phase circuit**

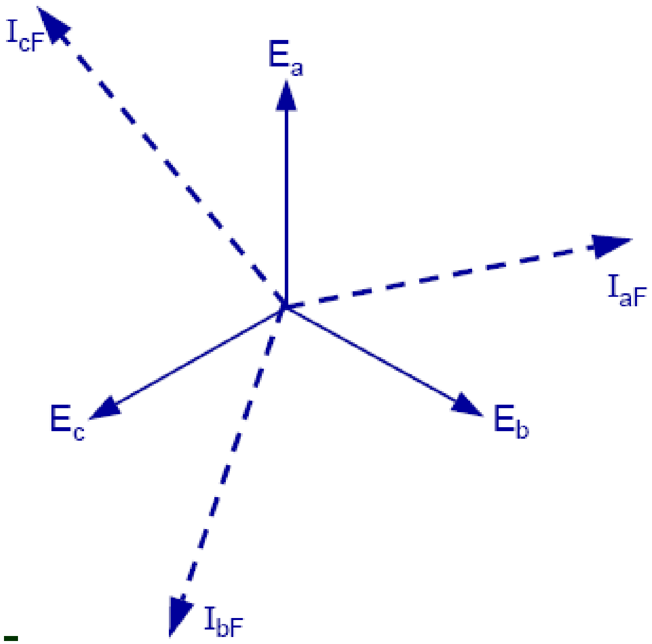
Power System Fault Analysis

Balanced 3-Phase Faults

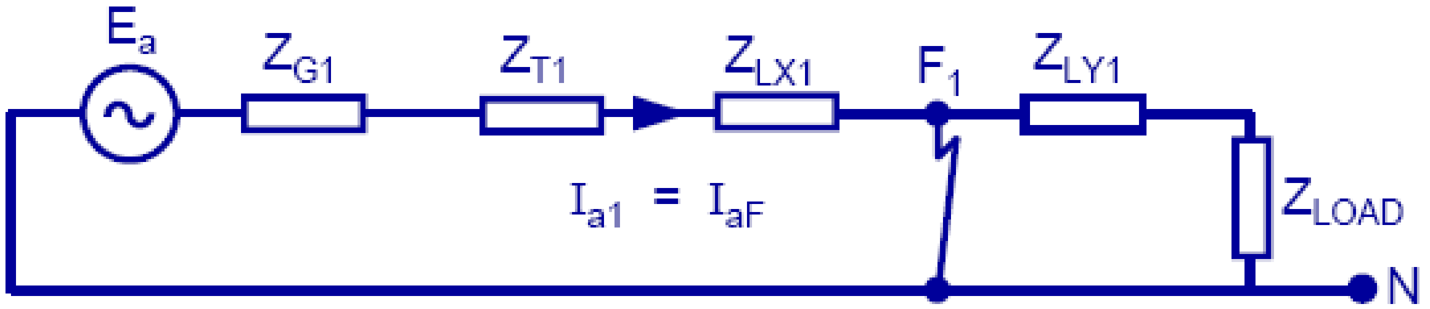


Power System Fault Analysis

Balanced 3-Phase Faults

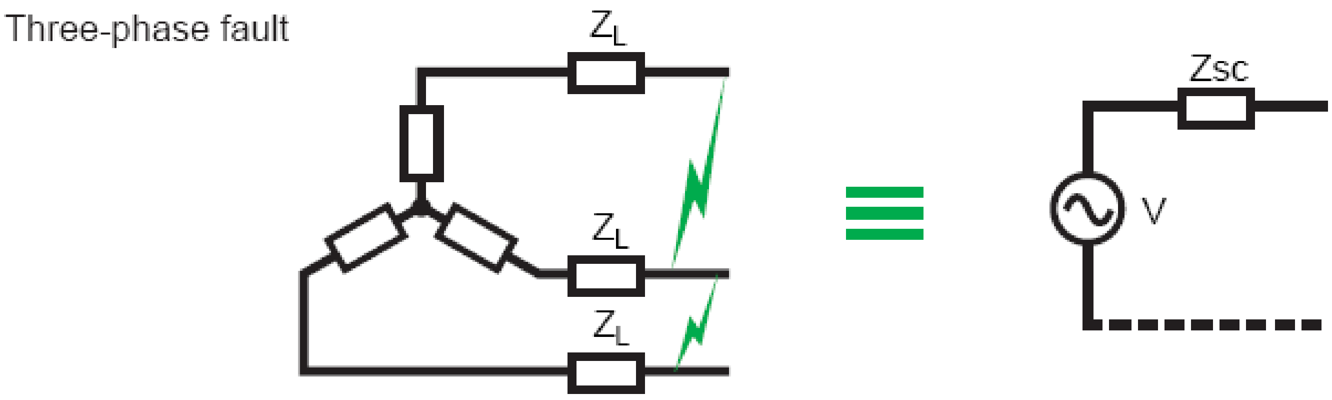


Positive Sequence (Single Phase) Circuit :-



Procedure For Calculating Maximum Fault Current
Using the Classical Method

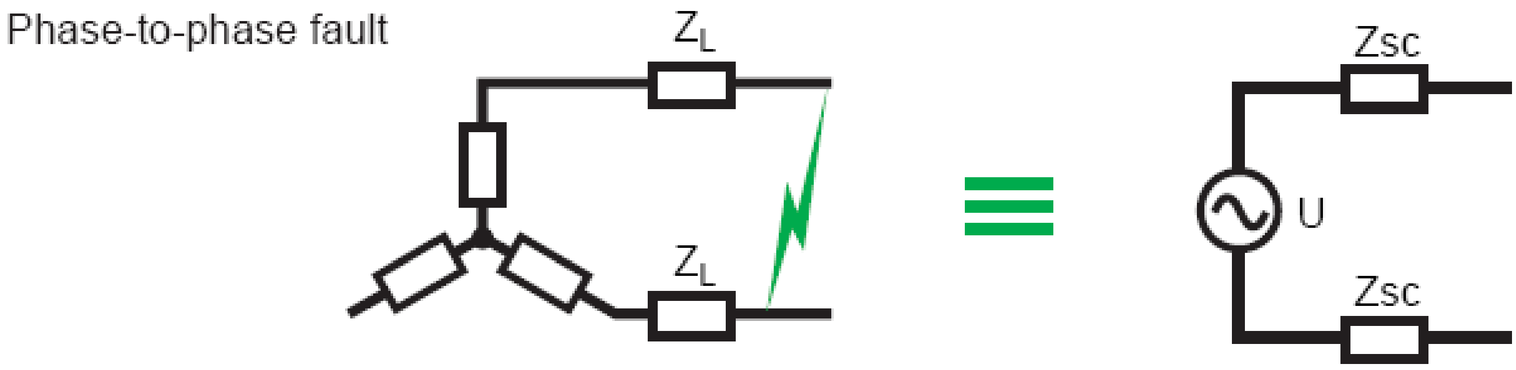
7. **Calculate the 3-PHASE FAULT CURRENT:**



$$I_{sc\text{-}3ph} = \frac{V_{ph}}{Z_{sc}}$$

Procedure For Calculating Maximum Fault Current
Using the Classical Method

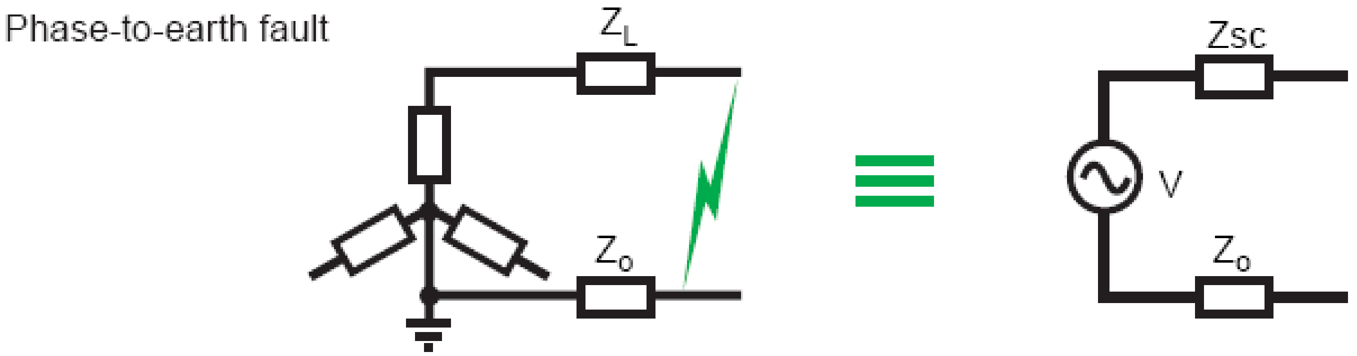
Calculate the PHASE-TO PHASE FAULT CURRENT:



$$I_{sc\text{-}2ph} = \frac{V_{ph\text{-}ph}}{2Z_{sc}} = \frac{\sqrt{3}}{2} I_{sc\text{-}3ph} = 0.866 I_{sc\text{-}3ph}$$

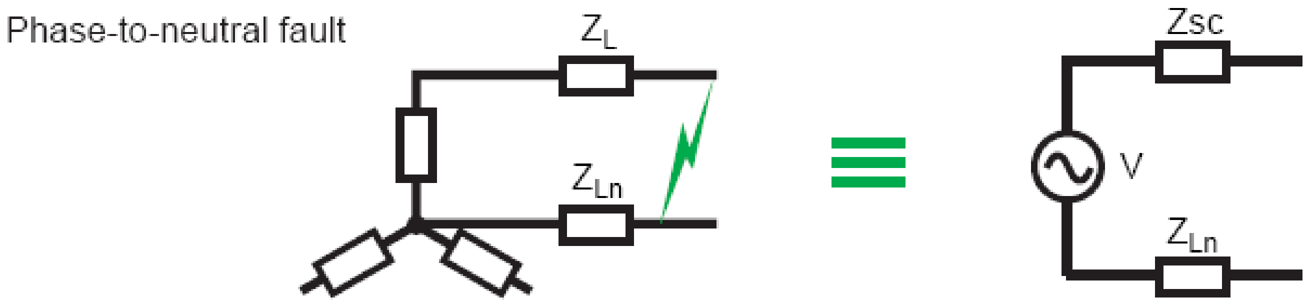
Procedure For Calculating Maximum Fault Current
Using the Classical Method

Calculate the PHASE-TO-GROUND FAULT CURRENT:



$$I_{sco} = \frac{V_{ph}}{Z_{sc} + Z_o}$$

Calculate the PHASE-TO-NEUTRAL FAULT CURRENT:



$$I_{sc1\text{-}ph} = \frac{V_{ph}}{Z_{sc} + Z_{LN}}$$

Procedure For Calculating Maximum Fault Current
Using the Classical Method

When using the PER-UNIT METHOD to calculate fault levels the following formulae are used to convert all impedances to per-unit values.

$$\text{SOURCE P.U. IMPEDANCE} = \frac{\text{Base MVA}}{\text{SOURCE S.C. MVA}} = \frac{V_s^2}{MVA_{SC}}$$

$$\text{TRANSFORMER P.U. IMPEDANCE } Z_{PU} = \frac{Z_T\,\%}{100} \times \frac{\text{Base MVA}}{\text{TRANSFORMER MVA}}$$

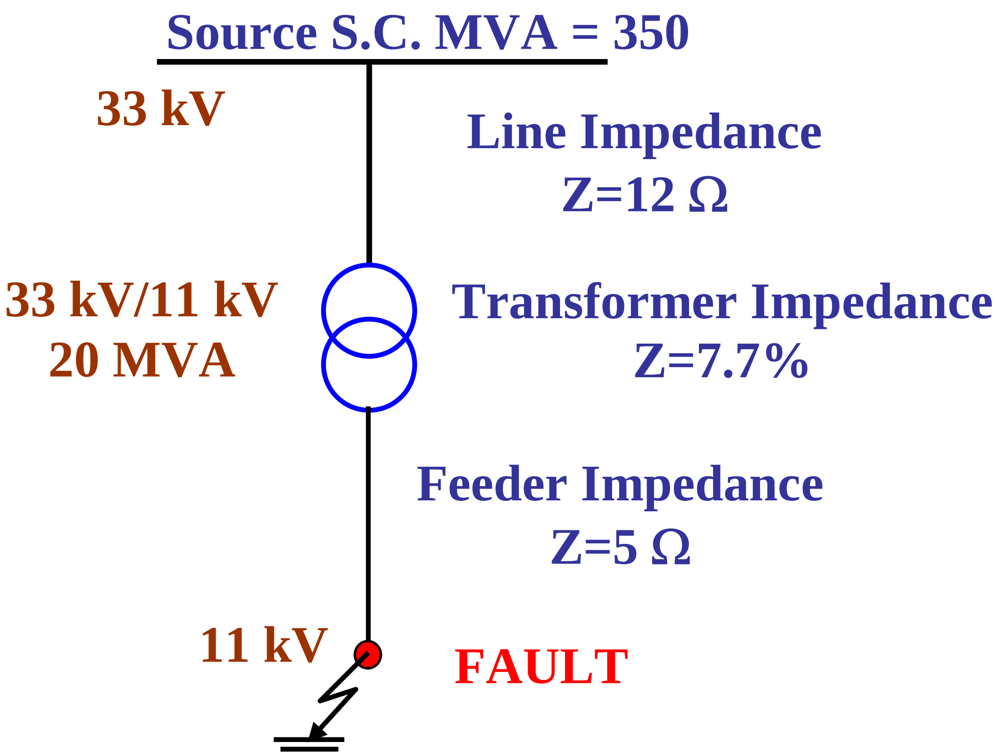
$$\text{FEEDER P.U. IMPEDANCE } Z_{PU} = \frac{Z_{OHMS}}{Z_{Base}} = \frac{Z_{OHMS}}{\dfrac{kV_{Base}^2}{\text{Base MVA}}} = Z_{OHMS} \times \frac{\text{Base MVA}}{kV_{Base}^2}$$

$$3-\text{Phase S.C. MVA at FAULT} = \frac{1}{\text{TOTAL } Z_{PU}} \times \text{Base MVA}$$

$$\text{RMS SYMM. S.C. CURRENT at FAULT} = \frac{1}{Z_{PU}} \times \frac{\text{Base MVA}}{\sqrt{3} \times kV}$$

$$= \frac{\text{S.C. MVA}}{\sqrt{3} \times kV}$$

Example of Maximum Fault Current



Base. MVA = 100

Assume HIGH *X*/*R*
resistances are ignored

$$I_{sym} = 889.6\ \text{A}$$

Example of Maximum Fault Current

$$\text{SOURCE P.U. Z} = \frac{100\ \text{MVA}}{350.\ \text{MVA}} \qquad = 0.286\ \text{pu}$$

$$33\ \text{kV Line}\ \ Z_{\text{PU}} = \frac{12 \times 100\ \text{MVA}}{(33\ \text{kV})^2} \qquad = 1.1\ \text{pu}$$

$$\text{TRANSFORMER P.U. IMPEDANCE}\ Z_{\text{PU}} = \frac{7.7\%}{100} \times \frac{100\ \text{MVA}}{20\ \text{MVA}} = 0.385\ \text{pu}$$

$$11\ \text{kV FEEDER P.U. Z} = 5 \times \frac{100\ \text{MVA}}{(11\ \text{kV})^2} \qquad = 4.13\ \text{pu}$$

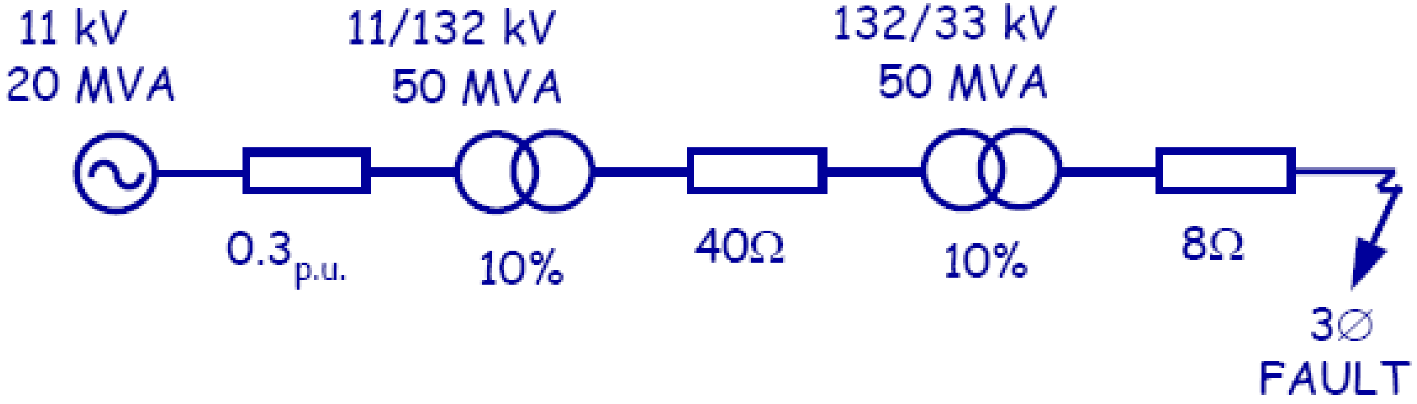
$$\text{Total Impedance from Source to FAULT} = \qquad = 5.90\ \text{pu}$$

$$3 - \text{Phase S.C. MVA at FAULT} = \frac{100\ \text{MVA}}{5.90\ \text{pu}} \qquad = 16.95\ \text{MVA}$$

$$\text{RMS SYMM. S.C. CURRENT at FAULT} == \frac{16.95\ \text{MVA}}{\sqrt{3} \times 11\ \text{kV}} \qquad = 889.6\ \text{A}$$

Example

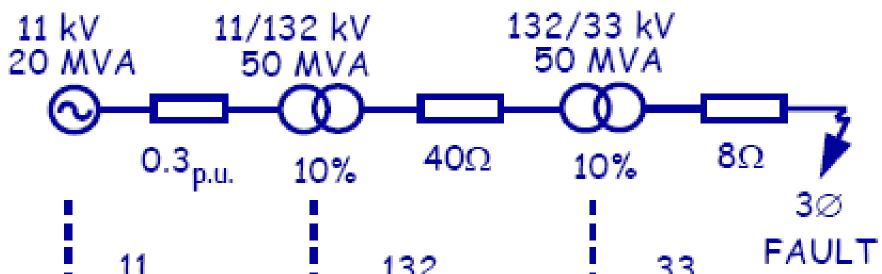
Calculate the fault currents in 11kV, 132kV and 33kV system for the three phase fault shown.

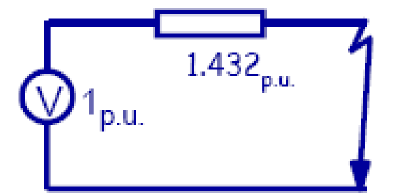


I_{sym}= 611 A

Example

Calculate the fault currents in 11kV, 132kV and 33kV system for the three phase fault shown.





kV_b	11		132		33
MVA_b	50		50		50
$Z_b = \frac{kV_b^2}{MVA_b}$	2.42 Ω		348.5 Ω		21.8 Ω
$I_b = \frac{MVA_b}{\sqrt{3}kV_b}$	2624 A		219 A		875 A
$Z_{p.u.}$	$0.3 \times \frac{50}{20} = 0.75_{p.u.}$	$0.1_{p.u.}$	$\frac{40}{348.5} = 0.115_{p.u.}$	$0.1_{p.u.}$	$\frac{8}{21.8} = 0.367_{p.u.}$

$$I_F = \frac{1}{1.432} = 0.698 \text{p.u.}$$

$$I_{11\,kV} = 0.698 \times I_b = 0.698 \times 2624 = 1832A$$

$$I_{132\,kV} = 0.698 \times 219 = 153A$$

$$I_{33\,kV} = 0.698 \times 875 = 611A$$

Unbalanced 3-Phase System

$$V_A = V_{A1} + V_{A2} + V_{A0}$$
$$V_B = V_{B1} + V_{B2} + V_{B0}$$
$$V_C = V_{C1} + V_{C2} + V_{C0}$$

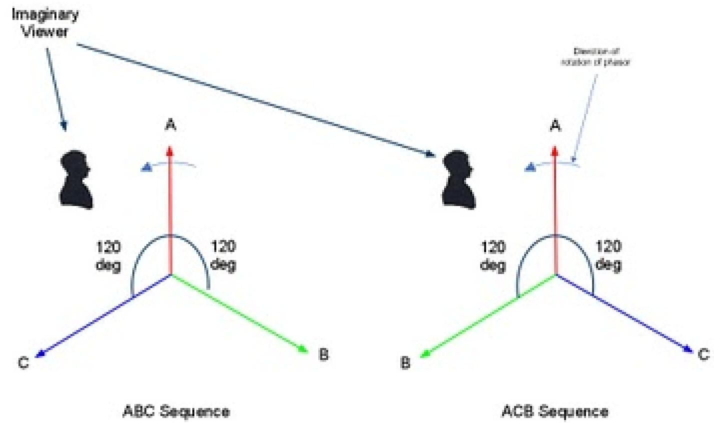
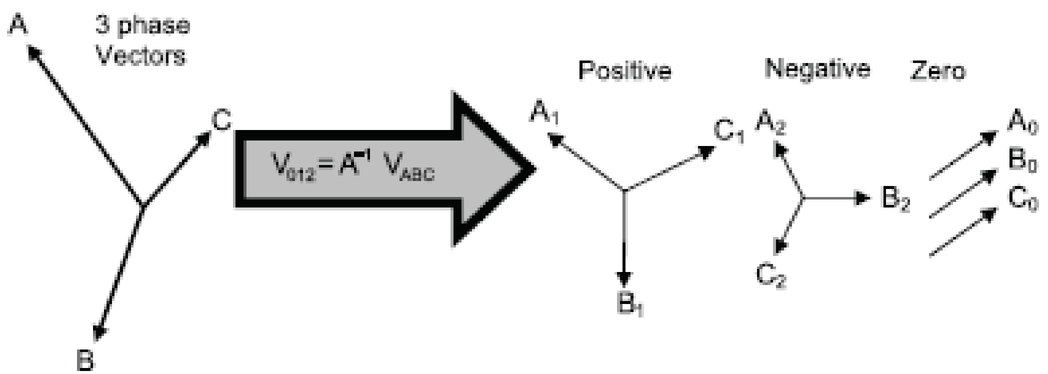
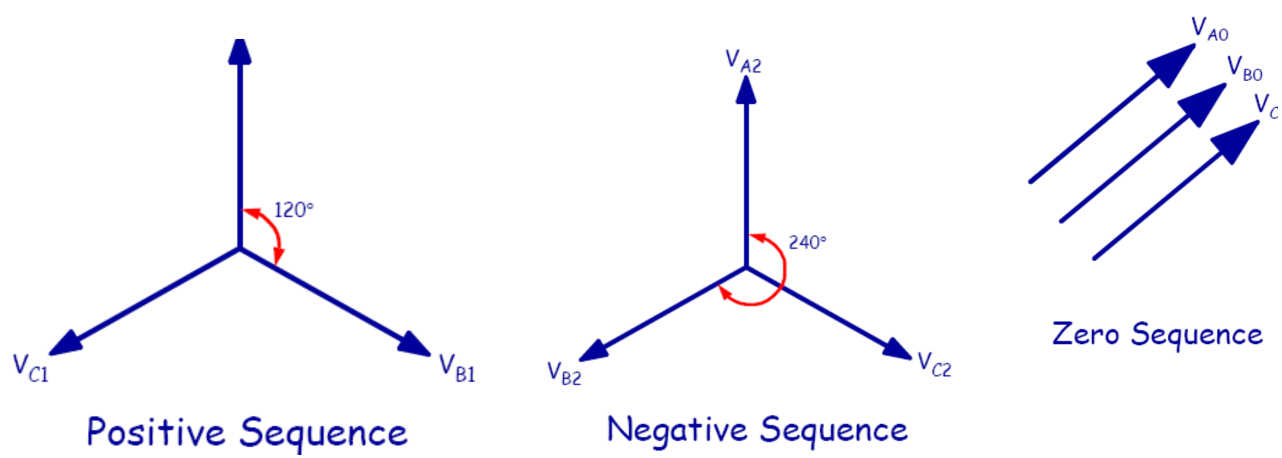


Fig A: ABC and ACB phase sequence, Anti clock wise Rotation

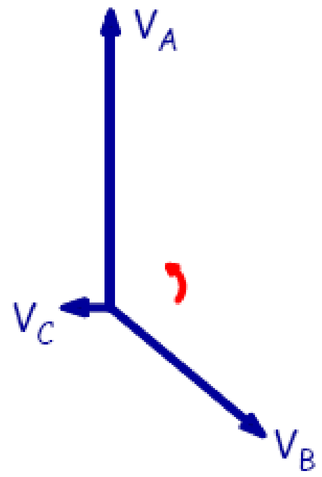


Symmetrical Component Conversion



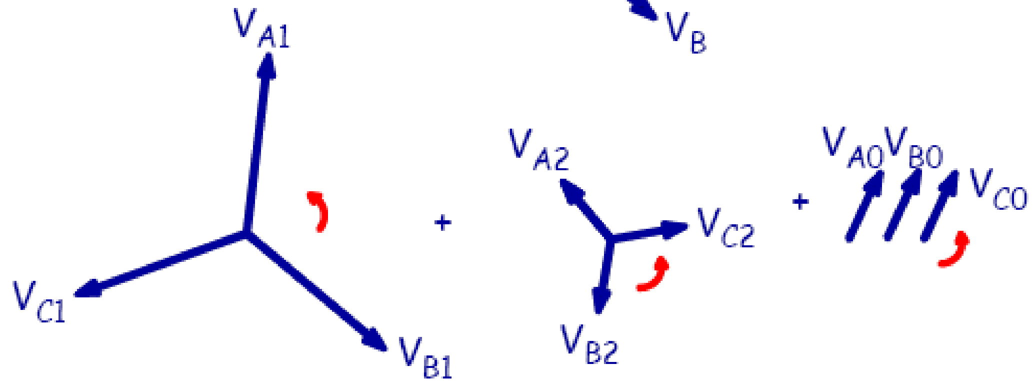
Symmetrical Components

Phase ≡ Positive + Negative + Zero



$$V_A = V_{A1} + V_{A2} + V_{A0}$$
$$V_B = V_{B1} + V_{B2} + V_{B0}$$
$$V_C = V_{C1} + V_{C2} + V_{C0}$$

$a = 1\angle 120^o$



$$V_{B1} = a^2 V_{A1} \qquad V_{B2} = a\, V_{A2} \qquad V_{B0} = V_{A0}$$
$$V_{C1} = a\, V_{A1} \qquad V_{C2} = a^2 V_{A2} \qquad V_{C0} = V_{A0}$$

Converting Sequence Components ⇔ Phase Values

$$V_A = V_{A1} + V_{A2} + V_{A0}$$

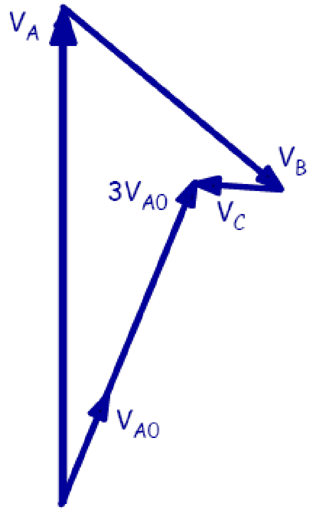
$$V_B = V_{B1} + V_{B2} + V_{B0} = a^2V_{A1} + a\,V_{A2} + V_{A0}$$

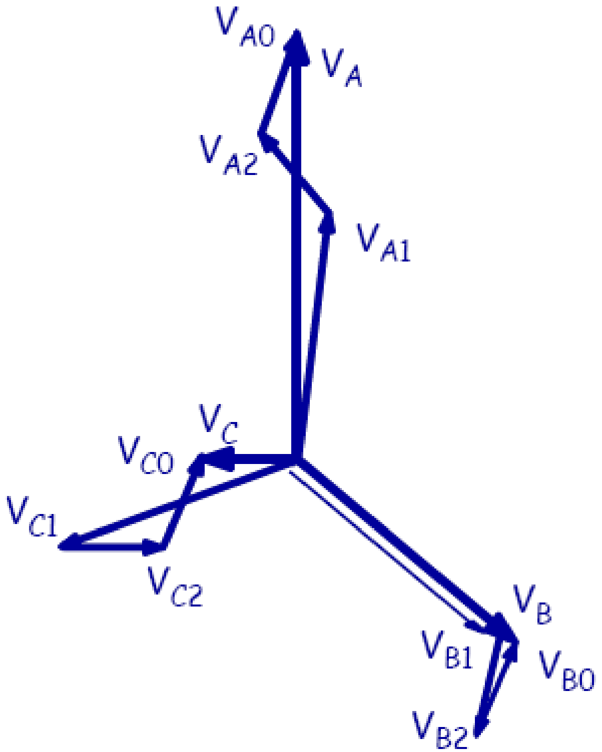
$$V_C = V_{C1} + V_{C2} + V_{C0} = a\,V_{A1} + a^2V_{A2} + V_{A0}$$

$$V_{A1} = 1/3\,\{V_A + a\,V_B + a^2V_C\}$$

$$V_{A2} = 1/3\,\{V_A + a^2V_B + a\,V_C\}$$

$$V_{A0} = 1/3\,\{V_A + V_B + V_C\}$$





Sequence Networks

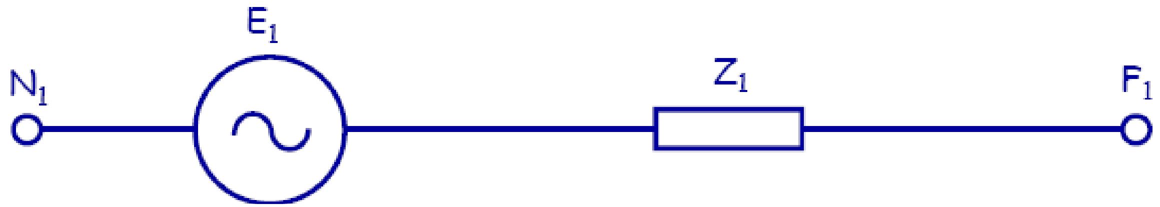
- It can be shown that provided the system impedances are balanced from the points of generation right up to the fault, each sequence current causes voltage drop of its own sequence only
- +ve, -ve and zero sequence networks are drawn for a 'reference' phase. This is usually taken as the 'A' phase.
- Faults are selected to be 'balanced' relative to the reference 'A' phase.

e.g. For Ø/E faults consider an A-E fault

For Ø/Ø faults consider a B-C fault

Positive Sequence Diagram

1. **Start with neutral point N_1**
   - ❖ **All generator and load neutrals are connected to N_1**
2. **Include all source voltages**
   - ❖ **Phase-neutral voltage**
3. **Impedance network**
   - ❖ **Positive sequence impedance per phase**
4. **Diagram finishes at fault point F_1**

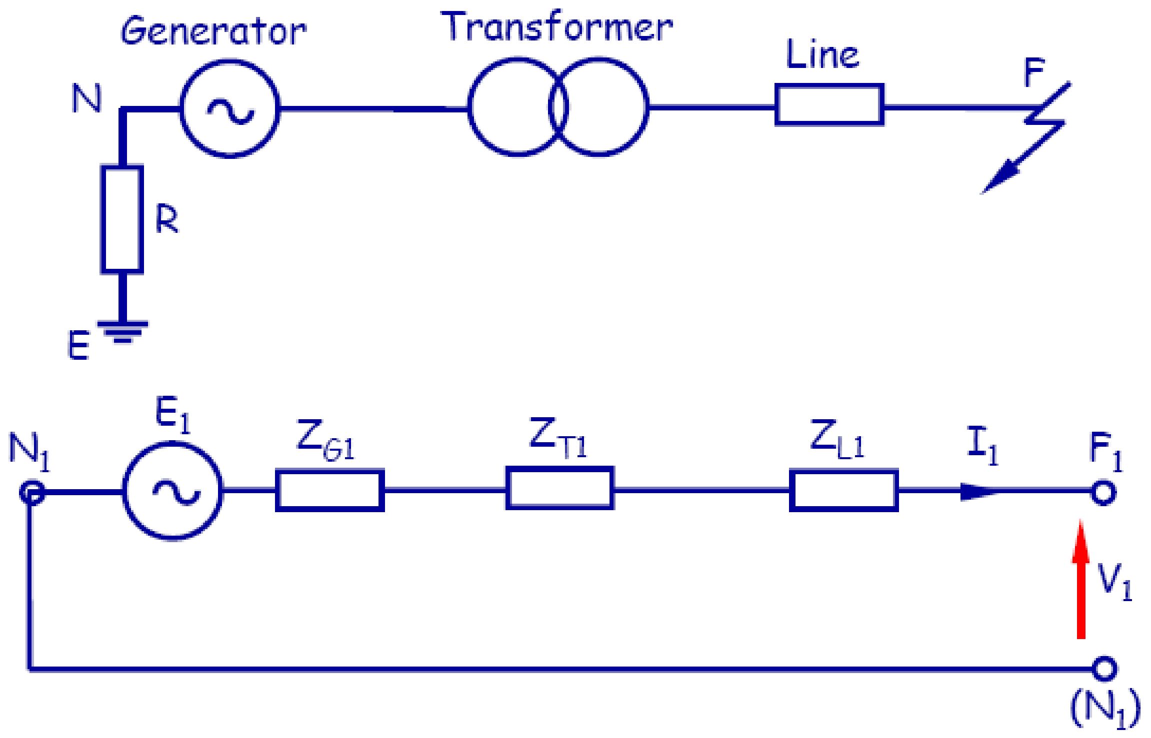


Positive Sequence Diagram

V_1 = Positive sequence Ph-N voltage at fault point

I_1 = Positive sequence phase current flowing into F_1

$$V_1 = E_1 - I_1 (Z_{G1} + Z_{T1} + Z_{L1})$$



Negative Sequence Diagram

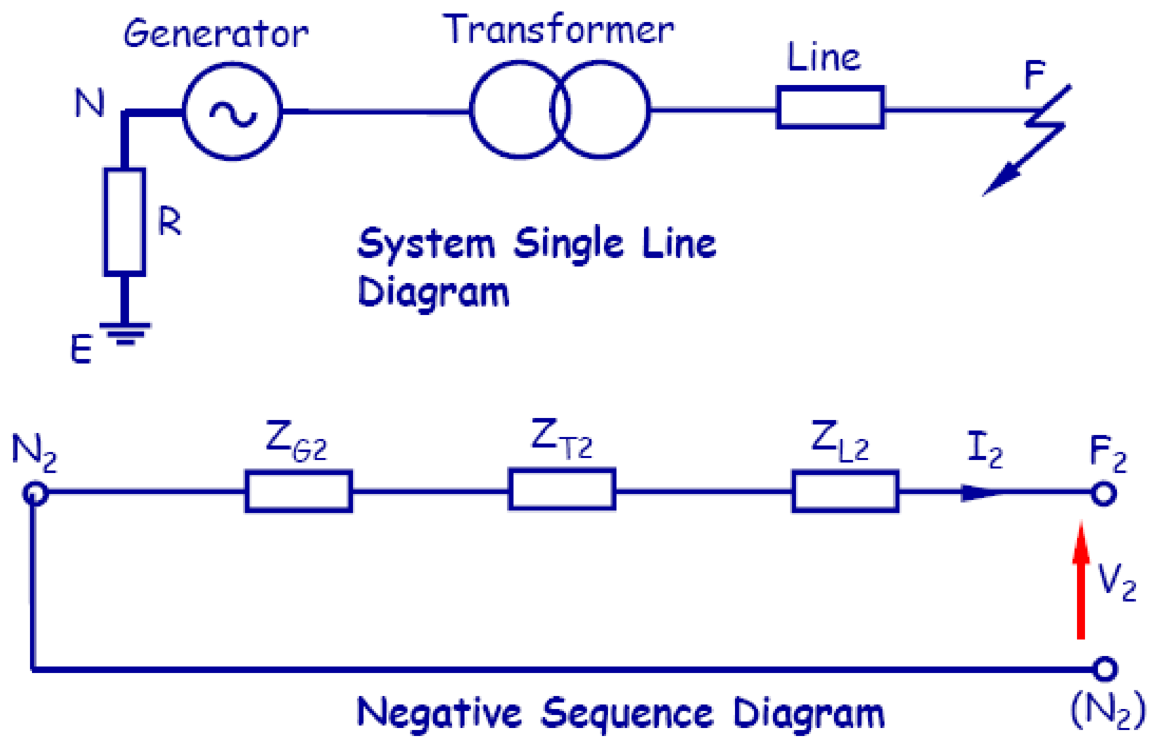
1. **Start with neutral point N_2**
   - **All generator and load neutrals are connected to N_2**
2. **No Voltages included**
   - **No negative sequence voltage is generated!**
3. **Impedance network**
   - **Negative sequence impedance per phase**
4. **Diagram finishes at fault point F_2**

Negative Sequence Diagram

V_2 = Negative sequence Ph-N voltage at fault point

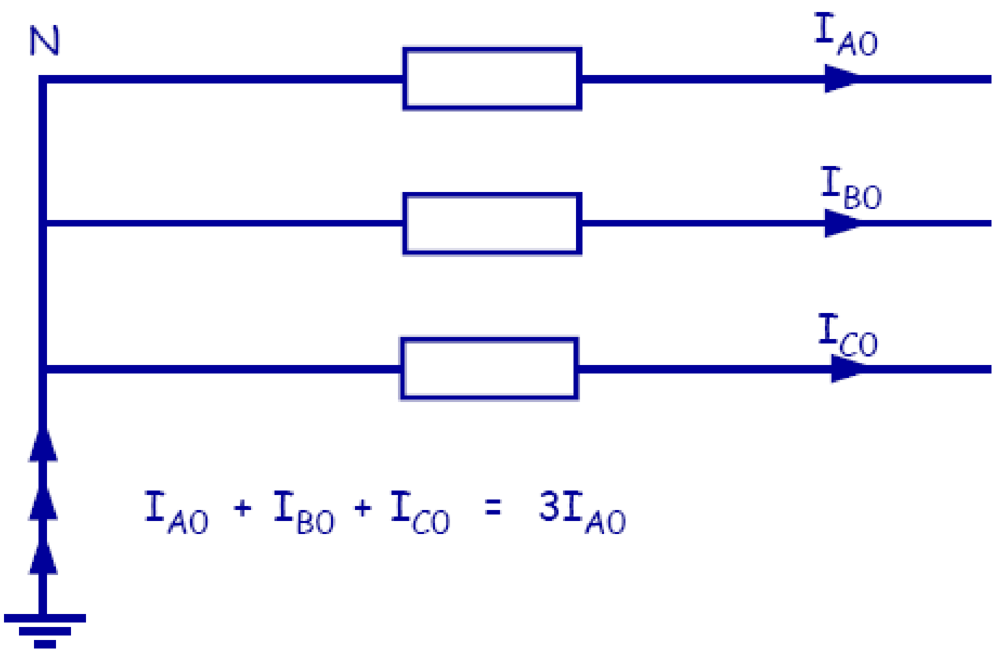
I_2 = Negative sequence phase current flowing into F_2

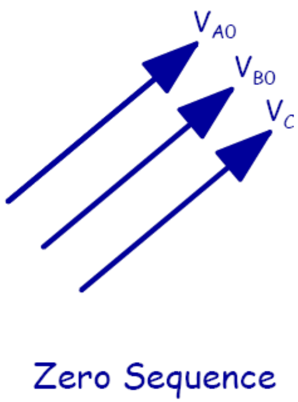
$V_2 = -I_2 \,(Z_{G2} + Z_{T2} + Z_{L2})$



Zero Sequence Diagram

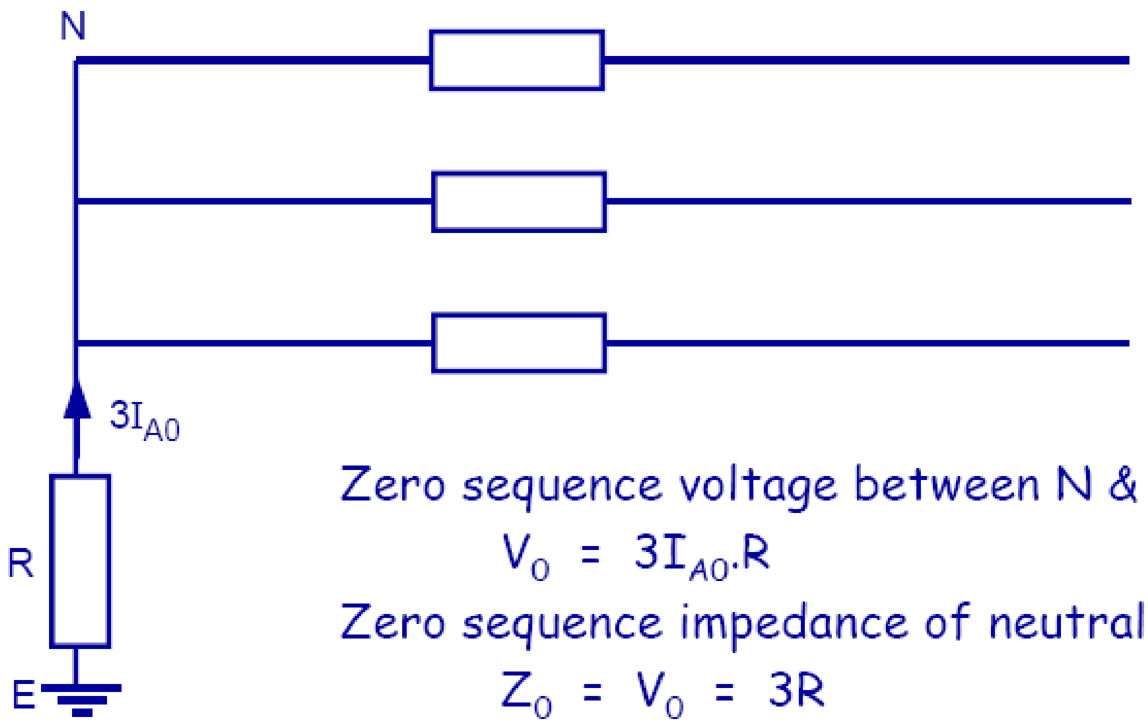
- For "In Phase" (Zero Phase Sequence) currents to flow in each phase of the system, there must be a fourth connection (this is typically the neutral or earth connection).





Zero Sequence Diagram

- **Resistance Earthed System :-**



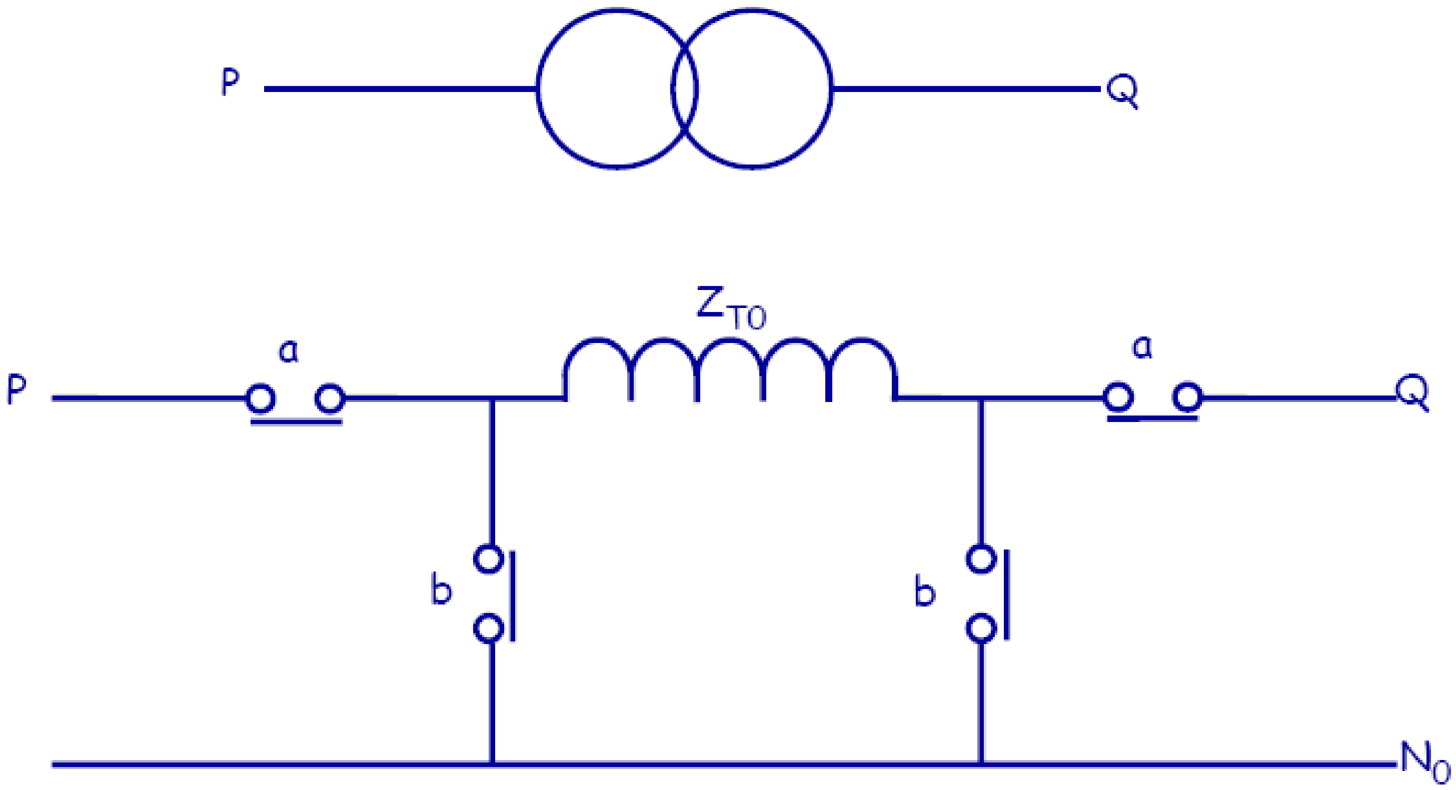
Zero sequence voltage between N & E given by

$$V_0 = 3I_{A0}.R$$

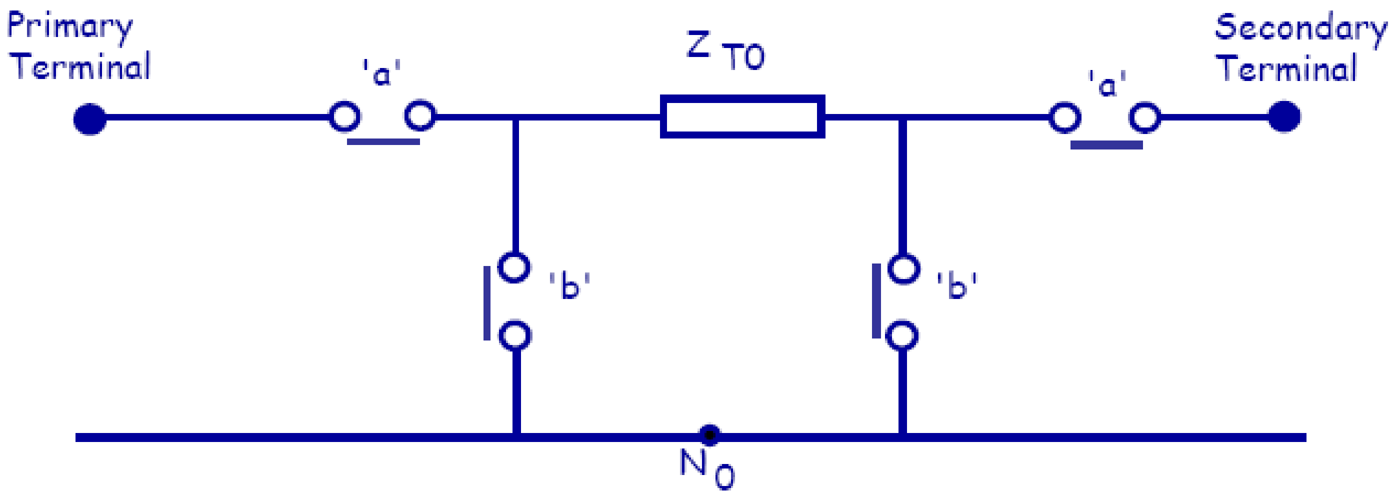
Zero sequence impedance of neutral to earth path

$$Z_0 = \frac{V_0}{I_{A0}} = 3R$$

Transformer Zero-Sequence Diagram



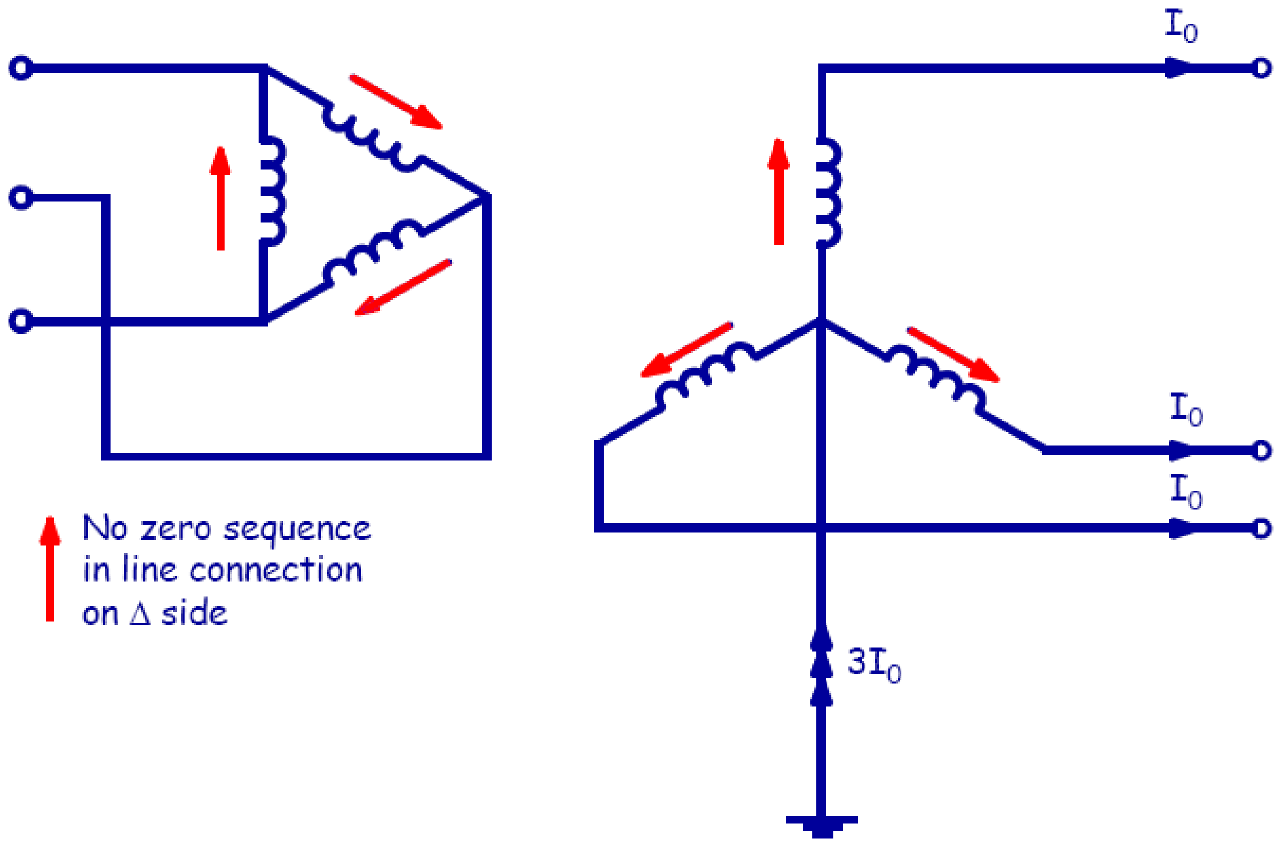
General Zero-Sequence Equivalent Circuit for Two Winding Transformer



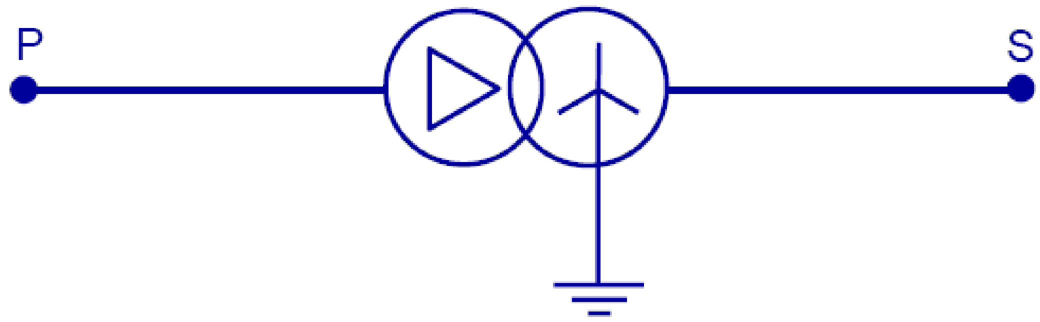
On appropriate side of transformer :

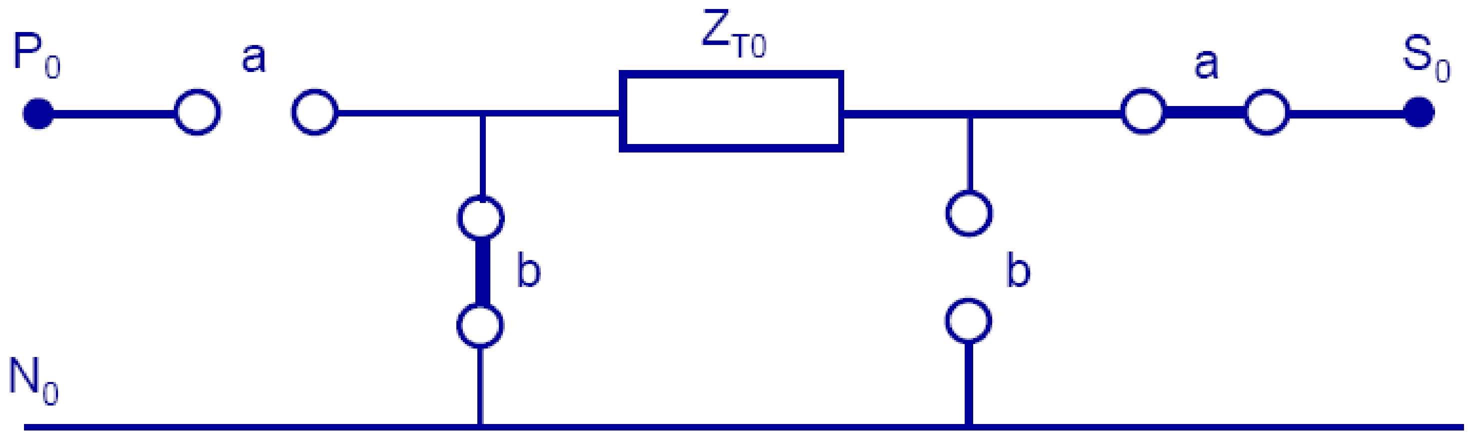
Earthed Star Winding	-	Close link 'a' Open link 'b'
Delta Winding	-	Open link 'a' Close link 'b'
Unearthed Star Winding	-	Both links open

Zero-Sequence Equivalent Circuit for "Dyn" Transformer



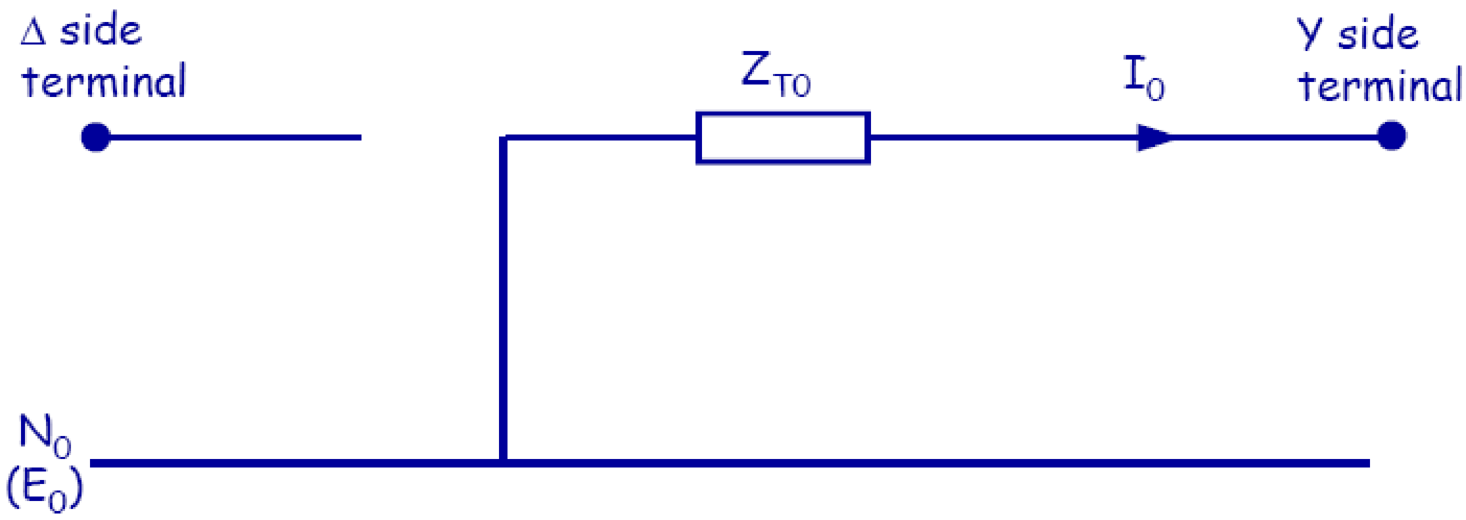
Zero-Sequence Equivalent Circuit for "Dyn" Transformer



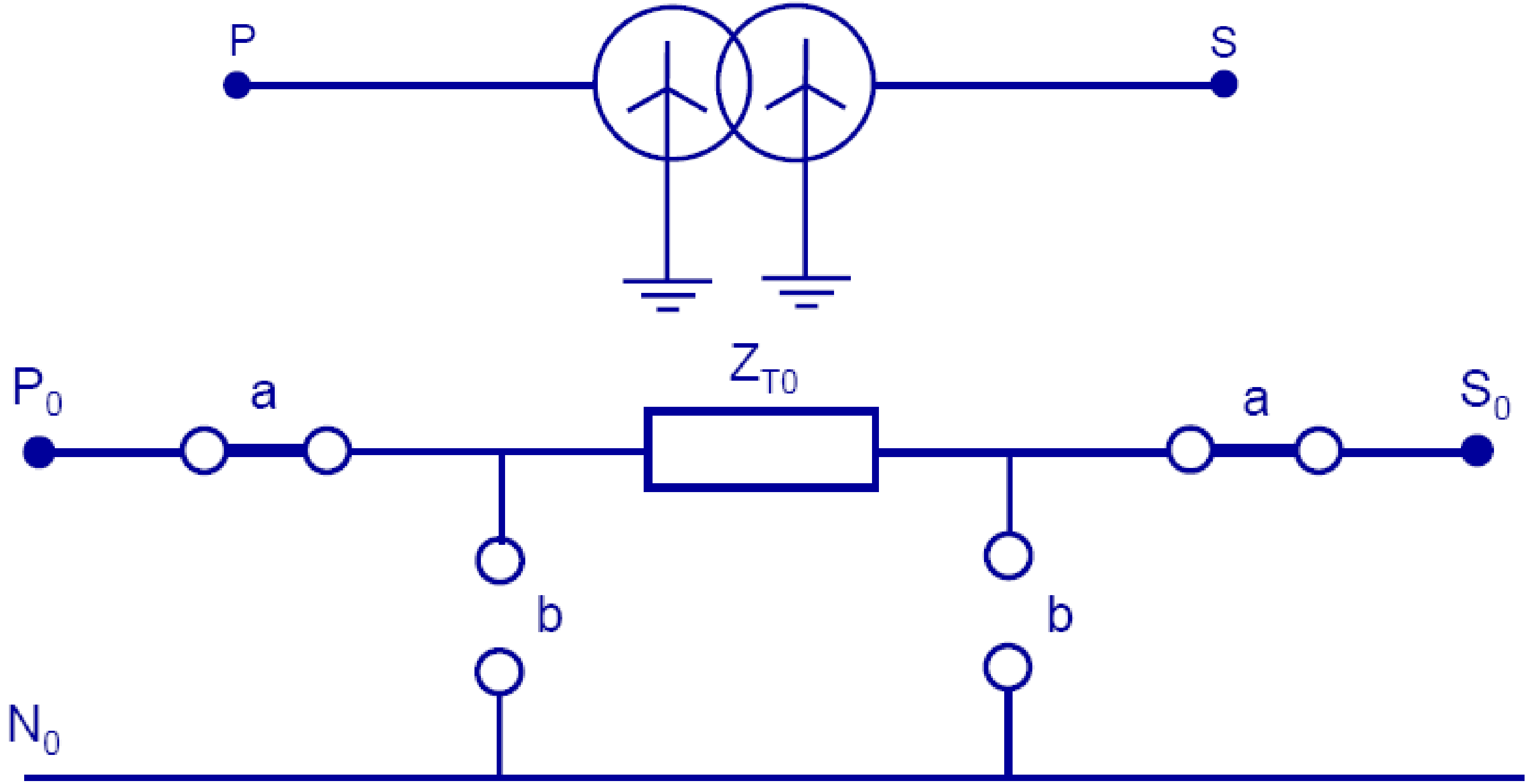


Zero-Sequence Equivalent Circuit for "Dy" Transformer

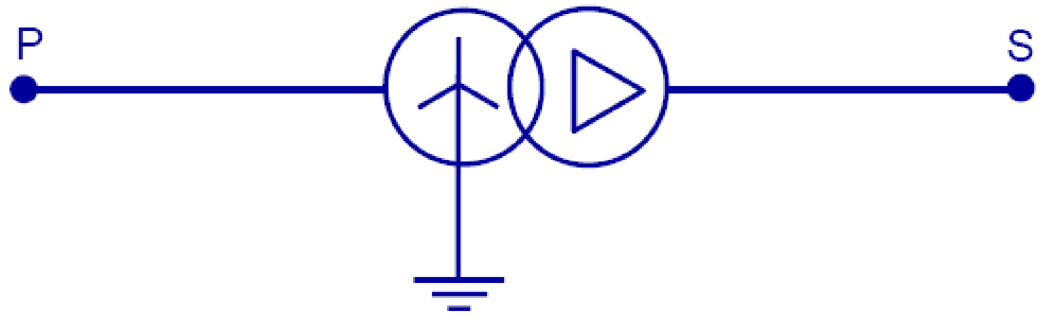
Thus, equivalent single phase zero sequence diagram :-

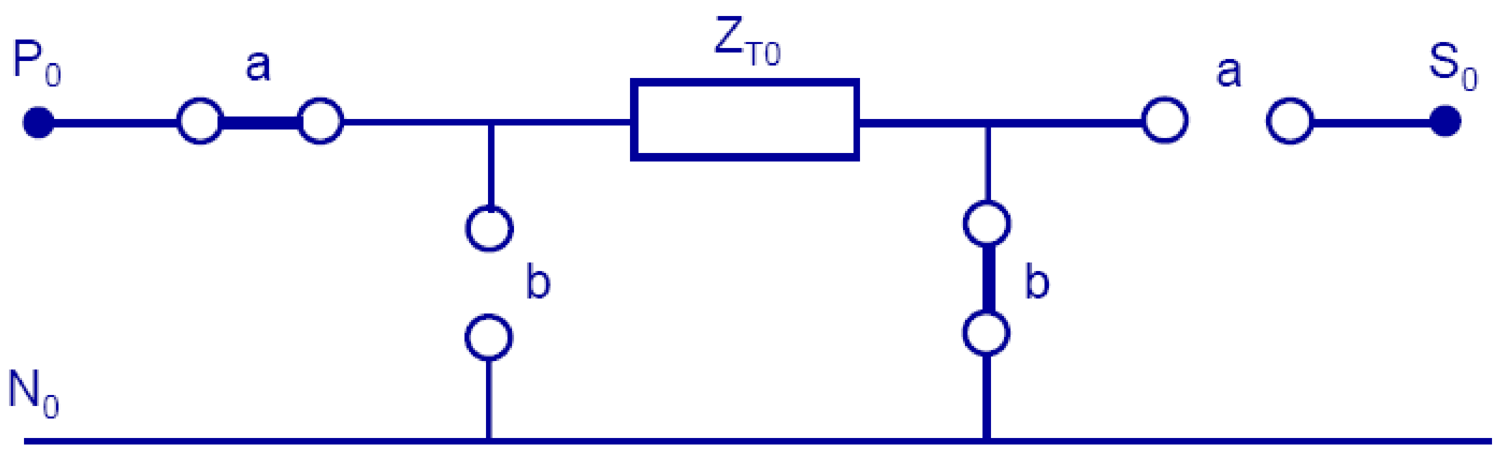


Zero-Sequence Equivalent Circuit for
“YNyn” Transformer

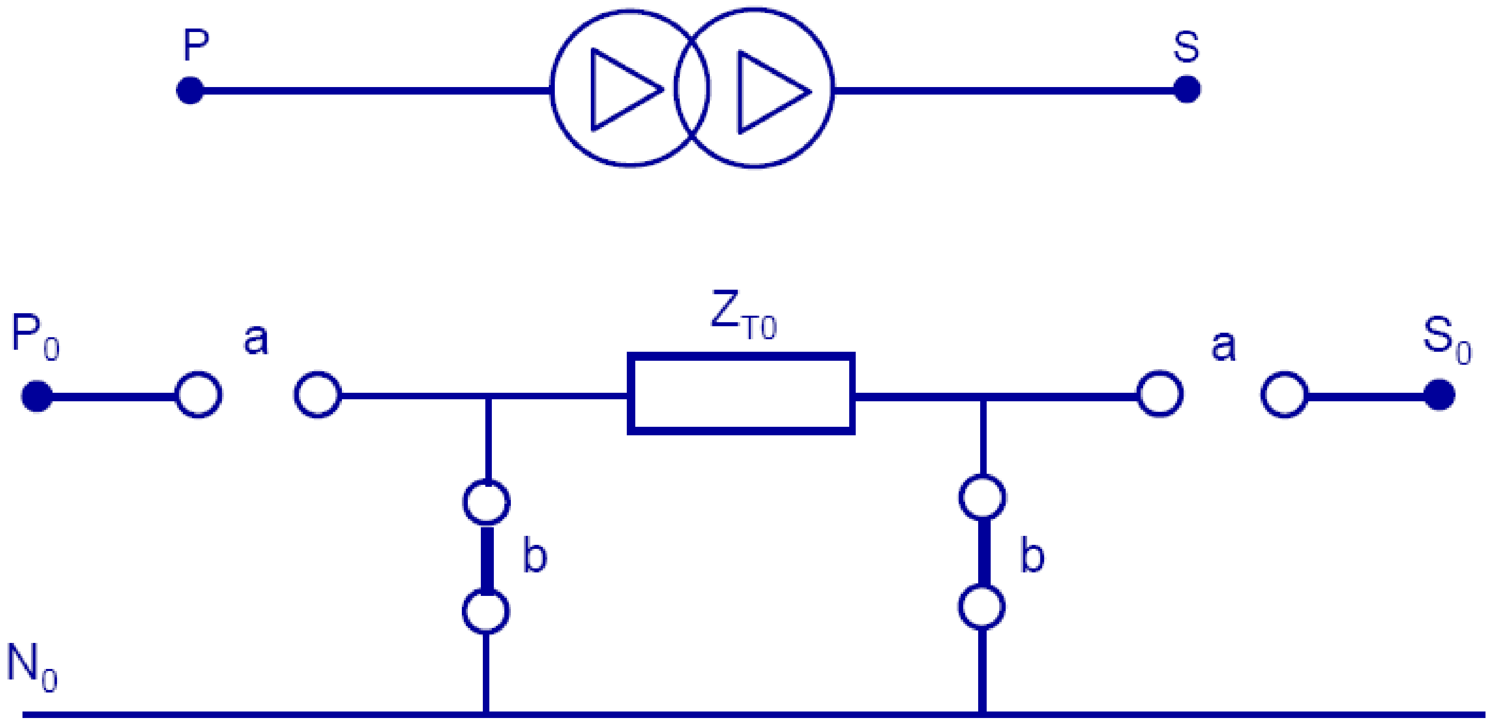


Zero-Sequence Equivalent Circuit for "YNd" Transformer

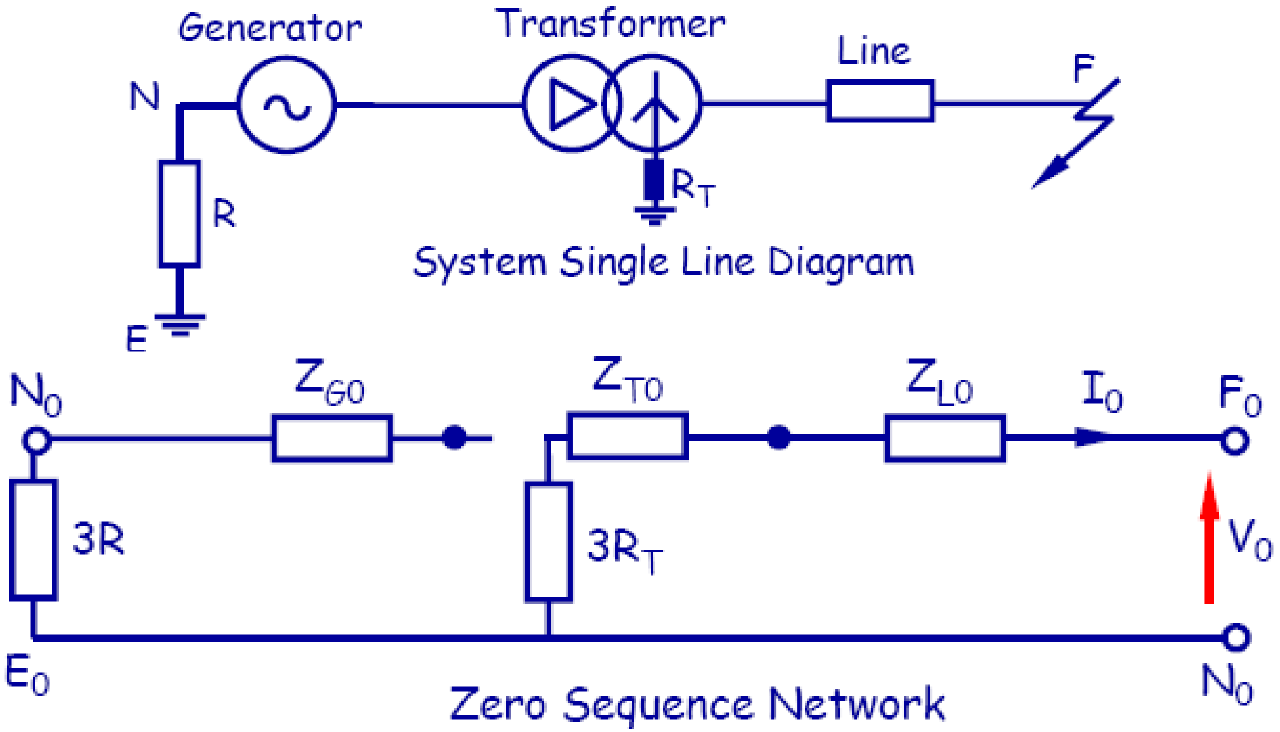




Zero-Sequence Equivalent Circuit for "Dd" Transformer



Zero-Sequence Equivalent Circuit
Example

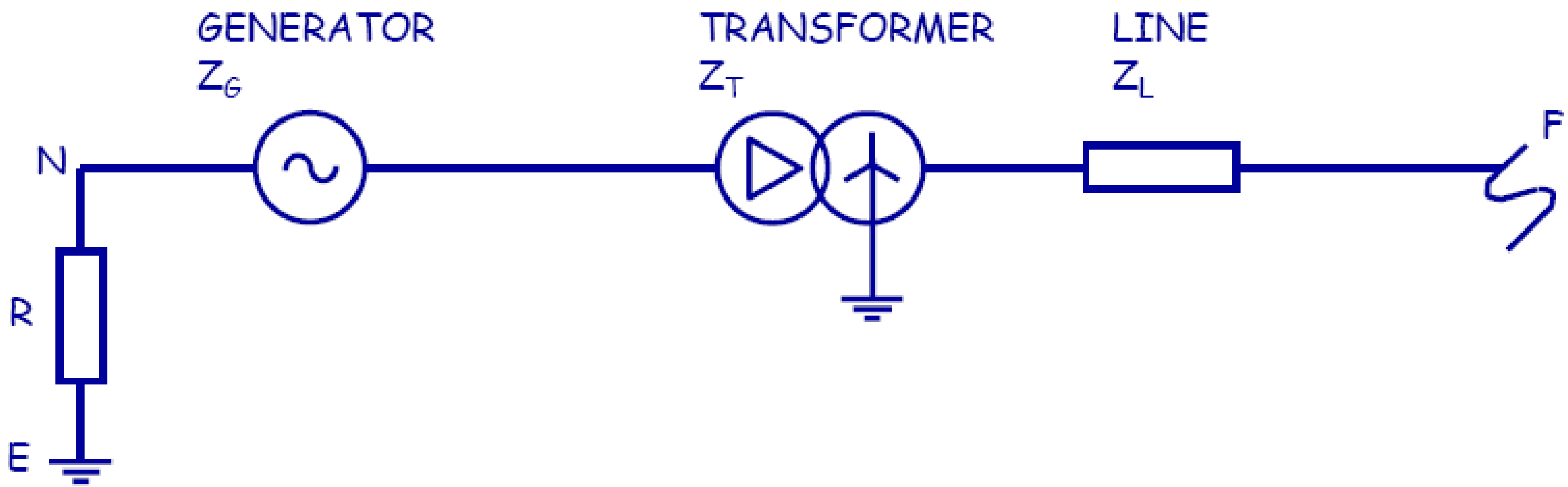


V_0 = Zero-sequence Ph-E voltage at fault point

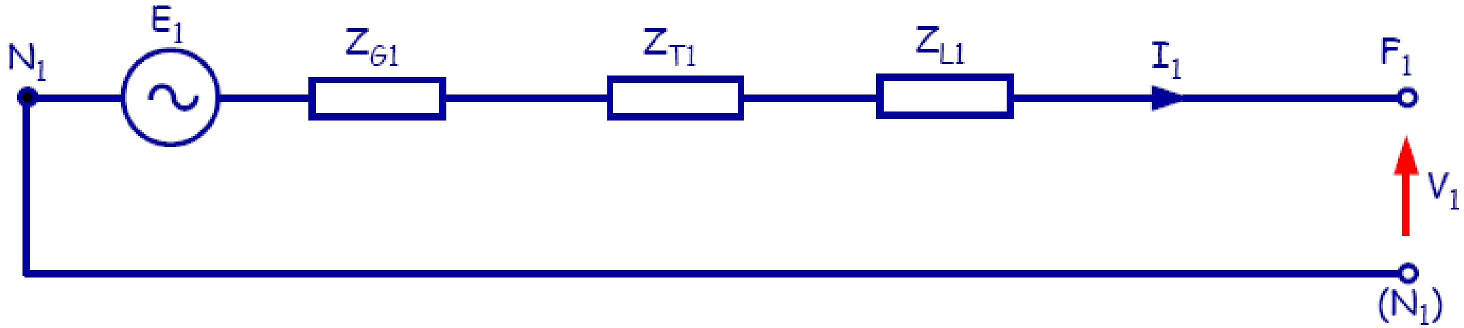
I_0 = Zer0-sequence current flowing into F_0

$V_0 = -I_0\,(Z_{T0} + Z_{L0})$

Summary of Sequence Diagrams

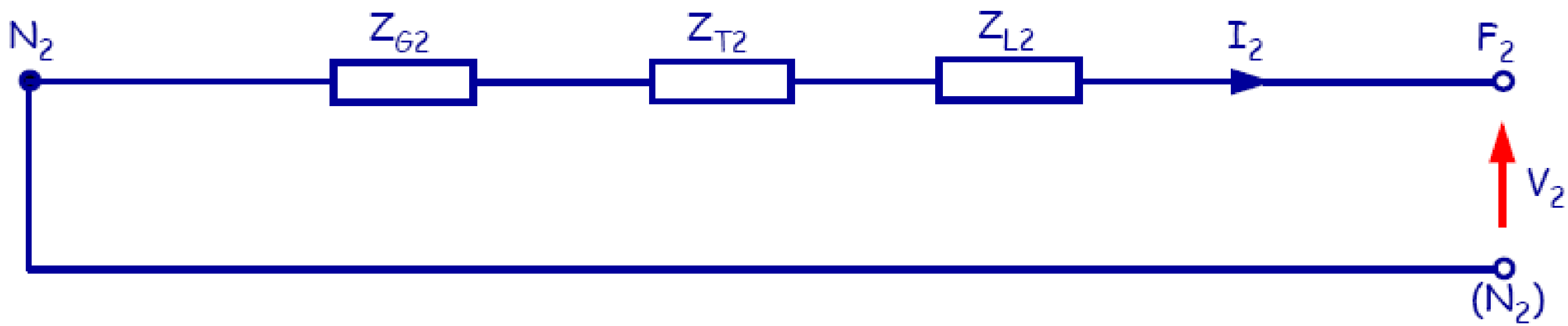


System Single Line Diagram

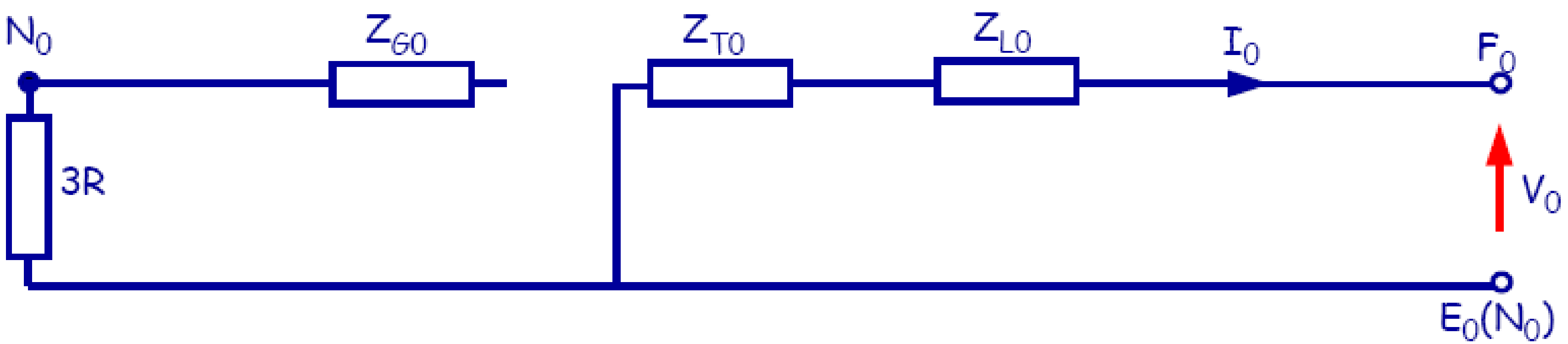


Positive-Sequence Diagram

Summary of Sequence Diagrams



Negative-Sequence Diagram



Zero-Sequence Diagram

Common Unbalanced Network Faults
Single Line to Ground Fault

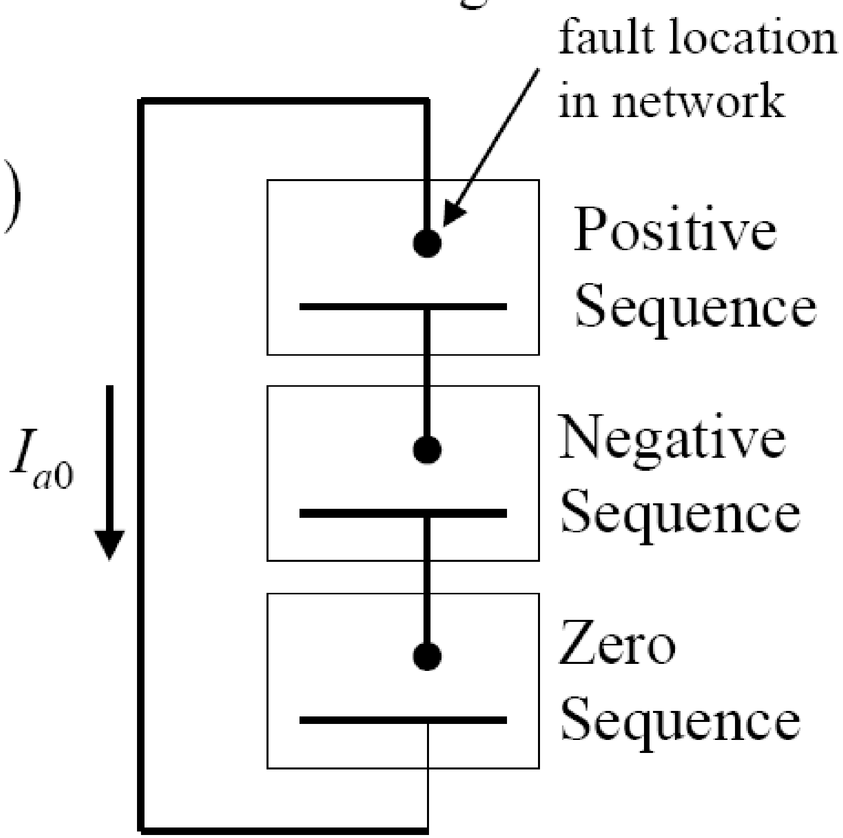
$$V_a = E_a - Z_a I_a = 0$$
$$= E_a - A\left(Z_{012} I_{012}\right)$$
$$= E_a - \left(Z_{a0} I_{a0} + Z_{a1} I_{a1} + Z_{a2} I_{a2}\right)$$
$$= E_a - \left(Z_{a0} + Z_{a1} + Z_{a2}\right) I_{a0}$$

$$I_{a0} = \frac{E_a}{\left(Z_{a0} + Z_{a1} + Z_{a2}\right)}$$

$$I_{a0} = I_{a1} = I_{a2}$$

$$I_f = 3\, I_{a0}$$

Network Diagram



Question # 1

For the distribution feeder, shown in Fig. Q1, use the per unit method to determine the magnitude of the fault current ($I_{f\text{-}3ph}$) in Amperes for a three phase fault at the feeder end. Use a system MVA base of 100 MVA and a voltage base of 13.2 kV at the feeder.

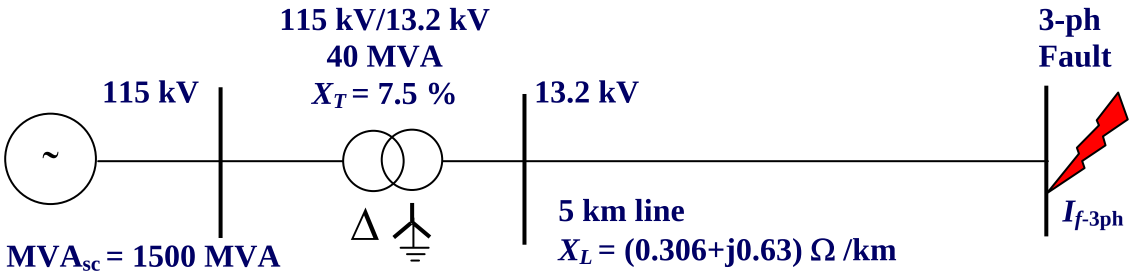


Fig. Q1

Solution:

$$X_{sc} = \frac{1}{MVA_{sc-pu}}, MVA_{sc-pu} = \frac{MVA_{sc}}{MVA_{base}} = \frac{1500}{100} = 15\ pu, X_{sc} = \frac{1}{15} = 0.067\ pu,$$

$$X_{sc\Omega} = X_{scpu} \times X_{base-13,2kV}, X_{base-13.2kV} = \frac{kV_{base}^2}{MVA_{base}} = \frac{13.2^2}{100} = 1.742\ \Omega, X_{sc\Omega} = 0.067 \times 1.742 = 0.117\ \Omega$$

$$X_{Tnew} = X_{Told} \times \frac{MVA_{basenew}}{MVA_{baseold}} = \frac{7.5}{100} \times \frac{100}{40} = 0.1875\ pu, X_{T\Omega} = X_{Tnew} \times X_{base-13.2kV} = 0.1875 \times 1.742 = 0.327\ \Omega$$

$$Z_{L1\Omega} = (0.306 + j0.63) \times 5 = 1.53 + j3.15\ \Omega, \Rightarrow Z_{L1pu} = \frac{Z_{L1\Omega}}{Z_{base-13.2kV}} = \frac{1.53 + j3.15\ \Omega}{1.742\ \Omega} = 0.88 + j1.81\ pu$$

$$Z_{L1\Omega} = 0.70\angle 64.1° \times 5 = 3.5\angle 64.1°\ \Omega \Rightarrow X_{L1pu} = 2.03\angle 64.1°\ pu$$

$$Z_{eq1\Omega} = jX_{sc1} + jX_{T1} + Z_{L1} = j0.117 + j0.327 + 1.53 + j3.15 = 1.53 + j3.59\ \Omega = 3.9\angle 66.9°\ \Omega$$

$$Z_{eq1pu} = jX_{sc1pu} + jX_{T1pu} + Z_{L1pu} = j0.067 + j0.188 + 0.88 + j1.81 = 0.88 + j2.07\ pu = 2.24\angle 66.9°\ pu$$

$$I_{f3phA} = \frac{E_1}{Z_{eq1}} = \frac{\frac{13.2 \times 10^3}{\sqrt{3}}\angle 0°\ V}{3.9\angle 66.9°\ \Omega} = \frac{7621\angle 0°\ V}{3.9\angle 66.9°\ \Omega} = 1954.1\angle -66.9°\ A$$

$$I_{f3phpu} = \frac{E_1}{Z_{eq1pu}} = \frac{1.0\angle 0°\ V}{2.24\angle 66.9°\ \Omega} = 0.446\angle -66.9°\ pu$$

$$I_{base-13.2kV} = \frac{MVA_{base}}{\sqrt{3}V_{LL}} = \frac{100 \times 1000}{\sqrt{3} \times 13.2} = 4373.9\ A$$

X_{sc} =	**0.067 *pu***
X_T =	**0.1875 *pu***
Z_{TL} =	**0.88 + *j* 1.81 *Pu***
Z_{eq} =	**0.88 + *j* 2.07 *Pu*** $2.24\angle 66.9°\ pu$
I_{f3ph} =	**0.446 *pu***
I_b =	**4373.9 *A***
I_{f3ph} =	**1954.1 *A***

Question # 2:

For the system shown in Fig. Q2, and starting from the data that are given, calculate the three-phase and single-line-ground fault in Amperes at the Buses 3, 2, and 1 using:

a. The Ohmic method referring the system to the 115 kV bus.
b. The per-unit method.

Solution:

a. The Ohmic method

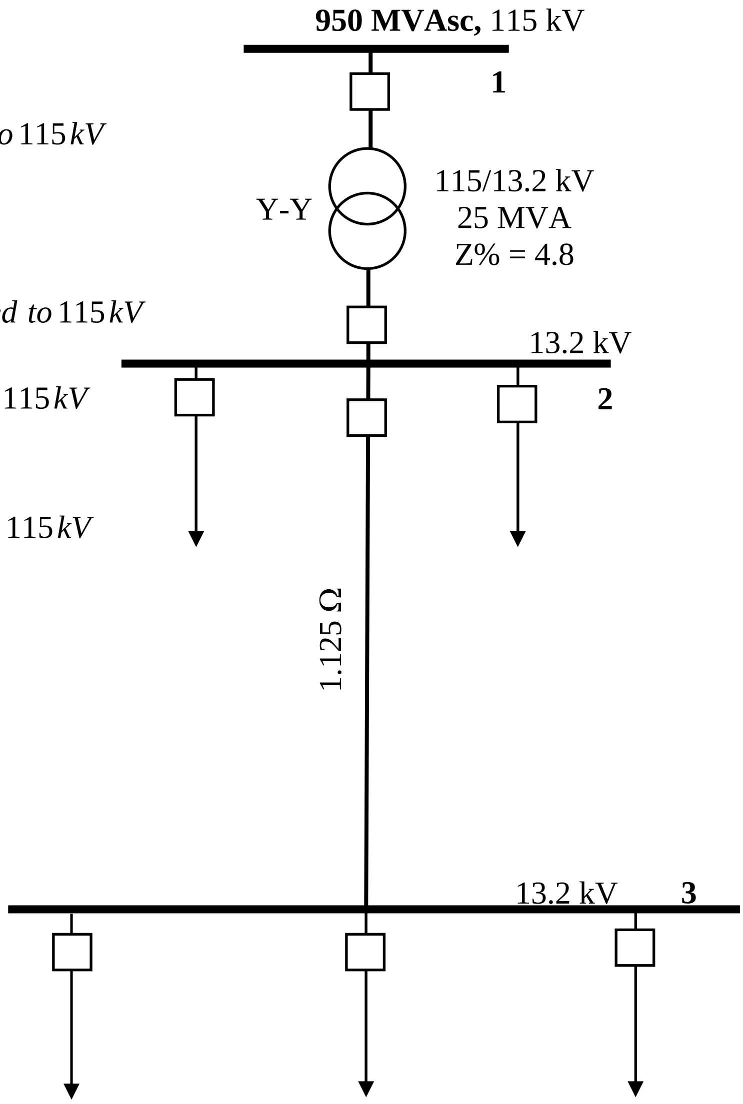


Fig. Q2

$$Z_{sc} = \frac{V_{LL}^2}{MVAsc} = \frac{(115)^2}{950} = 13.92\ \Omega \textit{ referred to } 115kV$$

$$Z_{transformer} = Z_{pu} \times Z_{base}$$

$$Z_{base} = \frac{V_{LL}^2}{MVA-TX} = \frac{(115)^2}{25} = 529\ \Omega \textit{ referred to } 115kV$$

$$Z_{transformer} = \frac{4.8}{100} \times 529 = 25.39\ \Omega \textit{ referred to } 115kV$$

$$Z_{TL} = 1.125 \times \left(\frac{115}{13.2}\right)^2 = 85.35\ \Omega \textit{ referred to } 115kV$$

a. Three-Phase Fault

$$I_{faultC} = \frac{115 \times 10^3}{\sqrt{3}(13.92 + 25.39 + 83.35)}$$
$$= 532.6\ A \textit{ referred to } 115kV$$
$$= 532.6 \times \left(\frac{115}{13.2}\right)$$
$$= 4640.2\ A \textit{ referred to } 13.2kV$$

$$I_{faultB} = \frac{115 \times 10^3}{\sqrt{3}(13.92 + 25.39)}$$
$$= 1689.0\ A \textit{ referred to } 115kV$$
$$= 1689.0 \times \left(\frac{115}{13.2}\right)$$
$$= 14714.8\ A \textit{ referred to } 13.2kV$$

$$I_{faultA} = \frac{115 \times 10^3}{\sqrt{3}(13.92)}$$
$$= 4769.8\ A \textit{ referred to } 115kV$$

The L-G Fault = 0, because the TX is not grounded

b. The per-unit method

Let MVAbase = 25 MVA
kVbase = 13.2 kV at the primary feeder.

$$Z_{base-HV} = \frac{V_{LL}^2}{MVAbase} = \frac{(115)^2}{25} = 529\ \Omega\ referred\ \ to\ 115\ kV$$

$$Z_{base-LV} = \frac{V_{LL}^2}{MVAbase} = \frac{(13.2)^2}{25} = 6.97\ \Omega\ referred\ \ to\ 13.2\ kV$$

$$Z_{TX-pu} = \frac{4.8}{100} = 0.048\ pu$$

$$MVA_{sc-pu} = \frac{MVA_{sc}}{MVA_{base}} = \frac{950}{25} = 38\ pu$$

$$Z_{sc-pu} = \frac{1}{MVA_{sc-pu}} = \frac{1}{38} = 0.0263\ \ pu$$

$$Z_{Line-pu} = \frac{1.125}{6.97} = 0.1614\ \ pu$$

a. Three-Phase Fault

$$V_{C-pu} = \frac{V_c}{V_{C-base}} = \frac{13.2}{13.2} = 1\,pu$$

$$I_{fault-C} = \frac{V_{C-pu}}{\sum Z} = \frac{V_{C-pu}}{Z_{Line} + Z_{TX} + Z_{sc}} = \frac{1.0}{0.1614 + 0.0263 + 0.048} = 4.24\,pu$$

$$I_{base-13.2kV} = \frac{MVA_{base}}{\sqrt{3} \times kV_{Base}} = \frac{25 \times 1000}{\sqrt{3} \times 13.2} = 1093.47\,\text{A}$$

$$I_{fault-C(A)} = I_{fault-C} \times I_{base-13.2kV} = 4.24\,pu \times 1093.47\,\text{A} = 4638.6\,\text{A}$$

$$V_{B-pu} = \frac{V_B}{V_{B-base}} = \frac{13.2}{13.2} = 1\,pu$$

$$I_{fault-B} = \frac{V_{B-pu}}{\sum Z} = \frac{V_{B-pu}}{Z_{TX} + Z_{sc}} = \frac{1.0}{0.0263 + 0.048} = 13.46\,pu$$

$$I_{base-13.2kV} = \frac{MVA_{base}}{\sqrt{3} \times kV_{Base}} = \frac{25 \times 1000}{\sqrt{3} \times 13.2} = 1093.47\,\text{A}$$

$$I_{fault-B(A)} = I_{fault-B} \times I_{base-13.2kV} = 13.46\,pu \times 1093.47\,\text{A} = 14713.8\,\text{A}$$

$$V_{A-pu} = \frac{V_A}{V_{A-base}} = \frac{115}{115} = 1\,pu$$

$$I_{fault-A} = \frac{V_{A-pu}}{\sum Z} = \frac{V_{A-pu}}{Z_{sc}} = \frac{1.0}{0.02638} = 38\,pu$$

$$I_{base-115kV} = \frac{MVA_{base}}{\sqrt{3} \times kV_{Base}} = \frac{25 \times 1000}{\sqrt{3} \times 115} = 125.5\,\text{A}$$

$$I_{fault-A(A)} = I_{fault-C} \times I_{base-13.2kV} = 38\,pu \times 125.5\,\text{A} = 4769.4\,\text{A}$$

Question # 3

A portion of an 11 kV radial system is shown in Fig.Q3. The system may be operated with one rather than two source transformers under certain operating conditions. Assume high voltage bus of transformer is an infinite bus. Protection system for three-phase and line-to-line faults has to be designed. Transformer and Transmission line reactances in ohms are referred to the 11 kV side as shown in the Fig. Q3. Calculate the maximum fault currents (I_{fmaxi}) and minimum fault currents (I_{fmini}) at bus 1-5.

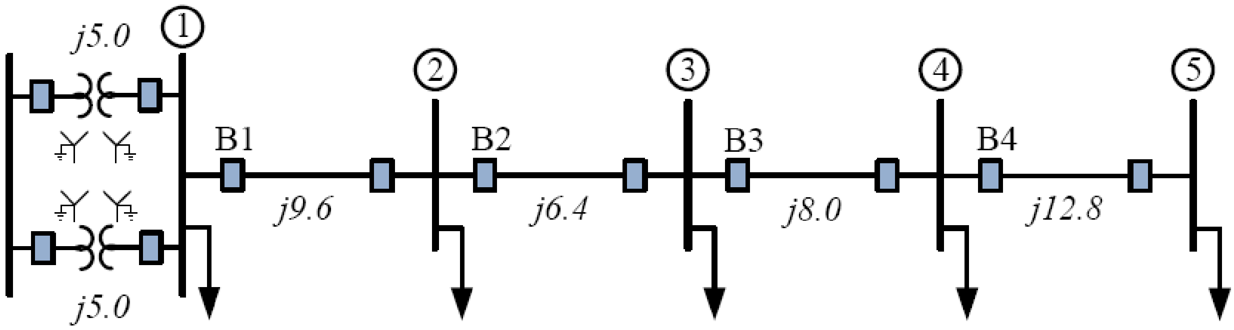


Fig. Q3

Solution Hints:

1. Maximum fault current will occur for a three-phase with both transformers in service.
2. Minimum fault in this case is assumed for a line-to-line fault. A line-to-line fault produces a fault current equal to $\sqrt{3}/2$ times the three-phase fault. Also the minimum fault current happens for line-to-line faults with one transformer in service.

The maximum and minimum fault currents are given below for faults at bus 1-5

Fault Level	Fault at Bus				
	1	2	3	4	5
Max Fault Current (A)	**2540**	**525**	**343**	**240**	**162**
Min Fault Current (A)	**1100**	**377**	**262**	**190**	**132**

Max Fault Current (A) **Min Fault Current (A)**

$$|I_{f3ph1}| = \left|\frac{V}{Z_{eq1}}\right| = \frac{11\times10^3/\sqrt{3}}{2.5} = 2540\ A \qquad |I_{fLL1}| = 0.866\times\left|\frac{V}{Z_{eq1}}\right| = 0.866\times\frac{11\times10^3/\sqrt{3}}{5} = 1100\ A$$

$$|I_{f3ph2}| = \left|\frac{V}{Z_{eq2}}\right| = \frac{11\times10^3/\sqrt{3}}{(2.5+9.6)} = 525\ A \qquad |I_{fLL2}| = 0.866\times\left|\frac{V}{Z_{eq2}}\right| = 0.866\times\frac{11\times10^3/\sqrt{3}}{(5+9.6)} = 377\ A$$

$$|I_{f3ph3}| = \left|\frac{V}{Z_{eq3}}\right| = \frac{11\times10^3/\sqrt{3}}{(2.5+9.6+6.4)} = 343\ A \qquad |I_{fLL3}| = 0.866\times\left|\frac{V}{Z_{eq3}}\right| = 0.866\times\frac{11\times10^3/\sqrt{3}}{(5+9.6+6.4)} = 262\ A$$

$$|I_{f3ph4}| = \left|\frac{V}{Z_{eq4}}\right| = \frac{11\times10^3/\sqrt{3}}{(2.5+9.6+6.4+8)} = 240\ A \qquad |I_{fLL4}| = 0.866\times\left|\frac{V}{Z_{eq4}}\right| = 0.866\times\frac{11\times10^3/\sqrt{3}}{(5+9.6+6.4+8)} = 190\ A$$

$$|I_{f3ph5}| = \left|\frac{V}{Z_{eq5}}\right| = \frac{11\times10^3/\sqrt{3}}{(2.5+9.6+6.4+8+12.8)} = 162\ A \qquad |I_{fLL5}| = 0.866\times\left|\frac{V}{Z_{eq5}}\right| = 0.866\times\frac{11\times10^3/\sqrt{3}}{(5+9.6+6.4+8+12.8)} = 132\ A$$

EE482-Power System Analysis II

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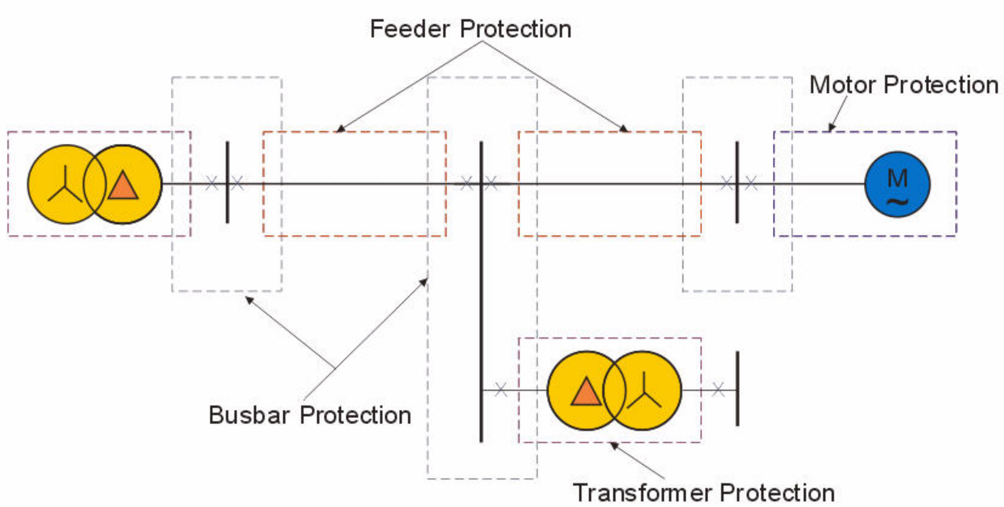
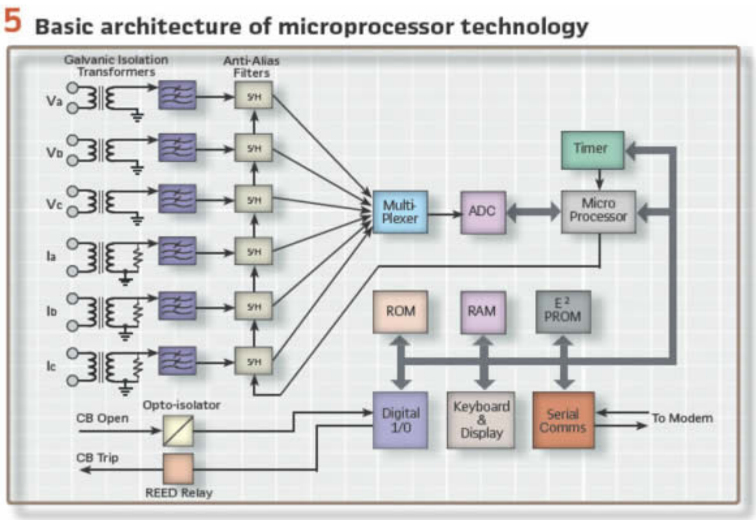
Electrical Engineering Department

School of Engineering

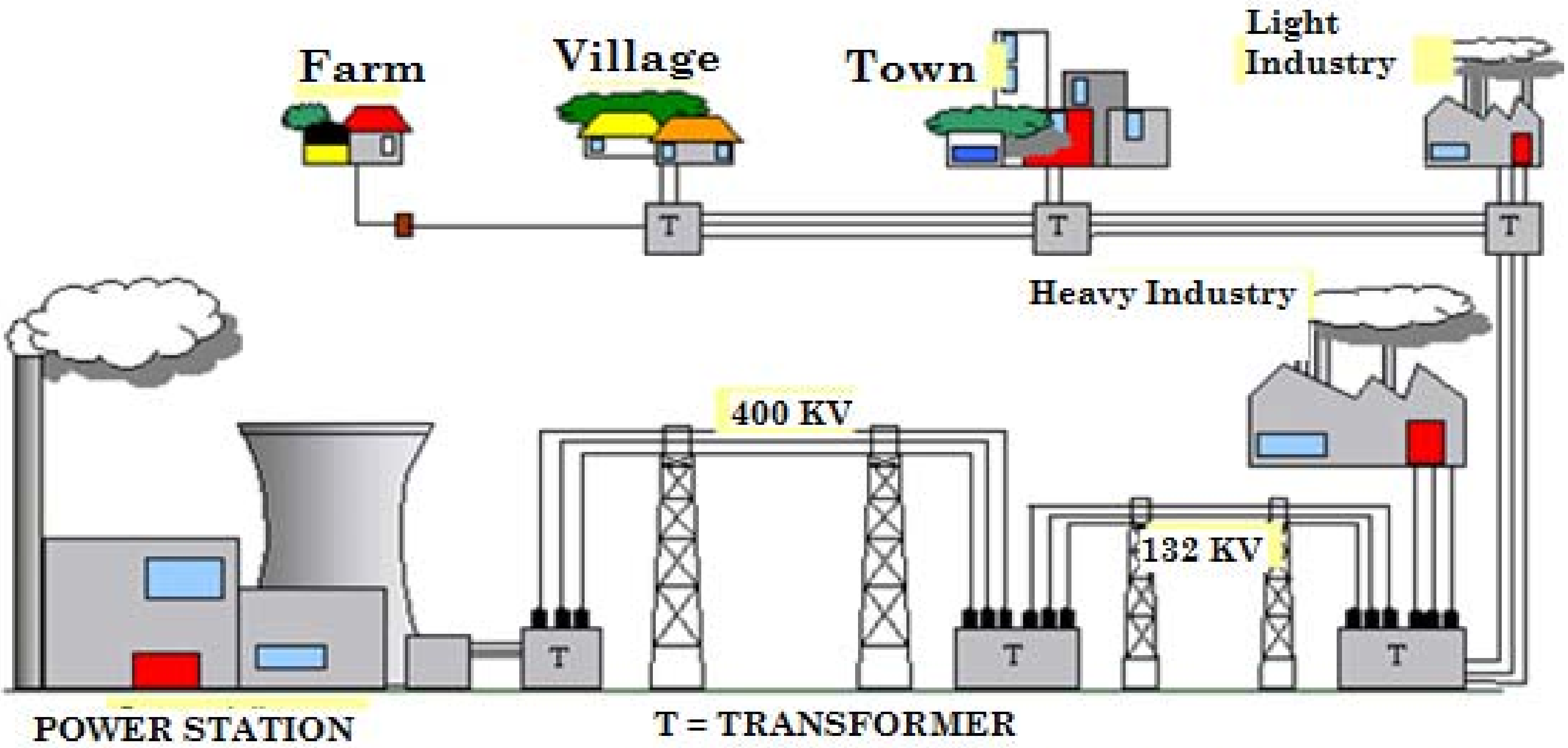
University of Jordan

Topic 1-2: Introduction





Electricity Generation, Transmission and Distribution



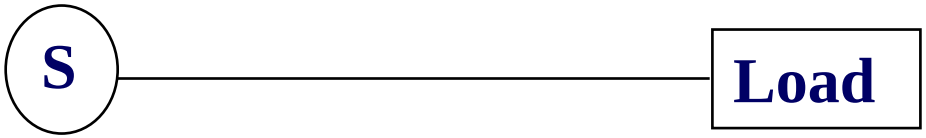
ELECTRICITY GENERATION ,TRANSMISSION AND DISTRIBUTION NETWORK

Definition

- Standard IEC 60038 defines voltage ratings as follows:
- ***Low voltage (LV):*** 100 V and 1,000 V, the standard ratings are: 400 V - 690 V - 1,000 V (at 50 Hz).
- ***Medium voltage (MV):*** between 1,000 V and 35 kV, the standard ratings are: 3.3 kV - 6.6 kV - 11 kV - 22 kV - 33 kV.
- ***High voltage (HV):*** between 35 kV and 230 kV, the standard ratings are: 45 kV - 66 kV - 110 kV - 132 kV - 150 kV - 220 kV.
- ***Extra High voltage (EHV):* > 132 kV**
- ***Ultra High voltage (EHV):* > 750 kV**

Simple Distribution Systems

- **Radial**



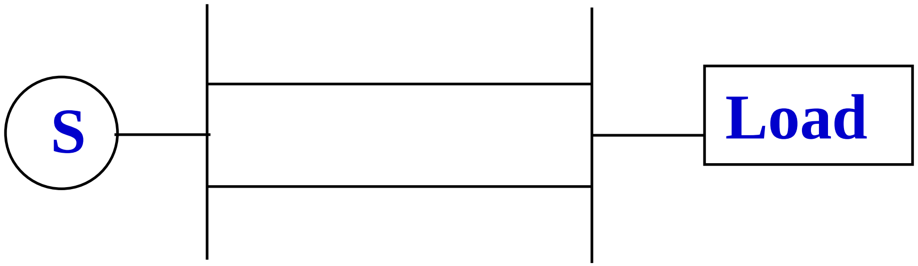
  - **Advantages**
    - **Simple (lowest capital cost)**
    - **Easy and simple to protect**
  - **Disadvantages**

Little security of supply for customer - a single fault will cause loss of supply

- **Parallel**
  - **Advantages**

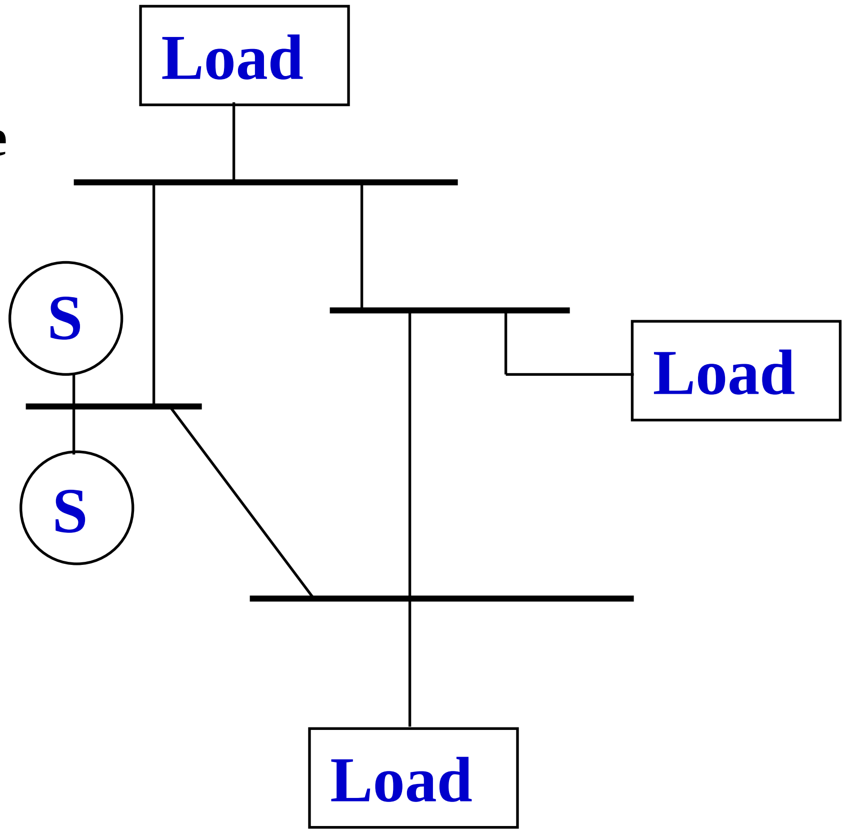
    Better power availability for customer

- **Disadvantages**
  - **More expensive**
  - **Increased fault currents**



Simple Distribution Systems

- ## Ring Main
  - ### Advantages
    - Maintains continuity even if one source fails.
    - Savings in Copper compared to parallel type.
  - ### Disadvantages
    - Lower impedance and Higher Fault current as in feed from two points
    - Requires better discrimination during faults due to alternate paths



The Need for Protection

Protective relaying is the *Science* or *Art* of detecting faults on power systems and clearing those faults from the power system as quickly as possible.

Protective equipment or protective relay is used in a power network to detect, discriminate and isolate the faulty equipment in the network.

Basic Requirements of Power System Protection

1. to ensure continuity of supply.
2. to minimize damage and repair costs.
3. to ensure safety of personnel.

Effects of Short Circuits

If short circuits are allowed to persist on a power system for an extended period, the following effects are likely to occur:

- **Reduced stability** margins for the power system.
- **Damage** to the equipment that is in the vicinity of the fault due to heavy currents, or low voltages produced by the short circuit.
- **Explosions** in equipment containing insulating oil, cause fire.
- **Disruption** in the entire power system service area.

Attributes of Power System Protection

Basic Qualities

Protective Relays must have the following characteristics:

1. ***Selectivity:*** To detect and isolate the faulty item only.
2. ***Sensitivity:*** To detect even the smallest values of fault current or system abnormalities and operate correctly at its setting before the fault causes irreparable damage.
3. ***Speed:*** To operate speedily when it is called upon to do so, thereby minimizing damage to the surroundings and ensuring safety to personnel.
4. ***Stability:*** To leave all healthy circuits intact to ensure continuity or supply.

Electrical Fault Energy

Why Speed is Important?

Energy released into fault = $I^2 \times R \times t$

where I = Fault Current

R = Resistance of Fault Arc

t = Time in seconds when fault is ON.

So, the faster the fault clearing time, the lesser is the energy released.

The Need for Speed

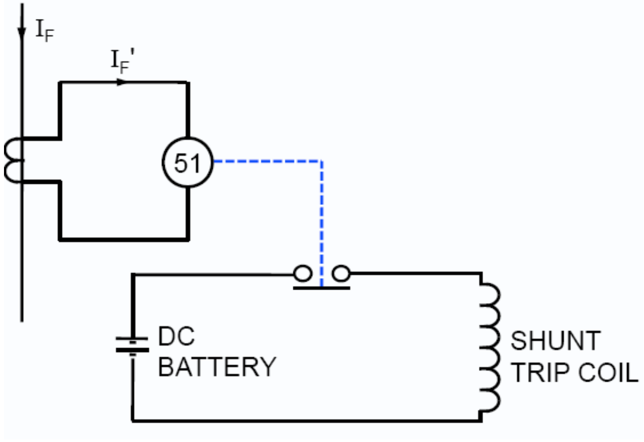
- Fault Current = 4000 Amps
- Clearance Time = 350 milliseconds (0.35 s)
- Assume ARC Resistance of 1 Ω
- Fault Energy = I^2 x R x t = 4000 x 4000 x 1 x 0.35

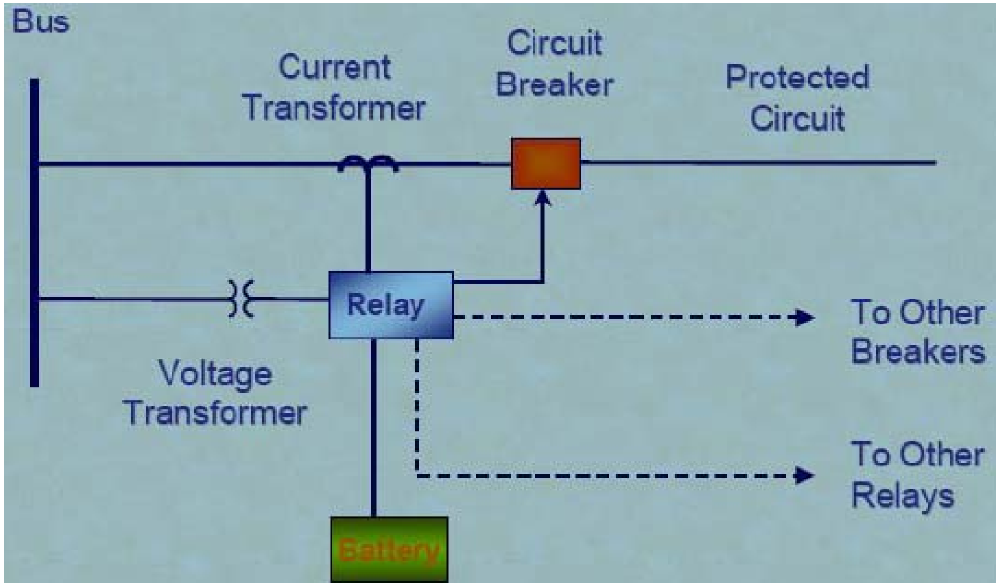
  = 5.6 Mega Joules
- If clearance time reduced to 100 milliseconds (0.1 s)
- Fault Energy = 4000 x 4000 x 1 x 0.1

  = 1.6 Mega Joules
- **HENCE A 70% REDUCTION !**
- If steps could be taken to also reduce level of fault current then major strides would be made.

Components of Protection Schemes

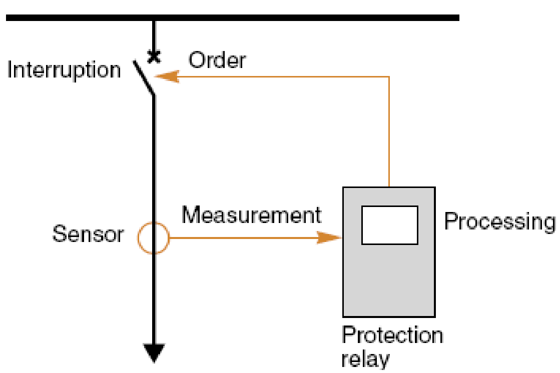
- Each power system protection scheme is made up from the following components:
  1. **Fault Detecting or Measuring Relays.**
  2. **Tripping and other Auxiliary Relays.**
  3. **Circuit Breakers.**
  4. **Current Transformers and Voltage transformers**
  5. **DC Batteries.**

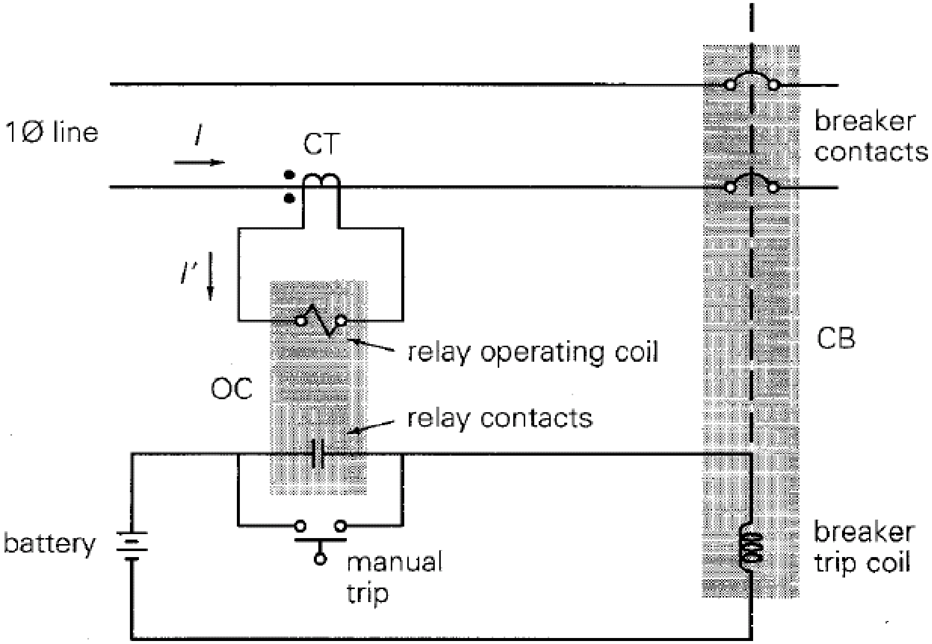




Components of Protection Schemes

- All power system elements are equipped with one or more protection schemes to detect faults on the system.
- When the protective relays have detected a fault, they send trip signals to the circuit breaker or breakers, which in turn clear the fault from the system.





Circuit Breaker and Relay Wiring Schematic
(1 of 3 phases)

Relay Technology

Protection Relay Technology Evolution

Electromechanical: A protection relay design which uses magnetomotive force in its decision making stage and has moving parts in it.

Static: A protection relay design which does not have any moving part in the decision making stage.

Protection Relay Technology Evolution

- PROTECTION RELAY
  - Electromechanical
    - Attractive force
    - Rotational torque
  - Static
    - Analog Electronic
    - Digital Electronic
    - Processor Based
      - μPs
      - DSPs

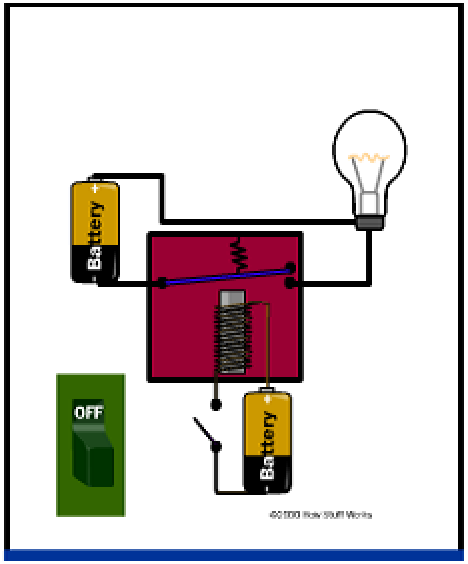
hardware

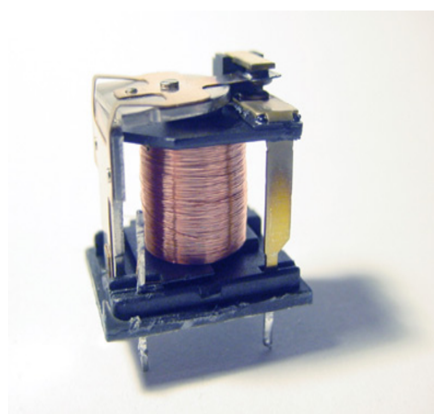
software

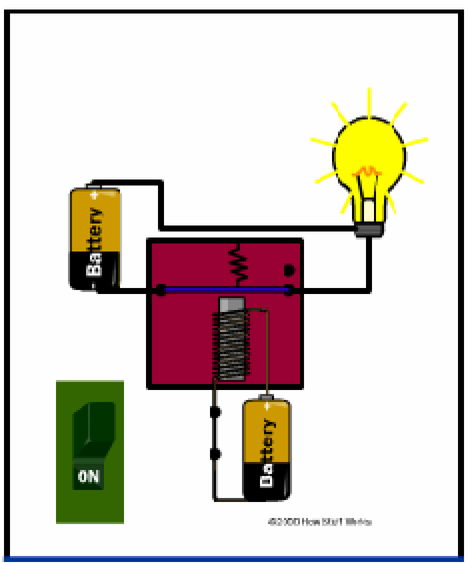
What are Relays?

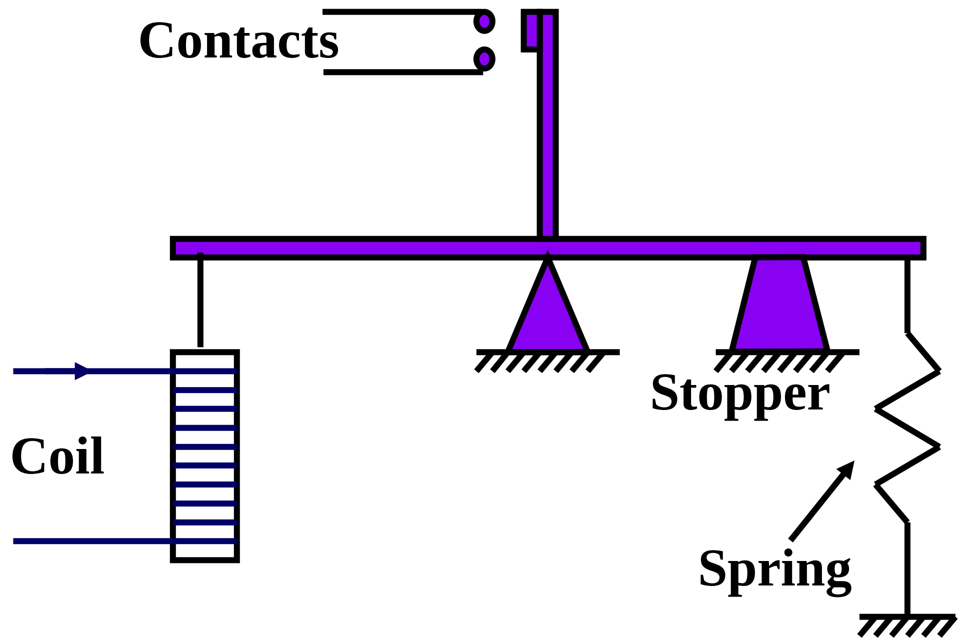
- Relays are electrical switches that open or close another circuit under certain conditions.
- **Electromagnetic Relays (EMRs)**
  - EMRs consist of an input coil that's wound to accept a particular voltage signal, plus a set of one or more contacts that rely on an armature (or lever) activated by the energized coil to open or close an electrical circuit.
- **Solid-state Relays (SSRs)**
  - SSRs use semiconductor output instead of mechanical contacts to switch the circuit. The output device is optically-coupled to an LED light source inside the relay. The relay is turned on by energizing this LED, usually with low-voltage DC power.
- **Microprocessor Based Relays**
  - Use microprocessor for switching mechanism. Commonly used in power system monitoring and protection.

How a Relay works?

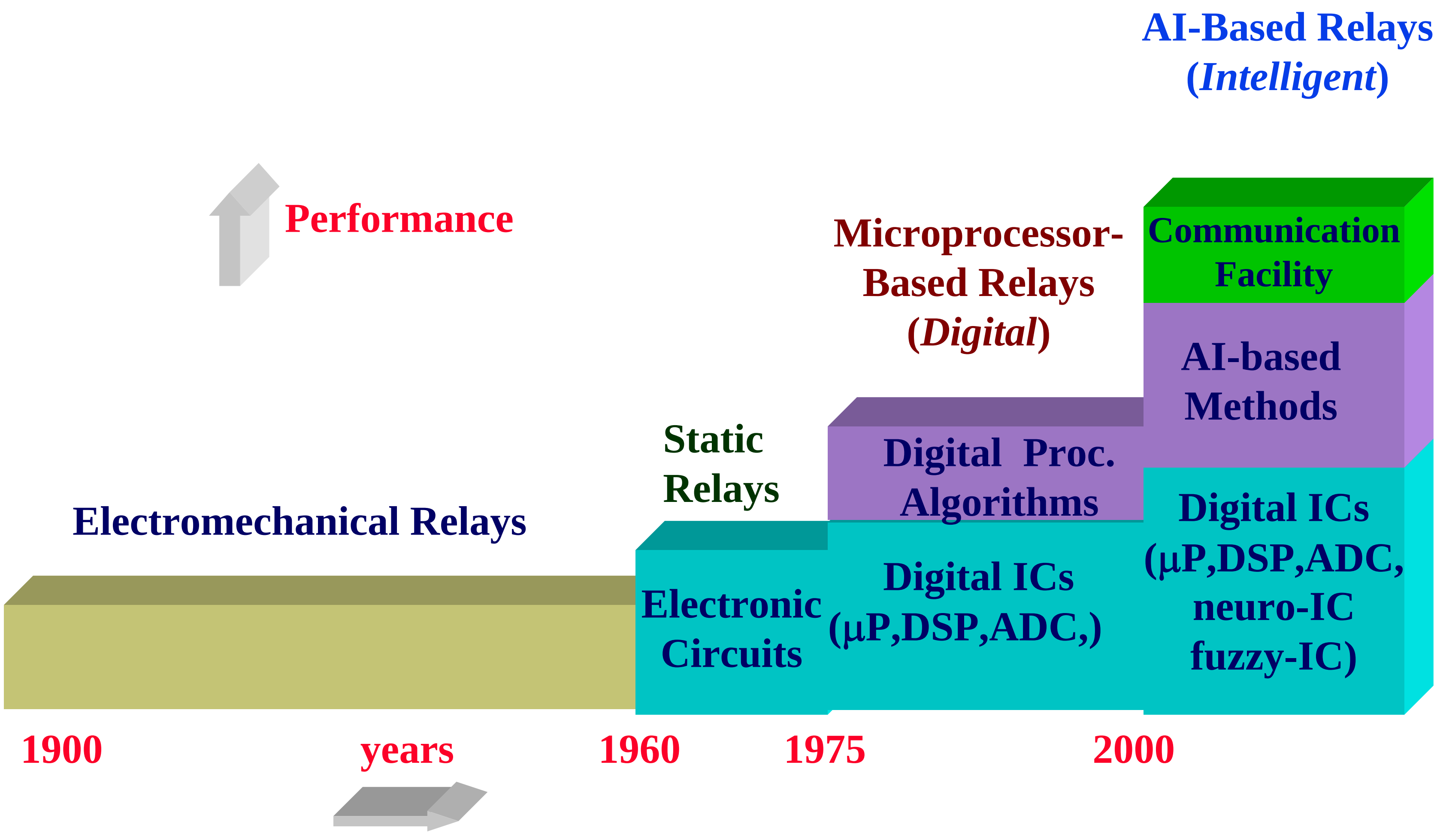








Development in Power System Relaying



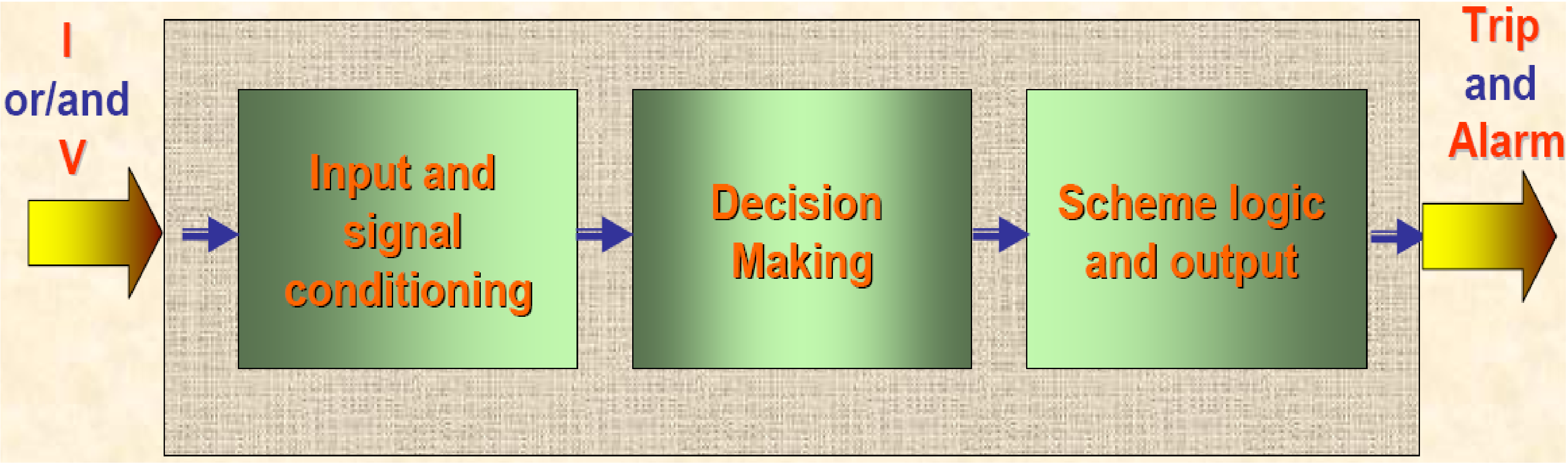
Examples of Relay Panels

Old Electromechanical

Microprocessor-Based Relay

Relay Technology

Protection Relay Functional Block Diagram

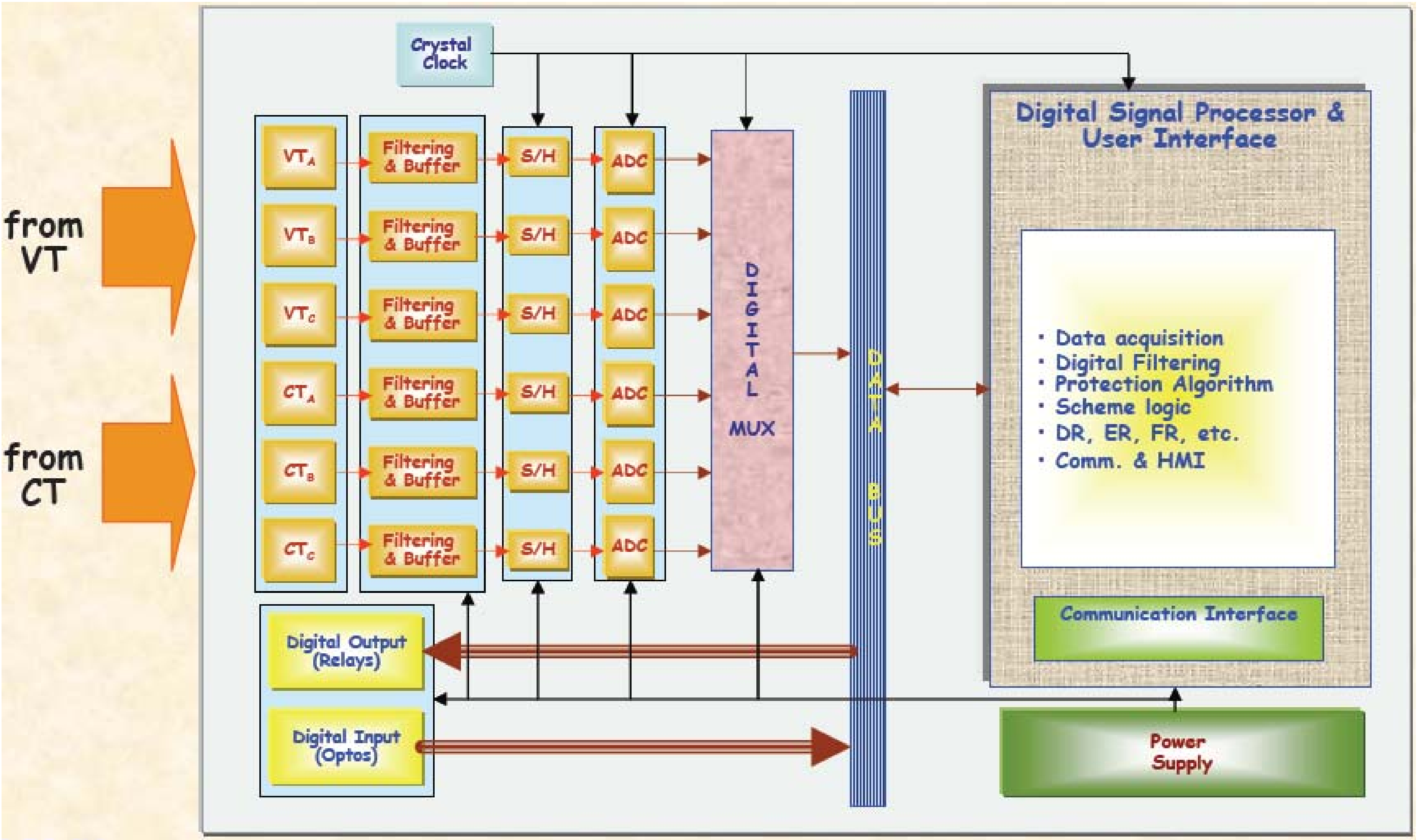


The voltage and/or current signal is first reduced to measurable quantities and necessary conditioning done .

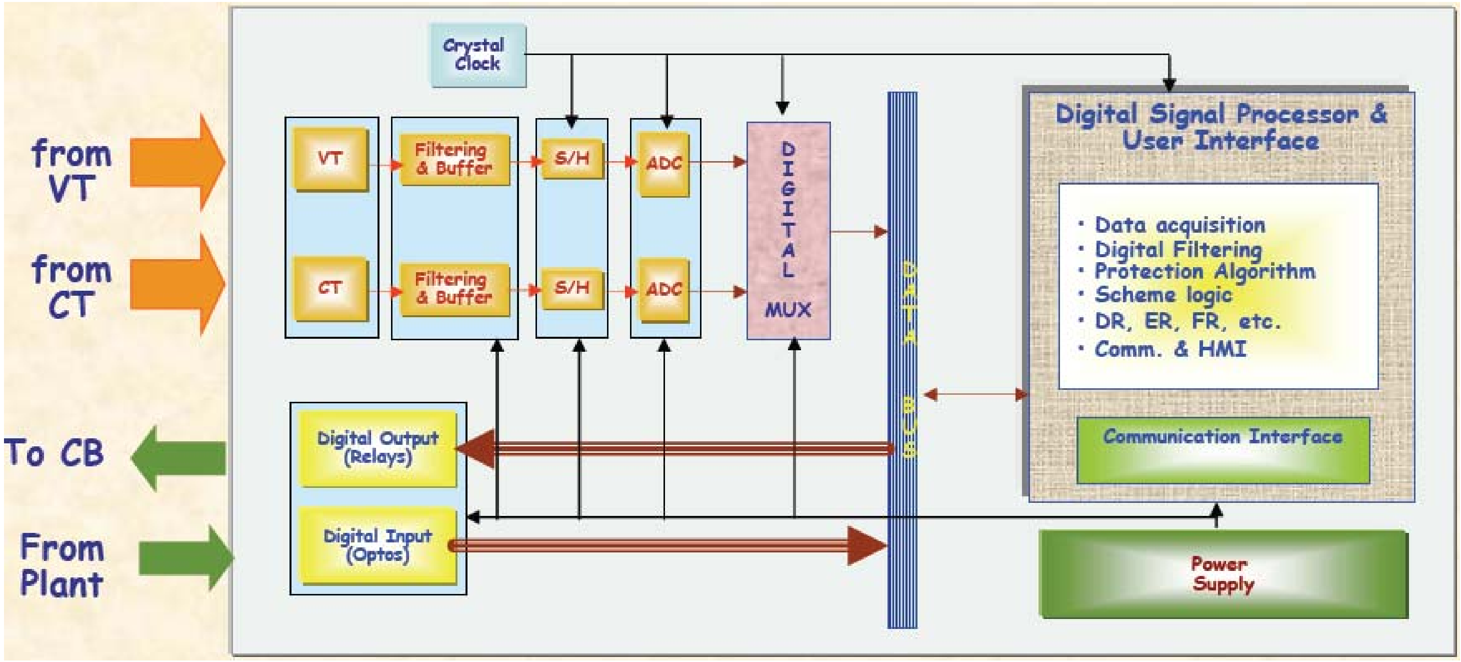
The decision making stage does the actual protection as per the set value.

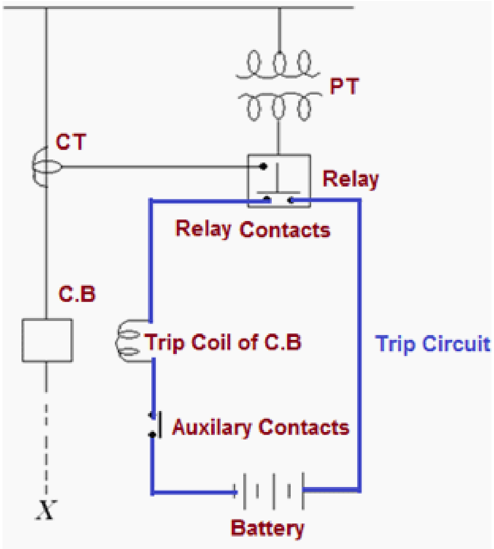
The output stage implements the necessary logic before issuing trip and alarm commands.

Protection Relay Technology-Numerical



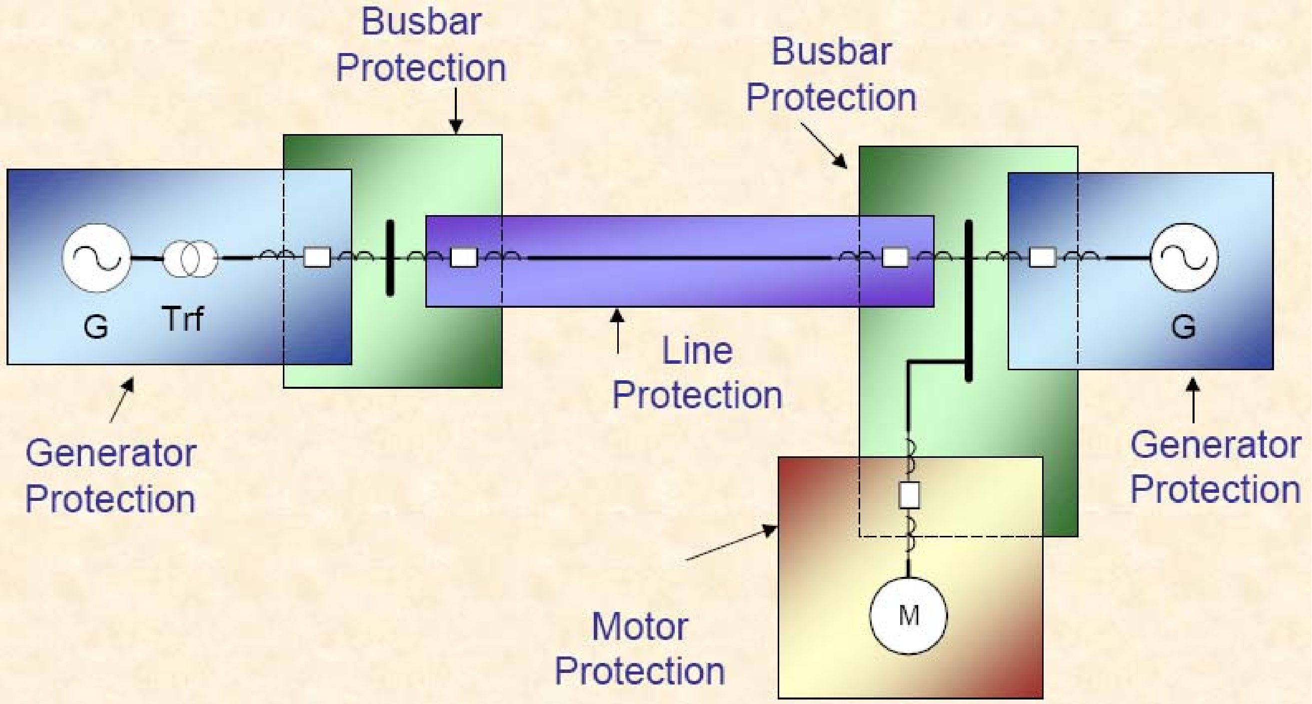
Single-Phase Impedance Relay-Numerical





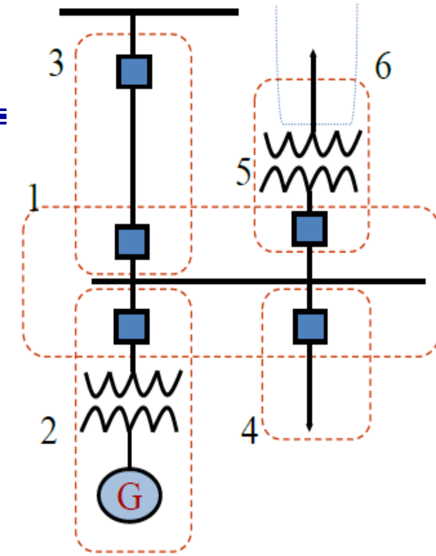
Zones of Protection

- For fault anyway within the zone, the protection system responsible to isolate everything within the zone from the rest of the system.
- Isolation done by CB
- Must isolate only the faulty equipment or section



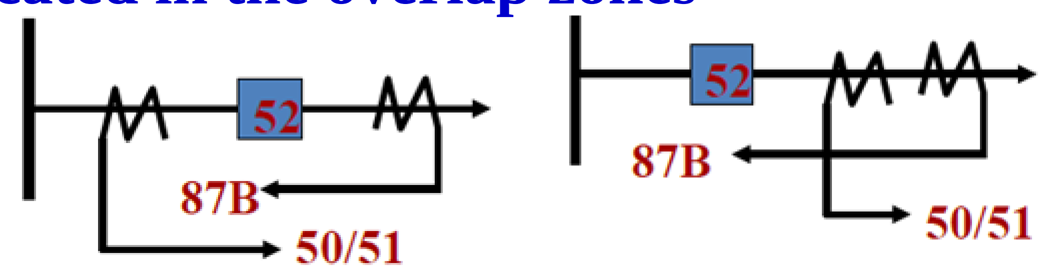
Protection Zones

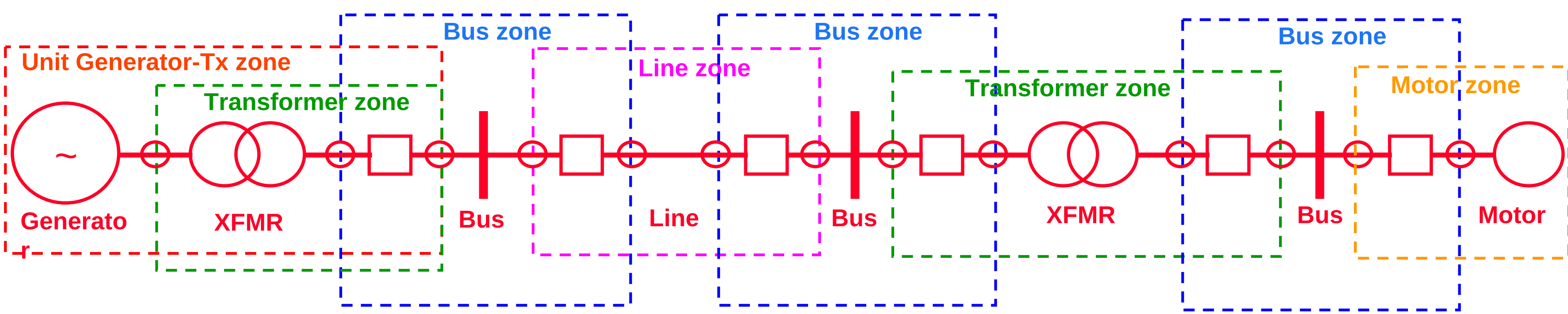
1. Generator or Generator-Transformer Units
2. Transformers
3. Buses
4. Lines (transmission and distribution)
5. Utilization equipment (motors, static loads, etc.)
6. Capacitor or reactor (when separately protected)



- **Circuit breakers are located in the overlap zones**

CT REQUIREMENTS FOR OVERLAPPING ZONES

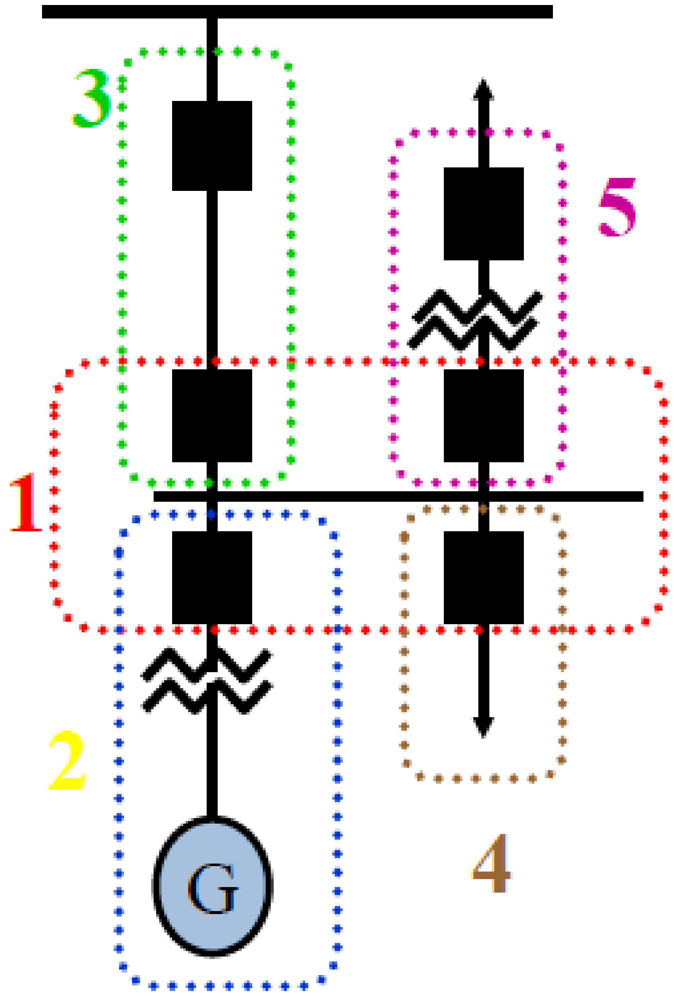




“Zones” of Protection

- Zones are defined for:
  - Generators
  - Transformers
  - Buses
  - Transmission and distribution lines
  - Motors

Zones of Protection



1 - Bus Protection

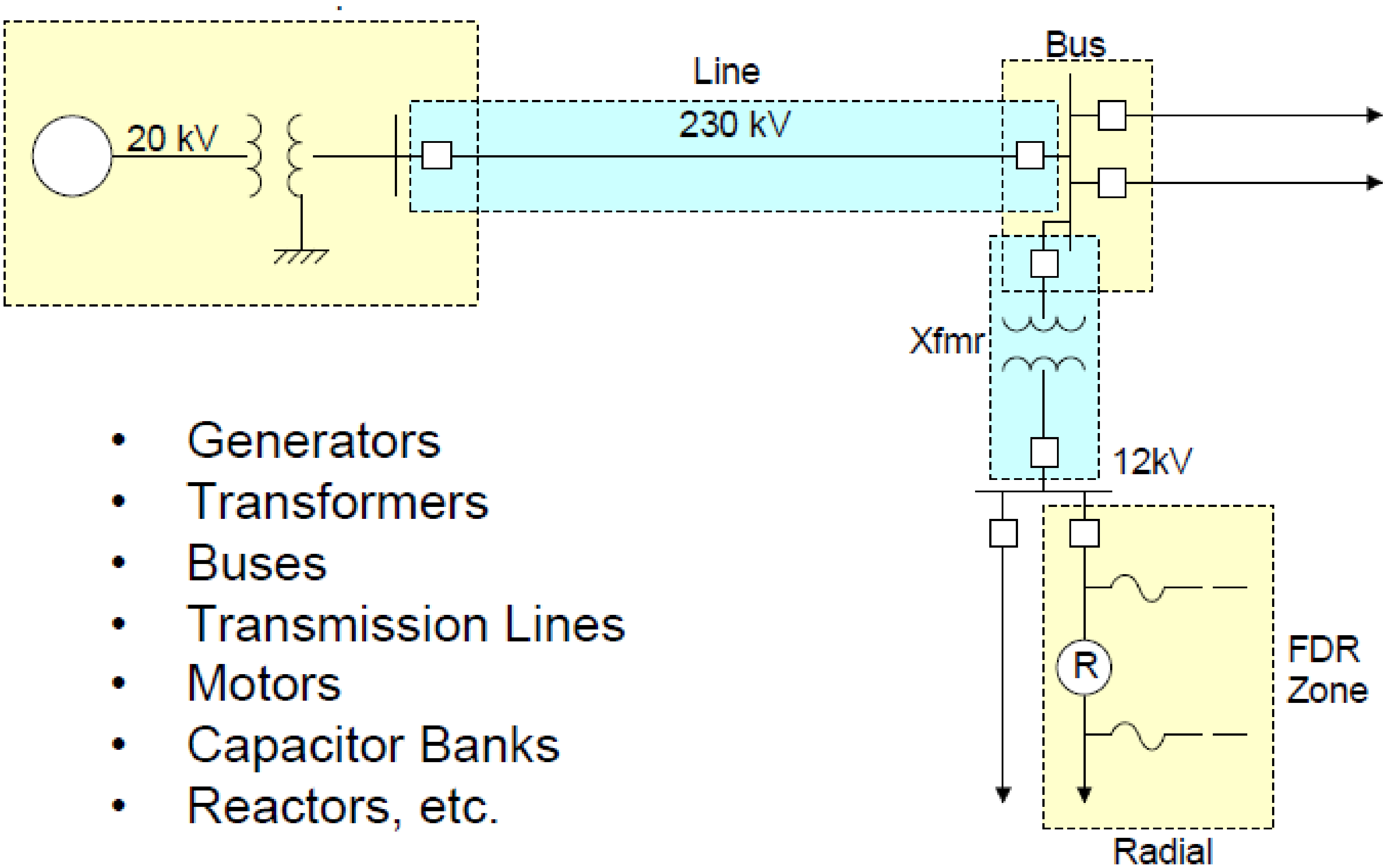
2 - Generator Protection

3 - Subtrans Line Protection

4 - Feeder Protection

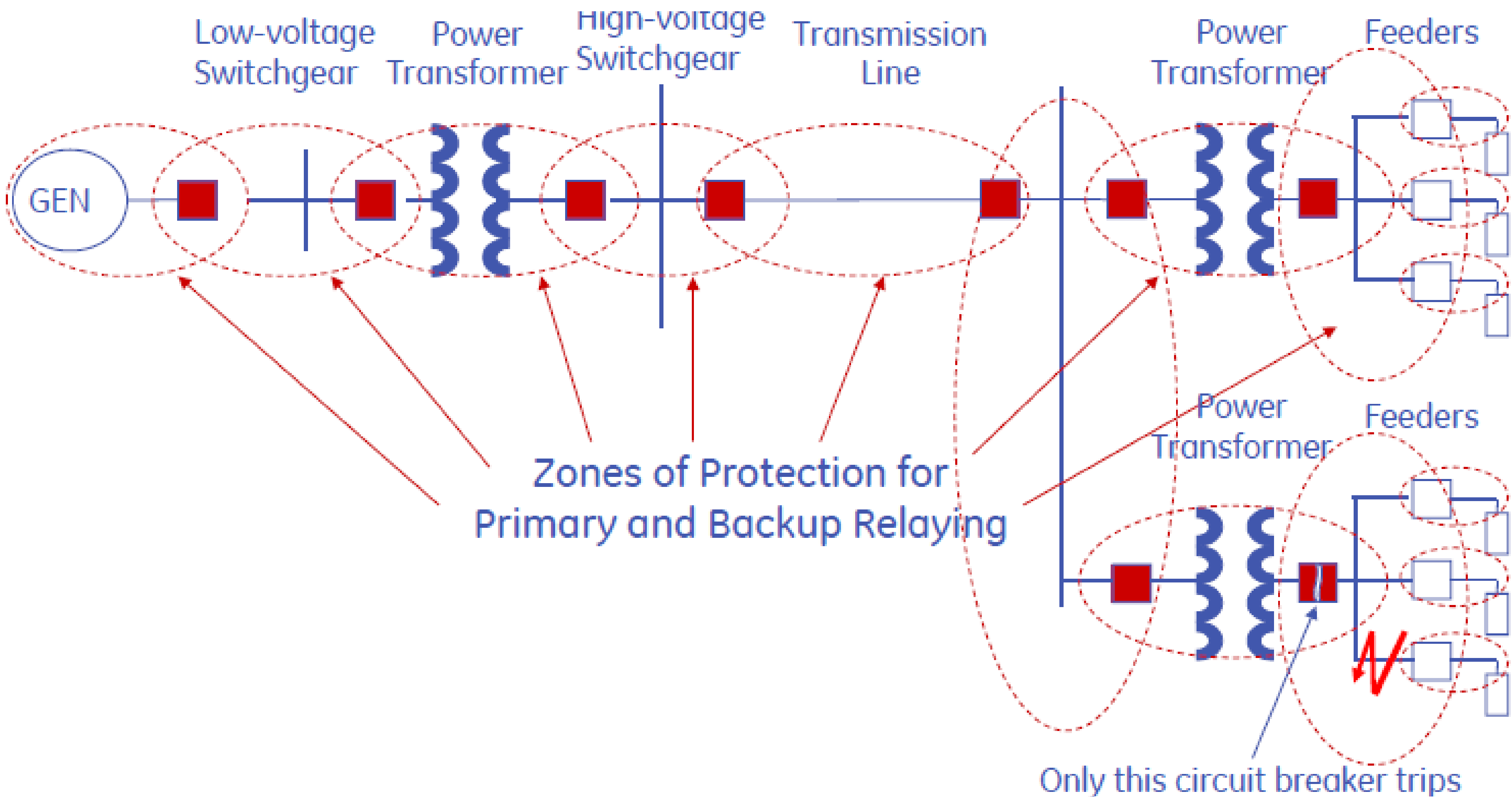
5 - Transformer Protection

Zones of Protection



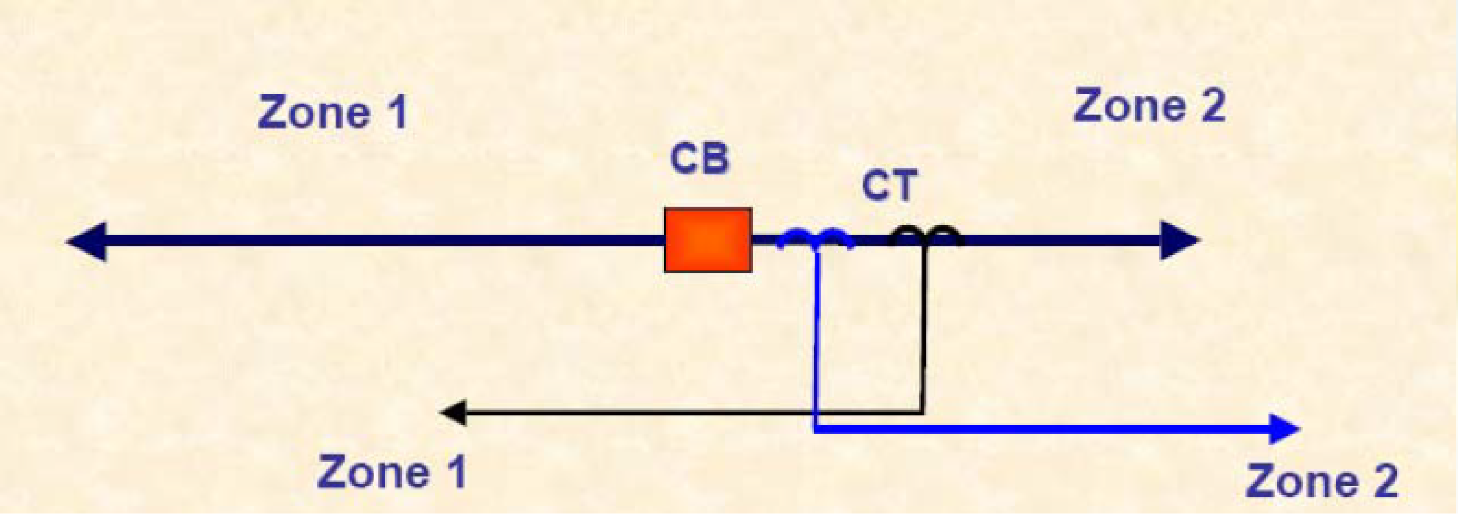
- Generators
- Transformers
- Buses
- Transmission Lines
- Motors
- Capacitor Banks
- Reactors, etc.

Zones of Protection



Overlapped Protection

- Overlap accomplish by having 2 sets of instrument transformers and relays for each CB.
- Achieved by the arrangement of CT and CB.



Types of Protection Schemes

A - Fuses

- For LV Systems, Distribution Feeders and Transformers, VT's, Auxiliary Supplies

B - Over current and earth fault

- Widely used in All Power Systems
  - Non-Directional
  - Directional

C - Differential

- For Distribution Feeders, Busbars, Transformers, Generators etc

Types of Protection Schemes

D - Distance

- For Transmission and Sub-transmission Lines and Distribution Feeders,
- Also used as back-up protection for transformers and generators without signaling with signaling to provide unit protection e.g.:
  - Time-stepped distance protection
  - Phase comparison for transmission lines
  - Directional comparison for transmission lines

Types of Protection Schemes

E - Miscellaneous:

- Under and over voltage
- Under and over frequency
- A special relay for generators, transformers, motors etc.
- Control relays: auto-reclose, tap change control, etc.
- Tripping and auxiliary relays

Which Relays are Used in What Applications?

It depends on the importance of the power system element being protected.

Protection Type	Application Areas
Fuse	Local LV distributor
HRC Fuse	Major LV feeder, local HV spur line, HV side of distribution substation
Overcurrent and Earth Fault relay	Major HV distribution feeder, backup to transformer differential protection and feeder impedance protection on sub-transmission lines
Impedance relay	Primary protection on transmission and sub-transmission lines
Differential relay	Primary protection on large distribution and all sub-transmission and transmission level transformers; large generators
Thermal Overload relay	Transmission and sub-transmission level transformers, large motors, large generators
Oil Surge relay	Transmission and sub-transmission level transformers
Under and Over Volts relay	Large motors, large generators
Under and Over Frequency relay	Large generators
Negative Sequence relay	Large generators
Loss of Excitation relay	Large generators

Relay Applications

1. Distribution

Overcurrent Relays (50/51)

- Non-directional
- Phase and Ground relays (typically 4 relays)
- Instantaneous Overcurrent (50)
- Time Overcurrent (51)

2. Transmission

Distance (21 or 21G)

Directional Overcurrent (67 or 67N)

Differential (87L)

Pilot Wire (85)

Relay Applications

3. Equipment

Transformer Differential (87T)

Transformer Overcurrent (50/51)

Bus Differential (87B)

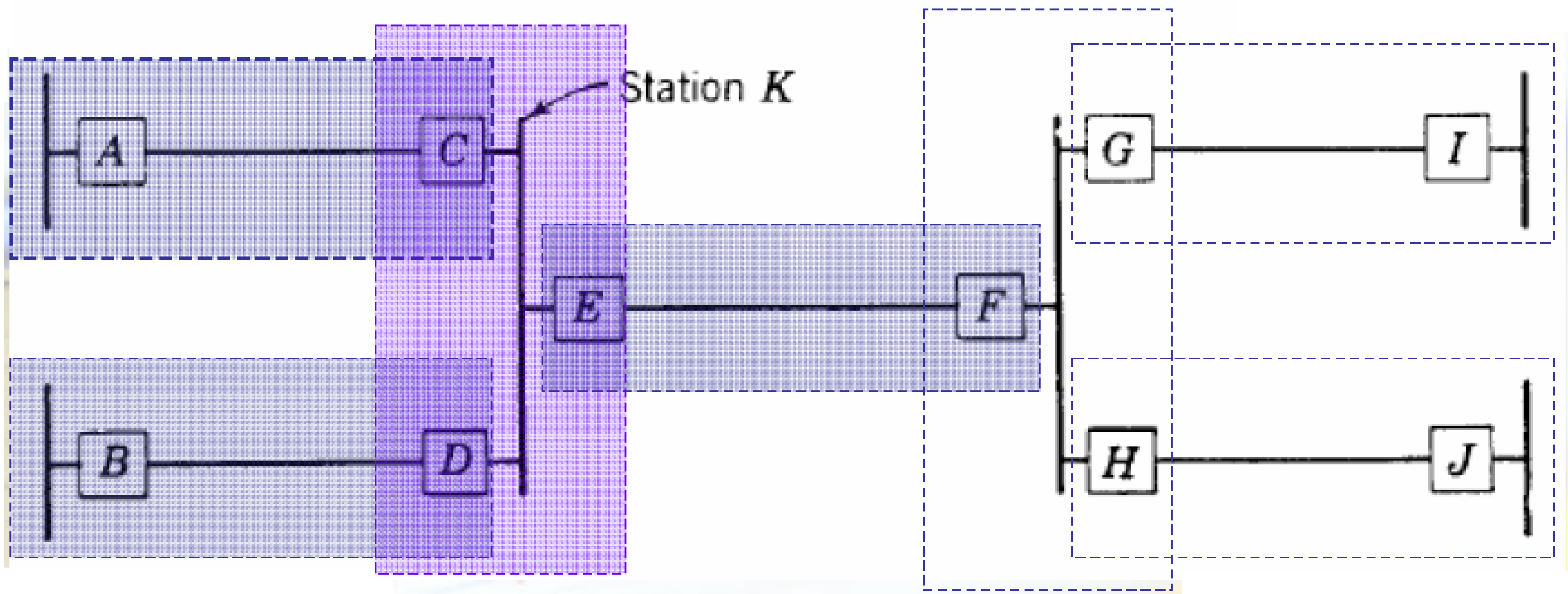
4. Under/Over Voltage (27/59)

- Undervoltage Load Shedding Scheme (27)
- Overvoltage Protection (59)

Primary & Backup Protection Schemes

- Primary protection is the protection provided by each zone to its elements.
- However, some component of a zone protection scheme fail to operate.
- Back-up protection is provided which take over only in the event of primary protection failure.

Primary & Backup Protection - Example

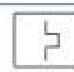


Fault	Primary	Back-up
K	C, D, E	A, B, F

ANSI Device Numbers – IEEE C37.2

01 Master Element
21 Distance Relay
25 Synchronizing or Synch Check Relay
27 Undervoltage Relay
30 Annunciator Relay
32 Directional Power
43 Selector Switch
50 Instantaneous Overcurrent Relay
51 AC Time Overcurrent Relay
52 AC Circuit Breaker
59 Overvoltage Relay
62 Time Delay Relay
63 Pressure Switch
64 Ground Detector Relay
67 AC Directional Overcurrent Relay
69 Permissive Control Device
71 Level Switch
74 Alarm Relay
79 Reclosing Relay
81 Frequency Relay
85 Carrier or Pilot Wire Relay
86 Lockout Relay
87 Differential Relay
88 Auxiliary Motor
89 Line Switch
90 Regulating Device
94 Tripping or Trip Free Relay

IEC Relay Symbols

Description	ANSI	IEC 60617	Description	ANSI	IEC 60617
Overspeed relay	12	$\omega >$	Inverse time earth fault overcurrent relay	51G	$I ⏚ >$
Underspeed relay	14	$\omega <$	Definite time earth fault overcurrent relay	51N	$I ⏚ >$
Distance relay	21	$Z <$	Voltage restrained/controlled overcurrent relay	51V	$U/ I >$
Overtemperature relay	26	$\theta >$	Power factor relay	55	$\cos\varphi >$
Undervoltage relay	27	$U<$	Overvoltage relay	59	$U>$
Directional overpower relay	32	$\rightarrow$ $P >$	Neutral point displacement relay	59N	$U_{rsd} >$
Underpower relay	37	$P <$	Earth-fault relay	64	$I ⏚ >$
Undercurrent relay	37	$I <$	Directional overcurrent relay	67	$\rightarrow$ $I >$
Negative sequence relay	46	$I_2 >$	Directional earth fault relay	67N	$\rightarrow$ $I ⏚ >$
Negative sequence voltage relay	47	$U_2 >$	Phase angle relay	78	$\varphi >$
Thermal relay	49		Autoreclose relay	79	$O \rightarrow I$
Instantaneous overcurrent relay	50	$I >>$	Underfrequency relay	81U	$f <$
Inverse time overcurrent relay	51	$I >$	Overfrequency relay	81O	$f >$
			Differential relay	87	$I_d>$

EE482-Power System Analysis II

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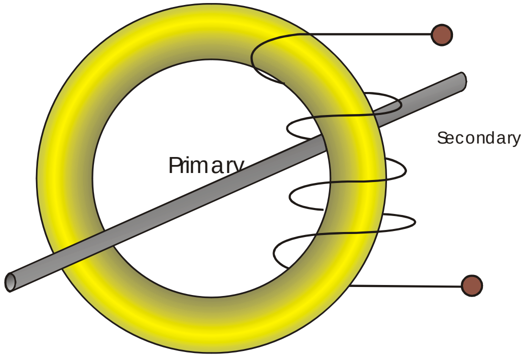
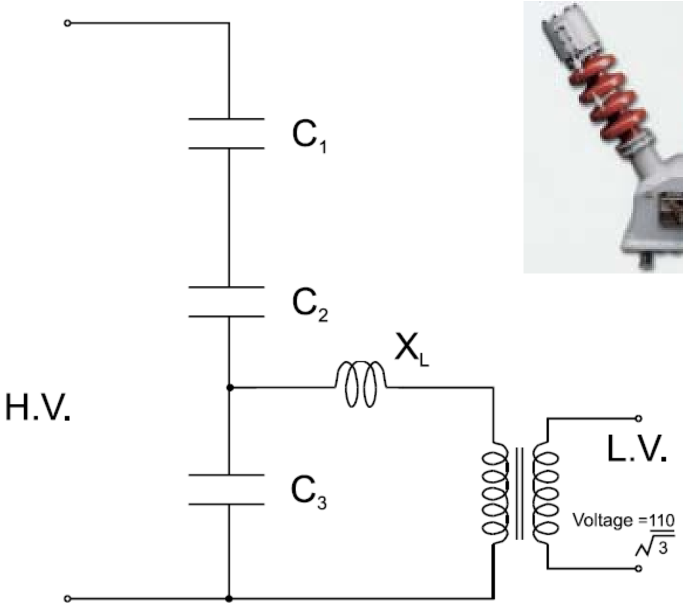
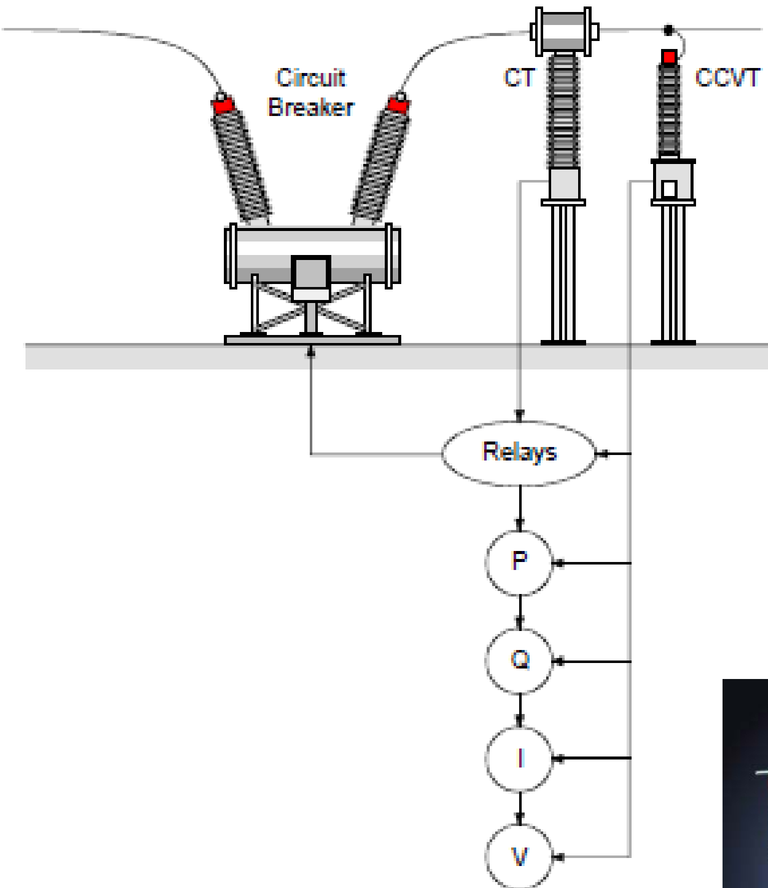
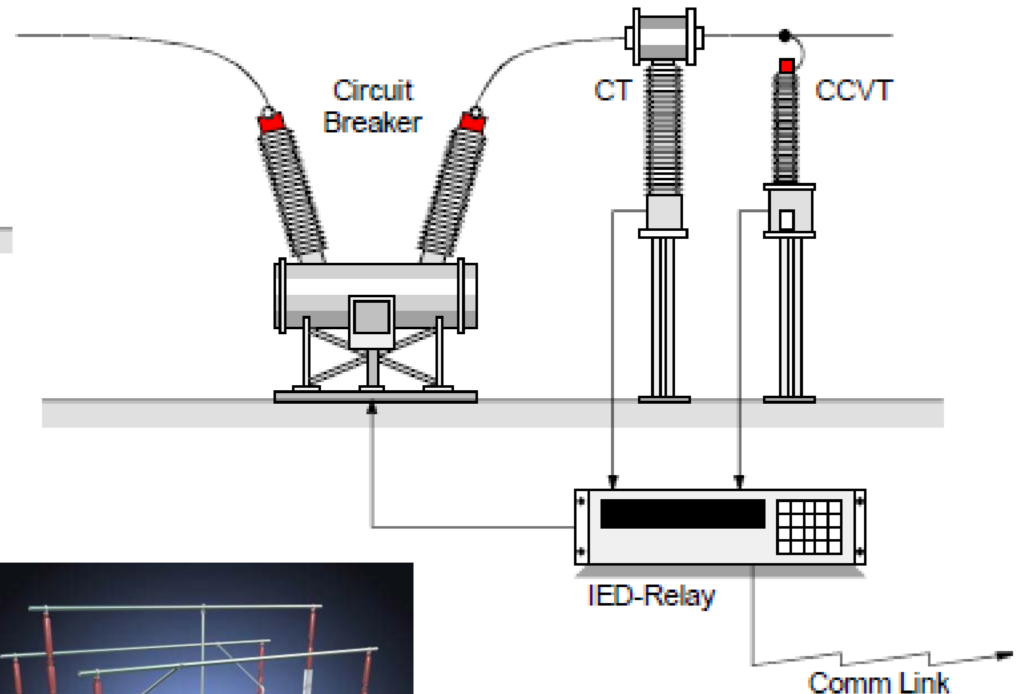


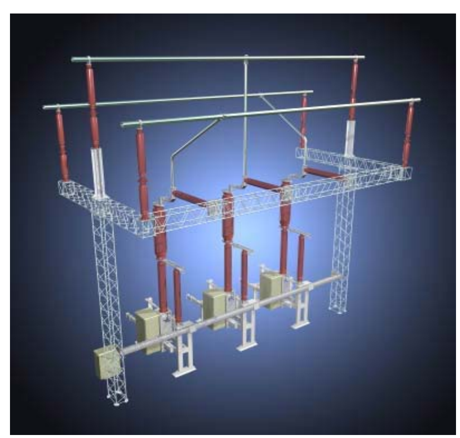
Topic 1-3: Instrument Transformers

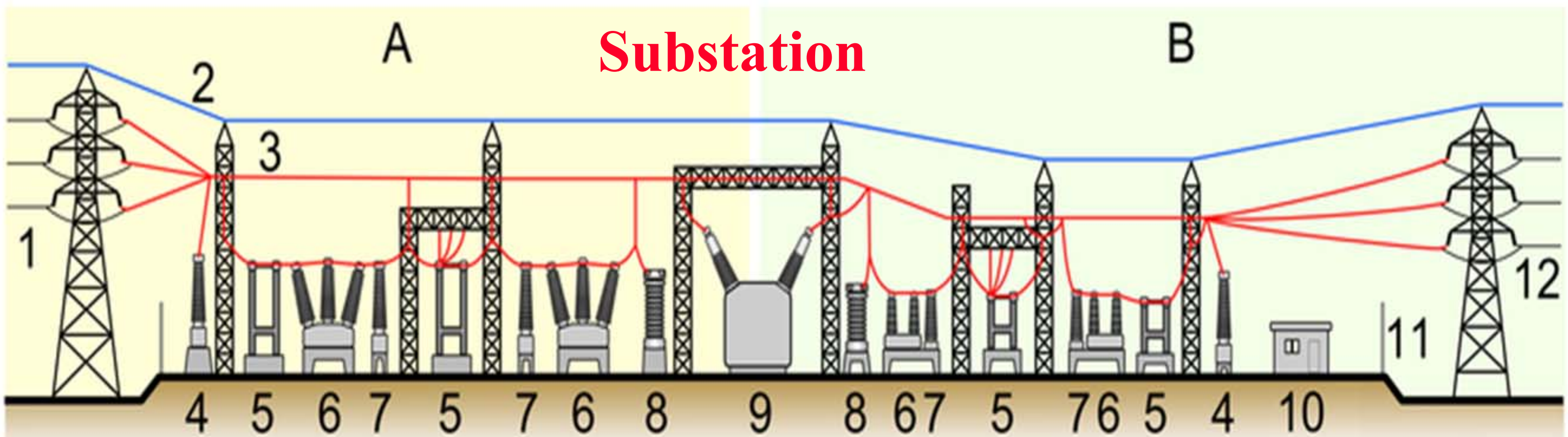


The OLD and the NEW



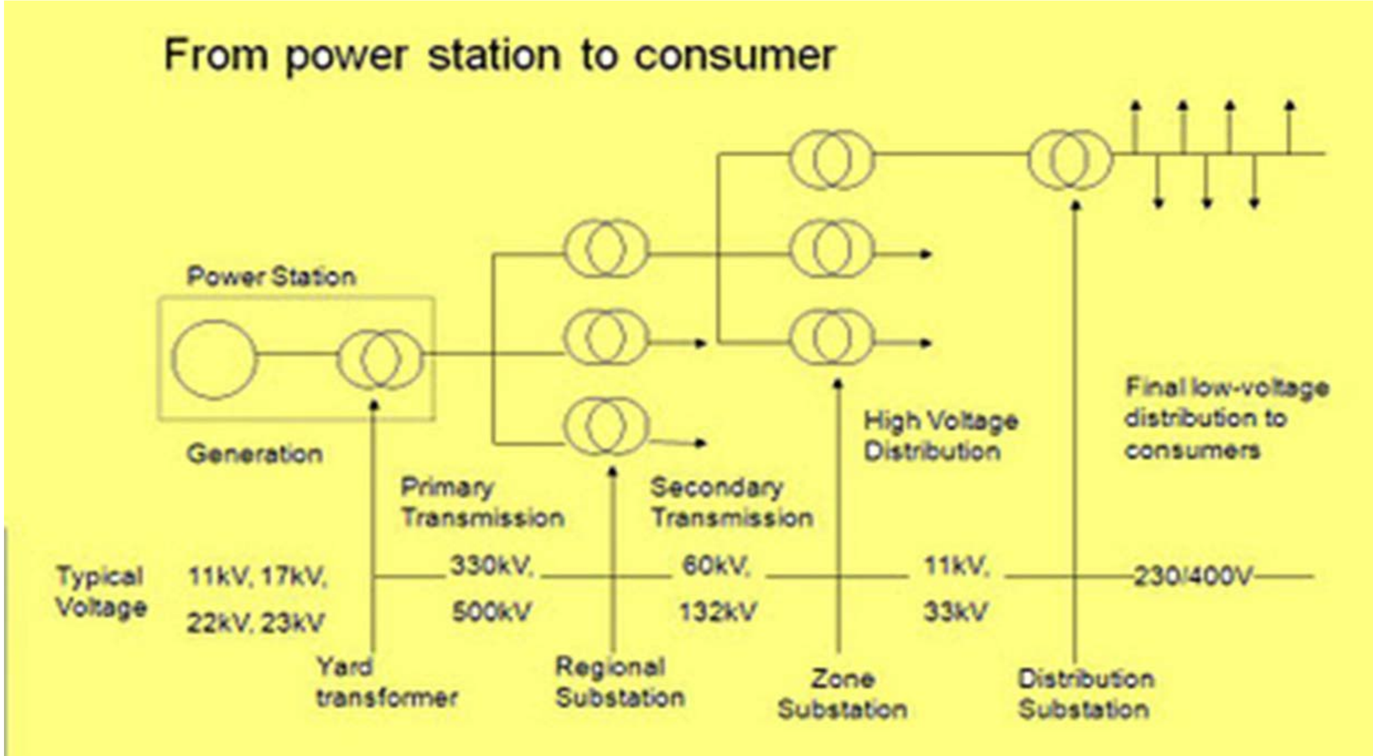




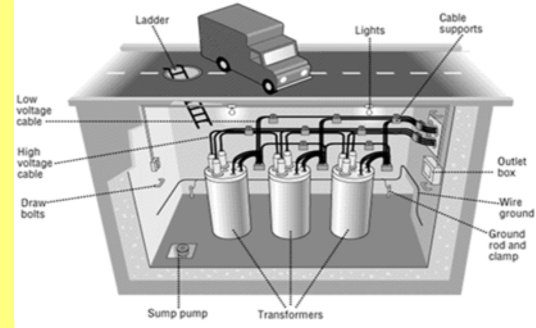


A:Primary power lines' side **B:Secondary power lines' side**

1.Primary power lines 2.Ground wire 3.Overhead lines 4.Transformer for measurement of electric voltage 5.Disconnect switch 6.Circuit breaker 7.Current transformer 8.Lightning arrester 9.Main transformer 10.Control building 11.Security fence 12.Secondary power lines

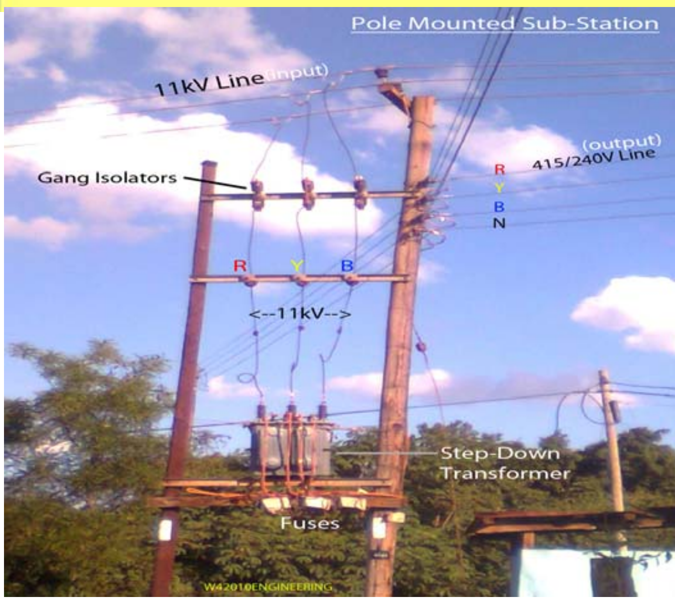


From power station to consumer
Power Station
Generation
Primary Transmission
Secondary Transmission
High Voltage Distribution
Final low-voltage distribution to consumers
Typical Voltage
11kV, 17kV, 22kV, 23kV
330kV, 500kV
60kV, 132kV
11kV, 33kV
230/400V
Yard transformer
Regional Substation
Zone Substation
Distribution Substation

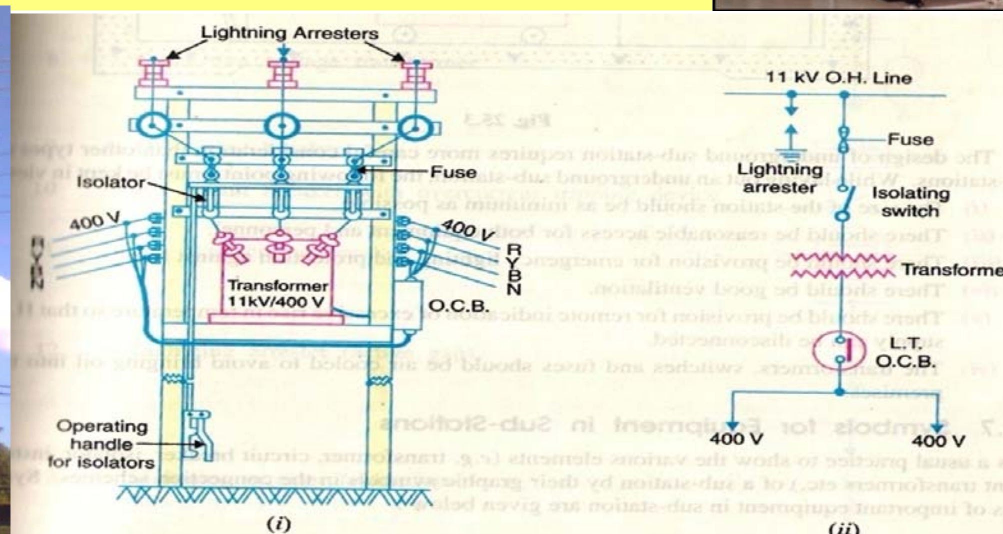


Ladder
Lights
Cable supports
Low voltage cable
High voltage cable
Outlet box
Draw bolts
Wire ground
Ground rod and clamp
Sump pump
Transformers

VT
BB
CT
CB
www.sayedsaad.com



Pole Mounted Sub-Station
11kV Line (input)
(output) 415/240V Line
Gang Isolators
R
Y
B
N
R Y B
<--11kV-->
Step-Down Transformer
Fuses



Lightning Arresters
Isolator
Fuse
400 V
400 V
R Y B N
Transformer 11kV/400 V
O.C.B.
Operating handle for isolators
(i)
11 kV O.H. Line
Fuse
Lightning arrester
Isolating switch
Transformer
L.T. O.C.B.
400 V
400 V
(ii)

Instrument Transformer (VT)

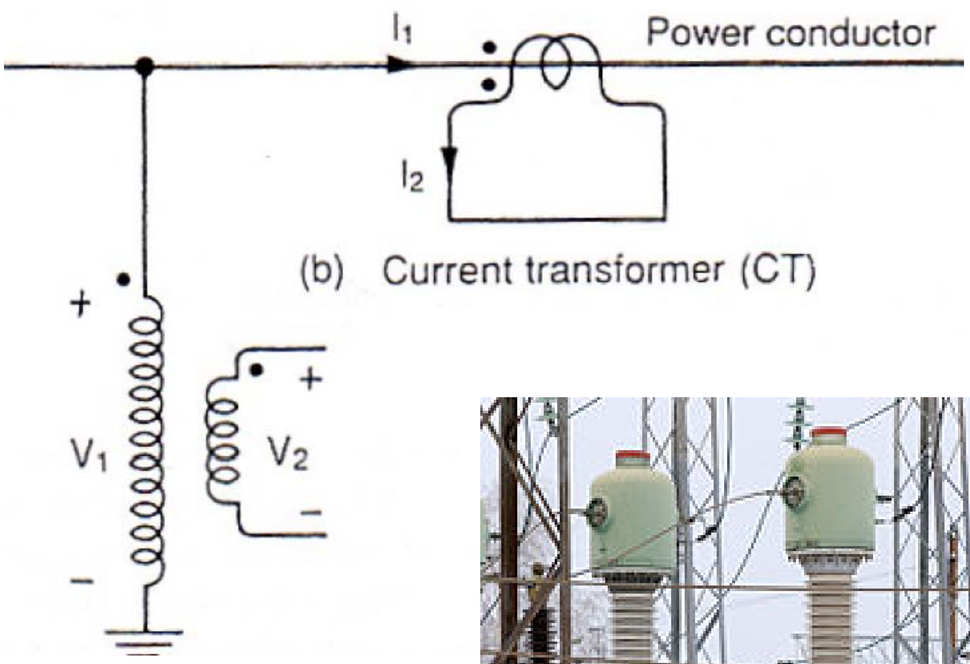
The main tasks of **instrument transformers** are:

- To transform ***currents*** *or* ***voltages*** from a usually high value to a value appropriate for relays and instruments (1 or 5 Amps)
- To insulate the relays, metering and instruments from the primary high voltage system.
- To provide possibilities of standardizing the relays and instruments etc. to a few rated currents and voltages.

Theory of Operation

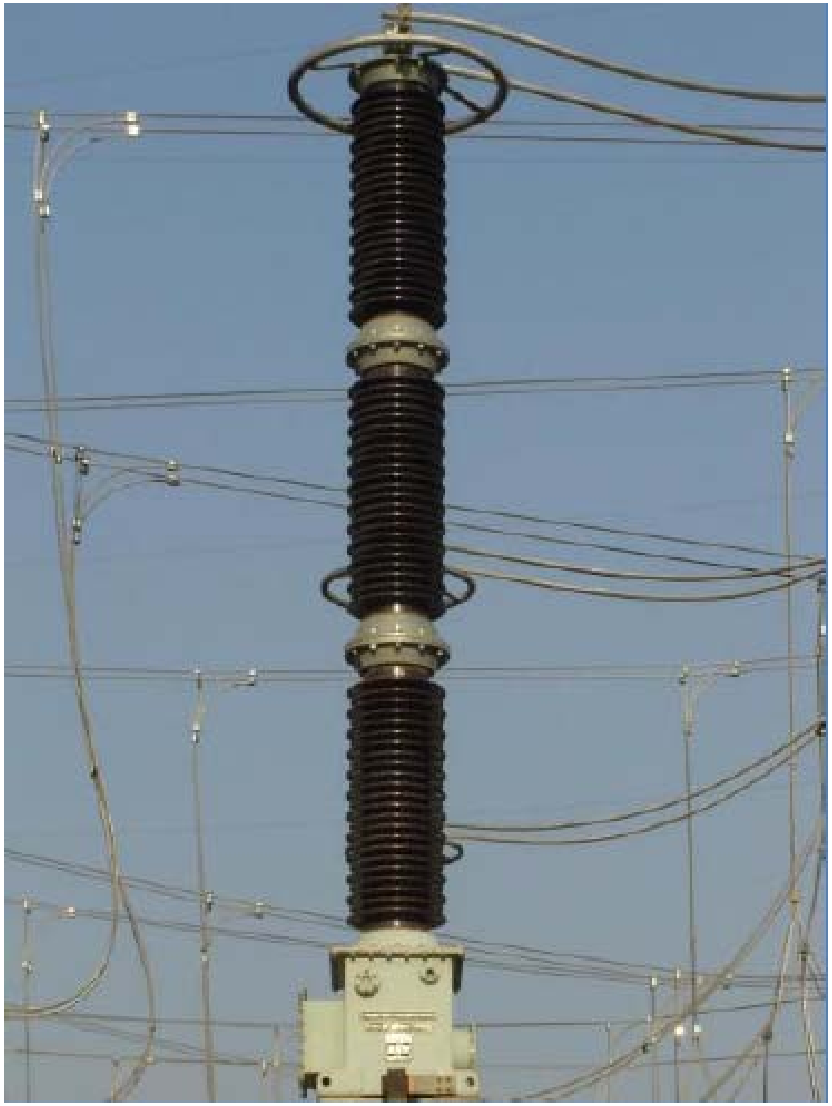
- Follows the basic Transformer principle to convert voltage on primary to an appropriate value on secondary through a common magnetic core.
- **Voltage Transformers** - Connected across the open circuit ends of the point of measurement
- **Current Transformers** - Connected in Series to carry the full rated / short circuit current of the circuit under measurement

CT and VT Schematic



SF6 110 kV current transformer

VT Schematic



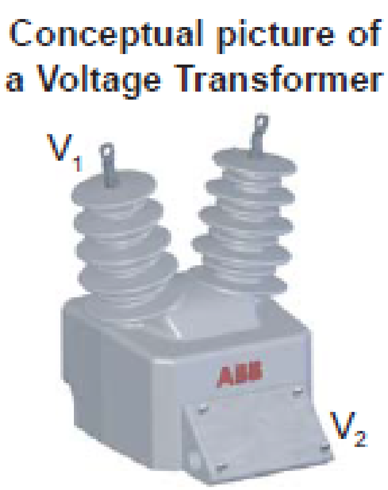
CVT

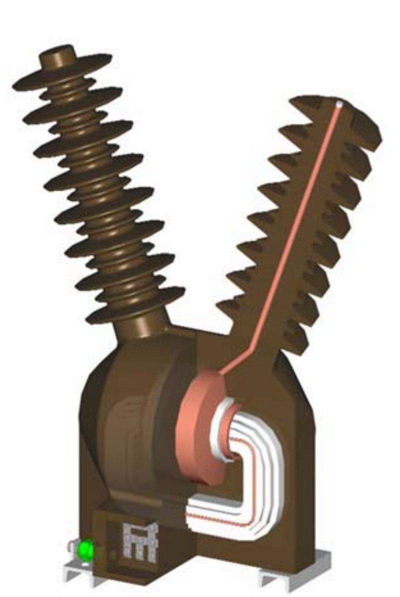
CT Schematic

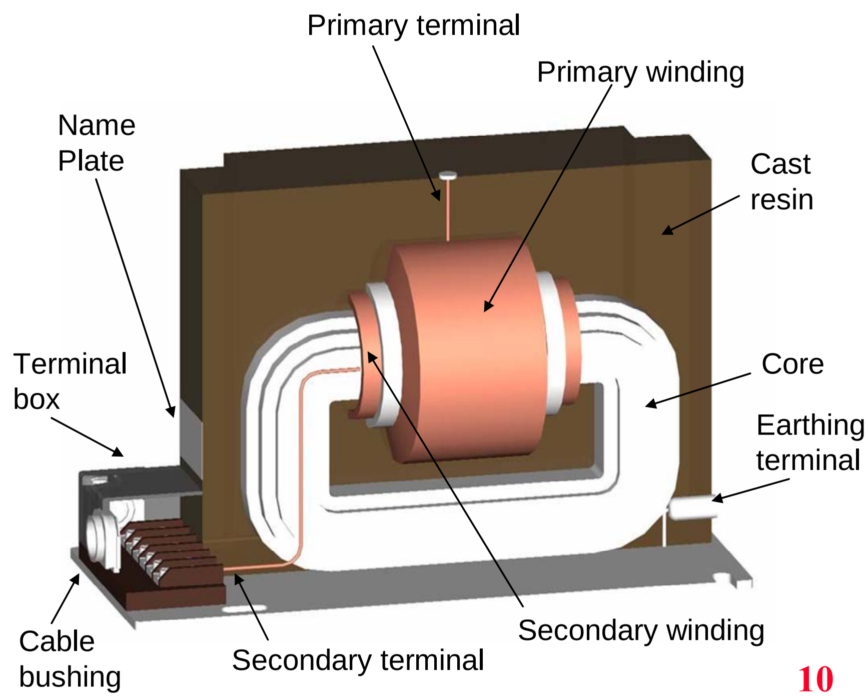
1. Voltage Transformers

Voltage Transformers (**VT**) are used to step the power system primary voltage from, say 132 kV or 33 kV to 120 volts phase-to-phase, or 69 volts phase-to-ground.

It is this secondary voltage that is applied to the fault detecting relays, and meters.



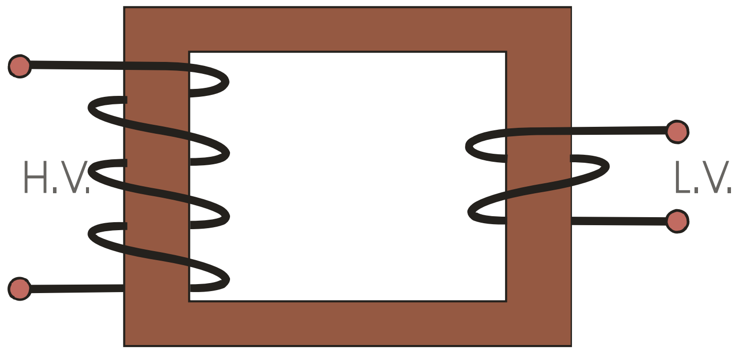


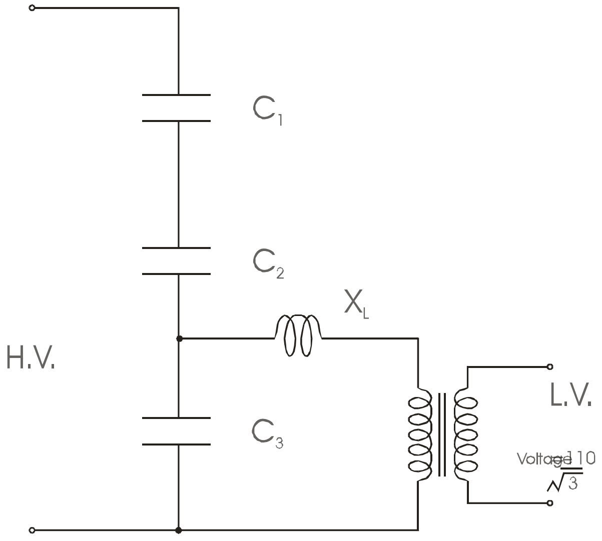


1. Voltage Transformers

The voltage transformers at these primary voltages of 50 kV and 33 kV are normally of the **WOUND** type. That is, a two winding transformer in an oil filled steel tank, with a turns ratio of 416.6:1 or 275:1.

On higher voltage systems, such as 132kV, 220kV and 400kV, **CAPACITOR VOLTAGE TRANSFORMERS** (**CVT's**) are normally used.

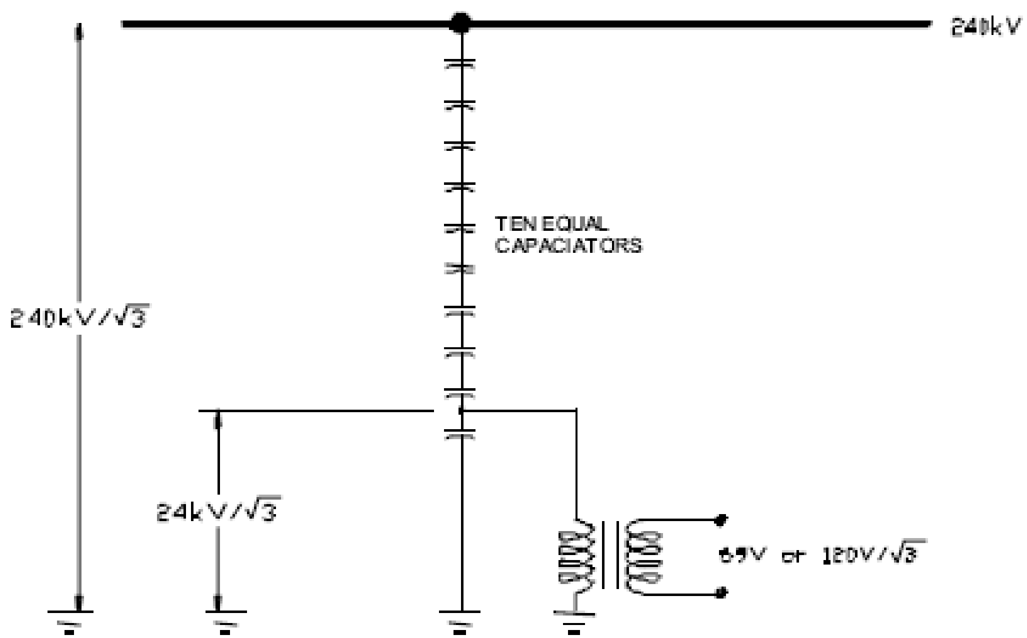




1. Voltage Transformers

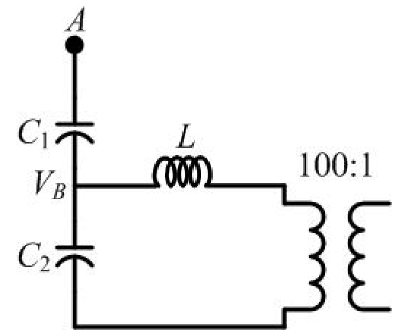
A **CVT** is comprised of a capacitor divider made up from typically 10 equal capacitors, connected in series from the phase conductor to ground, with a voltage transformer connected across the bottom capacitor.

This V.T. actually measures one-tenth of the line voltage, as illustrated in the diagram beside:



Electromagnetic Voltage Transformers

Values of C_1 and C_2 such that there is no phase displacement between the line voltage and the output of the CVT



Consider the circuit of CVT. The open-circuit voltage across C_2 is given by

$$V_B = \frac{V_A\left(1/j\omega C_2\right)}{1/j\omega C_1 + 1/j\omega C_2} = V_A \frac{C_1}{C_1 + C_2}$$

Also the short circuit current is

$$I_{sc} = \frac{V_A}{1/j\omega C_1} = j\omega C_1 V_A$$

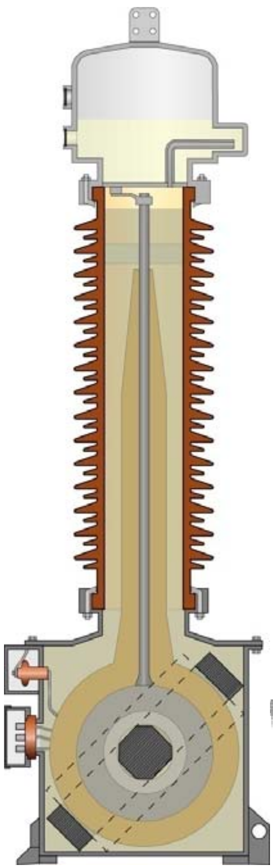
Thevenin impedance is given by

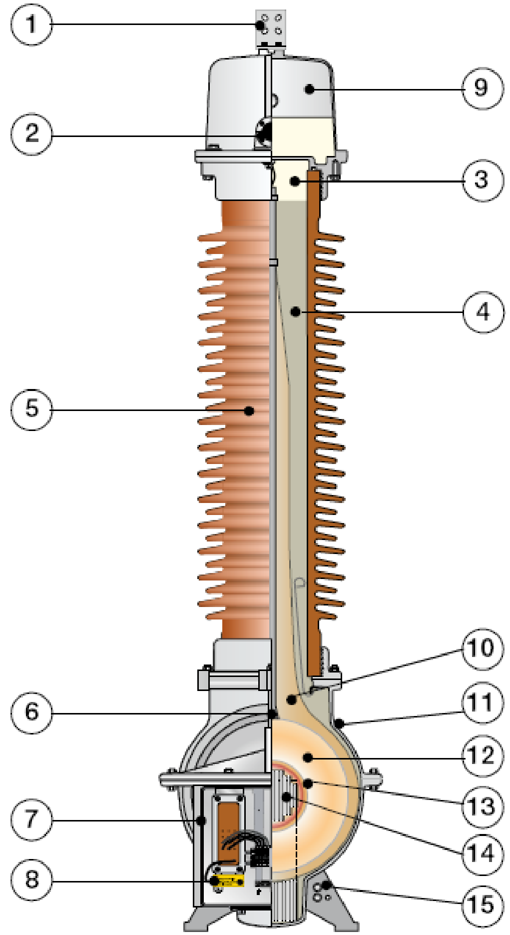
$$Z_{TH} = \frac{V_B}{I_{sc}} = \frac{1}{j\omega\left(C_1 + C_2\right)}$$

Let us assume L to be leakage impedance of the transformer. Let us now choose C_1 and C_2 such that

$$-\frac{1}{j\omega\left(C_1 + C_2\right)} = j\omega L \Rightarrow L = \frac{1}{\omega^2\left(C_1 + C_2\right)}$$

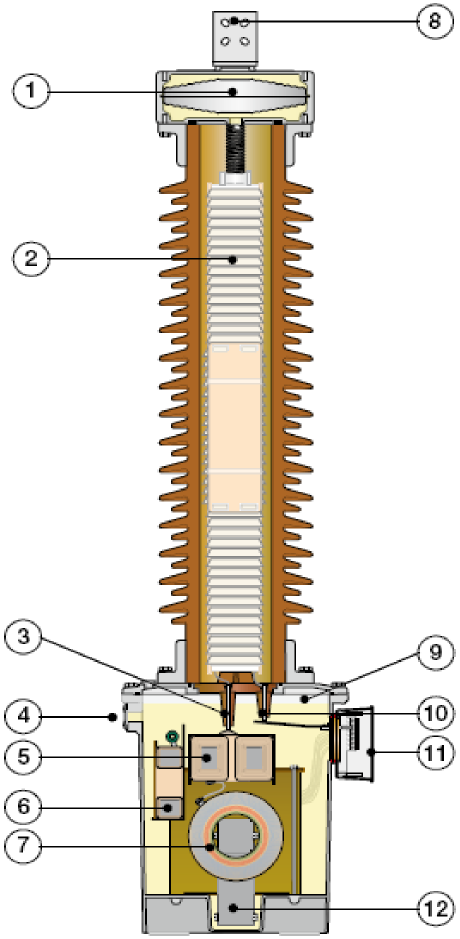
Electromagnetic Voltage Transformers





1. **Primary terminal**
2. Oil level sight glass
3. **Oil**
4. Quartz filling
5. **Insulator**
6. Lifting lug
7. Secondary terminal box
8. Neutral end terminal
9. Expansion system
10. Paper insulation
11. Tank
12. **Primary winding**
13. **Secondary windings**
14. Core

Capacitive Voltage Transformers



Capacitor Voltage Divider

1. Expansion system
2. **Capacitor elements**
3. Intermediate voltage bushing
8. **Primary terminal, flat 4-hole Al-pad**
10. **LV terminal (for carrier frequency use)**

Electromagnetic unit

4. Oil level glass
5. Compensating reactor
6. Ferroresonance damping circuit
7. **Primary and secondary windings**
9. Gas cushion
11. Terminal box
12. **Core**

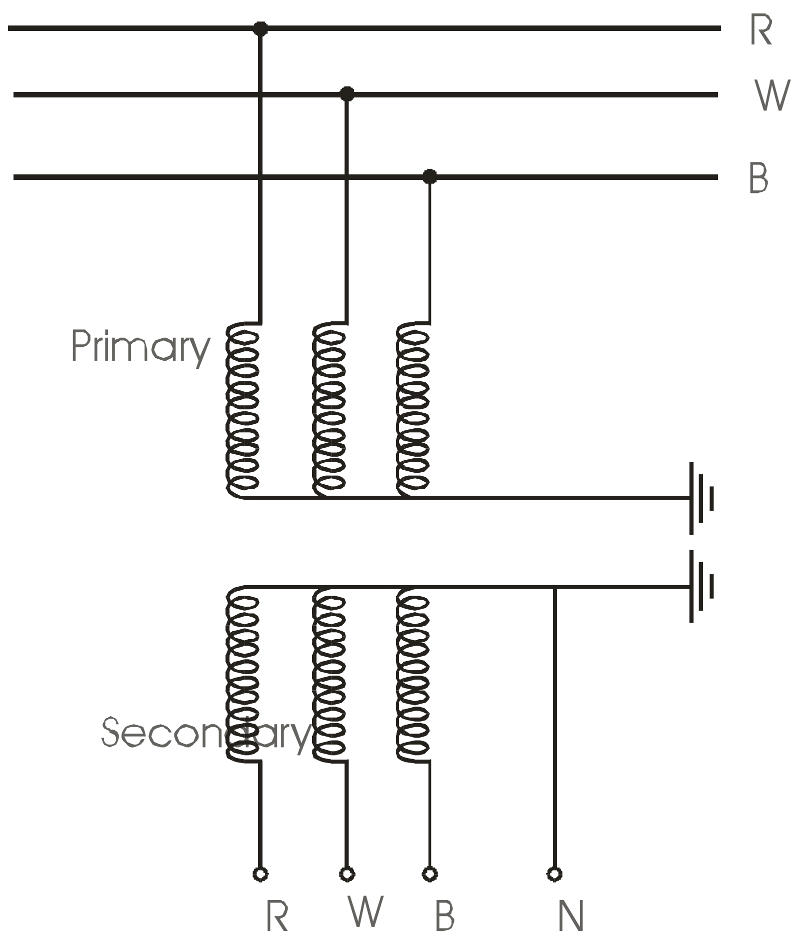
Voltage Transformers

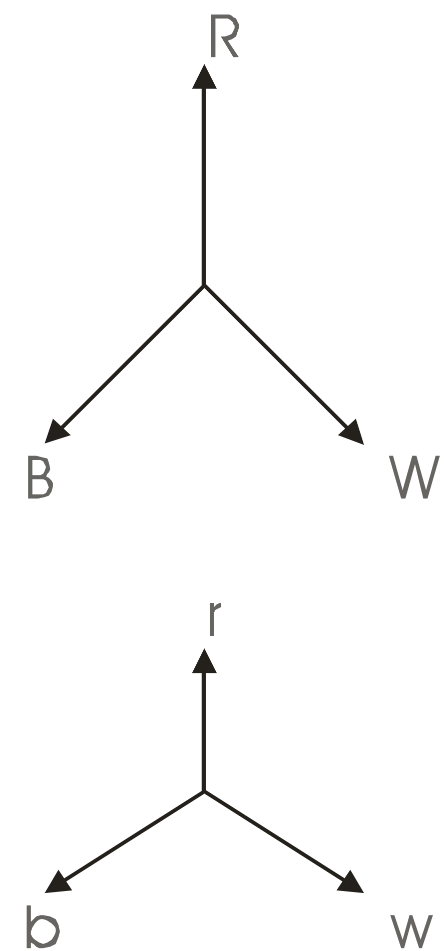
- VT ratios:
  - ratio of the high voltage/secondary voltage

1:1	2:1	2.5:1	4:1
5:1	20:1	40:1	60:1
80:1	100:1	200:1	300:1
400:1	600:1	800:1	1000:1
2000:1	3000:1	4500:1	

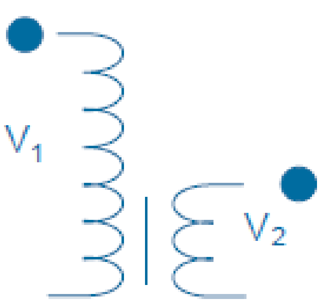
Connection of VT's

- VT's can be connected between phases or between phase and neutral (CVT's only phase - earth)



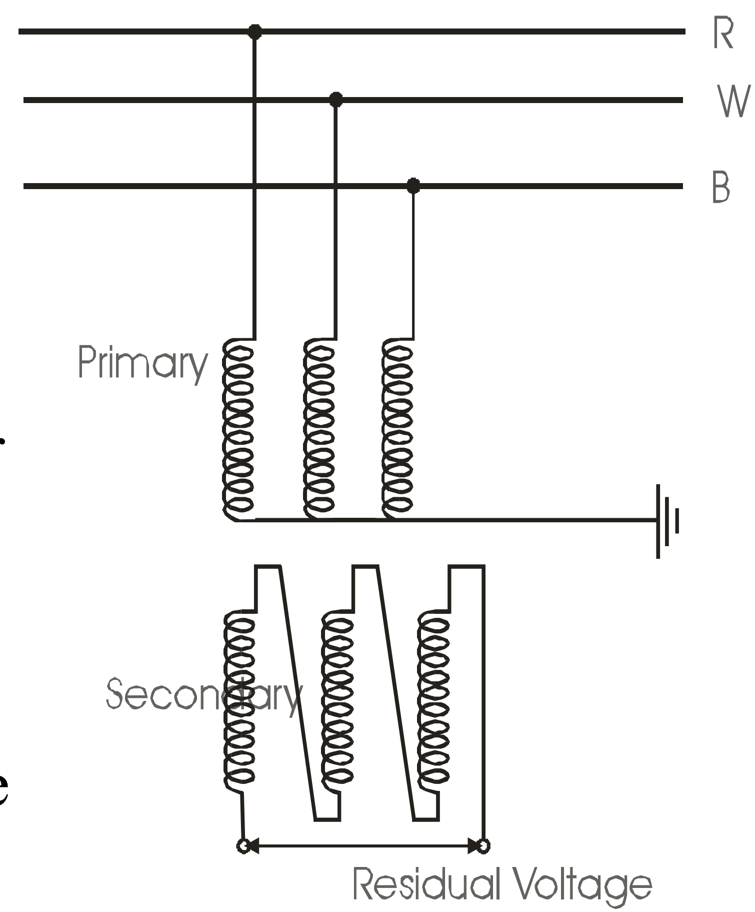


Symbol of a
Voltage Transformer

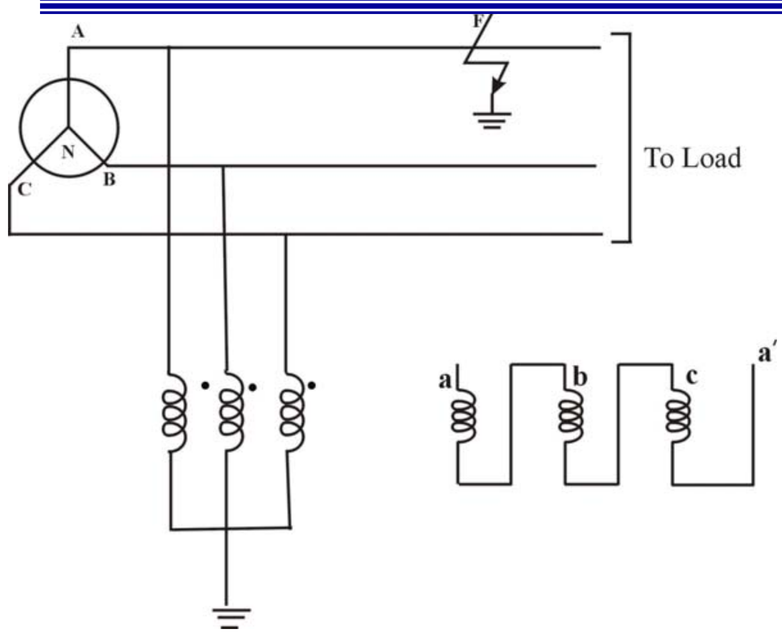


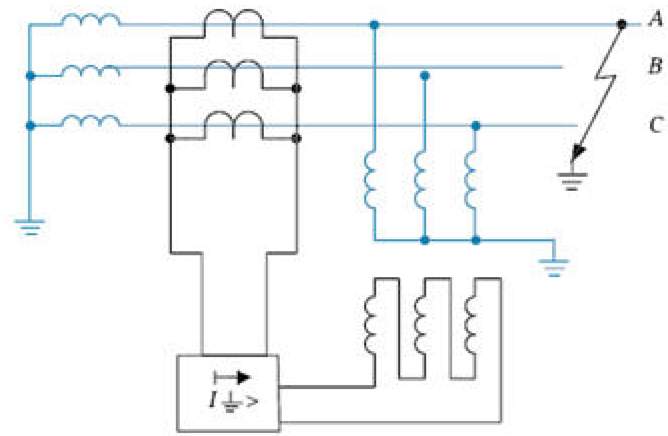
Residual voltage connection

- Normally used for earth fault detection
- VT's are connected in open-delta on primary and the vectorial sum of the 3 phase voltages will appear across the secondary output
- when there is an earth fault present the residual voltage will be non-zero



Residual voltage connection





When there is an Earth fault in line A it assumes Earth Potential.

Therefore Voltage across PT primary windings become

$V_A = 0$, $V_B = V_{BA,}$, $V_c = V_{CA}$

Thus Secondary Vectors are

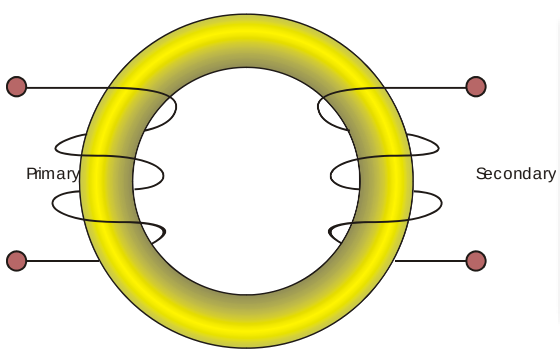
$V_a = 0$, $V_b = V_{ba}$, $V_c = V_{ca}$

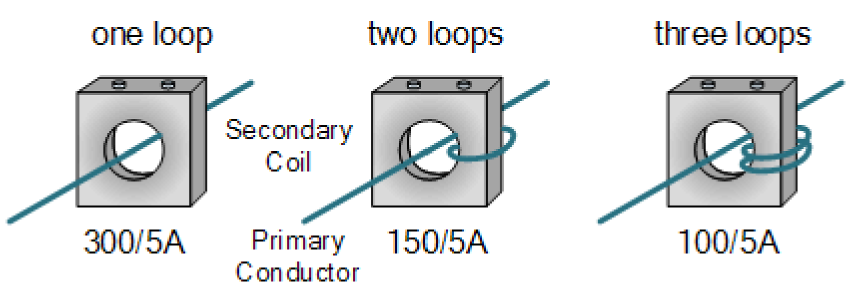
Types of Current Transformer Types

There are FOUR types of current transformers :
Wound, Toroidal, Bar and Bushing

The most common type of C.T. construction is the `**DOUGHNUT**' type. It is constructed of an iron toroid, which forms the core of the transformer, and is wound with secondary turns.

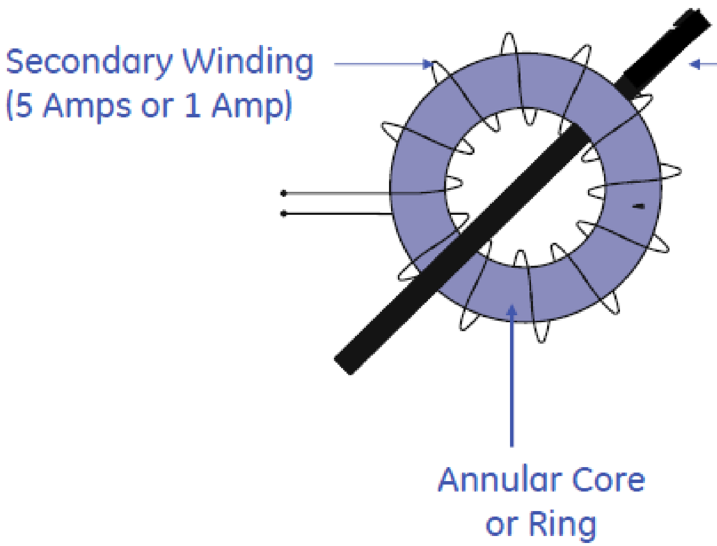
1. Wound Current Transformer – The transformers primary winding is physically connected in series with the conductor that carries the measured current flowing in the circuit. The magnitude of the secondary current is dependent on the turns ratio of the transformer.





Types of Current Transformer Types

2. Toroidal Current Transformer – These do not contain a primary winding. Instead, the line that carries the current flowing in the network is threaded through a window or hole in the toroidal transformer. Some current transformers have a “split core” which allows it to be opened, installed, and closed, without disconnecting the circuit to which they are attached.

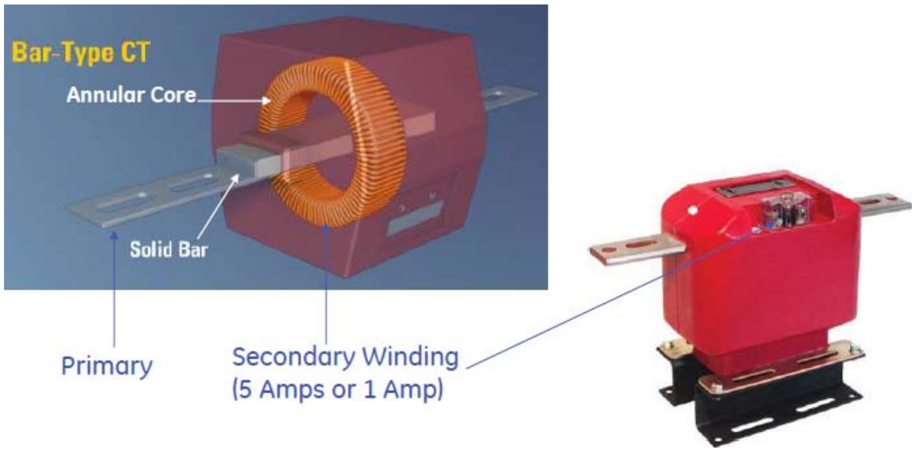


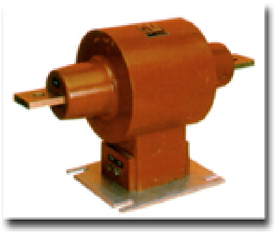
Types of Current Transformer Types

3. Bar-type Current Transformer – This type of current transformer uses the actual cable or bus-bar of the main circuit as the primary winding, which is equivalent to a single turn. They are fully insulated from the high operating voltage of the system and are usually bolted to the current carrying device.

The **toroid**, wound with secondary turns, is located in the **live tank** at the **top** of the C.T. High voltage insulation must, of course, be provided, between the H.V. primary conductor, and the secondary winding, which operates at essentially ground potential.

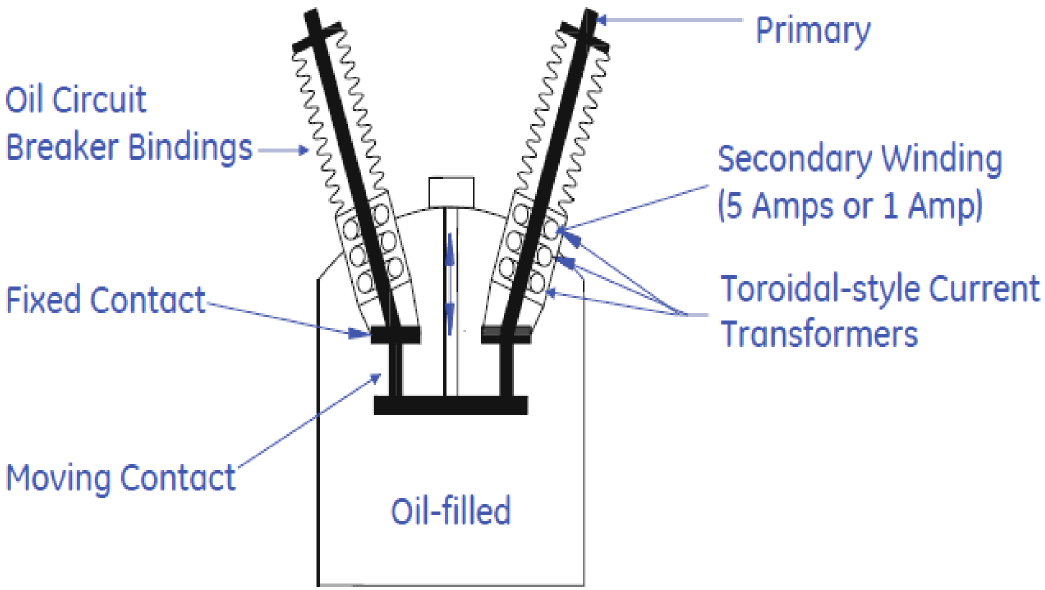
Current transformers of this type are often used at voltage levels of 44 kV, 33kV, and 13.8 kV.





Bushing Type Current Transformers

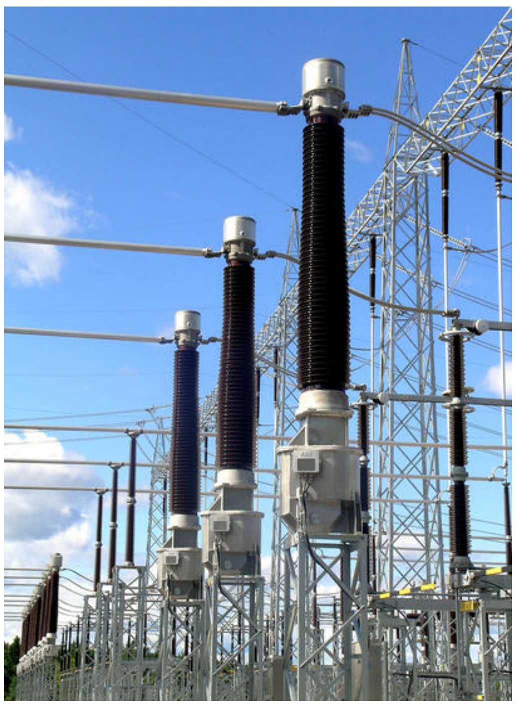
4. Bushing Current Transformer – This type of `*doughnut*' C.T. is most commonly used in circuit breakers and transformers. The C.T. fits into the bushing `*turret*', and the porcelain bushing fits through the centre of the `*doughnu*t'.

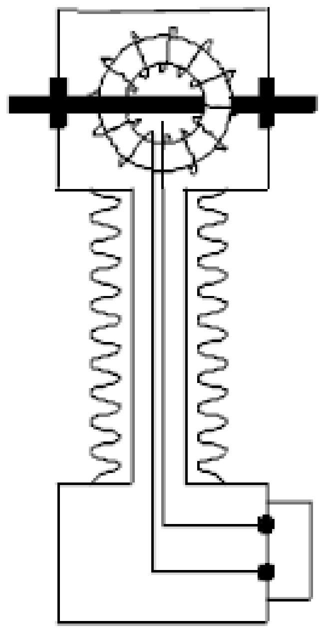


Bushing-type CTs installed on the bushings of 66kV Dead Tank Breaker

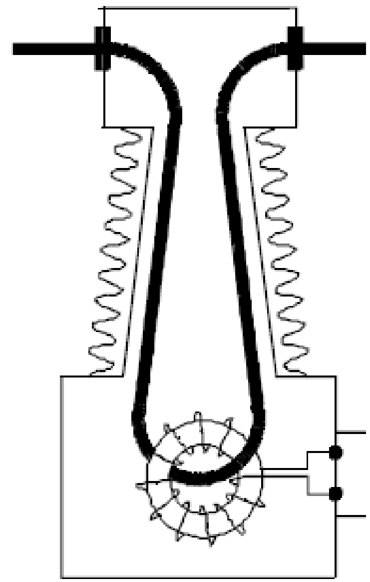
Other Types of CT Construction

The other principal type of C.T. construction is the ***Free Standing***, or ***Post*** type. These can be either ***Straight-Through*** or ***Hairpin*** construction.





Straight-Through

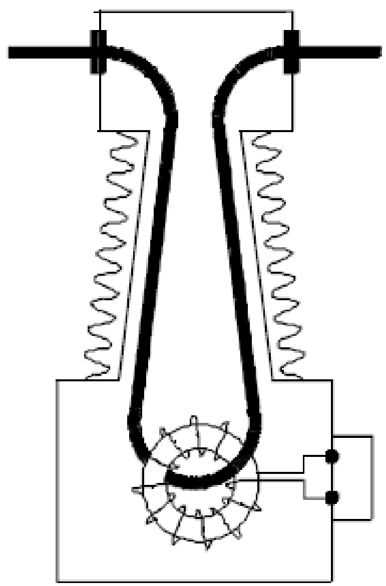


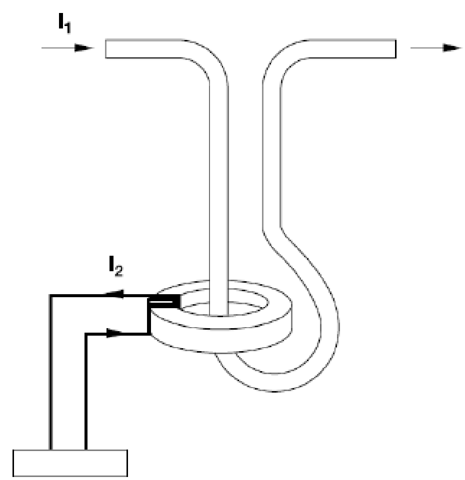
Hairpin

Hairpin - CT

The second kind of ***Free-Standing*** or ***Post*** type current transformer is the ***Hairpin*** construction as shown below:

The ***HAIRPIN*** C.T. gets it's name from the shape of the primary conductor within the porcelain. With this type, the tank housing the toroid is at ground potential. The primary conductor is insulated for the full line voltage as it passes into the tank and through the toroid.





CT-Instrument Transformer

Current transformers of this type are commonly used on H.V. transmission systems at voltage levels of 500kV and 230kV.

Free standing current transformers are very expensive, and are only used where it is not possible to install `Doughnut' C.T.'s in Oil Breakers or transformer bushing turrets.

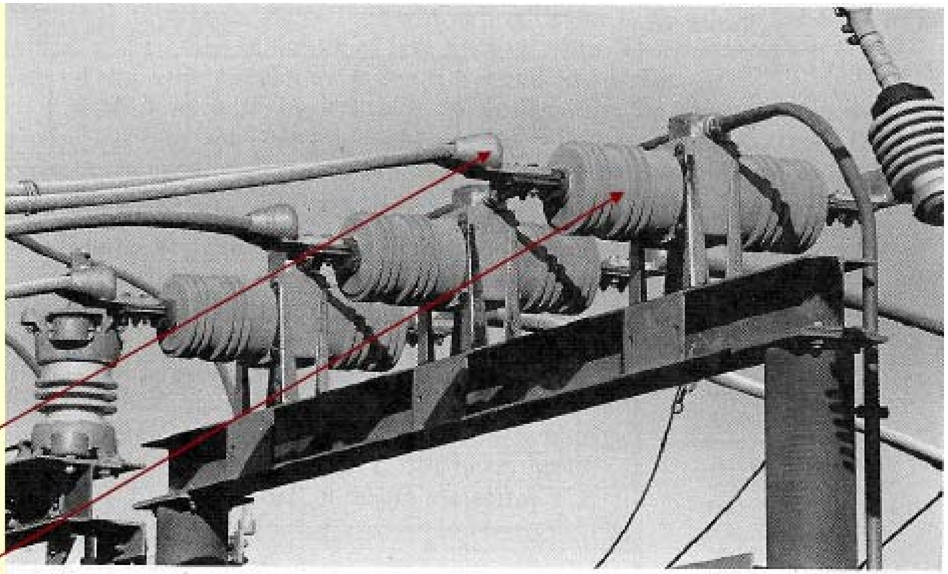
As an example, C.T.'s cannot easily be accommodated in Air Blast circuit breakers, or some outdoor SF6 breakers.

Free Standing current transformers must therefore be used with these types of switchgear. Current transformers often have multiple ratios. This is achieved by having taps on various points of the secondary winding, to provide the different turns ratios.

High Voltage CTs

High Voltage CTs

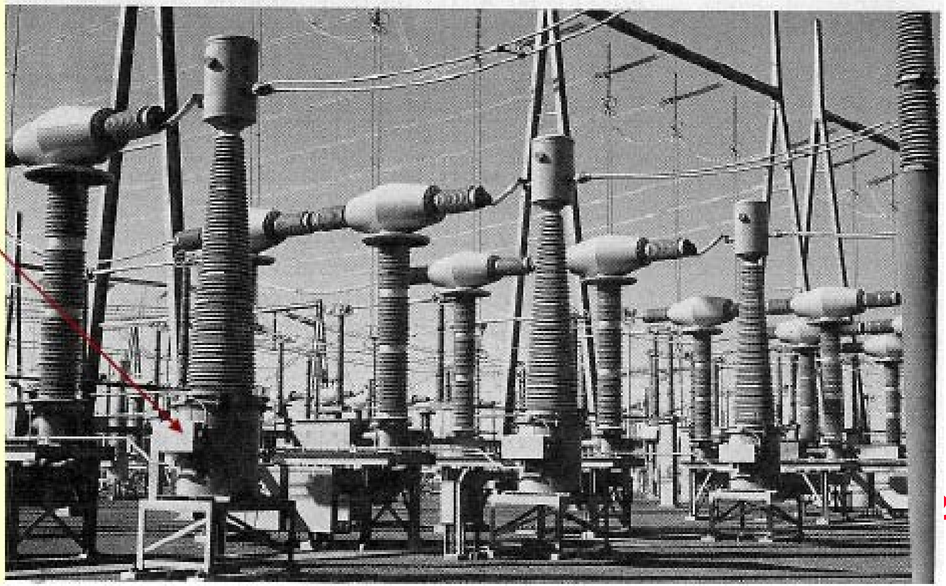
33kV single bar primary type CTs



Bar primary

Secondary coil

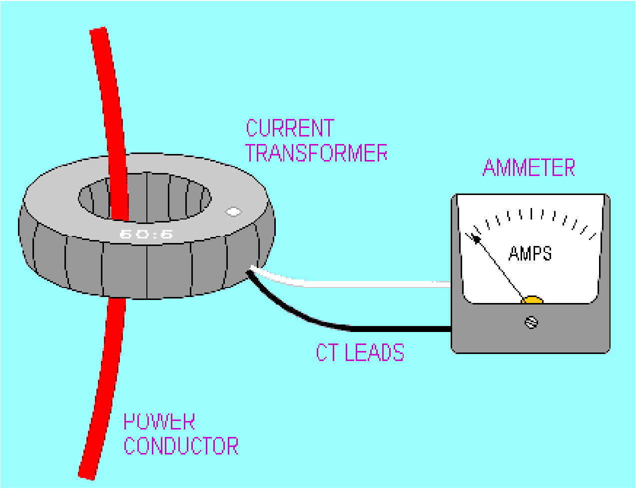
500kV "hairpin" single bar primary type CTs, with SF6 CB's behind

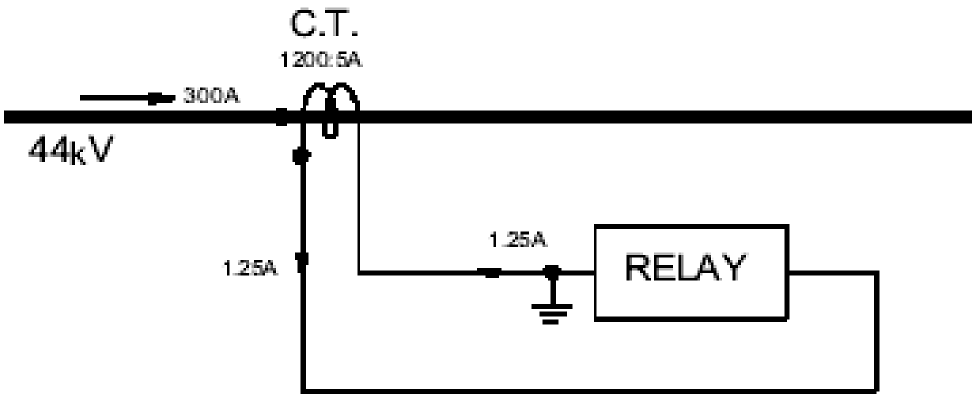


CT-Instrument Transformer

The `*doughnut*' fits over the primary conductor, which constitutes one primary turn. If the toroid is wound with 240 secondary turns, then the ratio of the C.T. is 240:1 or 1200:5A

The continuous rating of the secondary winding is normally **5 AMPS** in North America, and **1 AMP** or **0.5 AMP** in many other parts of the world.



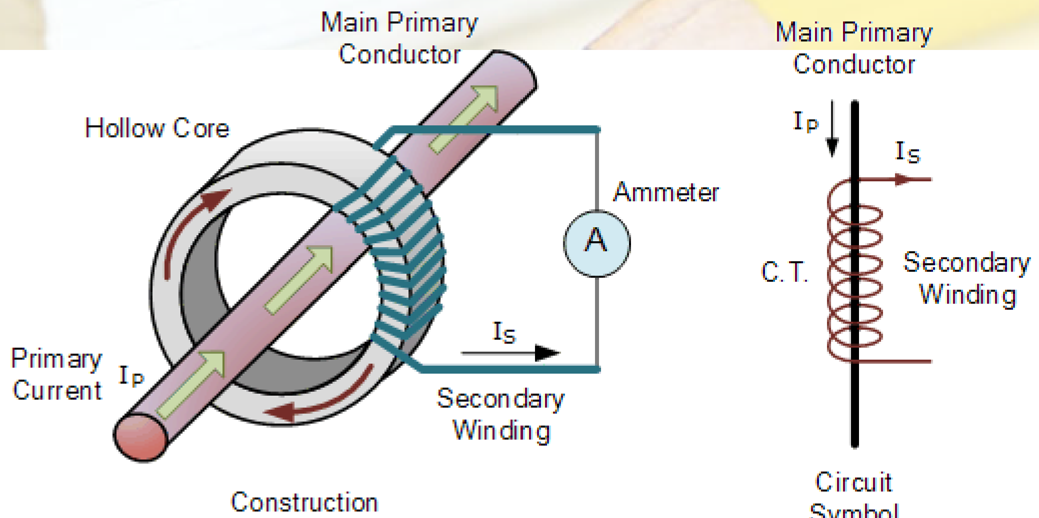


Current Transformers

- CTs ratio(secondary current rating is 5A)

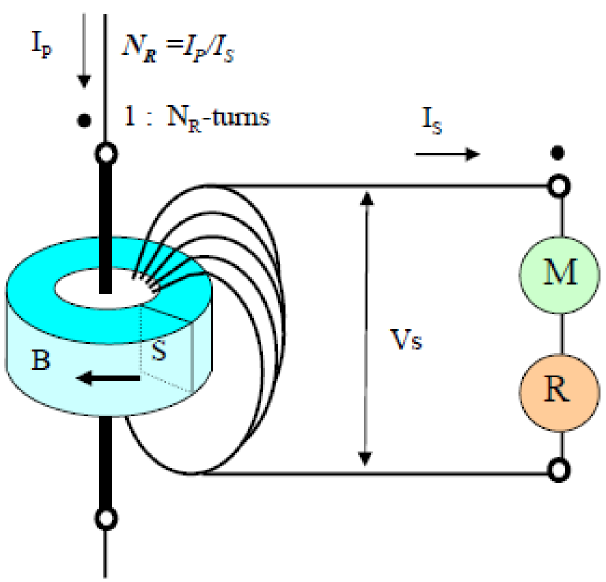
50:5	100:5	150:5	200:5
250:5	300:5	400:5	450:5
500:5	600:5	800:5	900:5
1000:5	1200:5		

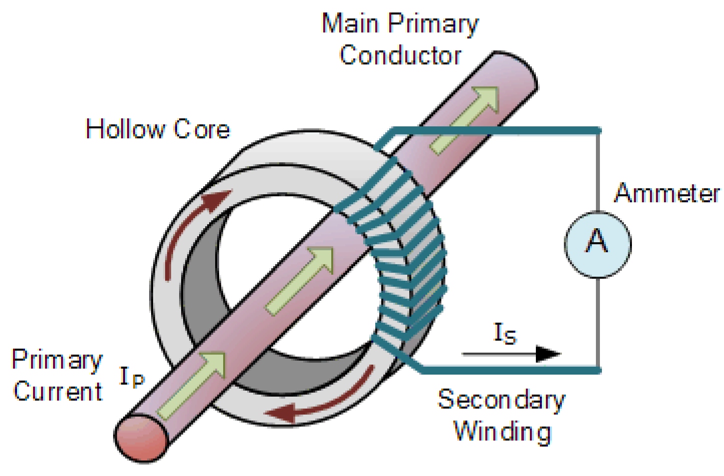
- CTs also available with the secondary rating of 1A



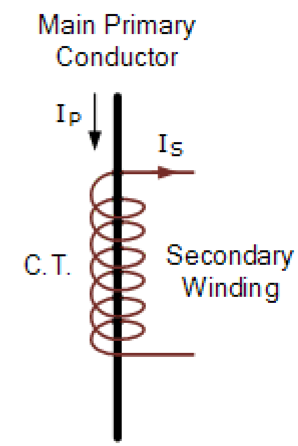
Current Transformers - CT

- As with all transformers-Ampere-turns balance must be achieved
- e.g. 1000Amps × 1 turn (bar primary) =
  1 Amp × 1000 turns (secondary side)
- Error introduced into measurement by magnetising current
- **Current Transformers for protective relaying purposes must reproduce the primary current accurately for all expected fault currents.**





Construction



Circuit Symbol

CT-Instrument Transformers

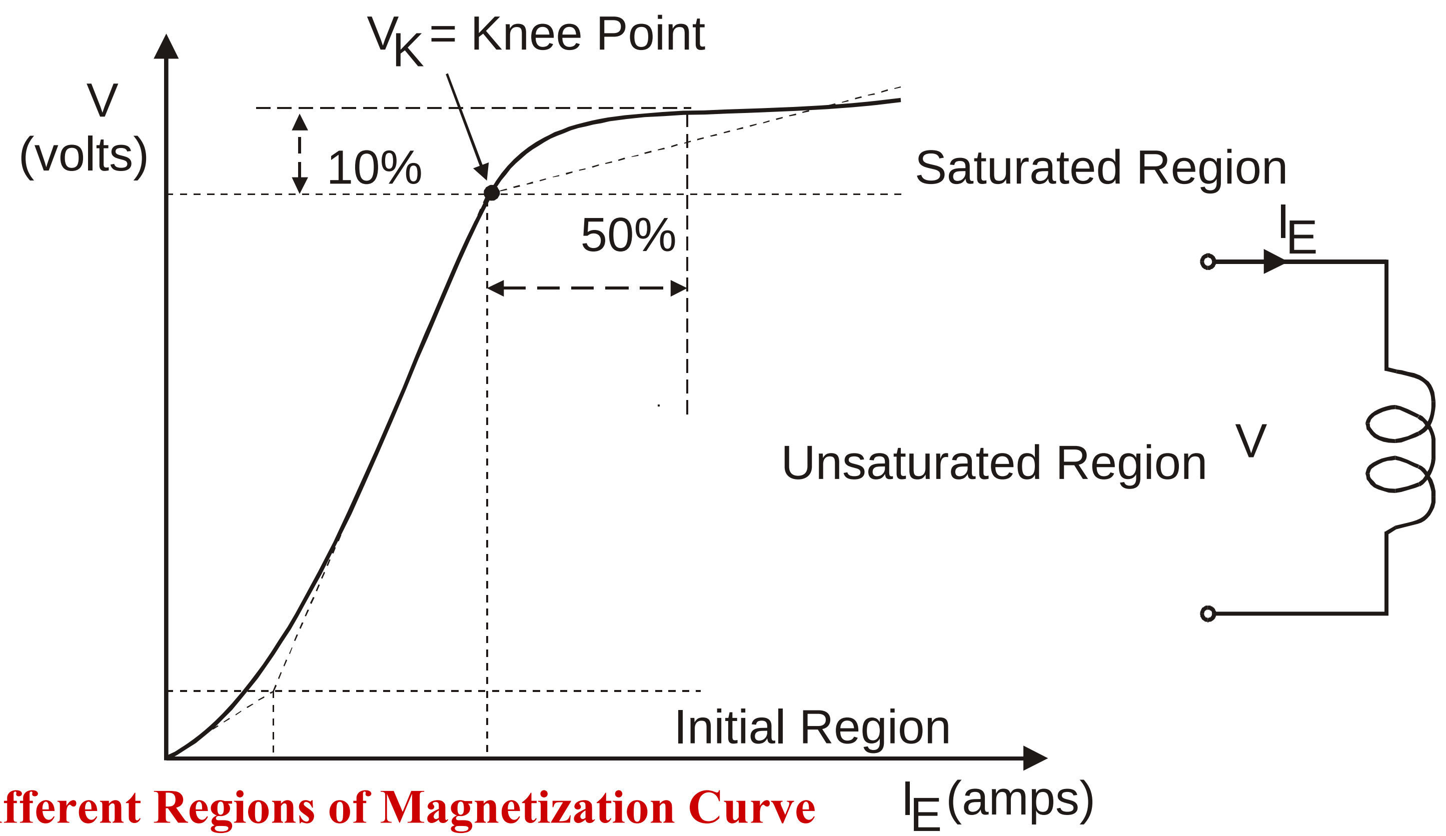
If we have a 33 kV C.T. with a ratio of 1200 : 5A, the secondary winding is continuously rated for 5 Amps.

If the maximum fault current that can flow through the C.T. is 12,000 Amps, then the C.T. must accurately produce a secondary current of 50 Amps to flow through the relay during this fault condition.

This current will, of course, flow for only about 0.2 seconds, until the fault current is interrupted by the tripping of the circuit breaker.

CT-Excitation Characteristics

The C.T. must be designed such that the iron core does not saturate for currents below maximum fault current. For a typical C.T.

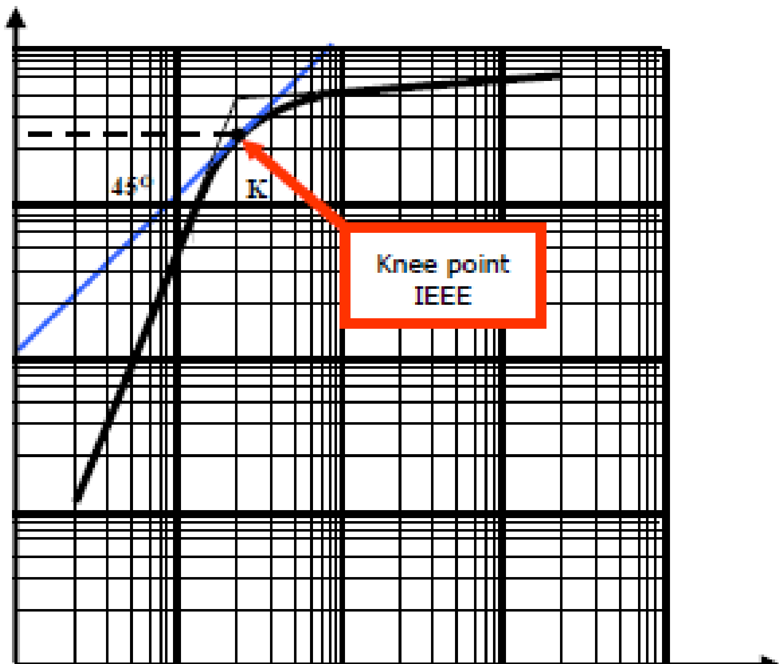


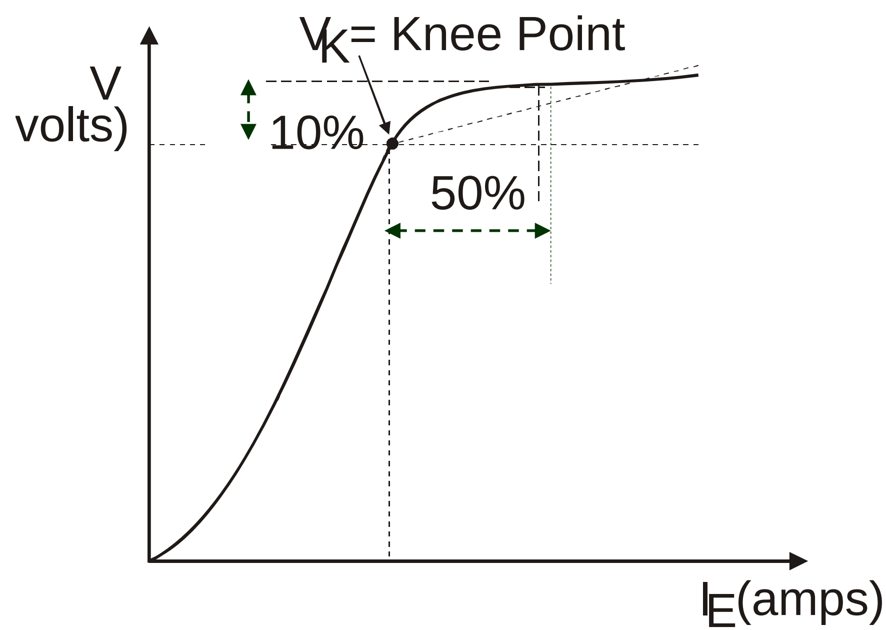
Different Regions of Magnetization Curve

CT-Excitation Characteristics

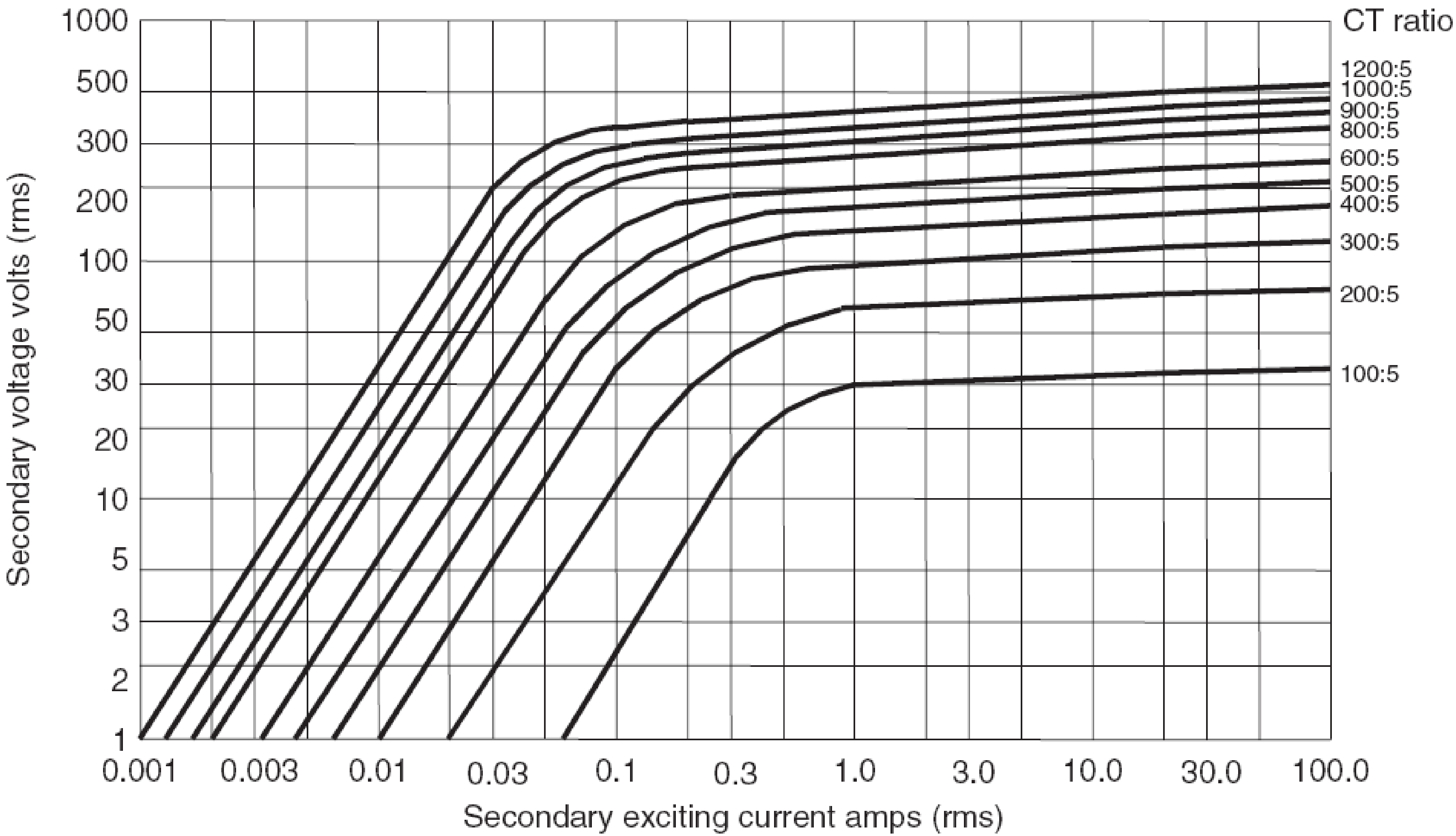
For a C.T. to operate satisfactorily at maximum fault currents, it must operate on the linear part of the magnetizing curve. i.e. Below the point at which saturation occurs, which is known as the **KNEE POINT.** The **KNEE POINT** is defined as:

<***the point at which a 10% increase in voltage produces a 50% increase in magnetizing current.***> *IEC.BS Standard*



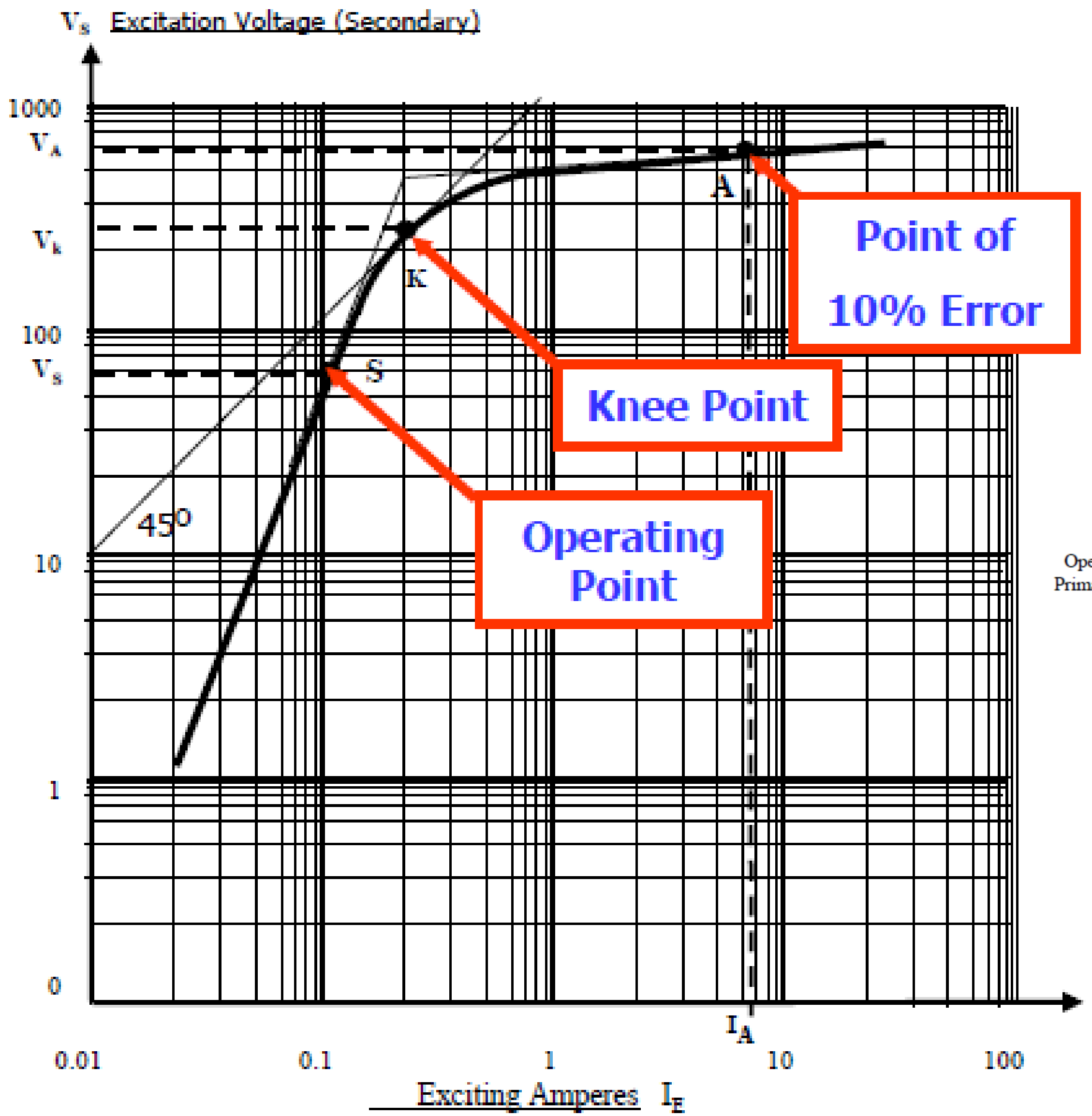


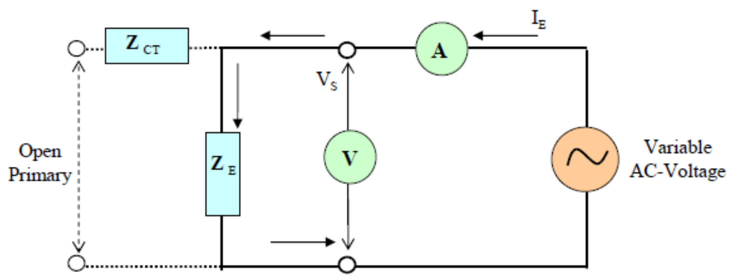
CT-Excitation Characteristics



Magnetizing characteristic of a typical CT

CT Typical Excitation Curve





Test Circuit of the CT

Two Main Types of CTs

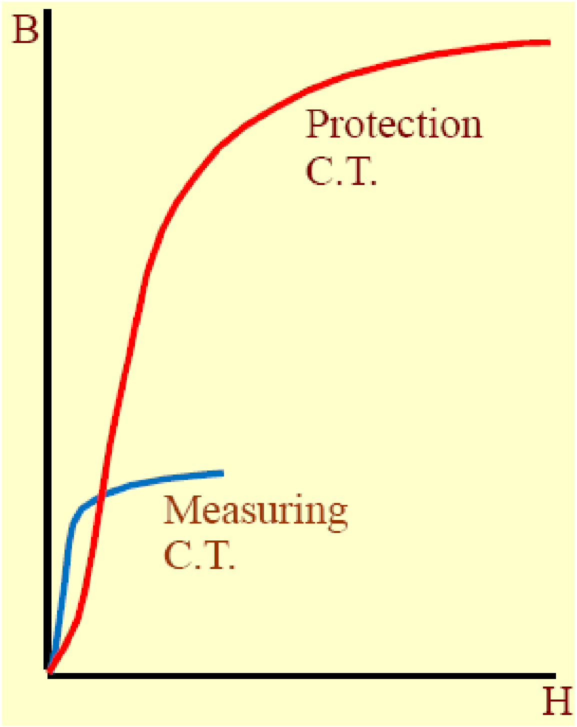
- Measuring CTs (type 'M')
- Protection CTS (type 'P')

Measurement class CT

- Designed to operate at rated current

Protection class CT

- Designed to operate at fault current

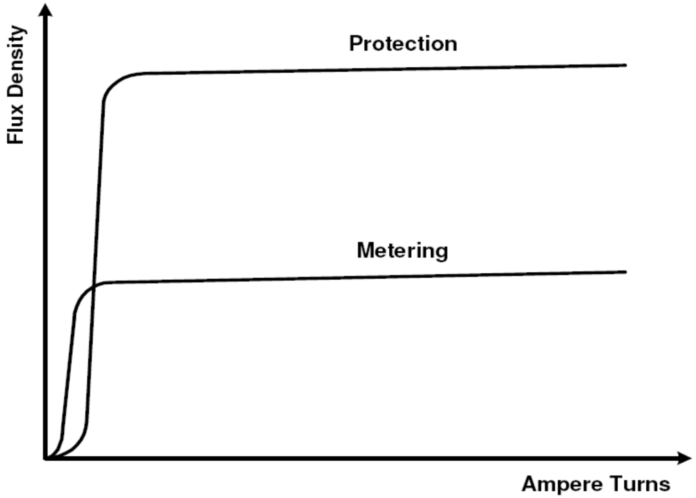


Characteristics and Specification are very different

Protection vs Metering CT's

- Protection CT's have to measure fault currents many times in excess of full load current without saturating to drive relays to trip - Accuracy is not as important

- Metering CT's have to be accurate as customers will be billed on the information provided by measuring the current from the metering CT's - special alloys are used for the cores so that they saturate quickly



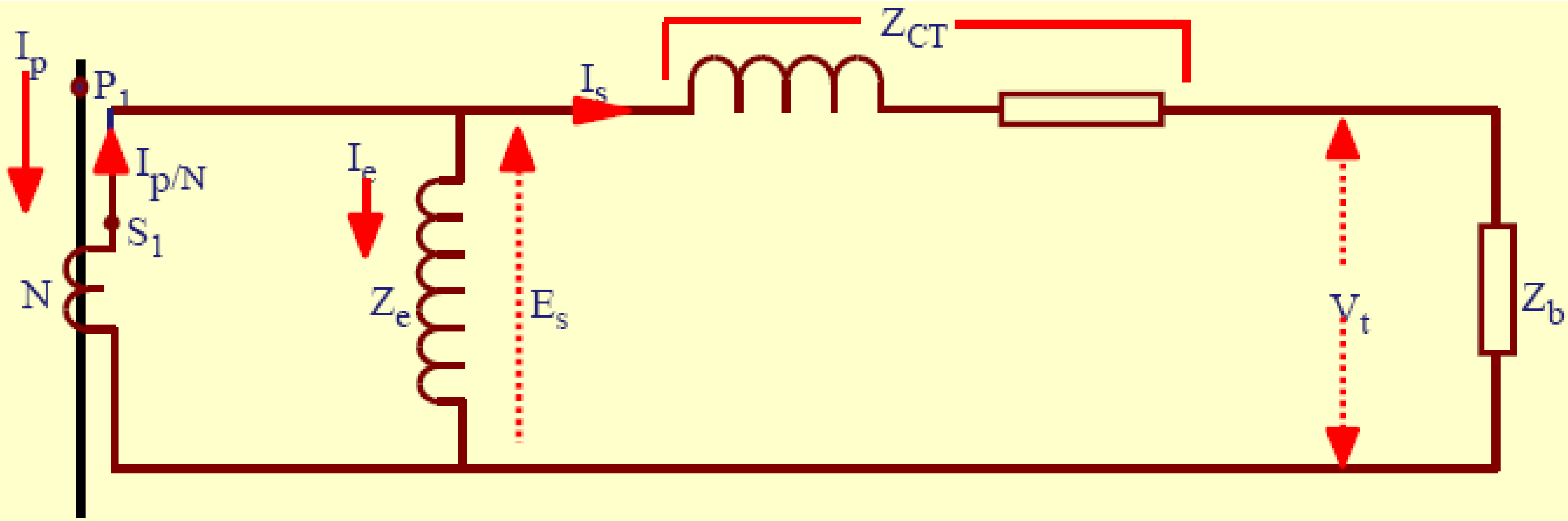
Protection vs Metering CT's

When C.T.'s are used for metering purposes, they must have a high degree of accuracy only at **LOAD** currents. i.e. 0 to 5 Amps secondary.

There is no need for a high degree of accuracy for fault currents, and it is quite acceptable for a metering C.T. to saturate when fault current flows through it.

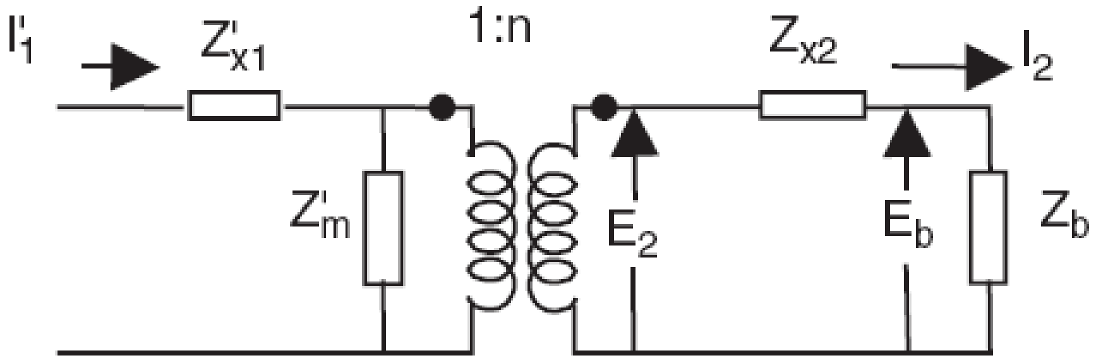
A C.T. for protective relaying purposes may typically have a ***knee point*** at 500 volts, whereas a metering C.T. may saturate at well below 100 volts.

CT- General Equivalent Circuit

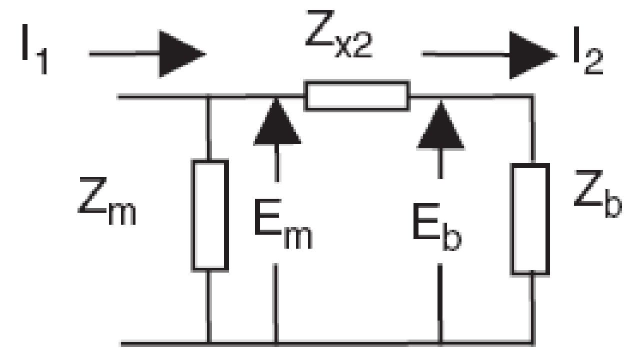


I_p	=	Primary rating of C.T.	I_e	=	Secondary excitation current
N	=	C.T. ratio	I_s	=	Secondary current
Z_b	=	Burden of relays in ohms (r+jx)	E_s	=	Secondary excitation voltage
Z_{CT}	=	C.T. secondary winding impedance in ohms (r+jx)	V_t	=	Secondary terminal voltage across the C.T. terminals
Z_e	=	Secondary excitation impedance in ohms (r+jx)			

CT- Equivalent Circuit and its Simplification



(a)



(b)

Since the primary winding of a CT is connected in series with the power network, its primary current I_1 is dictated by the network. Consequently, the leakage impedance of the primary winding $Z`x_1$ has no effect on the performance of the transformer, and may be omitted.

$$I_1 = \frac{I_1'}{n} \qquad E_m = E_b + Z_{x2} I_2$$

and the magnetizing current I_m is given by

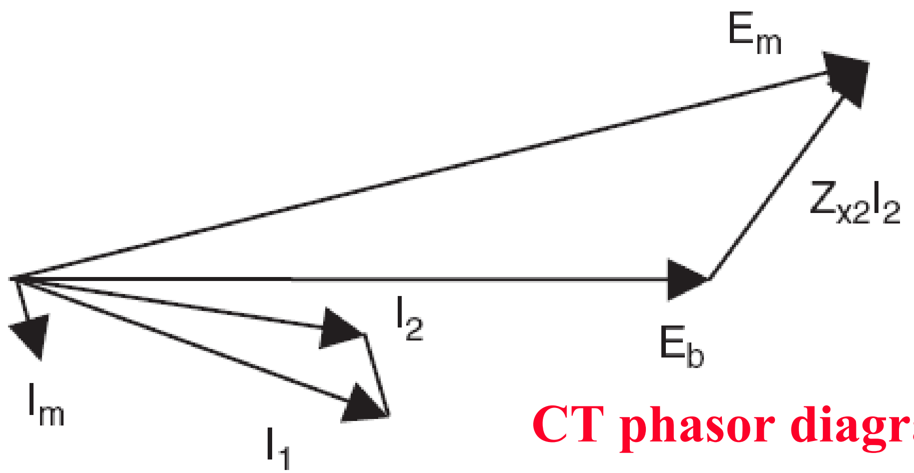
$$I_m = \frac{E_m}{Z_m}$$

$$Z_m = n^2 Z_m'$$

CT- Equivalent Circuit and its Simplification

The primary current I_1 (referred to the secondary winding) is given by

$$I_1 = I_2 + I_m$$



CT phasor diagram

For small values of the burden impedance, E_b and E_m are also small, and consequently I_m is small. The per unit current transformation error defined by

$$\varepsilon = \frac{I_1 - I_2}{I_1} = \frac{I_m}{I_1}$$

is, therefore, small for small values of Z_b. In other words, CTs work at their best when they are connected to very low impedance burdens.

CT- Equivalent Circuit and its Simplification

More often, the CT error is presented in terms of a ratio correction factor ***R*** instead of the per unit error **ε.**

The ratio correction factor ***R*** is defined as the constant by which the nameplate turns ratio ***n*** of a CT must be multiplied to obtain the effective turns ratio.

$$R = \frac{1}{1 - \varepsilon}$$

Example 1: CT Performance

Consider a current transformer with a turns ratio of 500:5, a secondary leakage impedance of (0.01+j0.1) and a resistive burden of 2.0. If the magnetizing impedance is (4.0+j15), then for a primary current (referred to the secondary) of I_1

$$E_{\mathrm{m}} = \frac{I_1(0.01 + \mathrm{j}0.1 + 2.0)(4.0 + \mathrm{j}15.0)}{(0.01 + \mathrm{j}0.1 + 2.0 + 4.0 + \mathrm{j}15.0)} = I_1 \times 1.922\angle 9.62^\circ$$

$$I_{\mathrm{m}} = \frac{I_1 \times 1.922\angle 9.62^\circ}{(4.0 + \mathrm{j}15.0)} = I_1 \times 0.1238\angle -65.45^\circ$$

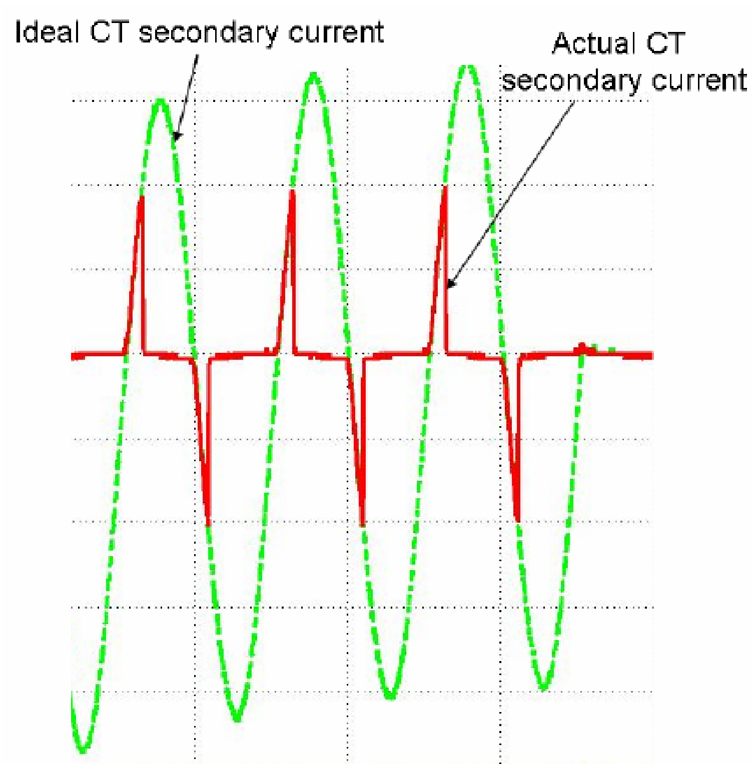
CT error $$\varepsilon = \frac{I_{\mathrm{m}}}{I_1} = 0.1238\angle -65.45^\circ$$

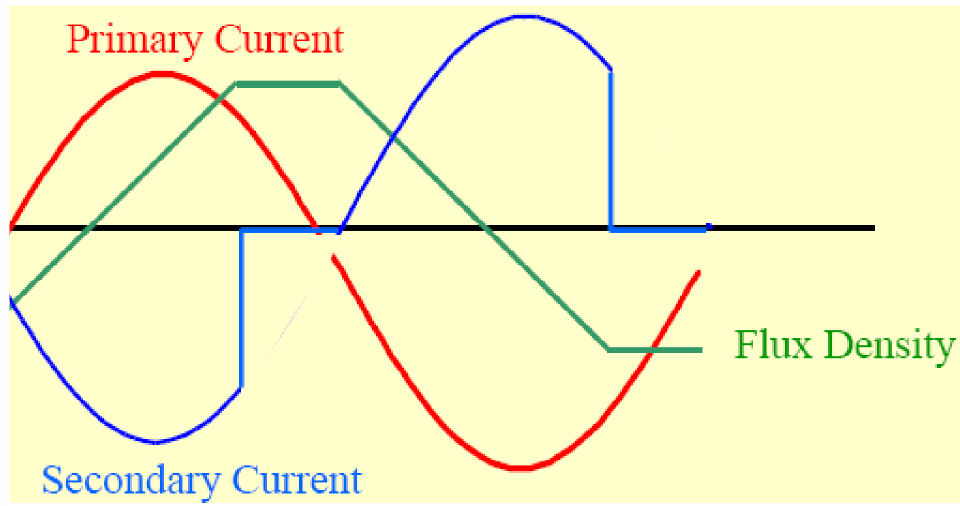
The corresponding ratio correction factor ***R*:**

$$R = \frac{1}{(1.0 - 0.1238\angle -65.45^\circ)} = 1.0468\angle -6.79^\circ \quad \text{for } Z_{\mathrm{b}} = 2 \text{ ohms}$$

Effect of CT Saturation

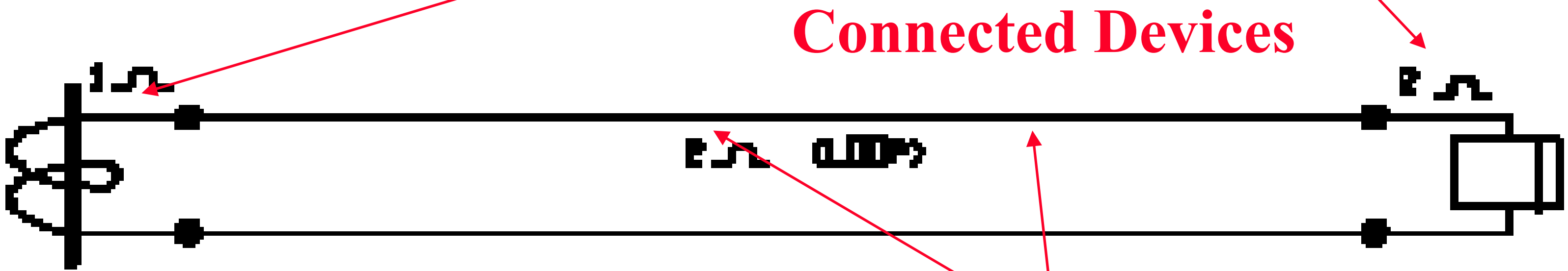
Output current drops to zero when flux is constant (core saturated)

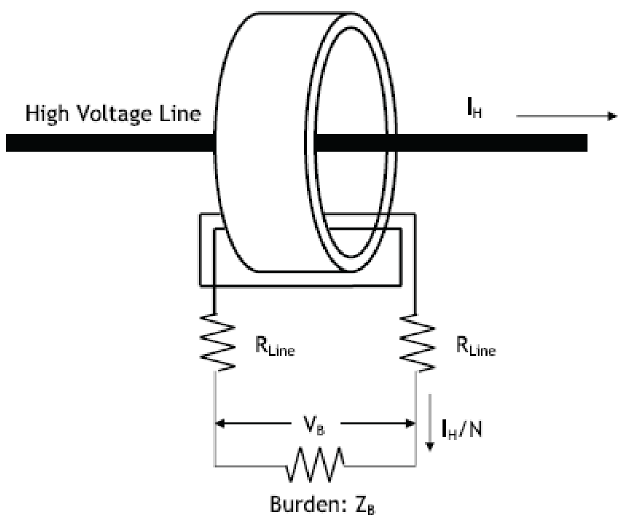




CT Knee-point Voltage

The point on the magnetizing curve at which the C.T. operates is dependent upon the resistance of the C.T. secondary circuit, as shown below:



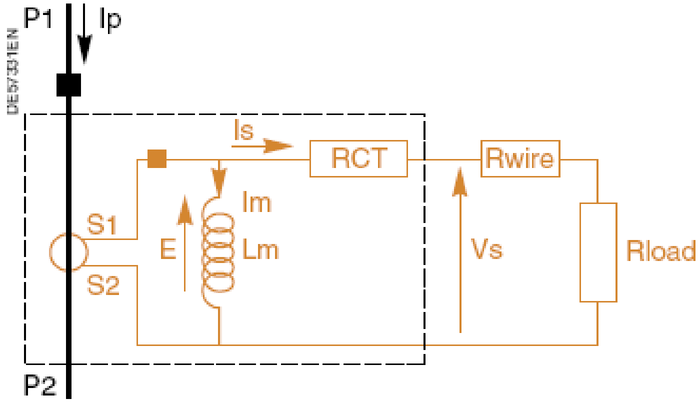


Current Transformer Circuit with Burden

CT Knee-point Voltage

Kee-point Voltage ➔ $V_k = (R_{ct} + R_b + 2{\times}R_L).I_{sn}$

- R_{ct}= CT secondary winding resistance
- R_L= secondary wiring (leads) resistance
- R_b = burden (load). (meter or relay)
- I_{sn} = rated secondary current
- V_k= required knee-point voltag

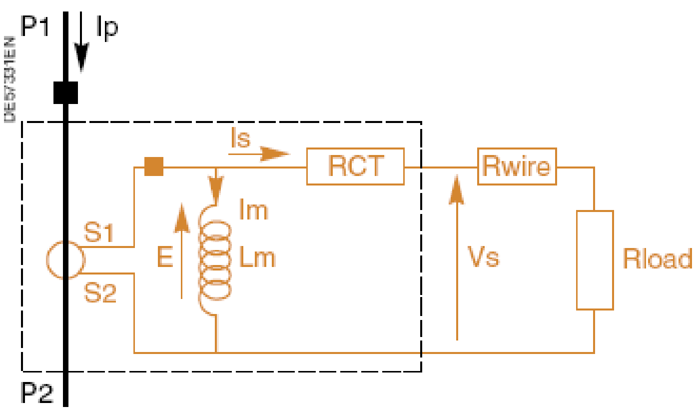


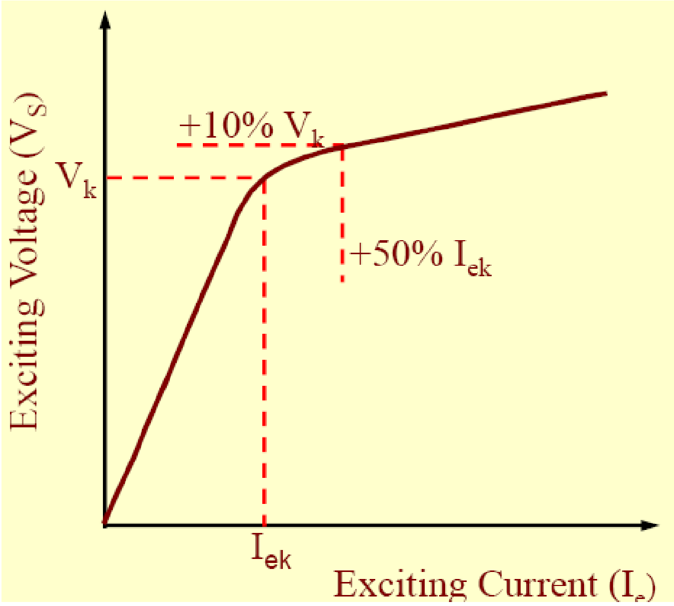
Example 2: CT Knee-point Voltage

In this example the resistance of the C.T. secondary circuit, or C.T. burden is:

C.T. Secondary Winding Resistance	= 1 OHM
Resistance of Cable from C.T. to Relay	= 2 OHMS
Resistance of Relay Coil	= 2 OHMS

Total Resistance of C.T. Secondary Circuit = 5 OHMS





Example 2: CT Performance

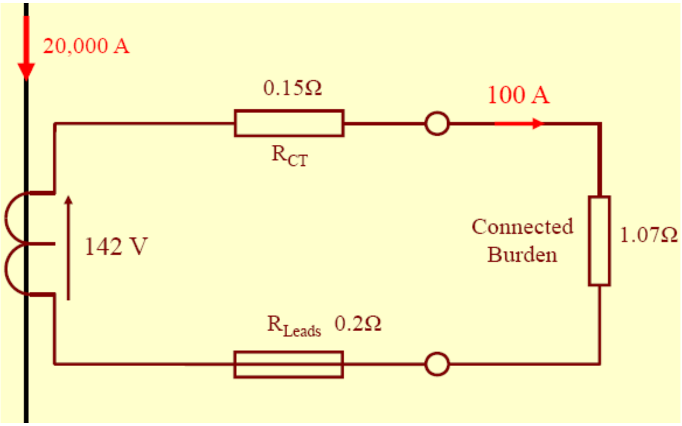
If the fault current is 12,000 Amps, and the C.T. ratio is 1200 : 5A, then the C.T. secondary current is 50 Amps. At this secondary current and the above C.T. burden of 5 OHMS, the C.T. must produce a terminal voltage of 250 volts.

For the C.T. to operate with good accuracy, without saturating for the maximum fault current, the ***knee point*** must be well above **250 volts**.

The importance of the C.T. maintaining good accuracy, and not saturating at the maximum fault current, is most critical on differential protection.

Example 3: CT Performance

- Find maximum allowable secondary burden
  - CT Ratio = 1000/5
  - R_{CT}= 0.15 Ω
  - R_{leads} = 0.1 Ω (R_{leads}= 2×0.1 Ω = 0.2 Ω)
  - Max Flux Density (B) = 1.6 Tesla
  - Core Cross Sectional Area (A) = 20cm^2



- **Solution:**
  - $V_k = 4.44 \times B \times A \times f \times N$ (the "transform erequation")
  - N = 1000/5 = 200 Turns
  - A = 20cm^2= 20×10^{-4} m^2
  - $V_k = 4.44 \times 1.6 \times 20 \times 10^{-4} \times 50 \times 200$ = 142 V (alternatively, if *A* not known, curves of V_k may be available)
  - Max Fault Current = 20kA (Primary) = 100 A (secondary)
  - Max resistance = 142V/100A = 1.42 Ω
  - R_{CT}= 0.15Ω; Lead resistance = 0.2 Ω
  - Maximum connected burden 1.42 - 0.15 - 0.2 = <u>1.07 Ω</u>

Example 4: CT Performance

- A current transformer is specified as being 600 A:5 A class C200. Determine it's characteristics. This designation is based on ANSI Std. C57.13-1978. 600 A is the continuous primary current rating, 5 A is the continuous secondary current rating, and the turns ratio is 600/5=120. C is the accuracy class, as defined in the standard. The number following the C, which in this case is 200,is the voltage that the CT will deliver to the rated burden impedance at 20 times rated current without exceeding 10 percent error. Therefore, the rated burden impedance is

$$Z_{\text{rated}} = \frac{\text{Voltage class}}{20 \cdot \text{Rated secondary current}} = \frac{200\ \text{V}}{20 \cdot 5\ \text{A}} = 2\ \Omega$$

- This CT is able to deliver up to 100 A secondary current to load burdens of up to 2 with less than 10 percent error. Note that the primary source of error is the saturation of the CT iron core and that 200 V will be approximately the knee voltage on the CT saturation curve.

Example 5: CT Performance

- The circuit of Fig.1 has 600:5 class C200 CTs. The peak-load current is a balanced 475 A per phase.

1. Determine the Relay Currents for the Peak-Load Conditions.

The A phase CT secondary current is

$$I_A = \frac{475\text{A}}{120} = 3.96\ \text{A}\underline{/0^\circ}$$

- Here, the A phase current is taken to be at 0°. The B and C phase currents are the same magnitude, shifted by 120°,

$$I_B = 3.96\ \text{A}\underline{/-120^\circ},\ I_C = 3.96\ \text{A}\underline{/120^\circ}$$

- The residual current is

$$I_R = I_A + I_B + I_C = 3.96\ \text{A}\underline{/0^\circ} + 3.96\ \text{A}\underline{/-120^\circ} + 3.9.6\ \text{A}\underline{/120^\circ} = 0\ \text{A}$$

Example 5: CT Performance

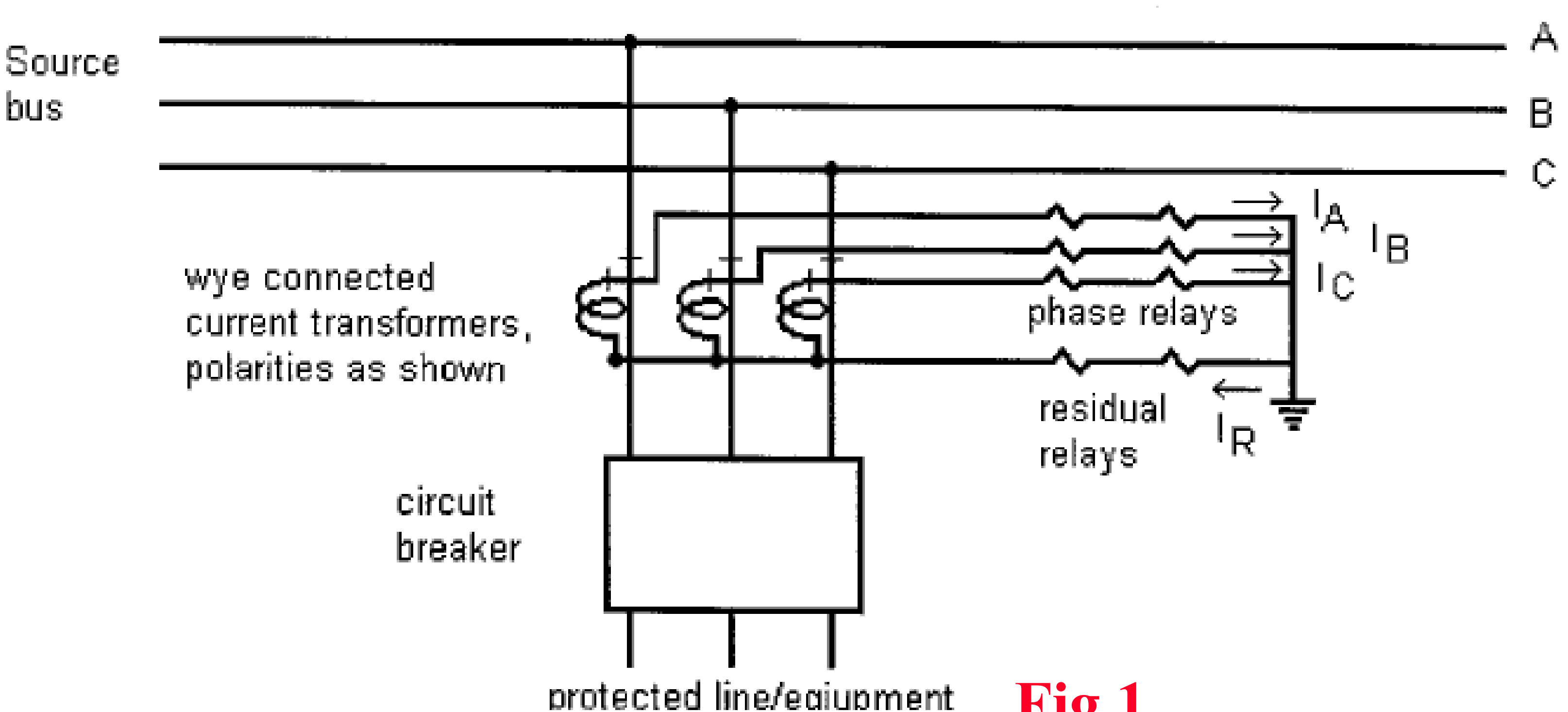


Fig.1

Typical setup for wye-connected CTs protecting a line or piece of equipment.

Example 5: CT Performance

2. The circuit has an A phase to ground fault on the line, with fault current magnitude of 9000 A. Find the phase and residual relay currents. Again, assume that the A phase current is at 0°.

$$I_A = \frac{9000\ \text{A}}{120} = 75\ \text{A}\underline{/0^\circ}$$

$$I_B = 0\ \text{A}$$

$$I_C = 0\ \text{A}$$

$$I_R = I_A + I_B + I_C = 75\ \text{A}\underline{/0^\circ} + 0\ \text{A} + 0\ \text{A} = 75\ \text{A}\underline{/0^\circ}$$

❖ The current path is therefore through the A phase lead and back through the residual lead.

Example 5: CT Performance

3. The circuit has a two-phase fault with 5000 amps going out B phase and back in on C phase. Choose B phase current to be at 0°.

$$I_A = \frac{0\text{ A}}{120} = 0\text{ A}$$

$$I_B = \frac{5000\text{ A}\angle 180^\circ}{120} = 41.7\text{ A}\angle 0^\circ$$

$$I_C = \frac{5000\text{ A}\angle 180^\circ}{120} = 41.7\text{ A}\angle 180^\circ = -I_B$$

$$I_R = I_A + I_B + I_C = 0\text{ A} + 41.7\text{ A}\angle 0^\circ + 41.7\text{ A}\angle 180^\circ = 0\text{ A}$$

- This current path involves the B and C phase leads, with no current in either the A phase lead or residual.

Example 5: CT Performance

4. The circuit has a three-phase fault with 8000 A per phase.

$$I_A = \frac{8000\ \text{A}\underline{/0^\circ}}{120} = 66.7\ \text{A}\underline{/0^\circ}$$

$$I_B = \frac{8000\ \text{A}\underline{/-120^\circ}}{120} = 66.7\ \text{A}\underline{/-120^\circ}$$

$$I_C = \frac{8000\ \text{A}\underline{/120^\circ}}{120} = 66.7\ \text{A}\underline{/-120^\circ}$$

$$I_R = I_A + I_B + I_C = 66.7\ \text{A}\underline{/0^\circ} + 66.7\ \text{A}\underline{/-120^\circ} + 66.7\ \text{A}\underline{/120^\circ} = 0\ \text{A}$$

- The phase currents sum to zero, so no current flows in the residual for this fault.
- The path of current flow for these various situations must be considered in calculating the CT excitation voltage and subsequent saturation.

Example 6: CT Performance

- For part 2, 3, and 4 of Example 5, calculate the CT voltage if the phase relay burden is 1.2Ω , the residual relay burden is 1.8 Ω , the lead resistance is 0.4 Ω ,and the CT resistance is 0.3 Ω. Neglect CT saturation in this calculation.

1. Single-Phase Fault

   The A phase CT will have an excitation voltage of

$$\begin{aligned} V_{exA} &= I_{Asec}(Z_{CT} + 2Z_{lead} + Z_{phase} + Z_{residual}) \\ &= 75\ \mathrm{A}(0.3\ \Omega + 2 \cdot 0.4\ \Omega + 1.2\ \Omega + 1.8\ \Omega) \\ &= 307\ \mathrm{V} \end{aligned}$$

- The impedances are primarily resistive, and phase angle is often neglected in the voltage calculations. The impedances can be determined by tracing the path of the current through the CT secondary circuit.

Example 6: CT Performance

2. Two-Phase Fault

The B phase CT will have an excitation voltage of

$$
\begin{aligned}
V_{exB} &= I_{Bsec}(Z_{\mathrm{CT}} + Z_{\mathrm{lead}} + Z_{\mathrm{phase}}) \\
&= 41.7\ \mathrm{A}(0.3\ \Omega + 0.4\ \Omega + 1.2\ \Omega) \\
&= 79.2\ \mathrm{V}
\end{aligned}
$$

❖ The C phase CT will see a similar voltage. Note that the A phase CT will also see a significant voltage, although it is carrying no current.

3. Three-Phase Fault

$$
\begin{aligned}
V_{exA} &= I_{Asec}(Z_{\mathrm{CT}} + Z_{\mathrm{lead}} + Z_{\mathrm{phase}}) \\
&= 66.7\ \mathrm{A}(0.3\ \Omega + 0.4\ \Omega + 1.2\ \Omega) \\
&= 126.7\ \mathrm{V}
\end{aligned}
$$

❖ The worst-case fault for this example is therefore the single-phase fault. It is clear that a CT with a saturation voltage of 200 V would experience substantial saturation for this fault. This saturation would cause a large reduction in the current delivered. In the other two cases, the CT remains unsaturated, so the CT will deliver the expected current at this voltage level.

Safety when working with CT’s

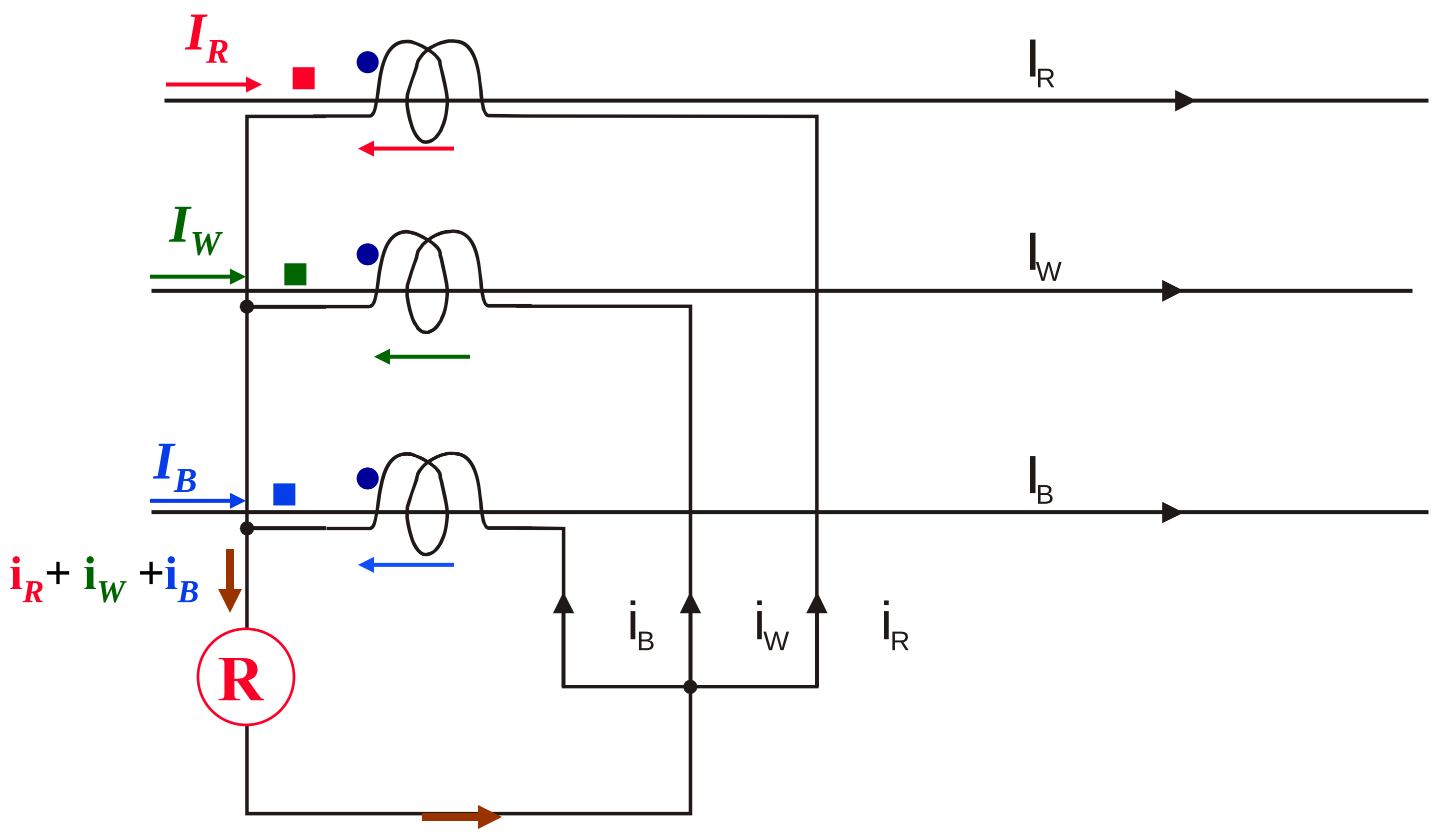
CAUTION:

When C.T.'s are in service they MUST have a continuous circuit connected across the secondary terminals. If the C.T. secondary is `open circuit' Whilst primary current is flowing, dangerously high voltages will appear across the C.T. secondary terminals.

Extreme care must be exercised when performing ‘on load' tests on C.T. circuits, to ensure that a C.T. is not inadvertently ‘open circuited'.

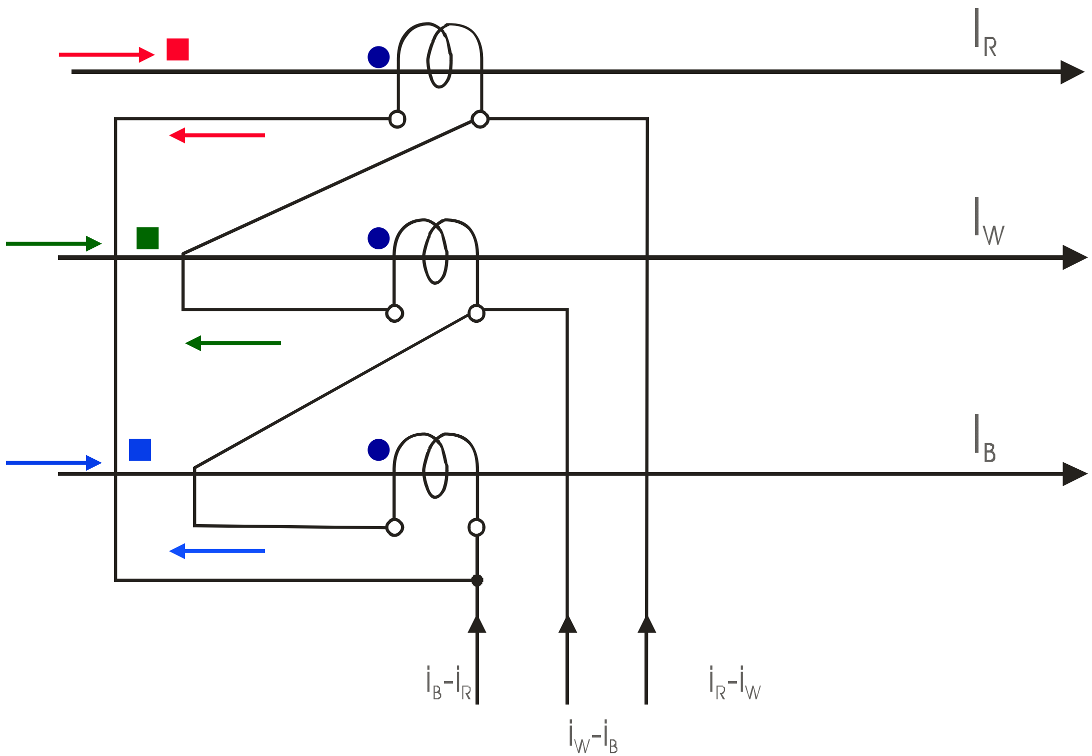
Connection of CT's in 3 phase systems

Most common connection is star (below) - residual current will spill through neutral and through relay **R** during faults



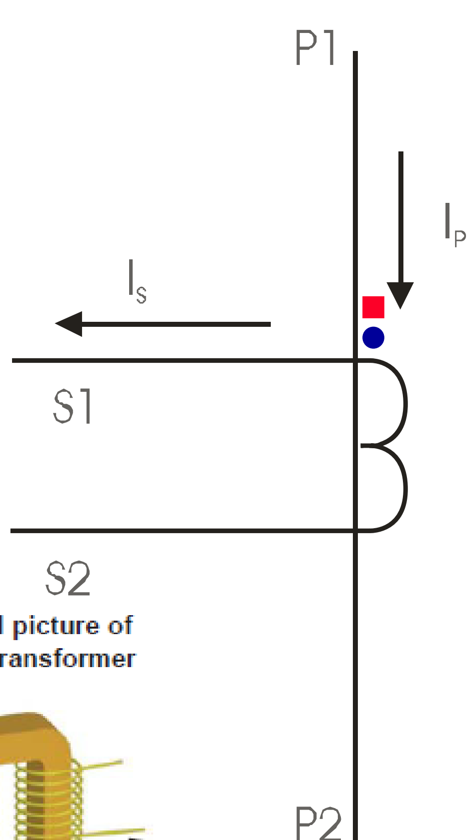
Connection of CT's in 3 phase systems

Delta connections are used when a phase shift with respect to the CT's on the other side of a ΔY transformer is required

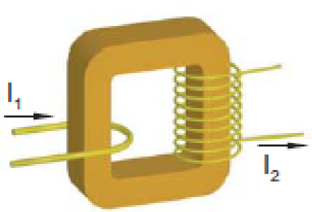


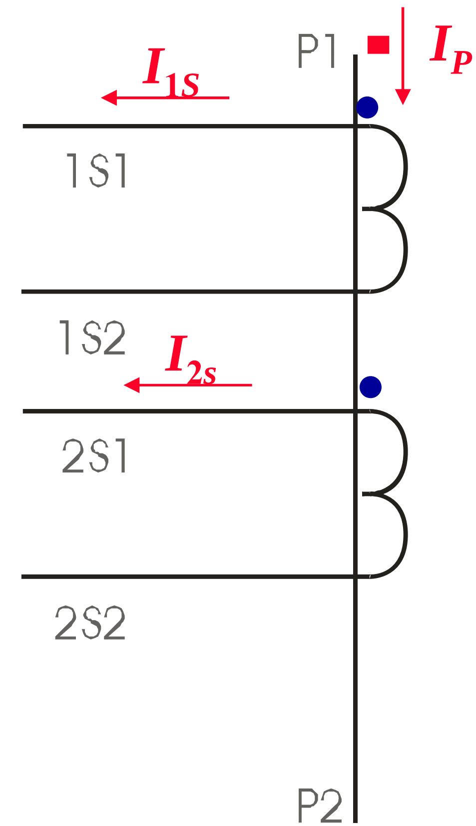
Terminal designations for CT’s

IEC185 - terminals to be marked as follows: P1, S1 to all have same polarity e.g. See below

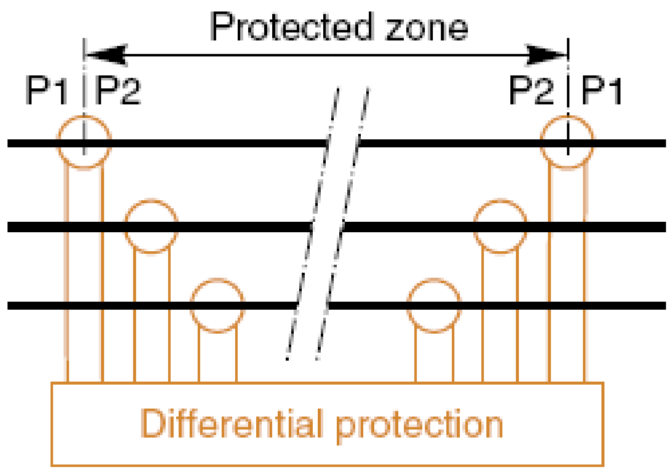


Conceptual picture of
a Current Transformer

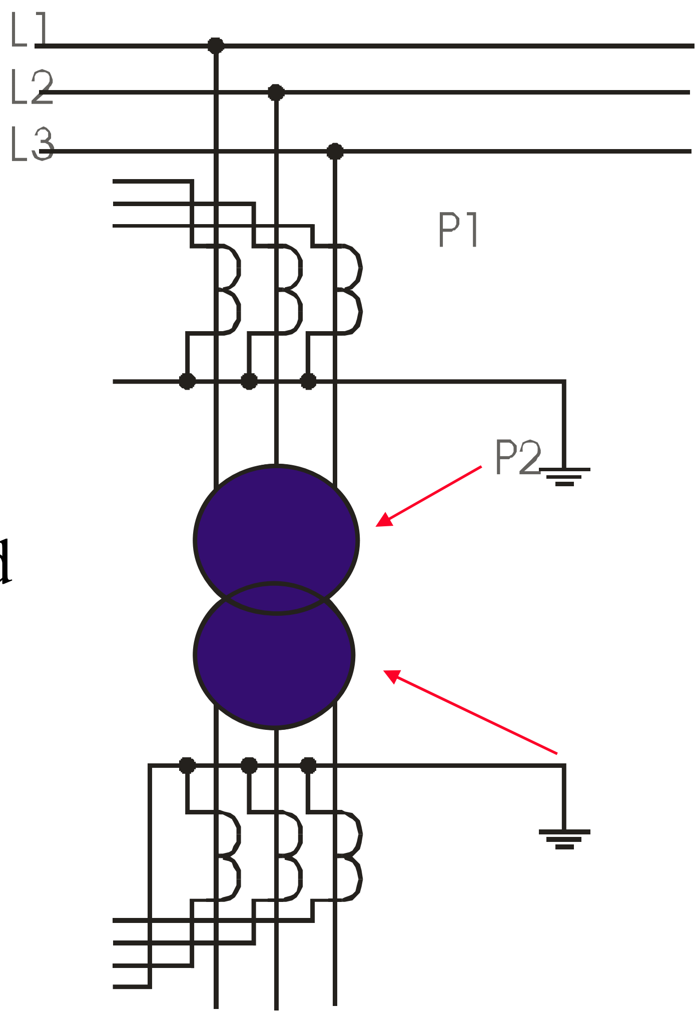




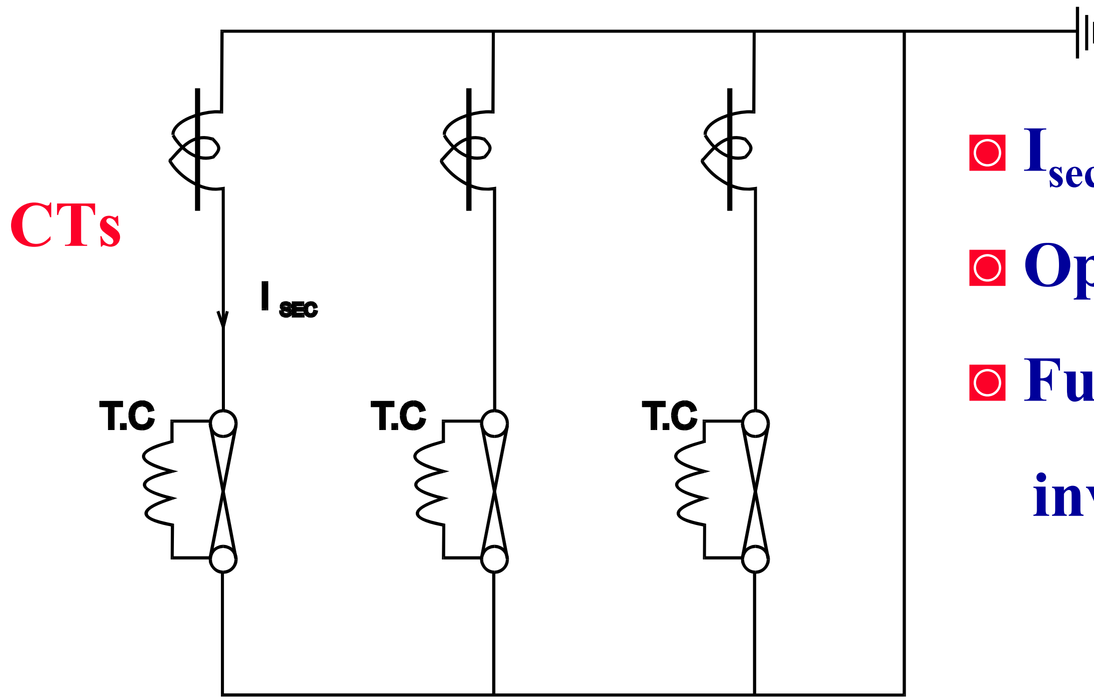
Earthing of CT's



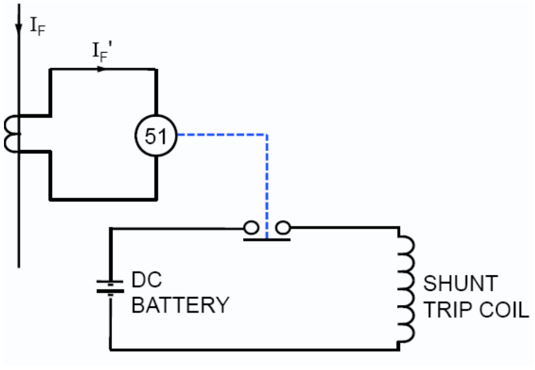
Star and earth closest to protected equipment



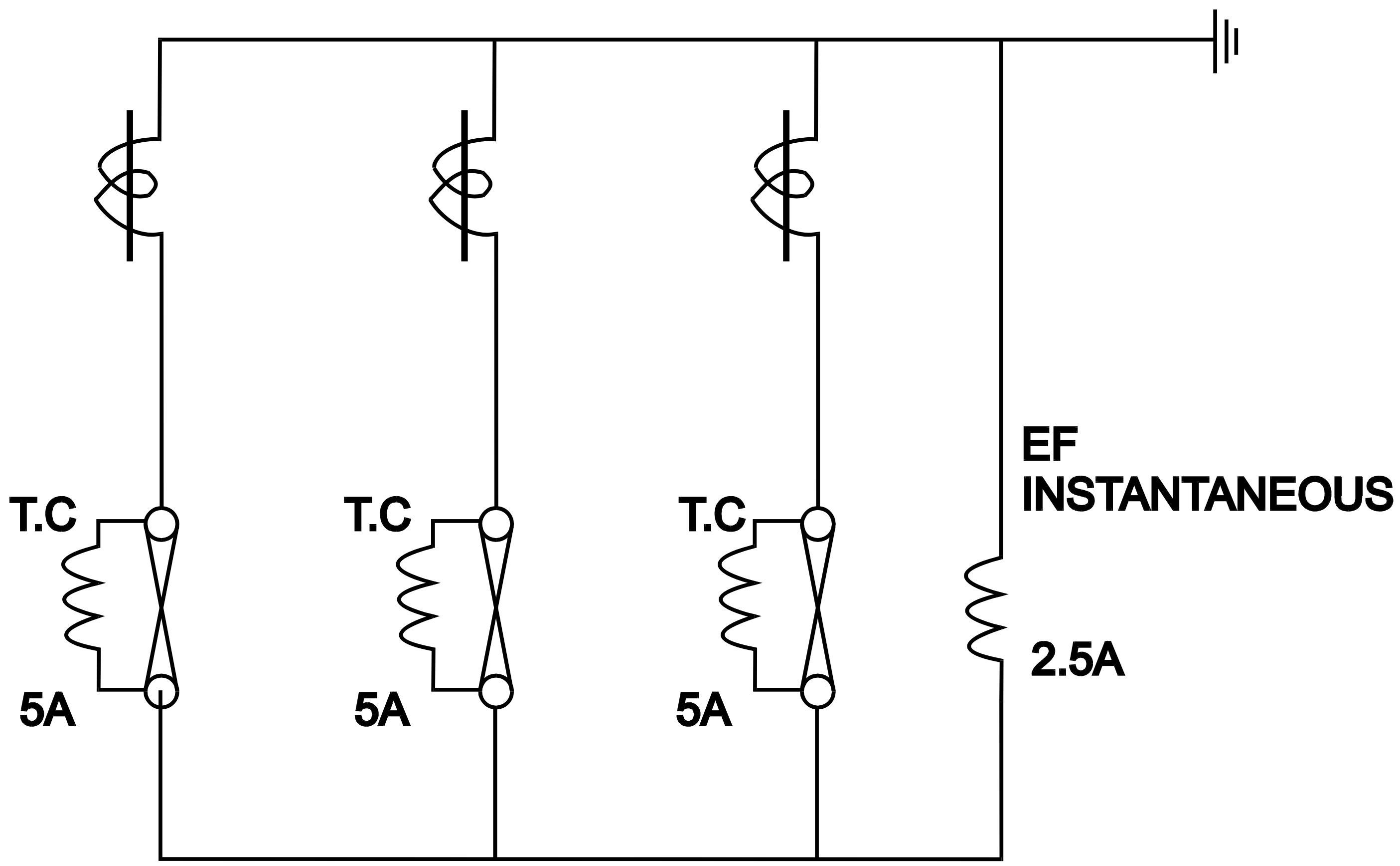
Application of CT's - Overcurrent



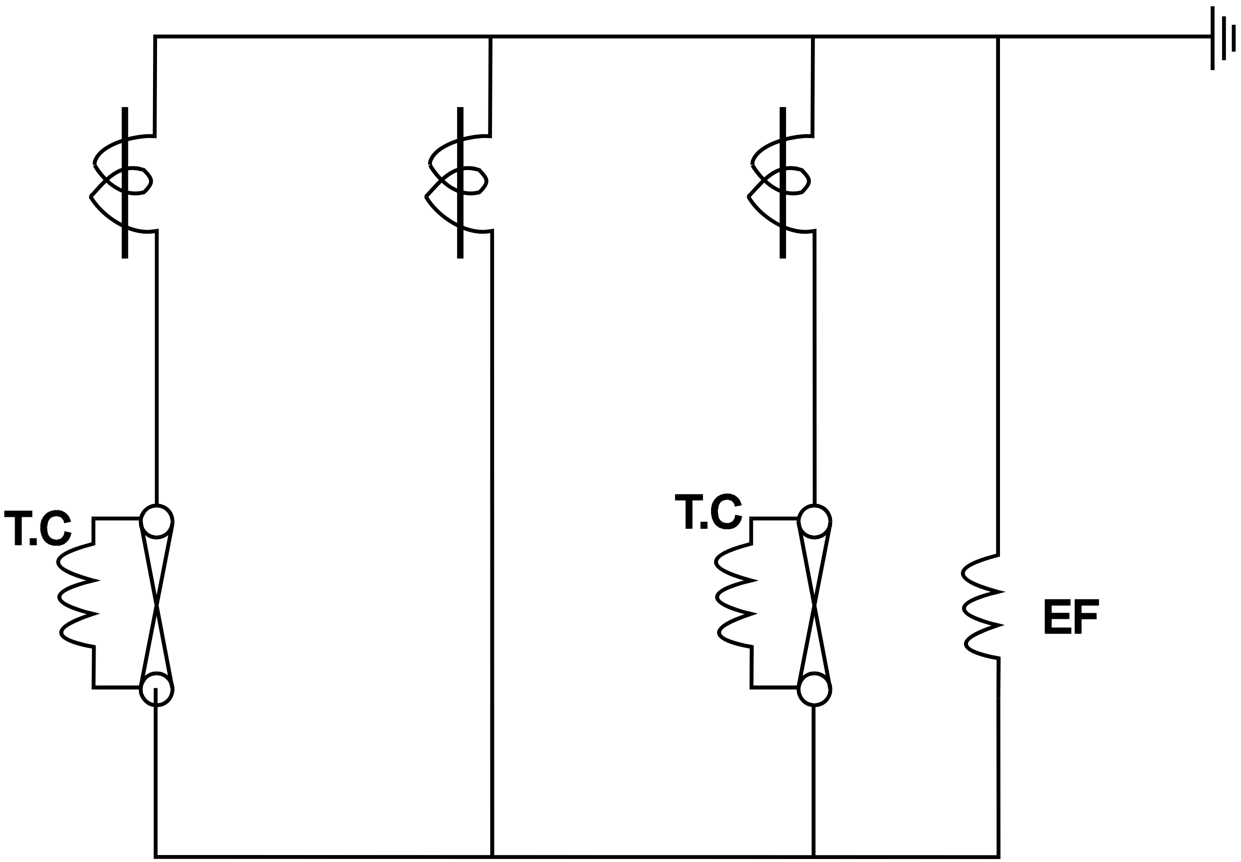
- I_{sec} blows fuse
- Operates trip coil (T.C.)
- Fuse characteristic is inverse



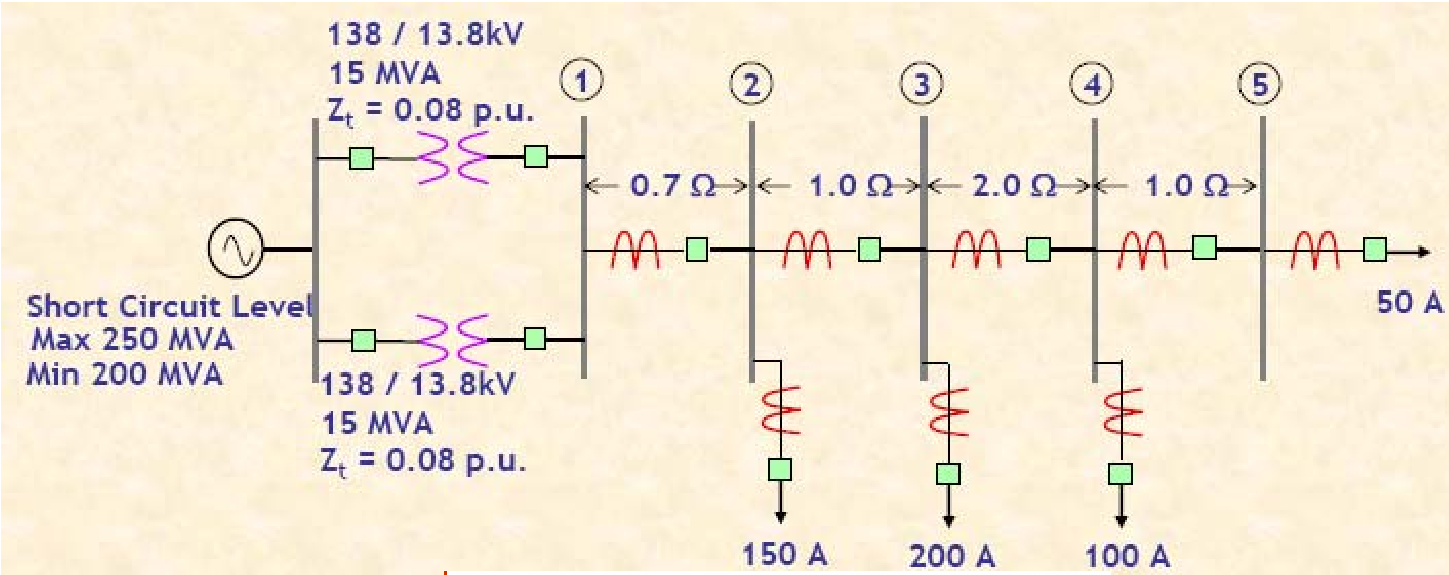
Overcurrent and Earth Fault



A more economical Arrangement



Example 7: Selection of CT Ratio



$I_{CT}=1.25\times I_L$

Relay Location Bus	Maximum Load Current (A)	CT Ratio Selected
1	500	800/5
2	350	500/5
3	150	200/5
4	50	100/5
5	50	100/5

Question # 1:

The conductor of one phase of a three-phase transmission line operating at 345 kV, 600 MVA, has a CT and a VT connected to it. The VT is connected between the line and ground as shown in Fig. 1. The CT ratio is 1200:5 and the VT ratio is 3000:1. Determine the CT secondary current and VT secondary voltage.

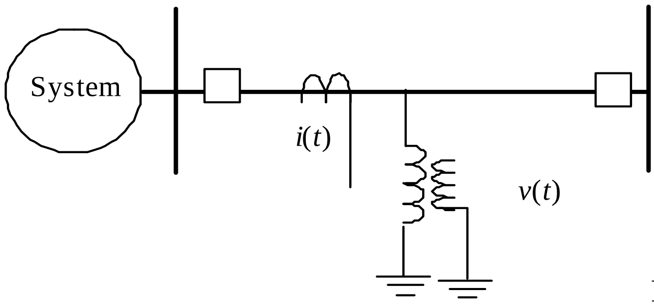


Fig. 1

The VT secondary voltage is:

$$V' = \frac{345/\sqrt{3}\times 10^3}{3000} = 66.4\,V$$

The current flowing through the line is:

$$I = \frac{600\times 10^6}{\sqrt{3}\times 345\times 10^3} = 1004.2.4\ A$$

Therefore the CT secondary current is:

$$I = \frac{1004.2.4\times 5}{1200} = 4.2\ A$$

Question # 2:

Consider the single-phase CVT shown in Fig. 2. The open circuit voltage requirement of the CVT is 100 V, while the line voltage connected across terminal A is 100 kV. Find the values of C1 and C2 such that there is no phase displacement between the line voltage and the output of the CVT. The leakage inductance (L) of the transformer is 1 mH and the supply frequency is 50 Hz.

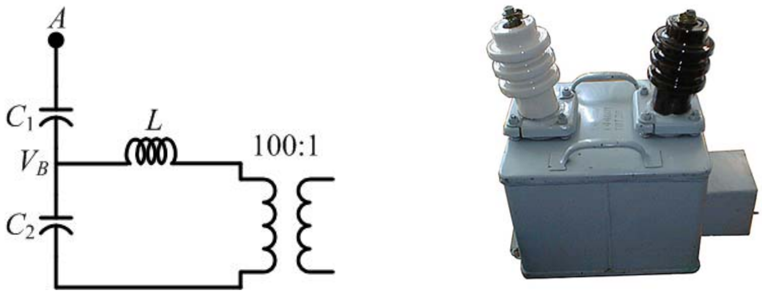


Fig. 2

Consider the circuit of Fig. 2. The open-circuit voltage across C_2 is given by

$$V_B = \frac{V_A(1/j\omega C_2)}{1/j\omega C_1 + 1/j\omega C_2} = V_A\frac{C_1}{C_1 + C_2}$$

Now we want 100 V at the output of the VT, which has a turns ratio of 100:1. Therefore,

$$100\times 100 = 100\times 10^3\frac{C_1}{C_1 + C_2} \Rightarrow C_1 + C_2 = 10C_1 \quad \Rightarrow C_2 = 9C_1$$

Again from the phase shift requirement, we have

$$L = \frac{1}{\omega^2(C_1 + C_2)} \Rightarrow C_1 + C_2 = \frac{1}{\omega^2 L} \Rightarrow 10C_1 = \frac{10^3}{(2\pi \times 50)^2} \Rightarrow C_1 = 1013.2\ \mu F$$

$$C_2 = 9C_1 = 9 \times 1013{,}2\,\mu F \Rightarrow C_2 = 9118.9\ \mu F$$

<u>Question # 3:</u>

The circuit has an A phase to ground fault on the line, with fault current magnitude of 16 kA at 0°. The circuit of Fig.1 has 1000:5 class C100 CTs. Given the following:

CT Winding Resistance R_C = 0.342 Ω

Burden resistance for phase relay R_{ph} = 0.50 Ω

Burden resistance for E/F relay R_E = 0.59 Ω

Leed Resistance (One leed) R_L = 0.224 Ω

Calculate,

a. the current seen by the secondary of the CT, I_{as}.
b. the total connected resistance seen by the phase CT, R_T.
c. the CT Secondary Voltage for phase to ground fault, V_s.
d. Does the CT get Saturated at the above LG fault current?

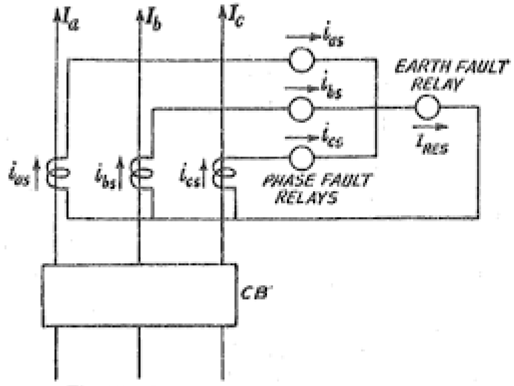


Fig. 1

Solution

a. Secondary Fault Current

   $I_{as} = I_f/CTR$ =16000/(1000/5) =16000/200 = 80 A.

b. Total connected resistance seen by the phase CT,

   $R_T = R_C + R_{ph} + R_E + R_L$

   R_T = 0.342 + 0.50 + 0.59 + 0.448 = 1.88 Ω.

c. CT secondary voltage for ground fault:

   $V_s = I_{as} R_T$

   V_s = 80×1.88=150.4V~150V

A CT with a saturation voltage of 100 V would experience substantial saturation for this fault. This saturation would cause a large reduction in the current delivered

Question #4:

A distribution feeder has 600 /5 C 100 CT with a knee point 100 Volt. A three phase fault of I_f = 10200 A occurs at ***F*** as shown in Fig. Q3.

a. Calculate the voltage developed across CT if the phase relay burden resistance Z_R =0.10Ω, the lead resistance R_L = 0.50Ω, and the CT resistance R_S = 0.40Ω.
b. Will this fault current lead to CT saturation?
c. If not, at what fault level, the CT will saturate?

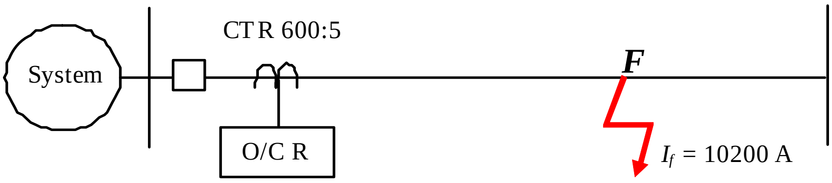


Fig. Q4

Solution:

Effective impedance seen by the CT

$$= R_S + R_L + Z_R$$
$$= (0.40 + (0.5) + 0.10)$$
$$= 1.0$$
$$V_S = I_S \times 1$$
$$= (10200 / 120) \times 1.0 = 85V$$

Not Saturated.

Since, the knee point is 100 V the CT will saturate at 100 V corresponding to I_f = 12000 A.

EE482-Power System Analysis II
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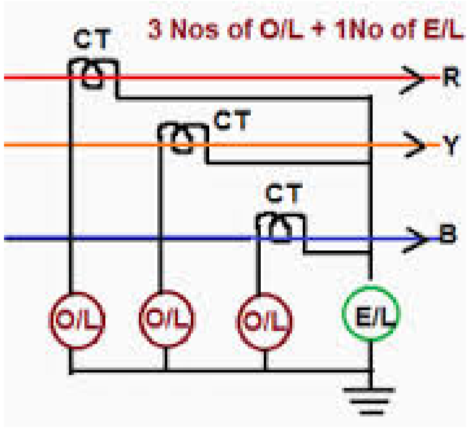
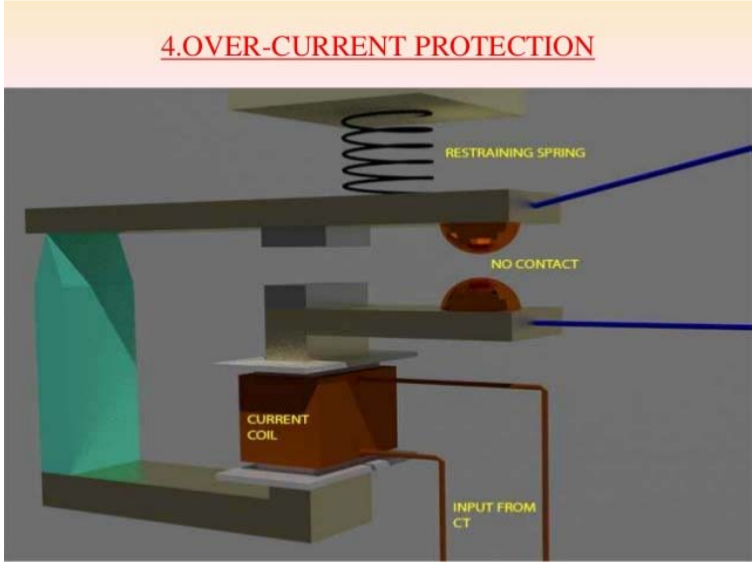
Electrical Engineering Department
School of Engineering
University of Jordan

Topic 1-4: Prt 1
IDMT Overcurrent Protection



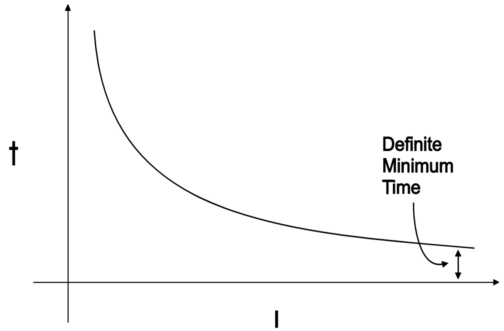


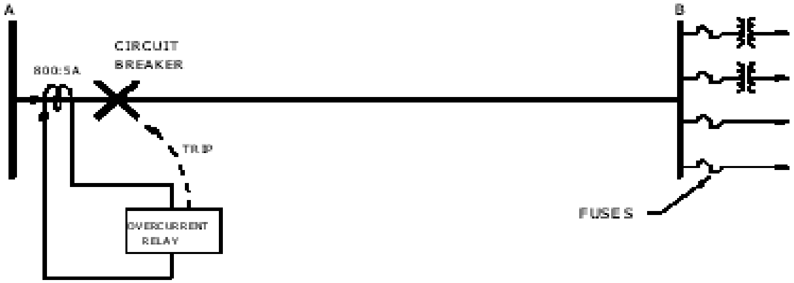
Over-Current Protection

- OC protection is that protection in which the relay picks up when the magnitude of current exceeds the pickup level.
- The basic element in OC protection is an OC relay.

- The OC relays are connected to the system, normally by means of **CT's**
- HRC fuses, drop out fuses, etc. are used in low voltage medium voltage and high voltage distribution systems, generally up to 11 kV.

- Thermal relays are used widely for over-current protection





Primary Requirements of OC Protection

- OC protection includes the protection from overloads which is generally provided by thermal relays.
- OC protection includes short-circuit protection. SC currents are generally several times (5 to 20) full load current. Hence fast fault clearance is always desirable on short-circuits

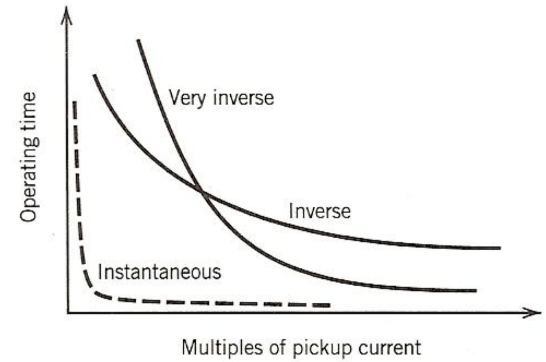
- OC protection should not operate for starting currents, permissible over-current, and current surges. To achieve this, the time delay is provided (in case of inverse relays). If time delay cannot be permitted, high-set instantaneous relaying is used.
- The protection should be coordinated with neighboring over-current protections so as to discriminate.

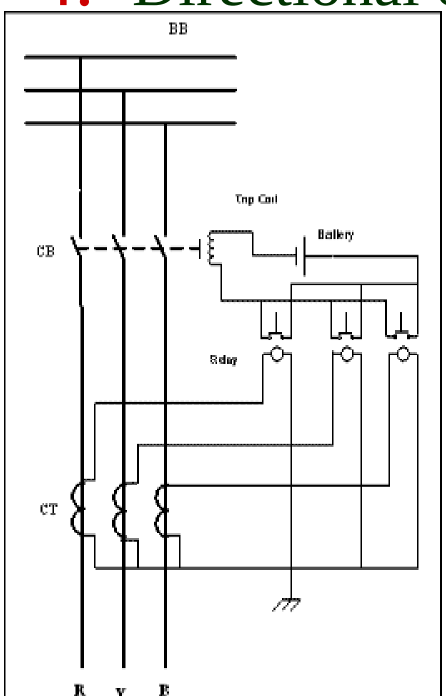
Applications of OC Protection

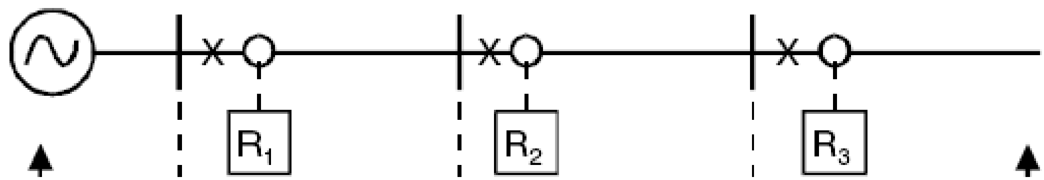
Line Protection

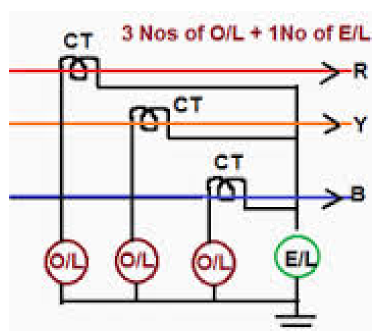
The lines (feeders) can be protected by

1. Instantaneous over-current relays.
2. Definite time Over-current relays
3. Inverse time over-current relays.
4. Directional over-current relay.



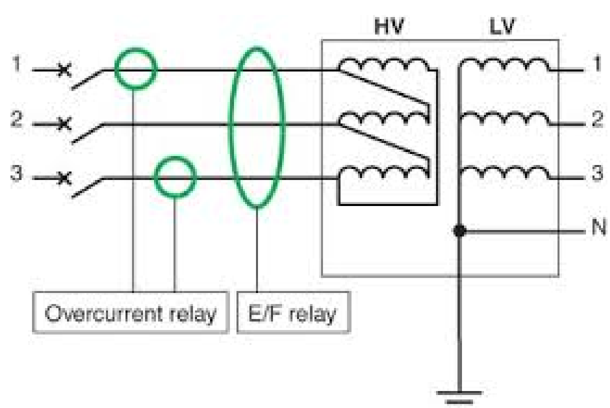


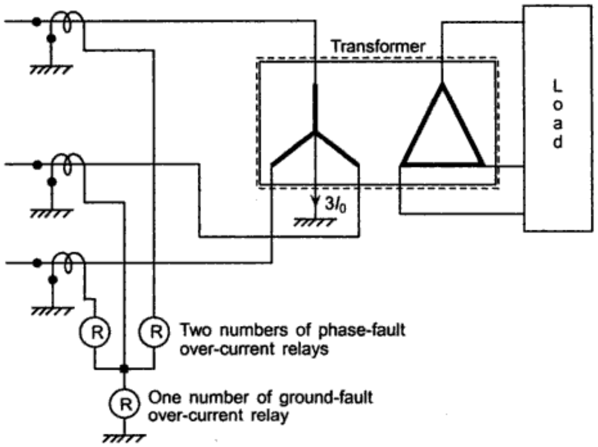




Applications of OC Protection

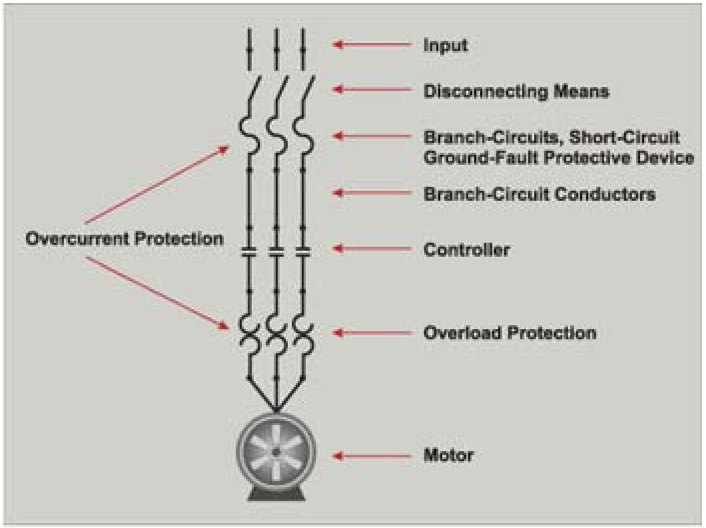
- **Transformer Protection**
  - Transformers are provided with OC protection against faults, only, when the cost of differential relaying cannot be justified.
  - OC relays are provided in addition to differential relays to take care of through faults. Temperature indicators and alarms are always provided for large transformers.
  - Small transformers below 500 kVA installed in distribution system are generally protected by fuses, as the cost of relays plus circuit-breakers is not generally justified.

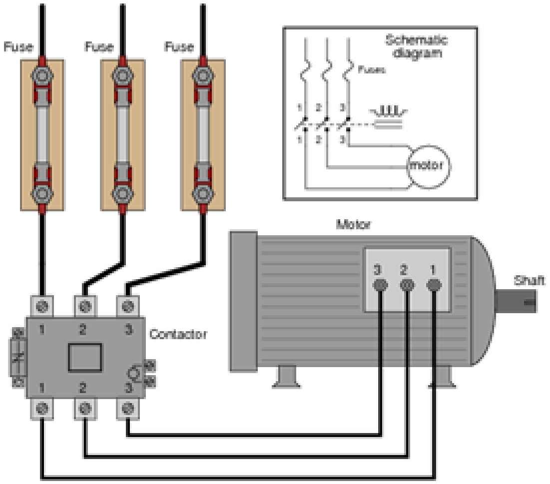




Applications of OC Protection

- **Motor Protection**
  - OC protection is the basic type of protection used against overloads and short-circuits in stator windings of motors. Inverse time and instantaneous phase and ground OC relays can be employed for motors above 1200 H.P.
  - For small/medium size motors where cost of CT's and protective relays is not economically justified, thermal relays and HRC fuses are employed, thermal relays used for overload protection and HRC fuses for short-circuit protection.

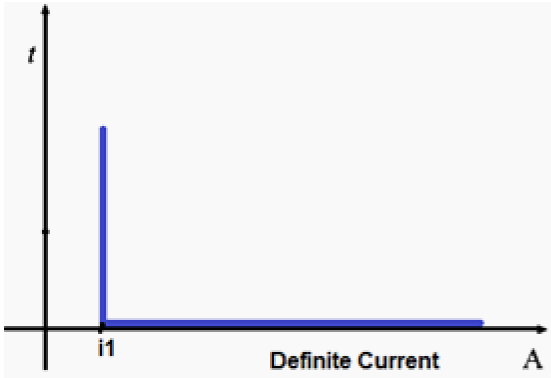




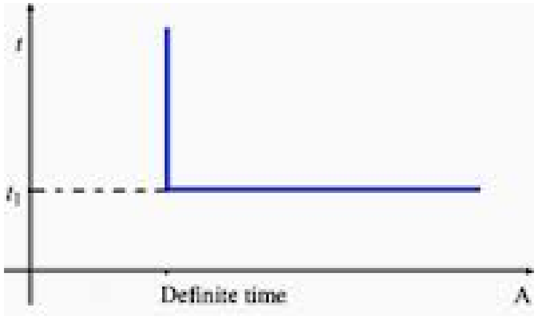
Types of Overcurrent Relays

a. Instantaneous Overcurrent Relays.

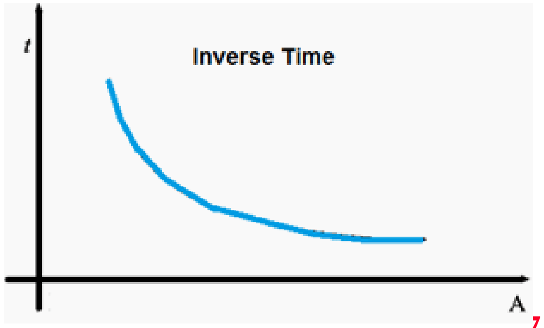
These relays operate, or *pick-up* at a specific value of current,
with no intentional time delay.



The *pick-up* setting is usually adjustable by means of a dial, or by plug settings.



b. Timed Overcurrent Relays.

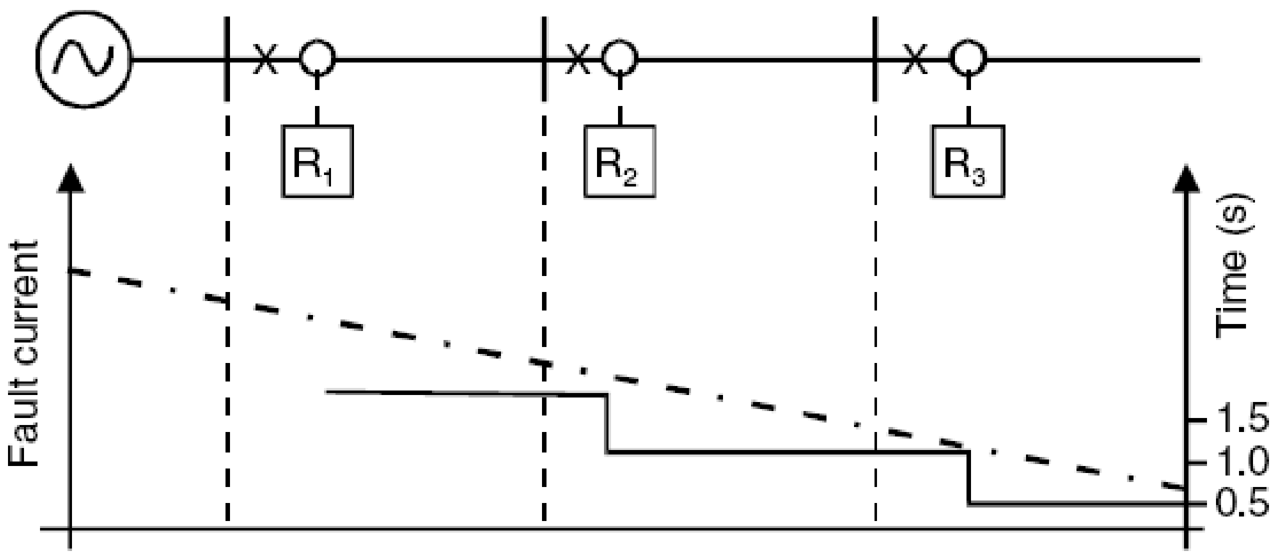


Two types:

1. Definite Time Lag
2. IDMT Relay
   (Inverse Definite Minimum Time)

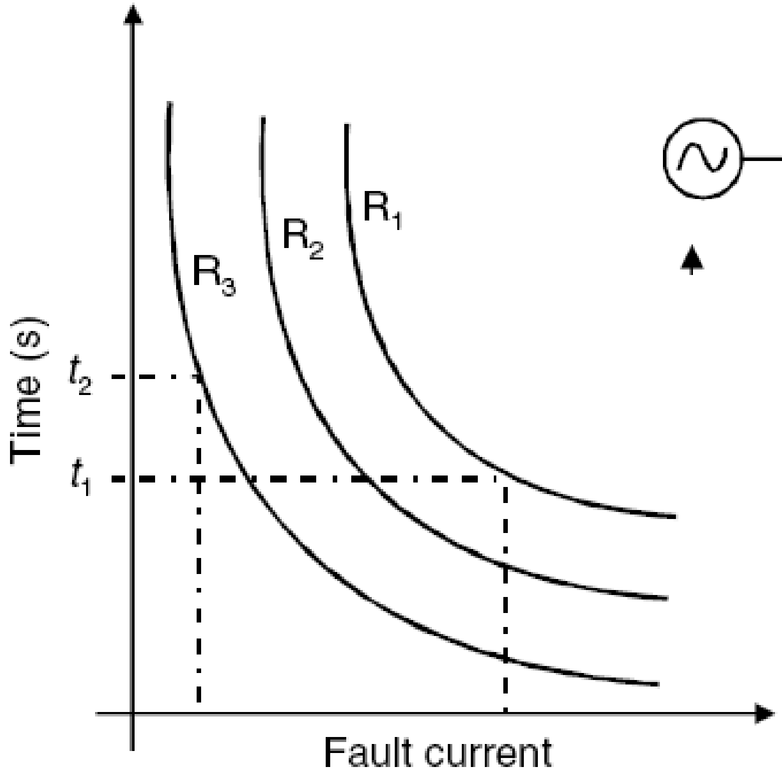
Definite Time Lag - O/C Relay

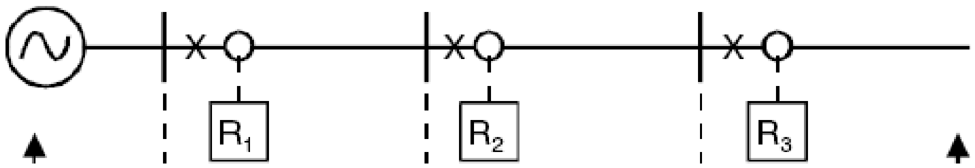
- For the first option, the relays are graded using a definite time interval of approximately 0.5 s. The relay R3 at the extremity of the network is set to operate in the fastest possible time, whilst its upstream relay R2 is set 0.5 s higher. Relay operating times increase sequentially at 0.5 s intervals on each section moving back towards the source as shown
- The problem with this philosophy is, the closer the fault to the source the higher the fault current, the slower the clearing time – exactly the opposite to what we should be trying to achieve.



IDMT - O/C Relay

- On the other hand, inverse curves as shown operate faster at higher fault currents and slower at the lower fault currents, thereby offering us the features that we desire. This explains why the **IDMT** philosophy has become standard practice throughout many countries over the years.

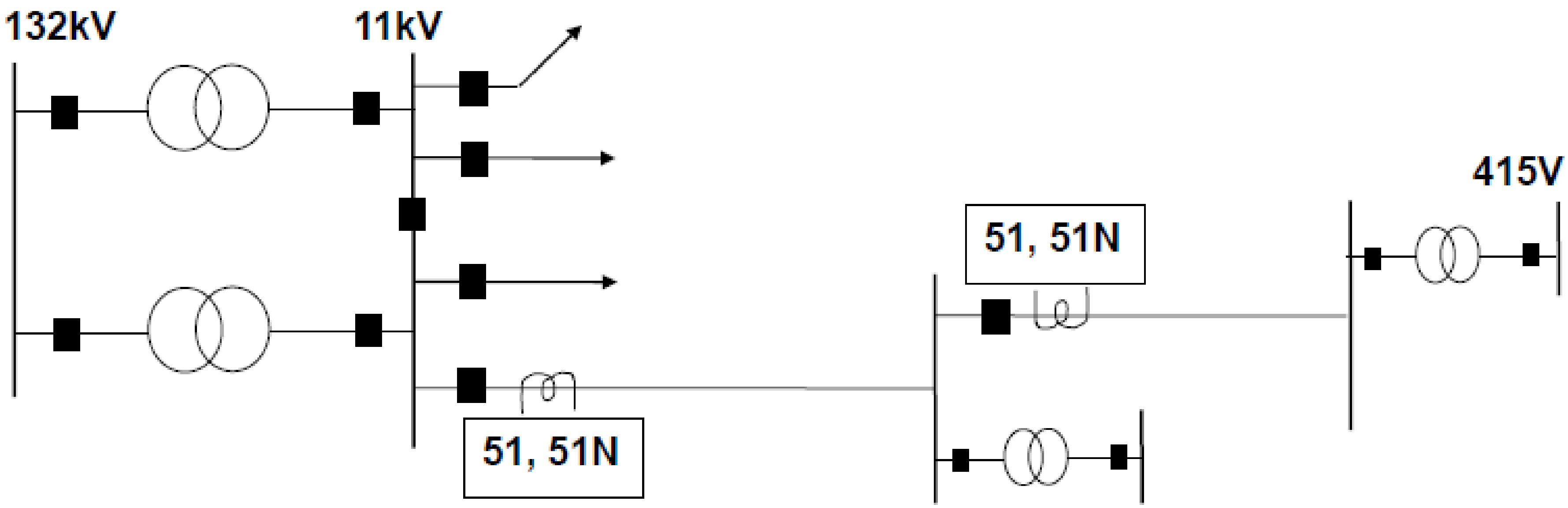




Time $t \propto \dfrac{1}{I^2}$

This gives an inverse characteristic.
(the higher the current - the shorter the rotating time)

IDMT - O/C Relay

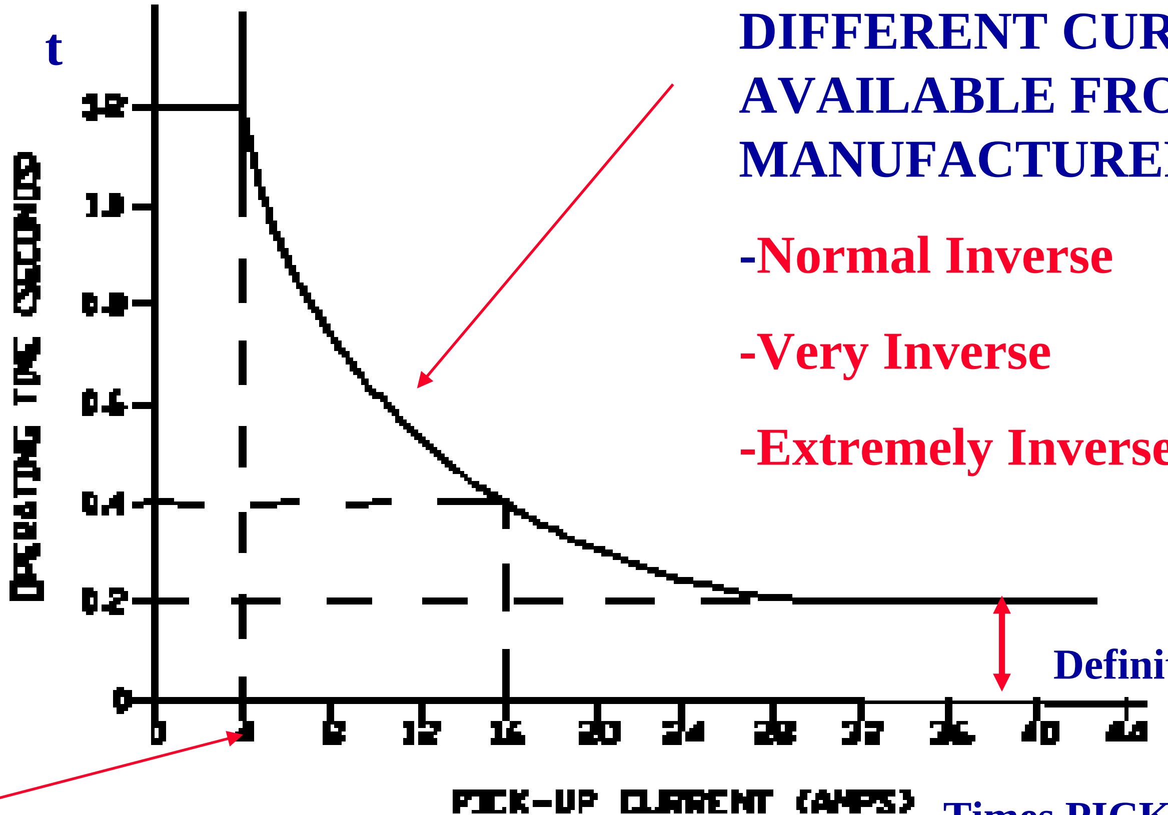


- Power flows from grid supply to LV network.
- Protection based on phase & earth inverse overcurrent relays.

OC IDMT Relay

Inverse Definite Minimum Time C/C

t



DIFFERENT CURVES AVAILABLE FROM MANUFACTURERS

-Normal Inverse

-Very Inverse

-Extremely Inverse

Definite Minimum Time

Times PICK-UP CURRENT

DEFINITE MINIMUM

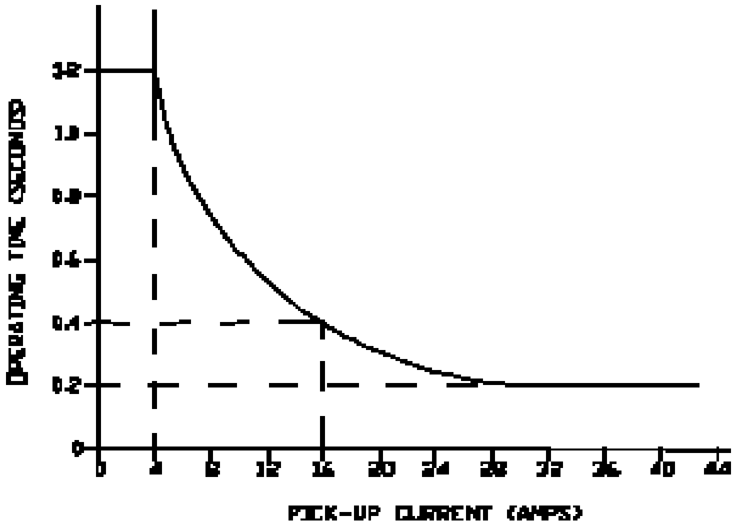
PICK-UP CURRENT

I

IDMT Relay

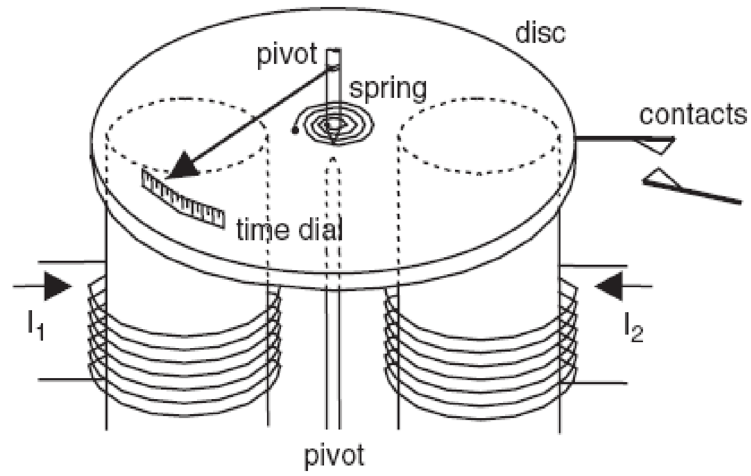
The relay characteristic is such that for very high fault currents the relay will operate in it's ***definite minimum time of 0.2*** seconds. For lower values of fault current the operating time is longer.

For example, at a relay current of 16 Amps, the operating time is 0.4 seconds. The relay has a ***definite minimum pick-up*** current of 4 Amps. This minimum ***pick-up*** current must, of course, be greater than the maximum load on the feeder.

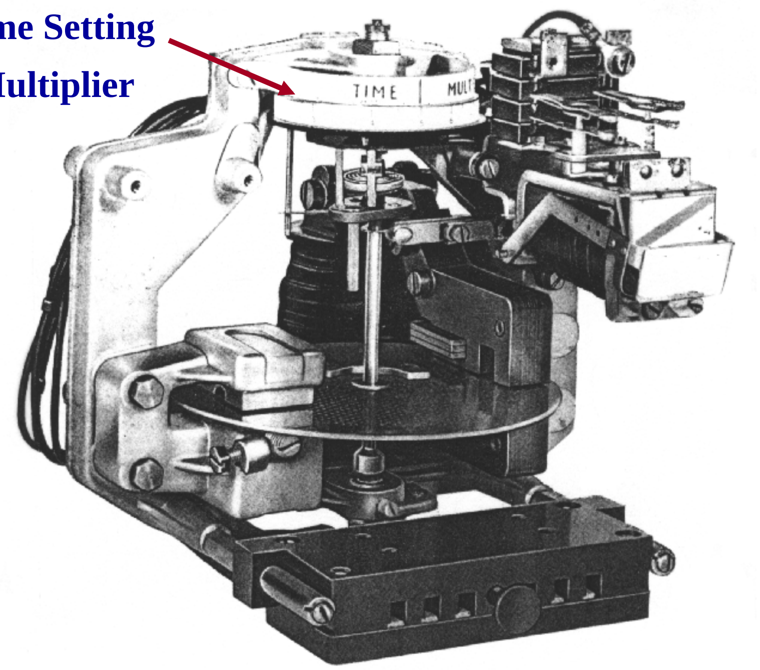


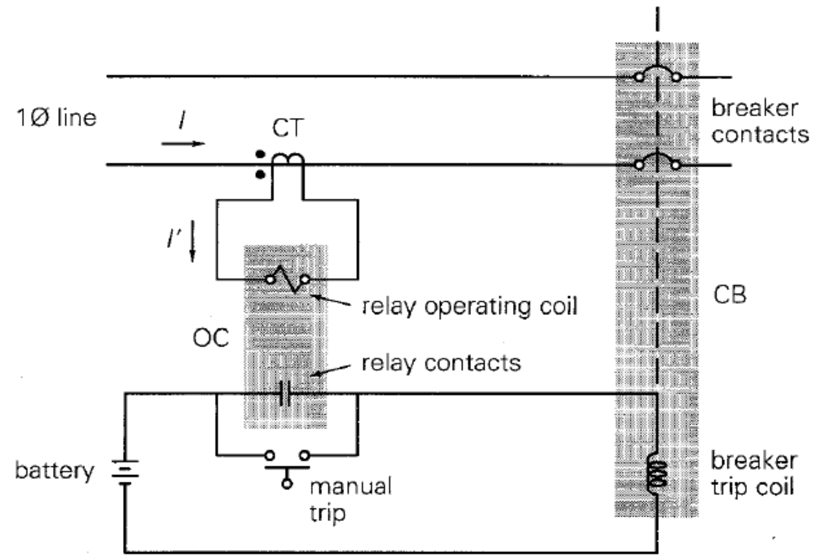
Overcurrent IDMT Relays

➢ The electro-mechanical version of the **IDMT** relay has an induction disc. The disc must rotate through a definite sector before the tripping contacts are closed.

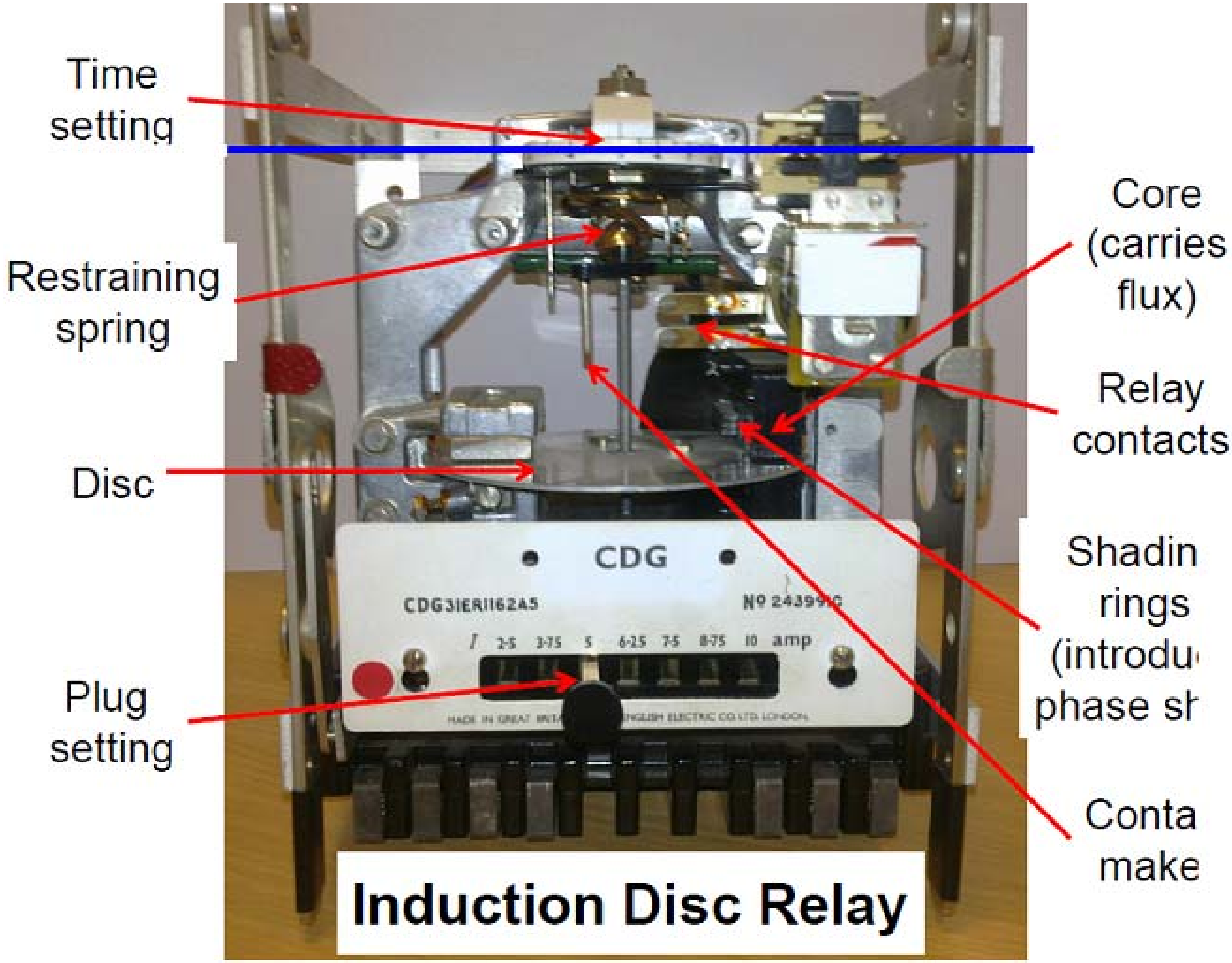


Time Setting Multiplier



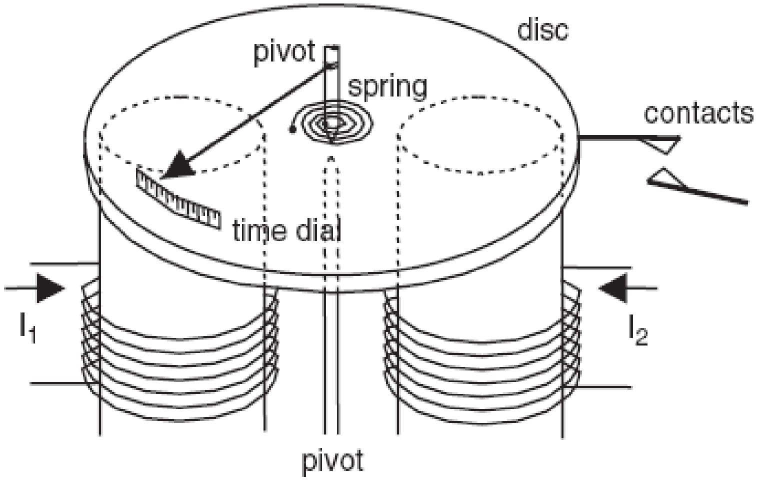


Overcurrent IDMT Relays



Adjustments of OC IDMT Relay

a) The ***time multiplier setting***:
This adjusts the operating time at a given ***multiple of setting current***, by altering by means of the torsion head, the distance that the disc has to travel before contact is made.



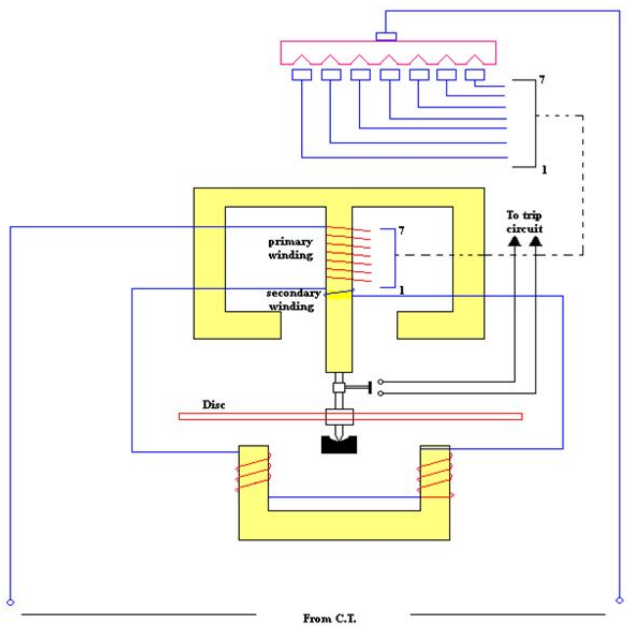
This dial rotates the disc and its accompanying moving contact closer to the fixed contact, thereby reducing the amount of distance to be traveled by the moving contact, hence speeding up the tripping time of the relay.

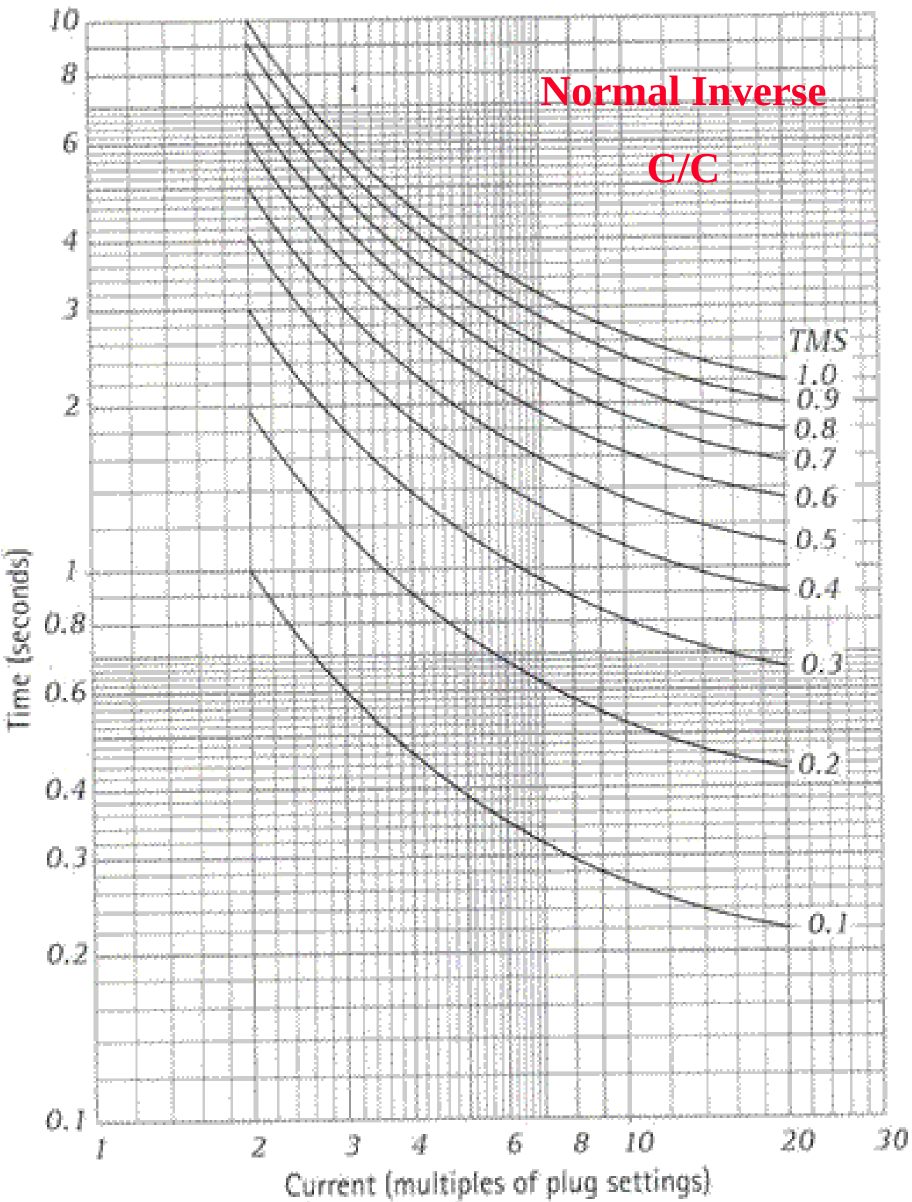
This has the effect of moving the inverse curve down the axis as shown below.

TM setting	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Degrees of travel before contacts operate	0°	18°	36°	54°	72°	90°	108°	126°	144°	162°	180°

OC IDMT Relay

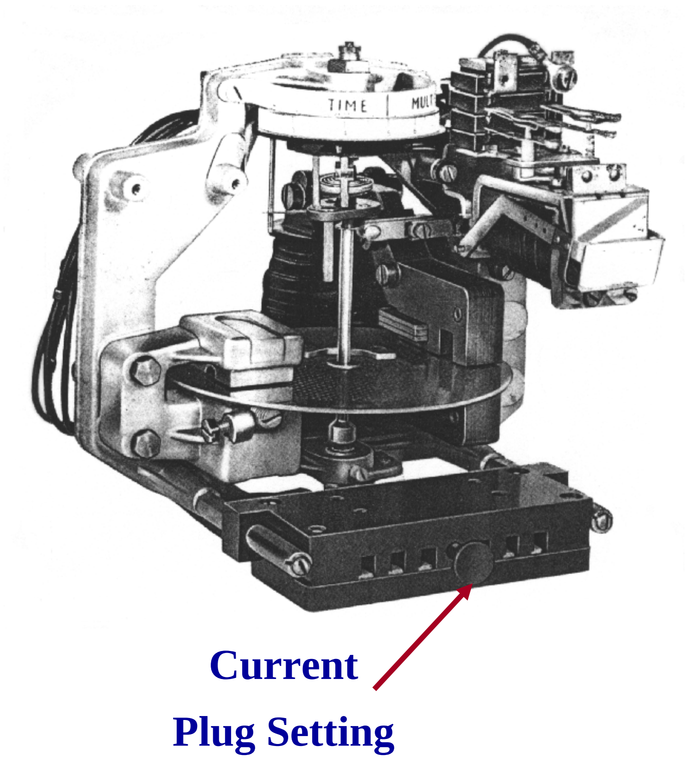
This gives the relay a very wide range of setting characteristics, and allows the relay setting to be coordinated with other protection devices, such as fuses, on adjacent power system elements

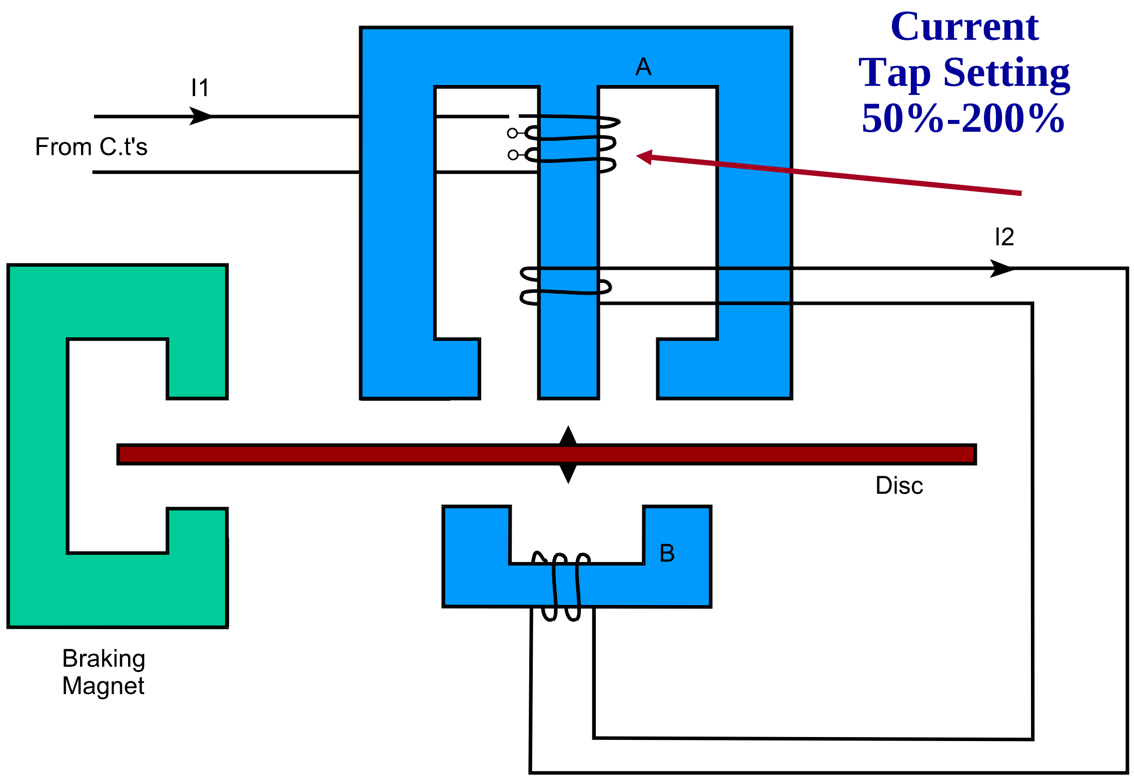




Adjustments of OC IDMT Relay

a) The ***current pick-up*** or ***plug setting***: This adjusts the setting current by means of a plug bridge, which varies the effective turns on the upper electromagnet.

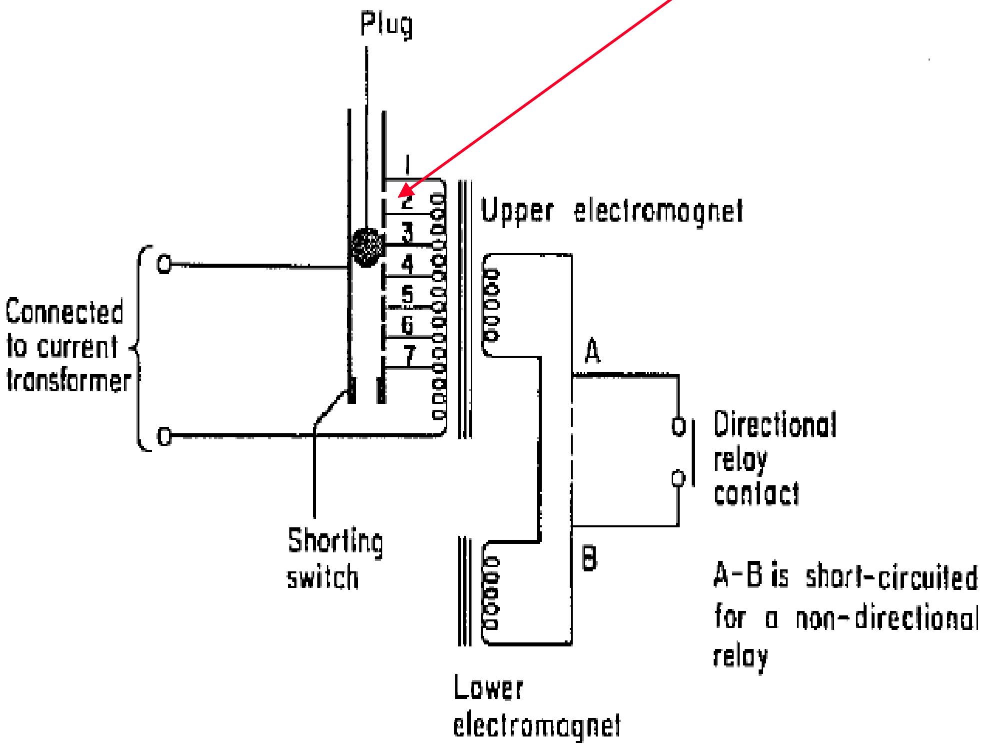


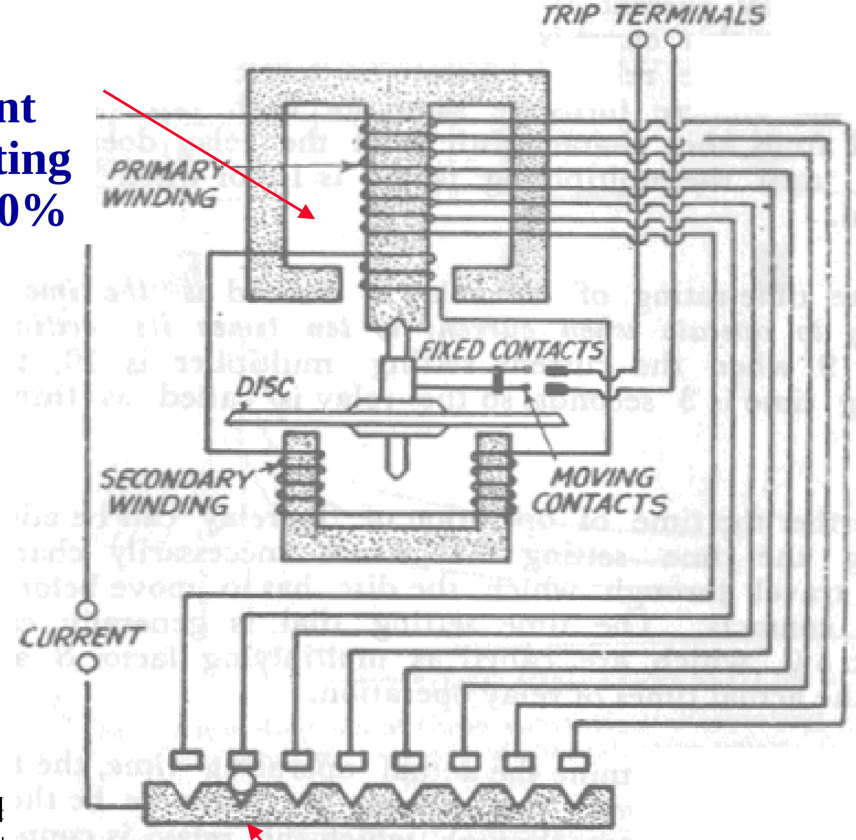


This setting determines the level of current at which the relay will start or pick-up

OC IDMT Relays

Current
Tap Setting
50%-200%

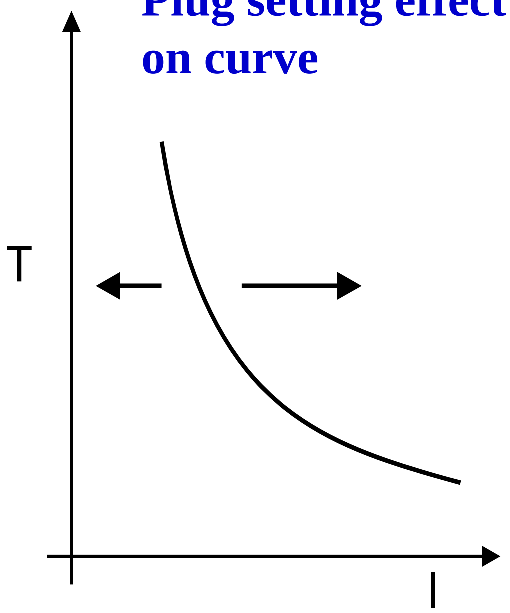




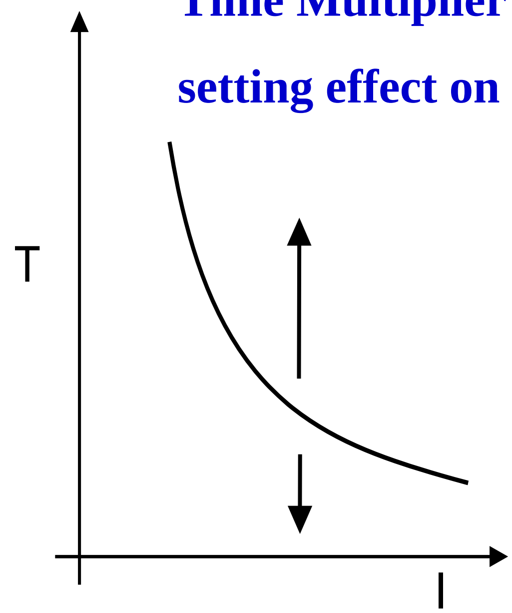
Plug Bridge

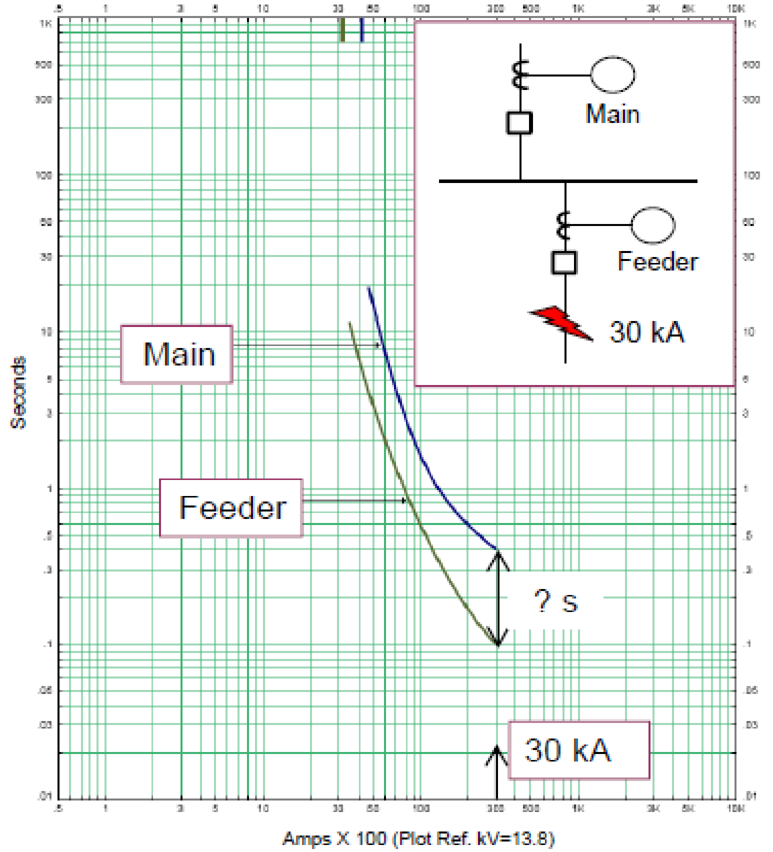
Effect of Settings and Coordination Curves

Plug setting effect on curve



Time Multiplier setting effect on curve





OC IDMT Relay

Percentage plug settings (Reyrolle)

Overcurrent:	50%	75%	100%	125%	150%	175%	200%
Earth fault:	20%	30%	40%	50%	60%	70%	80%
Or	10%	15%	20%	25%	30%	35%	40%

Current plug settings (GEC)–For 5 amp relay

Overcurrent:	1.5A	3.5A	5.0A	6.25A	7.5A	8.75A	10A
Earth fault:	1.0A	1.5A	2.0A	2.5A	3.0A	3.5A	4.0A
Or	0.5A	0.75A	1.0A	1.25A	1.5A	1.75A	2.A

Normally, the highest current tap is automatically selected when the plug is removed, so that adjustments can be made on load without open-circuiting the current transformer.

Magneto-motive-force*: *mmf = N.I

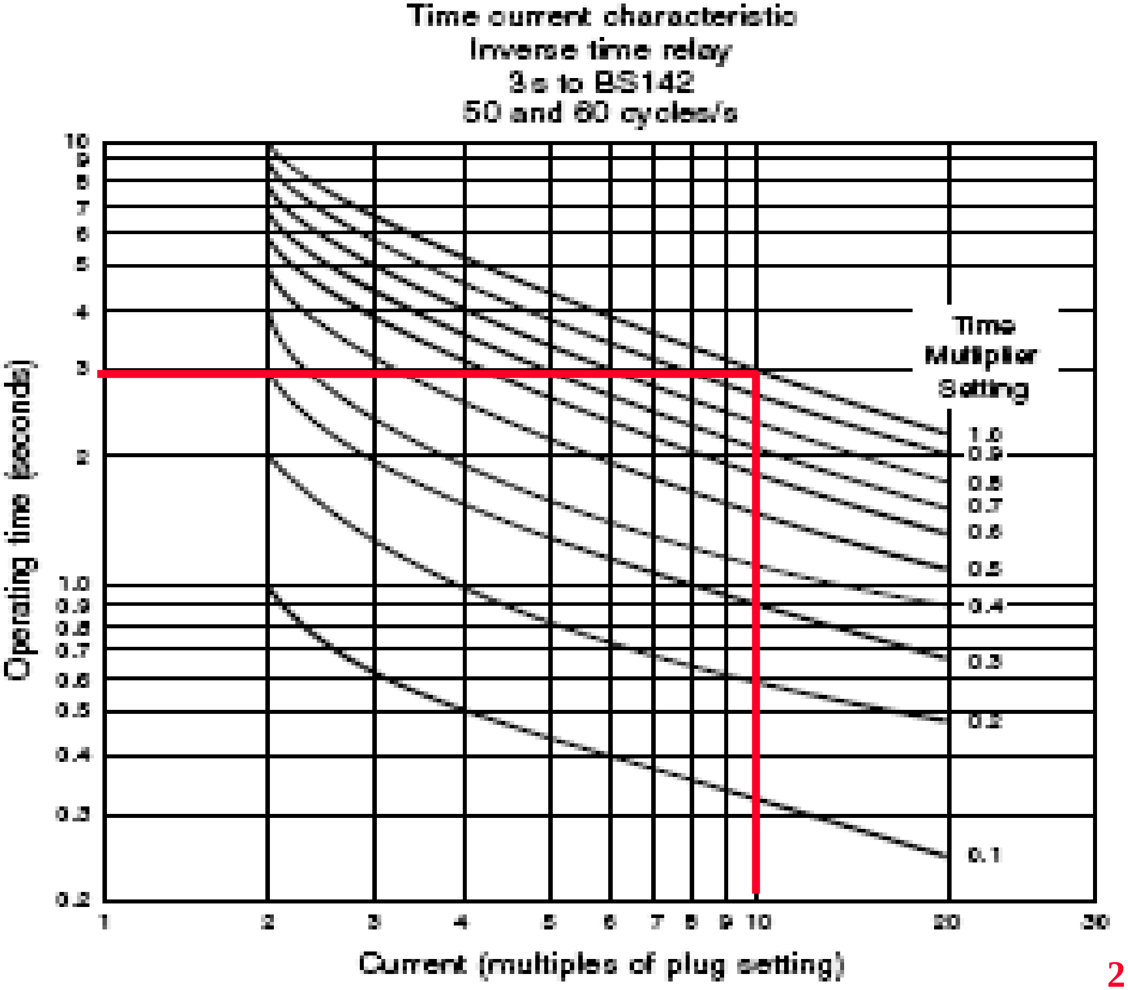
OC IDMT Relay

This curve shows the relay will operate in 3 seconds at 10 times the plug setting (with the time multiplier =1)

$$t = \frac{3.0}{\log PSM} \times TMS$$

$$t = \frac{3.0}{\log PSM}\bigg|_{TMS=1}$$

$$PSM = \frac{I_f}{I_{pickup}}$$



Different Curves

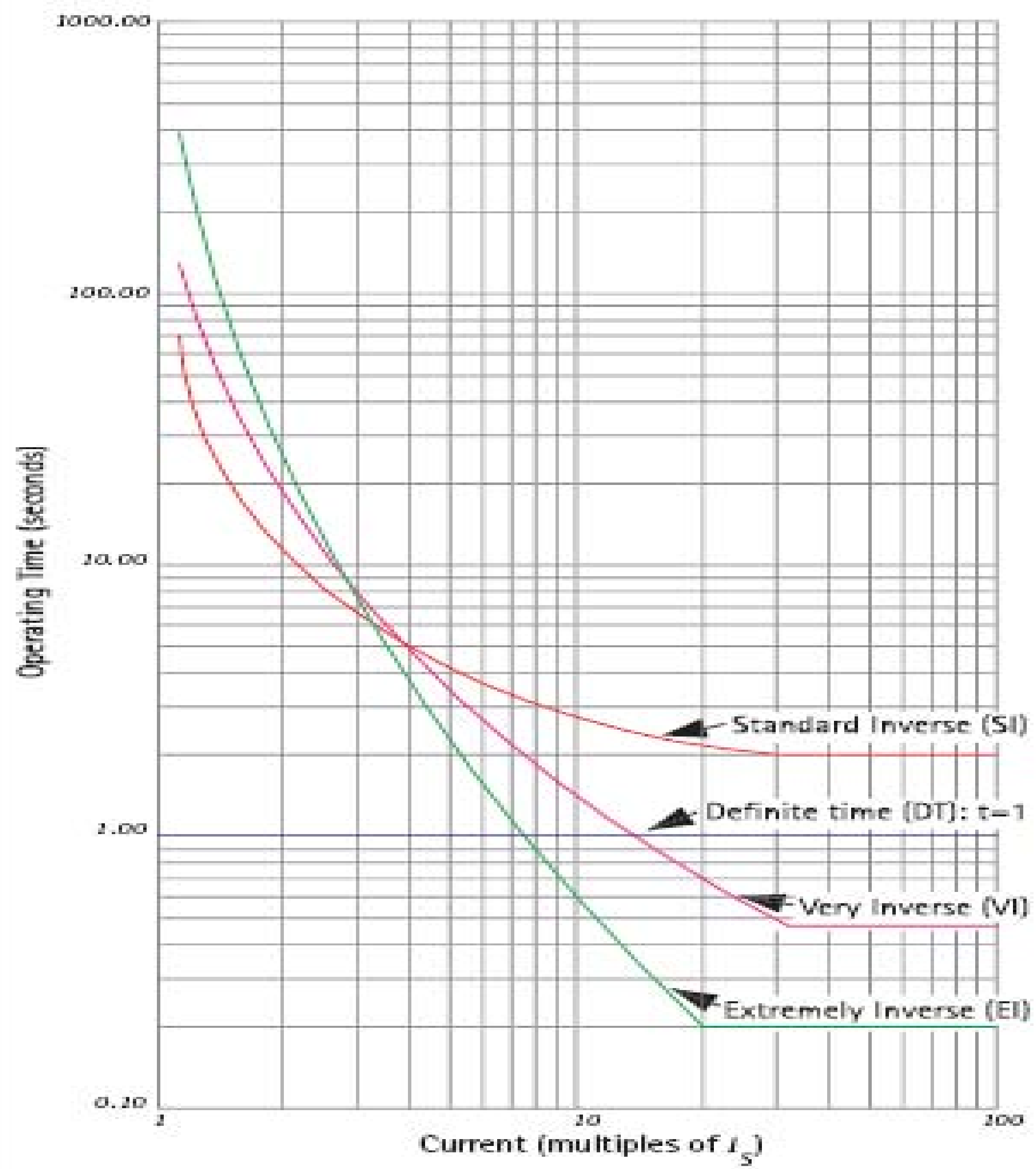
- The most common type used is:

  ➔ **NORMAL INVERSE CURVE**.
- Characteristic shows a **3 second operation at $10 \times$ the current plug setting**

  i.e. if the plug bridge is set at 1 A and when 10 A flows through, the relay will close its contacts after 3 seconds - sometimes called a 3/10 relay

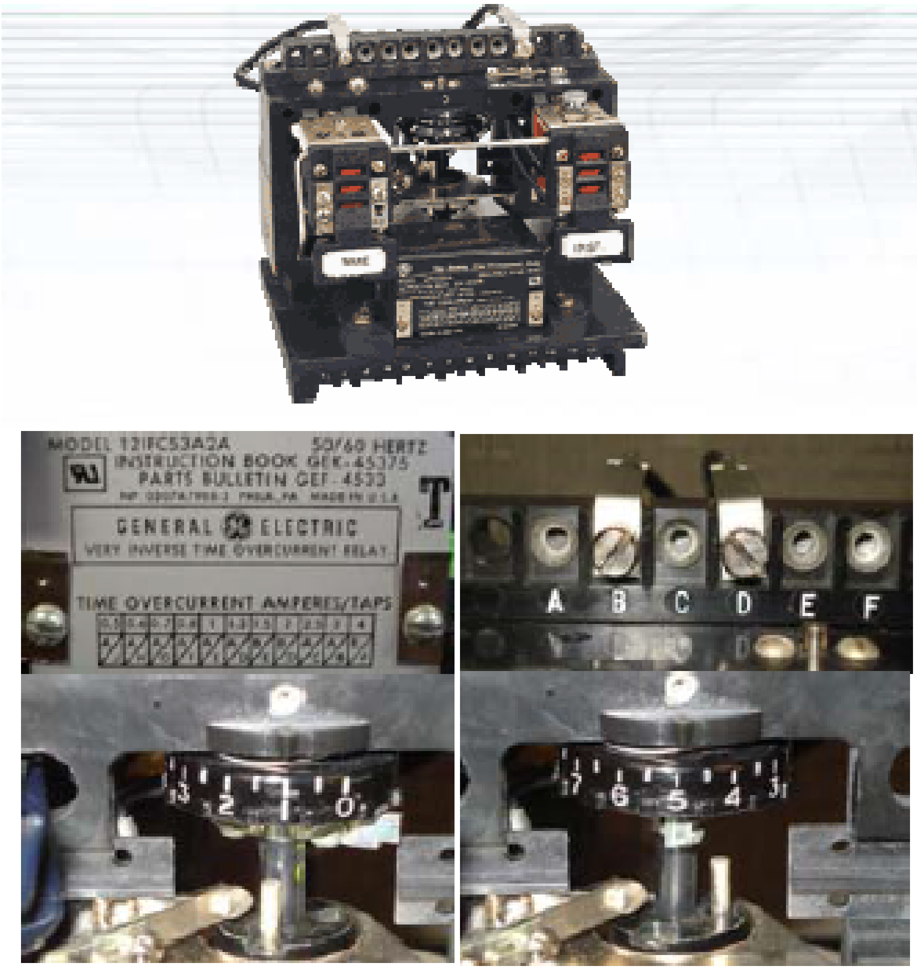
- Other characteristic curves are also available:
  - **Very Inverse**
  - **Extremely Inverse**

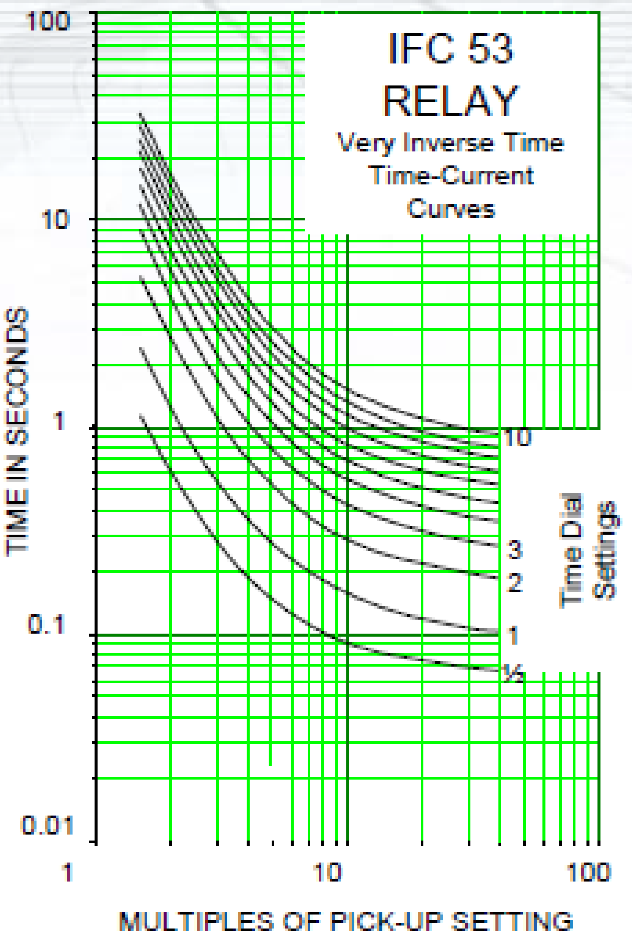
Relay characteristics to IEC 60255



(a) IEC 60255 characteristics ; TMS=1.0

IFC Relay (VI characteristics)



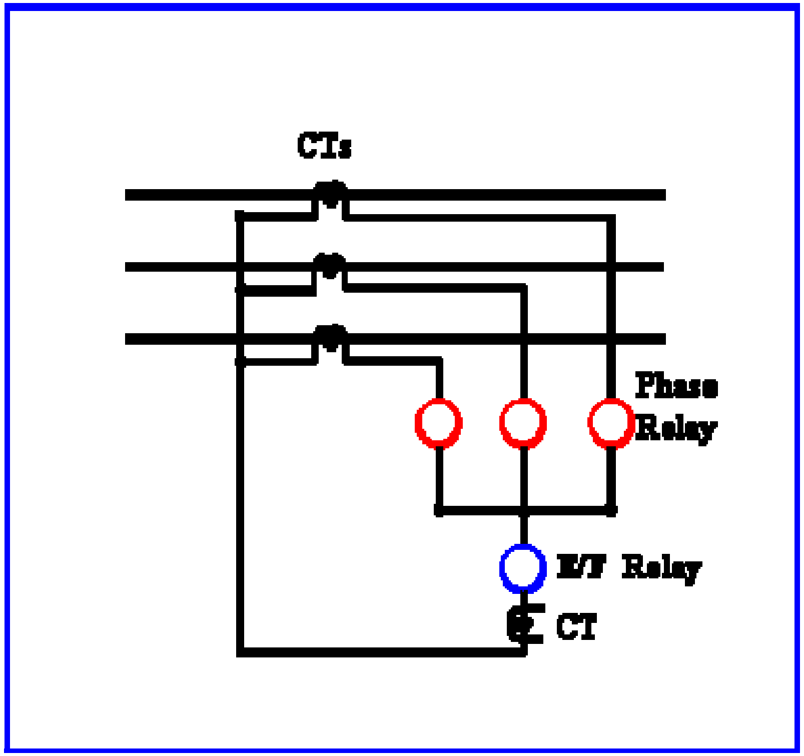


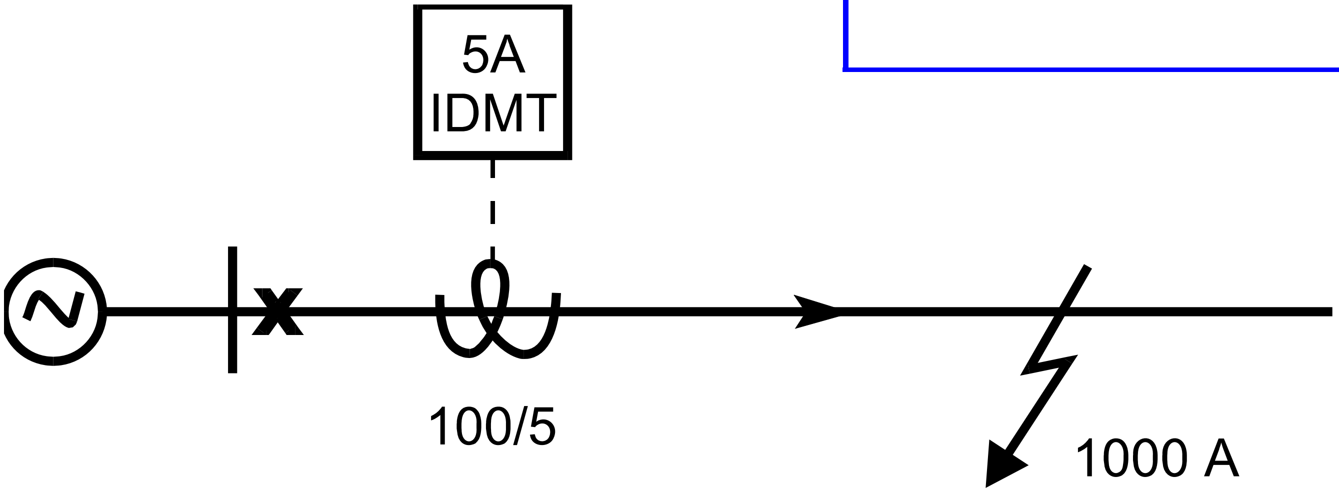
Example

Calculate:

- Plug setting (PS)
- Time multiplier setting (TMS)

for an IDMTL relay on the following network so that it will trip in 2.4 seconds.





Answer

- Fault Current = 1000 A
- CT Ratio = 100/5
- Hence current into relay = $\frac{I_f}{CTR}$ = 1000 × 5 /100 = 50A
- Choose plug setting (PS) of 5 A (100%) ➔PS=1.0
- Therefore current into relay as a multiple of plug setting times ➔PSM=10

$$PSM = \frac{I_f}{Ipickup} = \frac{50}{5} = 10$$

- Referring to curves on the next page, read off Time Multiplier setting where 10 times and 2.4 seconds cross …namely 0.8. ➔TSM=0.8

➔ Relay settings = Plug Setting PS = 5 A (PS = 100%)
= Time S ettingMultiplier (TSM = 0.8)

IDMT Relay

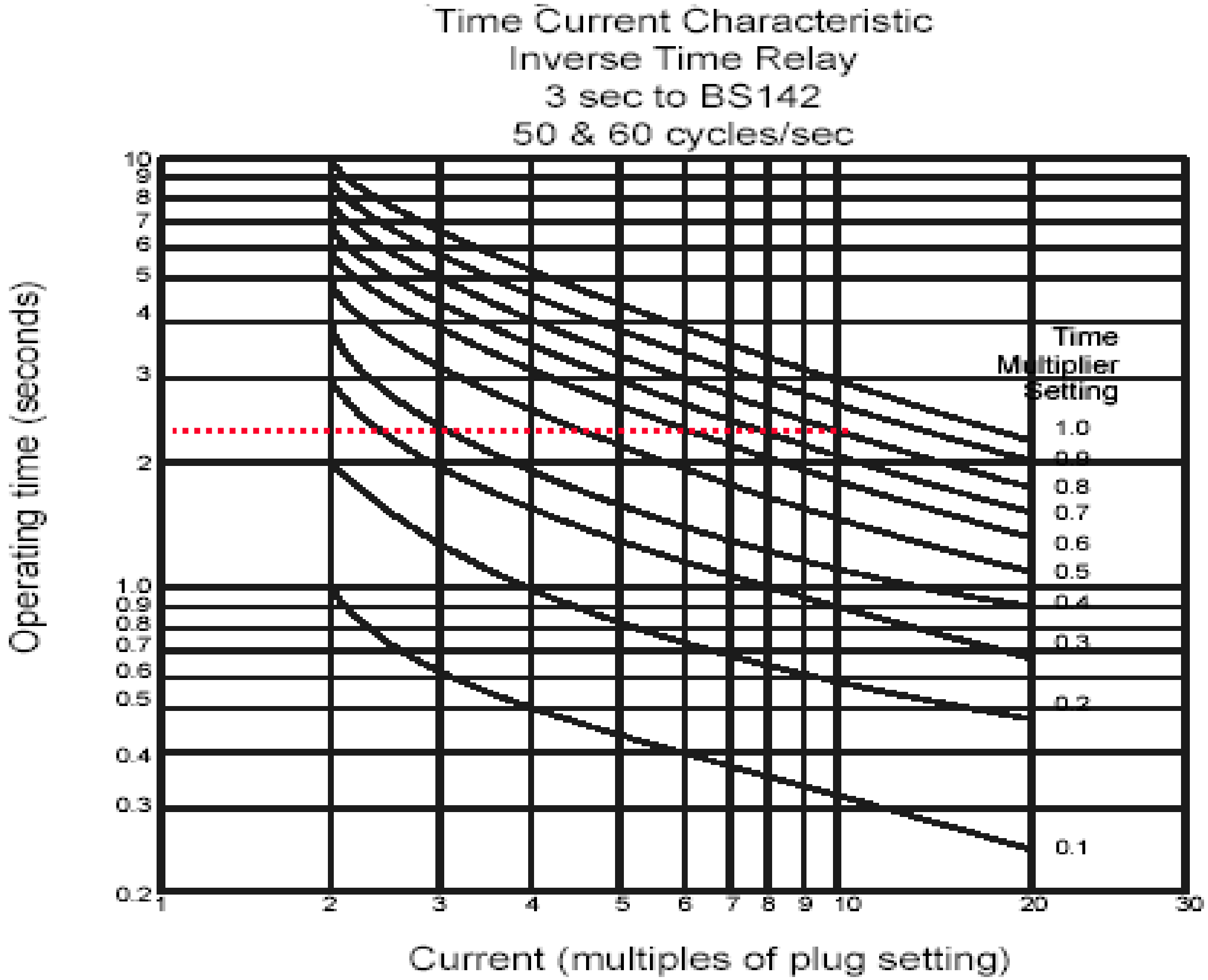


Figure 9.13
Multiples of plug setting current

IDMT Settings

- This technique is fine if the required setting falls exactly on the TM curve.
- If not….
- Go to the multiple of plug setting current and read off the seconds value corresponding to the 1.0 Time Multiplier curve. Then divide the desired time setting by this figure. This will give the exact Time Multiplier setting:
- Seconds figure at 10 times = 3 (TSM=1)
- Desired Setting = 2.4
- Therefore Time Setting Multiplier = 2.4/3 = 0.8

IDMT Relay

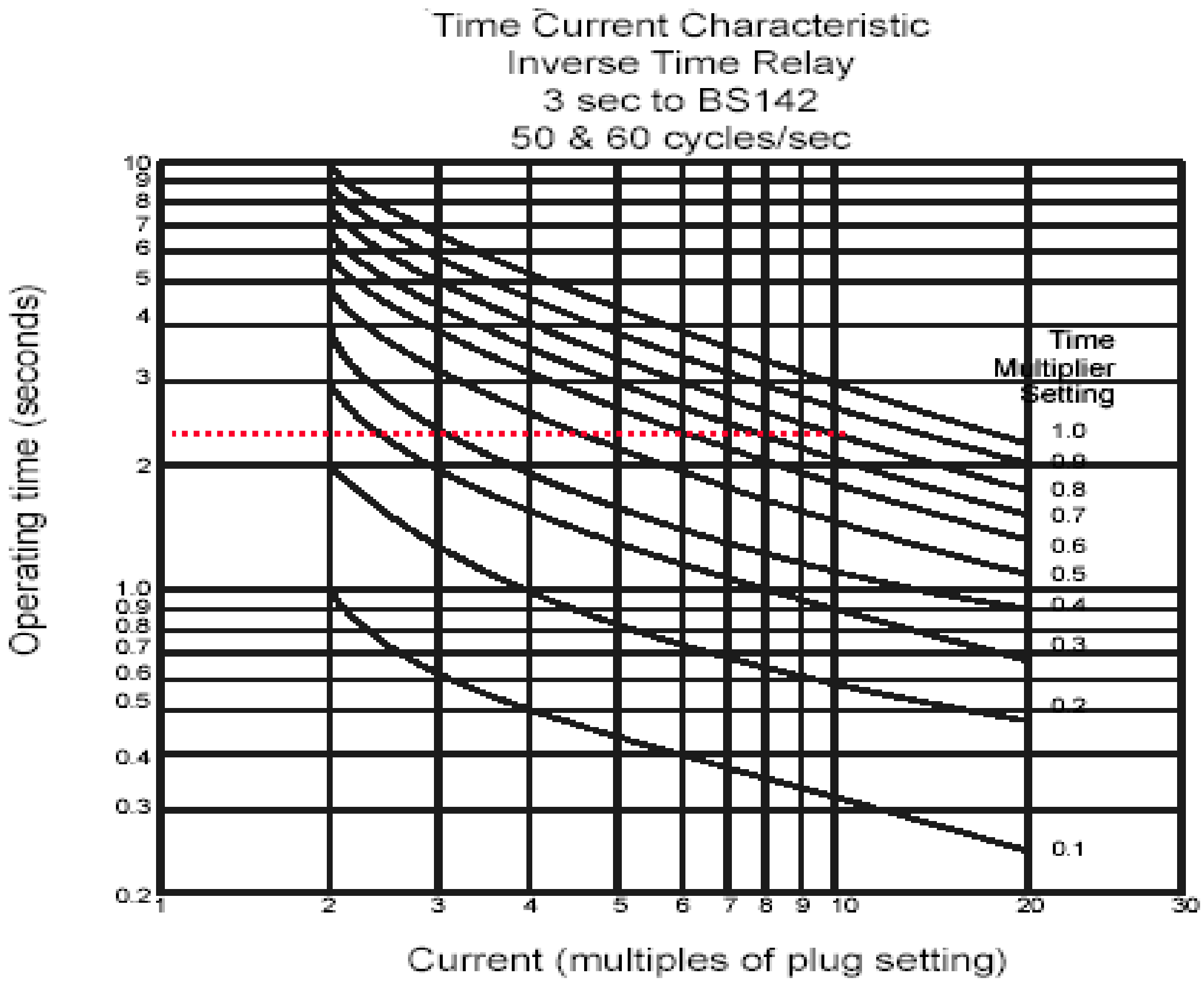


Figure 9.13
Multiples of plug setting current

IDMT Relay

- Alternatively, if the current plug setting is chosen as 125% (6.25 A), the PSM of the relay will be PSM=50/6.25=8 . The graph shows that 8 times plug setting to operate in 2.4 seconds, the time multiplier should be about 0.7.
- This technique is fine if the required setting falls exactly on the TM curve. However, if the desired setting falls between the curves, it is not easy to estimate the intermediate setting accurately as the scales of the graph are log/log. The following procedure is therefore recommended:
- Go to the multiple of plug setting current and read the seconds value corresponding to the 1.0 TM curve. Then divide the desired time setting by this figure. This will give the exact time multiplier setting:

Seconds value at 10 times =3 (at 8 times =3.4)

Desired time (setting) = 2.4

$\Rightarrow$TSM=2.4/3.0=0.8 or 2.4/3.4 =0.7 in the second case

Pickup Calculation - Electromechanical Relays

The relay should pick-up for current values above the motor FLC (~ 600 A).

For the IFC53, the available ampere-tap (AT) settings are 0.5, 0.6, 0.7, 0.8, 1, 1.2, 1.5, 2, 2.5, 3, & 4.

For this type of relay, the primary pickup current was calculated as:

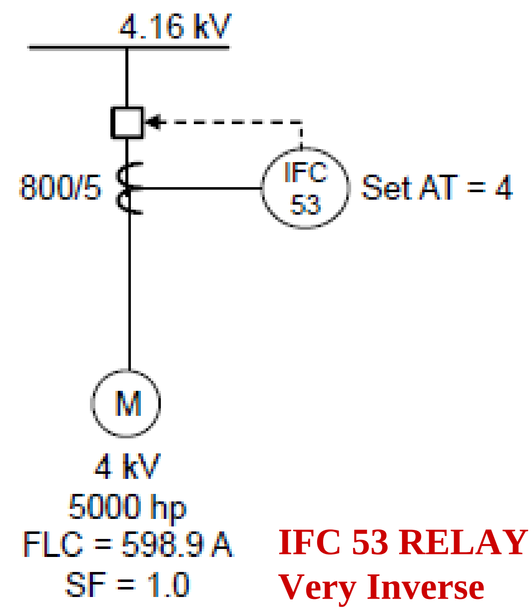
$I_{Pickup} = I_{FL}/CTR = 600/(800/5) = 3.75$

Set $I_{Pickup} = 4$ A (secondary)

$I_{Pickup} = 4 \times CTR$ (primary)

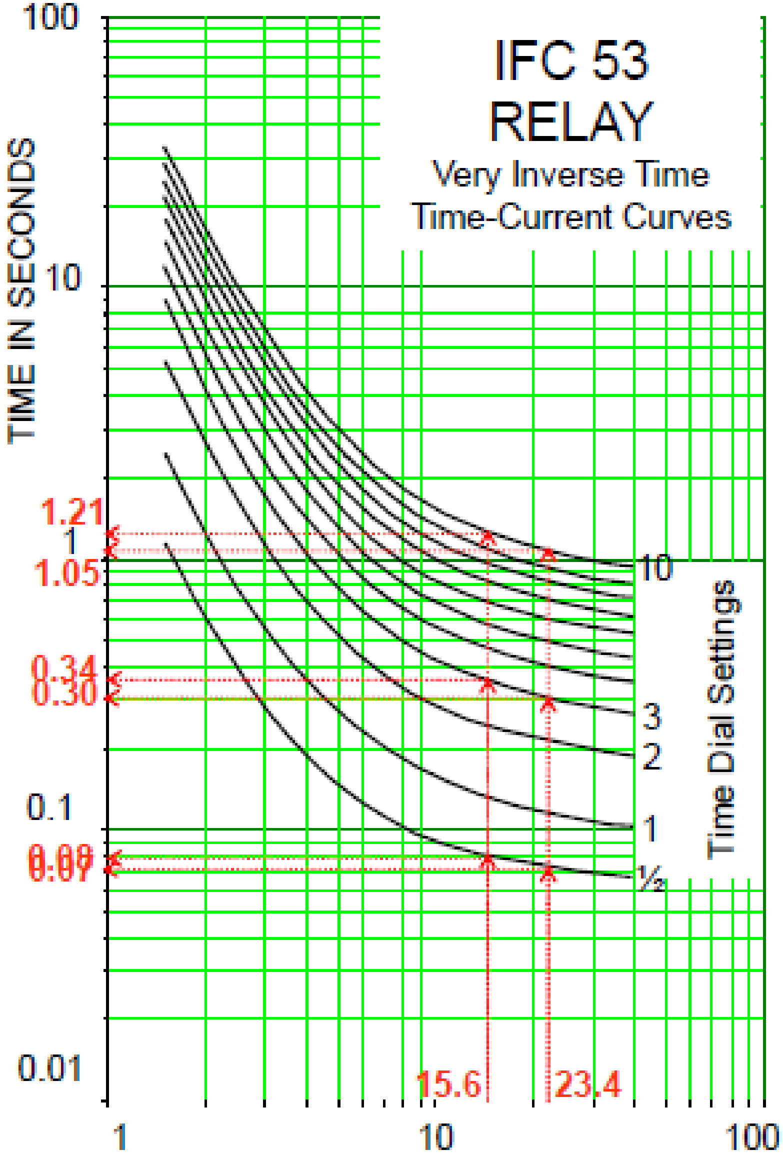
$= 4 \times (800/5)$

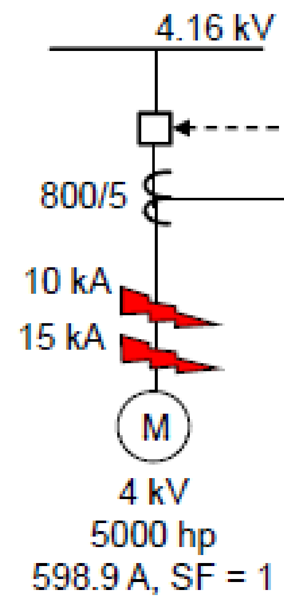
$= 640 \text{ A} > I_{FL}$



IFC 53 RELAY
Very Inverse

Pickup Calculation - EM Relays





IFC 53 Relay Operating Times		
Fault Current	15 kA	10 kA
Multiple of Pick-up	15000/640 = 23.4	10000/640 = 15.6
Time Dial ½	0.07 s	0.08 s
Time Dial 3	0.30 s	0.34 s
Time Dial 10	1.05 s	1.21 s

British Standard 142 and IEC 255 Inverse Curves

$$t = TMS \times \frac{\beta}{I_r^{\alpha} - 1}$$

C/C	α	β
Normal	0.02	0.14
Very	1.00	13.50
Extreme	2.00	80.00
Long Time	1.00	120.00

where:

- t = operating time in secs.
- TMS = time multiplier setting
- $I_r = (I/I_s)$
  - I=measured current
  - I_s= relay setting current
- α & β are constants for curve selection

Relay Characteristic	Equation (IEC 60255)
Standard Inverse (SI)	$t = TMS \times \frac{0.14}{I_r^{0.02} - 1}$
Very Inverse (VI)	$t = TMS \times \frac{13.5}{I_r - 1}$
Extremely Inverse (EI)	$t = TMS \times \frac{80}{I_r^2 - 1}$
Long time standard earth fault	$t = TMS \times \frac{120}{I_r - 1}$

IEC Standard Inverse Time Characteristic

Relay characteristics to IEC 60255

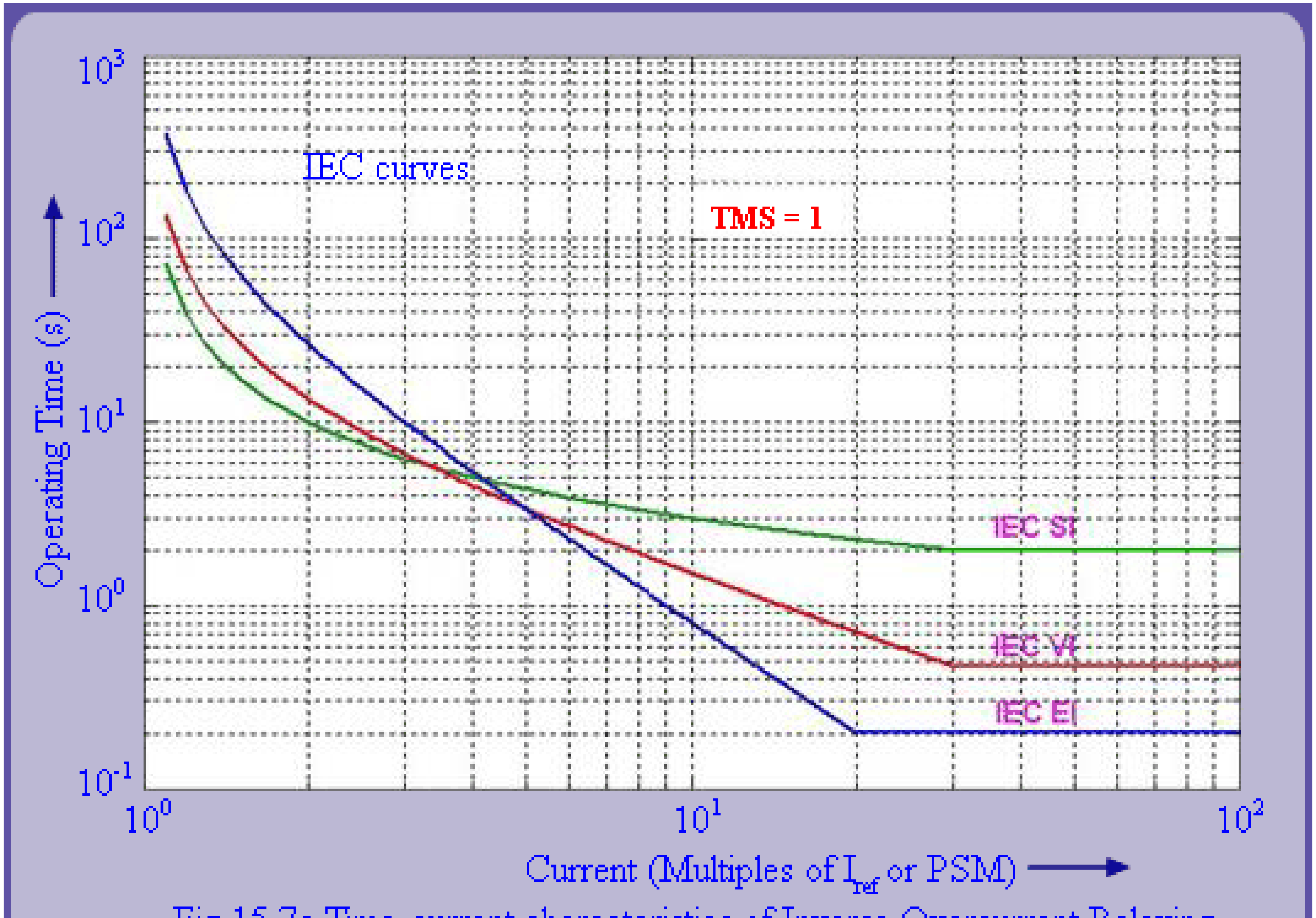


Fig 15.7a Time-current characteristics of Inverse Overcurrent Relaying

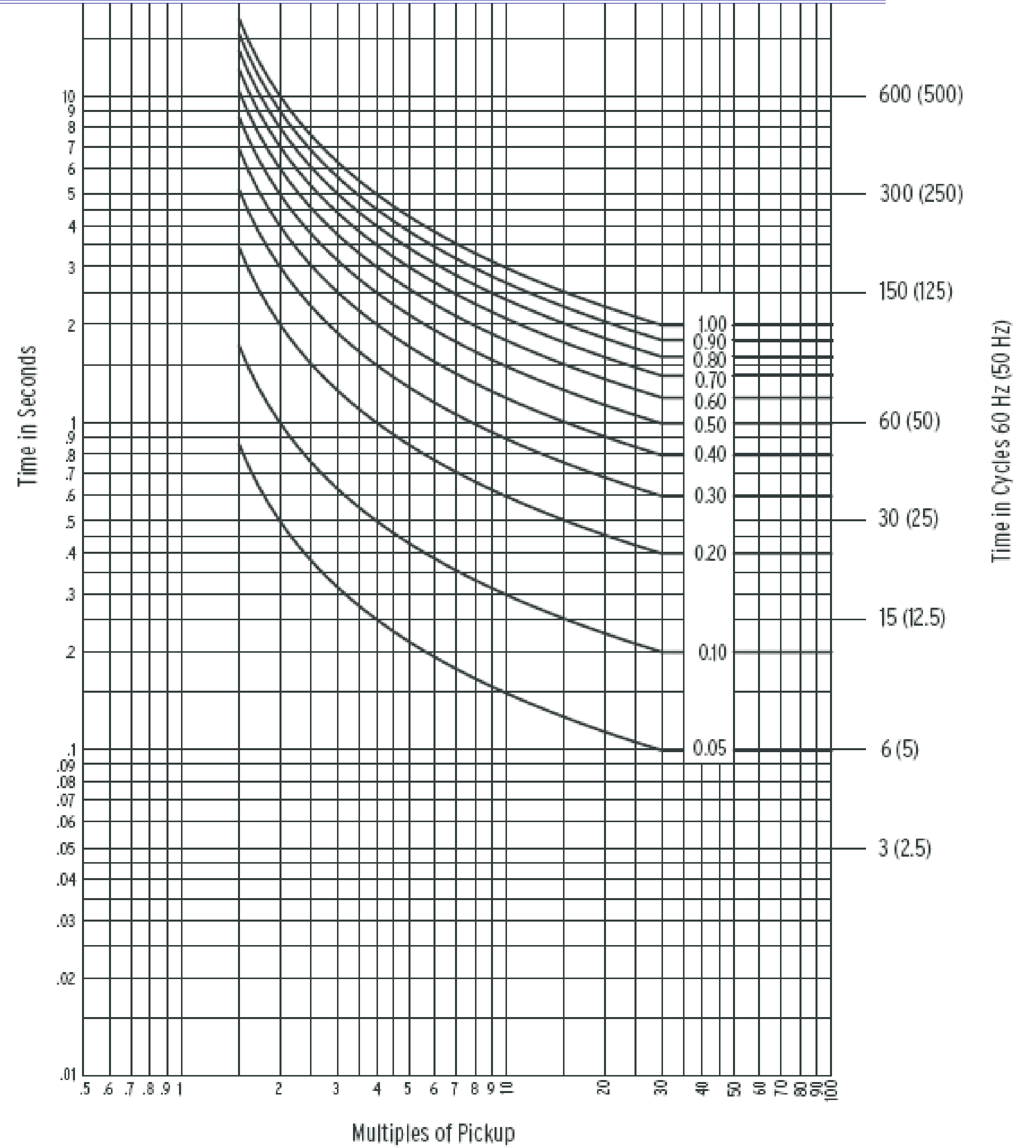
IEC Standard Inverse Time Characteristic

Relay characteristics to IEC 60255

Curve Type	Operating Time
C1 (Standard Inverse)	$T_p = TD \bullet \left(\frac{0.14}{M^{0.02} - 1}\right)$
C2 (Very Inverse)	$T_p = TD \bullet \left(\frac{13.5}{M - 1}\right)$
C3 (Extremely Inverse)	$T_p = TD \bullet \left(\frac{80}{M^2 - 1}\right)$
C4 (Long-Time Inverse)	$T_p = TD \bullet \left(\frac{120}{M - 1}\right)$
C5 (Short-Time Inverse)	$T_p = TD \bullet \left(\frac{0.05}{M^{0.04} - 1}\right)$

$$t = TMS \times \frac{0.014}{M^{0.02} - 1}$$

Standard Inverse C/C



IEEE Standard Inverse Time Characteristic

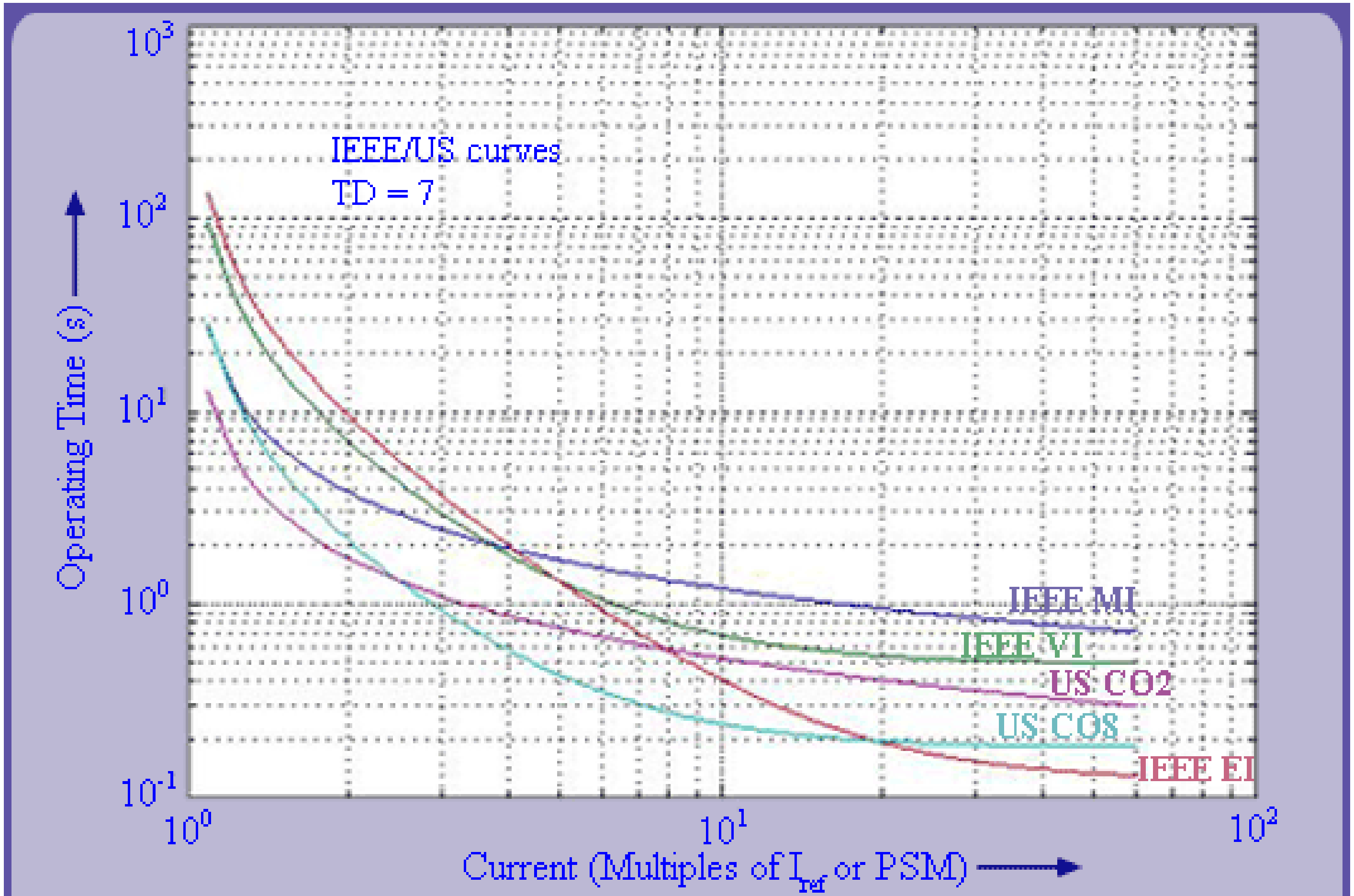
Pickup Time of an Inverse -Time Overcurrent Relay, for $M > 1$

$$t_p = \left(\frac{A}{I_r^p - 1} + B \right) \times \frac{TDS}{7}$$

- t_p: is the trip time in equation in seconds
- TDS: is the time dial setting
- I_r : is the I_{input}/I_{pickup}
  - (I_{pickup} is the relay current set point)
  - A, B, p : are constants to provide selected curve characteristics

C/C	*A*	*B*	*p*
Moderately Inverse	0.0515	0.1140	0.020
Very Inverse	19.61	0.491	2.000
Extremely Inverse	28.20	0.1217	2.00

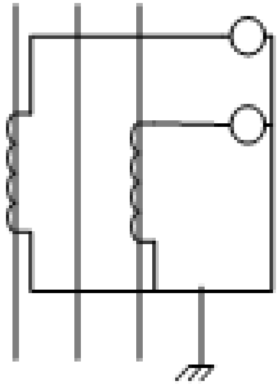
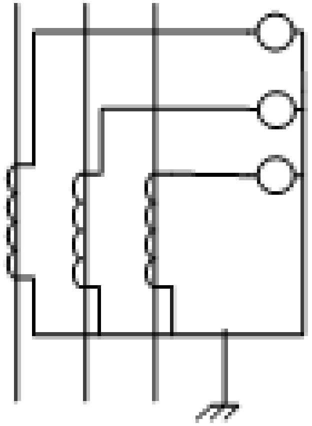
IEEE Standard Inverse Time Characteristic



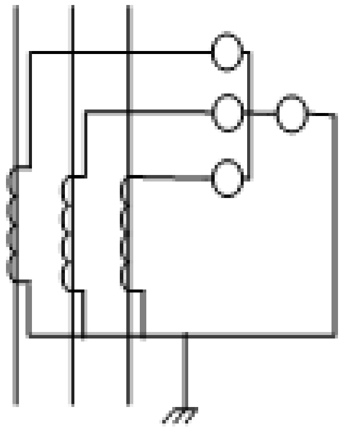
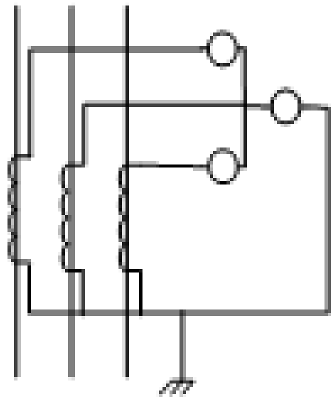
15.7b Time-current characteristics of Inverse Overcurrent Relaying (IEEE/US curves)

Methods of CT and Relay Connections in OC Protection of 3-Phase Circuits

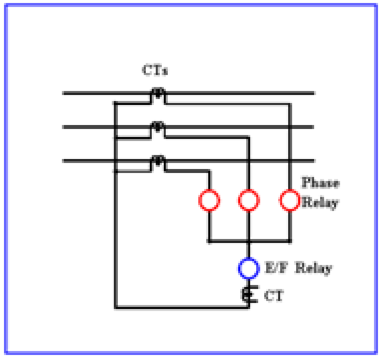
OC protection can be achieved by means of three **OC relays** or by two OC relays

1		Two OC relays with two CT's for phase to phase fault protection.
2		Three OC relays with three CT's for phase to phase fault protection.

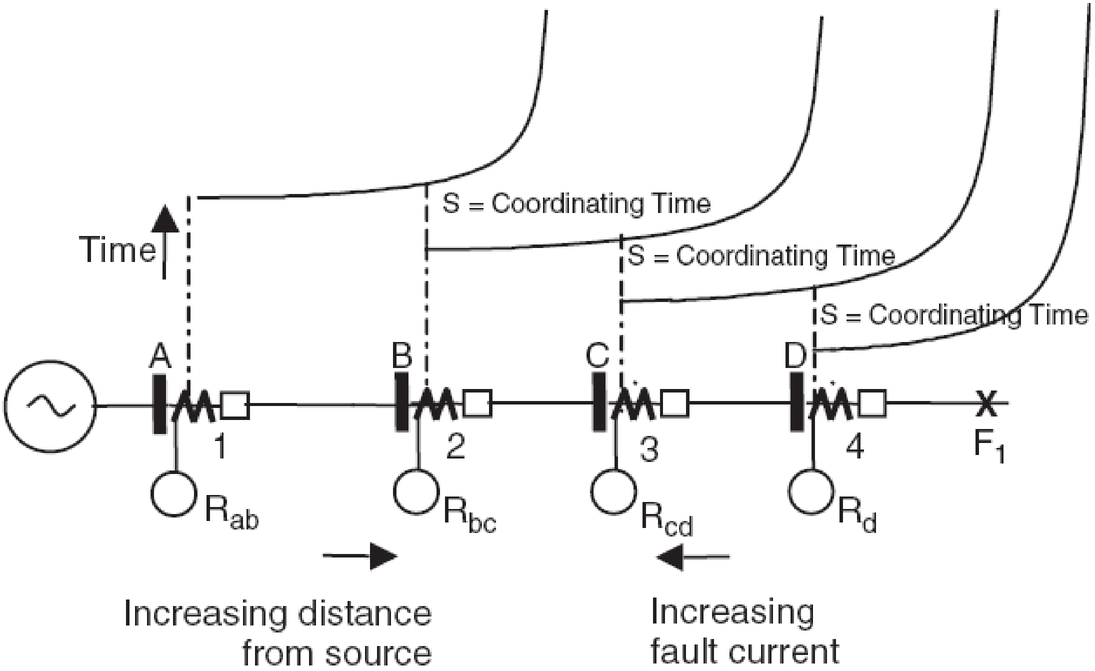
Methods of CT and Relay Connections in OC Protection of 3-Phase Circuits

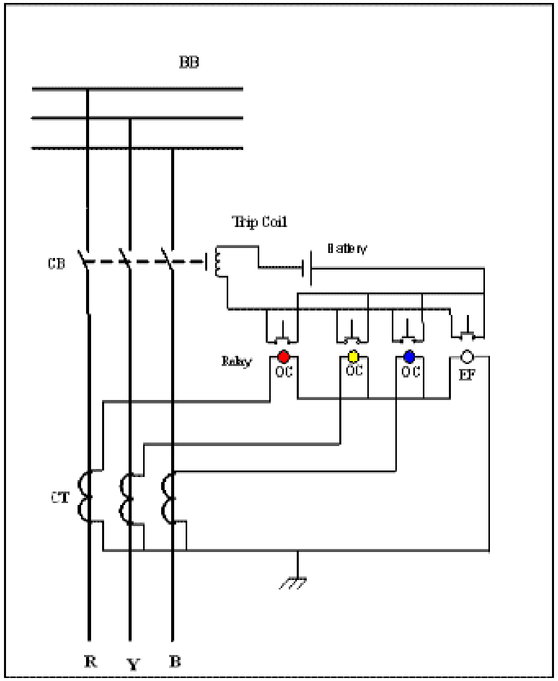
3		Three OC relays with three CT's for phase to phase fault protection and phase to earth fault. EF setting less than phase fault setting
4		Two OC relays for phase to phase protection One EF relays for phase to earth fault protection

EE482-Power System Analysis II
Dr. E. A. Feilat
Electrical Engineering Department
School of Engineering
University of Jordan



Topic 1-4:Prt 2: Feeder O/C Protection





Co-ordination by Time Grading

- Selectivity and coordination by time grading can be achieved by two philosophies:

1. Definite time lag (**DTL**)
2. Inverse definite minimum time (**IDMT**)

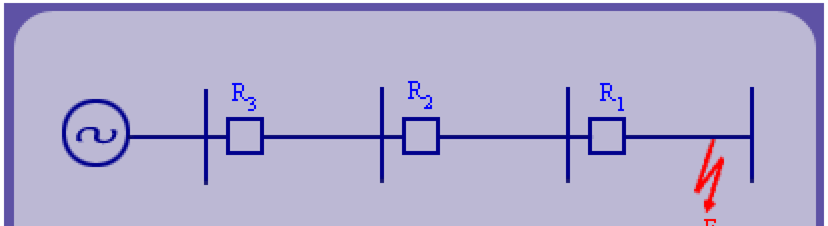
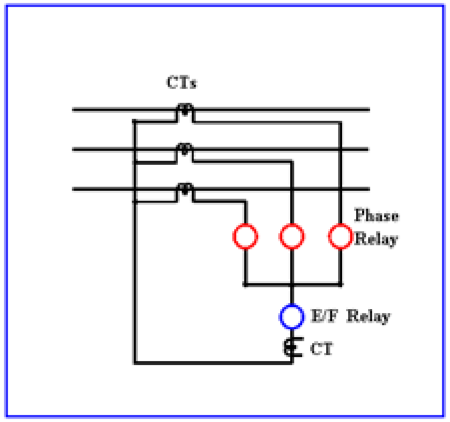


Fig 15.2 A Simple Radial Distribution System



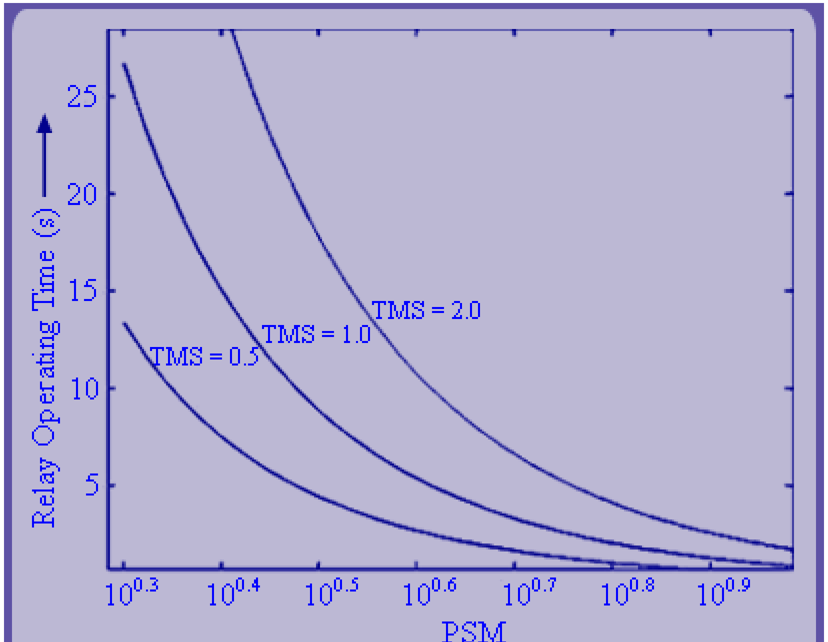


Fig 15.3 Role of TMS in Overcurrent Relay

Definite Time Lag (DTL) Philosophy

- Coordinate with a definite time of operation between successive relays (0.5 s)
- Relay R_1 is set to operate in the fastest possible time
- Relay operating times increase sequentially at 0.5 s intervals
- Disadvantage: the fault closer to source (higher fault current) cleared at the longest operating time.

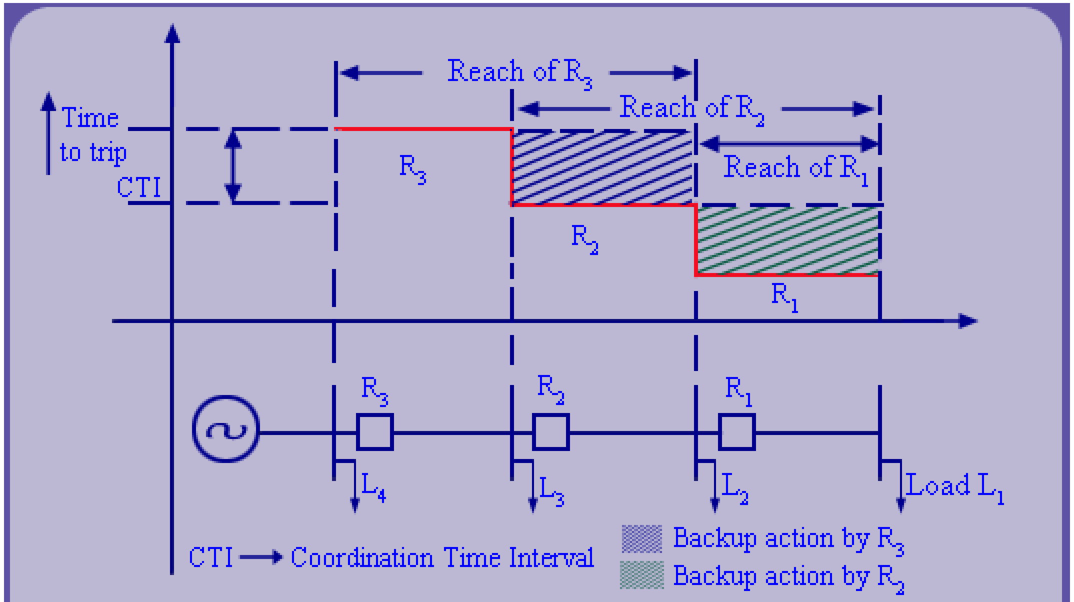
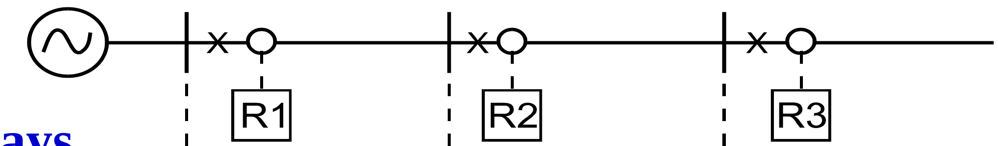


Fig 15.4 Protection of Radial Distribution System with Overcurrent Relay

Inverse Definite Minimum Time-(IDMT) Philosophy

- Use inverse *t-I* Characteristics (IDMT).
- Relay operates faster at the higher fault currents and slower at lower fault currents.
- IDMT relays have to be set to coordinate with both upstream and downstream relays



- **Use of Electromechanical Relays**
  - Need about 0.4 second interval between successive Relays due to acceptable errors
  - Imposes restrictions based on the network design
- **Use of Digital Relays**
  - Can get better coordination with considerably reduced time interval (0.3 second) due to better accuracies

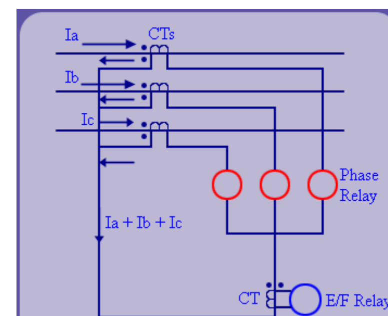
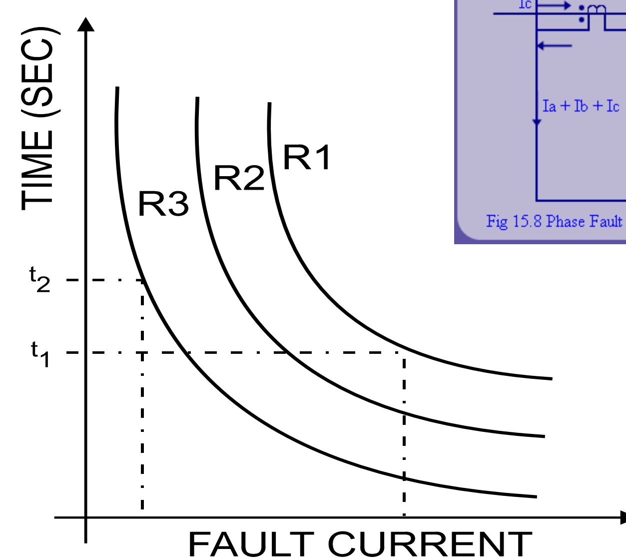
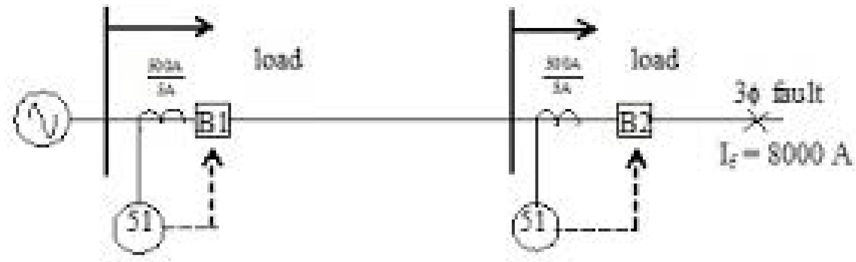


Fig 15.8 Phase Fault and Earth Fault Connection



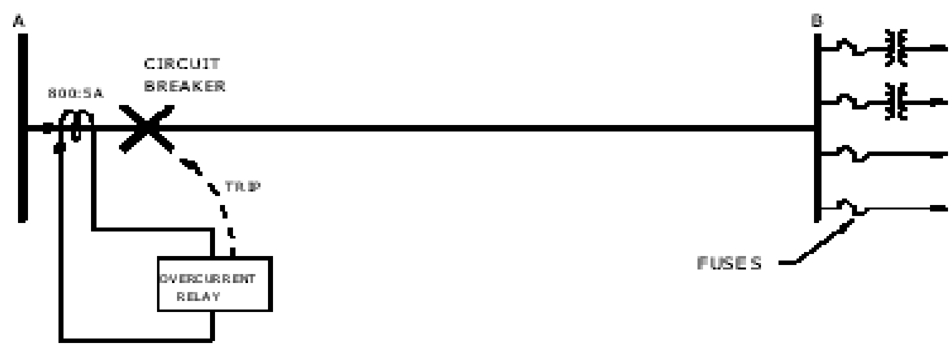
Feeder OC Protection

By far the most common type of protection for radial distribution feeders is **Overcurrent** protection.



Typical distribution system voltages are 33 kV & 11 kV

The point of supply is normally a few kilometers from the load.



Feeder O/C Protection

With ***Radial*** feeders there is only one possible point of supply, and the flow of fault current is in ***one direction only***.

Overcurrent protection can therefore be used to provide adequate protection.

The current entering the feeder at the circuit breaker is measured by means of a Current Transformer located at the base of the breaker bushing.

The C.T. secondary current is supplied to the OC relays. These OC relays must then operate and initiate tripping if a fault condition is detected on the feeder.

Feeder O/C Protection

<u>CRITERIA FOR SETTING THE INVERSE TIMED OVERCURRENT RELAY</u>

1. The relay must not operate for the maximum load current that will be carried by the feeder.

2. The relay setting must be sensitive enough for the relay to operate and clear faults at the very end of the feeder.

3. The relay operating characteristic must be set to coordinate with other protection devices, such as fuses, ‘downstream’ from the supply station.

Feeder O/C Protection

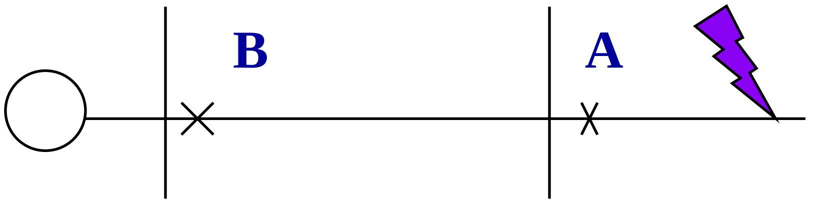
DIRECTIONAL OVERCURRENT PROTECTION

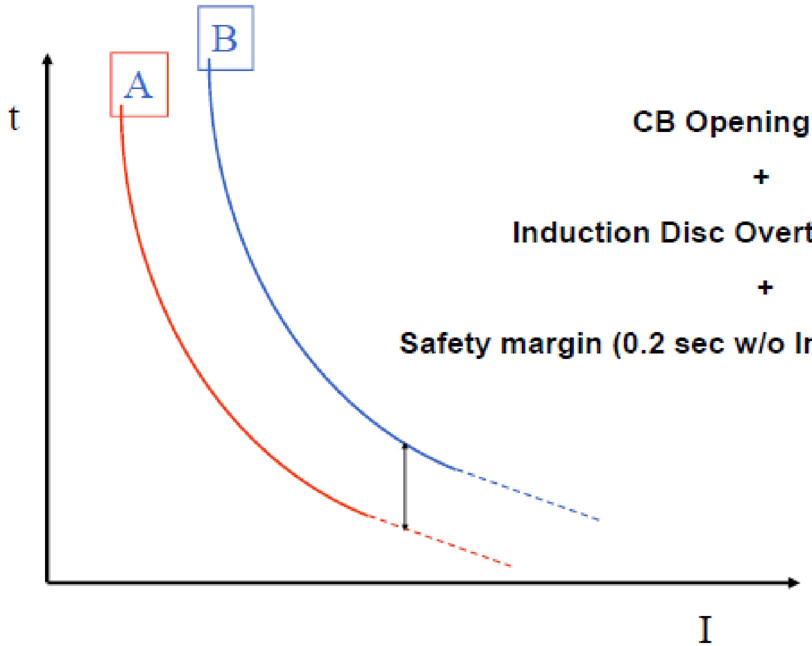
- If there is generation connected to a distribution feeder, the system is no longer **RADIAL.**

- Fault current can then flow in either direction – into the feeder from the power system **or** out of the feeder from the generator.

- A directional relay or element must be used to supervise the overcurrent relay elements to allow the overcurrent protection to trip **ONLY** if the fault current flows into the feeder from the power system.

Curves must not cross





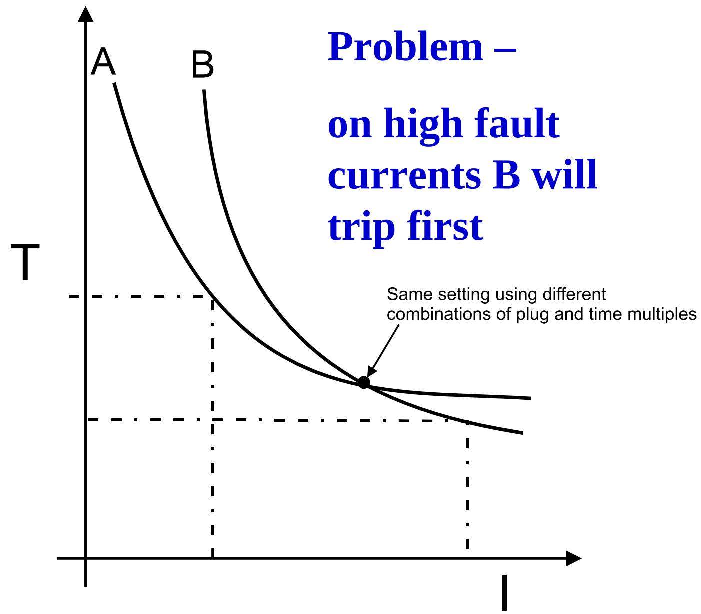
CB Opening Time

+

Induction Disc Overtravel (0.1 sec)

+

Safety margin (0.2 sec w/o Inst. & 0.1 sec w/ Inst.)



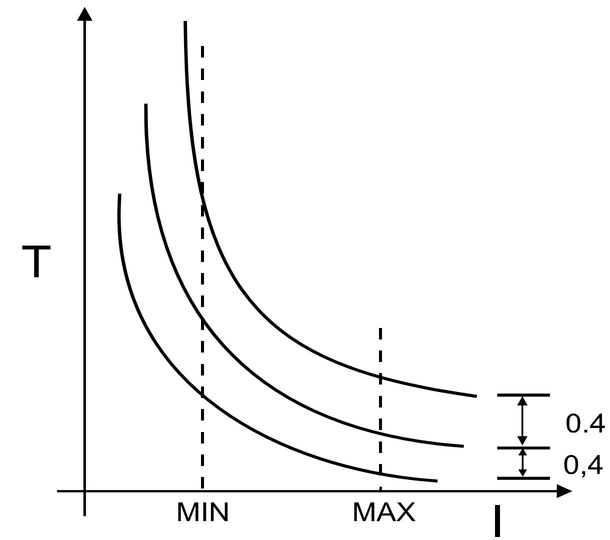
Problem –

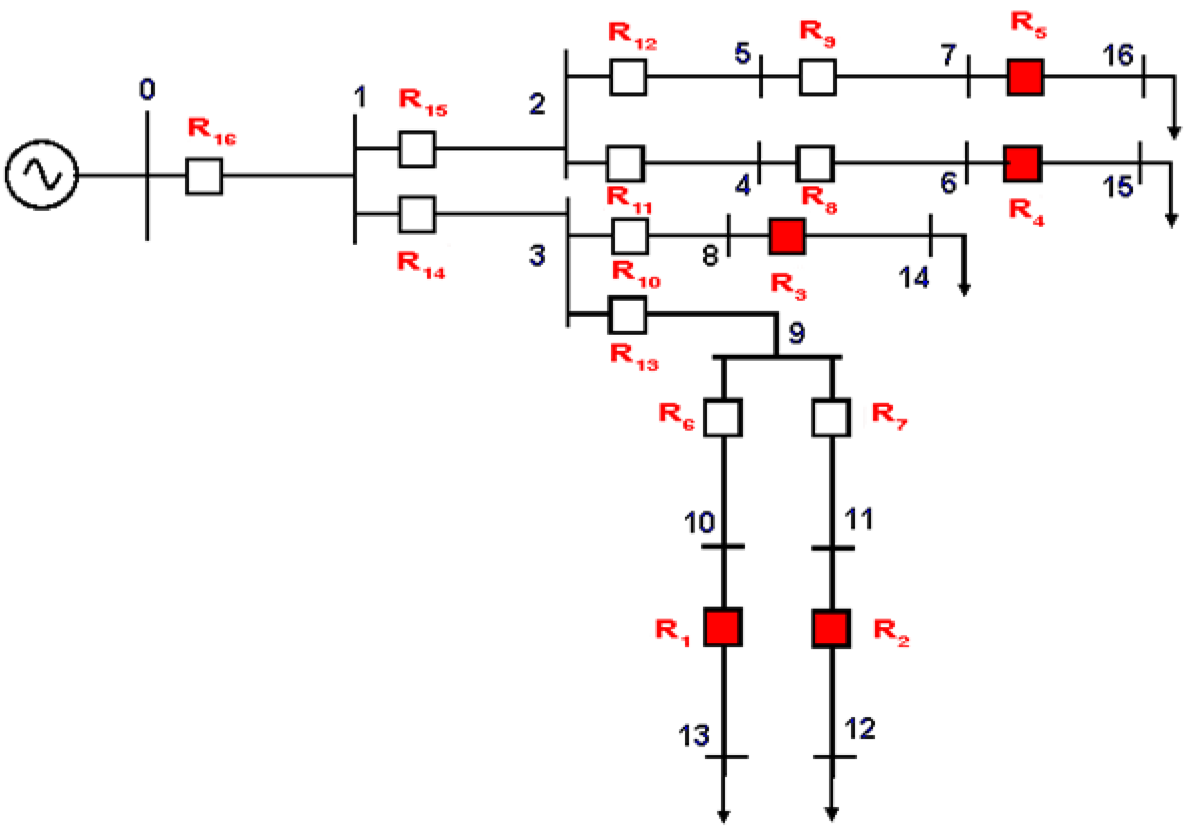
on high fault currents B will trip first

Ideal co-ordination of setting curves

Two Basic Rules !

- Pick up for lowest fault level (minimum)
- Must coordinate for highest fault level (maximum)





Ideal co-ordination of setting curves

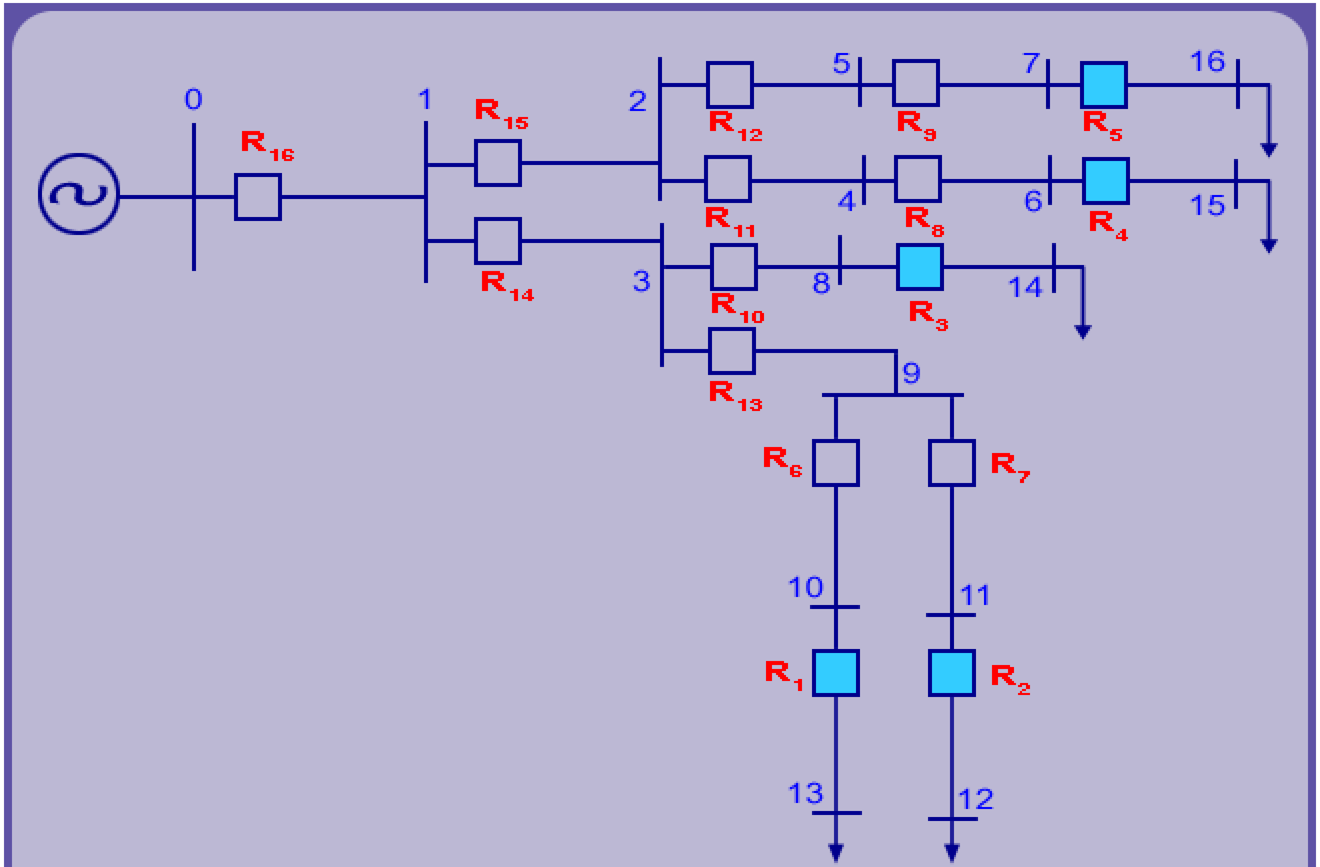


Fig 15.10 Interactive Relay Coordination Example (Primary and Backup Pairs)

Main Relay

Ideal co-ordination of setting curves

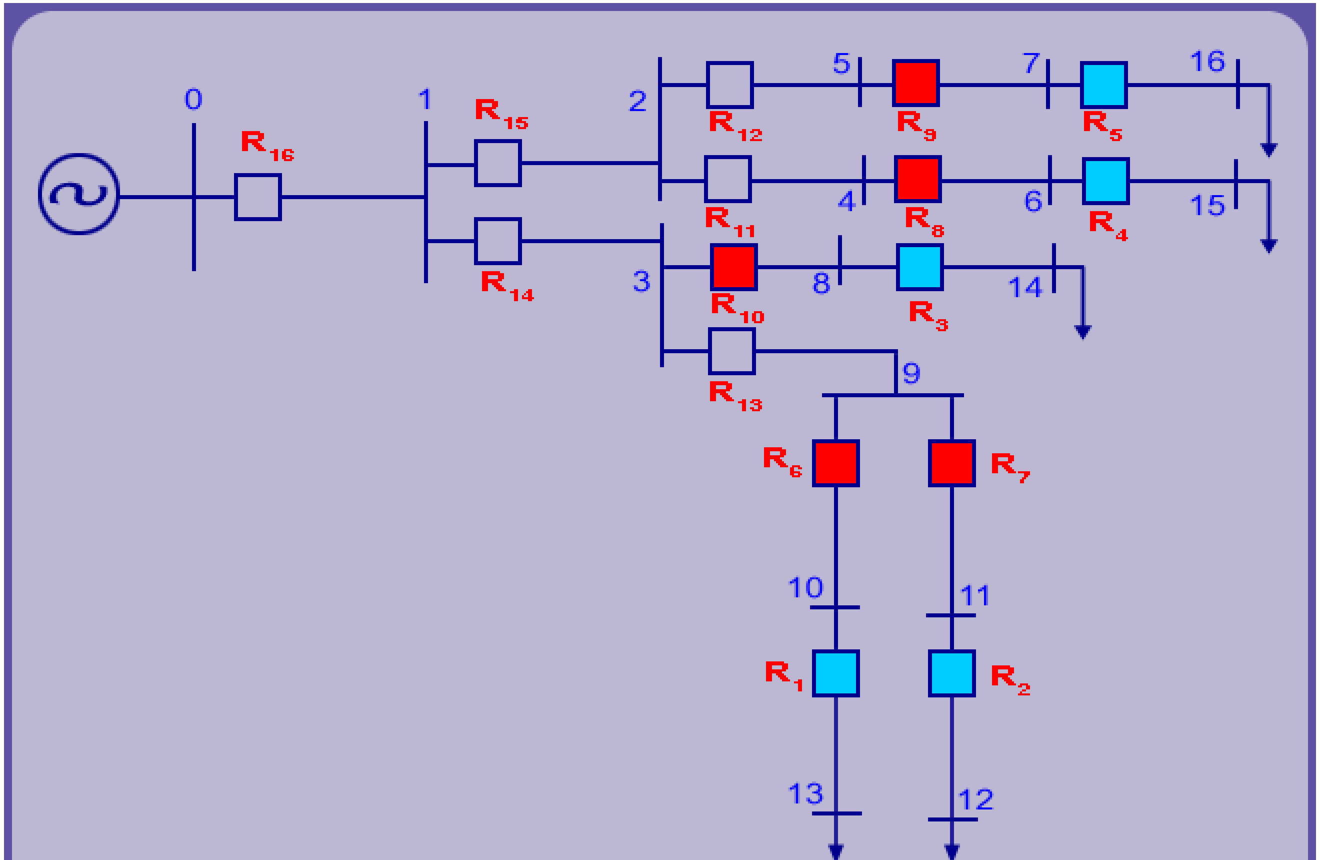


Fig 15.10 Interactive Relay Coordination Example (Primary and Backup Pairs)

Main Relay

Backup Relay

Ideal co-ordination of setting curves

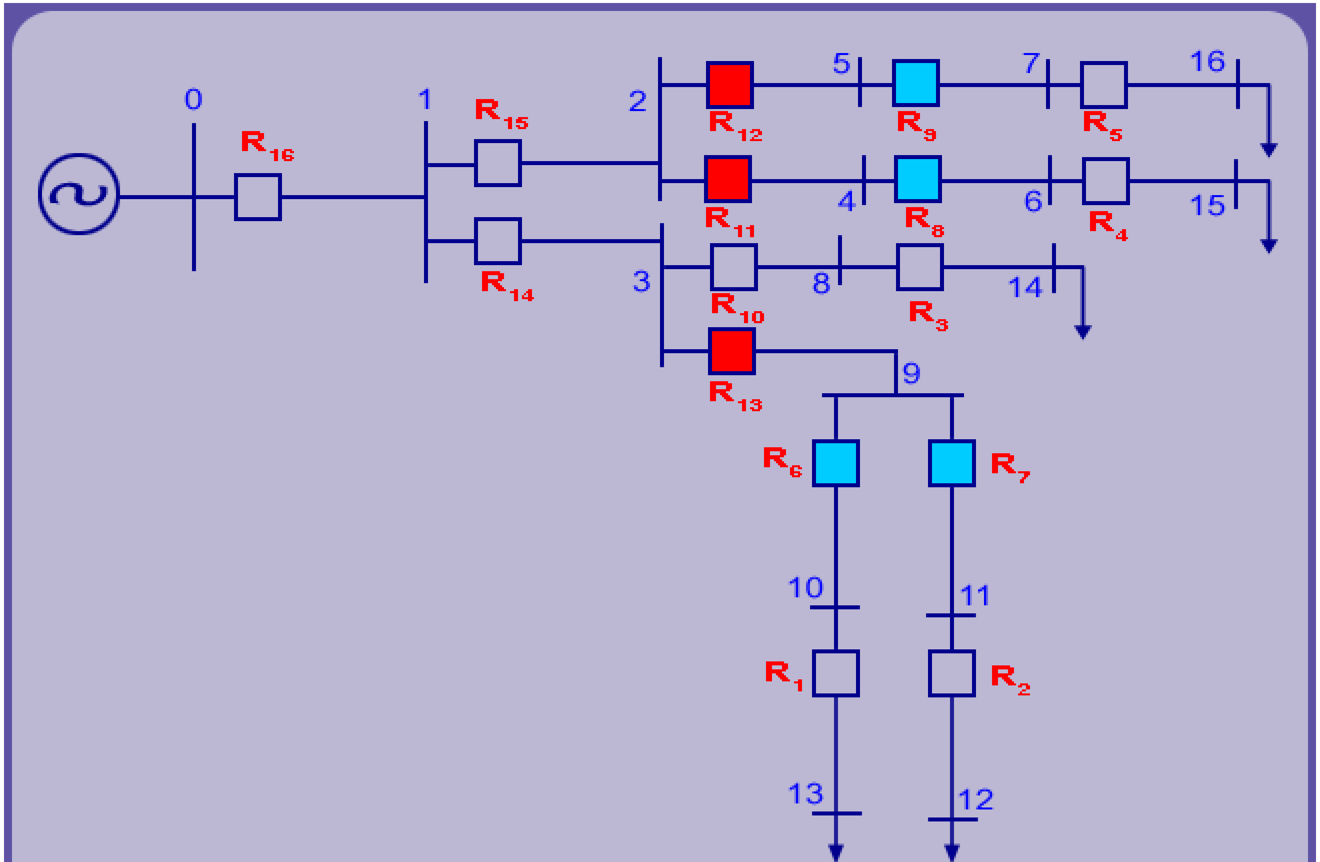


Fig 15.10 Interactive Relay Coordination Example (Primary and Backup Pairs)

Main Relay

Backup Relay

Ideal co-ordination of setting curves

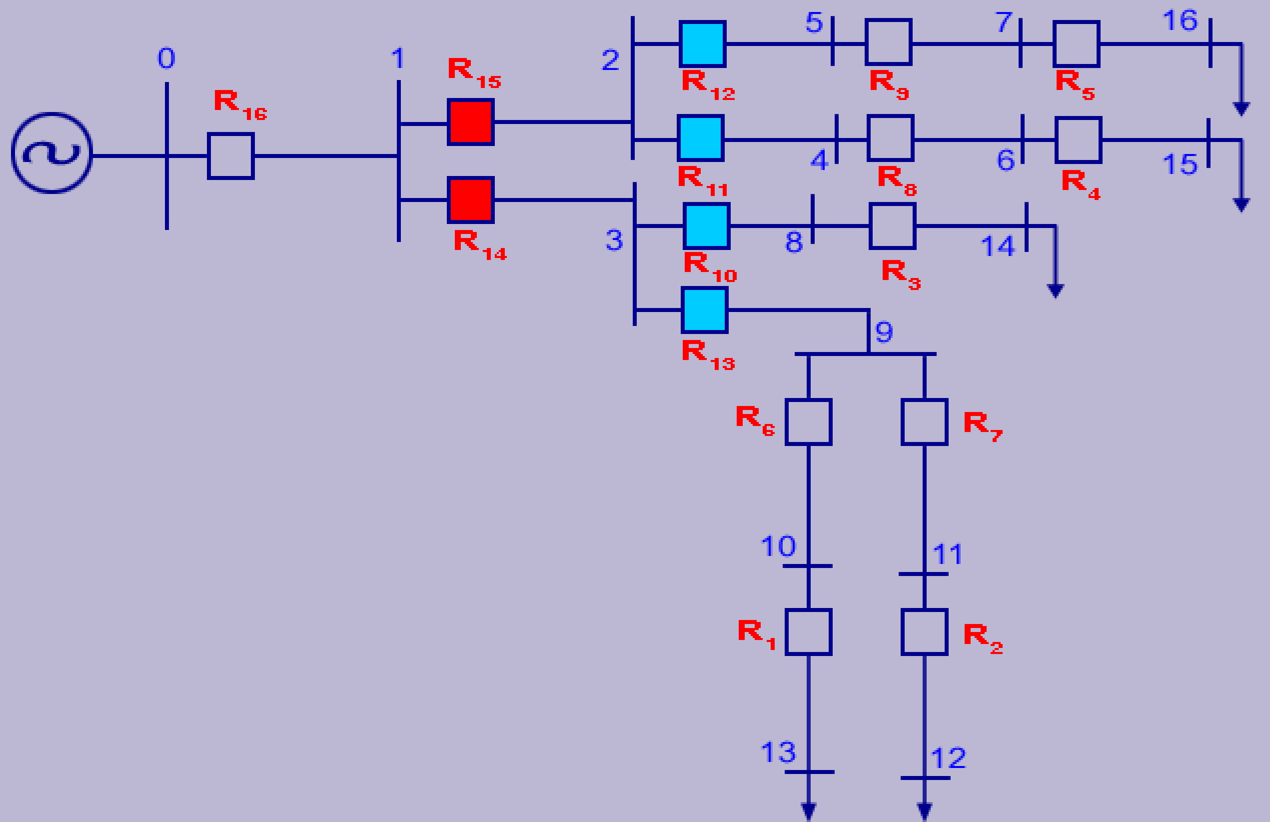


Fig 15.10 Interactive Relay Coordination Example (Primary and Backup Pairs)

Main Relay

Backup Relay

Ideal co-ordination of setting curves

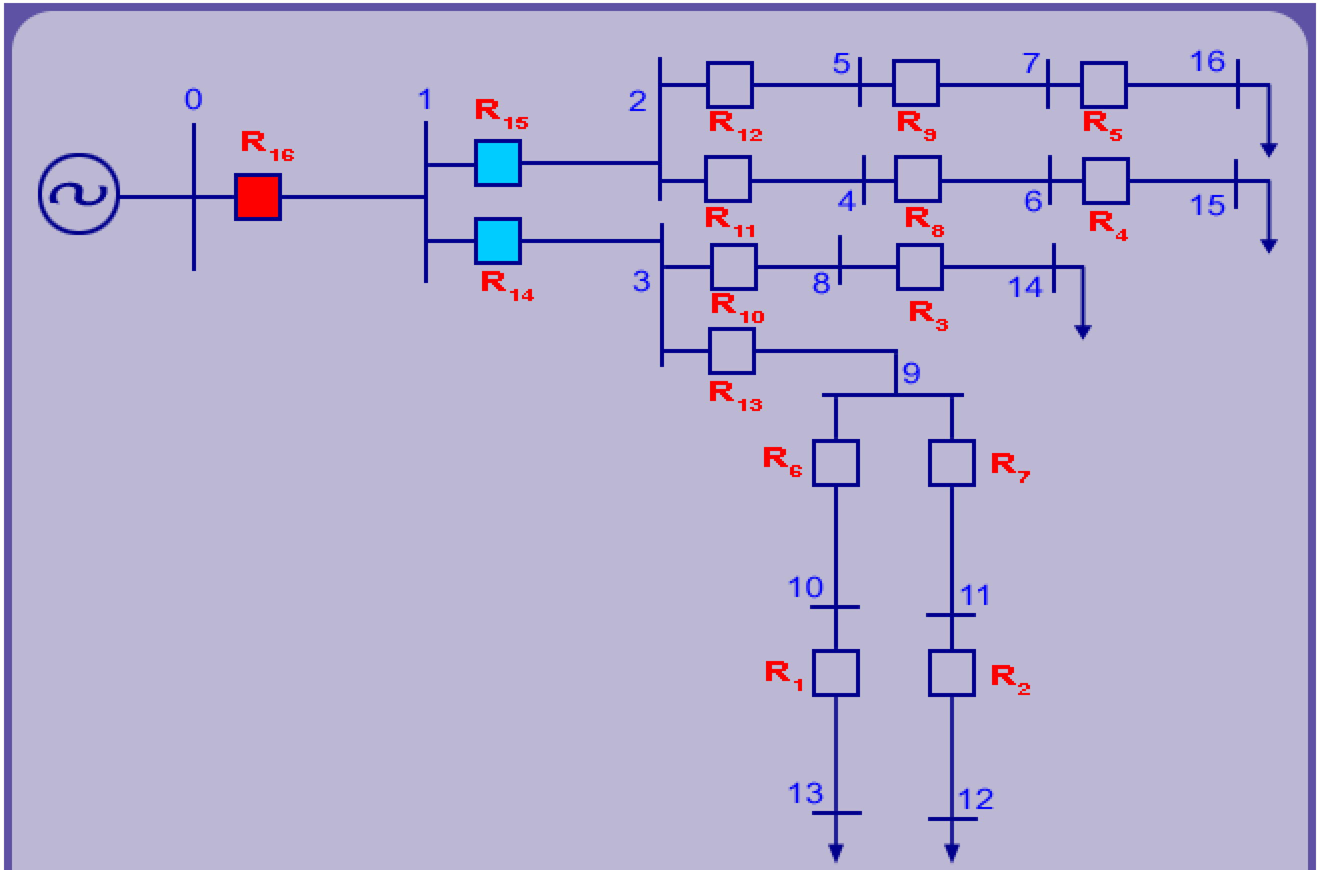
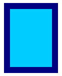
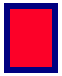


Fig 15.10 Interactive Relay Coordination Example (Primary and Backup Pairs)

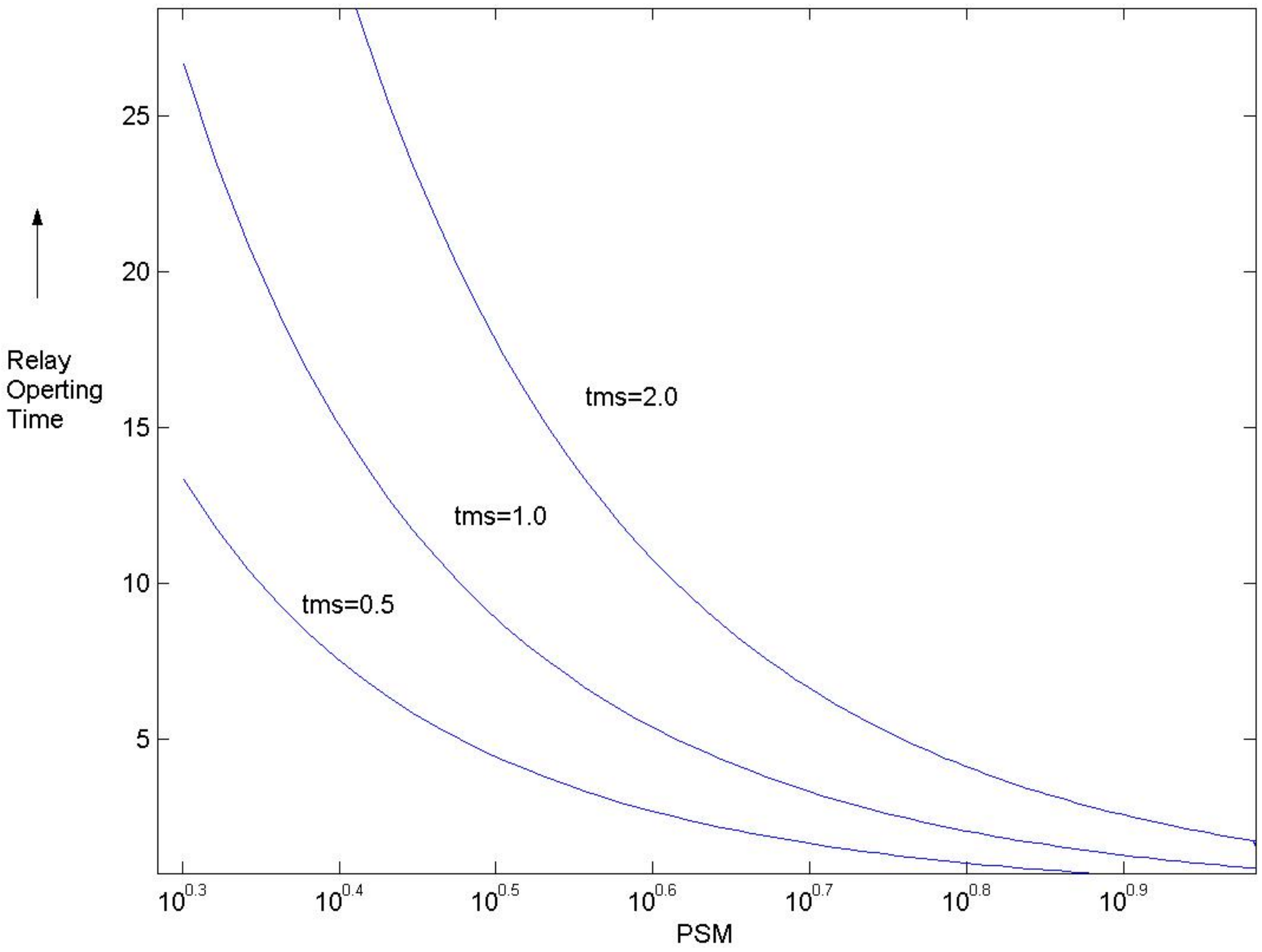


Main Relay



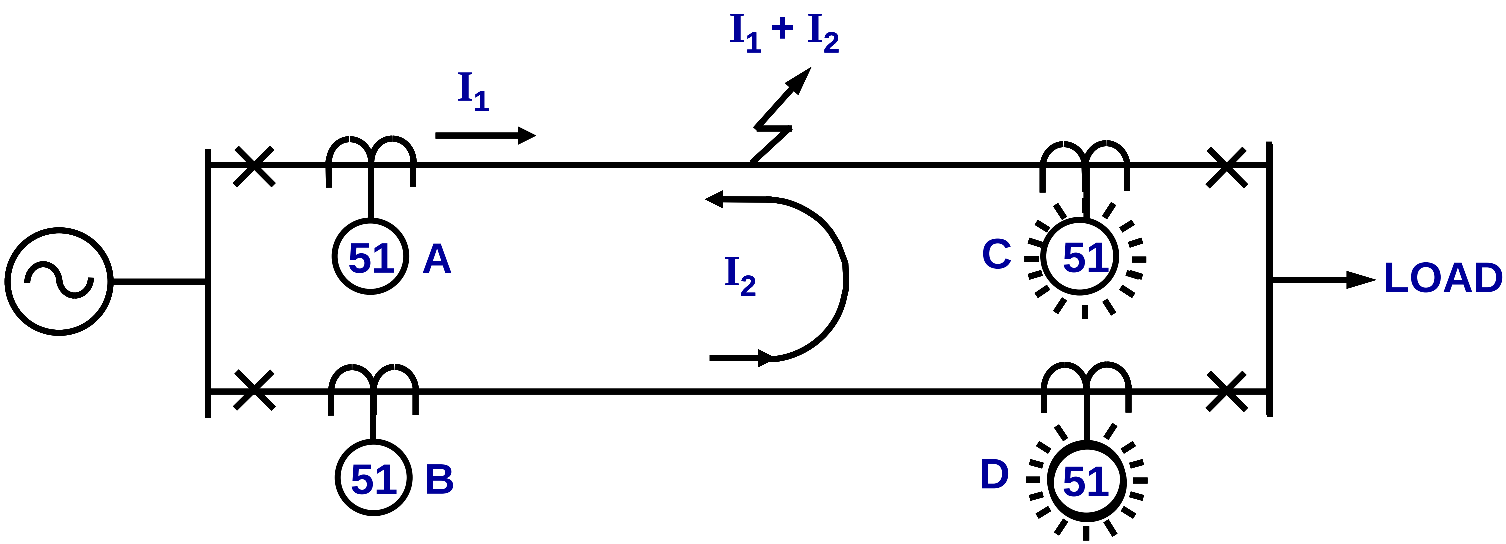
Backup Relay

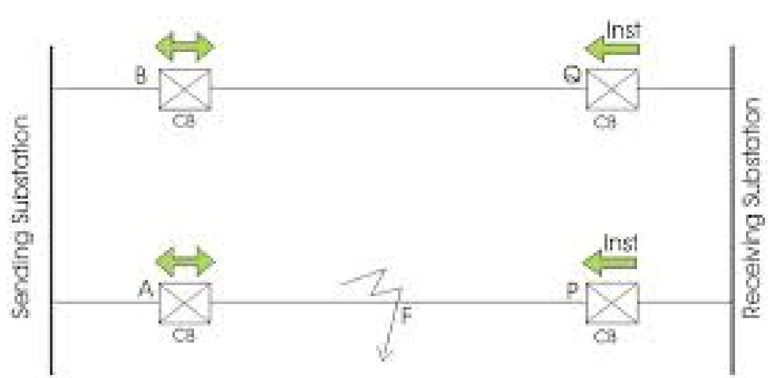
Ideal co-ordination of setting curves



Topic 3: Power System Protection

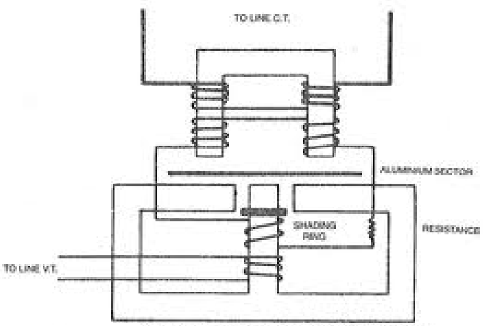
3.4-Prt 3: Directional Over Current Protection





Directional Overcurrent Relays

- When fault current can flow in both directions through the relay location, it may be necessary to make the response of the relay directional by the introduction of a directional control facility. The facility is provided by use of additional voltage inputs to the relay.
- Directional over-current protection comprises over-current relay and power directional relay- in a single relay casing. The power directional relay does not measure the power but is arranged to respond to the direction of power flow.
- The directional relay recognizes the direction in which fault occurs, relative to the location of the relay. It is set such that it actuates for faults occurring in one direction only. It does not act for faults occurring in the other direction.



Directional Element: Electro-magnetic System.

Directional Overcurrent Relays

◇ When fault current can flow in both directions through the relay location, it may be necessary to make the response of the relay directional by the introduction of a directional control facility. The facility is provided by use of additional voltage inputs to the relay.

◇ Directional over-current protection comprises over-current relay and power directional relay- in a single relay casing. The power directional relay does not measure the power but is arranged to respond to the direction of power flow.

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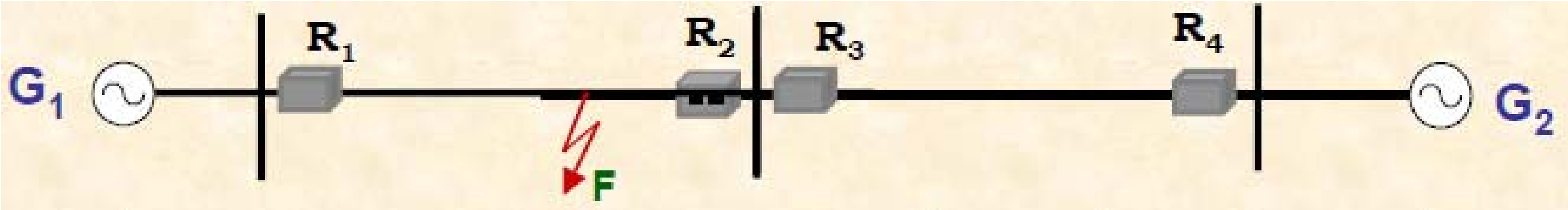


Directional Overcurrent Relays – Parallel Feeders

- If non-unit, non-directional relays are applied to **parallel feeders having a single generating source**, any faults that might occur on any one line will, regardless of the relay settings used, isolate both lines and completely disconnect the power supply.

- With this type of system configuration, it is necessary to apply **directional relays** at the **receiving end** and to grade them with the **nondirectional relays** at the **sending end**, to ensure correct discriminative operation of the relays during line faults.

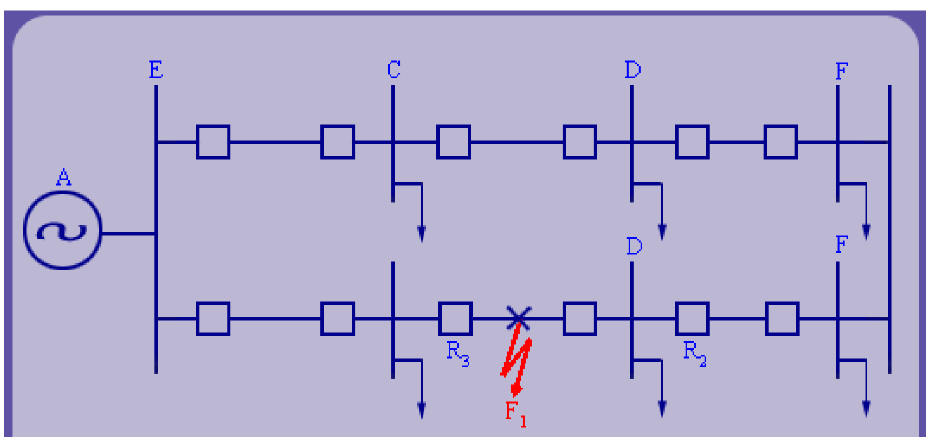
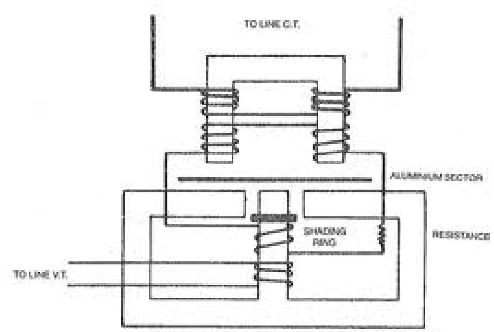


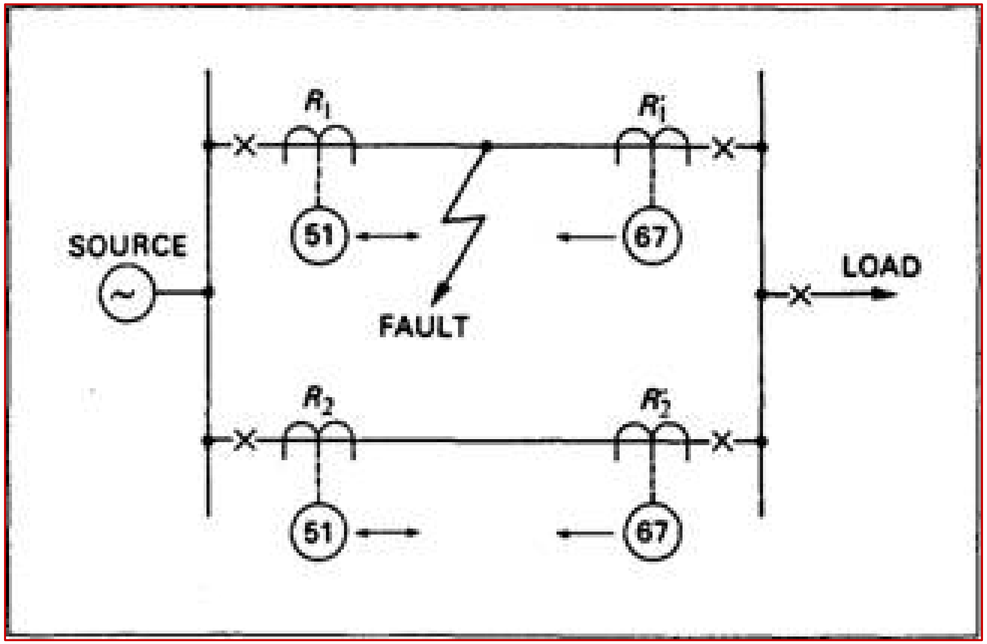
Fig 18.2 A Distribution System with Parallel Paths

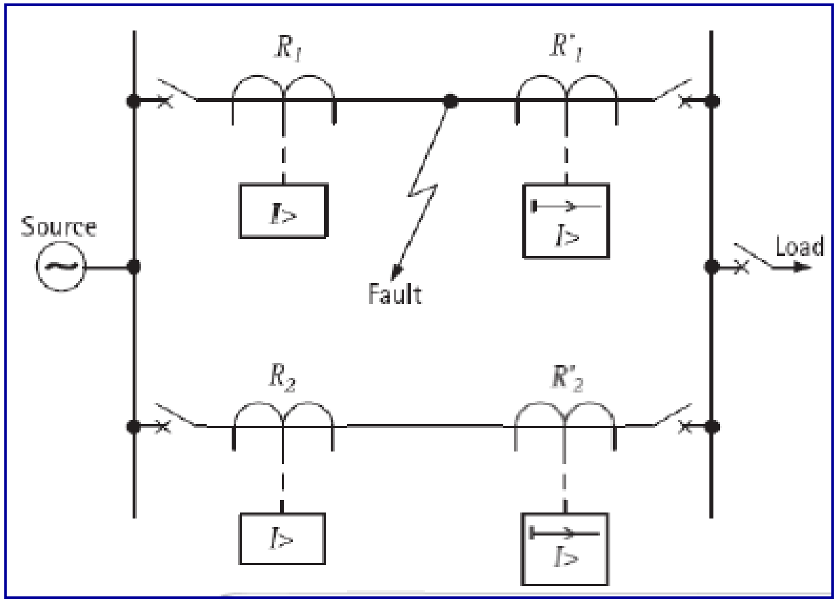


Directional Element: Electro-magnetic System.

Directional OC Relays-Parallel Feeders

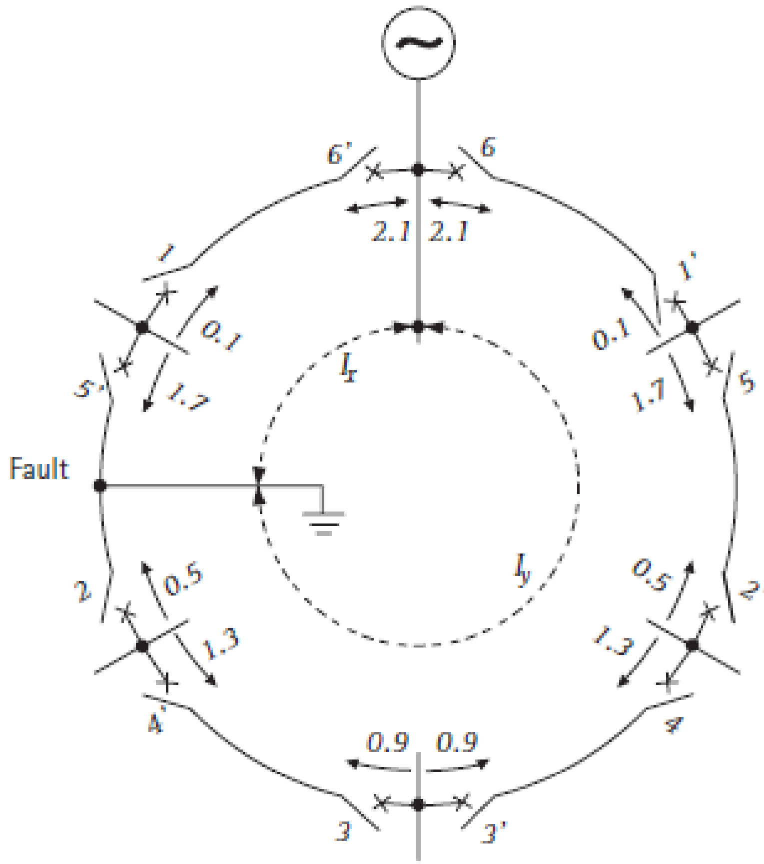
- This is done by setting the **directional relays R_1' and R_2'** in with their directional elements looking into the protected line, and giving them lower time and current settings than relays R_1 and R_2.
- The usual practice is to set relays **R_1' and R_2'** to **50% of the normal full load** of the protected circuit and **0.1TMS**.





Directional OC Relays- RING MAINS

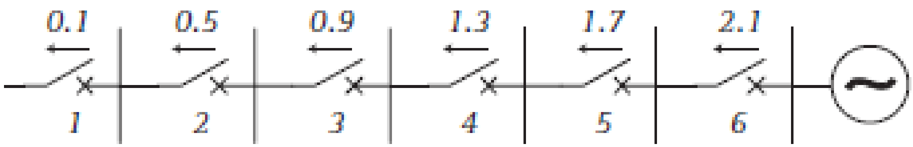
- A particularly common arrangement within distribution networks is the Ring Main. The primary reason for its use is to maintain supplies to consumers in case of fault conditions occurring on the interconnecting feeders.
- In a typical ring main with associated overcurrent protection, current may flow in either direction through the various relay locations, and therefore **directional overcurrent relays** are applied.
- With **modern numerical relays**, a directional facility is often available for little or no extra cost, so that it may be simpler in practice to apply **directional relays** at all locations



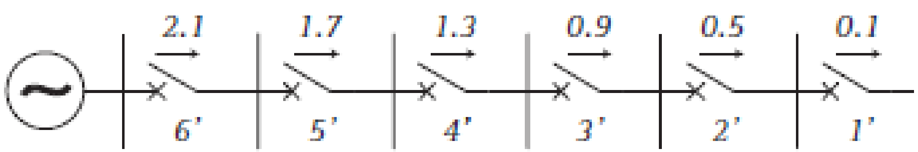
Grading of Ring Mains

- The usual grading procedure for relays in a ring main circuit is to open the ring at the supply point and to grade the relays first clockwise and then anti-clockwise.

- The relays looking in a **clockwise direction** around the ring are arranged to operate in the sequence 1-2-3-4-5-6



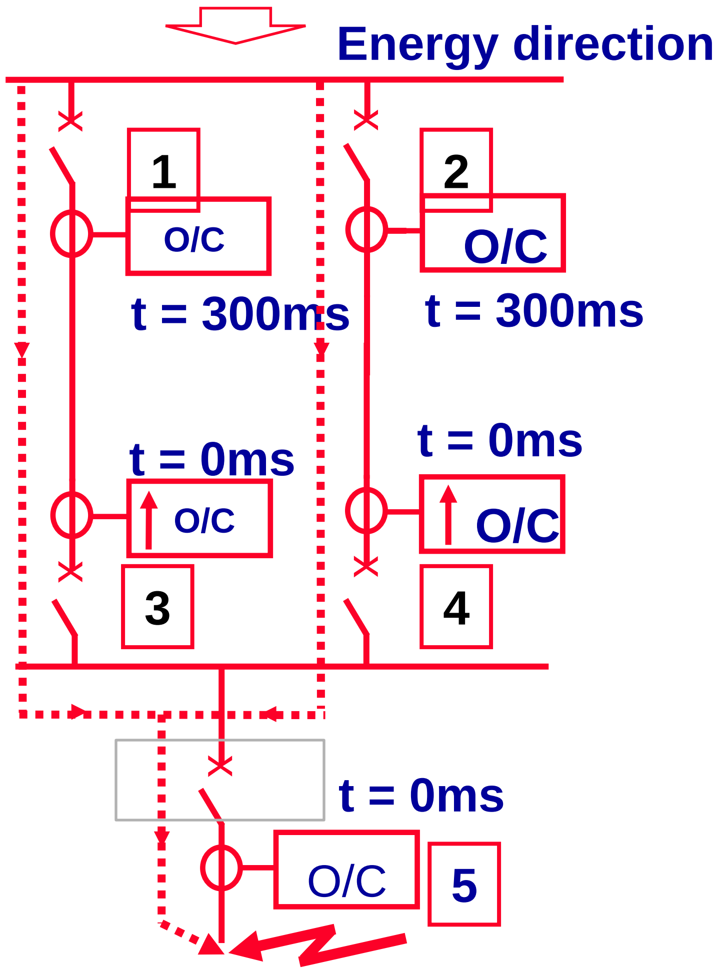
- The relays looking in the **anti-clockwise direction** are arranged to operate in the sequence 1’-2’-3’-4’-5’-6.



Grading of Ring Mains

- The arrows associated with the relaying points indicate the direction of current flow that will cause the relay to operate.
- A double-headed arrow is used to indicate a **non-directional relay**, such as those **at the supply point** where the **power can flow only in one direction**.
- A single-headed arrow is used to indicate a directional relay, such as those **at intermediate substations** around the ring where the **power can flow in either direction**.
- **The directional relays** are set in accordance with the invariable rule, applicable to all forms of directional protection, that the **current in the system must flow from the substation busbars into the protected line** in order that the relays may operate.

Directional Overcurrent Protection at parallel lines



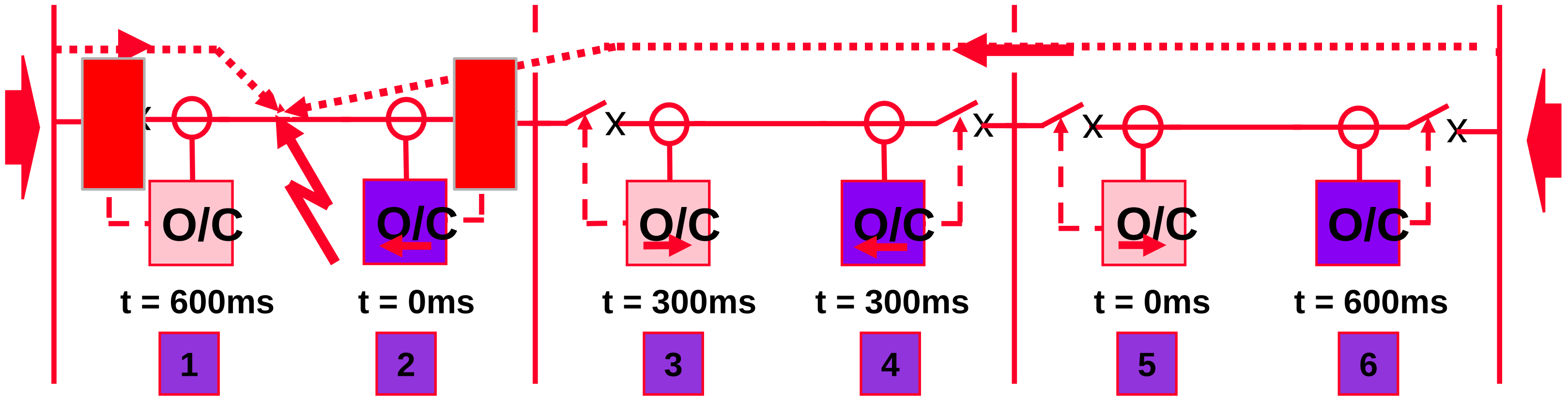
Devices 1 2 3 4 5 are started up

Devices 3 4 blocking (direction 'reverse')

Device 5 trips instantaneously

Devices 1 2 3 4 fall back.

Directional Overcurrent Time Protection by two-way supply



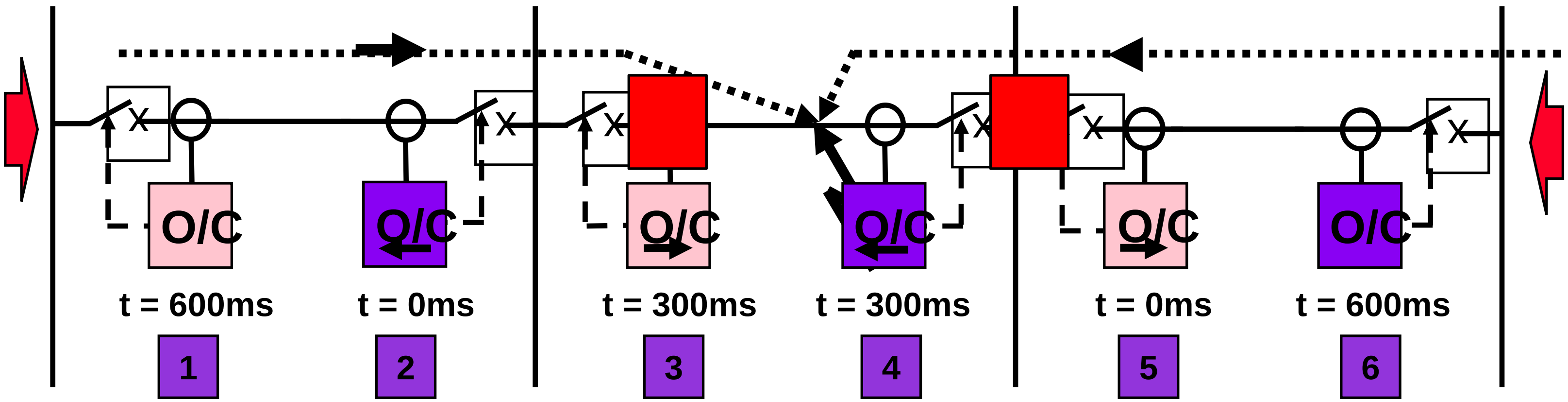
All devices are started

Device 2 trips instanteneously time

Device 3 4 5 6 fall back

Device 1 trips after 600ms

Directional Overcurrent Time Protection by two-way supply



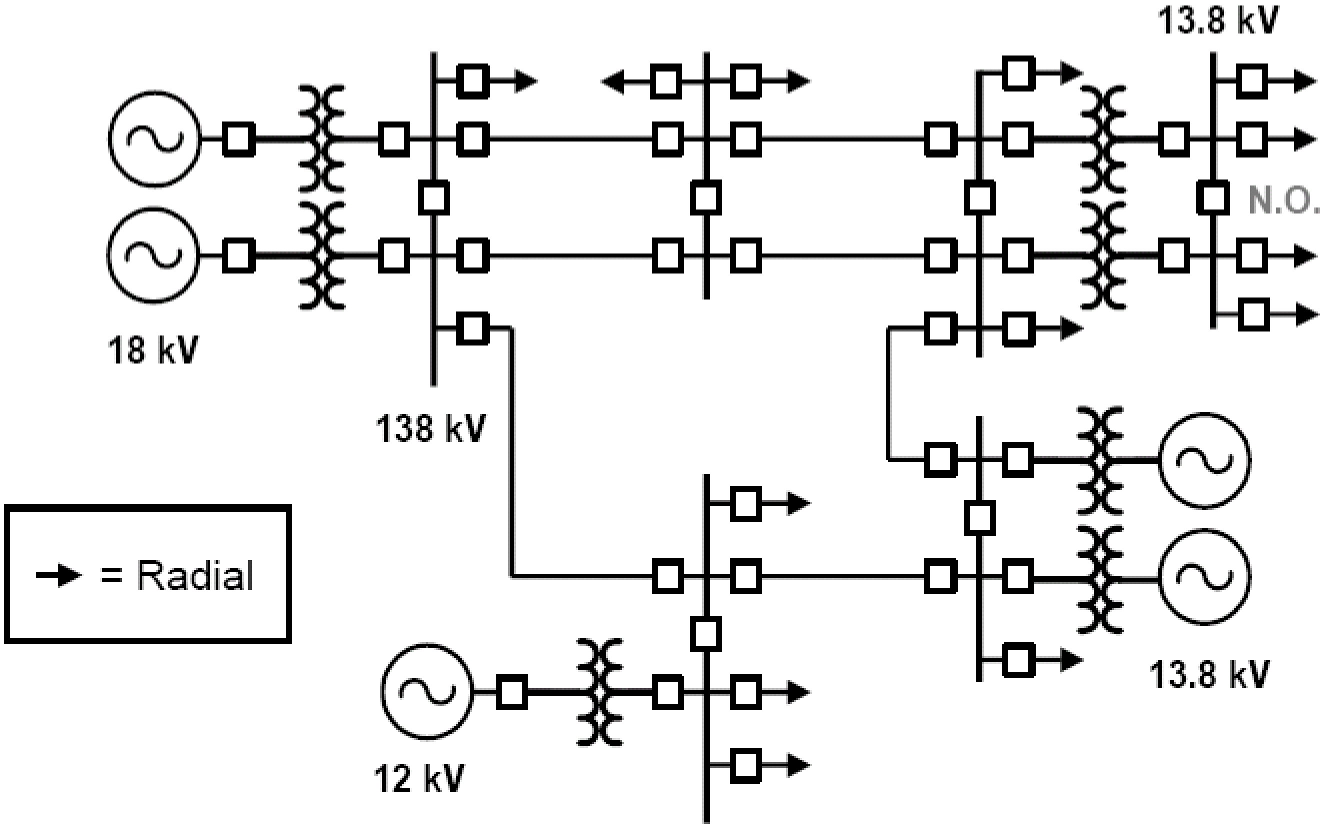
All devices are started up

Device 3 4 trips in 300ms

Devices 1 2 5 6 fall back

Directional Overcurrent Protection

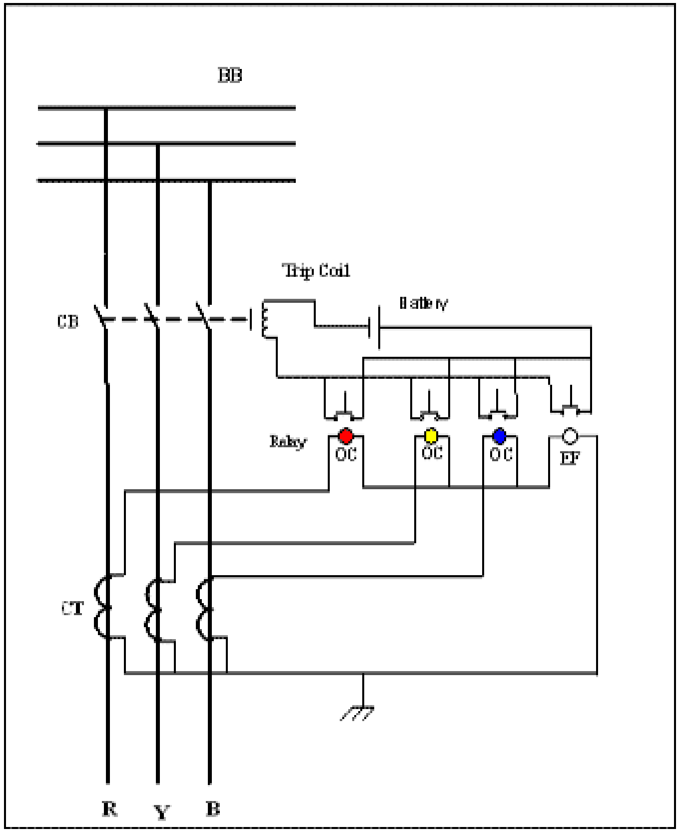
Exercise

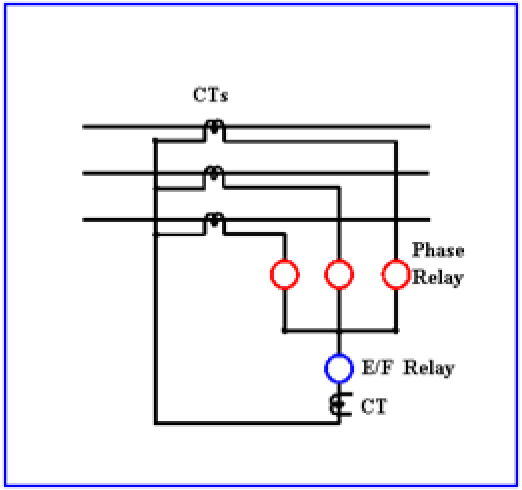


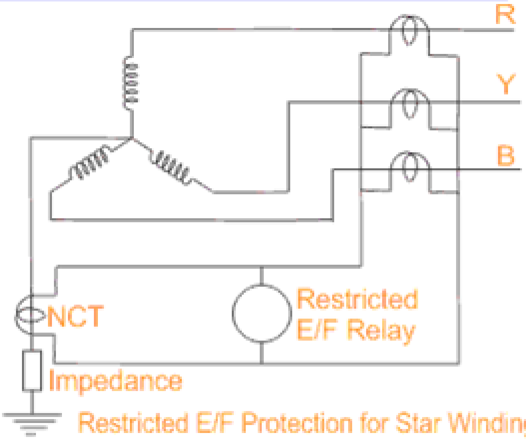
Which Relays Must be Directional?

Topic 3: Power System Protection

3.4-Prt 4: Earth-Fault Protection







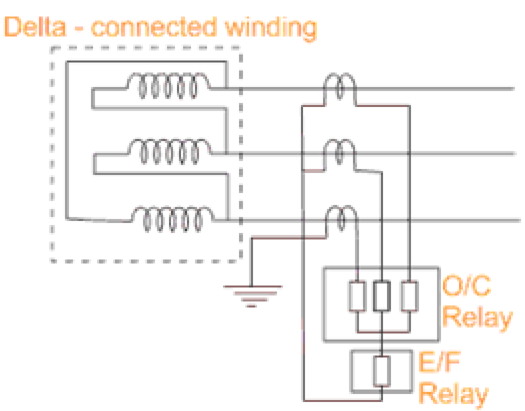
Restricted E/F Protection for Star Winding

Earth-Fault Protection

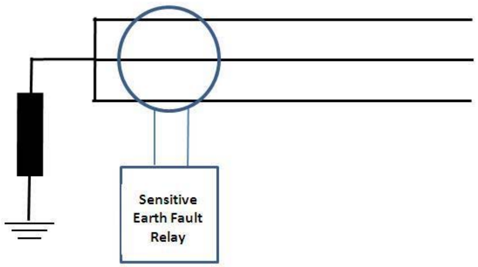
- When the fault current flows through earth return path, the fault is called ***Earth Fault***.
- Other faults which do not involve earth are called phase faults.
- Since earth faults are relatively frequent, earth fault protection is necessary in most cases.
- When separate earth fault protection is not economical, the phase relays sense the earth fault currents. However such protection lacks sensitivity. Hence separate earth fault protection is generally provided.
- Earth fault protection senses earth fault current. Following are the method of earth fault protection.

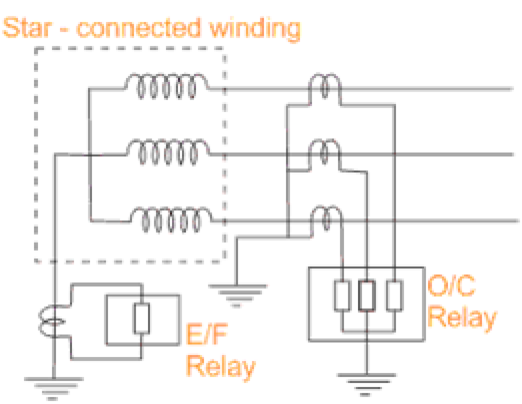
Methods of Earth-Fault Protection

1. Residually connected relay.
2. Relay connected in neutral-to-ground circuit.
3. Core-balance-scheme.
4. Distance relays arranged for detecting earth faults on lines.
5. Circulating current differential protection.



Backup O/C & E/F Protection Scheme





O/C & Unrestricted E/F Protection Scheme

Connections of CT's for Earth-Fault Protection

1. Residually connected Earth-fault Relay

- More sensitive protection against earth faults can be obtained by using a relay that responds only to the residual current of the system, since a residual component exists only when fault current flows to earth.
- In absence of earth-fault the vector sum of three line currents is zero. Hence the vector sum of three secondary currents is also zero.

$$\mathbf{I_R+I_Y+I_B=0}$$

The sum $(\mathbf{I_R+I_Y+I_B})$ is called residual current

- The earth-fault relay is connected such that the residual current flows through it (Fig.1 and Fig. 2).
- In the absence of earth-fault, therefore, the residually connected earth-fault relay does not operate.

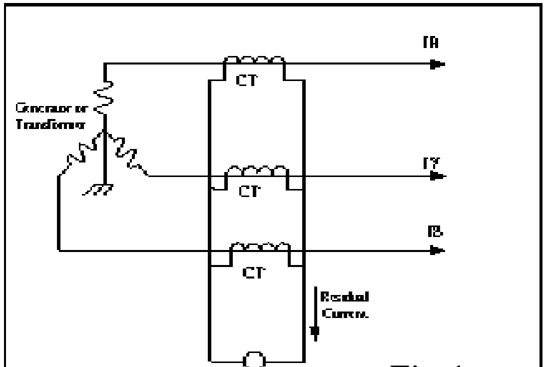


Fig.1

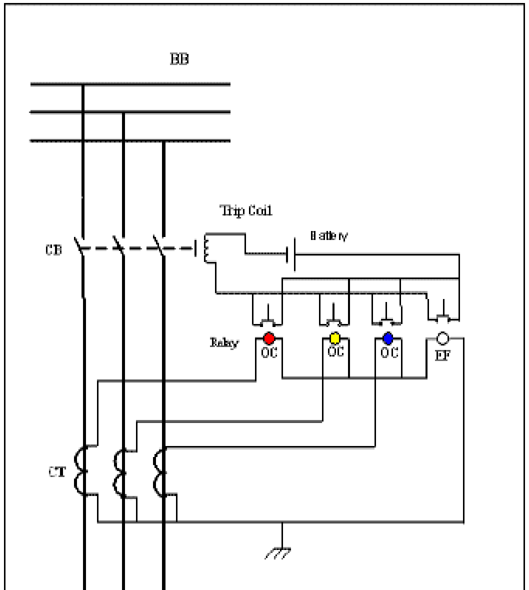


Fig.2

Connections of CT's for Earth-Fault Protection

1. Residually connected Earth-fault Relay

- More sensitive protection against earth faults can be obtained by using a relay that responds only to the residual current of the system, since a residual component exists only when fault current flows to earth.
- In absence of earth-fault the vector sum of three line currents is zero. Hence the vector sum of three secondary currents is also zero. $\mathbf{I_R+I_Y+I_B=0}$
- The sum $\mathbf{(I_R+I_Y+I_B)}$ is called residual current
- The earth-fault relay is connected such that the residual current flows through it (Fig.1 and Fig. 2).
- In the absence of earth-fault, therefore, the residually connected earth-fault relay does not operate.

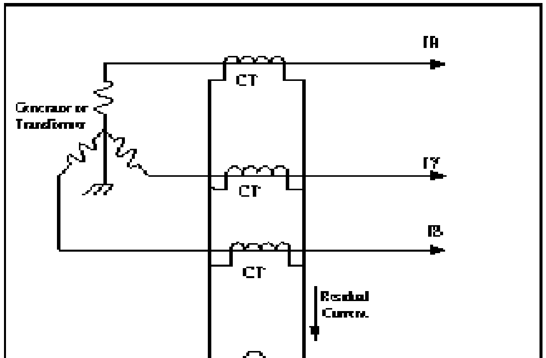


Fig.1

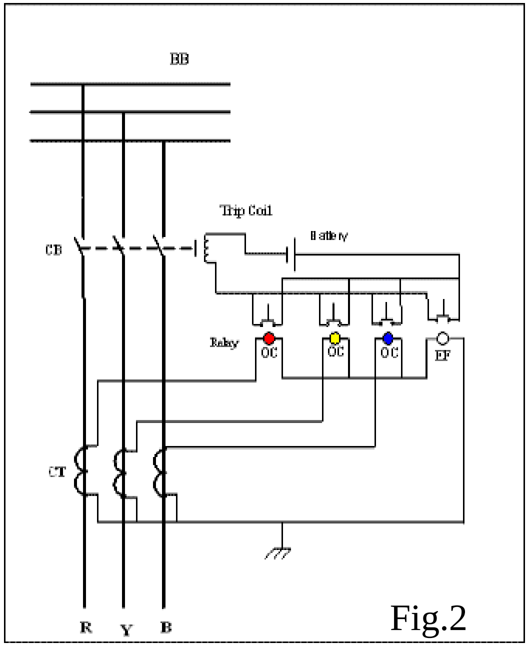
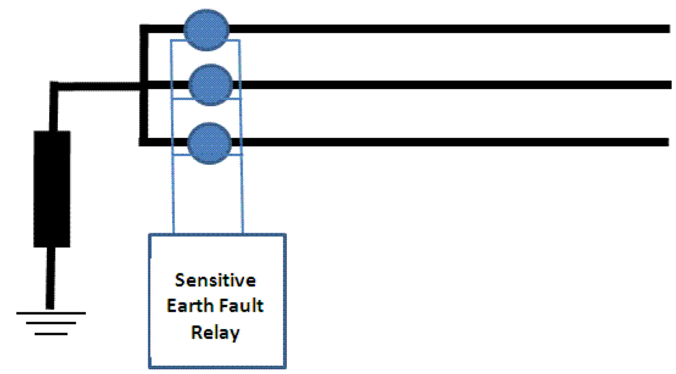


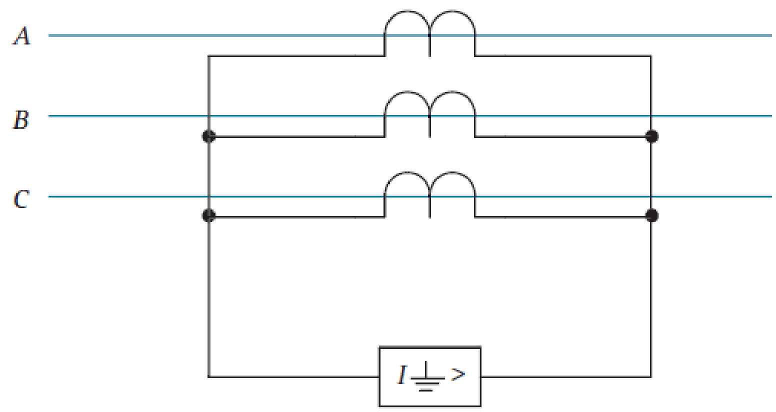
Fig.2

1. Residually connected Earth-fault Relay

- The residual component is extracted by connecting the line current transformers in parallel.



- However, in presence of earth fault the conditions is disturbed and **(I_R+I_Y+I_B)** is no more zero. Hence flows through the earth-fault relay. If the residual current is above the pick-up value, the earth-fault relay operates. (20%-40% of Full-Load Current)
- In the Residually Connection, the earth-fault at any location near or away from the location of CT's can cause the residual current flow. Hence the protected zone is not definite. Such protection is called **Unrestricted Earth-Fault Protection**



(a)

2. Earth-fault Relay connected in Neutral to Earth Circuit

- Another method of connecting an earth-fault relay is illustrated in Fig 3. The relay is connected to secondary of a CT whose primary is connected in neutral to earth connection.
- Such protection can be provided at various voltage levels by connecting earth-fault relay in the neutral-to-earth connection of that voltage level.
- The fault current finds the return path through the earth and then flows through the neutral-to-earth connection.

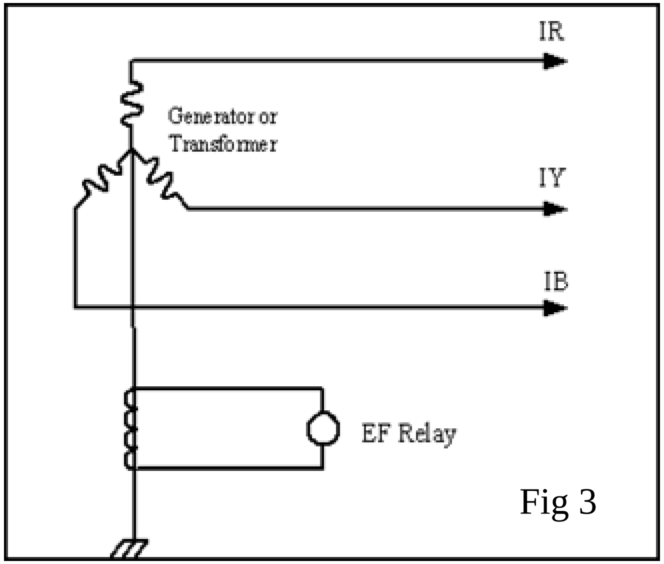


Fig 3

- The magnitude of earth fault current depends on type of earthing (resistance, reactance or solid) and location of fault. In this type of protection, the zone of protection cannot be accurately defined. The protected area is not restricted to the transformer/generator winding alone. The relay senses the earth faults beyond the transformer/generator winding hence such protection is called **Unrestricted Earth-Fault Protection.**

3. Combined Earth-Fault and Phase-Fault Protection

- It is convenient to incorporate phase-fault relays and earth-fault relay in a combined phase-fault and earth-fault protection. (Fig. 4)
- The increase in current of phase causes corresponding increase in respective secondary currents. The secondary current flows through respective relay-units Very often only two-phase relays are provided instead of three, because in case of phase faults current in any at least two phases must increase. Hence two relay-units are enough.

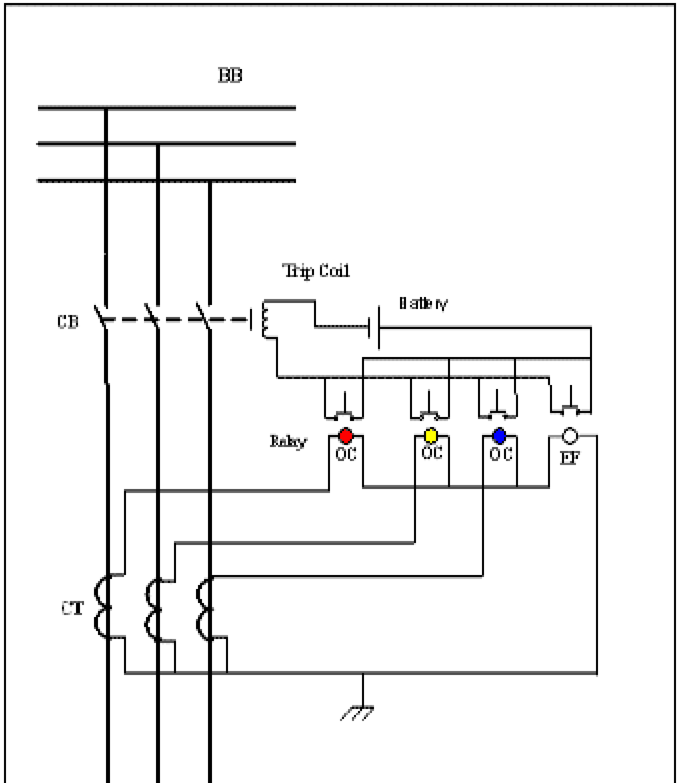
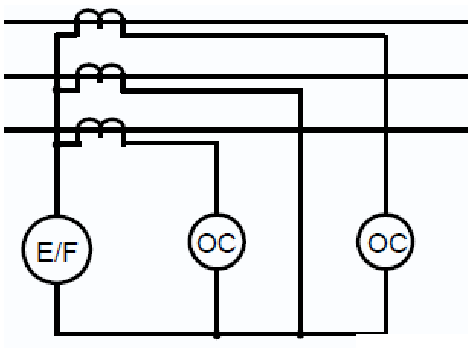
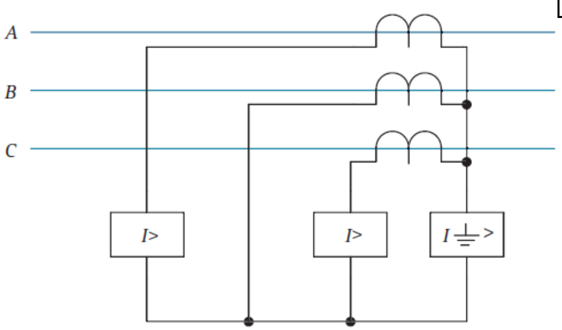
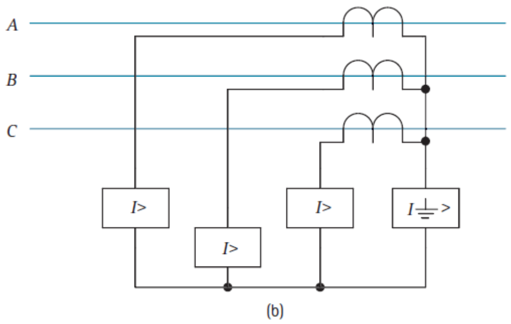


Fig. 4







Economize using 2x OC relays

4. Earth-fault Protection with Core Balance Current Transformers. Sensitive Earth-Fault Protection

- In this type of protection (Fig. 5) a single ring shaped core of magnetic material, encircles the conductors of all the three phases. A secondary coil is connected to a relay unit.
- The cross-section of ring-core is ample, so that saturation is not a problem.
- During no-earth-fault condition, the components of fluxes due to the fields of three conductors are balanced and the secondary current is negligible.
- During earth faults, such a balance is disturbed and current is induced in the secondary.
- Core-balance protection can be conveniently used for protection of low-voltage and medium voltage systems.

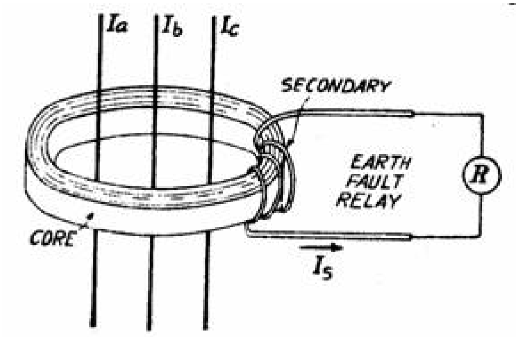
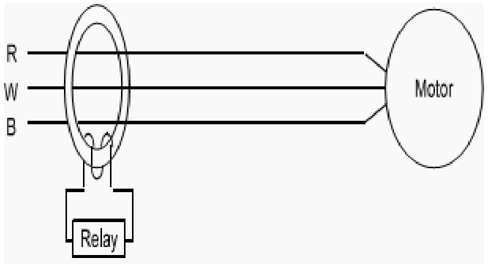


Fig. 5



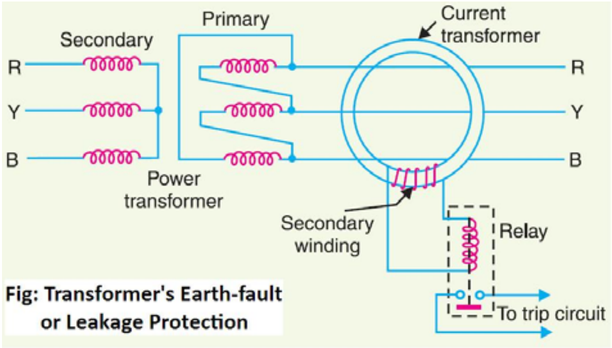
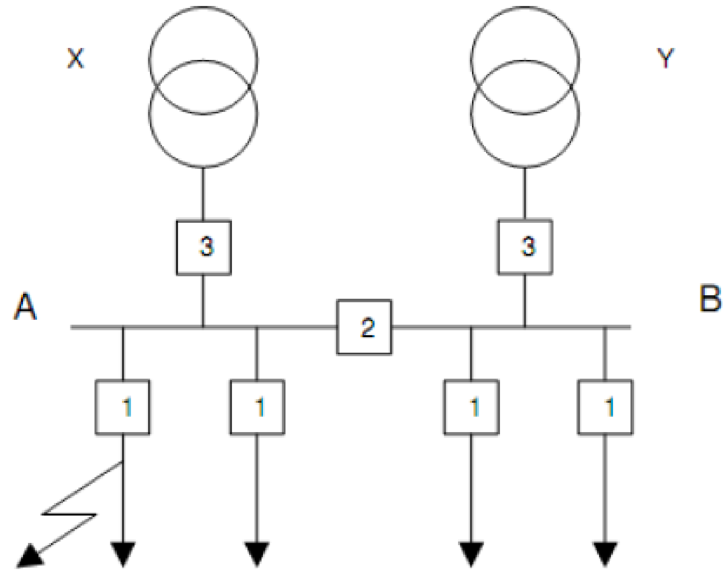


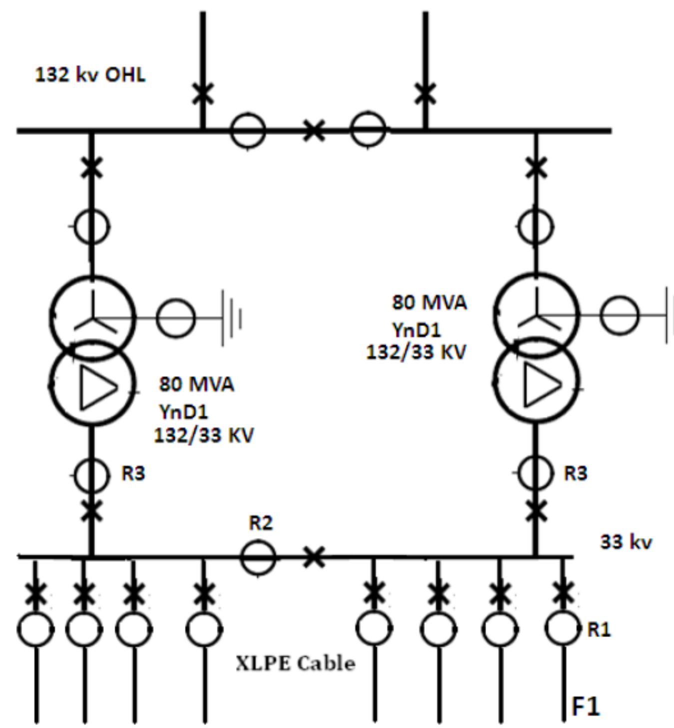
Fig: Transformer's Earth-fault or Leakage Protection

Overcurrent Coordination Example

➢ Coordination Criteria :

- In case of a fault along the feeder not cleared by relay 1 (for failure), relay 2 splits the busbar dividing the sound part of the network from the faulted one.
- In other words busbar section B continues feeding regularly its load through transformer Y whereas transformers X will open.
- Thus relay 3 should act only as back-up of relay 2 , otherwise in case of fault not cleared by relay 1, the operation of relays 3 causes the black out of the whole load fed by the substation.
- The setting of busbar and transformer protections should avoid possible superposition with downstream relays complying with a time margin higher than 250ms.





Question # 1:

The current plug (tap) settings (CTS) of a GEC 5-A overcurrent relay can be varied from 1 A to 12 A and the TMS can be varied from 0.5 to 10 as shown in Fig. 1. If the input current to the overcurrent relay is 10 A, determine the relay operating time for the following current tap setting (CTS) and time dial setting (TDS):

(a) CTS = 1.0 and TDS = ½;

(b) CTS = 2.0 and TDS = 1.5;

(c) CTS = 2.0 and TDS = 7;

(d) CTS = 3.0 and TDS = 7; and

(e) CTS = 12.0 and TDS = 1.

Use the overcurrent relay characteristics

$$t_p = TDS \times \left(\frac{A}{I_r^p - 1} + B \right)$$

t_p is the pickup or operating time

I_r is the ratio of $|I_f|/|I_p|$

$A = 28.2$, $B = 0.1217$ and $p = 2.0$

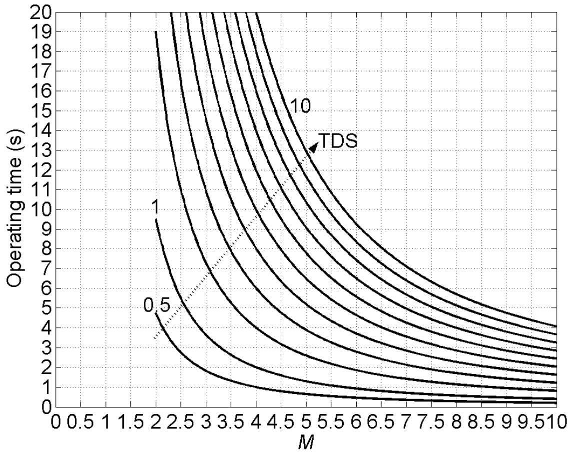


Fig. 1

Solution:

(a) CTS=1.0, then $I_r=|I_f|/|I_p|=10$. Therefore,

$$t_p = 0.5 \times \left(\frac{28.2}{10^2 - 1} + 0.1217 \right) = 0.2033 \sec$$

(b) CTS=2.0, then $I_r=|I_f|/|I_p|=5$. Therefore,

$$t_p = 1.5 \times \left(\frac{28.2}{5^2 - 1} + 0.1217 \right) = 1.945 \sec$$

(c) CTS=2.0, then $I_r=|I_f|/|I_p|=5$. Therefore,

$$t_p = 7 \times \left(\frac{28.2}{5^2 - 1} + 0.1217 \right) = 9.0769 \sec$$

(d) CTS=3.0, then $I_r=|I_f|/|I_p|=3.33$. Therefore,

$$t_p = 7 \times \left(\frac{28.2}{3.33^2 - 1} + 0.1217 \right) = 20.418 \sec$$

(e) CTS=12.0, then $I_r=|I_f|/|I_p|<1$. Therefore, the relay does not operate.

Question # 2:

The calculated short-circuit current through a feeder is 1200 A. An overcurrent relay of rating 5 A is connected for the protection of the feeder through a 600/5 A CT as shown in Fig. 2.

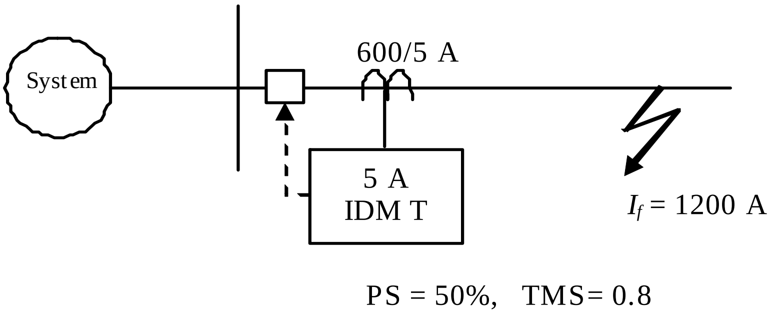


Fig. 2

Calculate the operating time of the relay when it has a plug setting (PS) of 50% and time multiple setting (TMS) of 0.8. The characteristic of the relay is as follows:

PSM	1.3	2	4	6	10	20	
Time (sec)	30	10	6.5	3.5	3	2.2	

Solution:

$$I_{pickup} = PS \times I_{rated} = 0.5 \times 5 = 2.5\ A$$

$$I_{f-relay} = \frac{I_f}{CTR} = \frac{1200}{600/5} = 10\ A$$

$$PSM = \frac{I_{f-relay}}{I_{pickup}} = \frac{10}{2.5} = 4$$ ➔ operating time at TMS = 1 is 6.5 s

Actual operating time $t_p = 6.5 \times TMS$ ➔ $t_p = 6.5 \times 0.8 = 5.2\ s$

Question #3:

Figure 3 shows a radial distribution system having identical IDMT overcurrent at A, B and C. For a time delay step (Δt) of 0.5 s, calculate the time multiplier settings (TMS) at A and B.

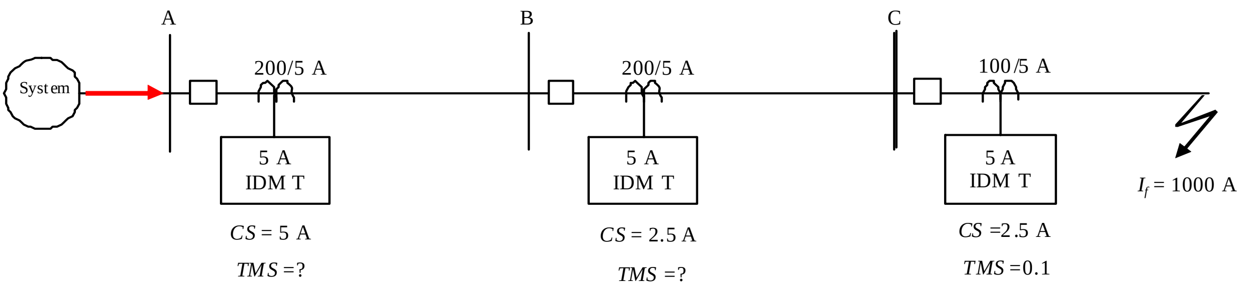


Fig. 3

The characteristic of the IDMT relay is as follows:

PSM	2	3	5	10	20
Time (sec)	10	6	4.5	3	2

Solution:

For relay C,

$$TMS_C = 0.1,\ I_{C-pickup} = 2.5\ A \quad I_{C-relay} = \frac{I_{Cf}}{CTR_C} = \frac{1000}{100/5} = 50\ A \quad PSM_C = \frac{I_{C-relay}}{I_{C-pickup}} = \frac{50}{2.5} = 20$$

➔ operating time at TMS = 1 is 2 s

Actual operating time of relay C is $t_p = 2 \times TMS$ ➔ $t_p = 2 \times 0.1 = 0.2\ s$

For relay B,

$$I_{B-pickup} = 2.5\ A \quad I_{C-relay} = \frac{I_f}{CTR_C} = \frac{1000}{200/5} = 25\ A \quad PSM_C = \frac{I_{C-relay}}{I_{C-pickup}} = \frac{25}{2.5} = 10$$

➔ operating time at TMS = 1 is 3 s

Actual operating time of relay B is t_p = 0.2 + 0.5 = 0.7 s = $3 \times TMS$ ➔ $\underline{TMS_B = \frac{0.7}{3} = 0.233}$

For relay A,

$$I_{A-pickup} = 5\ A \quad I_{A-relay} = \frac{I_f}{CTR_A} = \frac{1000}{200/5} = 25\ A \quad PSM_A = \frac{I_{A-relay}}{I_{A-pickup}} = \frac{25}{5} = 5$$

➔ operating time at TMS = 1 is 4.5 s

Actual operating time of relay A is t_p = 0.2 + 0.5+0.5 = 1.2 s = $4.5 \times TMS$ ➔ $\underline{TMS_A = \frac{1.2}{4.5} = 0.266}$

<u>Question #4:</u>

A 20 MVA Transformer which is used to operate at 30% overload feeds an 11 kV busbar through a circuit breaker (CB) as shown in Fig. 4. The transformer CB is equipped with a 1000/5 CT and the feeder CB with 400/5 CT and both CTs feed IDMT relays having the following characteristics

PSM	2	3	5	10	15	20
Time (sec)	10	6	4.1	3	2.5	2.2

The relay on the feeder CB has PS = 125% and TMS = 0.3. If a fault current of 5000 A flows from the transformer to the feeder, determine

a. operating time of feeder relay.

b. Suggest suitable PS and TMS of the transformer relay to ensure adequate discrimination of 0.5 s between the transformer relay and feeder relay.

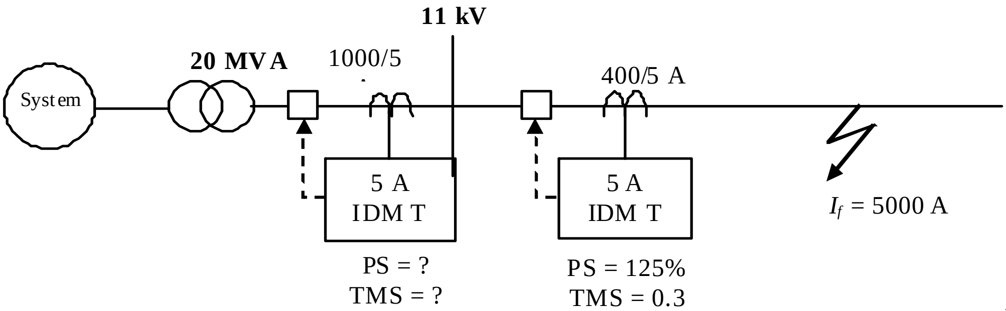


Fig. 4

Solution:

For Feeder relay

$TMS_{Feeder} = 0.3$, $PS = 125\%$ ➔ $I_{Feeder-pickup} = PS \times I_{rated} = 1.25 \times 5 = 6.25\ A$

$$I_{Feeder-relay} = \frac{I_f}{CTR_{Feeder}} = \frac{5000}{400/5} = 62.5\ A \quad PSM_{Feeder} = \frac{I_{Feeder-relay}}{I_{Feeder-pickup}} = \frac{62.5}{6.25} = 10$$

➔ operating time at TMS = 1 is 3 s

Actual operating time of the Feeder relay is $t_p = 3 \times TMS$ ➔ $t_p = 3 \times 0.3 = 0.9\ s$

For Transformer relay, $I_{Transformer-pickup} = PS \times I_{rated}$

Transformer overload current, $I_T = 1.3 \times \frac{S_{rated}}{\sqrt{3}V_{LL}} = 1.3 \times \frac{20 \times 10^6}{\sqrt{3} \times 11 \times 10^3} = 1365\ A$

$$I_{Transformer-relay} = \frac{I_{Transformer-overload}}{CTR_{Transformer}} = \frac{1365}{1000/5} = 6.825\ A$$

Since the transformer relay must not operate to overload current, $\mathrm{PS_{Transformer}} > \frac{\mathrm{I_{Transformer\text{-}relay}}}{\mathrm{I_{relay\text{-}rated}}}$

$\mathrm{PS_{Transformer}} > \frac{6.825}{5} > 1.365 \quad or\ 136.5\%$, the PS are restricted to standard values in steps of 25%, so the nearest value but higher than 136.5% is 150% ➔ $\mathrm{PS_{Transformer}} = 150\%$

$$I_{Transformer-pickup} = PS_{Transformer} \times I_{rated} = 1.5 \times 5 = 7.5\ A$$

$$PSM_{Transformer} = \frac{I_{f-Transformer-relay}}{I_{Transformer-pickup}} = \frac{5000/(1000/5)}{7.5} = \frac{25}{7.5} = 3.3$$

Operating time corresponding to $PSM_{Transformer} = 3.3$ and TMS=1 from the PSM-time curve is

t_p = 5.6 s, Actual operating time of transformer relay is t_p = 0.9 + 0.5 = 1.4 s = $3 \times TMS$

➔ $TMS_{Transformer} = \frac{1.4}{5.6} = 0.25$

EE482-Power System Analysis II

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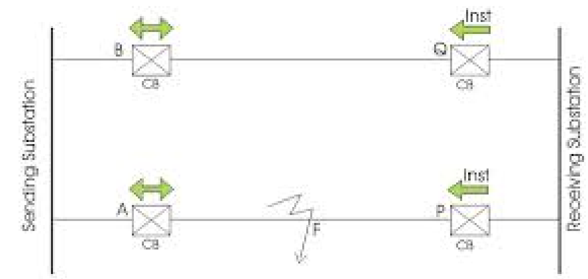
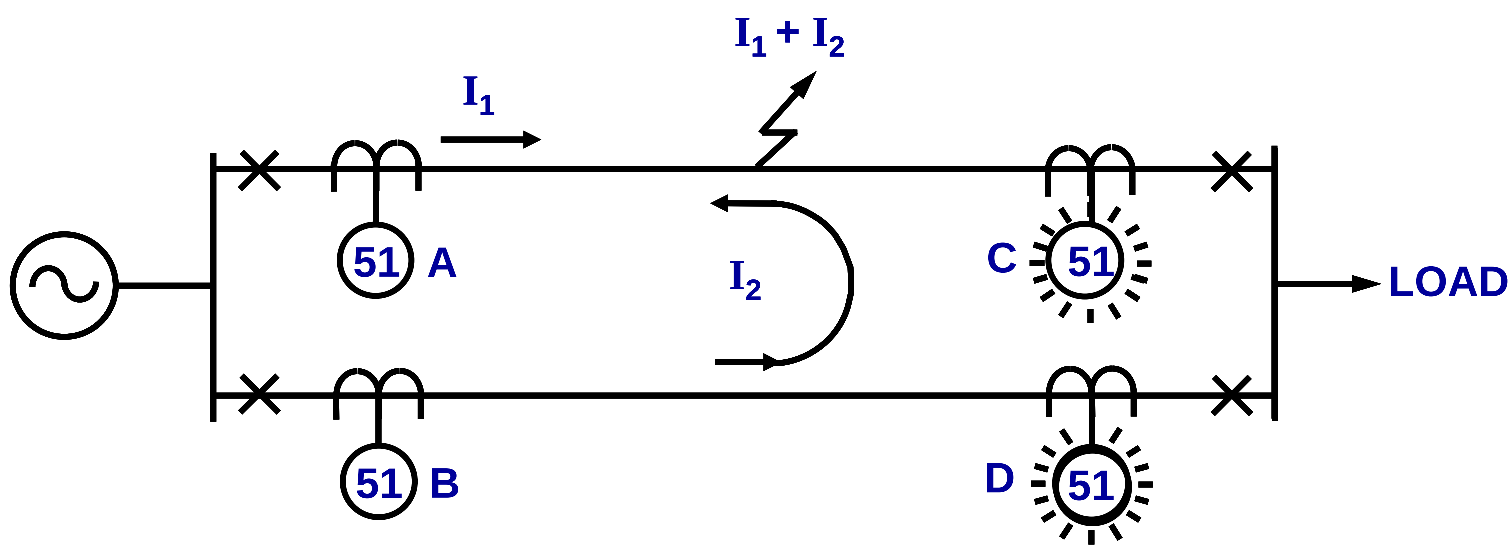
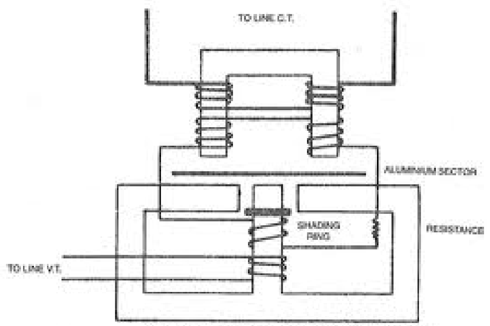


Topic 1-4: Prt3
Directional Over Current Protection



Directional Overcurrent Relays

- When fault current can flow in both directions through the relay location, it may be necessary to make the response of the relay directional by the introduction of a directional control facility. The facility is provided by use of additional voltage inputs to the relay.
- Directional over-current protection comprises over-current relay and power directional relay- in a single relay casing. The power directional relay does not measure the power but is arranged to respond to the direction of power flow.
- The directional relay recognizes the direction in which fault occurs, relative to the location of the relay. It is set such that it actuates for faults occurring in one direction only. It does not act for faults occurring in the other direction.



Directional Element: Electro-magnetic System.

Directional Overcurrent Relays

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- Directional over-current protection comprises over-current relay and power directional relay- in a single relay casing. The power directional relay does not measure the power but is arranged to respond to the direction of power flow.
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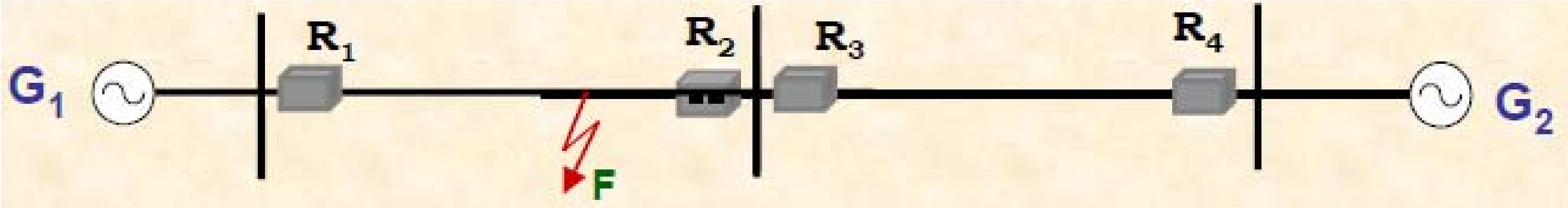


Directional Overcurrent Relays – Parallel Feeders

- If non-unit, non-directional relays are applied to **parallel feeders having a single generating source**, any faults that might occur on any one line will, regardless of the relay settings used, isolate both lines and completely disconnect the power supply.

- With this type of system configuration, it is necessary to apply **directional relays** at the **receiving end** and to grade them with the **nondirectional relays** at the **sending end**, to ensure correct discriminative operation of the relays during line faults.

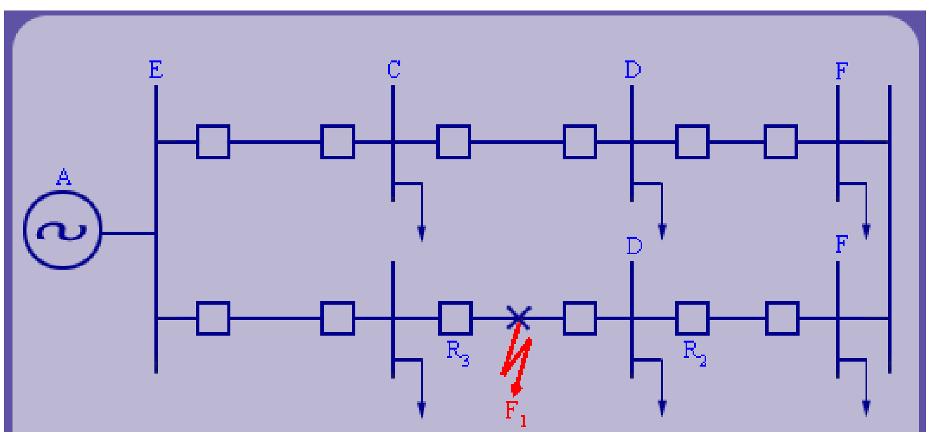
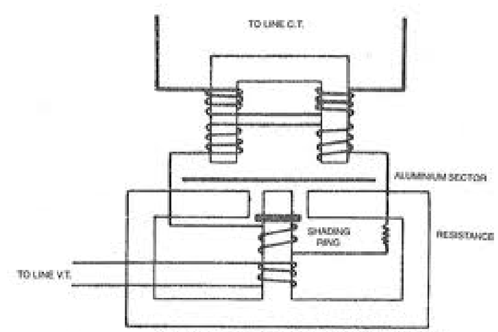


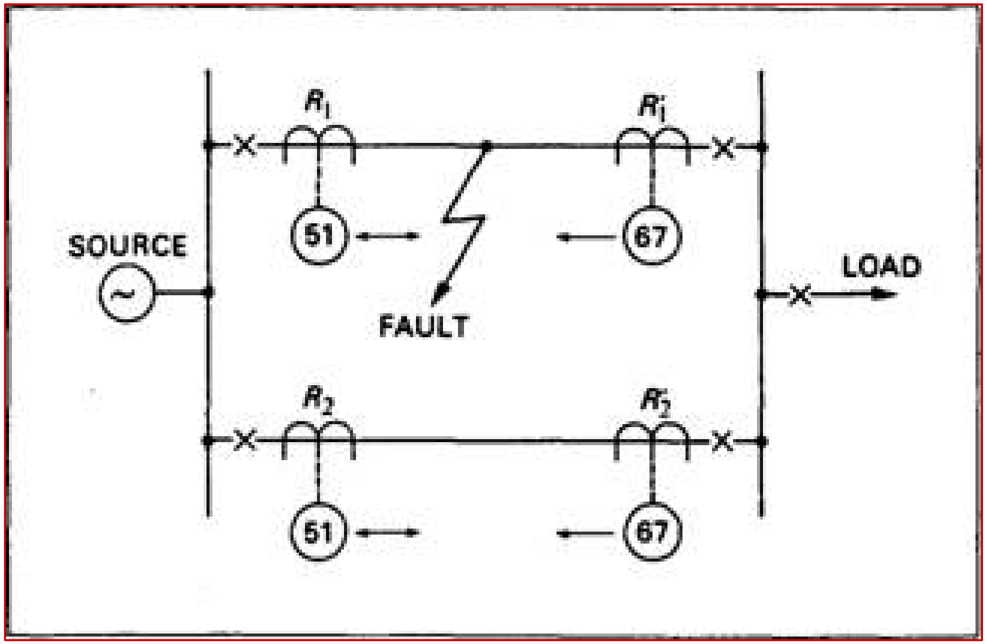
Fig 18.2 A Distribution System with Parallel Paths

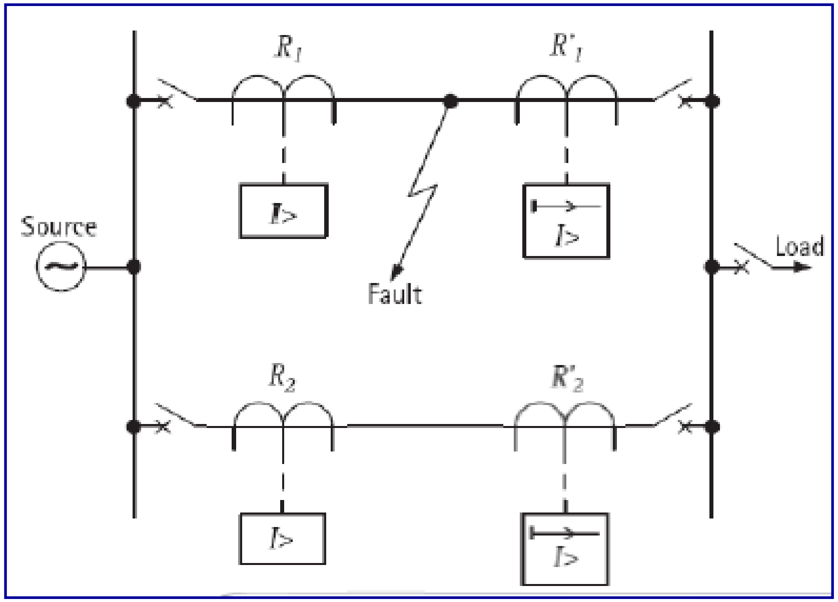


Directional Element: Electro-magnetic System.

Directional OC Relays-Parallel Feeders

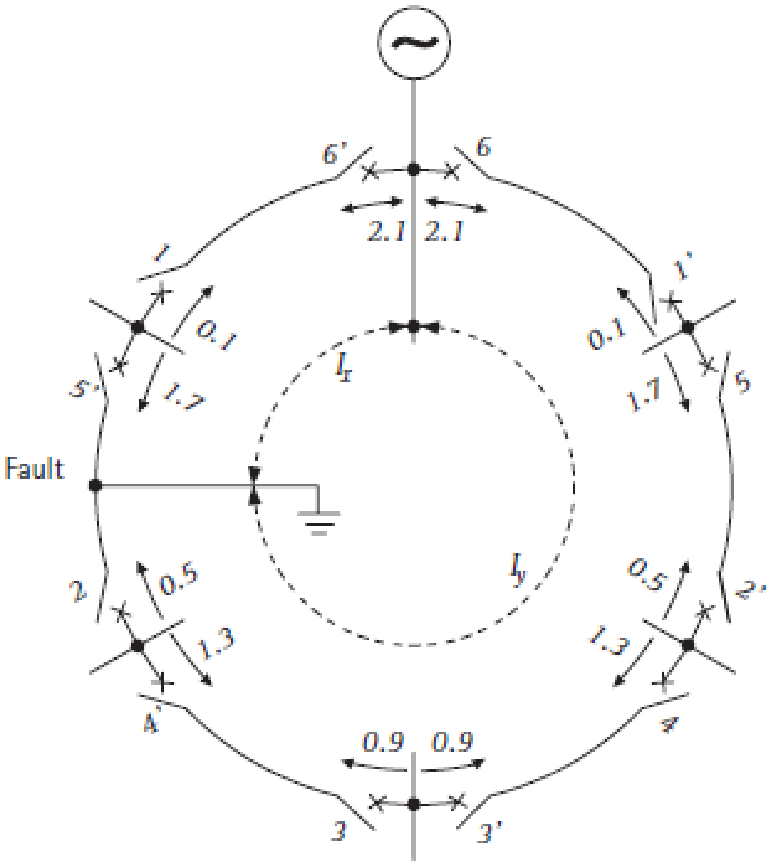
- This is done by setting the **directional relays R_1' and R_2'** in with their directional elements looking into the protected line, and giving them lower time and current settings than relays R_1 and R_2.
- The usual practice is to set relays **R_1' and R_2'** to **50% of the normal full load** of the protected circuit and **0.1TMS**.





Directional OC Relays- RING MAINS

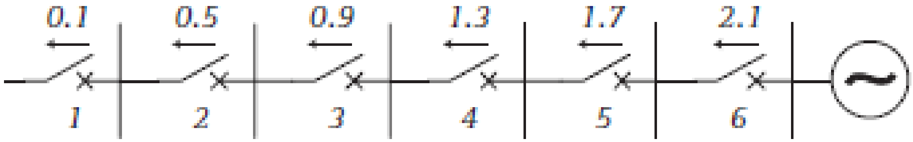
- A particularly common arrangement within distribution networks is the Ring Main. The primary reason for its use is to maintain supplies to consumers in case of fault conditions occurring on the interconnecting feeders.
- In a typical ring main with associated overcurrent protection, current may flow in either direction through the various relay locations, and therefore **directional overcurrent relays** are applied.
- With **modern numerical relays**, a directional facility is often available for little or no extra cost, so that it may be simpler in practice to apply **directional relays** at all locations



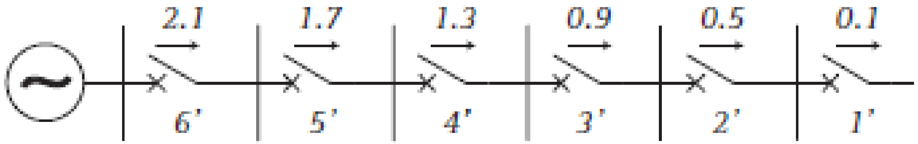
Grading of Ring Mains

◇ The usual grading procedure for relays in a ring main circuit is to open the ring at the supply point and to grade the relays first clockwise and then anti-clockwise.

◇ The relays looking in a **clockwise direction** around the ring are arranged to operate in the sequence 1-2-3-4-5-6



◇ The relays looking in the **anti-clockwise direction** are arranged to operate in the sequence 1'-2'-3'-4'-5'-6.



Grading of Ring Mains

- The arrows associated with the relaying points indicate the direction of current flow that will cause the relay to operate.
- A double-headed arrow is used to indicate a **non-directional relay**, such as those **at the supply point** where the **power can flow only in one direction**.
- A single-headed arrow is used to indicate a directional relay, such as those **at intermediate substations** around the ring where the **power can flow in either direction**.
- **The directional relays** are set in accordance with the invariable rule, applicable to all forms of directional protection, that the **current in the system must flow from the substation busbars into the protected line** in order that the relays may operate.

Problem # 1:
Referring to Fig. 1.1, determine the CT ratio, pickup and time dial settings for the relay at the C.B, assuming that no coordination with any other relay is required. Assume that the maximum load is 95 A, minimum fault is 600 A and the maximum fault is 1000 A. Select a CT ratio to give 5.0 A secondary current for maximum load.

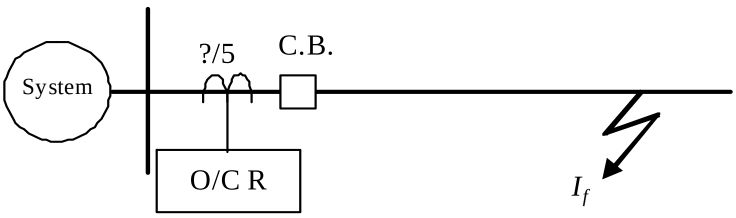


Fig. 1.1

Max. Load = 95 amperes
Min. Fault = 600 amperes
Max. Fault = 1000 amperes

Arreva P14x O/C protective devise.

Solution:

$$CTR \geq \frac{Maximum\,Load}{Desired\,CT\,Secondary\,Current}\frac{95}{5} = 19:1$$

Since this is not a standard CT ratio, select the nearest CT ratio of 20:1 or 100:5.

Table 1.1 Standard CT ratios.

Current Ratios							
50:5	100:5	150:5	200:5	250:5	300:5	350:5	400:5
450:5	500:5	600:5	800:5	900:5	1000:5	1200:5	1500:5
1600:5	2000:5	2400:5	2500:5	3000:5	3200:5	4000:5	5000:5

The relay pickup setting should be bracketed by twice the maximum load and one-third of the minimum fault. Using the actual CT ratio, twice maximum load is 190 A divided by 20, or a relay current of 9.5 A.

$$CS = I_{Pickup} = \frac{2I_{L\max}}{CTR} = \frac{2\times 95}{100/1} = \frac{190}{20} = 9.5\,A$$

Assuming the relay has taps 4.0, 6.0, 8.0, 10.0 and 12.0 A, then select the 10.0 A tap, giving a primary current relay pickup of 200A.

Effective pickup current $\Rightarrow$ $CS = I_{Pickup} = 20\times 10 = 200\,A(primary\,current)$
Dividing by 95 A load current results in a margin of 2.1×pu to prevent false operation (security). The minimum fault is 600 A divided by the relay pickup of 200A, which gives 3×pu to ensure correct operation (dependability).

For this configuration no coordination is required, so one can set the time delay at the lowest dial setting (fastest time) of ½.

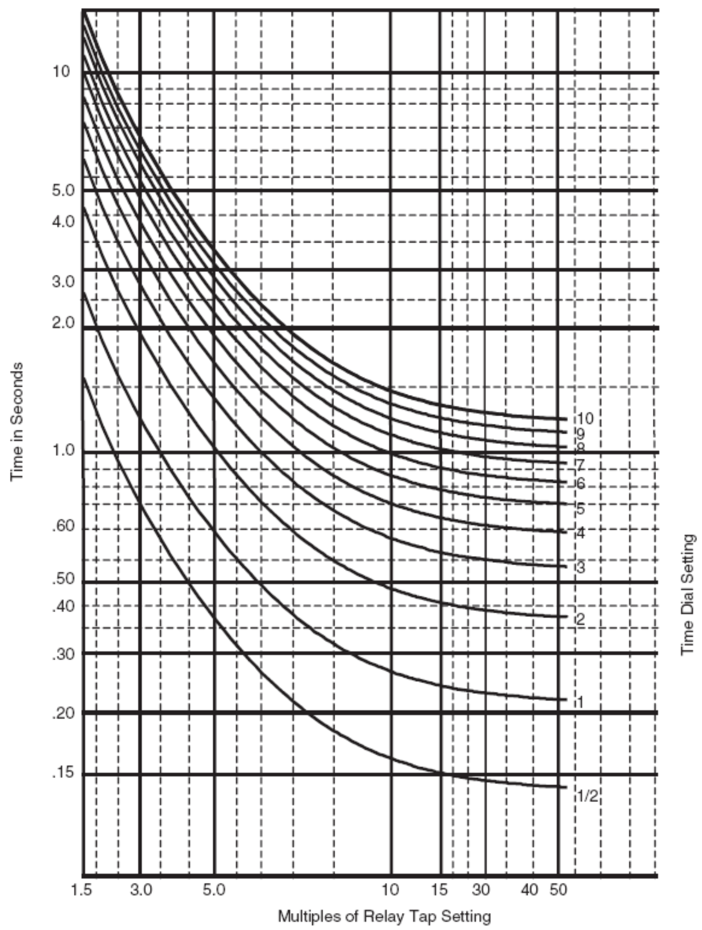


Fig. 1.2: Time-delay overcurrent relay operating characteristic

Problem # 2:

Consider the 11-kV radial system shown in Fig P2-a. Assume that all loads have the same power factor. Determine relay settings to protect the system assuming relay type CO-7 (with characteristics shown in Fig P2-b) is used.

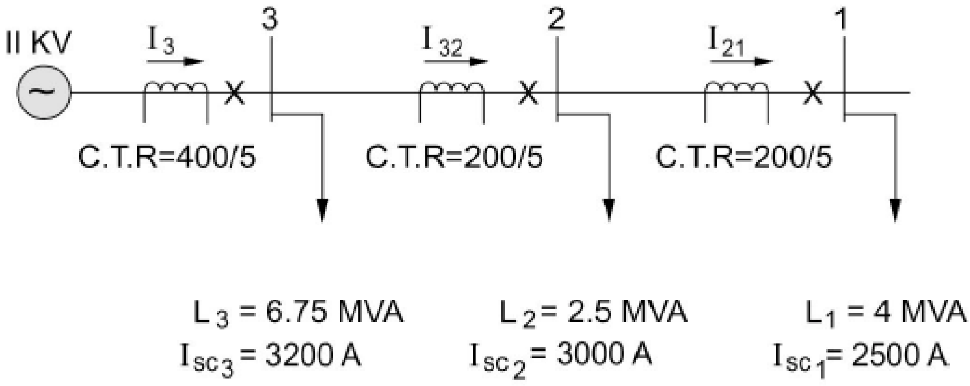


Fig.P2-a: An Example Radial System

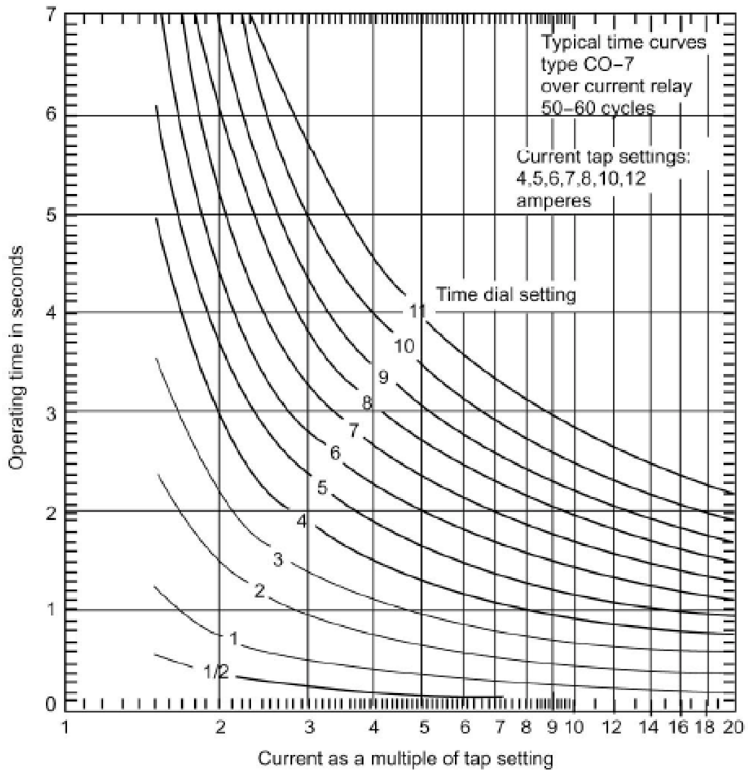


Fig. 2-b: CO-7 Time-Delay Overcurrent Relay Characteristics

Solution:

$$I_1 = \frac{4\times10^6}{\sqrt{3}\left(11\times10^3\right)} = 209.95\text{ A} \quad I_2 = \frac{2.5\times10^6}{\sqrt{3}\left(11\times10^3\right)} = 131.22\text{ A} \quad I_3 = \frac{6.75\times10^6}{\sqrt{3}\left(11\times10^3\right)} = 354.28\text{ A}$$

The normal currents through the sections are calculated as

$$I_{21} = I_1 = 209.95\text{ A} \quad I_{32} = I_{21} + I_2 = 341.16\text{ A} \quad I_S = I_{32} + I_3 = 695.44\text{ A}$$

With the current transformer ratios given, the normal relay currents are

$$i_{21} = \frac{209.92}{\frac{200}{5}} = 5.25\text{ A} \quad i_{32} = \frac{341.16}{\frac{200}{5}} = 8.53\text{ A} \quad i_S = \frac{695.44}{\frac{400}{5}} = 8.69\text{ A}$$

We can now obtain the current tap settings (C.T.S.) or pickup current in such a manner that the relay does not trip under normal currents. For this type of relay, the current tap settings available are 4, 5, 6, 7, 8, 10, and 12 amperes.

- For position 1, the normal current in the relay is 5.25 A; we thus choose $\underline{(\text{C.T.S.})_1 = 6\text{ A}}$,
- For position 2, the normal relay current is 8.53 A, and we choose $\underline{(\text{C.T.S.})2 = 10\text{ A}}$,
- Similarly for position 3, $\underline{(\text{C.T.S.})3 = 10\text{ A}}$.

Observe that we have chosen the nearest setting higher than the normal current.

The next task is to select the intentional delay indicated by the time dial setting (T.D.S.). We utilize the short-circuit currents calculated to coordinate the relays. The current in the relay at 1 on a short circuit at 1 is

$$i_{SC_1} = \frac{2500}{\left(\frac{200}{5}\right)} = 62.5\text{ A}$$

Expressed as a multiple of the pickup or C.T.S. value, we have

$$\frac{i_{SC_1}}{(\text{C.T.S.})_1} = \frac{62.5}{6} = 10.42$$

We choose the lowest T.D.S. for this relay for fastest action. Thus

$$\underline{(\text{T.D.S.})_1 = \frac{1}{2}}$$

By reference to the relay characteristic, we get the operating time for relay 1 for a fault at 1 as

$$T_{1_1} = 0.15\text{ s}$$

To set the relay at 2 responding to a fault at 1, we allow 0.1 second for breaker operation and an error margin of 0.3 second in addition to T_{11} . Thus,

$$T_{2_2} = T_{1_2} + 0.1 + 0.3 = 0.55\text{ s}$$

The short circuit for a fault at 1 as a multiple of the C.T.S. at 2 is

$$\frac{i_{SC_1}}{(\text{C.T.S.})_2} = \frac{62.5}{10} = 6.25$$

From the characteristics for 0.55-second operating time and 6.25 ratio, we get **$\underline{(\text{T.D.S.})_2 \approx 2}$**.
The final steps involve setting the relay at 3. For a fault at bus 2, the short-circuit current is 3000 A, for which relay 2 responds in a time $T22$ obtained as follows:

$$\frac{i_{SC_2}}{(\text{C.T.S.})_2} = \frac{3000}{\left(\frac{200}{5}\right)10} = 7.5$$

For the **$\underline{(\text{T.D.S.})_2 = 2}$**, we get from the relay's characteristic, $T_{22} = 0.50\text{ s}$.

Thus allowing the same margin for relay 3 to respond to a fault at 2, as for relay 2 responding to a fault at 1, we have

$$T_{32} = T_{22} + 0.1 + 0.3 = 0.90\text{ s}$$

The current in the relay expressed as a multiple of pickup is

$$\frac{i_{SC_2}}{(\text{C.T.S.})_3} = \frac{3000}{\left(\frac{400}{5}\right)10} = 3.75$$

Thus for $T3 = 0.90$, and the above ratio, we get from the relay's characteristic, **$\underline{(\text{T.D.S.})_3 \approx 2.5}$**

We note here that our calculations did not account for load starting currents that can be as high as five to seven times rated values. In practice, this should be accounted for.

Problem # 3:

Relay coordination on radial feeders using Use Extremely Inverse Relay Characteristics

For the radial power system shown i, Fig. 2.1 the CTR of the CTs and the relay current settings at buses 1-5 are given in Table 2.1. The relay current setting (CS) are given in % and in primary Amperes. Also, the minimum and maximum faults at buses 1-5 are given in Table 2.2.

Design an overcurrent protection for the above radial feeder using **Extremely Inverse Relay Characteristics**:

$$t = \left(\frac{28.2}{M^2 - 1} + 0.1217 \right) \times TDS$$

i.e. find the TDS considering a coordination time interval set to 0.4 s, and a TDS of relay at bus 5 (R5) set to TDS5 = 1.0.

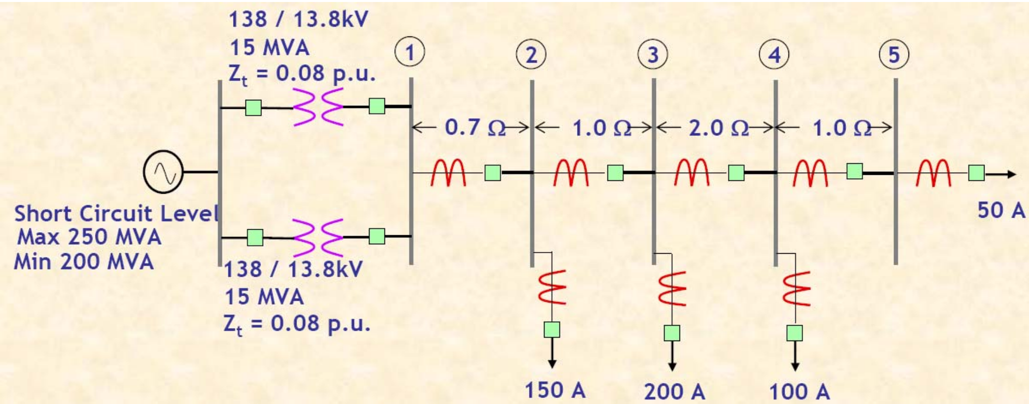


Figure 2.1: Radial system considered for relay overcurrent relay coordination study.

Table 2.1: CT Ratio and Relay Current Settings

Relay Location Bus	Maximum Load Current (A)	Selected CT Ratio	Relay Current Setting	
			Percent	Primary Current (A)
1	500	800/5	75 %	600
2	350	500/5	100 %	500
3	150	200/5	100 %	200
4	50	100/5	75 %	75
5	50	100/5	75 %	75

Table 2.2: Fault Current Calculations

Fault Location Bus	Minim Fault Current (A)	Maximum Fault Current (A)
1	4049	6274
2	2986	4045
3	2172	2683
4	1406	1603
5	1195	1335

Solution:

The principle of backup protection with O/C relays is that for any relay X backing up the next downstream relay Y, relay X must pick up:

a. **For one third of the minimum fault current seen by Y.**

b. **For the maximum fault current seen by Y but not sooner than 0.4 s after Y should have picked up for that current.**

Extremely Inverse overcurrent characteristic: $t_p = \left(\frac{28.2}{M^2-1}+0.1217\right)\times TDS$

1. Choosing relay 5 (R5) parameters

I_{5fmax} = 1335 A, I_{5fmin} = 1195 A, $I_{5pickup}$ = 75 A, TDS_5=1.0

$$t_{p5} = \left(\frac{28.2}{\left(\frac{1335}{75}\right)^2-1}+0.1217\right)\times 1.0 = \left(\frac{28.2}{(17.8)^2-1}+0.1217\right)\times 1.0 = 0.21s$$

2. Choosing relay 4 (R4) parameters

I_{4fmax} = 1603 A, I_{4fmin} = 1406 A, $I_{4pickup}$ = 75 A, TDS_4=?

The operating time of relay 5 (R5) for I_{5fmax} = 1335A is 0.21 s. The relay 4 (R4) will pickup for I_{5fmax}.
The operating time of relay 4 (R4) is:
t_{p4} = 0.21+0.4=0.61 s.

$$t_{p4} = \left(\frac{28.2}{\left(\frac{1335}{75}\right)^2-1}+0.1217\right)\times TDS_4 \quad \Rightarrow 0.61 = \left(\frac{28.2}{(17.8)^2-1}+0.1217\right)\times TDS_4$$

$$0.61 = 0.21\times TDS_4 \quad \Rightarrow TDS_4 = \frac{0.61}{0.21} = 2.9 \quad \Rightarrow TDS_4 = 3$$

The <u>actual</u> operating time of relay 4 (R4) for I_{5fmax} and TDS_4=3.0 is:

$$t_{p4} = \left(\frac{28.2}{(17.8)^2-1}+0.1217\right)\times 3 = 0.63s$$

The operating time of relay 4 (R4) for I_{4fmax} = 1603A is:

$$t_{p4} = \left(\frac{28.2}{\left(\frac{1603}{75}\right)^2-1}+0.1217\right)\times TDS_4 \quad = \left(\frac{28.2}{(21.37)^2-1}+0.1217\right)\times 3 = 0.55s$$

The operating time of relay 4 (R4) for I_{4fmin} = 1406A is:

$$t_{p4} = \left(\frac{28.2}{\left(\frac{1406}{75}\right)^2-1}+0.1217\right)\times TDS_4 \quad = \left(\frac{28.2}{(21.37)^2-1}+0.1217\right)\times 3 = 0.20\times 3 = 0.60s$$

3. Choosing relay 3 (R3) parameters

I_{3fmax} = 2683 A, I_{3fmin} = 2172 A, $I_{3pickup}$ = 200 A, TDS_3=?

The operating time of relay 4 (R4) for I_{4fmax} = 1603A is 0.55 s. The relay 3 (R3) will pickup for I_{4fmax}.
The operating time of relay 4 (R4) is:
t_{p3} = 0.55+0.4=0.95 s.

$$t_{p3}=\left(\frac{28.2}{\left(\frac{1603}{200}\right)^2-1}+0.1217\right)\times TDS_3 \quad \Rightarrow 0.95=\left(\frac{28.2}{(8.015)^2-1}+0.1217\right)\times TDS_3$$

$$0.95=0.568\times TDS_3 \quad \Rightarrow TDS_3=\frac{0.95}{0.568}=1.67 \quad \Rightarrow TDS_3=1.7$$

The <u>actual</u> operating time of relay 3 (R3) for I_{4fmax} and TDS_3=1.7 is:

$$t_{p3}=\left(\frac{28.2}{(8.015)^2-1}+0.1217\right)\times 1.7=0.568\times 1.70=0.965\,s$$

The operating time of relay 3 (R3) for I_{3fmax} = 2683A is:

$$t_{p3}=\left(\frac{28.2}{\left(\frac{2683}{200}\right)^2-1}+0.1217\right)\times TDS_3 \quad =\left(\frac{28.2}{(13.415)^2-1}+0.1217\right)\times 1.7=0.48\,s$$

The operating time of relay 3 (R3) for I_{3fmin} = 2172A is:

$$t_{p3}=\left(\frac{28.2}{\left(\frac{2172}{200}\right)^2-1}+0.1217\right)\times TDS_3 \quad =\left(\frac{28.2}{(21.37)^2-1}+0.1217\right)\times 1.7=0.363\times 1.7=0.62\,s$$

4. Choosing relay 2 (R2) parameters

I_{2fmax} = 4045 A, I_{2fmin} = 2986 A, $I_{2pickup}$ = 500 A, TDS_2=?

The operating time of relay 2 (R2) for I_{3fmax} = 2683A is 0.48 s. The relay 2 (R2) will pickup for I_{3fmax}.
The operating time of relay 2 (R2) is:
t_{p2} = 0.48+0.4=0.88 s.

$$t_{p2}=\left(\frac{28.2}{\left(\frac{2683}{500}\right)^2-1}+0.1217\right)\times TDS_2 \quad \Rightarrow 0.88=\left(\frac{28.2}{(5.366)^2-1}+0.1217\right)\times TDS_2$$

$$0.88=1.136\times TDS_2 \quad \Rightarrow TDS_2=\frac{0.88}{1.136}=0.77 \quad \Rightarrow TDS_2=0.8$$

The <u>actual</u> operating time of relay 2 (R2) for I_{3fmax} and TDS_2=0.8 is:

$$t_{p2}=\left(\frac{28.2}{(5.366)^2-1}+0.1217\right)\times 0.8=1.136\times 0.8=0.90\,s$$

The operating time of relay 2 (R2) for I_{2fmax} = 4045A is:

$$t_{p2}=\left(\frac{28.2}{\left(\frac{4045}{500}\right)^2-1}+0.1217\right)\times TDS_2 \quad =\left(\frac{28.2}{(8.09)^2-1}+0.1217\right)\times 0.9=0.50\,s$$

The operating time of relay 2 (R2) for I_{4fmin} = 2986A is:

$$t_{p2}=\left(\frac{28.2}{\left(\frac{2986}{500}\right)^2-1}+0.1217\right)\times TDS_2 \quad =\left(\frac{28.2}{(5.972)^2-1}+0.1217\right)\times 0.9=0.935\times 0.9=0.84\,s$$

5. Choosing relay 1 (R1) parameters

I_{1fmax} = 6274 A, I_{1fmin} = 4049 A, $I_{1pickup}$ = 600 A, TDS_1=?

The operating time of relay 2 (R2) for I_{2fmax} = 4045A is 0.5 s. The relay 1 (R1) will pickup for I_{2fmax}. The operating time of relay 1 (R1) is:

t_{p1} = 0.5+0.4=0.9 s.

$$t_{p1}=\left(\frac{28.2}{\left(\frac{4045}{600}\right)^2-1}+0.1217\right)\times TDS_1 \quad \Rightarrow 0.9=\left(\frac{28.2}{(6.74)^2-1}+0.1217\right)\times TDS_1$$

$$0.9=0.756\times TDS_1 \quad \Rightarrow TDS_1=\frac{0.9}{0.756}=1.19 \quad \Rightarrow TDS_1=1.2$$

The <u>actual</u> operating time of relay 1 (R1) for I_{2fmax} and TDS_1=1.2 is:

$$t_{p1}=\left(\frac{28.2}{(6.74)^2-1}+0.1217\right)\times 1.2=0.756\times 1.2=0.908s$$

The operating time of relay 2 (R2) for I_{1fmax} = 6274A is:

$$t_{p1}=\left(\frac{28.2}{\left(\frac{6274}{600}\right)^2-1}+0.1217\right)\times TDS_1 \quad =\left(\frac{28.2}{(10.46)^2-1}+0.1217\right)\times 1.2=0.46\,s$$

The operating time of relay 1 (R1) for I_{1fmin} = 4049A is:

$$t_{p1}=\left(\frac{28.2}{\left(\frac{4049}{600}\right)^2-1}+0.1217\right)\times TDS_1 \quad =\left(\frac{28.2}{(6.748)^2-1}+0.1217\right)\times 1.2=0.755\times 1.2=0.905\,s$$

Table 2.3: CT Ratio and Relay CS and TD Settings

Relay Location Bus	Selected CT Ratio	Relay Current Setting			Time Dial Setting
		Percent	Primary Current (A)	Secondary Current (A)	TDS
1	**800/5**	**75 %**	**600**	**3.75**	**1.2**
2	**500/5**	**100 %**	**500**	**5**	**0.8**
3	**200/5**	**100 %**	**200**	**5**	**1.7**
4	**100/5**	**75 %**	**75**	**3.75**	**3**
5	**100/5**	**75 %**	**75**	**3.75**	**1**

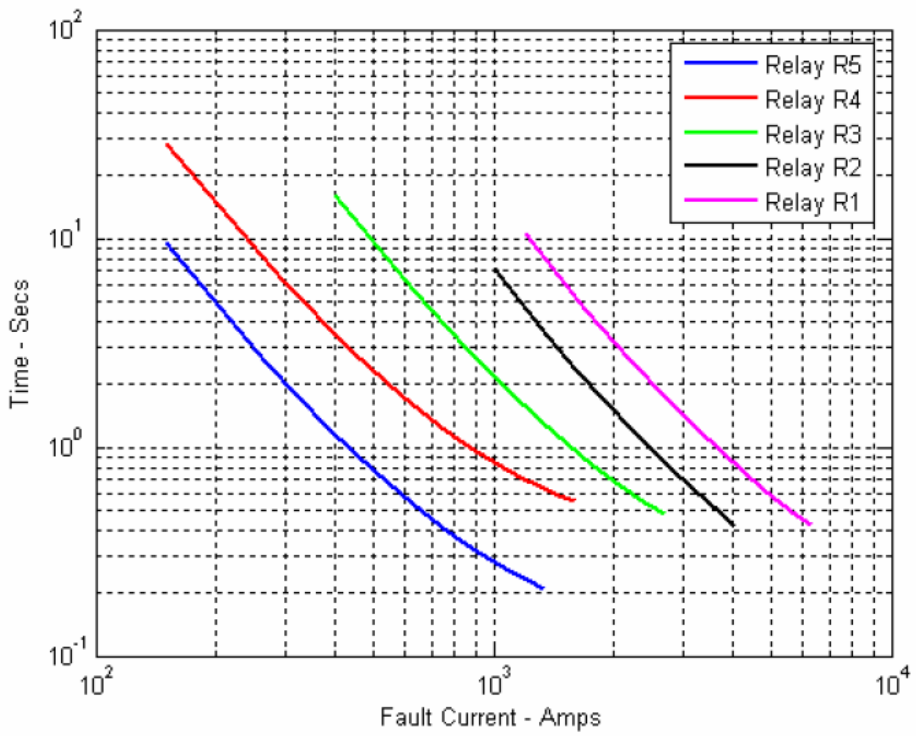
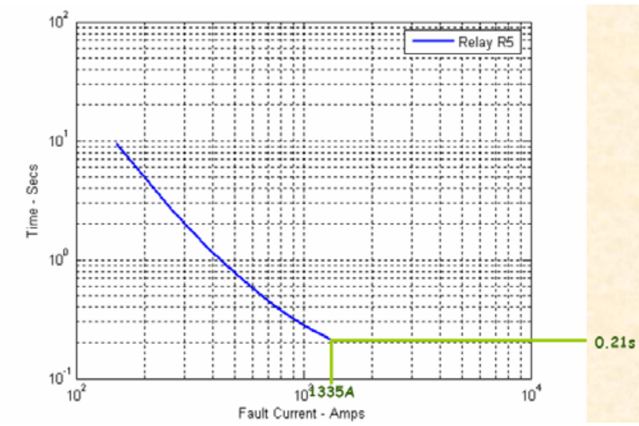
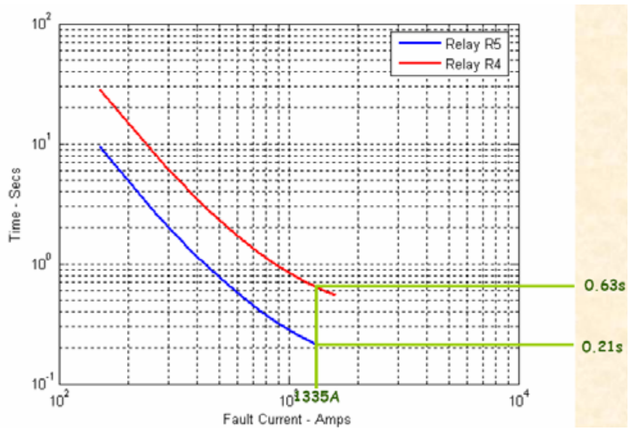
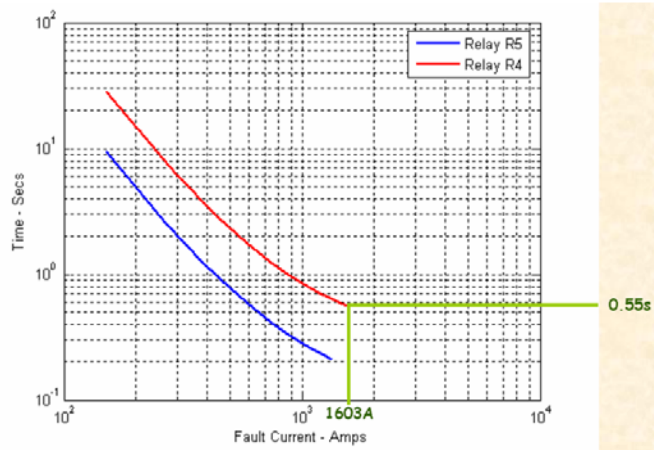
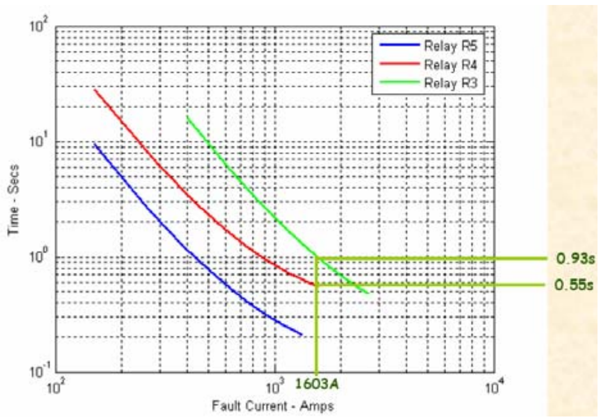


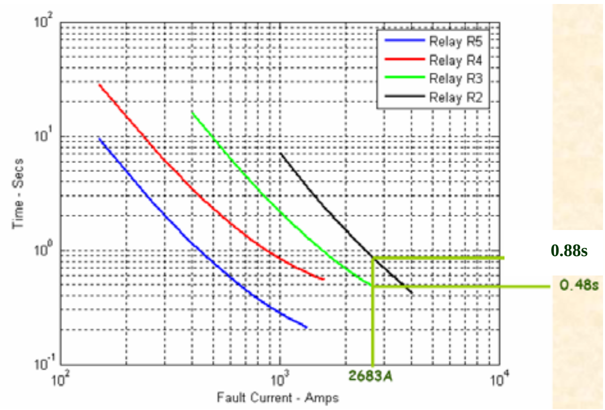
Fig. 2.2: overcurrent relay operating characteristic of R1-R5

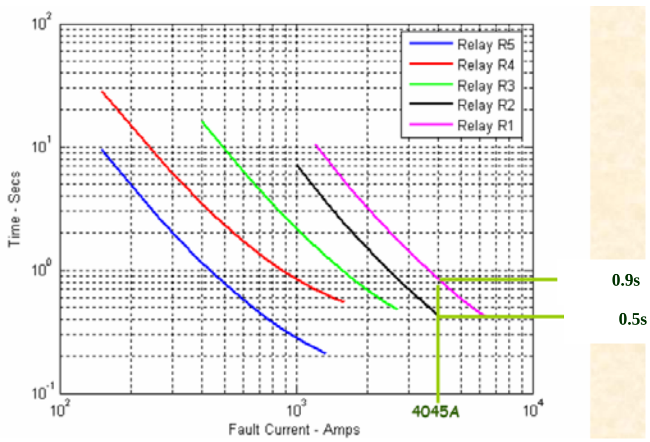

Relay R5
Time - Secs
Fault Current - Amps
0.21s
1335A


Relay R5
Relay R4
Time - Secs
Fault Current - Amps
0.63s
0.21s
1335A


Relay R5
Relay R4
Time - Secs
Fault Current - Amps
0.55s
1603A


Relay R5
Relay R4
Relay R3
Time - Secs
Fault Current - Amps
0.93s
0.55s
1603A


Relay R5
Relay R4
Relay R3
Relay R2
Time - Secs
Fault Current - Amps
0.88s
0.48s
2683A


Relay R5
Relay R4
Relay R3
Relay R2
Relay R1
Time - Secs
Fault Current - Amps
0.9s
0.5s
4045A

Problem # 4:

Figure 3 shows a network that is protected by Normal Inverse Overcurrent IDMT relays whose *t-I* relay characteristic and *PS%* are given by:

$$t = \frac{3}{\log(PSM)} \times TSM = \frac{3}{\left(\frac{I_f}{I_{pickup}}\right)} \times TSM$$

PS%: 50%, 75%, 100%, 125%, 150%, 175% and 200%.

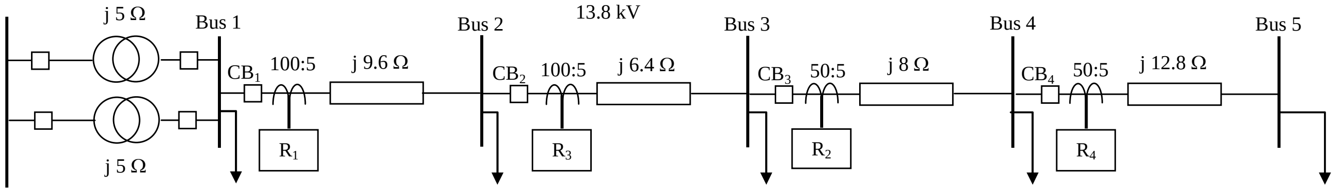


Fig. 3.

The minimum and maximum fault currents are given in Table 3-a.

Table 3-a: Minimum and Maximum Fault Currents

Bus No.	1	2	3	4	5
Minimum Fault Current (A), I_{fmin}	1380	472.6	328.6	237.9	165.1
Maximum Fault Current (A), I_{fmax}	3187	658	431	301	203

a. Select the plug setting multiplier (PSM) and time dial settings (TMS) for the relays R_4, R_3, R_2, and R_1 of the above system and fill the results in Table Q3-b, by evaluating the minimum CT pickup current (I_{pickup}), the plug setting multiplier (PSM), plug setting (*PS%*) and time setting multiplier (*TMS*). Set the *TMS* of R_4 at its minimum *TMS*=0.5. Use a grading margin (coordination time) of **0.3 seconds**.

b. Sketch the o/c characteristics of the four relays on a *t-I* characteristic.

Table Q3-b: PS and TSM of O/C Relays

Parameter	R_1	R_2	R_3	R_4
CT ratio	100:5	100:5	50:5	50:5
PS %				
TMS				0.5

***Setting for Relay* R_4:** This relay must operate for a current above 165.1 A (the minimum fault current at Bus-5). However for reliability, we must set this relay such that it picks up a current that is one third of the minimum, i.e.,

$$I'_{pickup4} = \frac{I_{f\,\min}}{3} = \frac{165.1}{3} = 55.03\ A$$

For a CT ratio of 50:5, the pickup current at the secondary of the CT will be

$$I_{pickup4} = \frac{I'_{pickup4}}{CTR_4} = \frac{55.03}{50/5} \times = 5.5\ A \Rightarrow\ PS\% = \frac{5.5}{5} \times 100 = 110\% \ \Rightarrow \text{Select } PS\%_4 = 125\%.$$

$\therefore$ Pick up Current (Effective Current) =Rated CT Primary Current × PS

$I'_{Pickup} = 50 \times 1.25 = 62.5\,A$ (Secondary current) or $I_{Pickup} = 5 \times 1.25 = 6.25\,A$ (Secondary current)

This means if the CT primary current exceeds 62.5 A (6.25 A secondary current), relay will start operating after some time delay.

Since the relay R_4 is located at the end of the network, i.e. does not coordinate with downstream relays. The *TMS* is restricted to a minimum of ½ for electro-mechanical relays $\Rightarrow TMS_4$=0.5.

***Setting for Relay* R_3:** This relay must provide backup for R_4. Therefore it must pick up the minimum current seen by relay R_4. We therefore choose the same CT ratio CTR_4=50:5 and pick up current $I_{pickup3}$= 6.25 A. To determine the TMS_3, we must provide a discrimination time of 0.3 s. This time is provided such that R_3 operates 0.3 s after the highest (not lowest) fault current seen by R_4. Therefore, R_3 operates in no less than 0.3 s after every possible fault seen by R_4.

For a maximum fault immediately after Bus 4, it will see a fault current that is equal to the fault current seen by Bus 4. Therefore the highest fault current seen by R_4 is 301 A (see Table 1). The current seen by both secondary of CTS of relay R_3 and R_4 for this fault will be

$$I_{f\,\max(\sec)4} = \frac{I'_{f\,\max 4}}{CTR_4} = \frac{301}{50/5} = 30.1\ A$$

Hence, $PSM_4 = \frac{I_{f\,\max 4}}{I_{pickup4}} = \frac{30.1}{6.25} = 4.82$. The tripping time with TMS_4= 0.5 is

$t_4 = \frac{3.0}{\log(PSM_4)} \times TMS_4 = \frac{3.0}{\log(4.82)} \times 0.5 = 2.2\ s$, therefore for any failure of R_4, relay R_3 must operate at 2.2 + 0.3 = 2.5 s.

Since relay R_3 also has PSM_3= 1.25 (6.25 A), we can calculate the TMS_3 from

$$t_3 = \frac{3.0}{\log(PSM_3)} \times TMS_3 = \frac{3.0}{\log(4.82)} \times TMS_3 = 4.39 \times TMS_3 = 2.5\ s$$

,

$$\Rightarrow TMS_3 = \frac{2.5}{4.39} = 0.569 \Rightarrow Let\ TMS_3 = 0.6$$

$\Rightarrow$ TMS_3= 0.6. This gives an operating time of

$t_3 = \frac{3.0}{\log(PSM_3)} \times TMS_3 = \frac{3.0}{\log(4.82)} \times 0.6 = 2.64\ s$. This maintains a minimum discrimination time of 0.3 s.

Setting for Relay R_2: This relay must provide a backup for relay R_3. The smallest fault current seen by R_2 to provide a backup for R_3 is 237.9 A (see Table 1). For reliable operation, we choose one-third of this current, i.e., 79.3 A. For a CTR_2=100:5.

$$I_{pickup(\sec)2} = \frac{I_{f\,\min 3}/3}{CTR_2} = \frac{237.9/3}{100/5} = \frac{79.3}{100/5} = 3.97\ A \Rightarrow PS\% = \frac{3.97}{5} \times 100 = 79.3\%$$

$\Rightarrow$ Select $PS\%_2$=100%.We now have to determine TMS_2 of R_2 from the maximum fault current seen by R_3.

The maximum current seen by R_3 is 431 A. Then, at R_3, for a CT ratio of CTR_3=50:5 and a PS_3= 6.25 A, we get a PSM_3 of

$PSM_3 = \frac{\left(\frac{I_{f\,\max 3}}{CTR_3}\right)}{I_{pickup3}} = \frac{\left(\frac{431}{50/5}\right)}{6.25} = 6.9$. For the above PSM_3=6.9 and a TMS_3 =0.6, the operating time of relay R_3 is

$$t_3 = \frac{3.0}{\log(PSM_3)} \times TMS_3 = \frac{3.0}{\log(6.9)} \times 0.6 = 2.15\ s$$

Thus, relay R_2 should add discrimination time of 0.3 s., i.e., the operating time should be 2.15 + 0.3 = 2.45 s.

Now relay R_2 is a backup for relay R_3 and therefore, it will see the same fault current of 431 A.

Then, PSM_2 for this fault is $PSM_2 = \dfrac{\left(\dfrac{I_{f\max 3}}{CTR_2}\right)}{I_{pickup2}} = \dfrac{\dfrac{431}{100/5}}{5} = 4.31$

For this value of PSM_2, we get a TMS_2 from

$$t_2 = \frac{3.0}{\log(PSM_2)} \times TMS_2 = \frac{3.0}{\log(4.31)} \times TMS_2 = 4.73 \times TMS_2 = 2.45\ s$$

$$\Rightarrow TMS_2 = \frac{2.45}{4.73} = 0.518 \Rightarrow Let\ TMS_2 = 0.6$$

This gives an operating time of $t_2 = \dfrac{3.0}{\log(PSM_2)} \times TMS_2 = \dfrac{3.0}{\log(4.31)} \times 0.6 = 2.84\ s$

Setting for Relay R_1: This relay must provide a backup for relay R_2. The smallest fault current seen by R_2 to provide a backup for R_3 is 328.6 A. For reliable operation, we choose one-third of this current, i.e., 109.5 A. A CT ratio CTR_1=100:5 is suitable.

For a CTR_1=100:5.

$$I_{pickup(\sec)1} = \frac{I_{f\min 3}/3}{CTR_1} = \frac{328.6/3}{100/5} = \frac{109.5}{100/5} = 5.48\ A \Rightarrow PS\% = \frac{5.48}{5} \times 100 = 109.5\%$$

$\Rightarrow$ Select $PS\%_1$=125% (PS_1 =5×1.25=6.25 A).

We now have to determine TMS_1 of R_1 from the maximum fault current seen by R_2. Thus we choose the same CT ratio and PS% for this relay as well. The maximum fault current seen by R_2 is 658 A. Then, at R_2, for a CT ratio of CTR_2=100/5 and a PS_2 of (5×100=5.00 A), we get

$PSM_2 = \dfrac{\left(\dfrac{I_{f\max 2}}{CTR_2}\right)}{I_{pickup2}} = \dfrac{\dfrac{658}{100/5}}{5} = 6.58$. For PSM_2=6.58 and TMS_2=0.6, the operating time of relay R_2 is

$$t_2 = \frac{3.0}{\log(PSM_2)} \times TMS_2 = \frac{3.0}{\log(6.58)} \times 0.6 = 2.2\ s\,.$$

Thus relay R_1 should add discrimination time of 0.3 s., i.e., the operating time should be 2.2 + 0.3 = 2.5 s.

Now relay R_1 is a backup for relay R_2 and therefore it will see the same maximum fault current of 658 A. Then for the same $PS\%_1$=6.25

$PSM_1 = \dfrac{\left(\dfrac{I_{f\max 2}}{CTR_1}\right)}{I_{pickup1}} = \dfrac{\dfrac{658}{100/5}}{6.25} = 5.26$. Then, TMS_1 can be calculated from

$$t_1 = \frac{3.0}{\log(PSM_1)} \times TMS_1 = \frac{3.0}{\log(5.26)} \times TMS_1 = 4.16 \times TMS_1 = 2.5\ s \Rightarrow TMS_1 = \frac{2.5}{4.16} = 0.60\,.$$

Table Q3-b: PS and TSM of O/C Relays

Parameter	**R_1**	**R_2**	**R_3**	**R_4**
CT ratio	**100:5**	**100:5**	**50:5**	**50:5**
PS %	125%	100%	125%	125%
TMS	0.6	0.6	0.6	**0.5**

Problem # 4:

Figure 4 shows a simple ring main, with a single infeed at bus **A** bus and three load busbars (**B**, **C** and **D**). The ring is protected with numerical (MiCOM P140 series) directional O/C relays $\mathbf{R_2 - R_7}$ and non-directional O/C relays $\mathbf{R_1}$ and $\mathbf{R_8}$ with standard inverse (SI) characteristics $t_p = \left(\dfrac{0.14}{I_r^{0.02} - 1}\right) \times TMS$

Based on the maximum load current in the ring CT's with CTR of 1000/1. The maximum fault current (I_{fmax}) at the buses **B**, **C** and **D** with open ring at $\mathbf{CB_1}$ and $\mathbf{CB_8}$,CTRs and percentage plug settings of relays $\mathbf{R_1}$ **-** $\mathbf{R_8}$ and the CTR of the CT's are shown in Table 1.

a. Draw the radial networks when $\mathbf{CB_1}$ and $\mathbf{CB_8}$ are open.
b. If the TMS of $\mathbf{R_2}$ and $\mathbf{R_7}$ are set at 0.05, find the **TMS** settings for relays $\mathbf{R_1}$, $\mathbf{R_3}$, $\mathbf{R_5}$, $\mathbf{R_4}$, $\mathbf{R_6}$ **and** $\mathbf{R_8}$**.** Use a **grading time ΔT** of **0.3 sec**. Fill the results in Table 2.

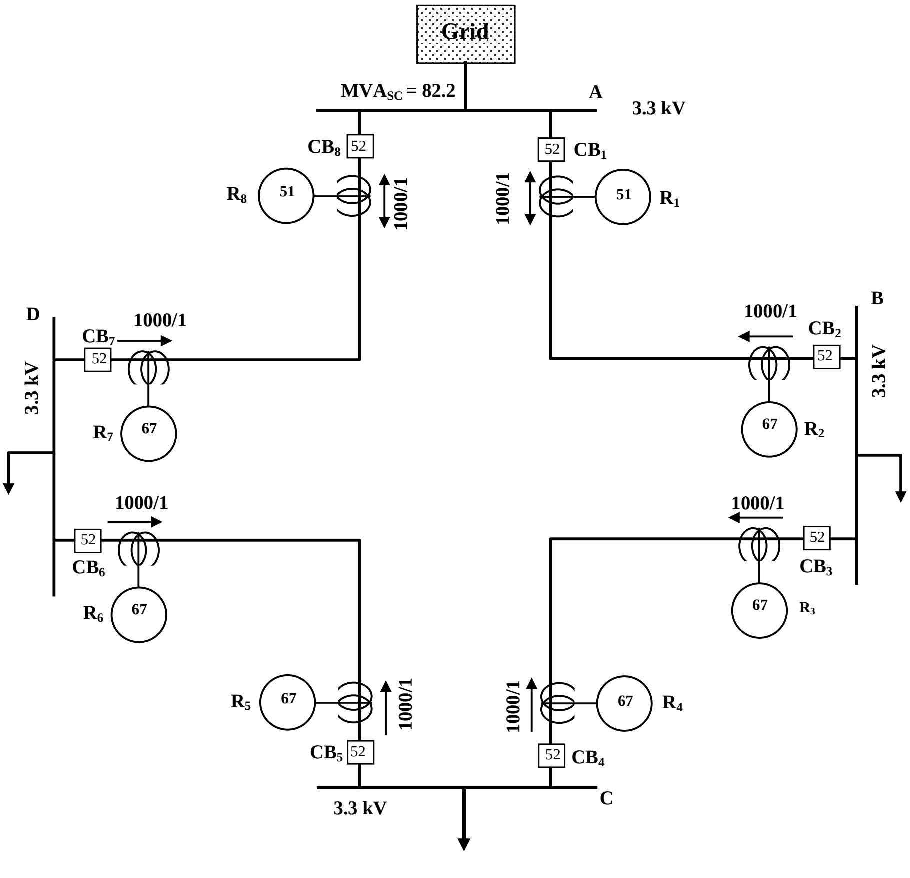


Fig. 4

Table 1: Maximum Fault Current with Ring Open

Clockwise (Open Point CB_8)					Anticlockwise (Open Point CB_1)				
Bus	I_{fmax} (kA)	CTR	%PS		Bus	I_{fmax} (kA)	CTR	%PS	
D	3376	1000/1	R_7	80	B	3376	1000/1	R_2	80
C	4259	1000/1	R_5	88	C	4259	1000/1	R_4	88
B	7124	1000/1	R_3	97	D	7124	1000/1	R_6	97
A	14387	1000/1	R_1	107	A	14387	1000/1	R_8	107

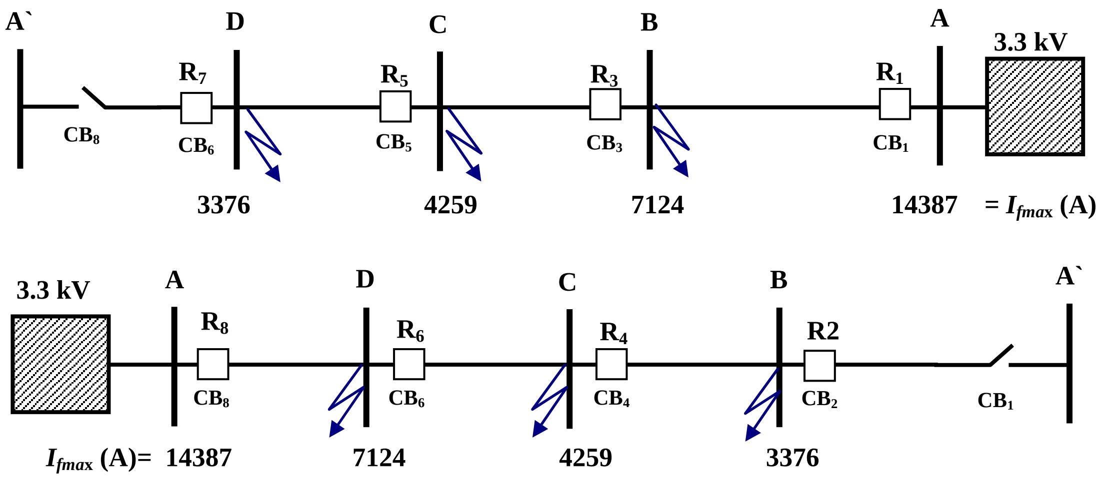


Table 2: Relay Settings

Relay	R_1	R_3	R_5	R_7	R_2	R_4	R_6	R_8
TMS	0.237	0.163	0.105	0.05	0.05	0.105	0.163	0.237

Directional Overcurrent Relay Coordination

Problem # 5

For the ring system shown in Fig 5, the remote bus fault currents seen by each primary and back up relay pairs are tabulated below in Table 5-a. For the relays in Table 4-a, if the pick up values are as tabulated in Table 5-b, find out the TMS of relays R1, R2, R3, R4, R5, R6, R7 and R8.

Table 4-a : Fault Current seen by Primary - Back up Relay Pairs				
	Anti clockwise loop		Clockwise loop	
Remote Bus Fault at	Current seen by primary relay	Current seen by back up relay	Current seen by primary relay	Current seen by back up relay
F_1	R_2 (639A)	R_1 (152A)	R_6 (1365A)	R_5 (272A)
F_2	R_1 (1652A)	R_4 (391A)	R_7(868A)	R_6 (240A)
F_3	R_4 (1097A)	R_3 (140A)	R_8 (1764A)	R_7 (287A)
F_4	R_3 (937A)	R_2 (142A)	R_5 (553A)	R_8 (197A)

Table 4-b : Pick up Values of Relays								
Relay	R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8
Pick up setting (A)	60	80	60	160	80	160	128	100

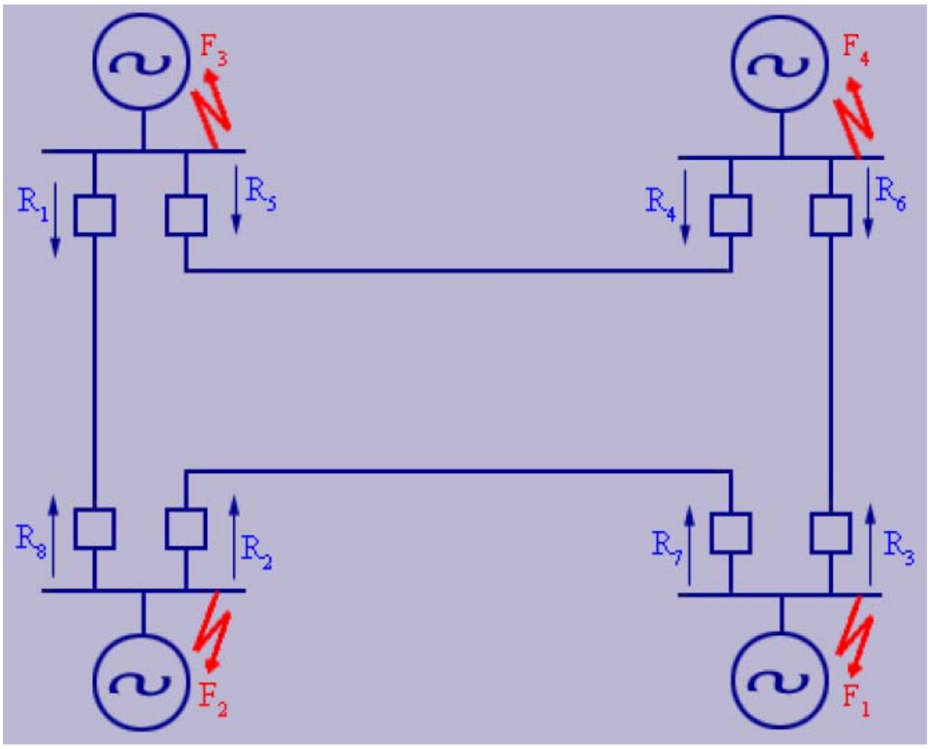


Fig. Q5: Relay Coordination in a Ring (Loop) System

Solution

Iteration 1 For relay R2, assume a TMS of 0.05 (Normal range is 0.025 to 1.2). The reason to initialize TMS to 0.05and not the minimum value i.e. 0.025 is that further iterations may reduce TMS. If to begin with 0.025then the problem becomes infeasible. For fault at F1 where R2 acts as primary, Time of operation of standard inverse relay,

$$t_{R_2} = \frac{TMS_{R_2} \times 0.14}{\left(I/_{Is}\right)^{0.02} - 1} = \frac{0.05 \times 0.14}{\left(639/_{80}\right)^{0.02} - 1} = 0.165\text{sec}$$

(where Is = 80A, I = 639A)

For fault at F1, R1 will back up R2. Hence time of operation R1 = t_{R2}+ CTI (where CTI is the coordination time interval and CTI = 0.3sec.) = 0.165 + 0.3 = 0.465sec

$$0.465 = \frac{TMS_{R_1} \times 0.14}{\left(I/_{Is}\right)^{0.02} - 1} = \frac{TMS_{R_1} \times 0.14}{\left(152/_{60}\right)^{0.02} - 1}$$

where I = 152A, Is = 60A),

$TMS_{R_1} = 0.0623$

For fault at F_2, where R_1 acts as primary,

$$t_{R_1} = \frac{0.0623 \times 0.14}{\left(I/Is \right)^{0.02} - 1}$$ (where I = 1652A, Is = 60A)

= 0.127sec

Relay R_4 will back up R_1 for fault at F_2. Hence, time of operation of R_4 = t_{R_1} + CTI = 0.127 + 0.3 = 0.427sec

i.e., $$0.427 = \frac{TMS_{R_4} \times 0.14}{\left(I/Is \right)^{0.02} - 1}$$ (where I = 391A, Is = 160A)

Then, TMS_{R_4} = 0.055

For fault at F_3, where R_4 acts as primary relay, we have

$$t_{R_4} = \frac{TMS_{R_4} \times 0.14}{\left(I/Is \right)^{0.02} - 1}$$ (where I = 1097A, Is = 160A)

= 0.196sec

Since relay R_3 has to back up R_4, time of operation of relay R_3 = t_{R_4} + CTI = 0.496sec

For a fault at F_3

i.e., $$0.496 = \frac{TMS_{R_3} \times 0.14}{\left(I/Is \right)^{0.02} - 1}$$ (where I = 140A, Is = 60A)

TMS_{R_3} = 0.0605

Now for fault at F_4, where R_3 acts as primary,

$$t_{R_3} = \frac{TMS_{R_3} \times 0.14}{\left(I/Is \right)^{0.02} - 1}$$ (where I = 937A, Is = 60A)

= 0.15sec

For fault F_4, R_2 has to back up R_3

i.e., Time of operation of R_2 = t_{R_3} + CTI = 0.45sec

$$0.45 = \frac{TMS_{R_2} \times 0.14}{\left(I/Is \right)^{0.02} - 1}$$ (where I = 142A, Is = 80A)

TMS_{R_2} = 0.037

We had assumed a value of 0.05 for TMS_{R_2}, but now the value has changed to 0.037. Therefore, let us update the TMS of R_2 to 0.037.

Iteration 2 Repeating the same process as above,

For fault at F_1, time of operation $t_{R_2} = \dfrac{0.037 \times 0.14}{\left(639/80\right)^{0.02} - 1}$

= 0.122sec

Time of operation of R_1 = t_{R_2} + CTI

= 0.3 + 0.122 = 0.422sec

i.e., $0.422 = \dfrac{TMS_{R_1} \times 0.14}{\left(152/60\right)^{0.02} - 1}$ or TMS_{R_1} = 0.0565

For fault at F_2, where R_1 acts as primary,

$t_{R_1} = \dfrac{0.0565 \times 0.14}{\left(1652/60\right)^{0.02} - 1}$ = 0.1154

R_4 backs up R_1 for fault at F_2

Time of operation of R_4 = t_{R_1} + CTI = 0.1154 + 0.3

= 0.4154

i.e. $0.4154 = \dfrac{TMS_{R_4} \times 0.14}{\left(391/160\right)^{0.02} - 1}$

TMS_{R_4} = 0.0535

Now, for fault at F_3, where R_4 acts as primary,

$t_{R_4} = \dfrac{0.0535 \times 0.14}{\left(1097/160\right)^{0.02} - 1}$ = 0.191sec

Since, relay R_3 backs up R_4, time of operation of relay R_3 = t_{R_4} + CTI = 0.191 + 0.3 = 0.491

i.e. 0.491 = $\dfrac{TMS_{R_3} \times 0.14}{\left(140/60\right)^{0.02} - 1}$

TMS_{R_3} = 0.0599

For fault at F_4, where R_3 acts as primary,

Time of operation $t_{R_3} = \dfrac{0.0599 \times 0.14}{\left(937/60\right)^{0.02} - 1}$ = 0.1484sec

R_2 backs up R_3; Therefore,

Time of operation of R_2 = t_{R_3} + CTI = 0.3 + 0.1484

= 0.4484sec

i.e. $0.4484 = \dfrac{TMS_{R_2} \times 0.14}{\left(142/80\right)^{0.02} - 1}$

TMS_{R_2} = 0.0369

Now, let us update the TMS of R_2 to this new value, i.e., 0.0369 and repeat iteration.

Iteration 3

For fault at F_1, $t_{R_2} = \dfrac{0.0369 \times 0.14}{\left(639/80\right)^{0.02} - 1}$

= 0.1217sec

For relay R_1, which has to back up R_2

Time of operation = 0.3 + 0.1217 = 0.4217sec

i.e. $0.4217 = \dfrac{TMS_{R_1} \times 0.14}{\left(152/60\right)^{0.02} - 1}$

TMS_{R_1} = 0.0565

Then for fault at F_2, $t_{R_1} = \dfrac{0.0565 \times 0.14}{\left(1652/60\right)^{0.02} - 1}$ = 0.1154sec

Since R_4 backs up R_1, time of operation of R_4

= 0.1154 + 0.3 = 0.4154sec

i.e. $0.4154 = \dfrac{TMS_{R_4} \times 0.14}{\left(391/160\right)^{0.02} - 1}$

TMS_{R_4} = 0.0535

For fault at F_3, where R_4 acts as primary, we have

$t_{R_4} = \dfrac{0.0535 \times 0.14}{\left(1097/160\right)^{0.02} - 1}$ = 0.191sec

R_3 backs up R_4

Time of operation of R_3 = 0.3 + 0.191 = 0.491sec

i.e. $0.491 = \dfrac{TMS_{R_3} \times 0.14}{\left(140/60\right)^{0.02} - 1}$

TMS_{R_3} = 0.0599

For fault at F_4, $t_{R_3} = \dfrac{0.0599 \times 0.14}{\left(937/60\right)^{0.02} - 1}$

= 0.1484sec

Now R_2 backs up R_3

i.e. time of operation of R_2 = 0.3 + 0.1484 = 0.4484 = $\dfrac{TMS_{R_2} \times 0.14}{\left(142/80\right)^{0.02} - 1}$

TMS_{R_2} = 0.0369 which is same as the result of iteration 2.

Therefore no more iteration is required. Hence, setting and coordination of all the four anticlockwise relays are complete.

Setting and Coordination of Clockwise Relays

Iteration 1

Now let us start setting all the clockwise relays. Let us start from relay R_5 for fault at F_4.

Assume a TMS of 0.05 for relay R_5. Then, time of operation of relay R_5, $t_{R_5} = \dfrac{0.05 \times 0.14}{\left(553/80\right)^{0.02} - 1} = 0.1775$

i.e. Time of operation of back up relay R_8 = t_{R_5} + CTI

= 0.1775 + 0.3
= 0.4775sec

Now, 0.4775 = $\dfrac{TMS_{R_8} \times 0.14}{\left(197/100\right)^{0.02} - 1}$

= 0.04656

TMS_{R_8}

For a fault at F_3, where R_8 acts as primary,

$$t_{R_8} = \frac{0.0465 \times 0.14}{\left(1764/100\right)^{0.02} - 1} = 0.11\text{sec}$$

Now relay R_7 will back up R_8. Then time of operation of R_7 = 0.11 + 0.3 = 0.41sec

i.e., $0.41 = \dfrac{TMS_{R_7} \times 0.14}{\left(287/128\right)^{0.02} - 1}$

TMS_{R_7} = 0.0477

R_7 acts as primary relay for fault at F_2.

$$t_{R_7} = \frac{0.0477 \times 0.14}{\left(868/128\right)^{0.02} - 1} = 0.1711\text{sec}$$

R_6 backs up R_7,

i.e. Time of operation for R_6

= 0.1711 + 0.3 = 0.4711

i.e. $0.4711 = \dfrac{TMS_{R_6} \times 0.14}{\left(240/160\right)^{0.02} - 1}$

TMS_{R_6} = 0.0274

For fault at F_1, R_6 acts as primary,

i.e., $t_{R_6} = \dfrac{0.0274 \times 0.14}{\left(1365/160\right)^{0.02} - 1}$ = 0.0875sec

R_5 backs up R_6

i.e. Time of operation of R_5 = 0.0875 + 0.3 = 0.3875

i.e., $0.3875 = \dfrac{TMS_{R_5} \times 0.14}{\left(272/80\right)^{0.02} - 1}$

TMS_{R_5} = 0.0686

i.e. after 1^{st} iteration TMS of R_5 has been changed from 0.05 to 0.0686. Let us update TMS of R_5 to 0.0686 and begin iteration 2.

Iteration 2

TMS_{R_5} = 0.0686

For fault F_4, $t_{R_5} = \dfrac{0.0686 \times 0.14}{\left(553/80\right)^{0.02} - 1}$ = 0.2436

For fault at F_4, R_8 backs up R_5

i.e. Time of operation of R_5 = t_{R_5} + CTI = 0.2436 + 0.3

= 0.5436sec

i.e. 0.5436 = $\dfrac{TMS_{R_8} \times 0.14}{\left(197/100\right)^{0.02} - 1}$

TMS_{R_8} = 0.053

For fault F_3, where R_8 acts as primary,

$t_{R_8} = \dfrac{0.053 \times 0.14}{\left(1764/100\right)^{0.02} - 1}$ = 0.1256sec

Relay R_7 backs up R_8

Time of operation of R7 = 0.1256 + 0.3 = 0.4256sec

i.e. 0.4256 = $\dfrac{TMS_{R_7} \times 0.14}{\left(287/128\right)^{0.02} - 1}$

TMS_{R_7} = 0.0495

For fault at F_2, R_7 acts as primary,

i.e. $t_{R_7} = \dfrac{0.0477 \times 0.14}{\left(868/128\right)^{0.02} - 1}$ = 0.1776sec

R_6 backs up R_7,

i.e. Time of operation for R_6 = 0.1776 + 0.3 = 0.4776sec

i.e. 0.4776 = $\dfrac{TMS_{R_6} \times 0.14}{\left(240/160\right)^{0.02} - 1}$

TMS_{R_6} = 0.0278

For fault at F_1, R_6 acts as primary,

i.e. $t_{R_6} = \dfrac{0.0278 \times 0.14}{\left(1365/160\right)^{0.02} - 1}$ = 0.0888sec

R_5 backs up R_6,

i.e. Time of operation of R_5 = 0.0888 + 0.3

= 0.3888sec

i.e. 0.3888 = $\dfrac{TMS_{R_5} \times 0.14}{\left(272/80\right)^{0.02} - 1}$

TMS_{R_5} = 0.0688

Now let us set TMS of R_5 to 0.0688 and repeat iteration.

Setting and Coordination of Clockwise Relays
Iteration 3

TMS_{R_5} = 0.0688

For fault at F_4 $t_{R_5} = \dfrac{0.0688 \times 0.14}{\left(553/80\right)^{0.02} - 1}$ = 0.2443

R_8 backs up R_5,
i.e. Time of operation of R_8 = t_{R_5} + CTI = 0.2443 + 0.3
= 0.5443sec

0.5443 = $\dfrac{TMS_{R_8} \times 0.14}{\left(197/100\right)^{0.02} - 1}$

i.e. TMS_{R_8} = 0.0531

For fault at F_3, R_8 acts as primary,

Then $t_{R_8} = \dfrac{0.0531 \times 0.14}{\left(1764/100\right)^{0.02} - 1}$ = 0.1258sec

Relay R_7 backs up R_8
i.e. Time of operation of R_7 = 0.3 + 0.1258 = 0.4258sec

0.4258 = $\dfrac{TMS_{R_7} \times 0.14}{\left(287/128\right)^{0.02} - 1}$

TMS_{R_7} = 0.0495

For fault at F_2, R_7 acts as primary,

i.e. $t_{R_7} = \dfrac{0.0495 \times 0.14}{\left(868/128\right)^{0.02} - 1}$ = 0.1776sec

R_6 backs up R_7, Time of operation of R_6,
= 0.3 + 0.1776 = 0.4776sec

i.e. 0.4776 = $\dfrac{TMS_{R_6} \times 0.14}{\left(240/160\right)^{0.02} - 1}$

TMS_{R_6} = 0.0278

For fault at F_1, R_6 acts as primary,

$t_{R_6} = \dfrac{0.0278 \times 0.14}{\left(1365/160\right)^{0.02} - 1}$ = 0.0888sec

Since R_5 backs up R_6 for fault at F_1, time of operation of R_5 = 0.3 + 0.0888sec = 0.3888sec i.e., $0.3888 = \dfrac{TMS_{R_5} \times 0.14}{\left(272/80\right)^{0.02} - 1}$

TMS_{R_5} = 0.0688.

Since, the result of iterations 2 and 3 are the same, the iteration is complete.

Answer

Setting and Coordination of Clockwise Relays

Table 3 TMS Setting for Relay

Relay	1st Iteration	2nd Iteration	3rd Iteration
R_1	0.623	0.0565	0.0565
R_2	0.05	0.0369	0.0369
R_3	0.0605	0.0599	0.0599
R_4	0.055	0.0535	0.0535
R_5	0.05	0.0686	0.0688
R_6	0.0274	0.0278	0.0278
R_7	0.0477	0.0495	0.0495
R_8	0.04656	0.053	0.0531

EE482-Power System Analysis II

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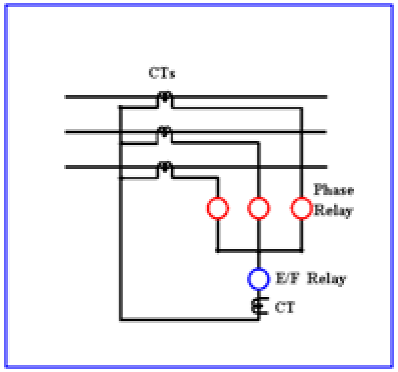
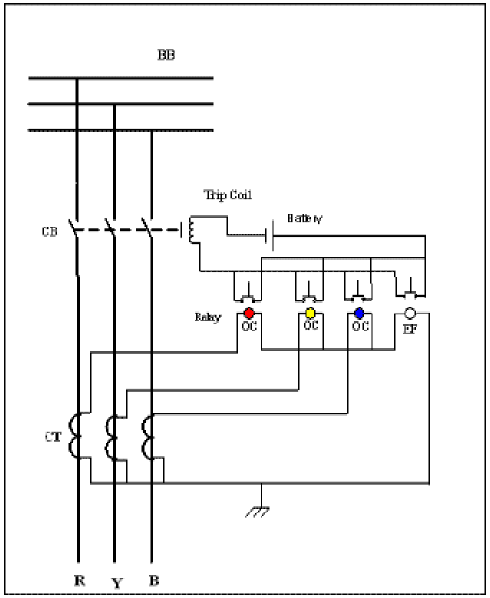
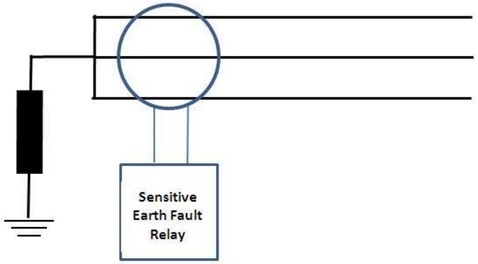
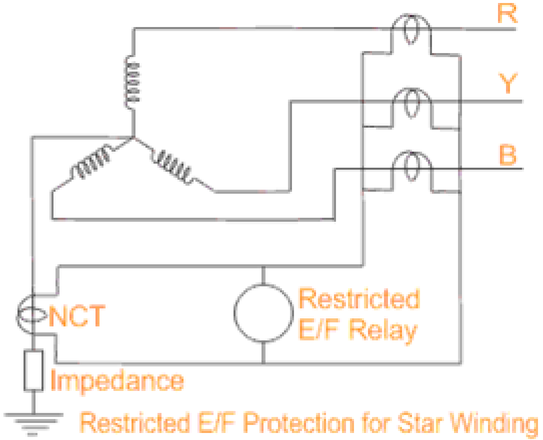


Topic 1-4: Prt 4- Earth-Fault Protection





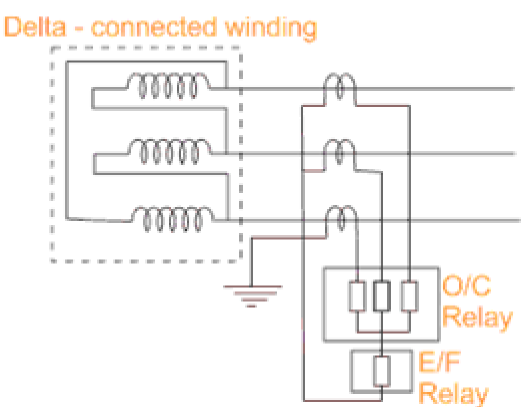


Earth-Fault Protection

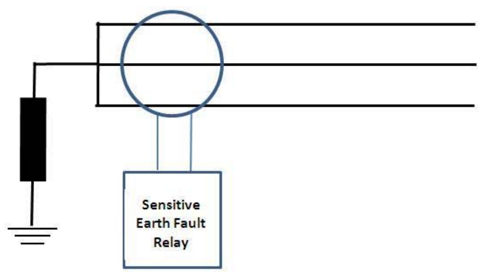
- When the fault current flows through earth return path, the fault is called ***Earth Fault***.
- Other faults which do not involve earth are called phase faults.
- Since earth faults are relatively frequent, earth fault protection is necessary in most cases.
- When separate earth fault protection is not economical, the phase relays sense the earth fault currents. However such protection lacks sensitivity. Hence separate earth fault protection is generally provided.
- Earth fault protection senses earth fault current. Following are the method of earth fault protection.

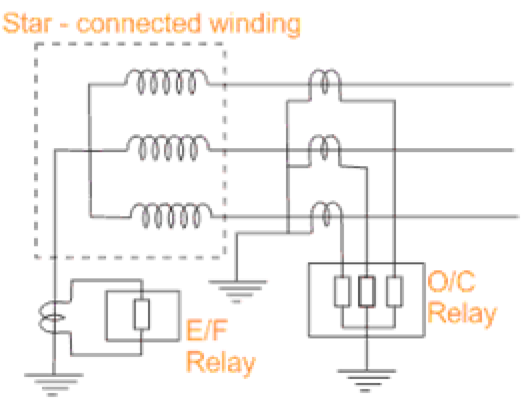
Methods of Earth-Fault Protection

1. Residually connected relay.
2. Relay connected in neutral-to-ground circuit.
3. Core-balance-scheme.
4. Distance relays arranged for detecting earth faults on lines.
5. Circulating current differential protection.



Backup O/C & E/F Protection Scheme





O/C & Unrestricted E/F Protection Scheme

Connections of CT's for Earth-Fault Protection

1. Residually connected Earth-fault Relay

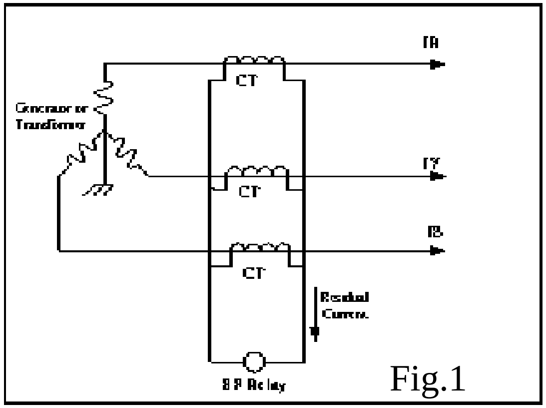


Fig.1

- More sensitive protection against earth faults can be obtained by using a relay that responds only to the residual current of the system, since a residual component exists only when fault current flows to earth.
- In absence of earth-fault the vector sum of three line currents is zero. Hence the vector sum of three secondary currents is also zero.

$$\mathbf{I_R+I_Y+I_B=0}$$

The sum $(\mathbf{I_R+I_Y+I_B})$ is called residual current

- The earth-fault relay is connected such that the residual current flows through it (Fig.1 and Fig. 2).
- In the absence of earth-fault, therefore, the residually connected earth-fault relay does not operate.

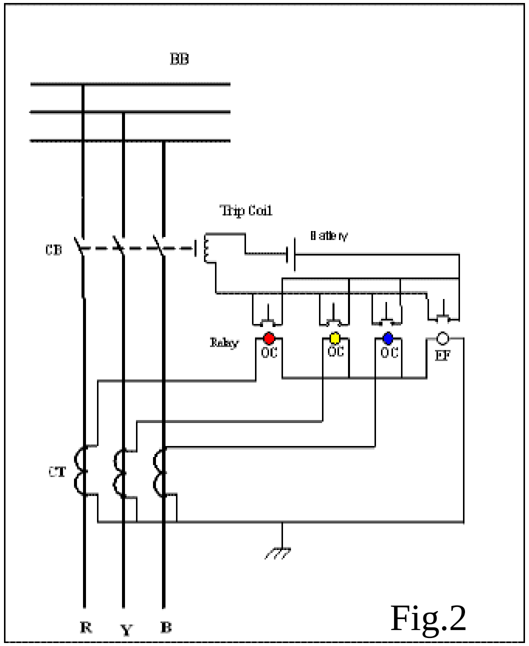
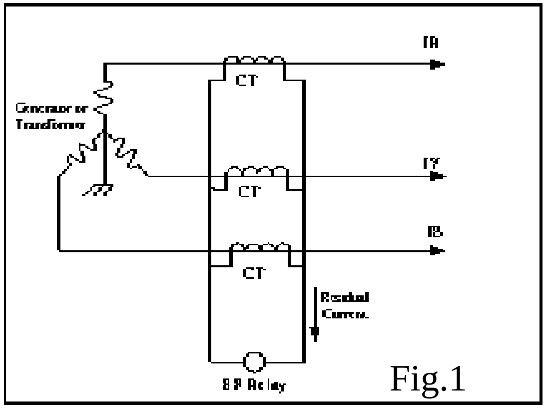


Fig.2

Connections of CT's for Earth-Fault Protection

1. Residually connected Earth-fault Relay



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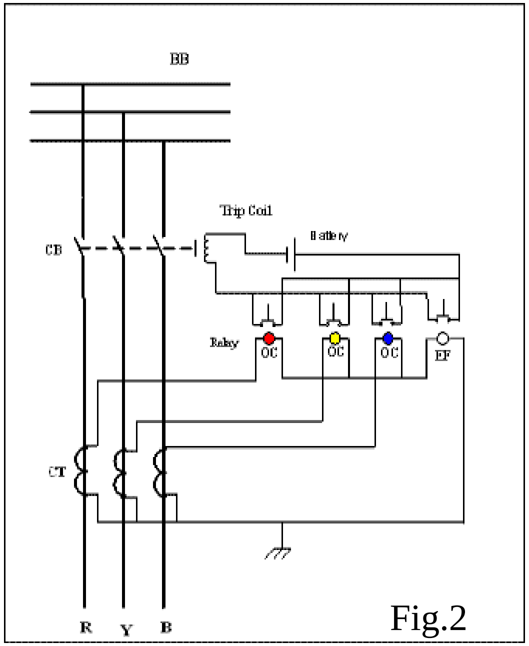
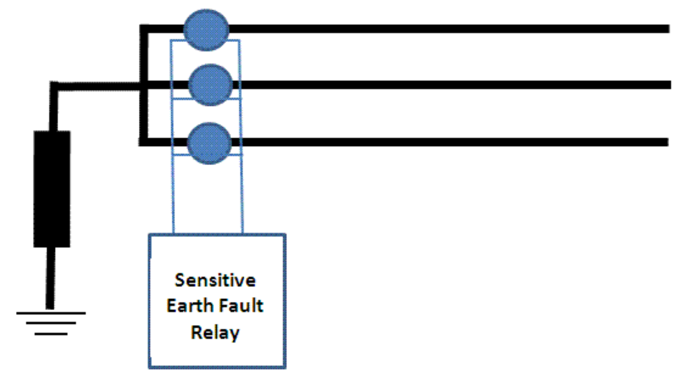


Fig.2

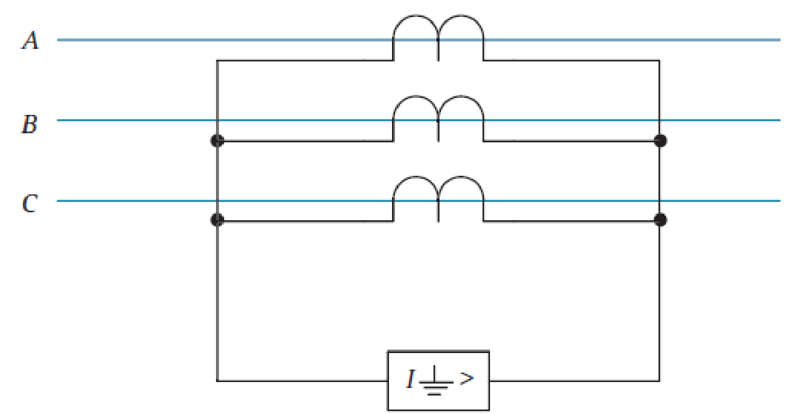
1. Residually connected Earth-fault Relay

- The residual component is extracted by connecting the line current transformers in parallel.



- However, in presence of earth fault the conditions is disturbed and **$(I_R+I_Y+I_B)$** is no more zero. Hence flows through the earth-fault relay. If the residual current is above the pick-up value, the earth-fault relay operates. (20%-40% of Full-Load Current)

- In the Residually Connection, the earth-fault at any location near or away from the location of CT's can cause the residual current flow. Hence the protected zone is not definite. Such protection is called **Unrestricted Earth-Fault Protection**



(a)

2. Earth-fault Relay connected in Neutral to Earth Circuit

- Another method of connecting an earth-fault relay is illustrated in Fig 3. The relay is connected to secondary of a CT whose primary is connected in neutral to earth connection.
- Such protection can be provided at various voltage levels by connecting earth-fault relay in the neutral-to-earth connection of that voltage level.
- The fault current finds the return path through the earth and then flows through the neutral-to-earth connection.

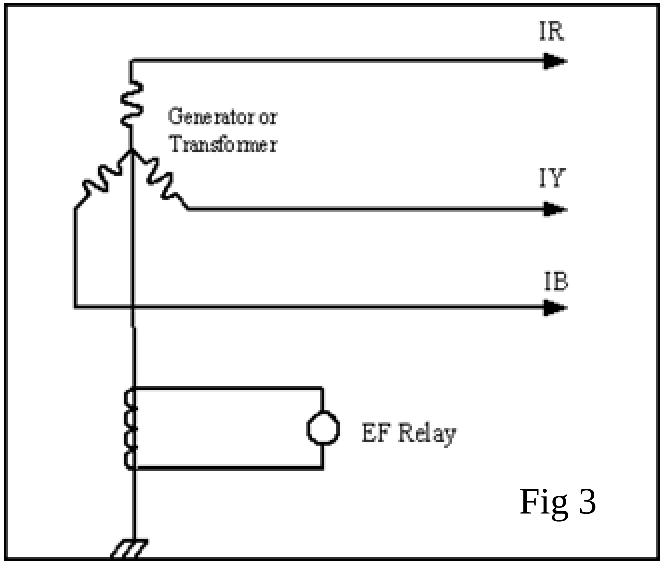


Fig 3

- The magnitude of earth fault current depends on type of earthing (resistance, reactance or solid) and location of fault. In this type of protection, the zone of protection cannot be accurately defined. The protected area is not restricted to the transformer/generator winding alone. The relay senses the earth faults beyond the transformer/generator winding hence such protection is called **Unrestricted Earth-Fault Protection.**

3. Combined Earth-Fault and Phase-Fault Protection

- It is convenient to incorporate phase-fault relays and earth-fault relay in a combined phase-fault and earth-fault protection. (Fig. 4)
- The increase in current of phase causes corresponding increase in respective secondary currents. The secondary current flows through respective relay-units Very often only two-phase relays are provided instead of three, because in case of phase faults current in any at least two phases must increase. Hence two relay-units are enough.

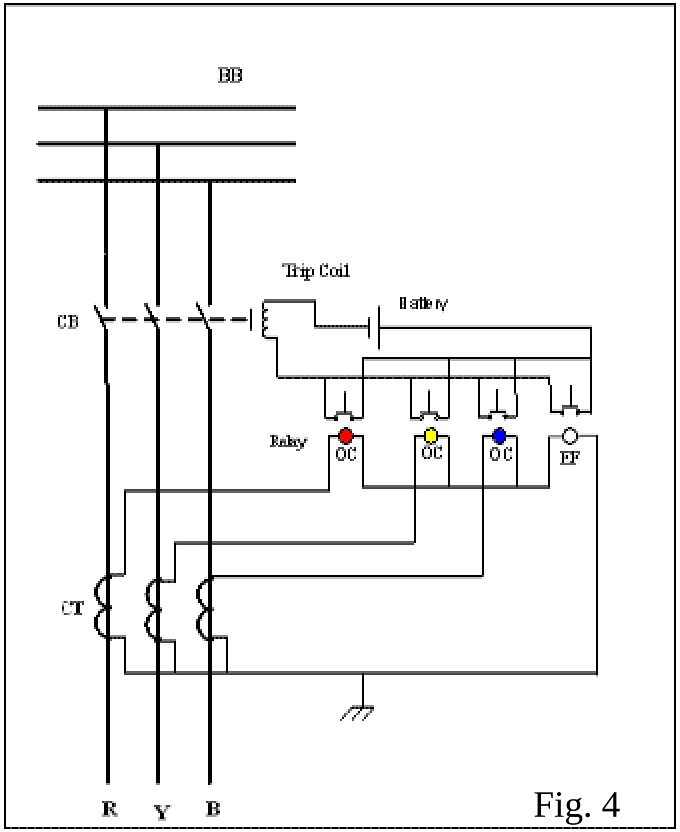
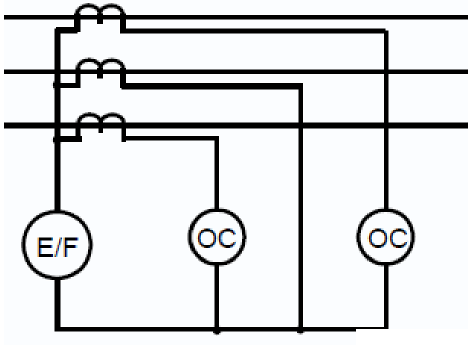
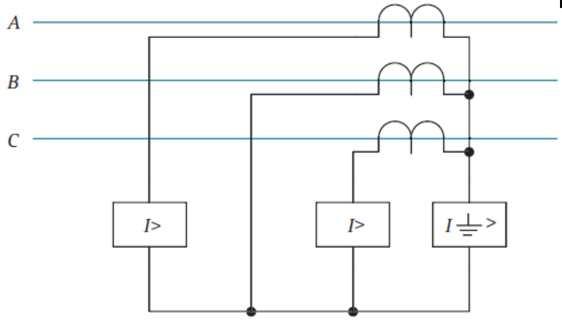
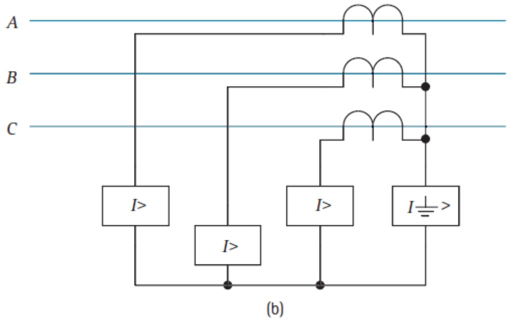


Fig. 4







Economize using 2x OC relays

4. Earth-fault Protection with Core Balance Current Transformers. Sensitive Earth-Fault Protection

- ◇ In this type of protection (Fig. 5) a single ring shaped core of magnetic material, encircles the conductors of all the three phases. A secondary coil is connected to a relay unit.
- ◇ The cross-section of ring-core is ample, so that saturation is not a problem.
- ◇ During no-earth-fault condition, the components of fluxes due to the fields of three conductors are balanced and the secondary current is negligible.
- ◇ During earth faults, such a balance is disturbed and current is induced in the secondary.
- ◇ Core-balance protection can be conveniently used for protection of low-voltage and medium voltage systems.

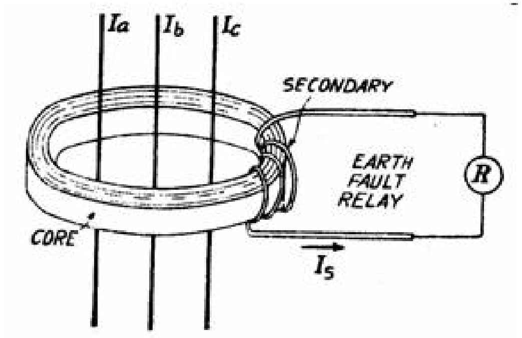
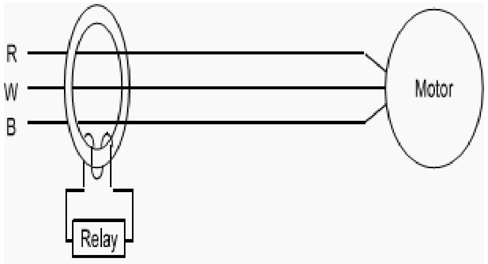


Fig. 5



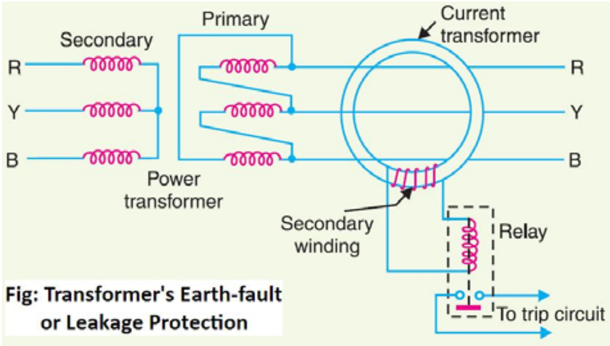


Fig: Transformer's Earth-fault or Leakage Protection

EE482-Power System Analysis II
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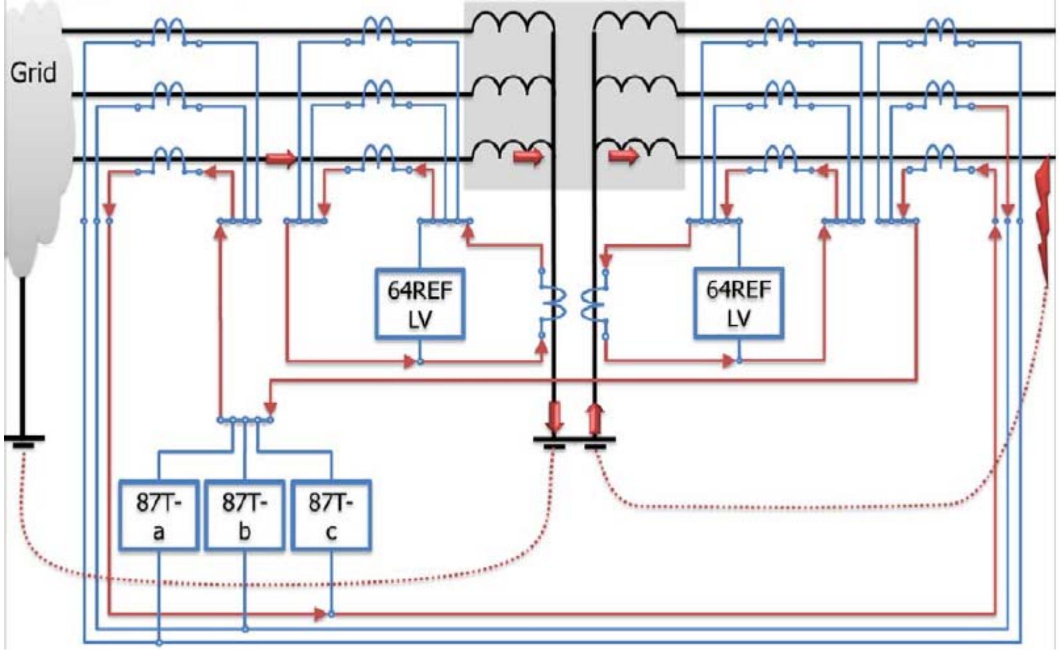
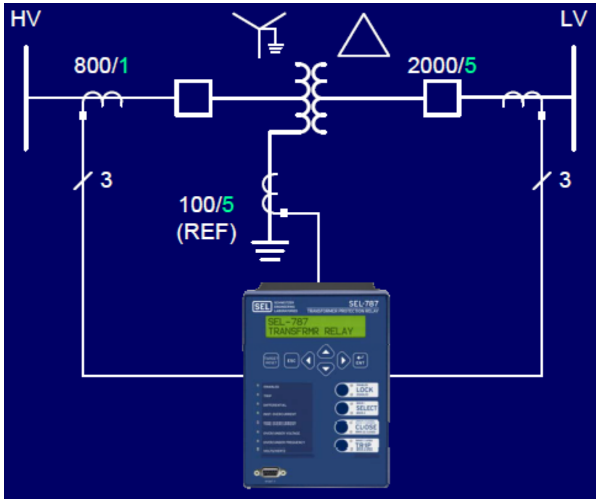
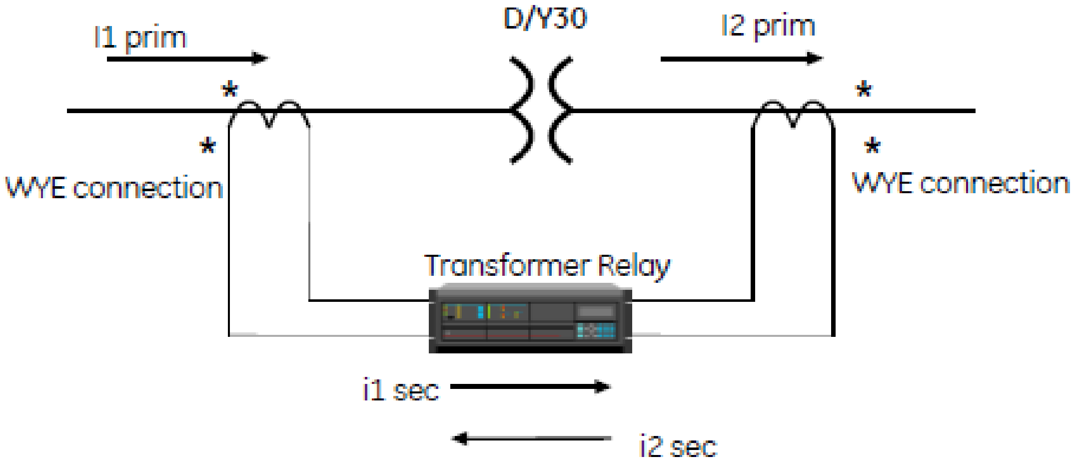
Topic 1-5: Transformer Protection

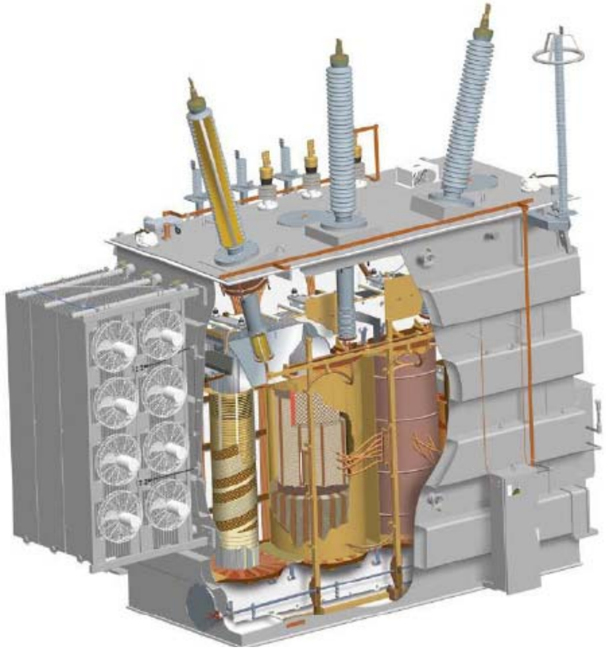




Transformer Protection

- Power transformers are expensive.
  - ➔Protection must be effective
- What can go wrong?
  - **Phase-to-phase faults**
  - **Three-phase faults**
  - **Ground faults**
  - **Core faults**
  - **Tank faults**
  - **Bushing faults**

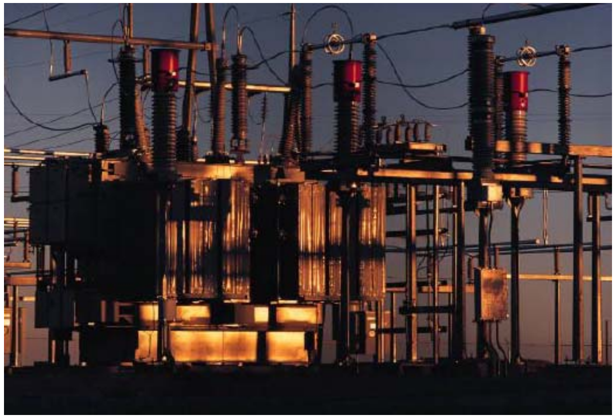




Causes of Transformer Failures

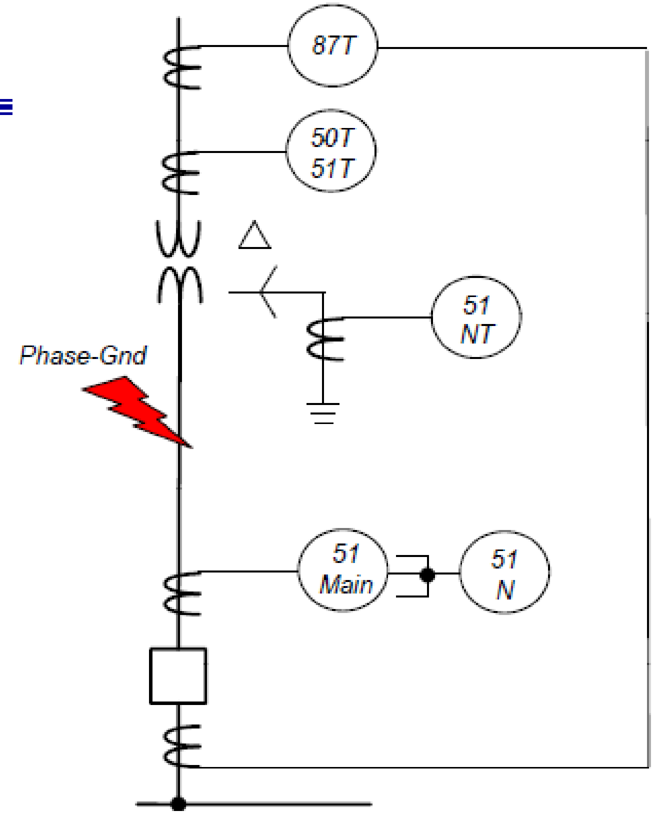
- Winding failures **51%**
- Tap changer failures **19%**
- Bushings failures **9%**
- Terminal board failures **6%**
- Core failures **2%**
- Miscellaneous failures **13%**

Differential protection can detect all of the types of failures above

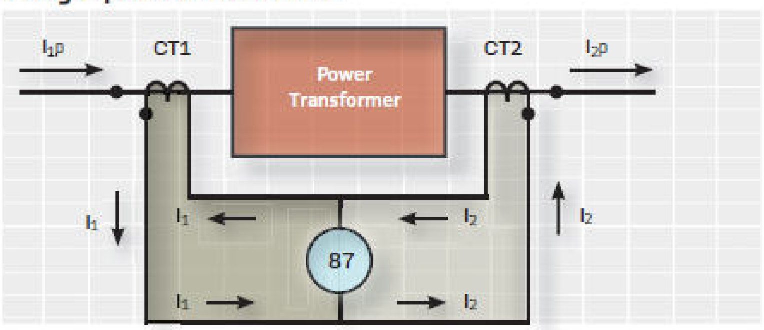


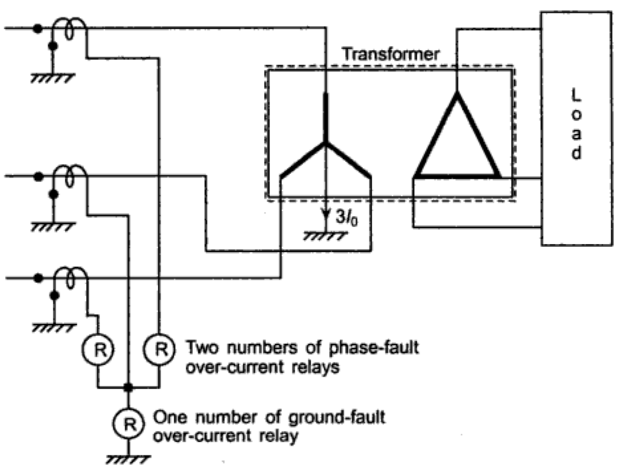
Transformer Protection

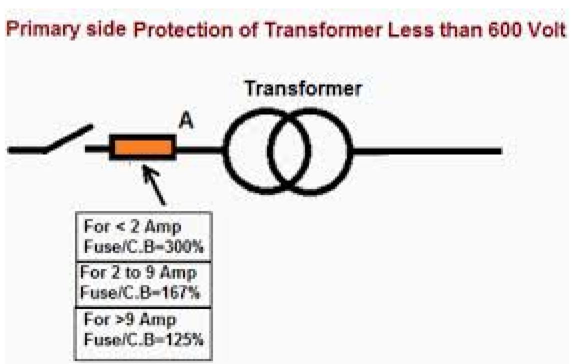
- Protection Methods
  - Protection must be effective
  - Fuse Protection
  - Overcurrent Protection
  - Differential Protection



3 Typical connections of a differential relay applied to a single-phase transformer



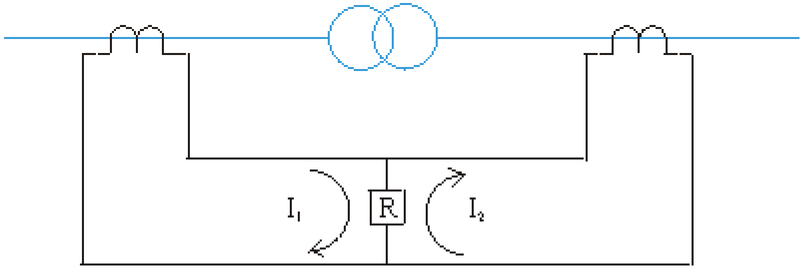




Protective Relay Systems

- Basic function of protection is to detect faults and to clear them as soon as possible.
- Minimum number of items of equipment should be disconnected.
  - Called **SELECTIVITY**.
- **Speed** and **Selectivity** are the most desirable features of Protection
- But Cost also Decides the selection

Unit Protection ⇒

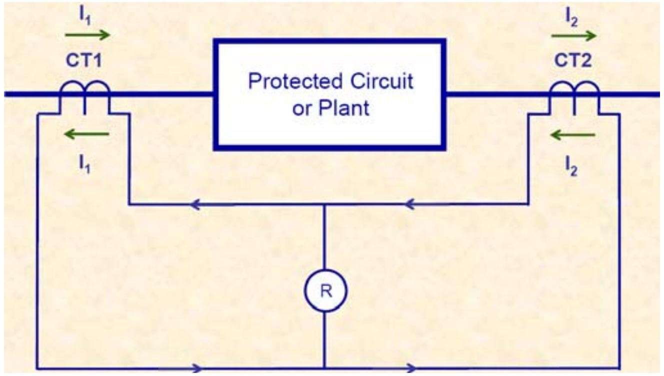


- Internal Fault Protection (87T)

Applications of Unit Protection

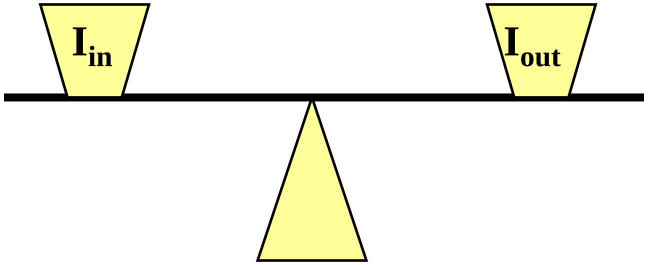
➔ **Circulating current** systems generally used for

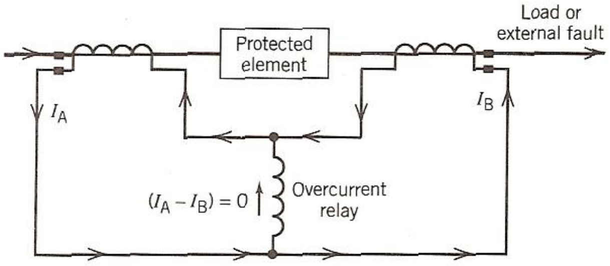
- Transformer,
- Generator,
- Busbars



➔ CT's are situated in same sub-station with common relay

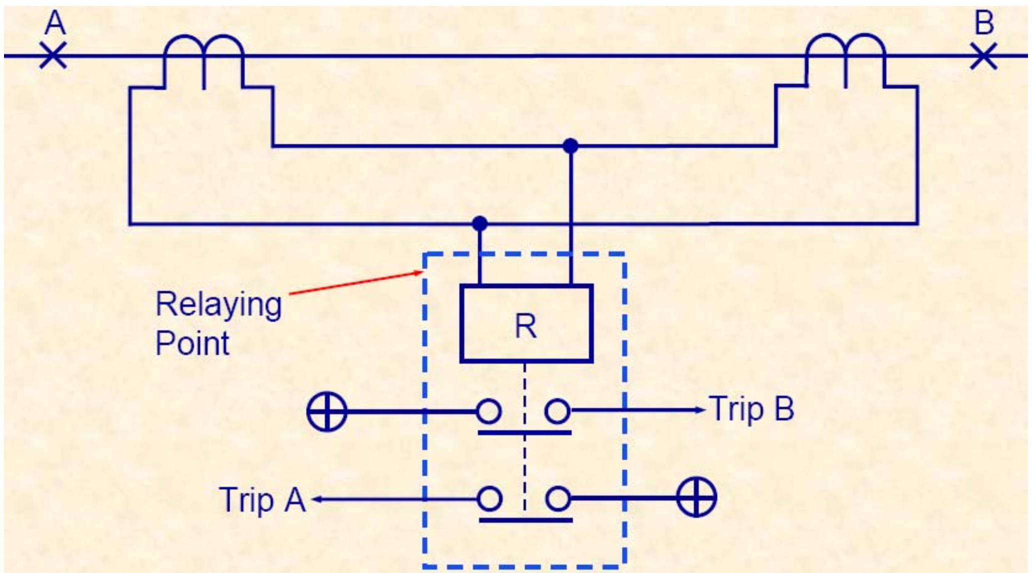
- Compares currents flowing into and leaving a protected zone
- Operates when a set value of differential (difference) currents is reached
- Analog is a balancing beam

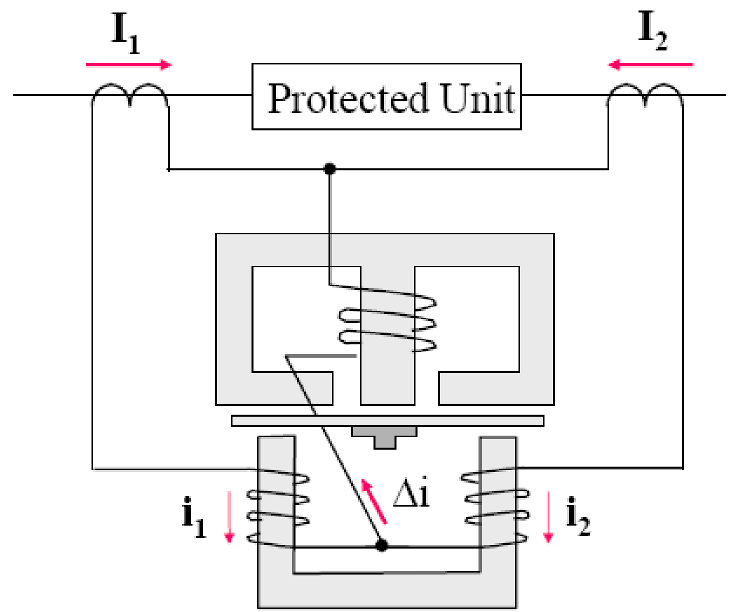




Balanced Circulating current

- Compares currents flowing into and leaving a protected zone
- Use Two sets of CTs at two ends with relay in between
- Require Matching CT's at both the ends



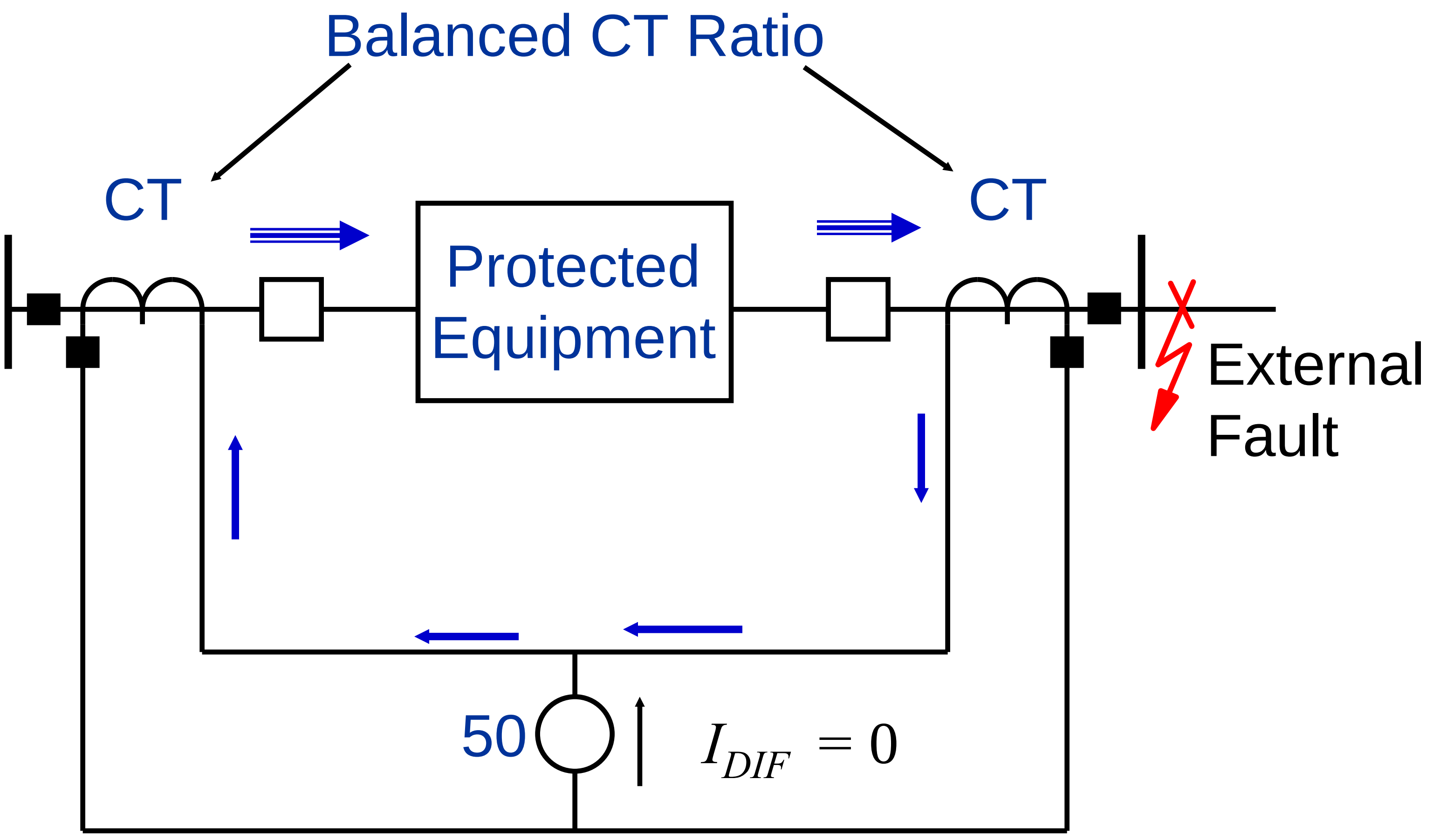


Current balance

circulating current scheme

Balanced Circulating current

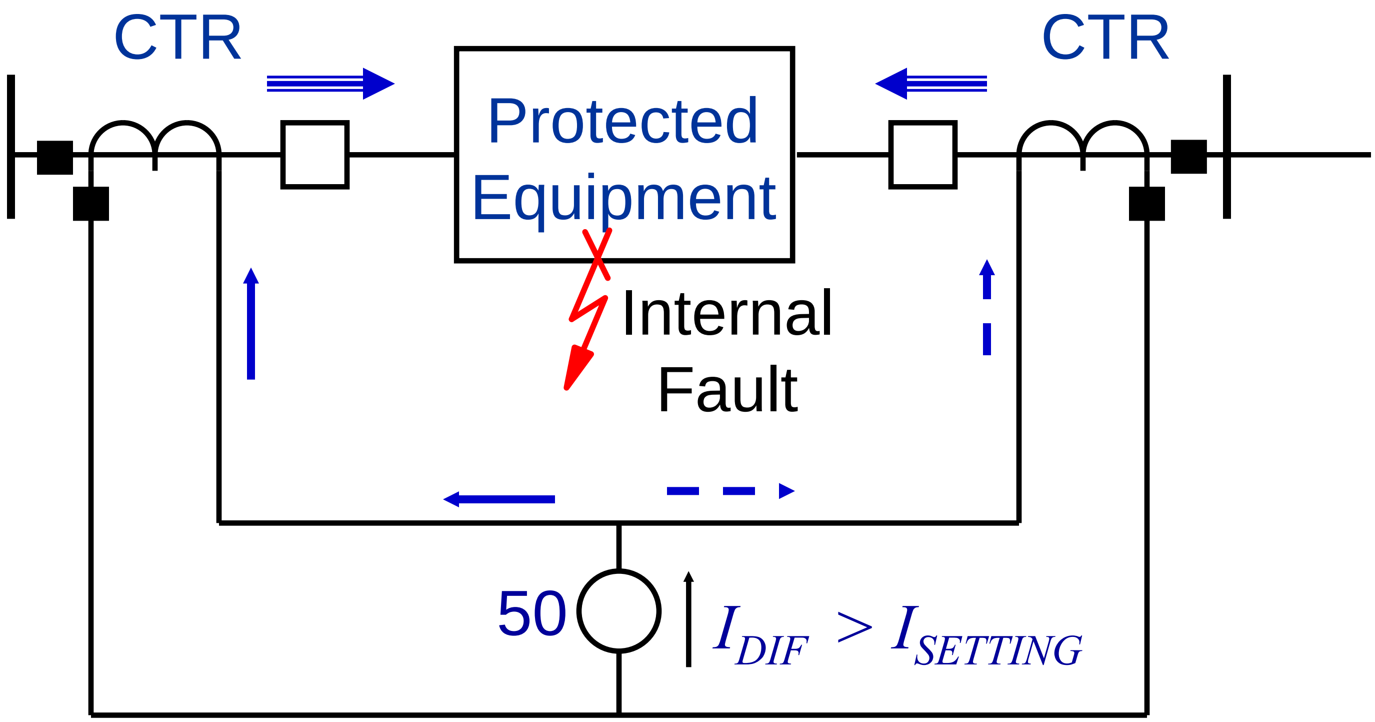
- **External Faults - Stable**



No Relay Operation if CTs Are Considered Ideal

Balanced Circulating current

- **Internal Faults - Operates**



Relay Operates

Protection of Transformers

- Transformers are expensive and important.
- IDMTL relays are not for Overload.
- Recommended protection
  - **Differential protection (optional)**
  - **HV and LV restricted earth fault.**
  - **Buchholz gas and surge relay.**
  - **Oil and winding temperature.**

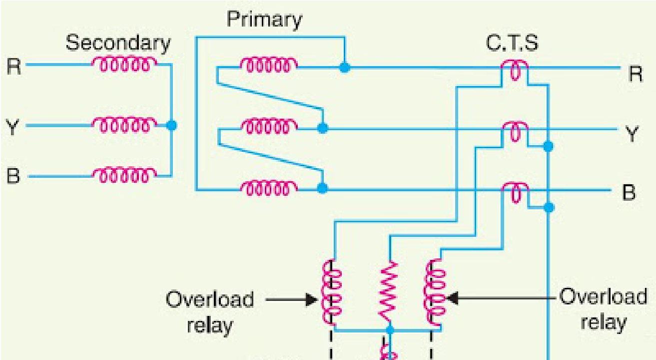
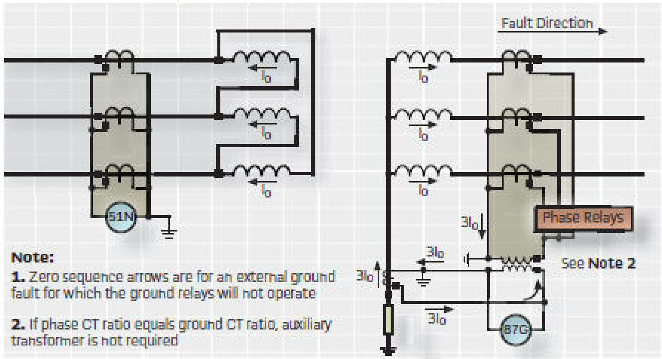


Fig: Transformer's Combined Leakage and Over-load Protection

8 Ground fault protection for a delta-wye transformer



Note:
1. Zero sequence arrows are for an external ground fault for which the ground relays will not operate
2. If phase CT ratio equals ground CT ratio, auxiliary transformer is not required

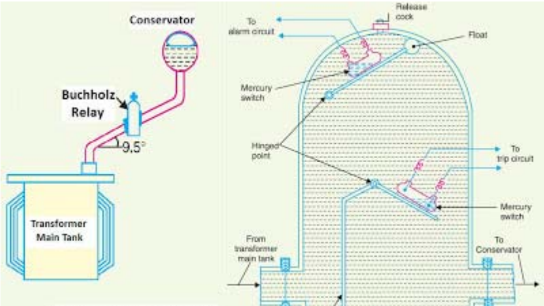


Fig: Buchholz Relay with Constructional Details

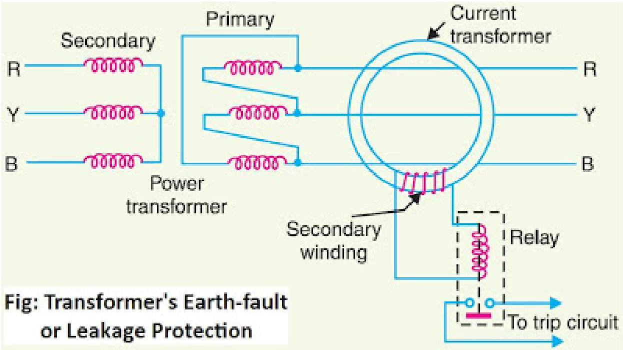


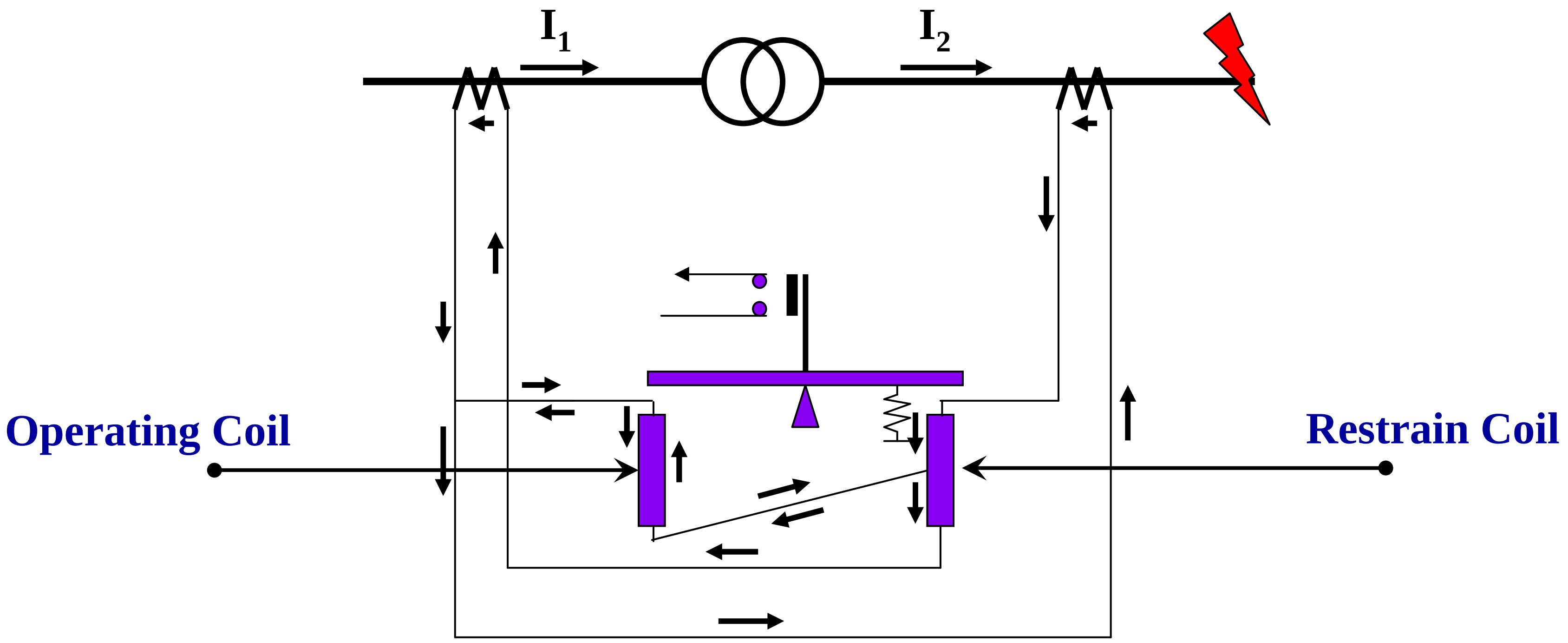
Fig: Transformer's Earth-fault or Leakage Protection

Balanced Circulating current

External fault

$(I_{diff} = I_1 - I_2) < (RST = I_1 + I_2)$

The restraining current increased . So no tripping for the relay

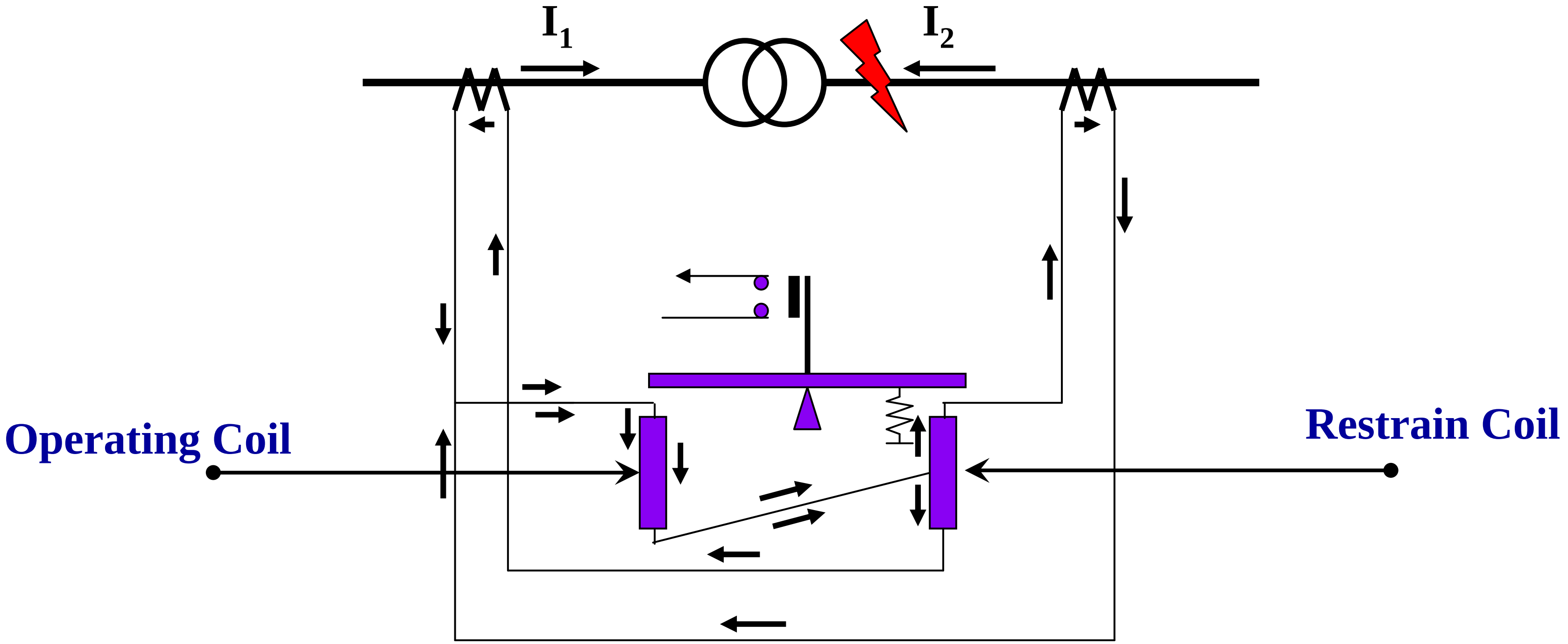


Balanced Circulating current

Internal fault

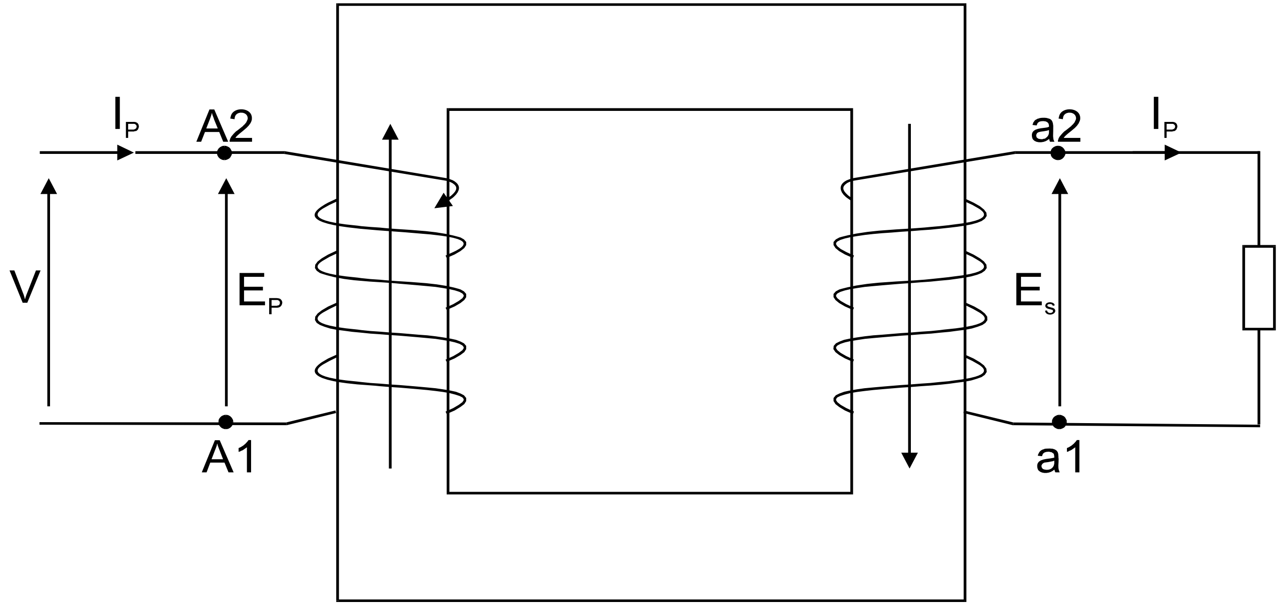
$(I_{diff} = I_1+I_2) >(\ RST= \ I_1+-I_2)$

The restraining current decreased . So the tripping is assured. The percentage differential relay is insensitive to internal faults, with this, the internal faults the restraining current is small. So as we said the tripping is assured.



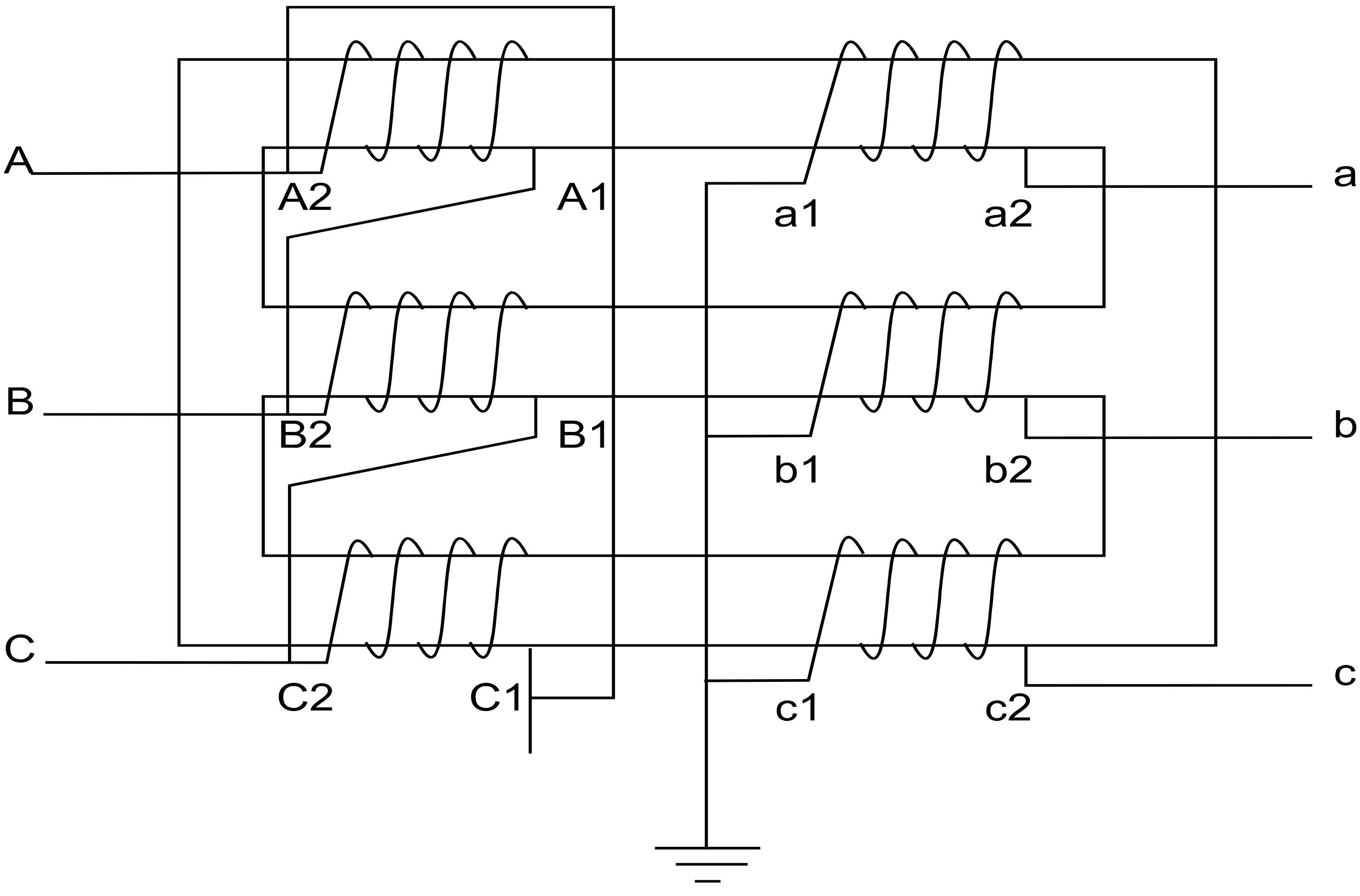
Winding Polarity

- International standards define polarity
- Current in towards **A2** on primary - Then current out from **a2** on secondary because $\boldsymbol{E_s}$ is from **a1** to **a2**

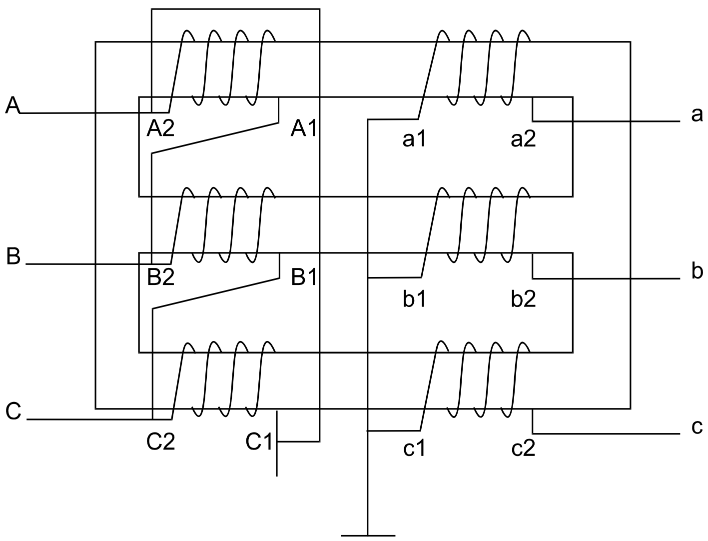


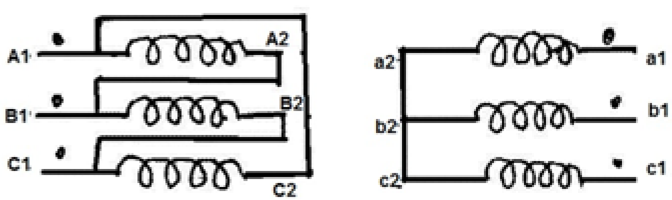
Transformer Connections

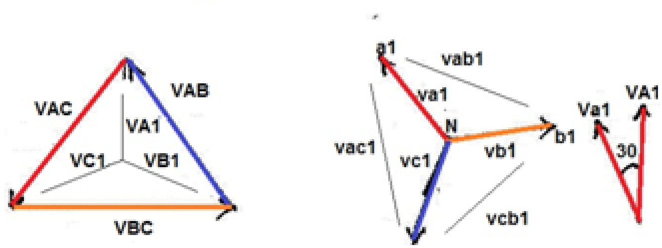
3 Phase -Typical Delta Star (Δ-Y) Connection



Vector Representation







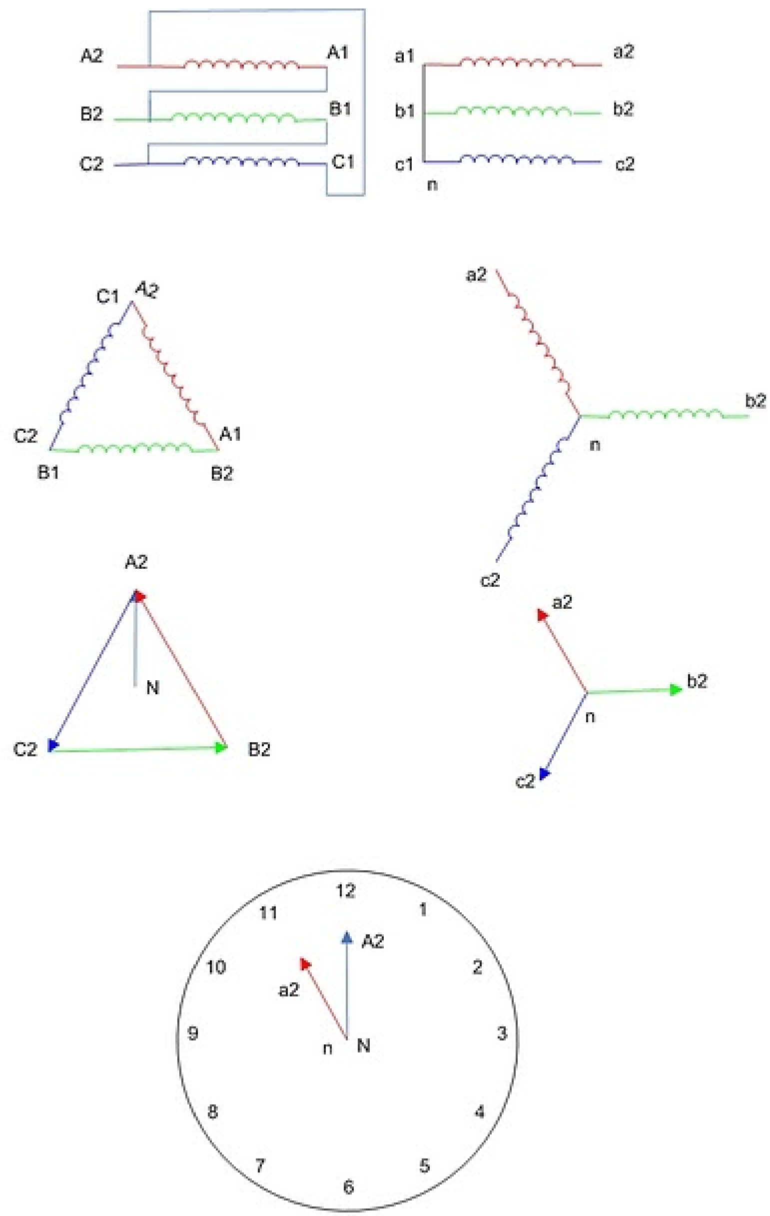
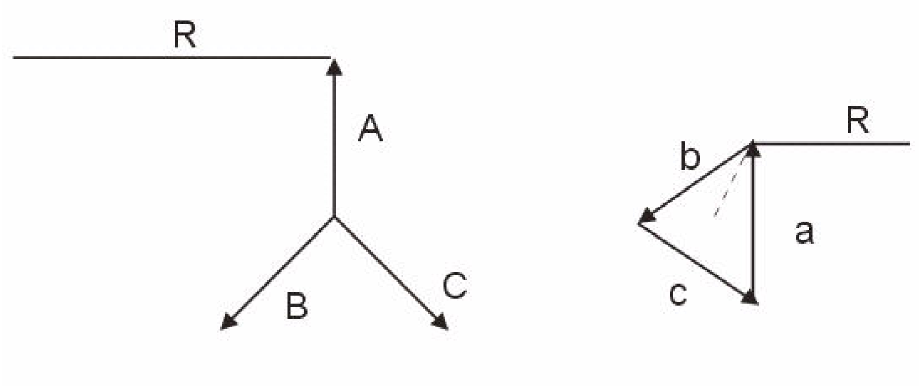


Fig D: Delta - Wye connection for Dy11 vector group

Phase shift

➢ Depending on how the windings in the transformer are arranged - the secondary voltages may be phase - shifted from the primary voltages e.g. Ynd1



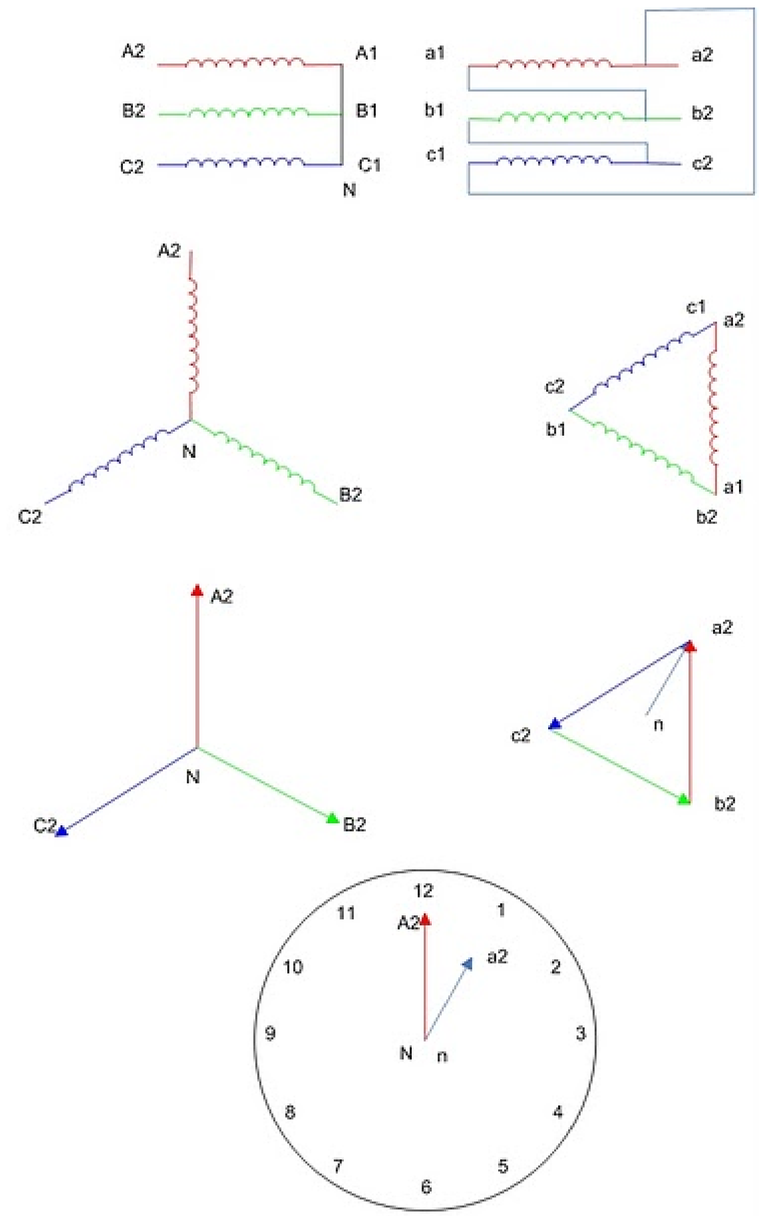


Fig B: Wye - Delta connection for Yd1 vector group

Transformer Vector Group Representation

- Phase Shift Represented by 12 hour Clock positions
- Each 30 degree corresponds to 1 hour shift
- Knowledge of Vector Group and Polarities MUST for Correct Protection

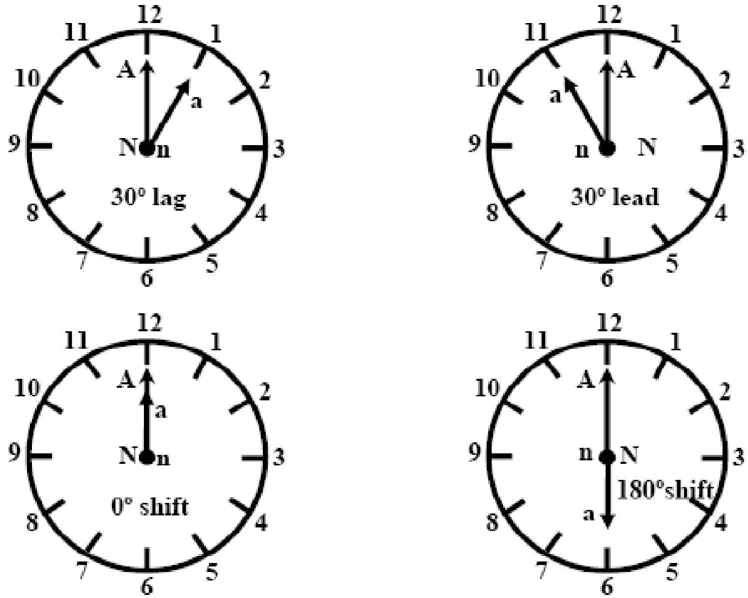


Figure2: Clock convention representing vector groups.

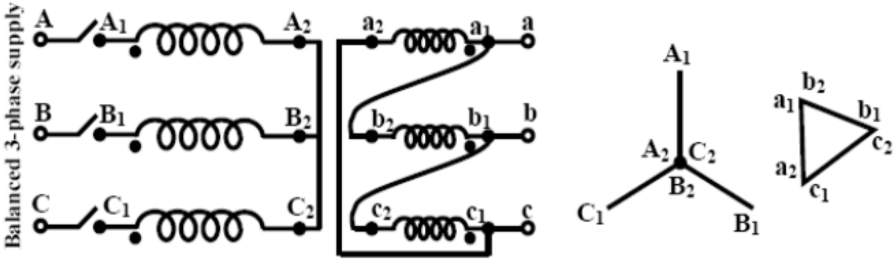


Figure 3: Connection and phasor diagram for *Y d*11.

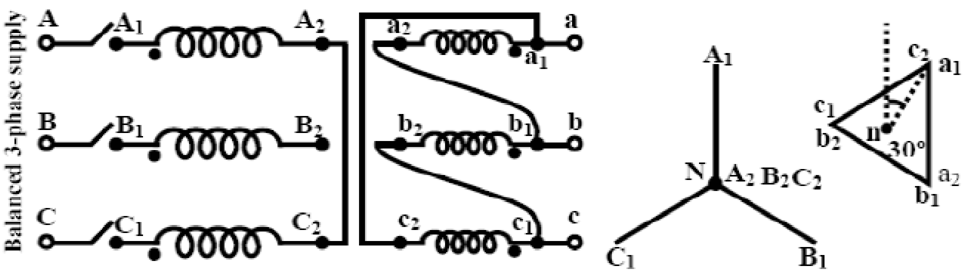
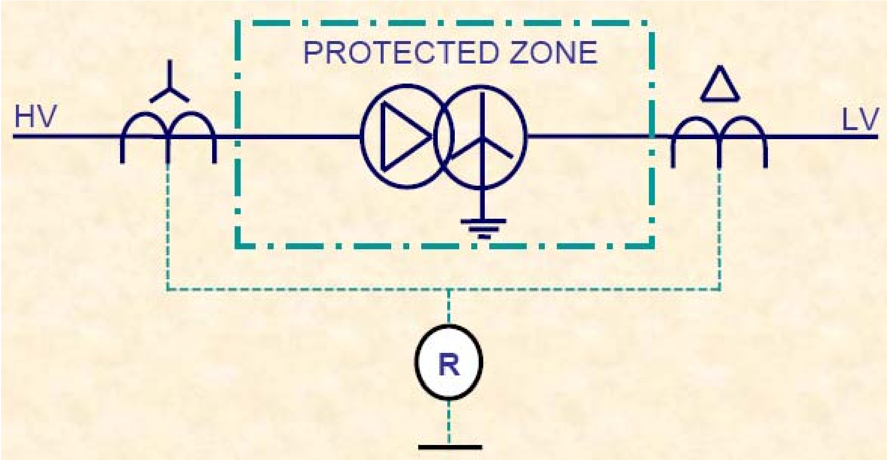


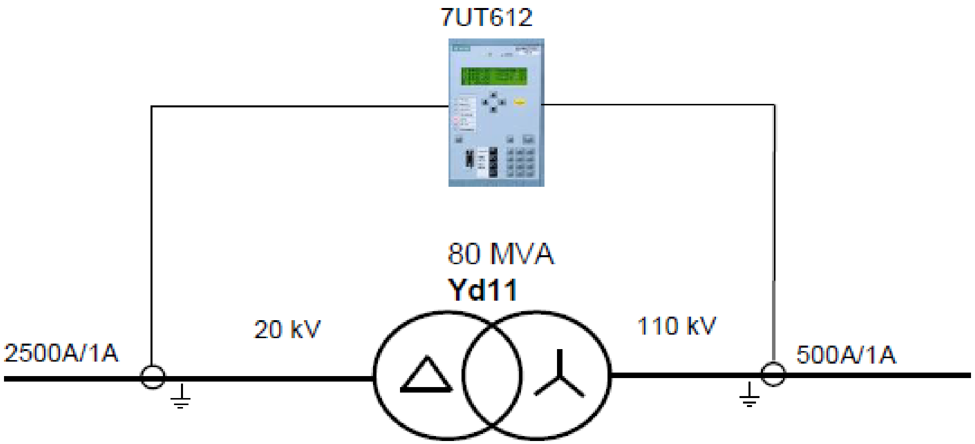
Figure 4: Connection and phasor diagram for *Y d*1.

Delta-Star transformer Differential Connection

Correct application of differential protection requires CT ratio and winding connections to match those of transformer.
CT secondary circuit should be a “replica”of primary system.
Consider:

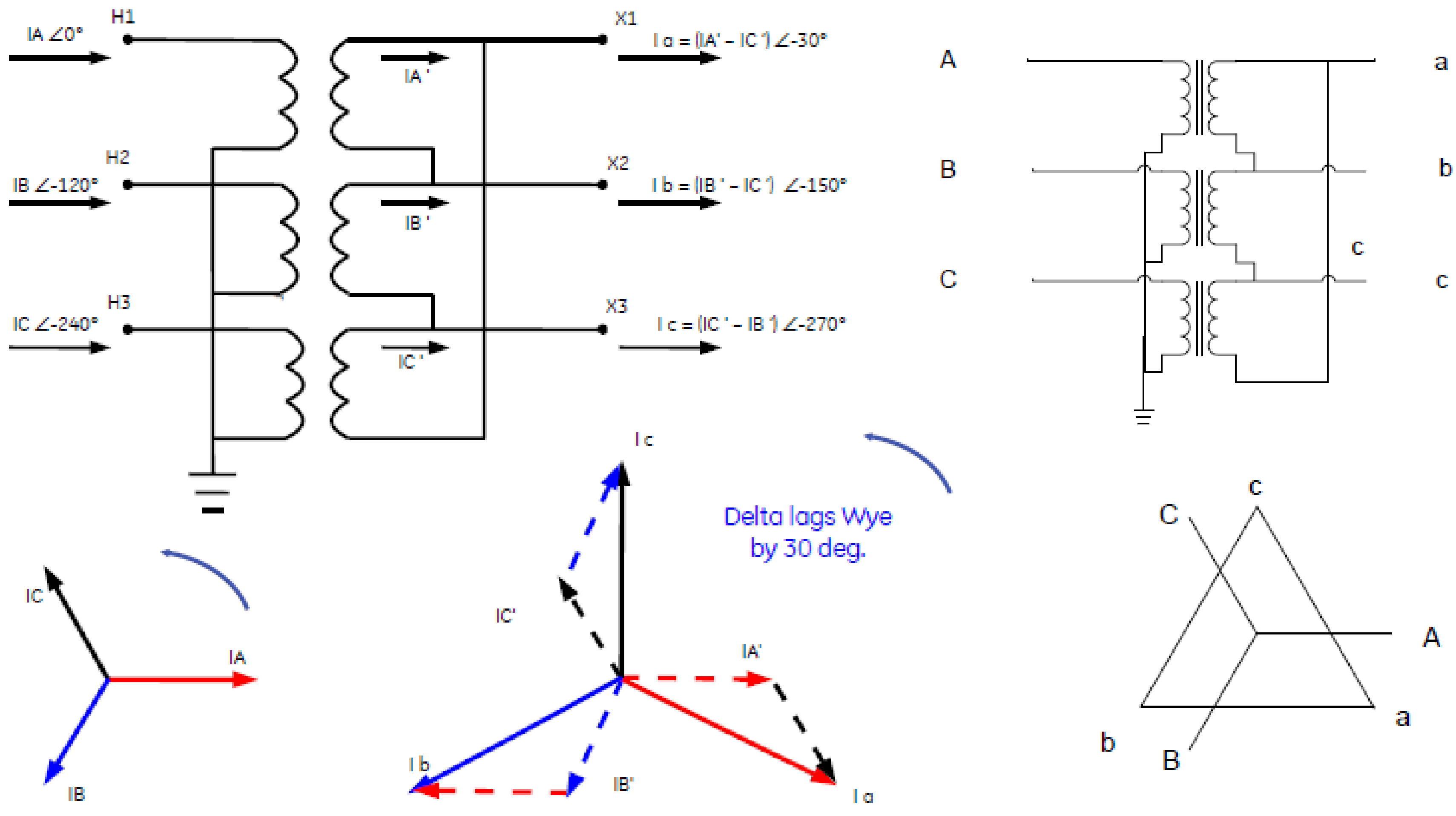
(1)Difference in current magnitude
(2)Phase shift
(3)Zero sequence currents



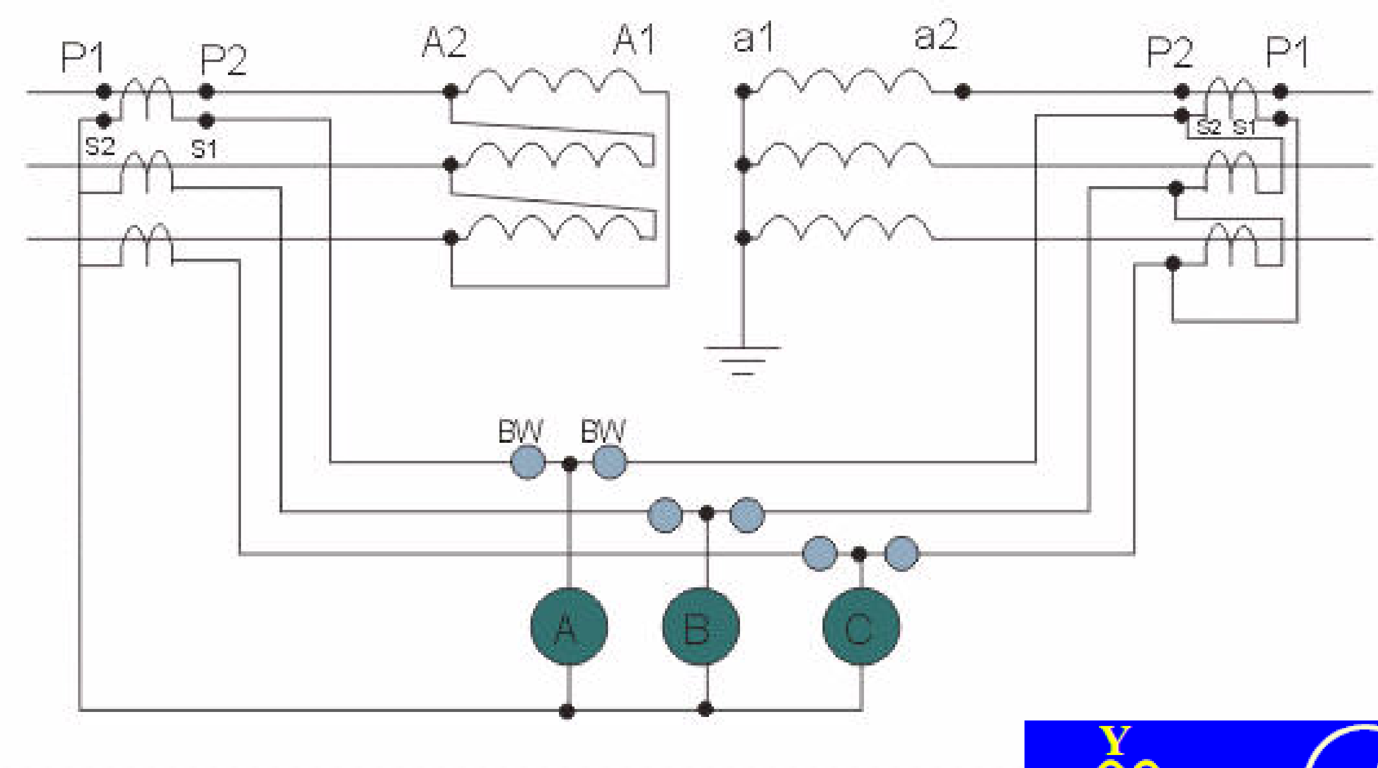


Phase Compensation

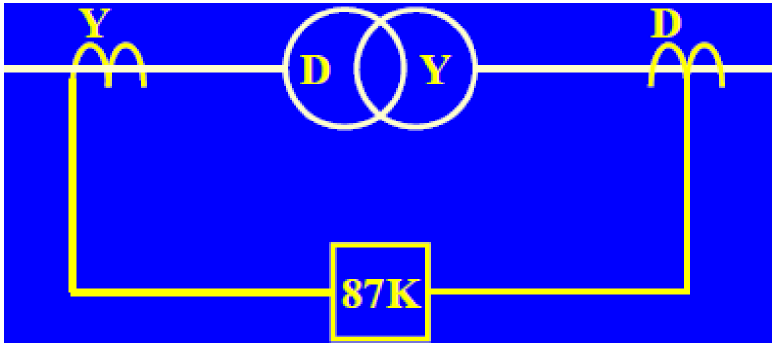
ABC rotation : compensation angle = - 30º – 0º = 30º lag



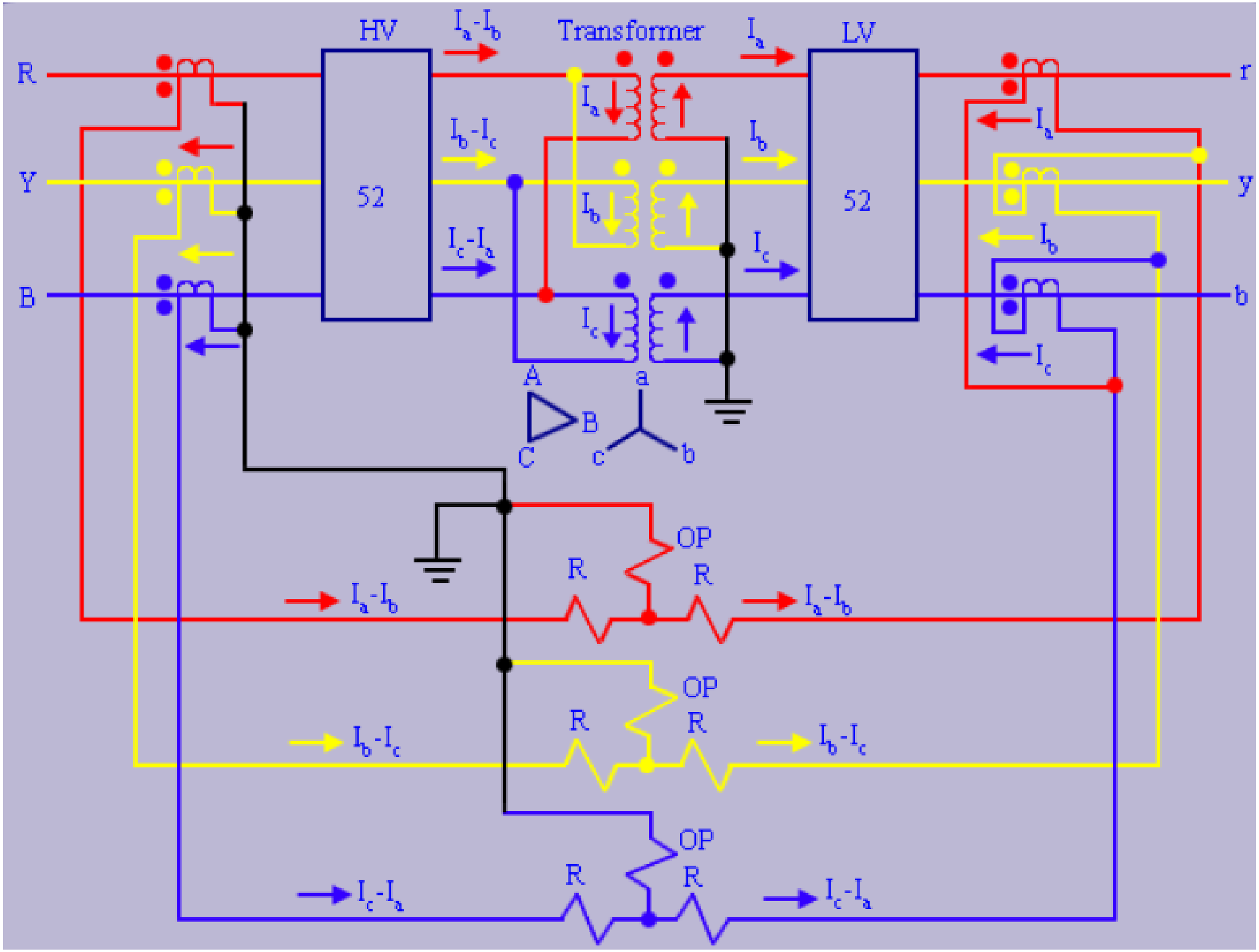
Delta-Star transformer Differential Connection



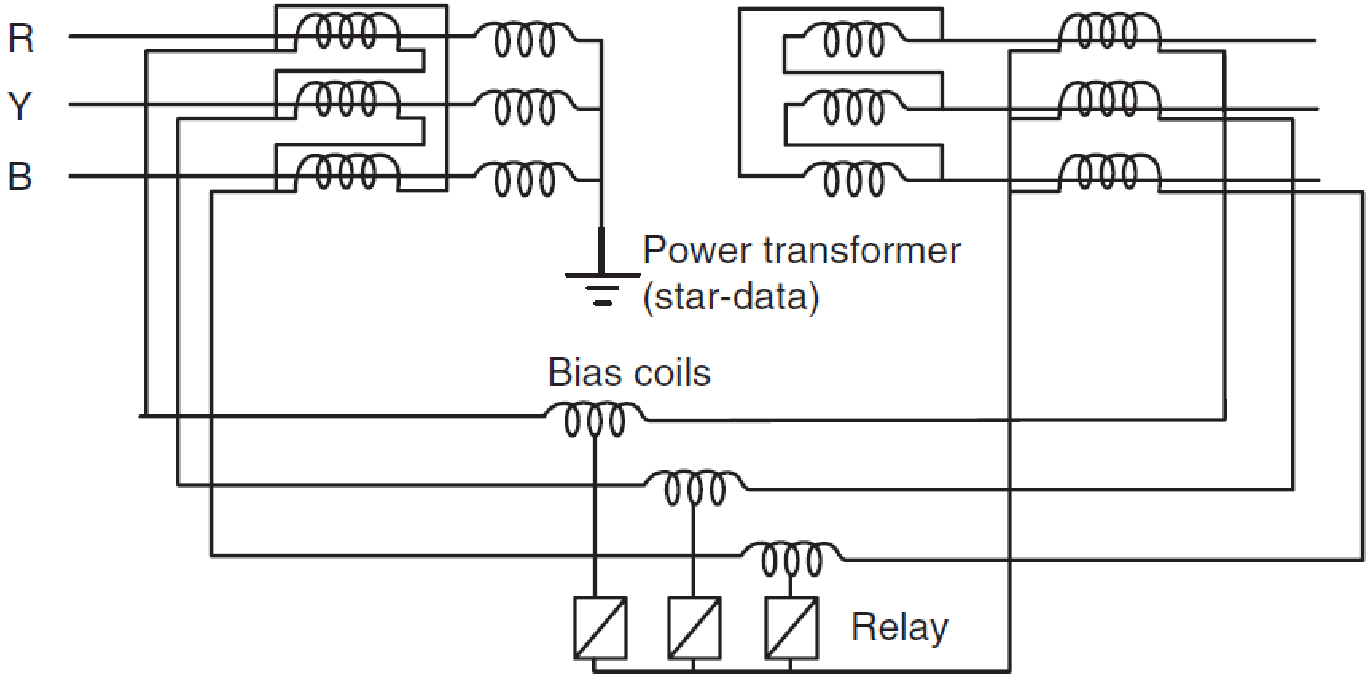
Connect HV and LV CT's in Star delta opposite to the vector group connections of the primary windings



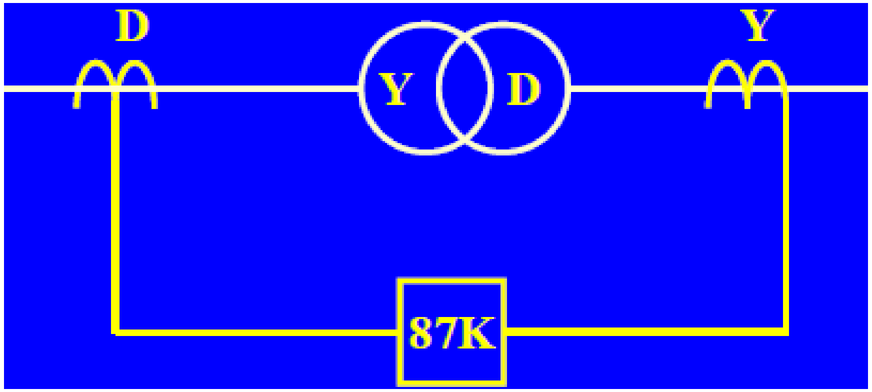
Delta-Star transformer Differential Connection



Star- Delta transformer Differential Connection



Connect HV and LV CT's in delta Star opposite to the vector group connections of the primary windings



Delta-Delta transformer Differential Connection

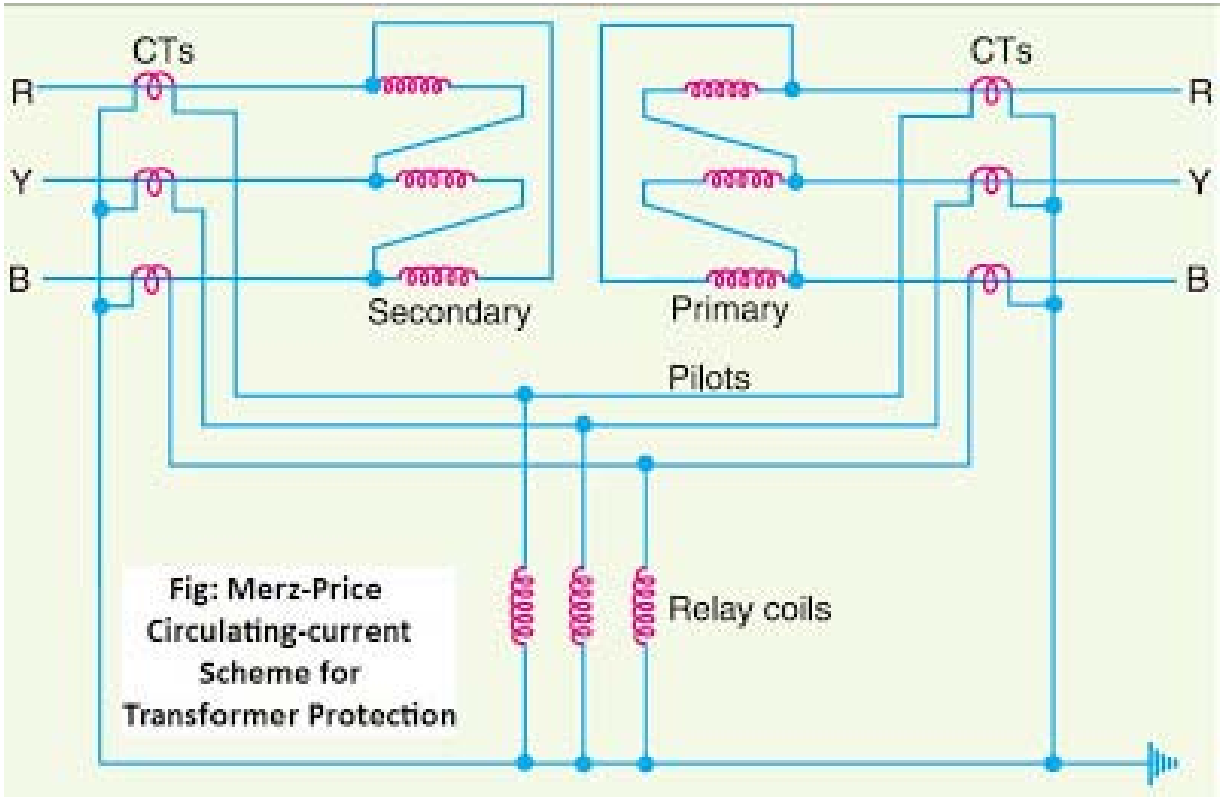
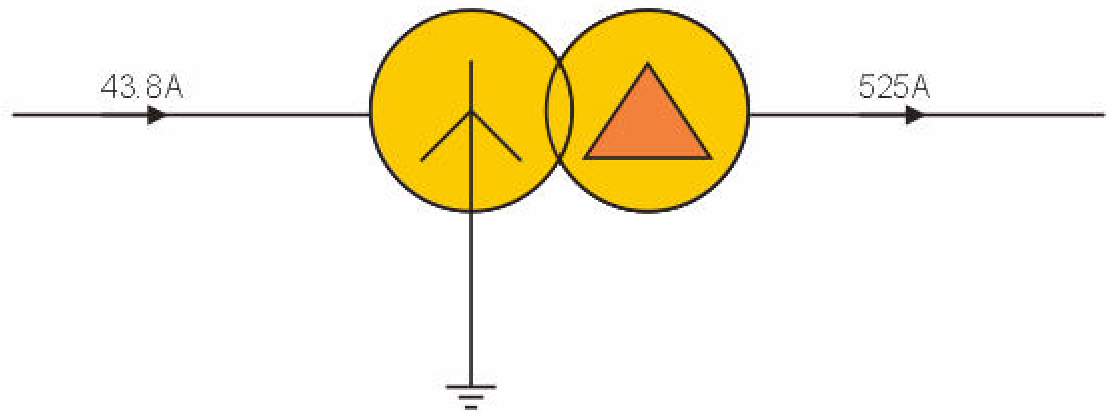


Fig: Merz-Price Circulating-current Scheme for Transformer Protection

Mis-match of CT's

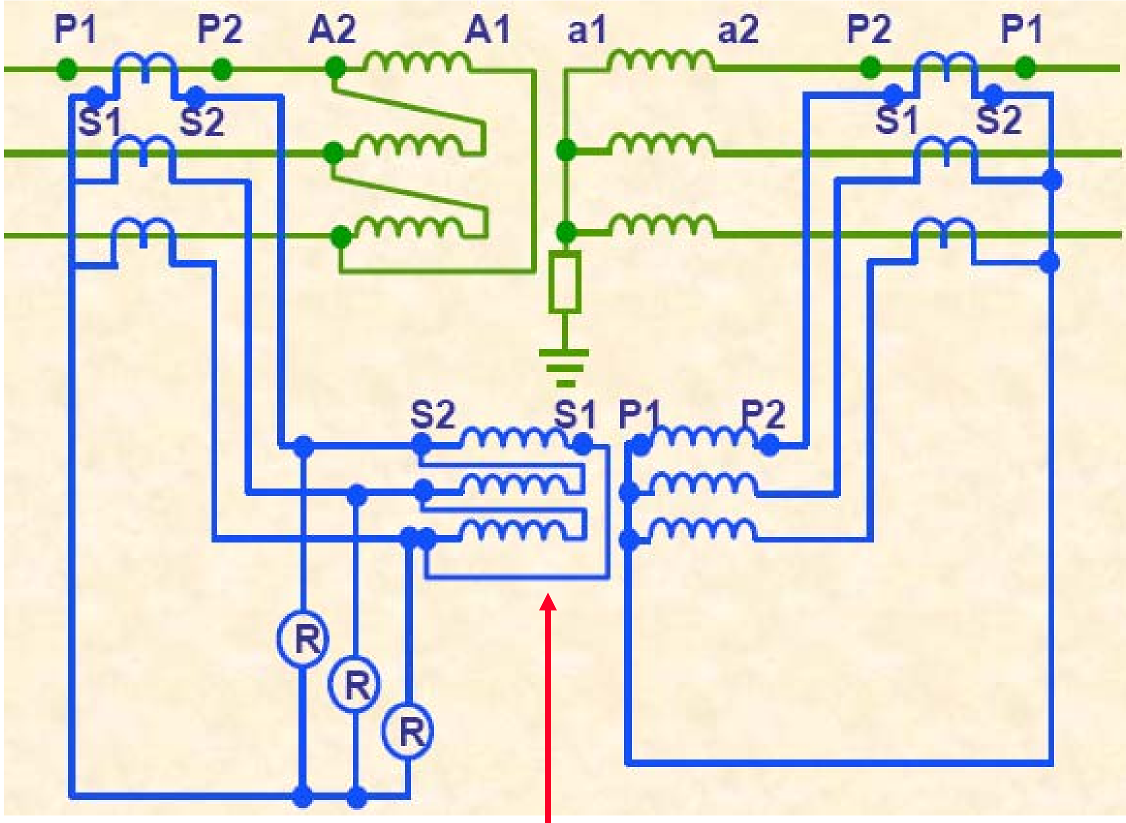
132/11 kV

10 MVA



- CT's Required on primary and secondary for Protection
- The closest ratio available to 43.8 amps is 50/1 ➔ 0.876 secondary Amps
- the closest ratio to 525 Amps is probably 500/1 ➔ 1.05 secondary Amps
- Also, the CT's could be from different manufacturers
- Auxilairy or Matching Transformer is required

Transformer Differential Protection Relay



Matching Transformer

Interposing CT provides :

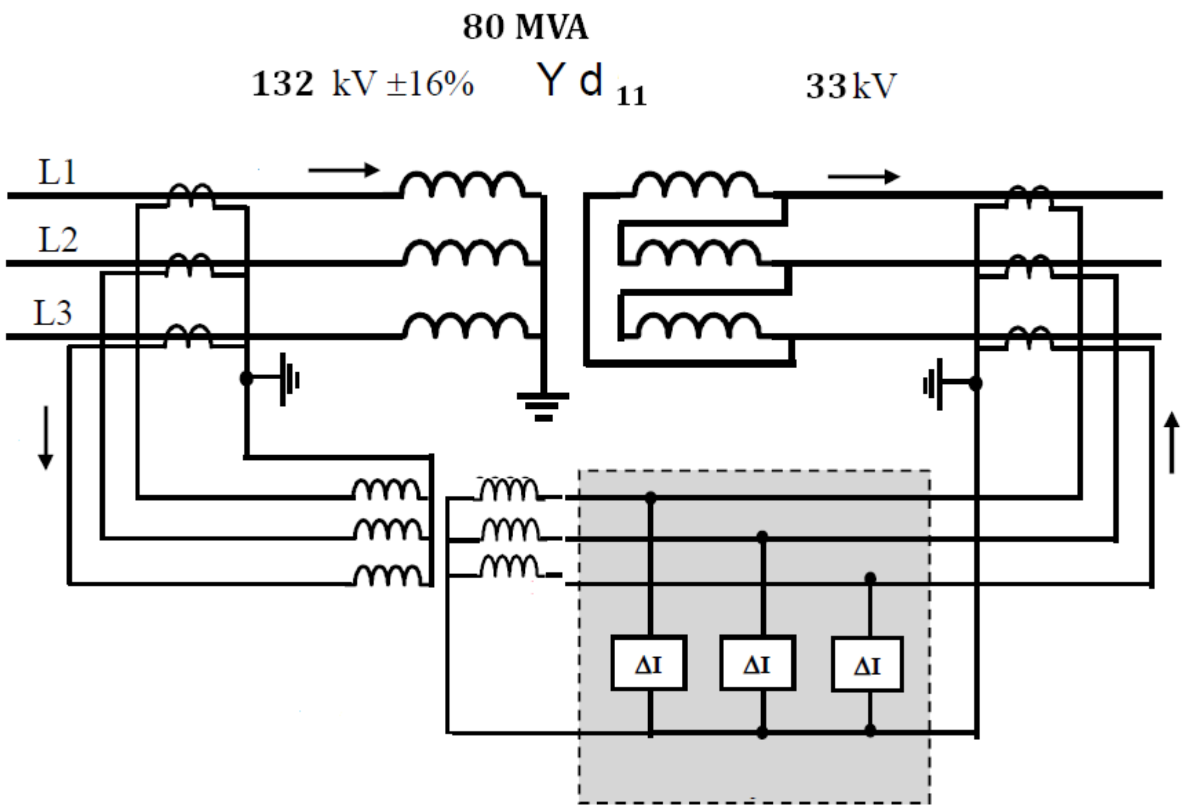
- Vector correction
- Ratio correction
- Zero sequence compensation

Transformer Differential Protection Relay

1. Current Mismatch

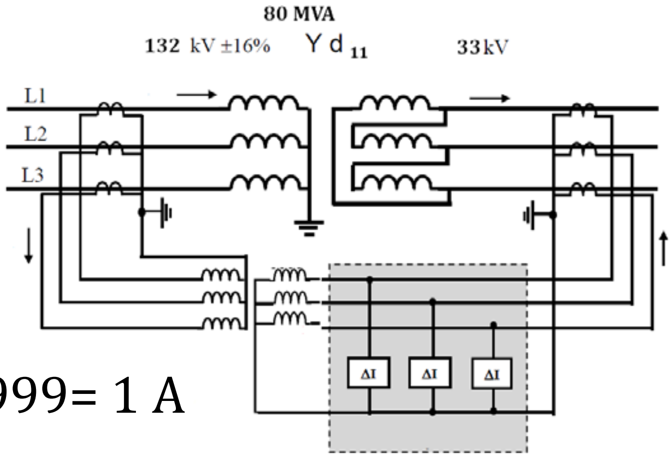
Interposing CT’s can easily overcome the current mismatches.

Let us look at this example:



Transformer Differential Protection Relay

At the current at the secondary side (33 kV)



$$I2 = \frac{80\ MVA}{\sqrt{3} \times 33\ KV} = 1399.63$$

So we choose a CT with 1400/1

So the secondary current = 1399.63/1400 = .9999= 1 A

At the 132 kV side important note. In this example we will not consider the tap changer changing current.

$$I1 = \frac{80\ MVA}{\sqrt{3} \times 132\ KV} = 349.9 = 350\ A$$

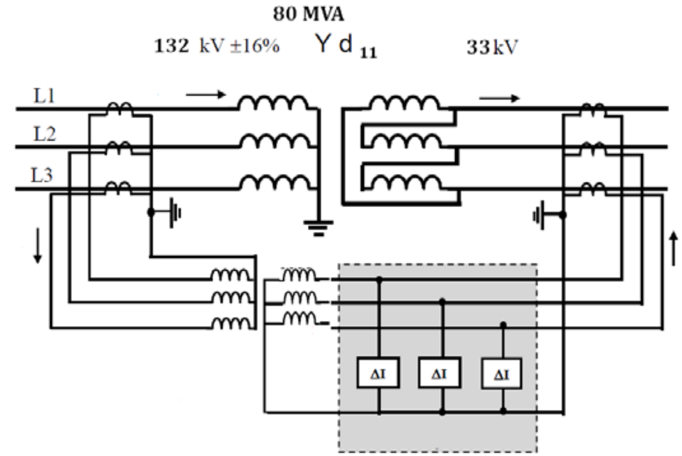
So we choose a ct ratio with 400/1
So the secondary current will be = 0.875 A
So we need to correct the 0.875 A into 1 amps

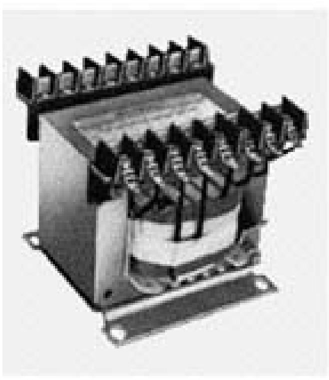
So we need an interposing CT with a ratio = $\frac{1}{0.875} \approx 1.1$

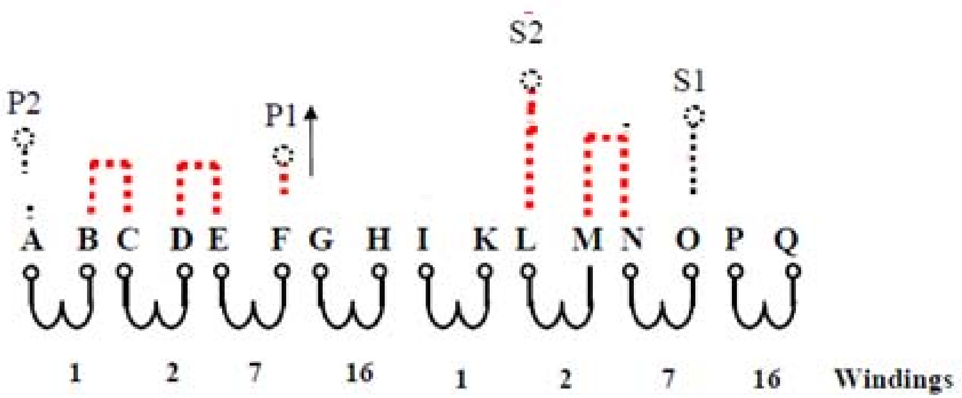
How this can be done?

Transformer Differential Protection Relay

Use Interposing CT







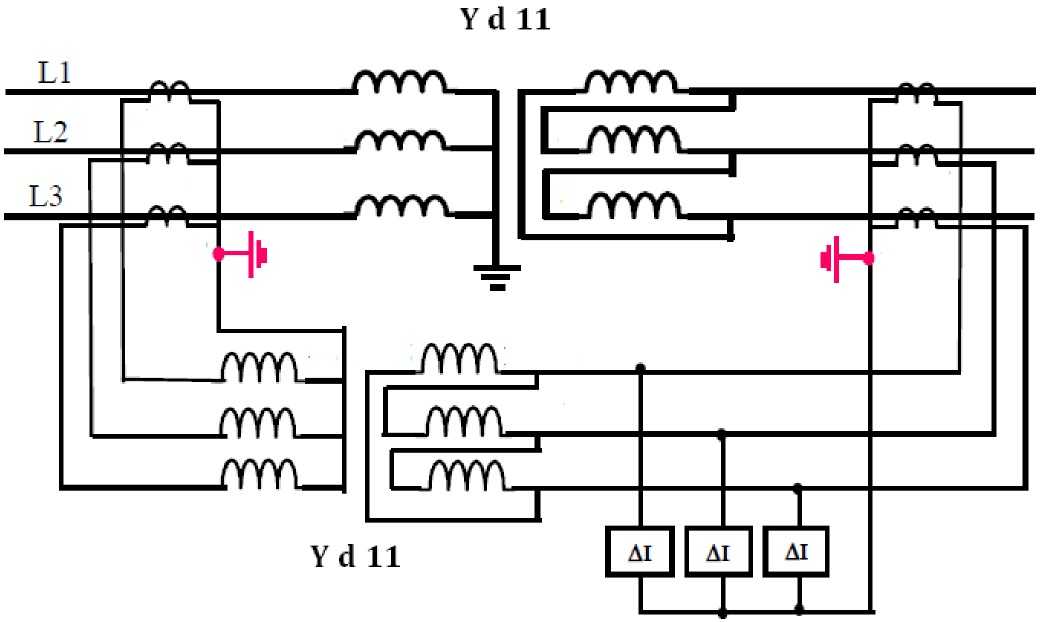
$$\text{chosen}: \frac{w_1}{w_2} = \frac{1+2+7}{7+2} = \frac{10}{9} = 1.1$$

Connection and links as shown.

Transformer Differential Protection Relay

2. Phase Shift Correction:

Phase shift can be easily overcome using interposing CT's. Let us look at this example:

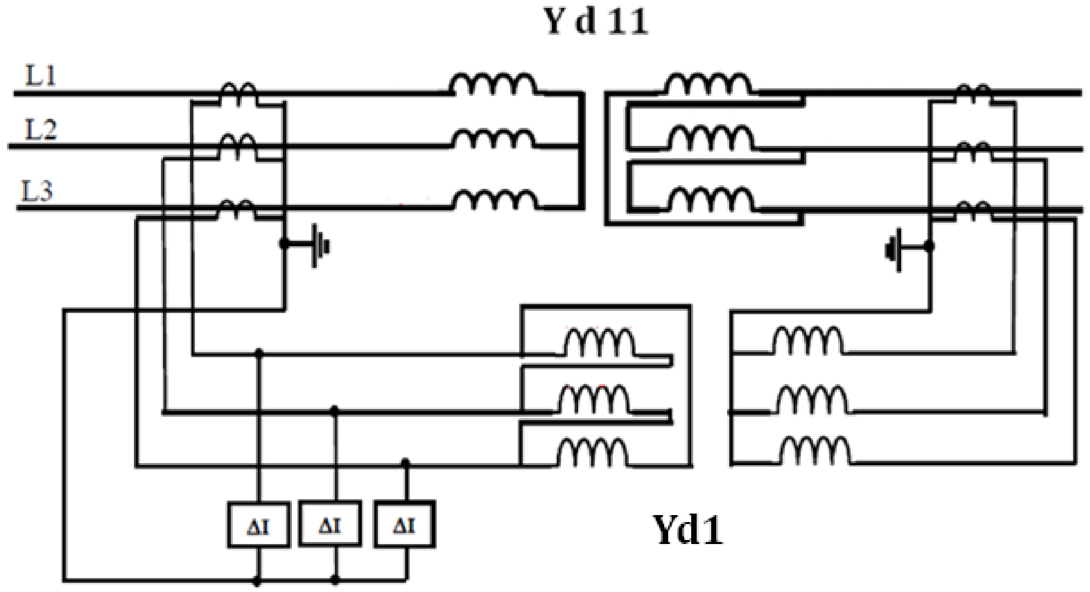


The secondary of the transformer is phase shifted by 330^{o} or its leading the primary by 30^{o}.

Any way we want to phase shift the primary side by the same shift so we change it at the interposing CT by Yd11 so at the interposing Ct the current no is phase shifted by 330^{O} or also leading by 30^{O} i.e. the same at the secondary so the phase shift is resolved .

Transformer Differential Protection Relay

Suppose we want to overcome the secondary winding ….?

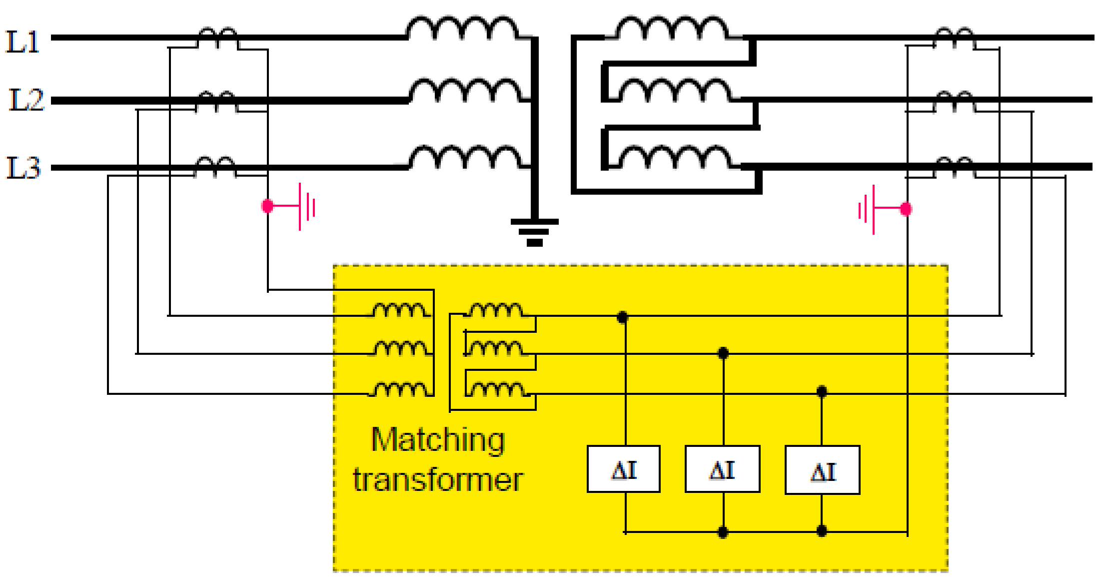


The secondary winding is phase shifted by 330^{o} leading the primary by 30^{o}, so we need to phase shift the secondary into 0^{o} (the angle of the primary) so we connect the interposing CT into Yd1,

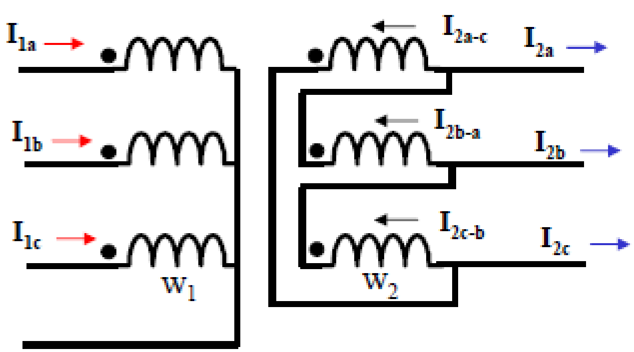
so the secondary winding now phase shifted by 30^{o} lagging, so 30^{o} lagging + 30^{o} leading = 0^{o} which is the same as the primary current.

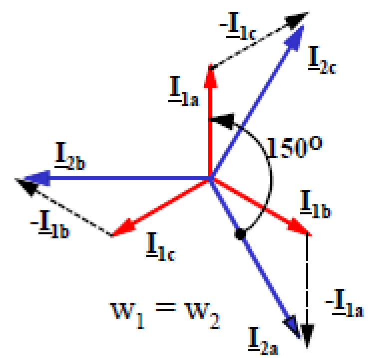
Transformer Differential Protection Relay

Y d 5



Yd5



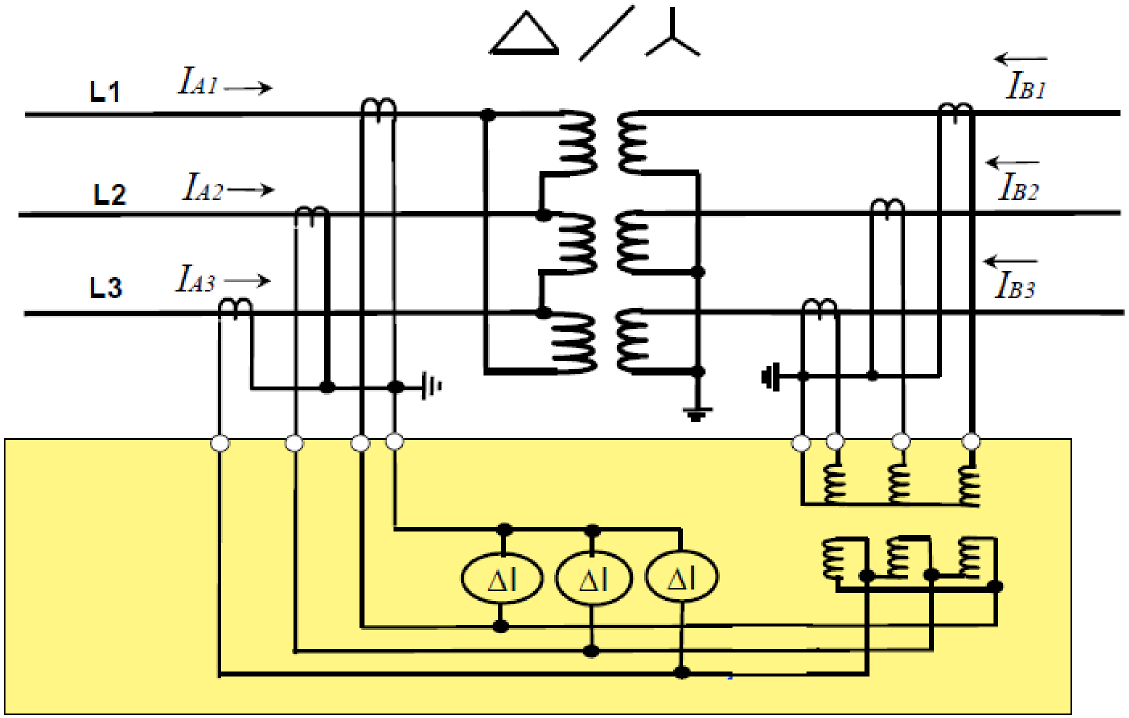


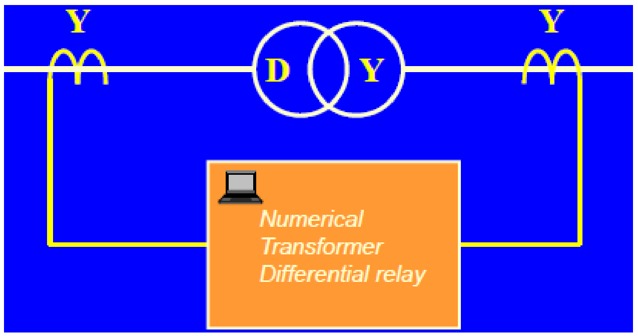
$$\underline{I}_1 = \frac{1}{\sqrt{3}} \cdot \underline{I}_2 \cdot \frac{w_2}{w_1} \cdot e^{j\cdot(n\cdot 30^\circ)}$$

Transformer Differential Protection Relay

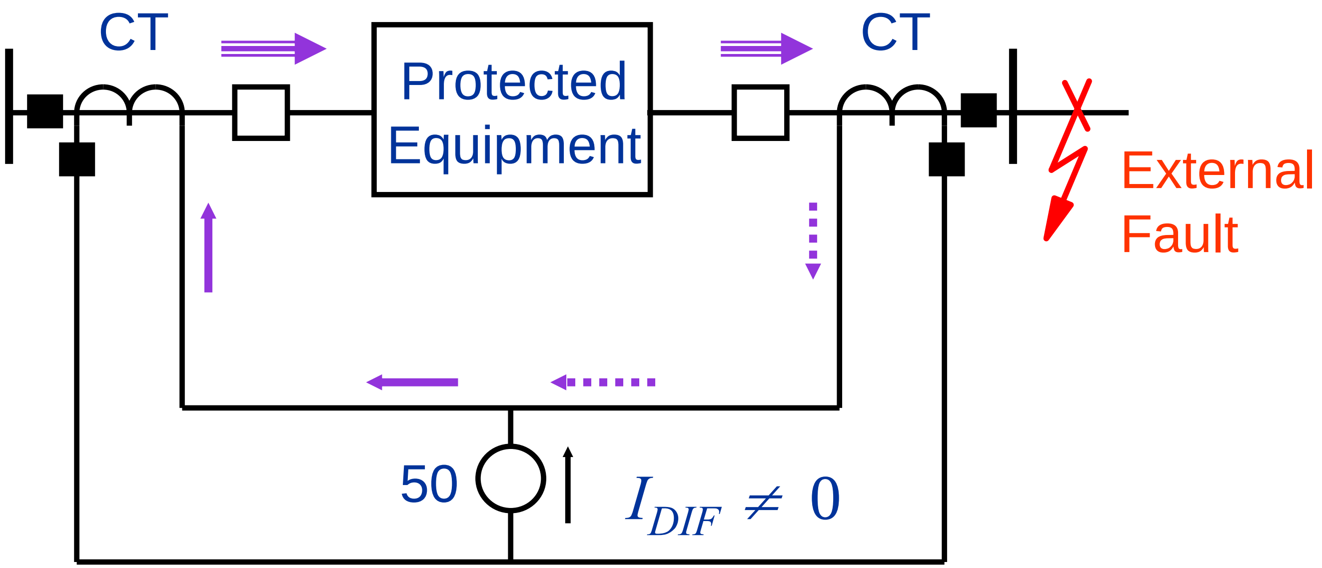
Transformer Digital differential protection:

Digital relays in these days do internally the phase shift correction + zero sequence correction + current mismatches.
We just add the transformer information into the relay , the relay itself do all the correction



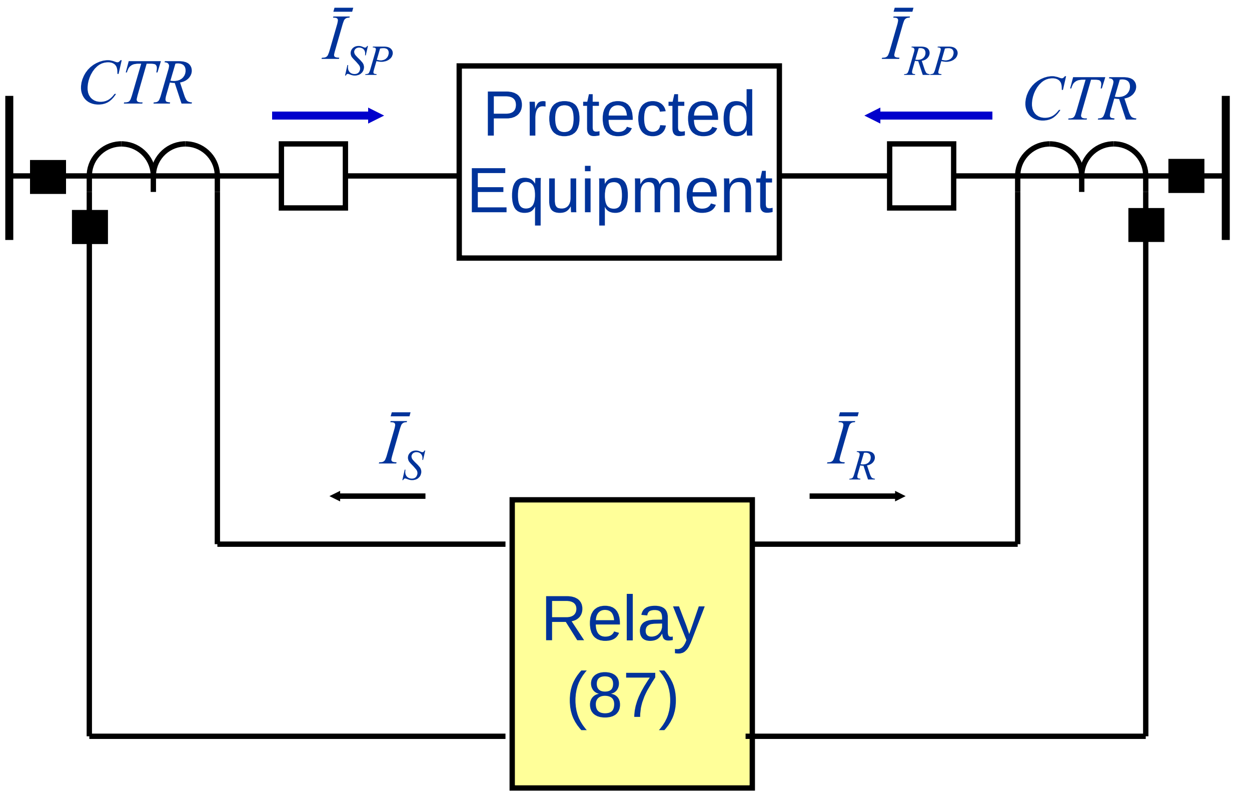


Problem of Unequal CT Performance



- False differential current can occur if a CT saturates during a through-fault
- Use some measure of through-current to desensitize the relay when high currents are present

Possible Scheme – Percentage Differential Protection

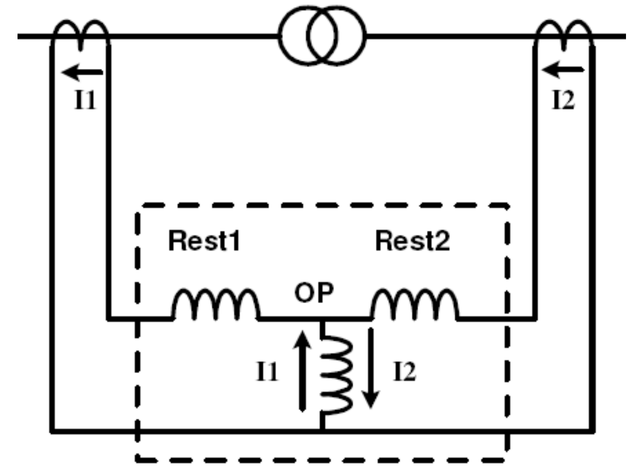


Compares:

$$\begin{cases} \left|\bar{I}_{OP}\right| = \left|\bar{I}_S + \bar{I}_R\right| \\ k \cdot I_{RT} = k \cdot \dfrac{|\bar{I}_S| + |\bar{I}_R|}{2} \end{cases}$$

Biased Differential Relay

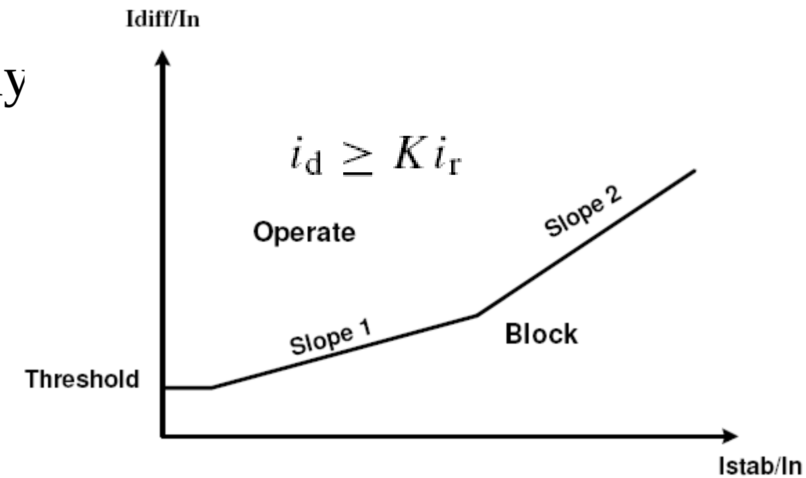
- Large external fault may cause false operation of simple differential relay (because of CT Saturation) .
- To make the differential relay more stable to external faults and improve relay quality, restraining coils were inserted.
- Two restraining (Biasing) coils and one operating are used as shown. Restraining coils will opposite the operation of operating coil. The relay will operate only when the operating force is higher than restraining force.



Biased Differential Relay

Measurement

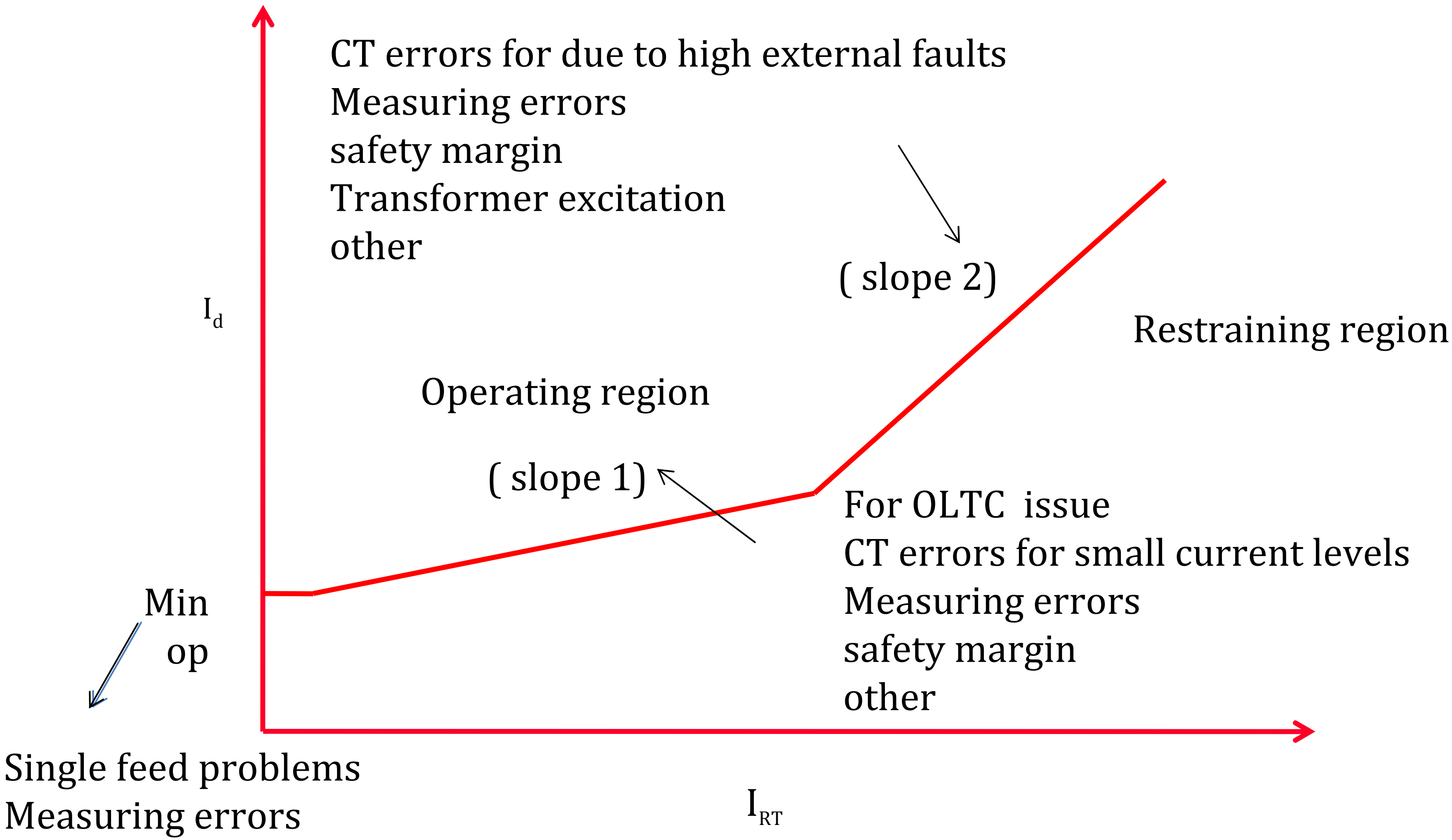
- $I_{bias} = (|I_1| + |I_2|)/2$
- $I_{diff} = |I_1 + I_2|$



Tripping Characteristics of Biased Differential Relay

Stability provided by Biasing

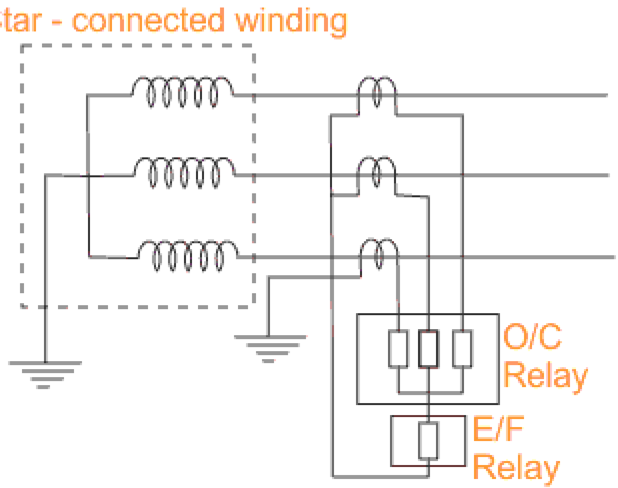
Biased Differential Relay



Over Current and Earth Fault Protection of Transformer

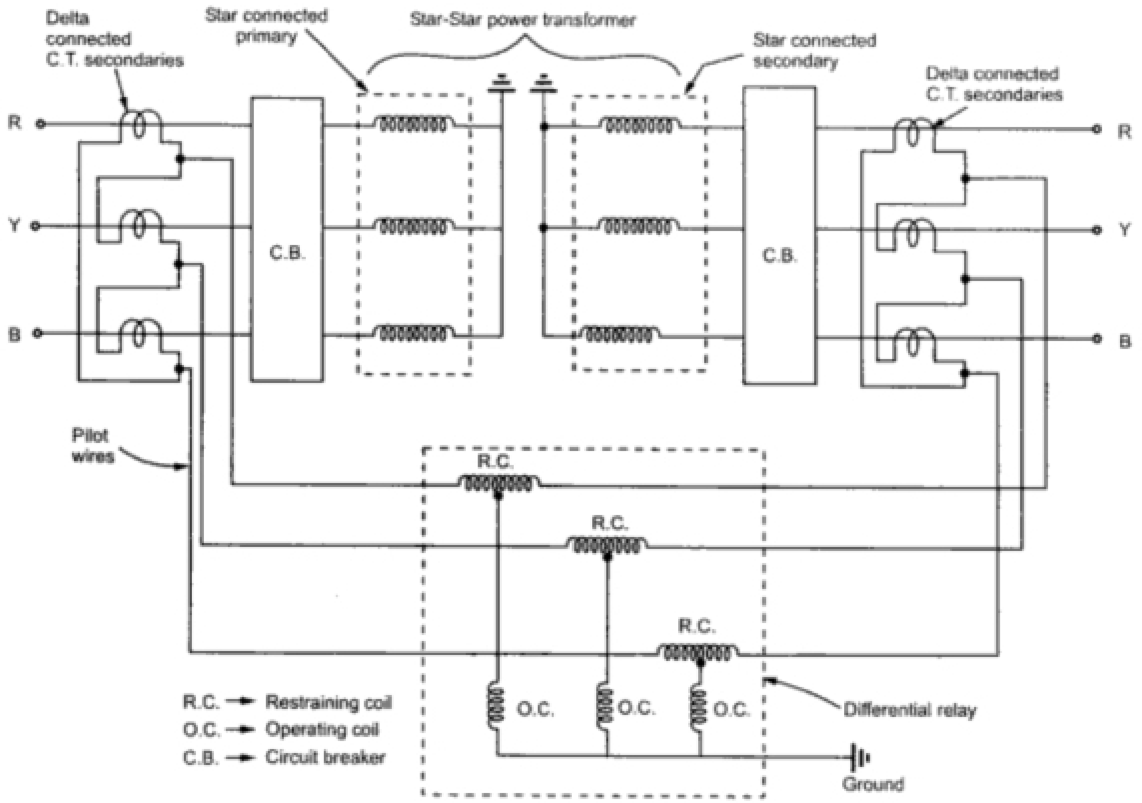
- **Backup protection of electrical transformer** is simple **Over Current and Earth Fault protection** applied against external short circuit and excessive over loads.
- These overcurrent and earth Fault relays may be of Inverse Definite Minimum Time (IDMT) or Definite Time type relays.
- Generally IDMT relays are connected to the in-feed side of the transformer .

Over Current and Earth Fault protection relays may be also provided in load side of the transformer too, but it should not interrupt the primary side Circuit Breaker



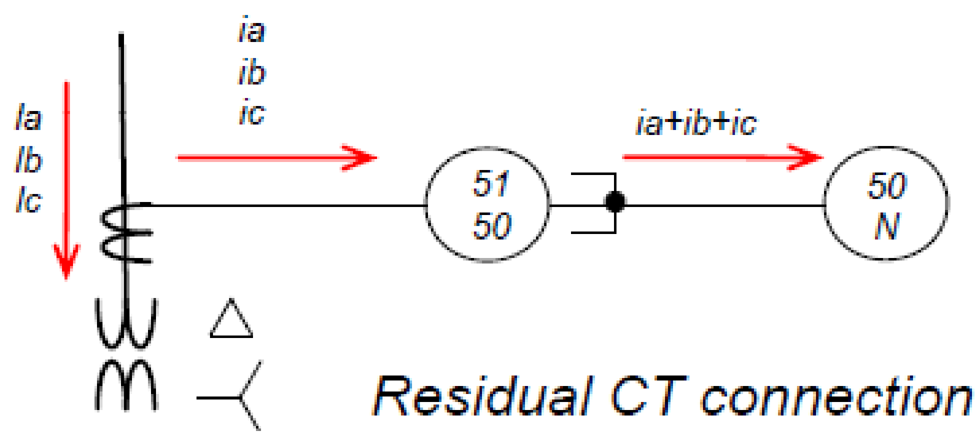
Backup O/C & E/F Protection Scheme

Star-Star transformer Differential Connection



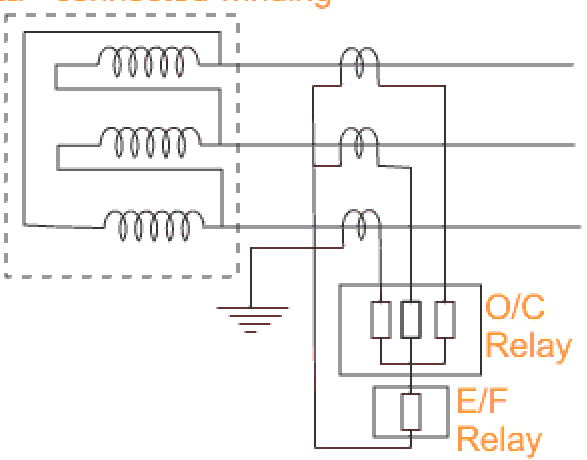
Over Current and Earth Fault Protection of Transformer

- **Backup protection of transformer** has four elements, three **OC relays** connected each in each phase and one **EF relay** connected to the common point of three **OC relays**.
- The normal range of current settings available on IDMT **OC relays** is 50% to 200% and on **EF relay** 20 to 80%.



Residual CT connection

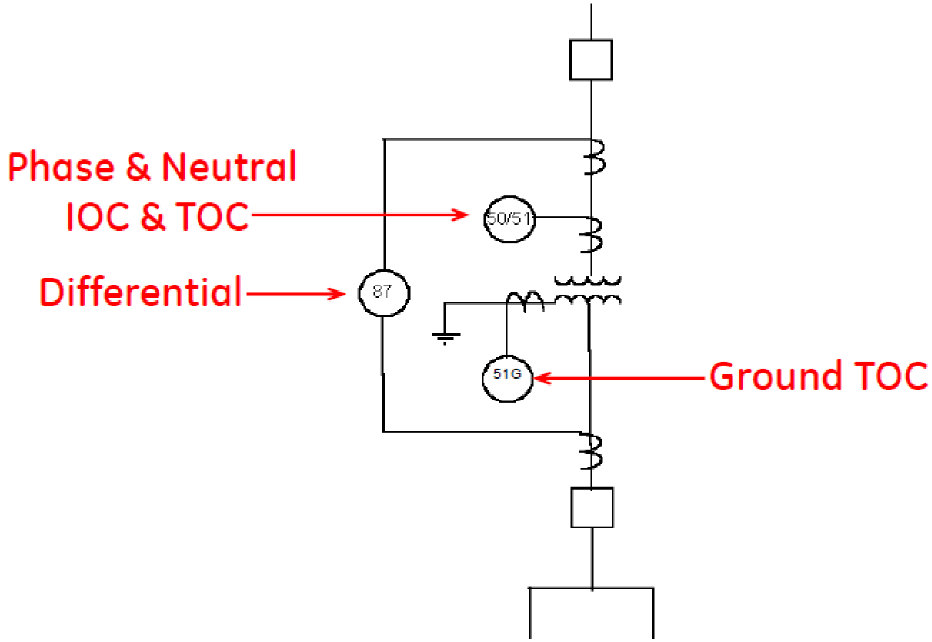
Delta - connected winding

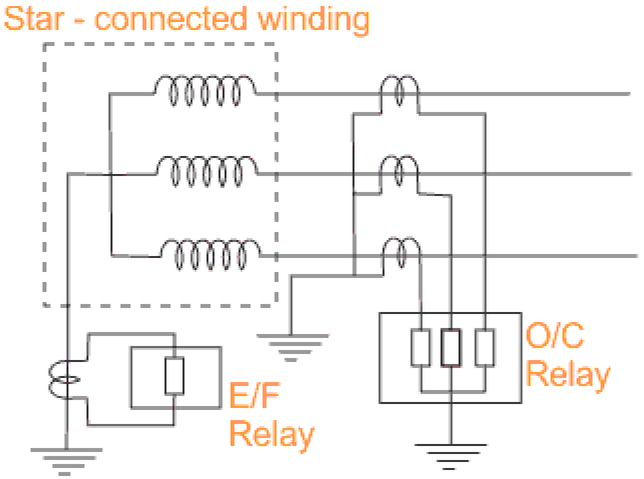


Backup O/C & E/F Protection Scheme

Over Current and Earth Fault Protection of Transformer

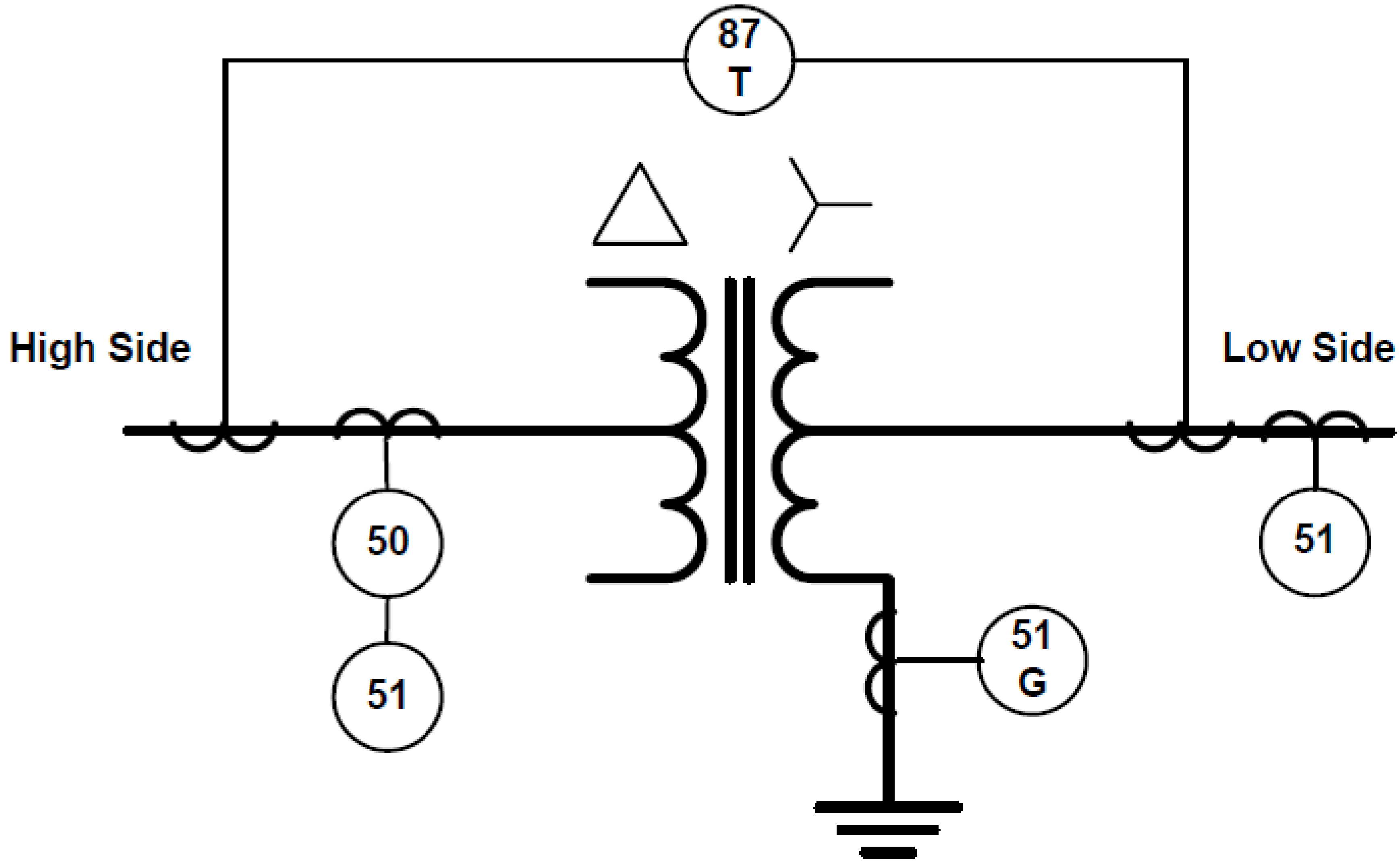
- In the case of transformer winding with **neutral earthed**, **unrestricted earth fault protection** is obtained by connecting an ordinary earth fault relay across a neutral current transformer.
- The unrestricted **OC relays** and **EF relay** should have proper time lag to coordinate with the protective relays of other circuit to avoid





O/C & Unrestricted E/F Protection Scheme

Restricted Earth Fault System

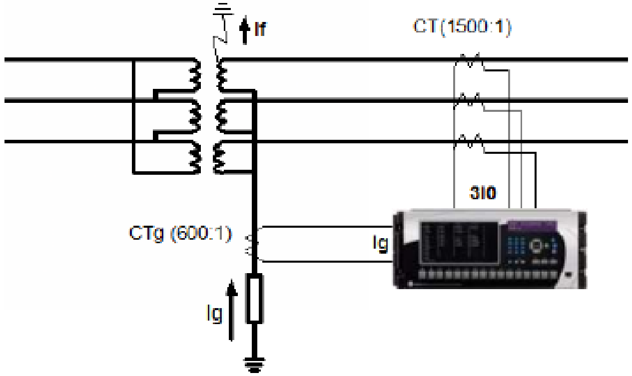


Restricted Earth Fault Protection

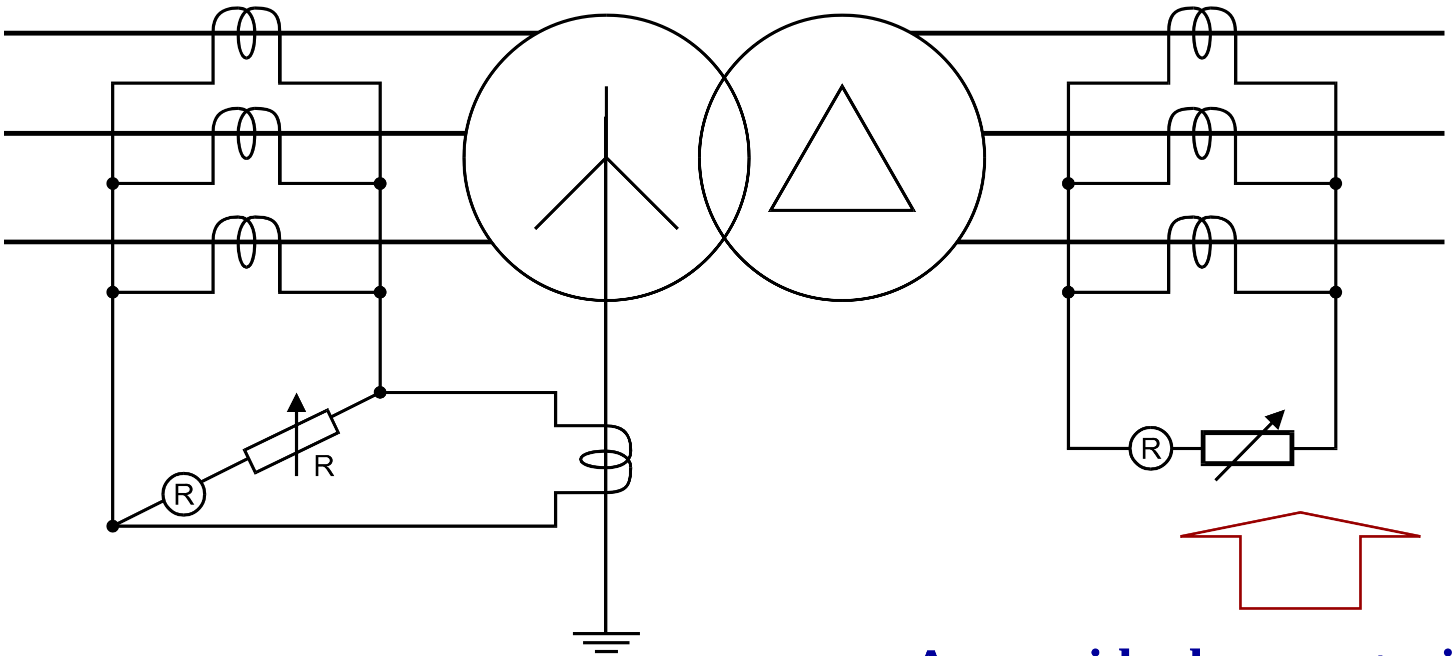
Differential relay provides a sensitive protection for internal faults, including phase to phase and earth faults, but the relay is not sensitive for small internal short faults, so another protection is applied in order to protect from earth faults,

Restricted earth fault relay is based on comparison of measured variables. By comparing residual current of the phase current transformer of a given winding with the current of the associated grounded star potential.

REF may be referred as unit earth fault protection and the restricted part of the earth fault protection refers to an area defined between the 2 CT's.

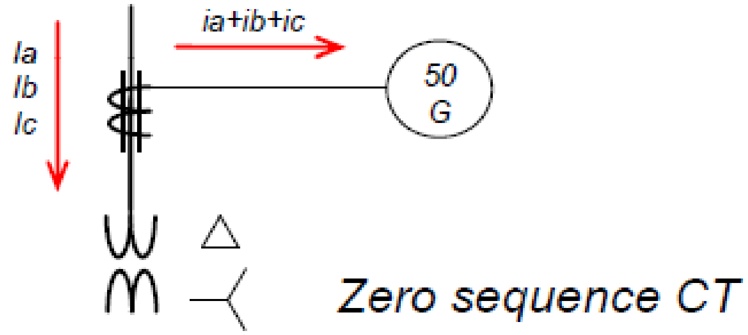


Restricted Earth Fault System



Any residual current will cause relay to operate for E/F in zone

CT currents balance - no operating voltage to relay



Restricted Earth Fault Protection

The restricted earth fault relay is a high impedance differential scheme which balances zero sequence current flowing in the transformer neutral against zero sequence current flowing in
the transformer phase windings.

Any unbalance for in-zone fault will result in an increasing voltage on the CT secondary and thus will activate the REF protection.

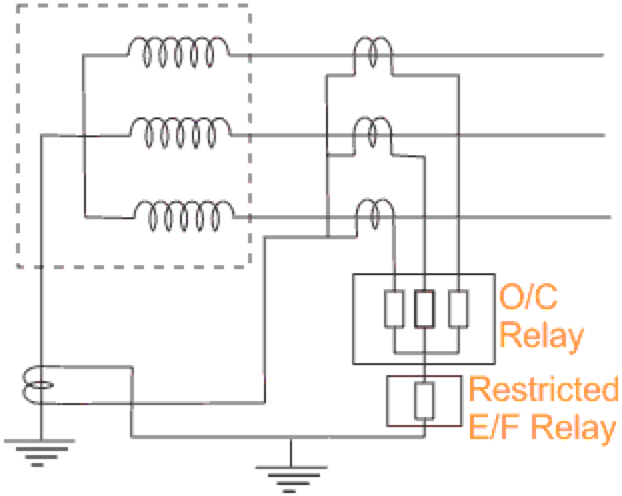
This scheme is very sensitive and can then protect against low levels of fault current in resistance grounded systems where the earthling impedance and the fault voltage limit the fault current.

In addition, this scheme can be used in a solidly grounded system.

Restricted Earth Fault (REF) Protection

- On the HV side, the **residual current** of the 3 line CT's is balanced against the output current of the CT in the **neutral conductor**.
- The **REF relay** will not be actuated for **external earth fault**. But during **internal fault** the neutral current transformer only carries the **unbalance fault** **current** and operation of **REF Relay** takes place.
- Both windings of the transformer can thus be protected separately with restricted **REF Relay** .
- Provide high speed protection against earth faults over the whole of the transformer windings.
- Relay used is an instantaneous type.

Restricted E/F Protection Scheme with O/C



Restricted Earth Fault Principle

To apply the ref protection on a Y connected transformer, you connect three CT's in parallel for all CT's then connect this combination with the CT in the neutral .

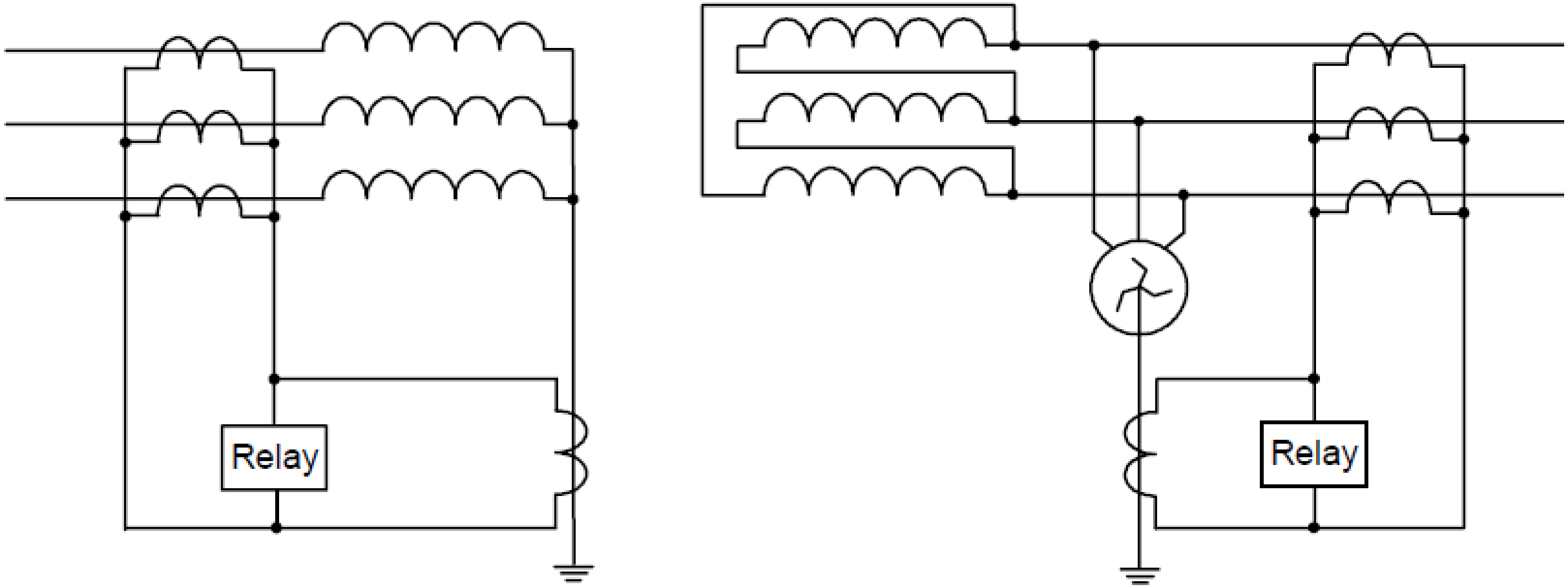
The CT connection provides a path for all zero sequence currents to circulates in the CT's the REF provides a protection for all earth fault in the area between the phase CT's and the neutral CT For the Delta connected transformers the phase CT's connected in parallel as the Y connected transformers,

but as there is no neutral for the Delta connection, an artificial neutral used, taken from the zigzag transformer. So the CT is connected on the artificial neutral which may connected through NGR.

Restricted Earth Fault Principle

To apply REF protection on star-connected transformers, connect the three-phase CTs in star, and connect this combination to a CT in the neutral leg of the transformer,

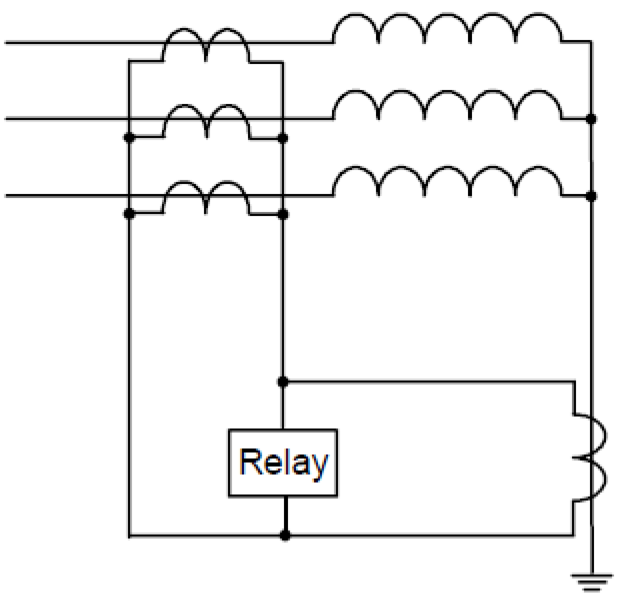
In the case of delta winding to perform the REF protection we connect the phase CT’s in parallel and we connect the CT for using earthing transformer to get the artificial neutral

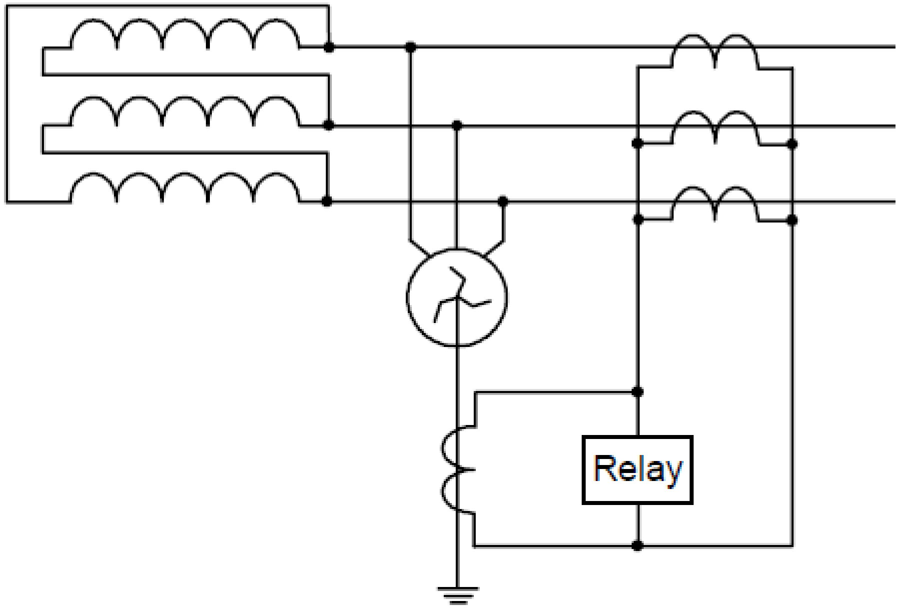


Restricted Earth Fault Principle

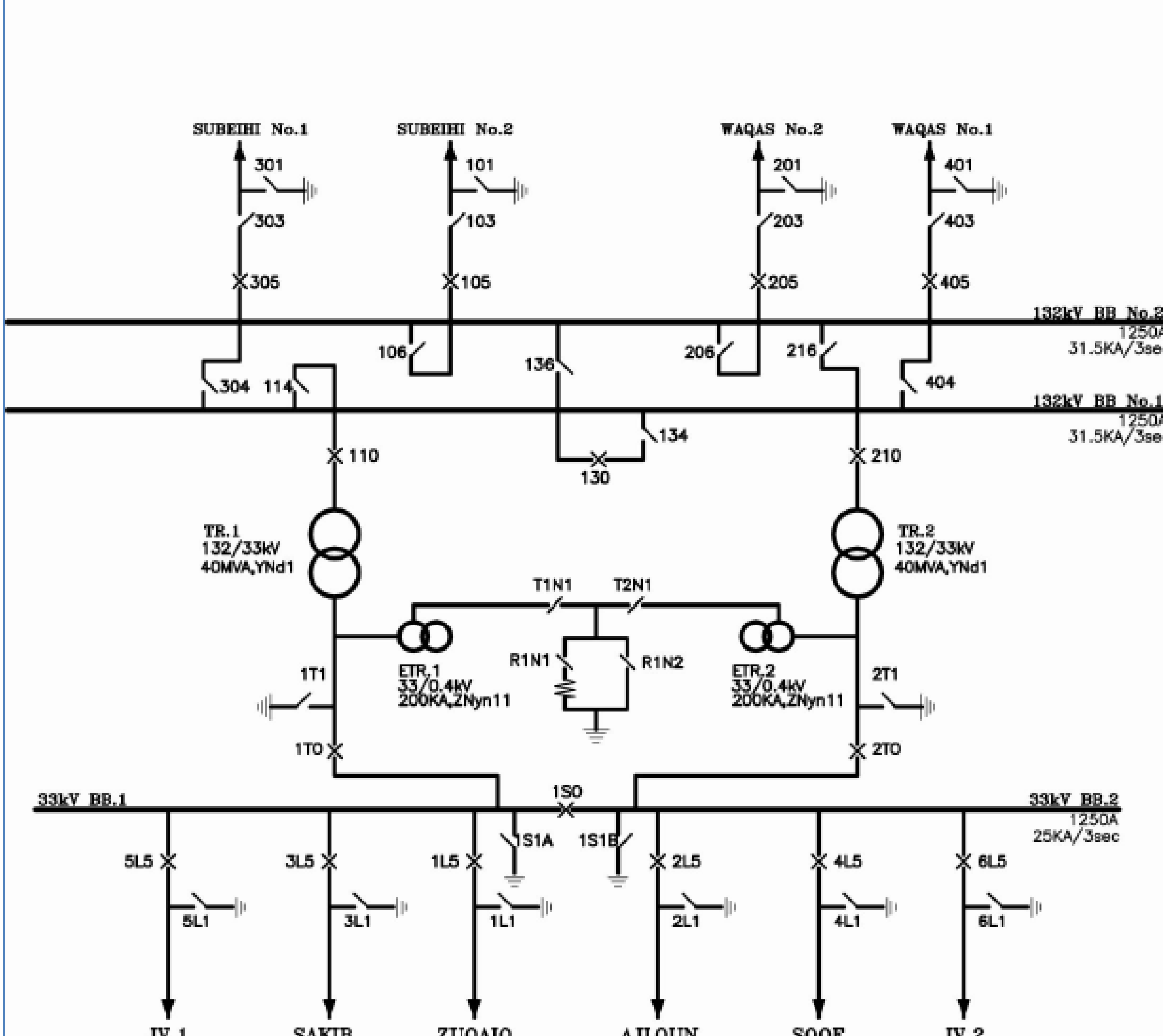
For unearthed delta winding or even star winding REF excludes the neutral CT from the circuit and the three phase CTs are all connected in parallel with the relay element.

This is called a balanced earth-fault connection. The zone of protection is still only the Delta winding of the transformer

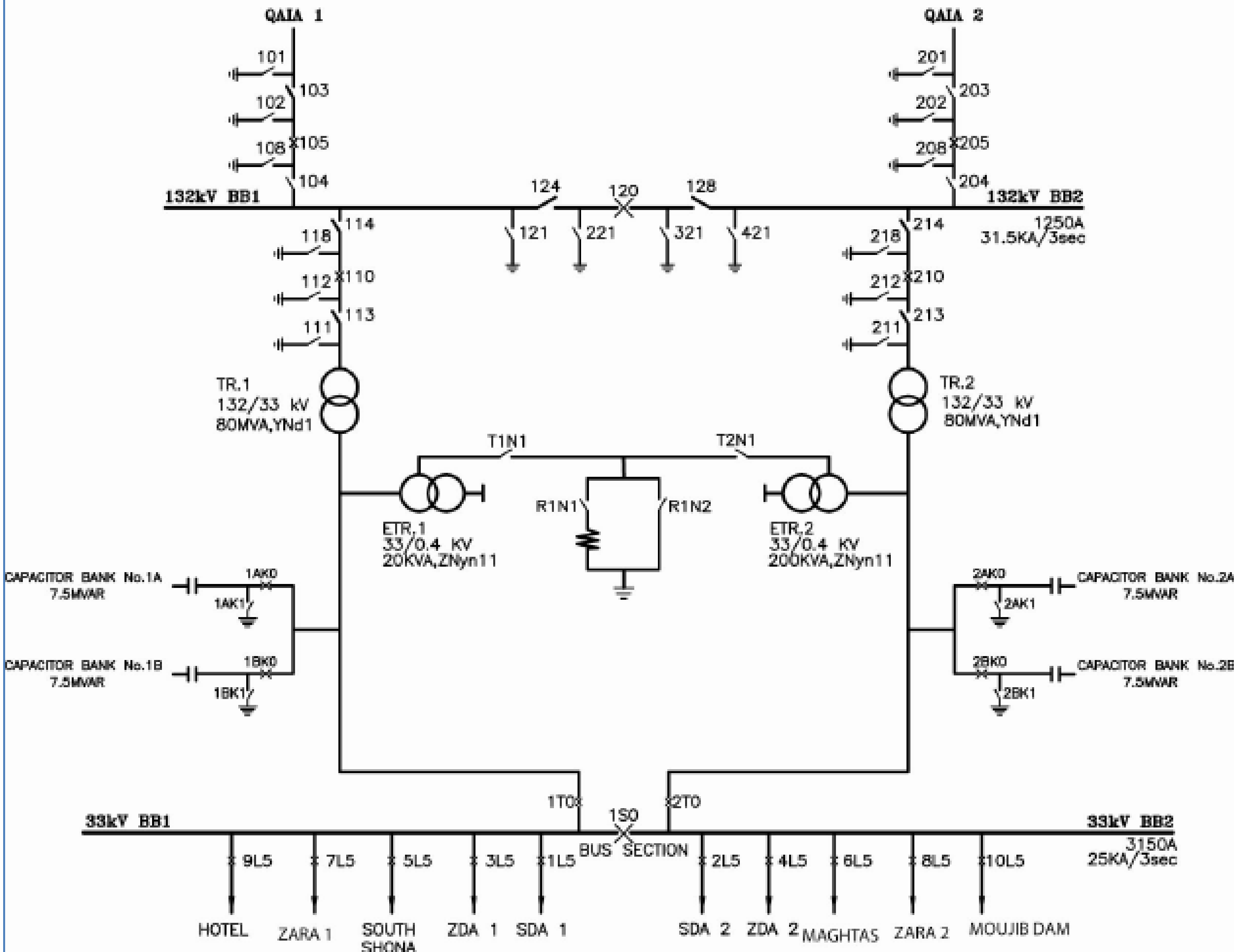




ISHTAFINA 132/33 kV SUBSTATION

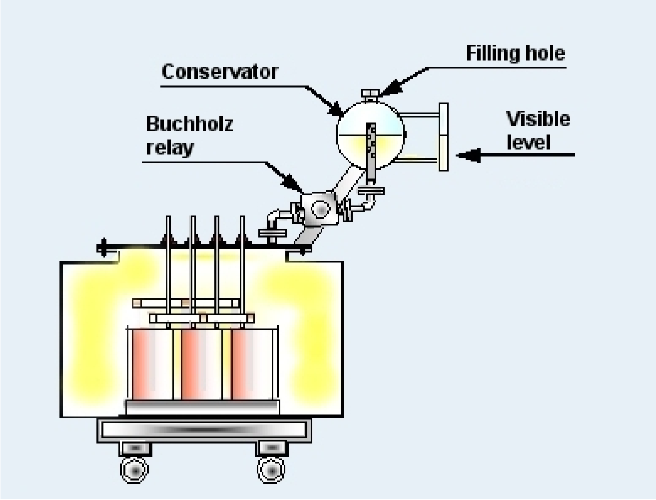


SWEIMEH 132/33 kV SUBSTATION



Buchholz Protection

- Failure of the winding insulation will result in some form of arcing which can decompose the oil into Hydrogen, acetylene and methane.
- Localized heating can precipitate a breakdown in the oil into gas.
- Severe arcing will cause a rapid release of a large volume of gas as well as oil vapor. The action can be so violent that the build-up of pressure can cause an oil surge from the tank to the conservator.
- Buchholz relay can detect both gas and oil surges as it is mounted in the pipe to the conservator.



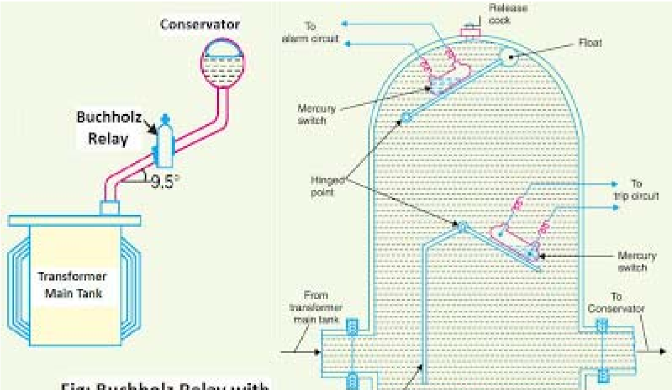
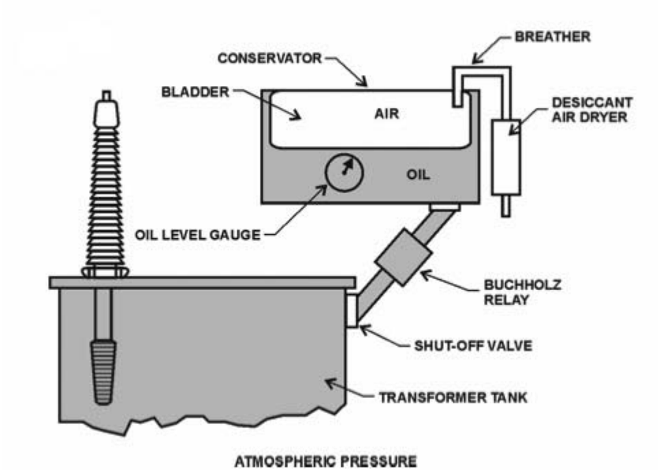


Fig: Buchholz Relay with Constructional Details

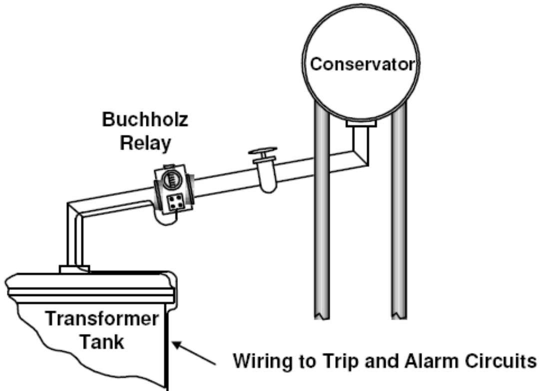
Buchholz Protection

Buchholz relay is a gas operated relay, it protect the transformer from all internal faults. Its name came from the inventor Buchholz.



A **Buchholz relay, is a safety device mounted on some oil-filled power** transformers equipped with an external overhead oil reservoir called a *conservator*.

The Buchholz Relay is used as a protective device sensitive to the effects of dielectric failure inside the equipment.



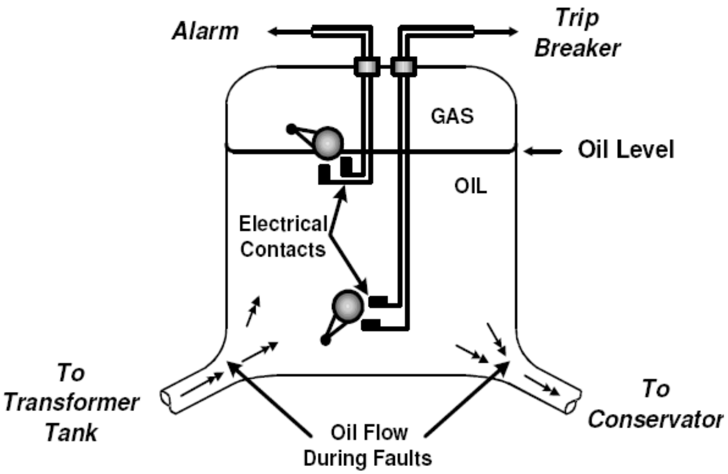
Buchholz Protection

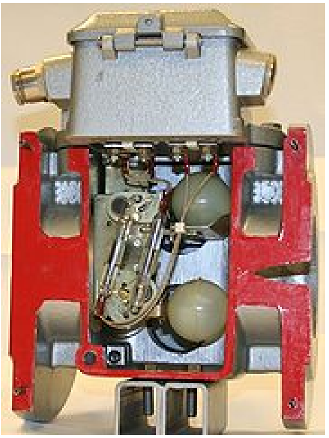
- **Gas discharge: The Buchholz relay detects gas bubbles**
- **Excess tank pressure: The Buchholz relay detects a rapid flow of dielectric fluid from the transformer's tank to the expansion tank**

In Buchholz relays:
A **first mercury contact** detects gas discharge and **initiate an alarm signal**.

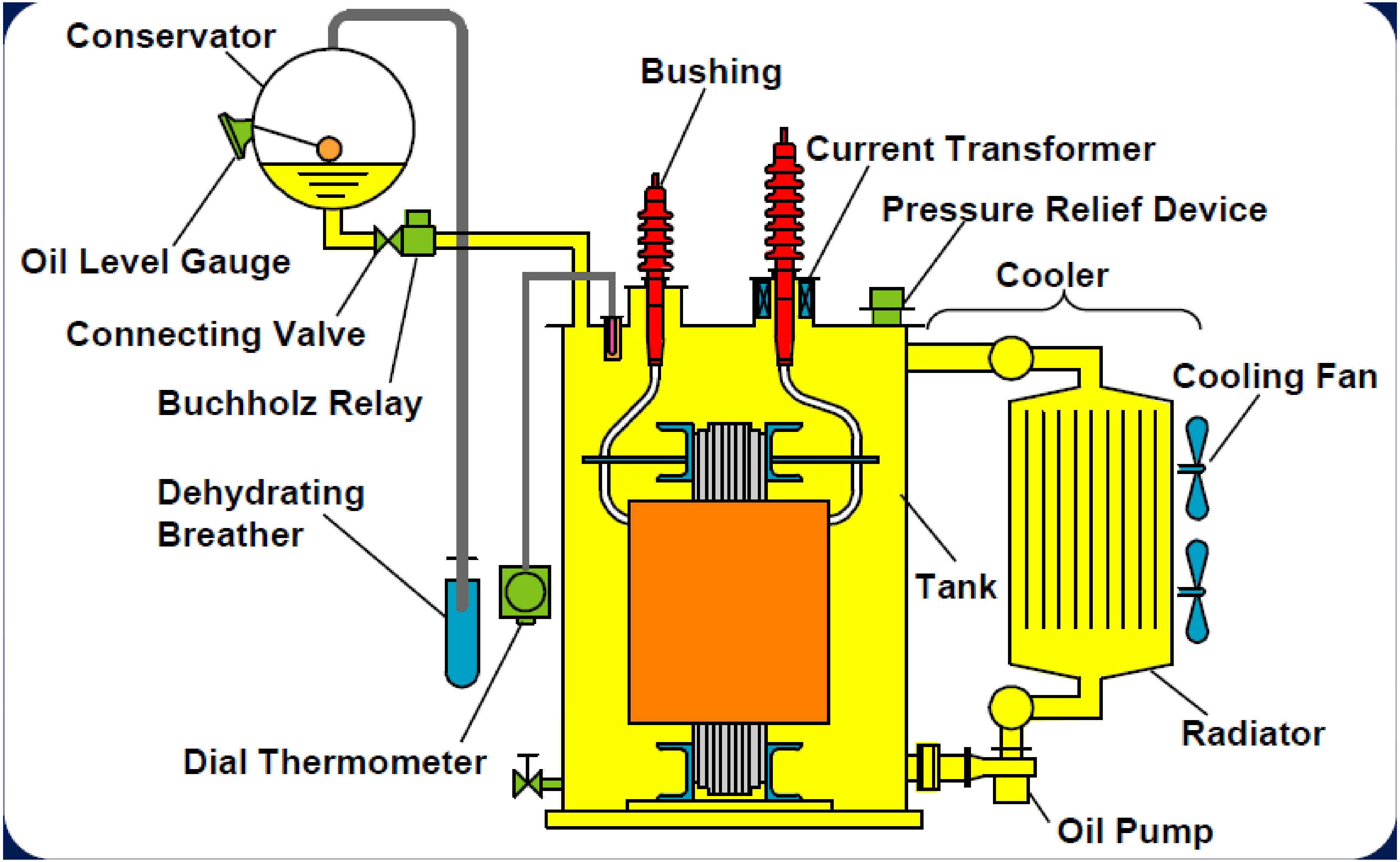
A **second mercury contact** detects rapid flow of dielectric fluid from the transformer to the expansion tank and **initiate a trip signal**.

Transformer manufacturers usually mount Buchholz relays as standard equipment on expansion-tank transformers.

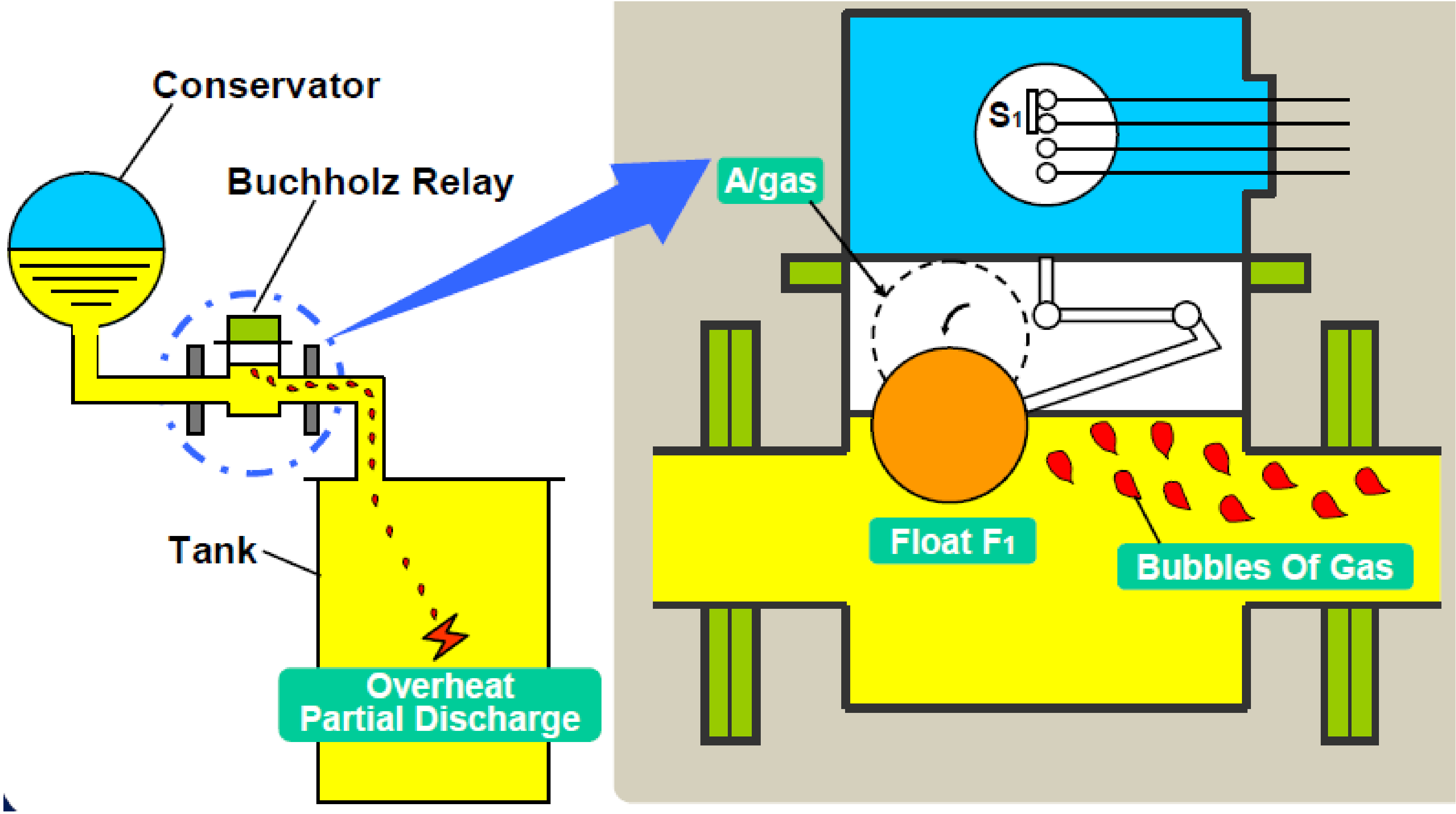




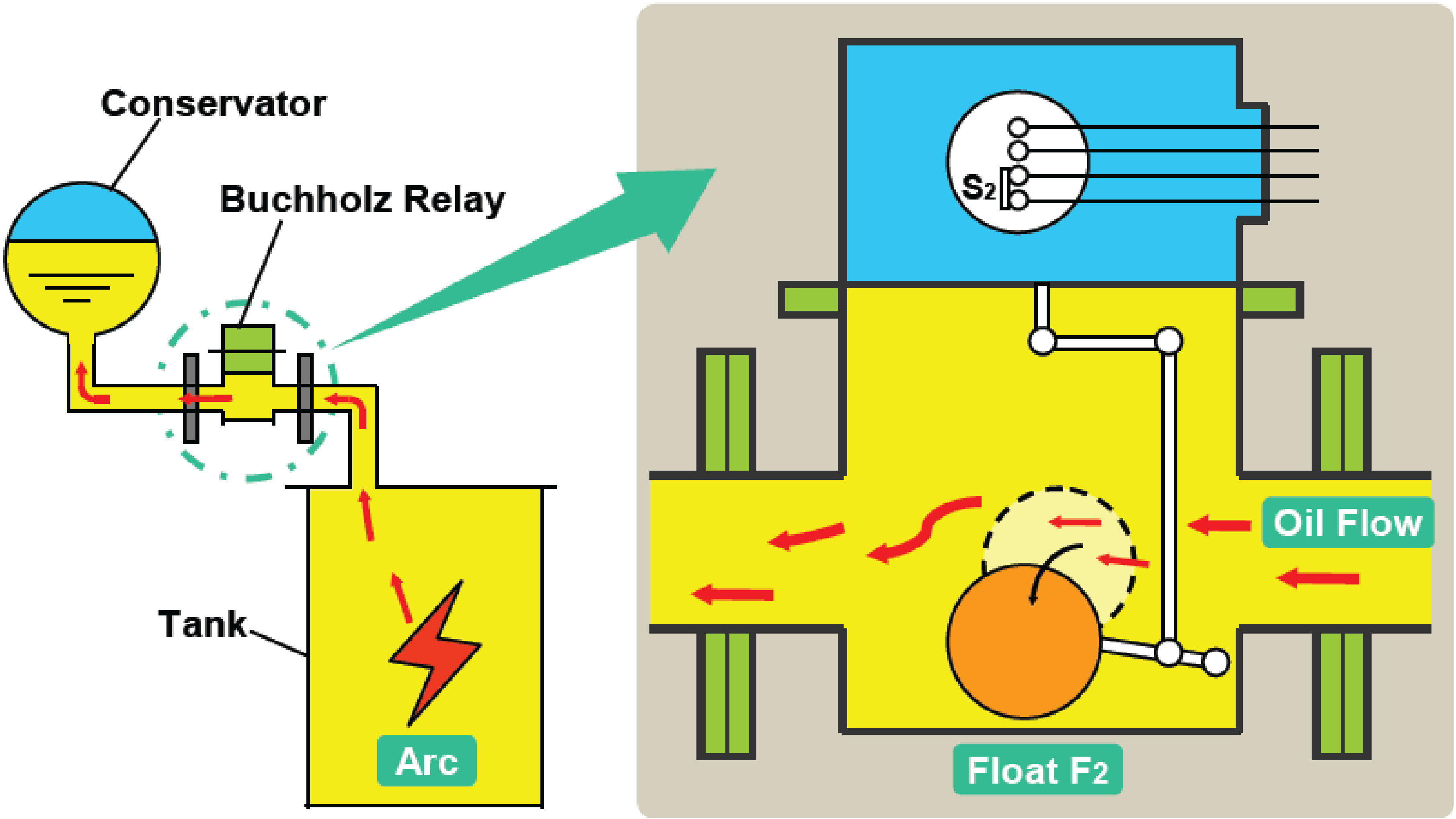
Buchholz Protection



Buchholz Relay (Alarm)



Buchholz Relay (Trip)



Transformer Overloading

- Sustained overloading reduces transformer life
- Operating Temperatures also decide the transformer oil life

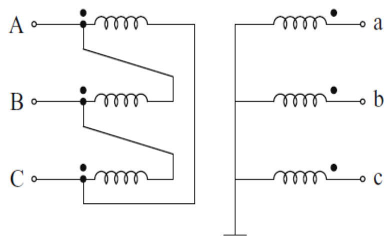
Operating Temperature	Oil Life
60 deg C	20 years
70 deg C	10 years
80 deg C	5 years
90 deg C	2.5 years
100 deg C	13 months
110 deg C	7 months

Problem # 1

A 3-phase 5 MVA 11/3.3 kV Dy1 transformer is subjected to L-L-L, LL and L-G faults at the LV side of the transformer.

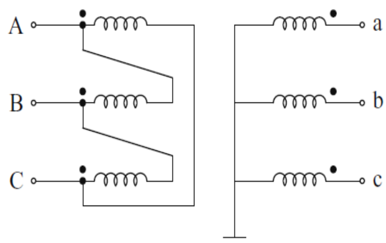
a. If the 3-ph fault phasor currents $\boldsymbol{I_a}$, $\boldsymbol{I_b}$ and $\boldsymbol{I_c}$, are $I_a = 12235\angle -30°\ A$, $I_b = 12235\angle -150°\ A$ and $I_c = 12235\angle 90°\ A$. Find the phasor phase currents $\boldsymbol{I_A}$, $\boldsymbol{I_B}$ and $\boldsymbol{I_C}$, and phasor line currents $\boldsymbol{I_{LA}}$, $\boldsymbol{I_{LB}}$ and $\boldsymbol{I_{LC}}$ at the HV side of the transformer. Show the directions of currents on both sides.

$\boldsymbol{I_A}$	$\boldsymbol{I_B}$	$\boldsymbol{I_C}$
$I_A = 2119\angle -30°\ A$	$I_B = 2119\angle -150°\ A$	$I_C = 2119\angle 90°\ A$
$\boldsymbol{I_{LA}}$	$\boldsymbol{I_{LB}}$	$\boldsymbol{I_{LC}}$
$I_{LA} = 3671\angle 0°\ A$	$I_{LB} = 3671\angle -120°\ A$	$I_{LC} = 3671\angle 120°\ A$



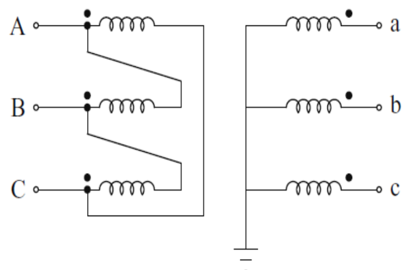
b. If the L-L fault phasor currents $\boldsymbol{I_a}$, $\boldsymbol{I_b}$ and $\boldsymbol{I_c}$, are $I_a = 0\angle 0°\ A$, $I_b = 10596\angle 0°\ A$ and $I_c = 10596\angle 180°\ A$. Find the phasor currents $\boldsymbol{I_A}$, $\boldsymbol{I_B}$ and $\boldsymbol{I_C}$, and line currents $\boldsymbol{I_{LA}}$, $\boldsymbol{I_{LB}}$ and $\boldsymbol{I_{LC}}$ at the HV side of the transformer. Show the directions of currents on both sides.

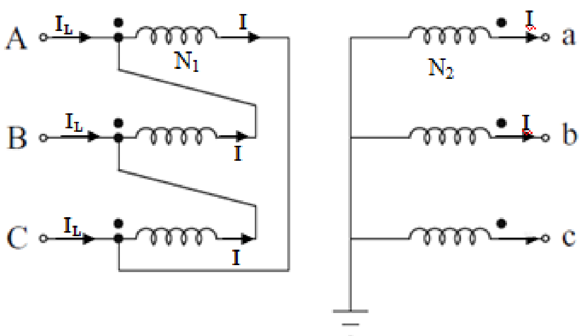
$\boldsymbol{I_A}$	$\boldsymbol{I_B}$	$\boldsymbol{I_C}$
$I_A = 0\angle 0°\ A$	$I_B = 1835\angle 0°\ A$	$I_C = 1835\angle 180°\ A$
$\boldsymbol{I_{LA}}$	$\boldsymbol{I_{LB}}$	$\boldsymbol{I_{LC}}$
$I_{LA} = 1835\angle 180°\ A$	$I_{LB} = 3670\angle 0°\ A$	$I_{LC} = 1835\angle 180°\ A$



c. If the L-G fault phasor currents $\boldsymbol{I_a}$, $\boldsymbol{I_b}$ and $\boldsymbol{I_c}$, are $I_a = 12235\angle -30°\ A$, $I_b = 0\angle 0°\ A$ and $I_c = 0\angle 0°\ A$. Find the phasor currents $\boldsymbol{I_A}$, $\boldsymbol{I_B}$ and $\boldsymbol{I_C}$, and line currents $\boldsymbol{I_{LA}}$, $\boldsymbol{I_{LB}}$ and $\boldsymbol{I_{LC}}$ at the HV side of the transformer. Show the directions of currents on both sides.

$\boldsymbol{I_A}$	$\boldsymbol{I_B}$	$\boldsymbol{I_C}$
$I_A = 2119\angle -30°\ A$	$I_B = 0\angle 0°\ A$	$I_C = 0\angle 0°\ A$
$\boldsymbol{I_{LA}}$	$\boldsymbol{I_{LB}}$	$\boldsymbol{I_{LC}}$
$I_{LA} = 2119\angle -30°\ A$	$I_{LB} = 0\angle 0°\ A$	$I_{LC} = 2119\angle 150°\ A$





$$\frac{N_1}{N_2} = \frac{V_{ph1}}{N_{ph2}} = \frac{V_{L1}}{V_{L2}/\sqrt{3}} = \sqrt{3}\frac{V_{L1}}{V_{L2}}$$

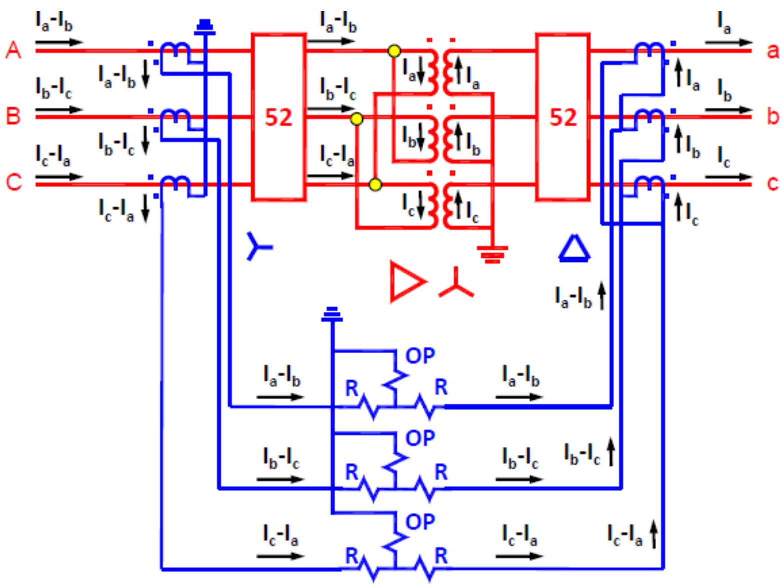
$$\frac{N_1}{N_2} = \sqrt{3}VR = \sqrt{3}\frac{11}{3.3} = 5.77$$

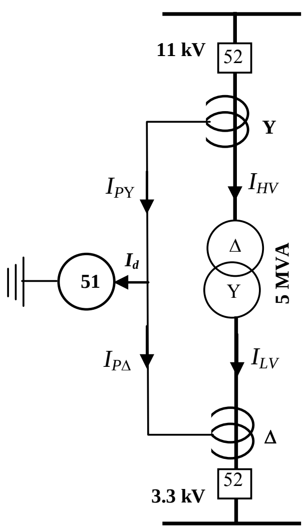
Problem # 2

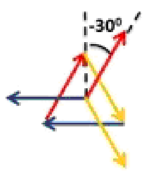
For the 3-ph 5 MVA, 11/3.3 kV ΔY transformer, shown below, **design a differential protection** scheme using available standard CT ratios with Y-connected CT on the HV side and Δ-connected CT on the LV side.

a. Specify the vector group of the ΔY transformer.
b. Show the appropriate **CT vector group** and the connections of the CTs and the OC relay on both sides of the power transfer.
c. Considering a **10% allowable overload**, find the overload line current on the HV and LV sides of the transformer (I_{HVOL}, I_{LVOL}).
d. Find the appropriate CTR, with **1 A secondary** current, of the CTs on both the HV and LV sides of the transformer CTR_{HV} and CTR_{LV}.
e. Calculate the pilot wire currents I_{PY} and $I_{P\Delta}$ on both left- and right-hand sides of the OC relay.
f. If the OC relay has plug settings of 50% -200% of the relay rated current with 25% increment, find the appropriate %PS of the OC relay.

		11 kV Δ-Side	**3.3 kV Y-Side**
a.	**ΔY** transformer Vector Group	Dy1	
b.	**CT** Vector Group	Yd11	
c.	Line current allowing 10% OL	$I_{HV\,OL} = 289\,A$	$I_{LV\,OL} = 962\,A$
d.	**CT** ratio	$CTR_{HV} = \frac{300}{1} = 300$	$CTR_{LV} = \frac{1000}{1} = 1000$
e.	Pilot wire	$I_{PY} = 0.96$ A	$I_{P\Delta} = 1.67$ A $(0.96\times\sqrt{3})$
f.	Relay Differential current	$I_d = 0.7$ A	
g.	OC Relay Plug Setting	%PS = 75%	





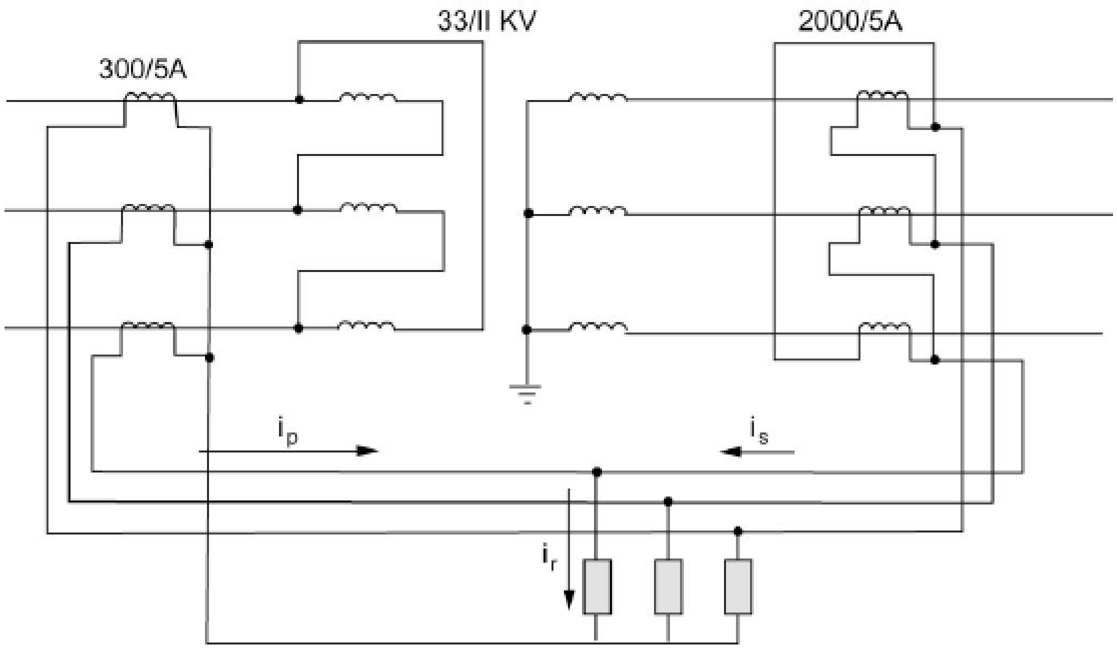


Dy1

Problem # 3

Consider a Δ/Y-connected, 20-MVA, 33/11-kV transformer with differential protection applied, for the current transformer ratios shown below. Calculate:

a. the relay currents on full load.
b. the minimum relay current setting to allow 125 percent overload.



Solution:

The primary line current is given by

$$I_p = \frac{20\times10^6}{\left(\sqrt{3}\right)\left(33\times10^3\right)} = 349.91\,\text{A}$$

$$i_p = 349.91\left(\frac{5}{300}\right) = 5.832\,\text{A}$$

The secondary line current is

$$I_s = \frac{20\times10^6}{\left(\sqrt{3}\right)\left(11\times10^3\right)} = 1049.73\,\text{A}$$

The C.T. current in the secondary side is

$$i_s = 1049.73\left(\frac{5}{2000}\right)\sqrt{3} = 4.545\,\text{A}$$

Note that we multiply by √3 to obtain the values on the line side of the Δ-connected C.T.'s. The relay current on normal load is therefore

$$i_r = i_p - i_s = 5.832 - 4.545 = 1.287\,\text{A}$$

With 1.25 overload ratio, the relay setting should be

$$I_r = (1.25)(1.287) = 1.61\,\text{A}$$

Problem # 4

For the Δy11 transformer shown below, there is a phase angle difference between primary and secondary equal to -30°. So, an auxiliary current transformer (matching) is installed in the secondary circuit of 11 kV current transformer side to compensate the magnitude and phase. Determine:

a. the primary (I_{L66P}) and secondary (I_{L11S}) currents of the ΔY-connected transformer when the transformer is delivering its rated MVA.
b. the currents seen by the CTs on the Δ-connected primary ($I_{CTA\Delta\text{-}S}$, $I_{CTB\Delta\text{-}S}$, and $I_{CTC\Delta\text{-}S}$) side and the currents seen by the Y-connected secondary ($I_{CTAY\text{-}S}$, $I_{CTBY\text{-}S}$, and $I_{CTCY\text{-}S}$) side of the transformer.
c. the line current of the primary Y-side of the matching transformer ($I_{P\text{-}match\text{-}L}$) and of the line current of the secondary Δ-side of the matching transformer ($I_{S\text{-}match\text{-}L}$).
d. the turns ratio of the matching transformer $\dfrac{N_{P-match}}{N_{S-match}}$.
e. the currents seen by each relay (I_{relayA}, I_{relayB}, and I_{relayC}) under normal conditions.

Solution:

$$I_{L66P} = (MVA \times 1000)/\left(\sqrt{3} \times 66\,kV\right) = (25 \times 1000)/\left(\sqrt{3} \times 66\right) = 218.7\ A$$

$$I_{CTA\Delta-S} = \frac{I_{L66P}}{CTR_P} = \frac{218.7}{400/5} = \frac{(25 \times 1000)/\left(\sqrt{3} \times 66\right)}{80} = \frac{\quad}{80} = 2.73\ A$$

$I_{CTA\Delta\text{-}S} = I_{CTB\Delta\text{-}S} = I_{CTC\Delta\text{-}S} = 2.73$ A

$$I_{L11S} = (MVA \times 1000)/\left(\sqrt{3} \times 11\,kV\right) = (25 \times 1000)/\left(\sqrt{3} \times 11\right) 1312.2\ A$$

$$I_{CTAY-S} = \frac{I_{L11S}}{CTR_S} = \frac{1312.2}{1500/5} = 4.37\ A$$

$I_{CTAY\text{-}S} = I_{CTBY\text{-}S} = I_{CTCY\text{-}S} = 4.37$ A – (Input to the matching transformer)

For equilibrium of differential relay:-
Current of 11 kV of differential relay must be equal to current of 66 kV side of differential relay.
$\Rightarrow I_{CT\Delta\text{-}S} = I_{CTY\text{-}S} = 2.73$ A.
But, input current of matching transformer is $I_{match\text{-}P} = 4.37$ A. Therefore, the output current of the matching transformer (input to the differential relay) must be equal $I_{match\text{-}S} = 2.73$ A.

Note: the connection of the matching transformer must be YΔ1 to compensate the original angle of the power transformer.

The turns ratio of the matching transformer $N_{Pmatch}/N_{Smatch} = I_{Smatch\text{-}ph}/I_{Pmatch\text{-}ph}$.

$$\frac{N_{P-match}}{N_{S-match}} = \frac{I_{S-match-ph}}{I_{P-match-ph}} = \frac{I_{Smatch-L}/\sqrt{3}}{I_{Pmatch-L}} = \frac{2.73/\sqrt{3}}{4.37} = \frac{1.58}{4.37} \cong 0.36$$

$$\Rightarrow I_{Smatch-L} = 1.58\ A.$$

$$I_{relayA} = I_{CTA\Delta-S} - I_{S-mactch-L} = 2.73 - 2.73 = 0\ A$$

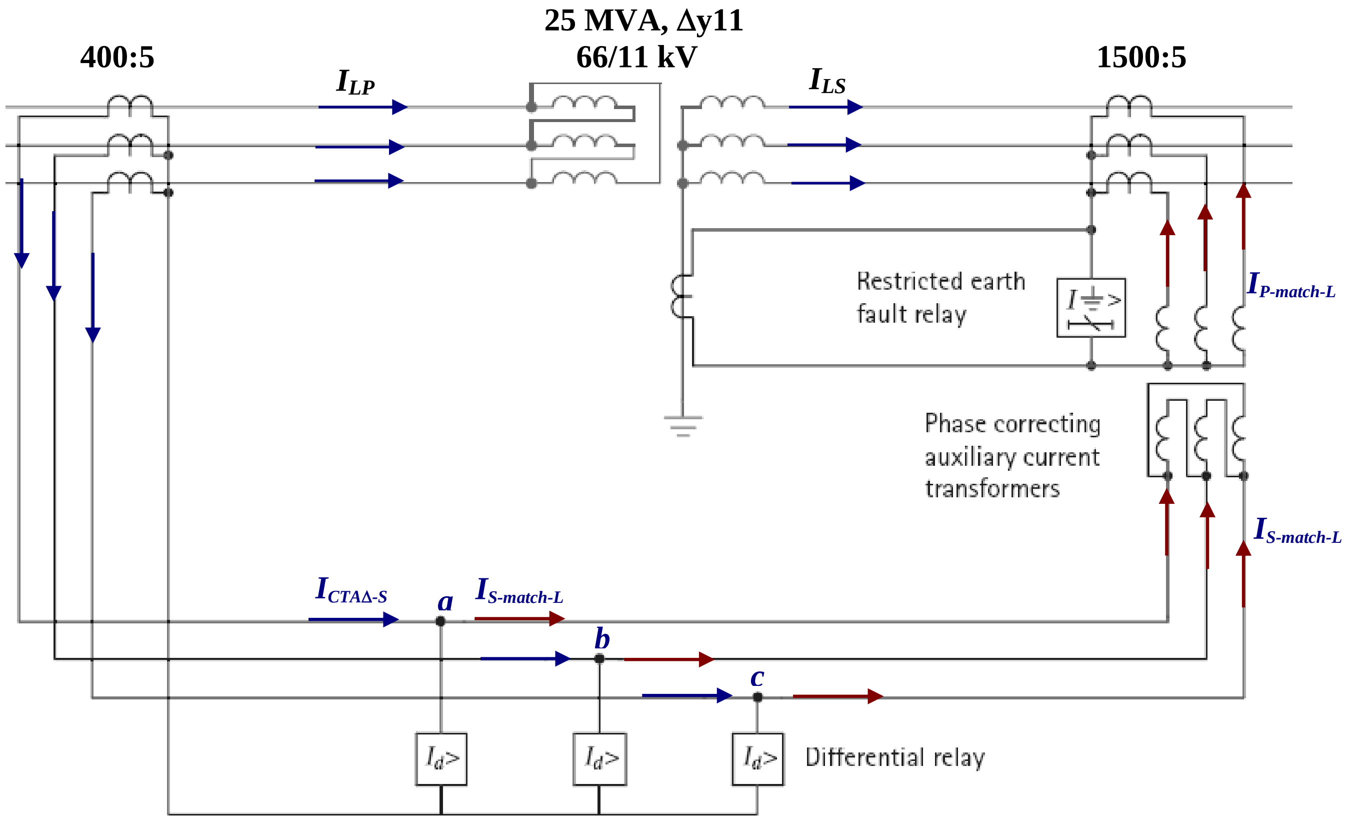
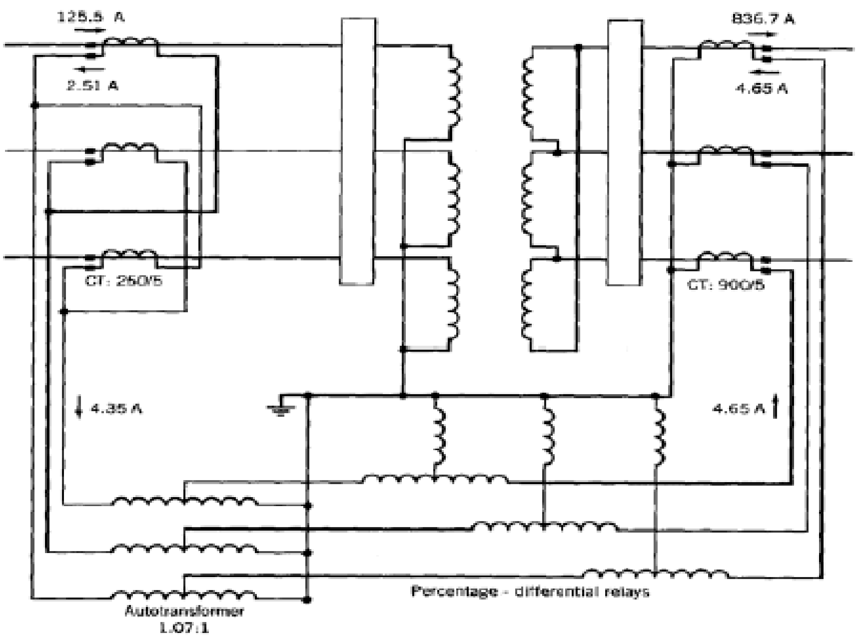


Fig. P4: Transformer Differential Protection

Problem # 5

Design the protection of a three-phase, 50-MVA, 230/34.5 kV power transformer using available standard CT ratios. The high-voltage side is Y-connected and the low-voltage side is Δ-connected. Specify the CT ratios, and show the three phase wiring diagram indicating the CT polarities. Determine the currents in the transformer and the CTs. Specify the rating of an autotransformer, if one is needed.



Y- Δ transformer protection

Solution:

When the transformer is carrying rated load, the line currents on the high-voltage side and low-voltage side are

$$I_{\mathrm{HV}} = \frac{50{,}000}{\sqrt{3}(230)} = 125.5\ \mathrm{A}$$

$$I_{\mathrm{LV}} = \frac{50{,}000}{\sqrt{3}(34.5)} = 836.7\ \mathrm{A}$$

The CTs on the low-voltage side are Y-connected, and the CT ratio selected for this side is 900/5. The current in the leads flowing to the percentage-differential relay on this side is equal to the CT secondary current and is given by

$$I_{\mathrm{LV\ lead}} = 836.7\left(\frac{5}{900}\right) = 4.65\ \mathrm{A}$$

The current in the leads to the relay from the low-voltage side must be balanced by an equal current in the leads connected to the Δ-connected CTs on the high-voltage side. This requires a CT secondary current equal to

$$I_{\mathrm{CT\ sec}} = \frac{4.65}{\sqrt{3}} = 2.68\ \mathrm{A}$$

To obtain a CT secondary current of 2.68 A, the CT ratio of the high-voltage CTs is chosen as

$$\mathrm{CT\ ratio} = \frac{125.5}{2.68} = 46.8$$

The nearest available standard CT ratio is 250/5. If this CT ratio is selected, the CT secondary currents will actually be

$$I_{\mathrm{CT\ sec}} = 125.5\left(\frac{5}{250}\right) = 2.51\ \mathrm{A}$$

Therefore, the currents in the leads to the Δ-connected CTs from the percentage-differential relays will be

$$I_{\mathrm{HV\ lead}} = \sqrt{3}(2.51) = 4.35\ \mathrm{A}$$

It is seen that the currents in the leads on both sides of the percentage-differential relay are not balanced. This condition cannot just be ignored because it could lead to improper tripping of the circuit breaker for an external fault. This problem can be solved by using an autotransformer as shown in Fig. P3. The autotransformer should have a turns ratio of

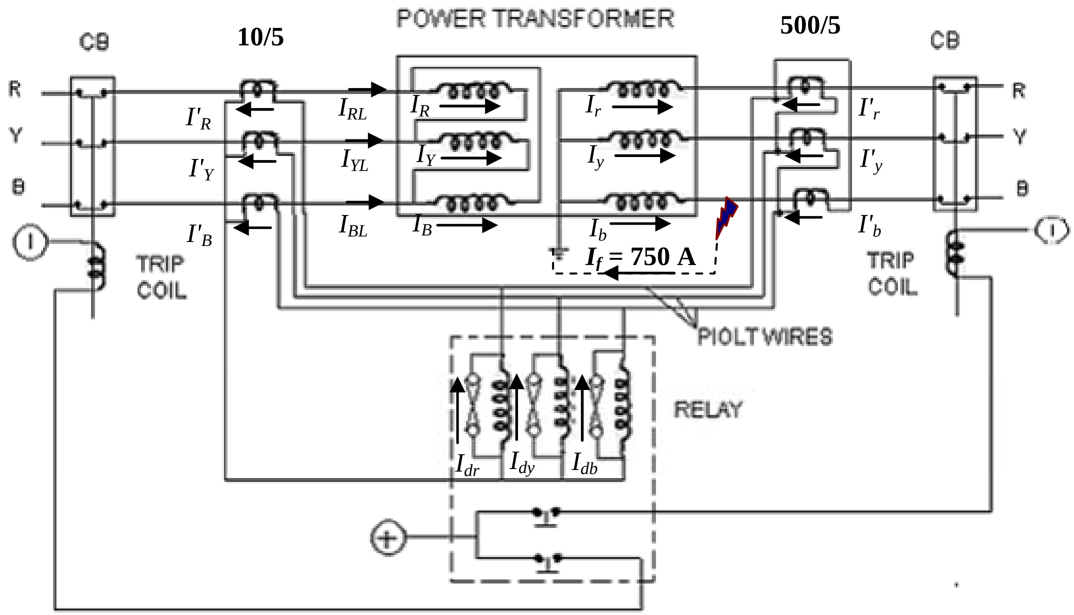
$$N_{\mathrm{autotransformer}} = \frac{4.65}{4.35} = 1.07$$

In the design of the transformer protection of Problem 3, the magnetizing current of the transformer has been assumed to be negligible. This is a reasonable assumption during normal operating conditions because the magnetizing current is a small percentage of the rated load current. However, when a transformer is being energized, it may draw a large magnetizing inrush current that soon decays with time to its normal value. The inrush current flows only in the primary, causing an unbalance in current, and the differential relay will interpret this an internal fault and will pick up to trip the circuit breakers. To prevent the protection system from operating and tripping the transformer during its energization, percentage-differential relaying with harmonic restraint is recommended. This is based on the fact that the magnetizing inrush current has high harmonic content, whereas the fault current consists mainly of fundamental frequency sinusoid. Thus, the current supplied to the restraining coil consists of the fundamental and harmonic components of the normal restraining current of $(I_A + I_B)/2$, plus another signal proportional to the harmonic content of the differential current $(I_A - I_B)/2$. Only the fundamental frequency of the differential current is supplied to the operating coil of the relay.

Problem # 6

A 3-phase 200 kVA, 11/0.4 kV 3-phase transformer is connected as ΔY as shown below. The CT on the 0.4 kV side has a CTR of 500/5 and the CT on the 11 kV side has a CTR of 10/5.

An earth fault of I_f =750 A fault current occurred on the **blue phase** within the protection zone. If the load current is negligible, find the following:



1.	the LV red phase winding current, I_r	0	A
2.	the LV yellow phase winding current, I_y	0	A
3.	the LV blue phase winding current, I_b	750	A
4.	the HV red phase winding current, I_R	0	A
5.	the HV yellow phase winding current, I_Y	0	A
6.	the HV blue phase winding current, I_B	15.75	A
7.	the HV red phase line current, I_{RL}	-15.75 A	
8.	the HV yellow phase line current, I_{YL}	0	A
9.	the HV blue phase line current, I_{BL}	15.75 A	
10.	the HV red phase CT current, I'_R	-7.87 A	
11.	the HV yellow phase CT current, I'_Y	0	A
12.	the HV blue phase CT current, I'_B	7.87 A	
13.	the LV red phase CT current, I'_r	0	A
14.	the LV yellow phase CT current, I'_y	0	A
15.	the LV blue phase CT current, I'_b	0	A
16.	the red phase differential current, I'_{dr}	(7.87)	A
17.	the yellow phase differential current, I'_{dy}	0	A
18.	the blue phase differential current, I'_{db}	(7.87)	A

Problem # 7

A 115/13.2 kV Dy1 transformer rated at 25MVA has differential protection as indicated below. The transformer is connected to a radial system, with the source on the 115 kV side. The minimum operating current of the relays is 1 A. The transformer 13.2 kV winding is earthed via a resistor which is set so that the current for a single-phase fault on its secondary terminals is equal to the nominal load current. Draw the complete three-phase diagram and indicate on it the current values in all the elements for:

(i) Find the value of the grounding resistance R.

(ii) When a fault occurs at the middle of the winding on phase C, on the 13.2 kV side, assuming that the transformer is not loaded. For both cases indicate if there is any relay operation.

Solution:

Full load conditions

The full load conditions for the maximum load of the transformer are as follows:

$$I_{FL(13.2kV)} = \frac{25\times10^6}{\sqrt{3}\times13.\times10^3} = 1093.5\,A, \qquad R = \frac{13.2\times10^3/\sqrt{3}}{1093.46} = 6.97\,\Omega$$

Fault at the middle of 13.2 kV winding C

Since the transformer is earthed through a resistor that limits the current for faults at the transformer 13.2 kV bushings to the rating of the winding, and since the fault is at the middle of the winding, the fault current is then equal to half the rated value as follows:

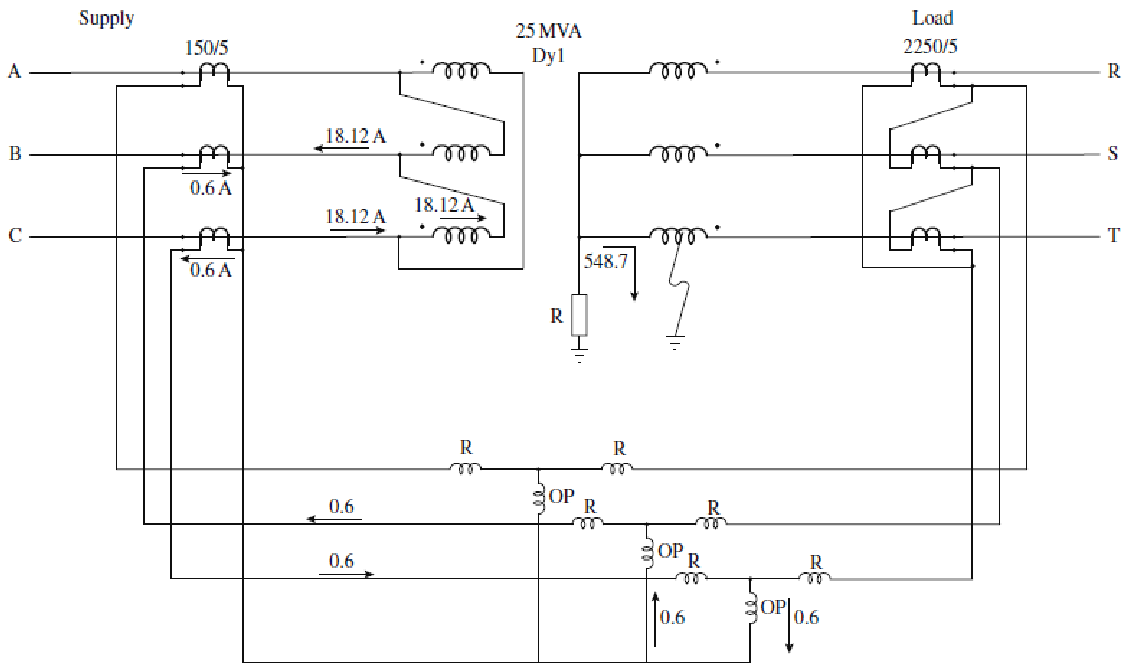
$$I_{fault} = (I_{nom(13.2\text{kV})})/2 = 1093.47/2 = 546.7\text{ A}$$

The primary current within the delta winding is

$$I_{prim} = I_{fault} \times \frac{(N_2/2)}{N_1}, \quad \frac{N_2}{N_1} = \frac{V_2/\sqrt{3}}{V_1}$$

$$I_{prim} = \frac{1}{\sqrt{3}\times VR} = 546.5\times\frac{\left(13.2/\sqrt{3}\right)/2}{115} = 18.1\,A$$

The differential relays **do not operate** since the current through their operating coils is only 0.6 A, which is less than the 1A required for relay operation.



Conditions for a fault at the middle of the winding on phase C on the 13.2 kV side

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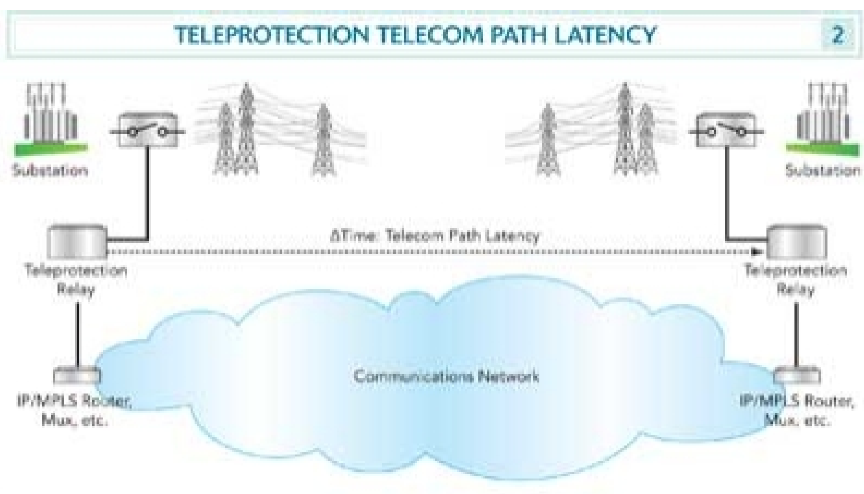
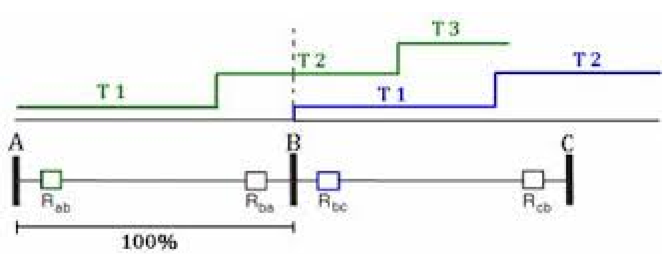
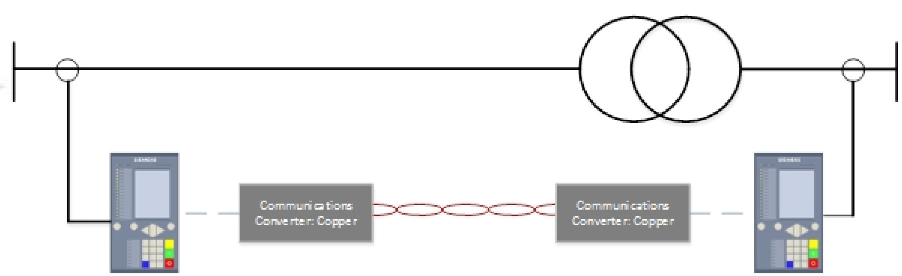
Topic 1-6: Distance Protection



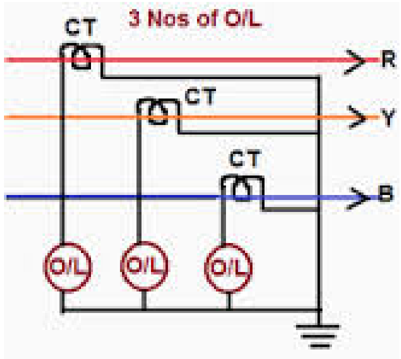


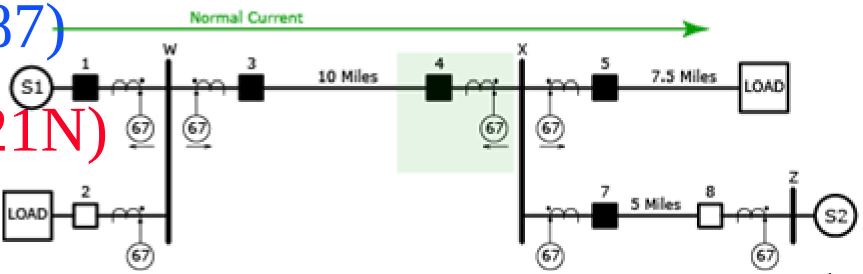


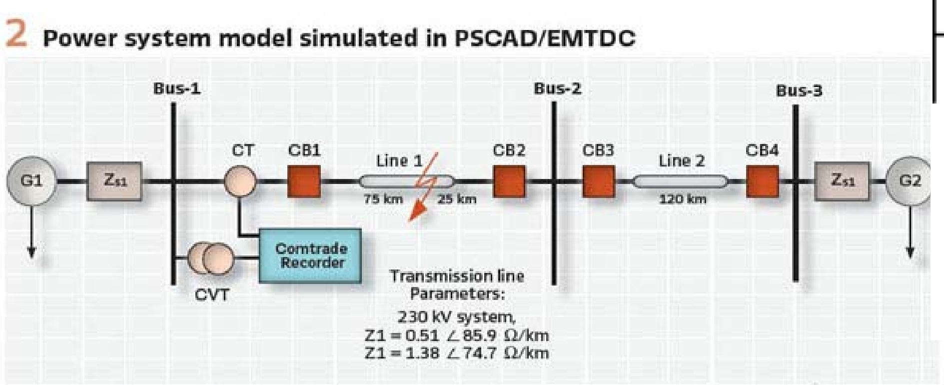
Transmission Line Protection

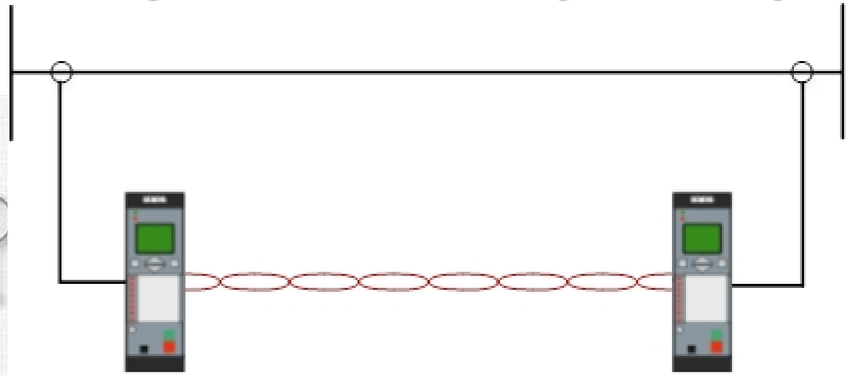
How Do We Protect Transmission Lines?

- ➔ **Overcurrent** (50, 51, 50N, 51N)
- ➔ **Directional Overcurrent** (67, 67N)
- ➔ **Pilot Wire Protection**
- ➔ **Line Current Differential** (87)
- ➔ **Distance (Impedance)** (21, 21N)









Transmission Line Protection

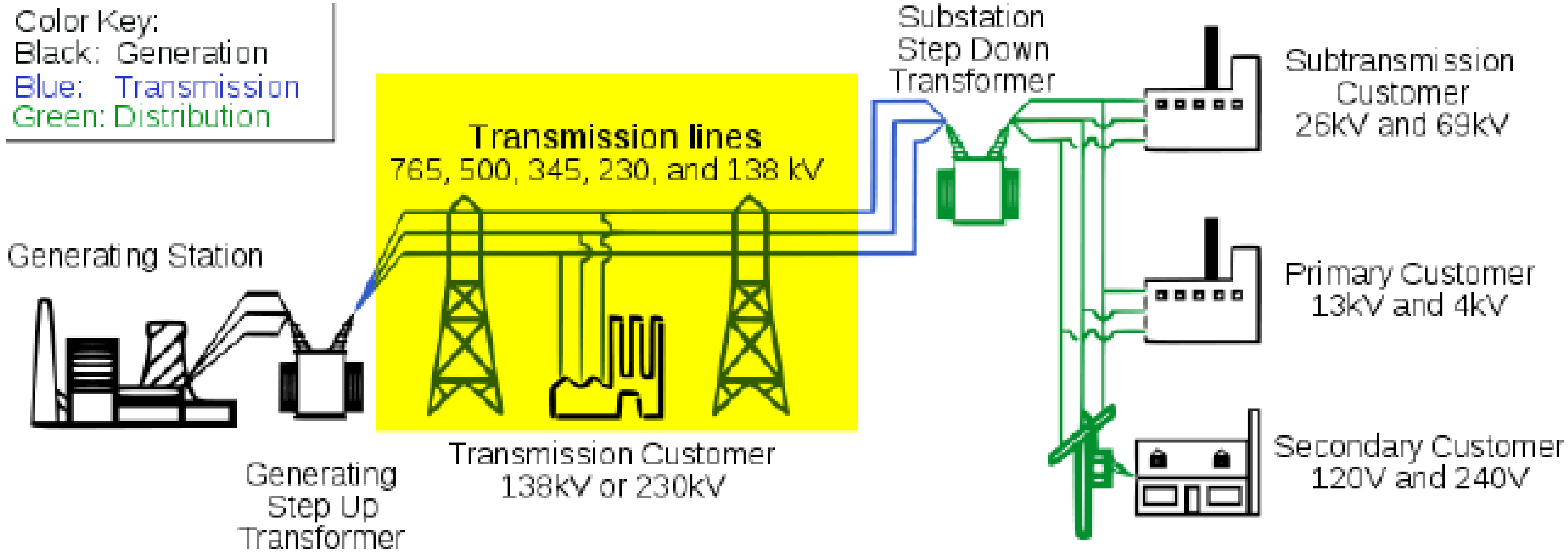
➔ Overcurrent Protection

- Non-Directional
- Relay responds to overcurrent condition

➔ Instantaneous (IOC) device #50

- No intentional time delay

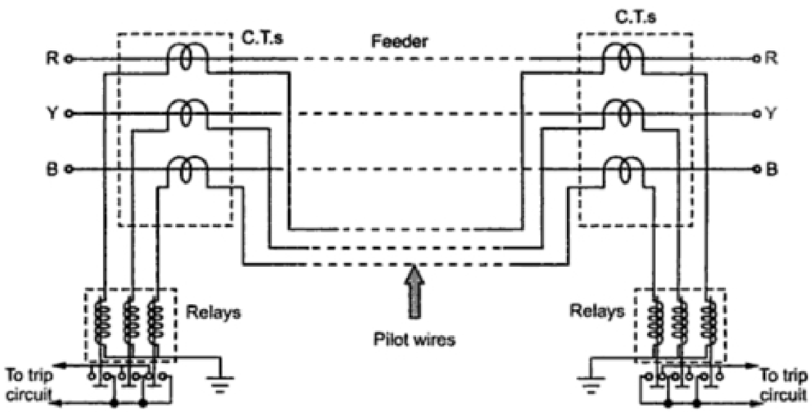
➔ Time Overcurrent (TOC) device #51



Pilot wire differential Protection

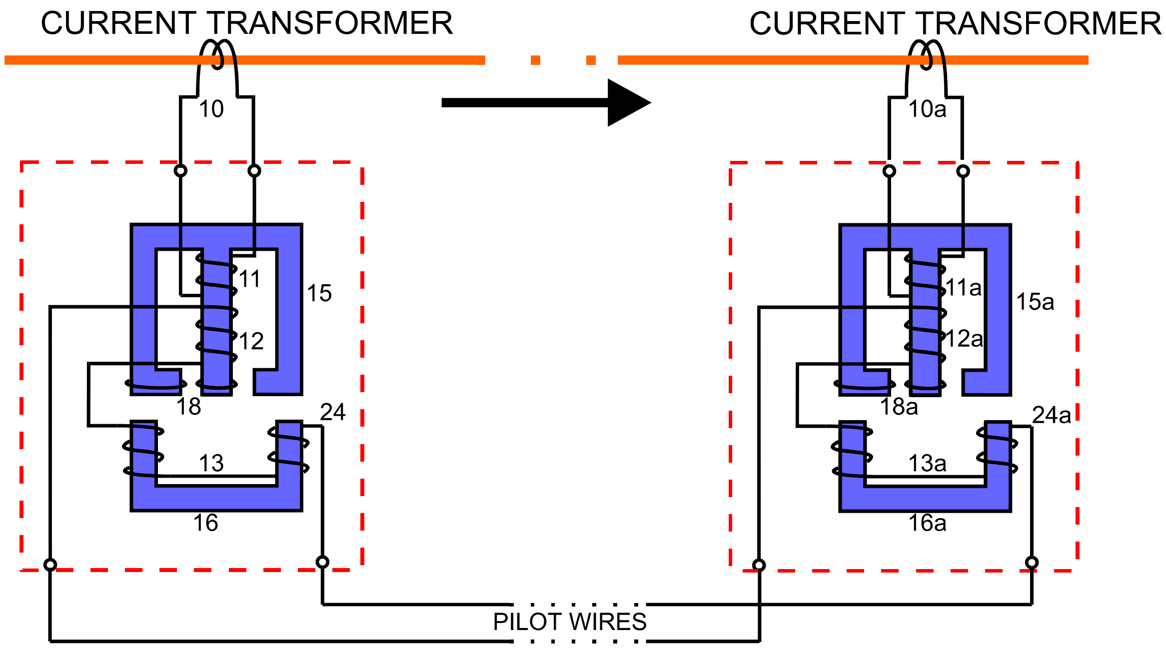
- **Protection of Cables and Short Lines**
- **Pilot protection schemes** use communication channels to send information from the local relay terminal to the remote relay terminal, thereby allowing high-speed tripping for faults occurring within 100% of the protected line.

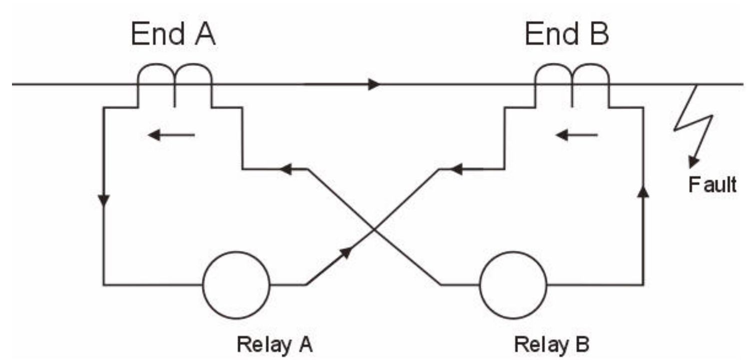
- **Pilot wire protection**: Pilot protection in which a metallic circuit is used for the communications channel between relays at the circuit terminals.



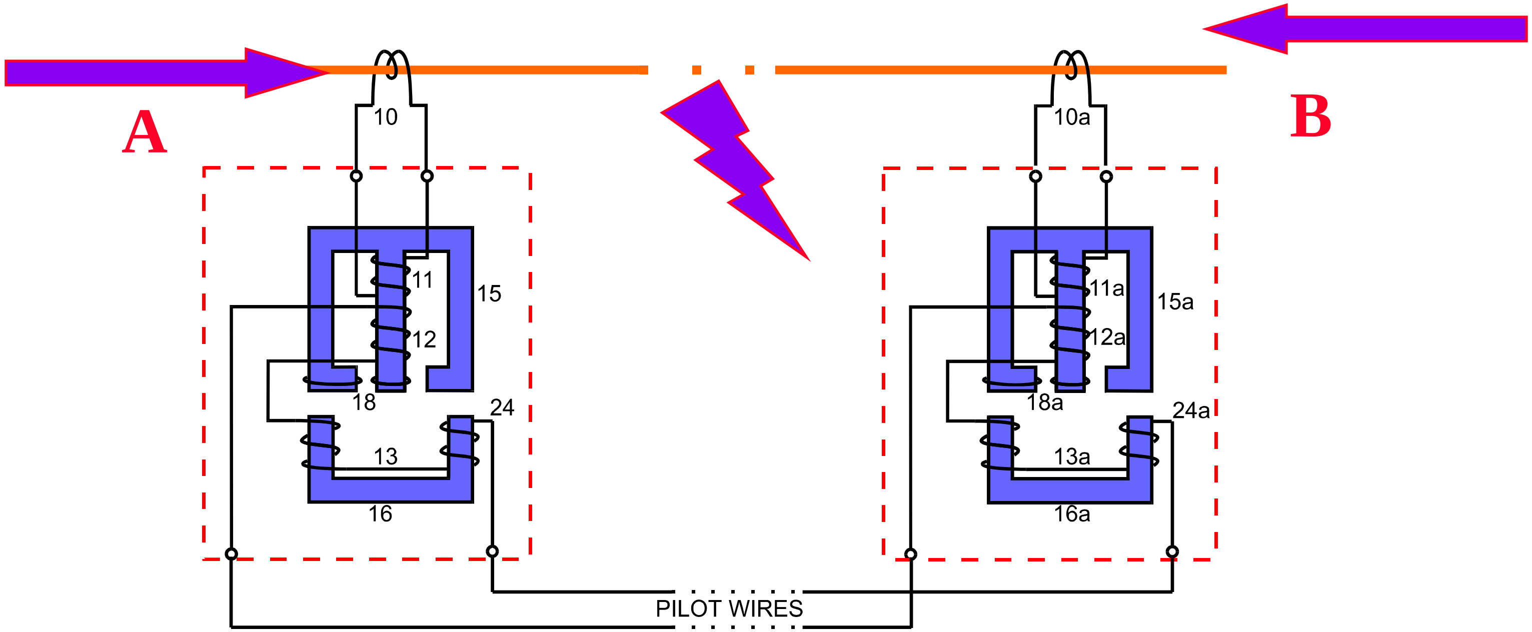
Applications of Unit Protection

➔ **Balanced Voltage** Systems are used on feeder systems where CT's are away from one another with independent relays at both ends





Fault fed from Both ends

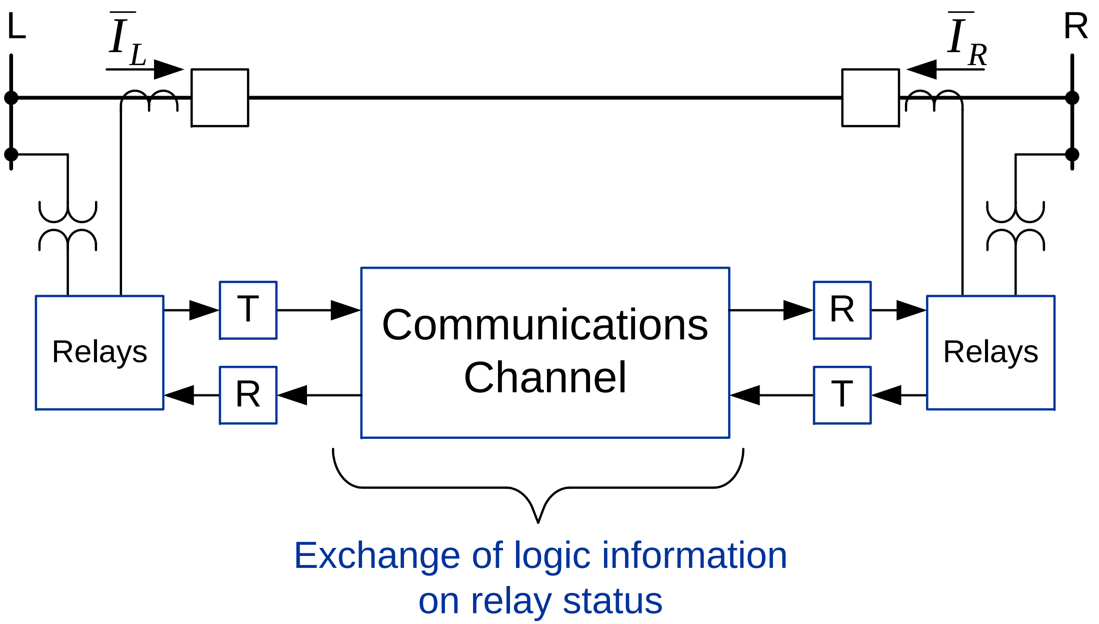


- An internal fault fed from A and B increases the current in primary winding (11) and (11a) with a corresponding current reversal in (11a)
- This results in the induced voltage in (12a) adding to that in (12) - producing an operating torque in both discs - tripping both ends

Differential Protection of Transmission Lines

- The ideal way of protecting any piece of power system equipment is to compare the current entering that piece of equipment, with the current leaving it.
- Under normal healthy conditions the two are equal. If the two currents are not equal, then a fault must exist. This is the principle of "Differential Protection", which is commonly used in TX Protection.

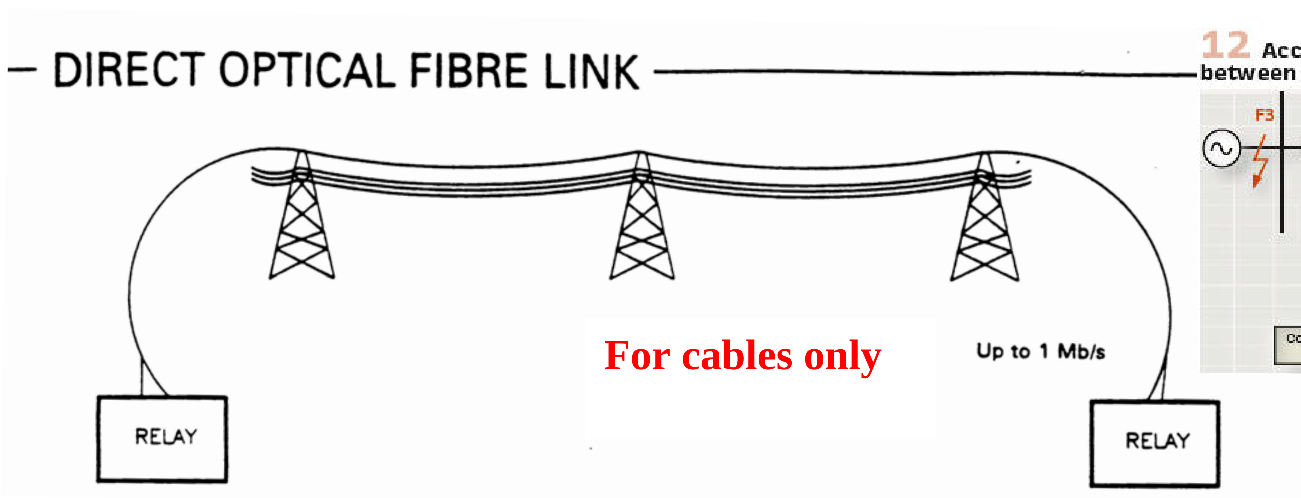
- The current differential relay is a unit protection intended for overhead lines and underground cables

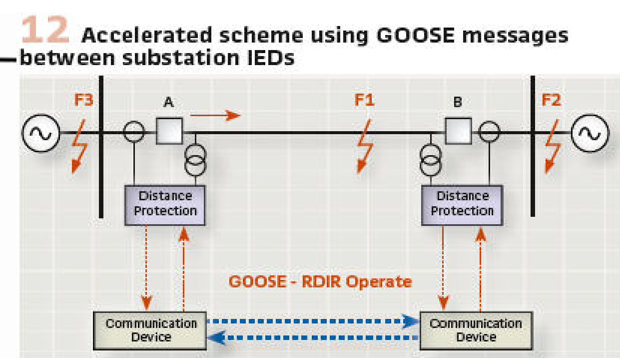


Channel of Communication

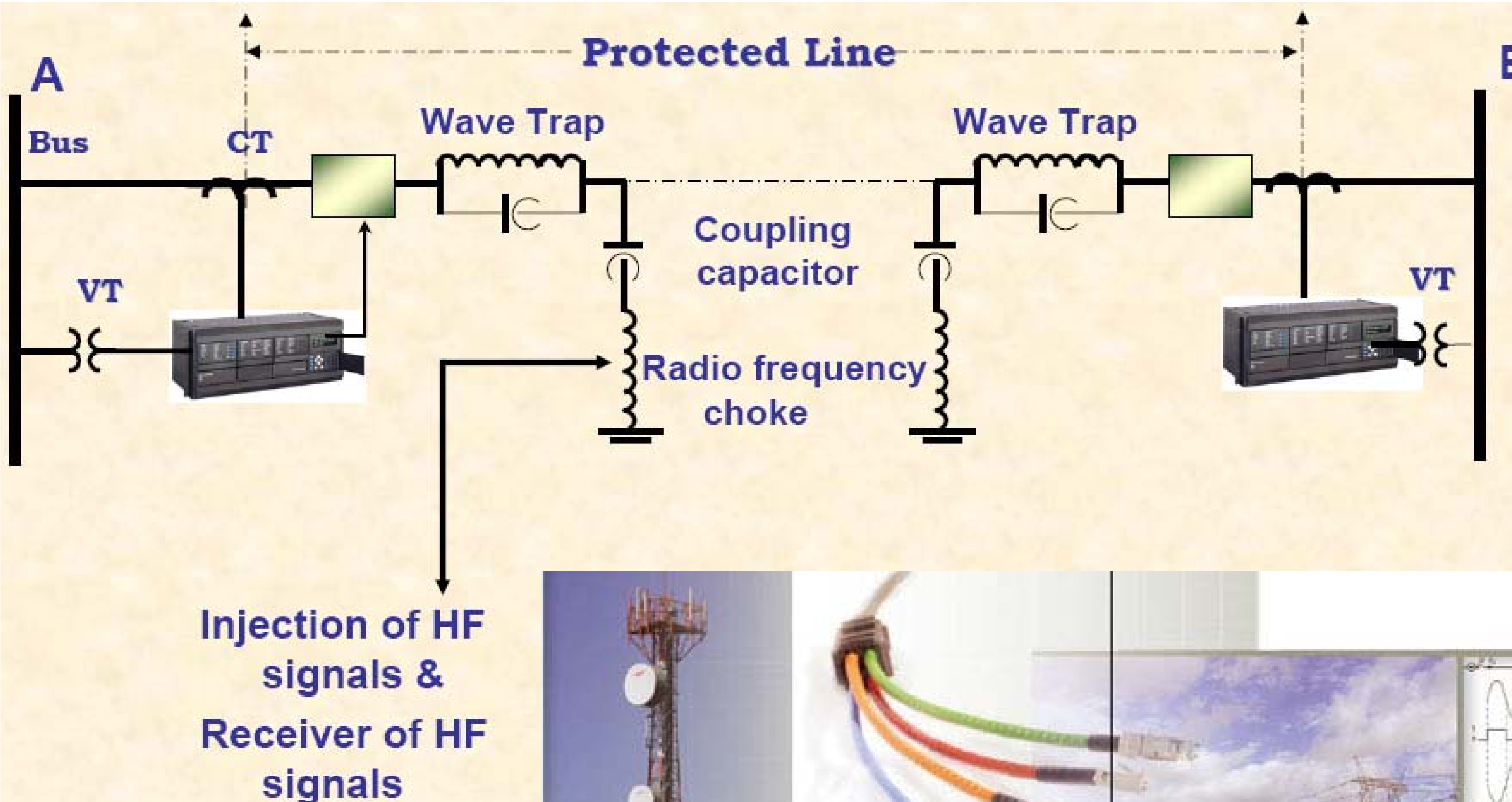
- The data is digitised before being sent over an optical fibre.
- The comparison is on a per phase basis.

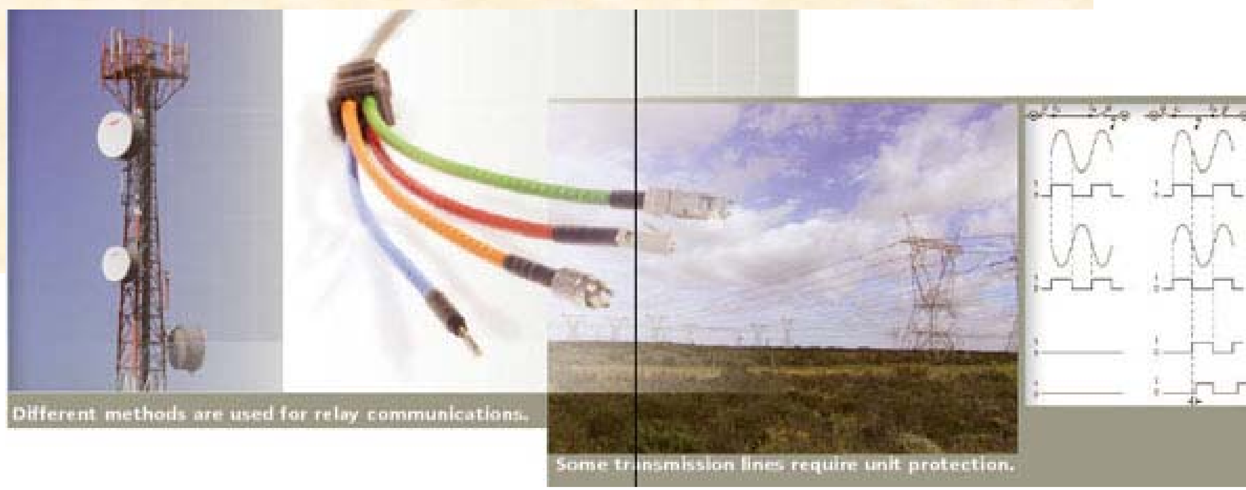
Direct optical fibre link – up to about 2 – 20 km, depending on the type of optical fibre. Used mainly for cables





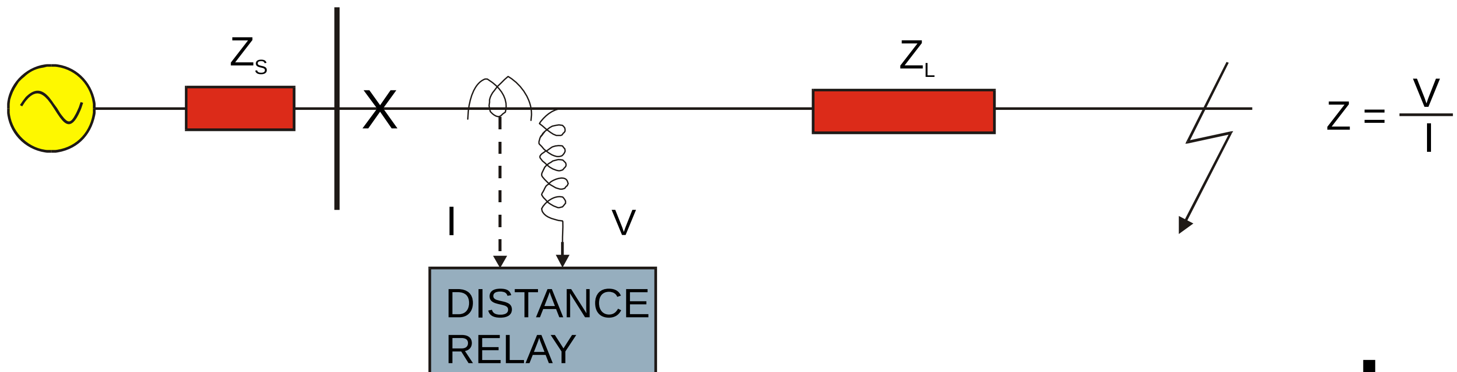
Power Line Carrier

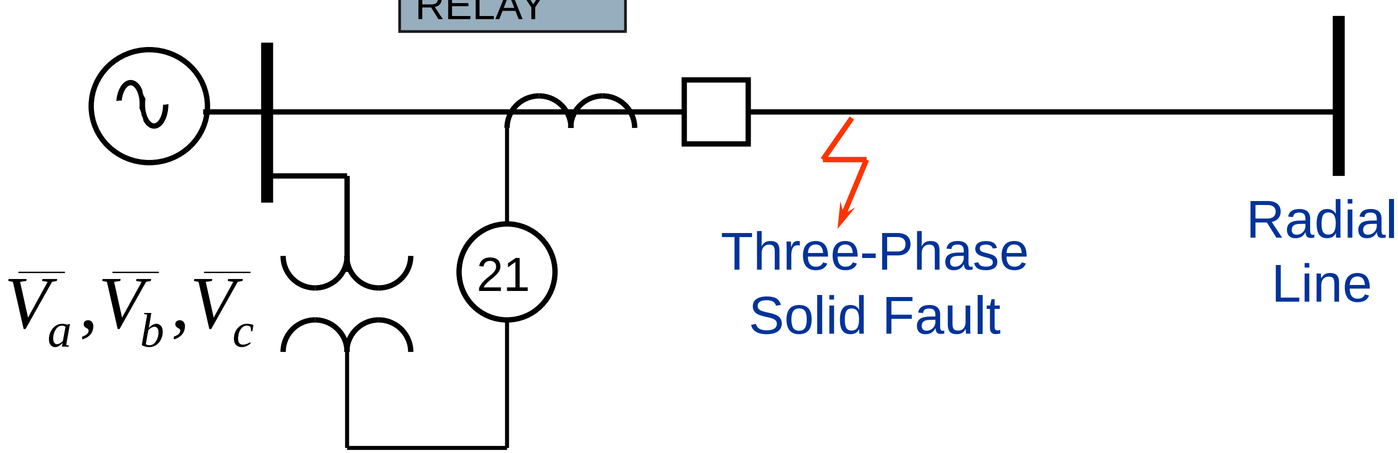




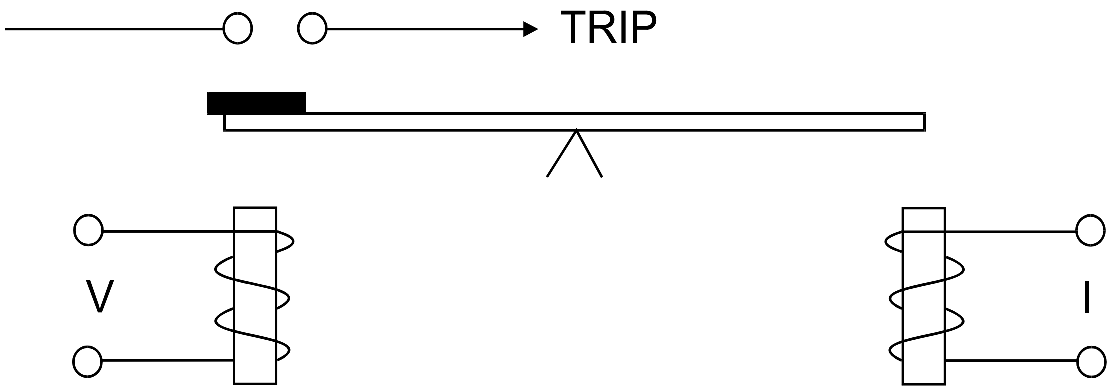
Distance Protection

- A distance relay has the ability to detect a fault within a pre-set distance along a transmission line or power cable from its location.
- Total impedance of power cable is a function of its length.
- A distance relay looks at current and voltage and compares these two quantities on the basis of Ohms Law.

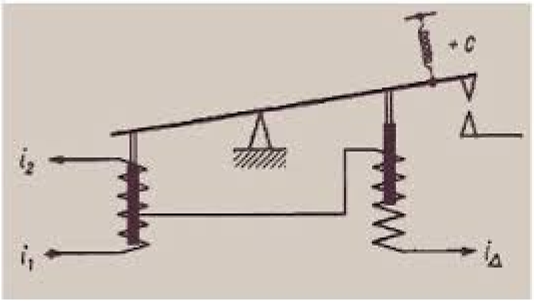




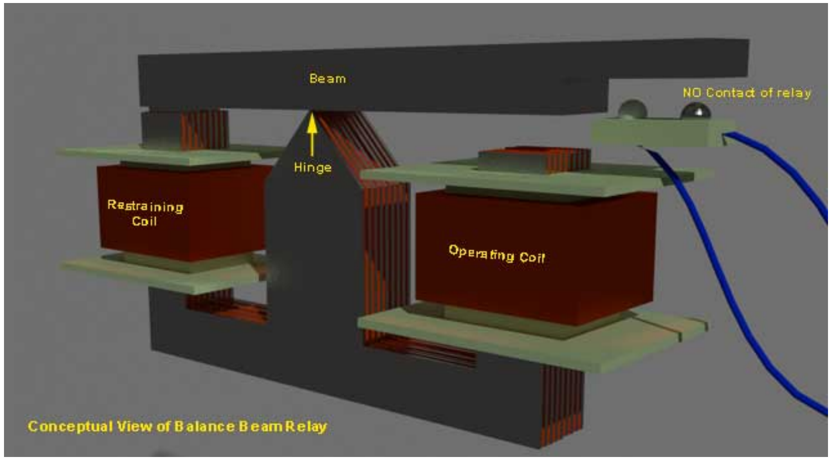
Pioneer-Type Balanced Beam Relay



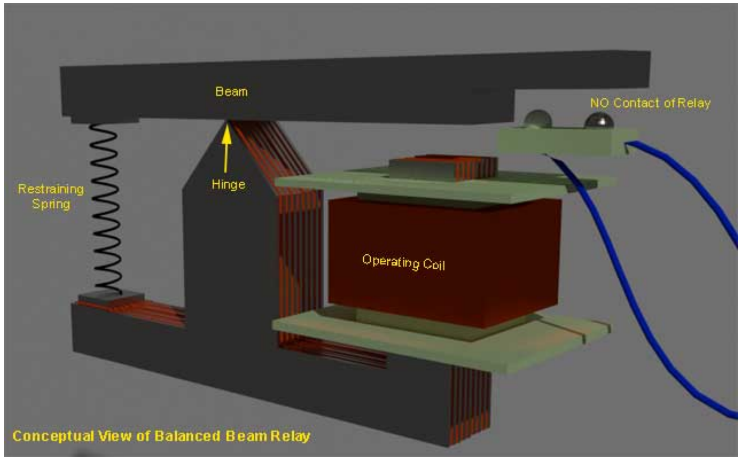
14 Percentage differential relay, McColl, 1917



- Voltage is fed into one coil to provide restraining torque.
- Current is fed into the other coil to provide the operating torque.



Conceptual View of Balance Beam Relay



Conceptual View of Balanced Beam Relay

Pioneer-Type Balanced Beam Relay

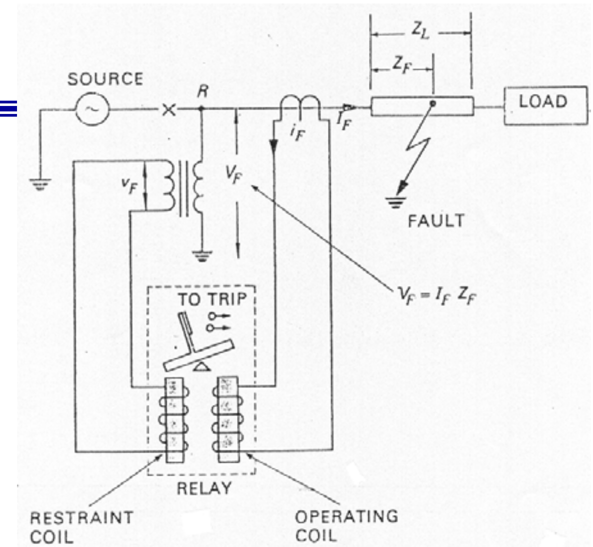
- Under healthy conditions the voltage will be high (at full rated level) whilst the current will be low (at normal load level) thereby balancing the beam so that the contacts remain open

- Under fault conditions, the voltage collapses and the current increase dramatically causing the beam to unbalance and close the contacts.

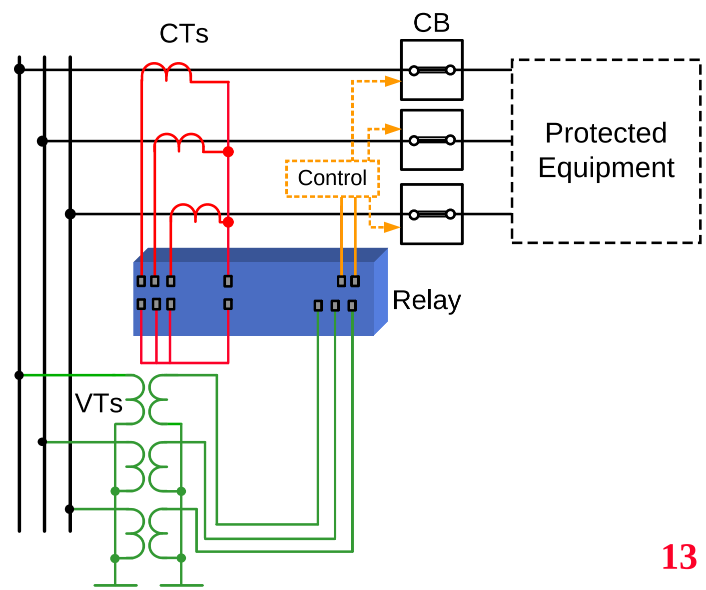
- By changing the **ampere-turns** relationship of the **current coil** to the voltage coil the ohmic reach of the relay can be adjusted.

Pioneer-Type Balanced Beam Relay

- This relay uses voltage and current inputs to measure "electrical distance" or impedance from relay to fault.
- Typically, the relay is set to 80% of the line impedance, $Z_S = 0.80 \times Z_L$. Z_s is the relay setting and Z_L -the line impedance.
- If the fault impedance, $Z_F < Z_S$, then the fault is within the relay setting and the relay trips instantaneously.
- If $Z_F > Z_S$, then the fault is outside the relay setting.

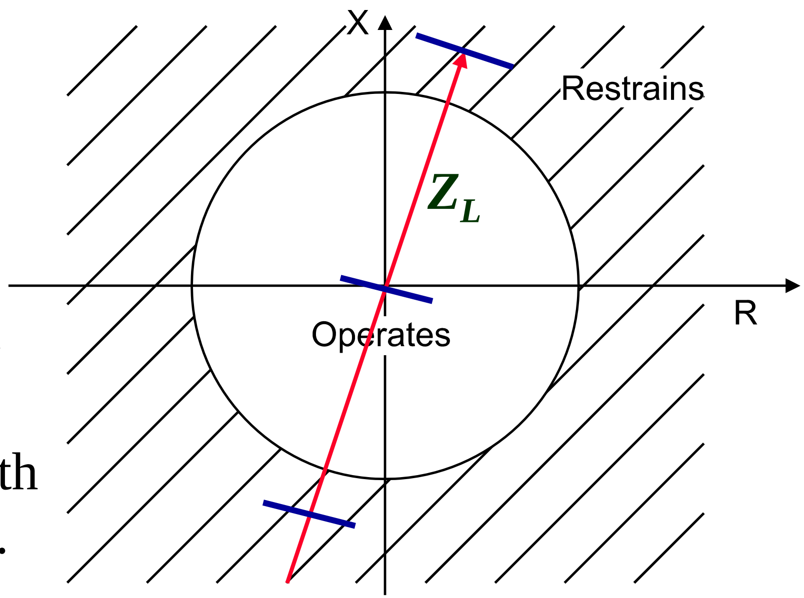


Z_F	=	V_F/I_F	
Z_F	<	Z_L	INTERNAL LINE FAULT
Z_F	>	Z_L	EXTERNAL LINE FAULT



Plain Impedance Relay

- Transmission line has resistance and reactance proportional to length.
- Represented on an R-X diagram as shown below.



- Plot the relay's operating boundary on an R-X diagram.
- Its impedance characteristic is a circle with its centre at the origin of the co-ordinates.
- The radius will be the setting in ohms.

$$Z_L = \sqrt{R^2 + X^2}$$

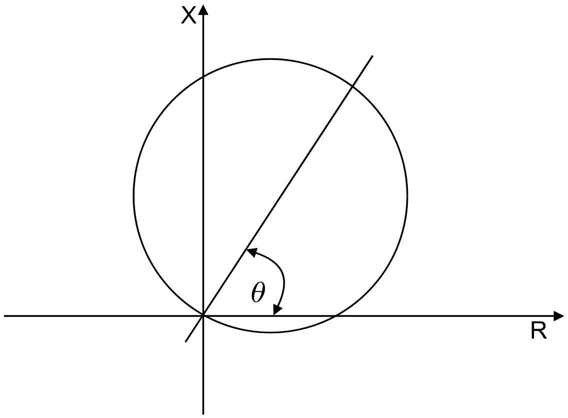
- The relay will operate for all values less than its setting … i.e. for all points within the circle.

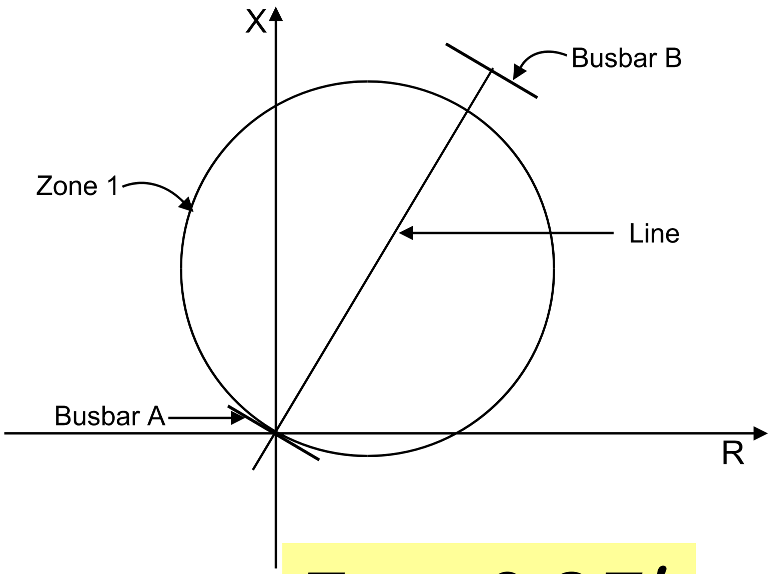
$$Z_r = 0.8Z'_L$$

Non-directional….it can operate for faults behind the relaying point

MHO Relay Characteristic

- Additional voltages are fed into the comparator in order to compare the relative phase angles of voltage and current; so providing a directional feature.
- This moves the circle so that the circumference of the circle passes through the origin.
- Angle θ is called characteristic angle.
- Called MHO relay.

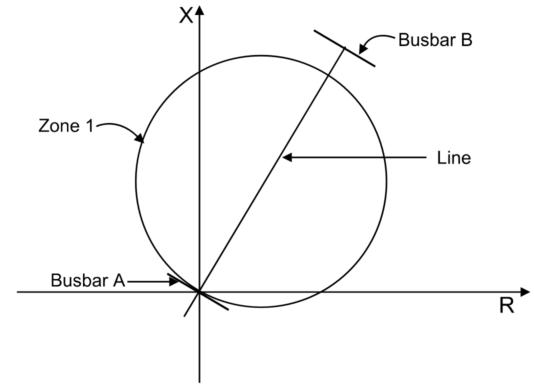




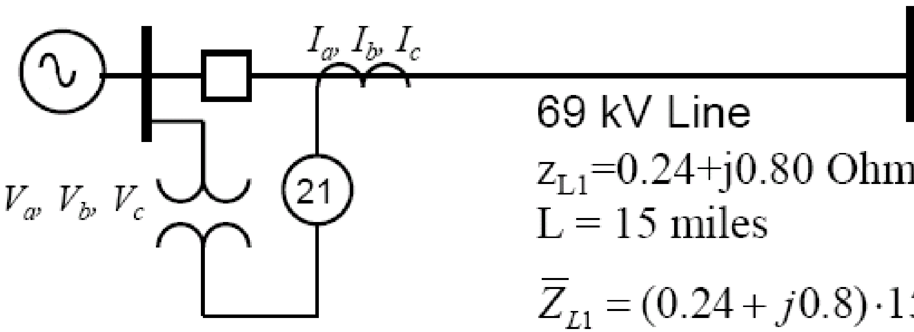
$$Z_{r1} = 0.8 Z_L'$$

MHO Relay Characteristic

This example shows the calculations involved in the determination of a simple impedance relay setting.



$$Z_{r1} = 0.8 Z_L'$$



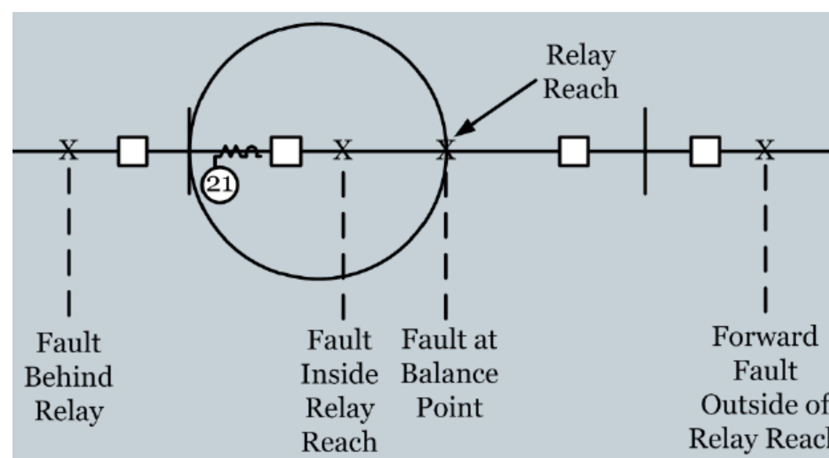
z_{L1}=0.24+j0.80 Ohms/mile
L = 15 miles

$$\bar{Z}_{L1} = (0.24 + j0.8) \cdot 15 =$$
$$= 3.6 + j12 \text{ Ohms}$$

Relay setting: $Z_{r1} = (0.8)\,|3.6 + j12| = 10.02 \text{ Ohms}$

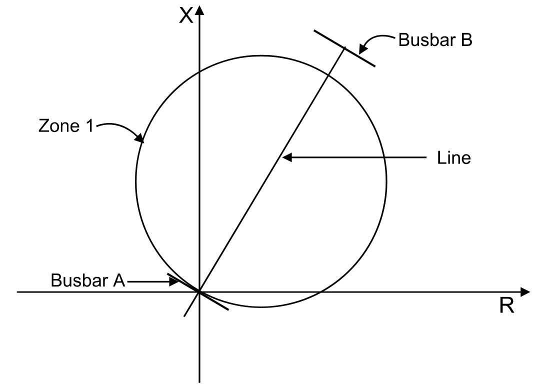
Relay will operate for:

$$R^2 + X^2 \leq 10.02^2 = 100.4 \text{ Ohms}$$

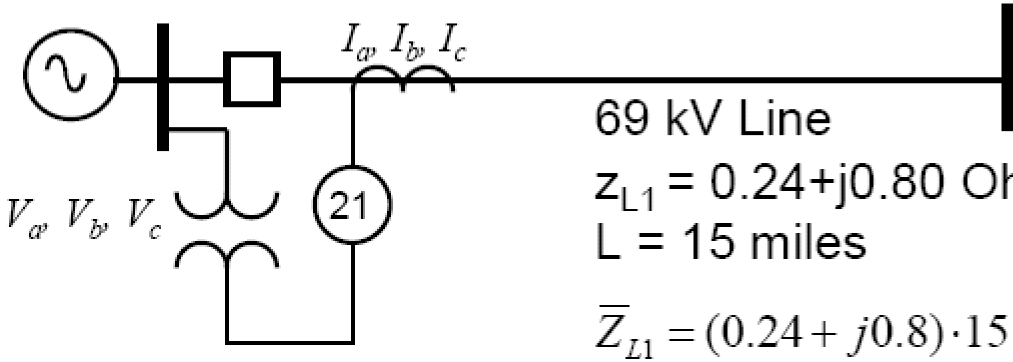


MHO Relay Characteristic

This example shows the calculations involved in the determination of a simple impedance relay setting referred to the secondary circuit.



$$Z_{r1} = 0.8 Z_L'$$

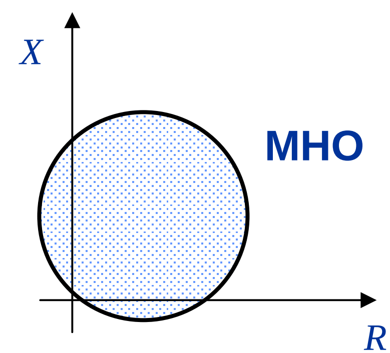


69 kV Line

z_{L1} = 0.24+j0.80 Ohms/mile

L = 15 miles

$$\overline{Z}_{L1} = (0.24 + j0.8)\cdot 15 =$$
$$= 3.6 + j12 \text{ Ohms}$$



$$VTR = 40000/120$$
$$CTR = 600/5 \qquad \Rightarrow ZTR = VTR/CTR = 2.77$$

Relay setting at 80% of Line Z:

$$Z_{r1} = 10.02/2.77 = 3.62 \text{ Secondary Ohms}$$

Distance Protection

- **Problem?**

1. What happens for a fault on the protected line that is beyond the reach of the relay?
2. If the relay operates instantaneously, it cannot be used as a remote back-up for a relay protecting a line adjacent to the remote substation.

- **Solution:**

These two problems are overcome by adding time-delay distance relays. This is accomplished by using the distance relay to start a definite time timer. The output of the timer can then be used as a tripping signal.

Distance Protection

2 Power system model simulated in PSCAD/EMTDC

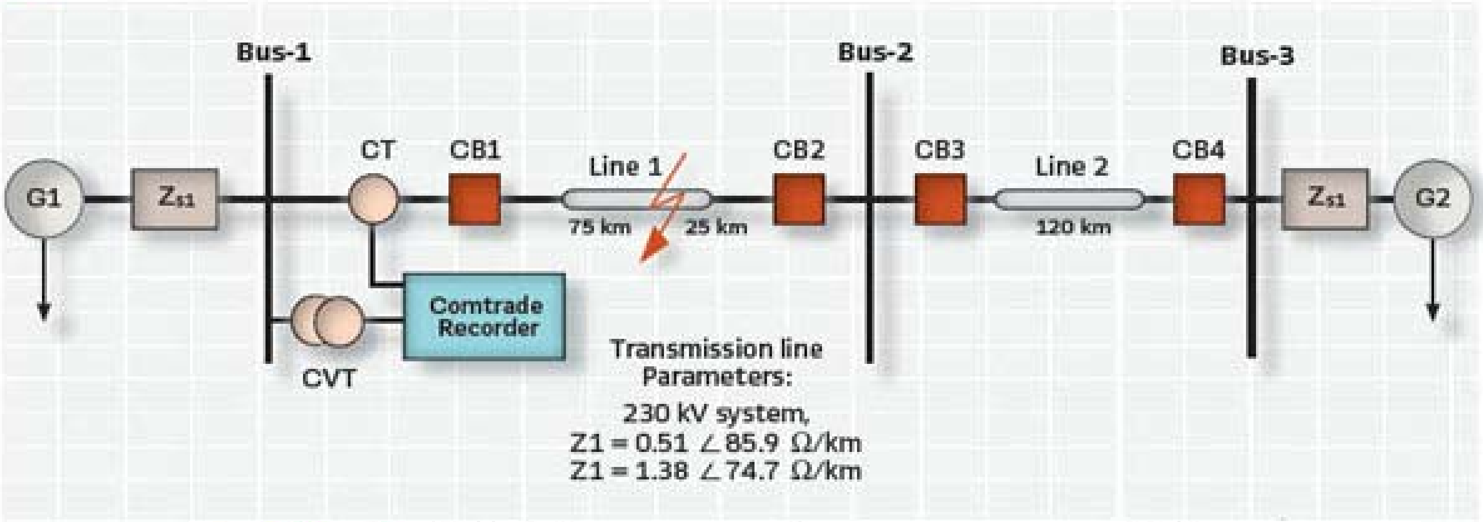
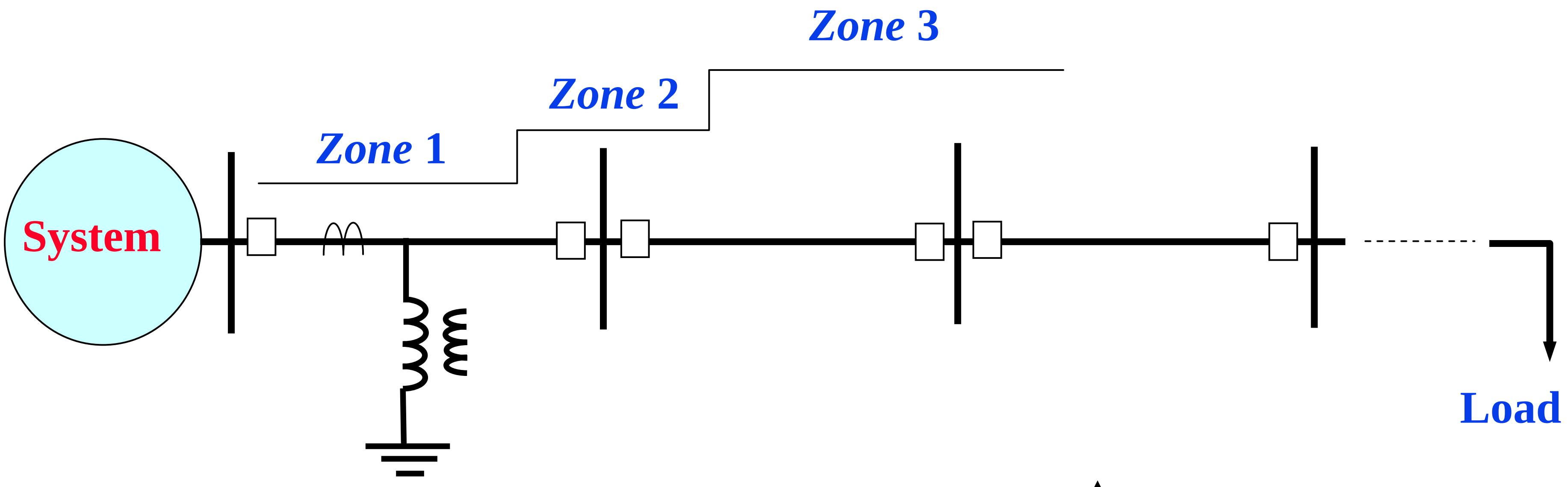


table 1 **Distance relay zone settings**

Serial Number	Zones of Protection	Zone Coverage	Time Delay (msec)
1	Zone - 1	80% of Line - 1	0
2	Zone - 2	100% of Line - 1 + 20% of Line - 2	300
3	Zone - 3	100% of Line - 1 + 100% of Line - 2	1,000

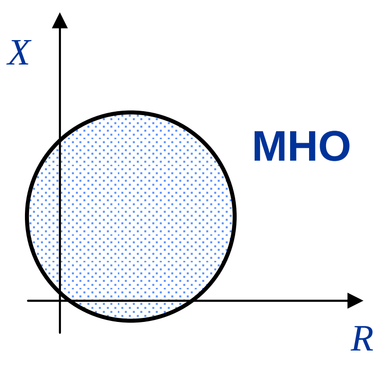
Conventional Distance Protection



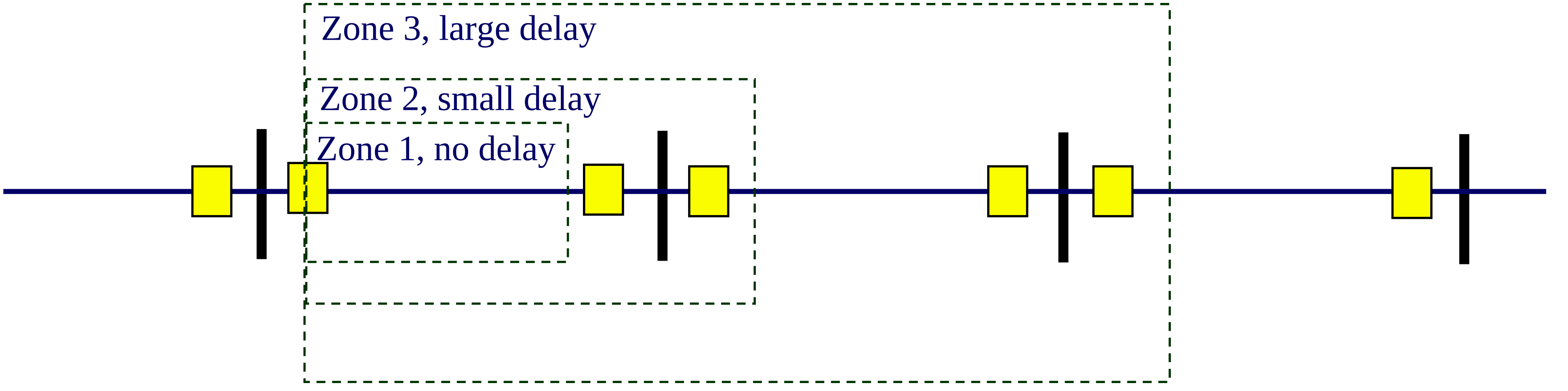
Zone 1: 0-80% of line length

Zone 2: 80% - 120% of line length

Zone 3: 120%- 220% of line length



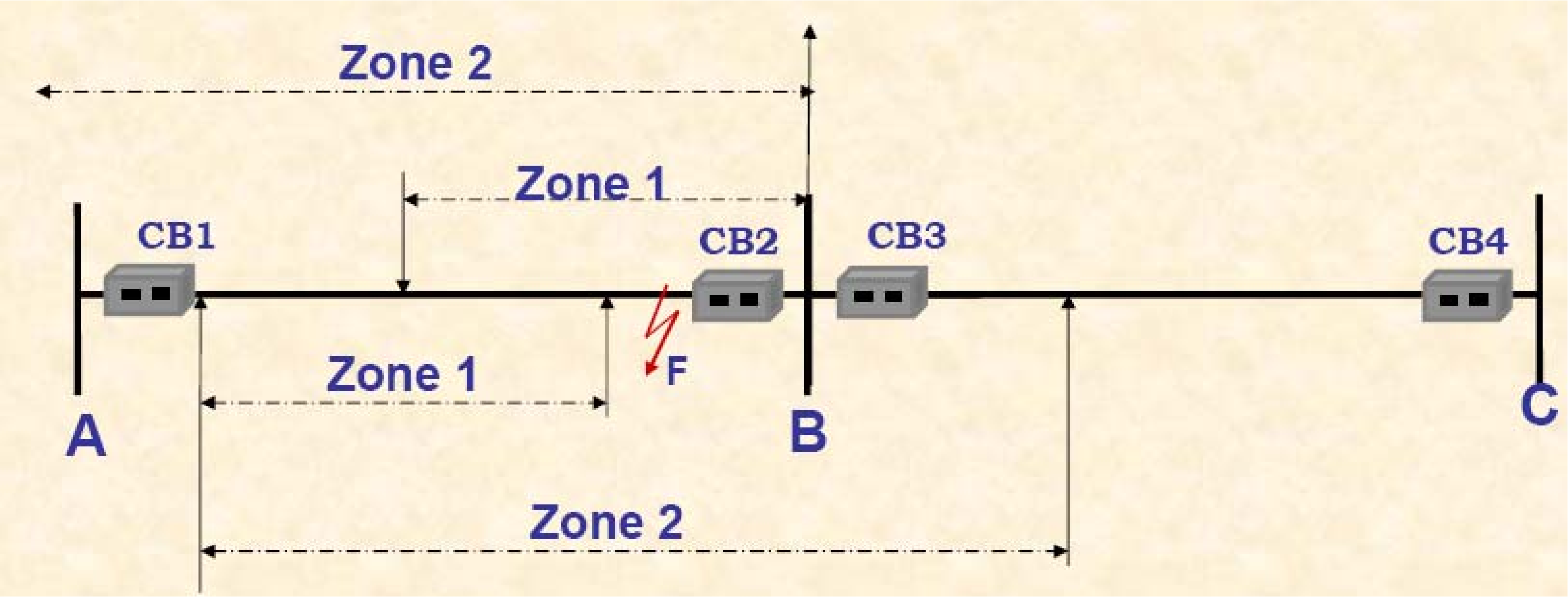
Conventional Distance Protection



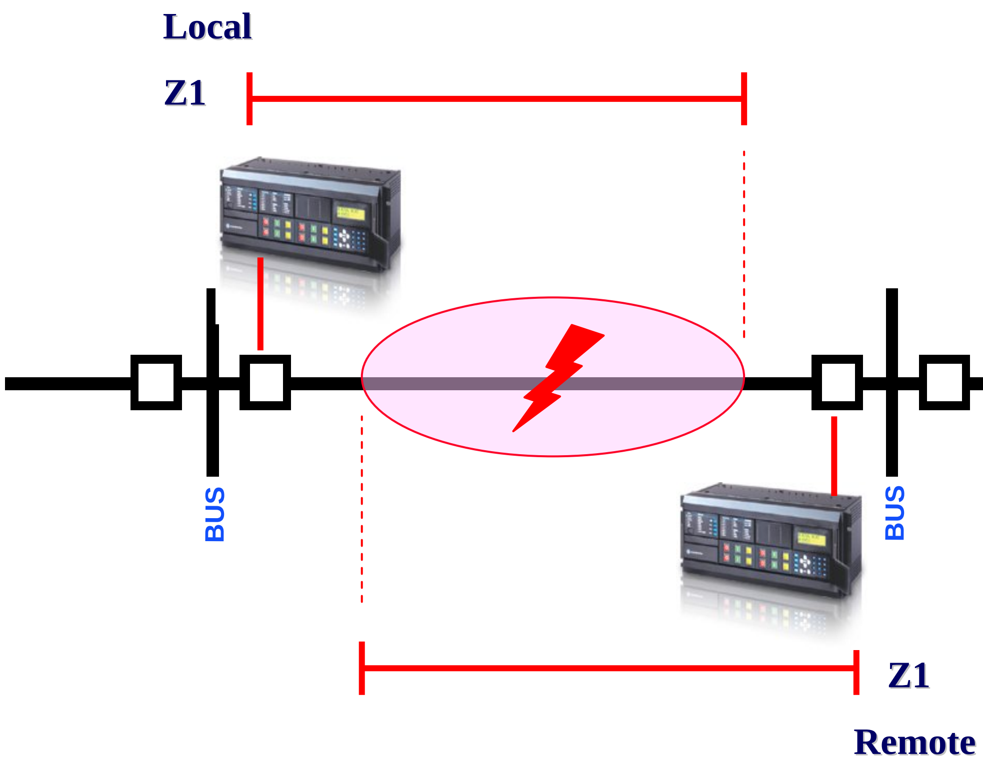
- Relays sense V/I and trip if it is too low; good approach because fault conditions are low voltage, high current.
- Relays are directional; trip only for faults “looking” in one direction.
- Zone 1 trips instantly; trip zone for primary protection
- Zone 2 has small delay.
- Zone 3 has large delay; these are trip zones for “backup” protection

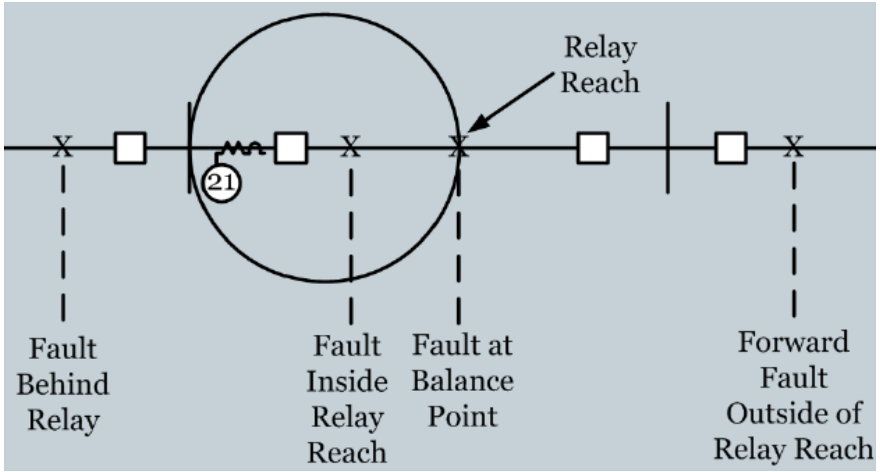
Zone 1

- Relay characteristic has been added.
- **Reach** of the measuring element is approximately **80% of the line length**.
- **Under reach** setting purposely chosen to avoid **over-reaching** into the next line section to ensure sound **selectivity**.

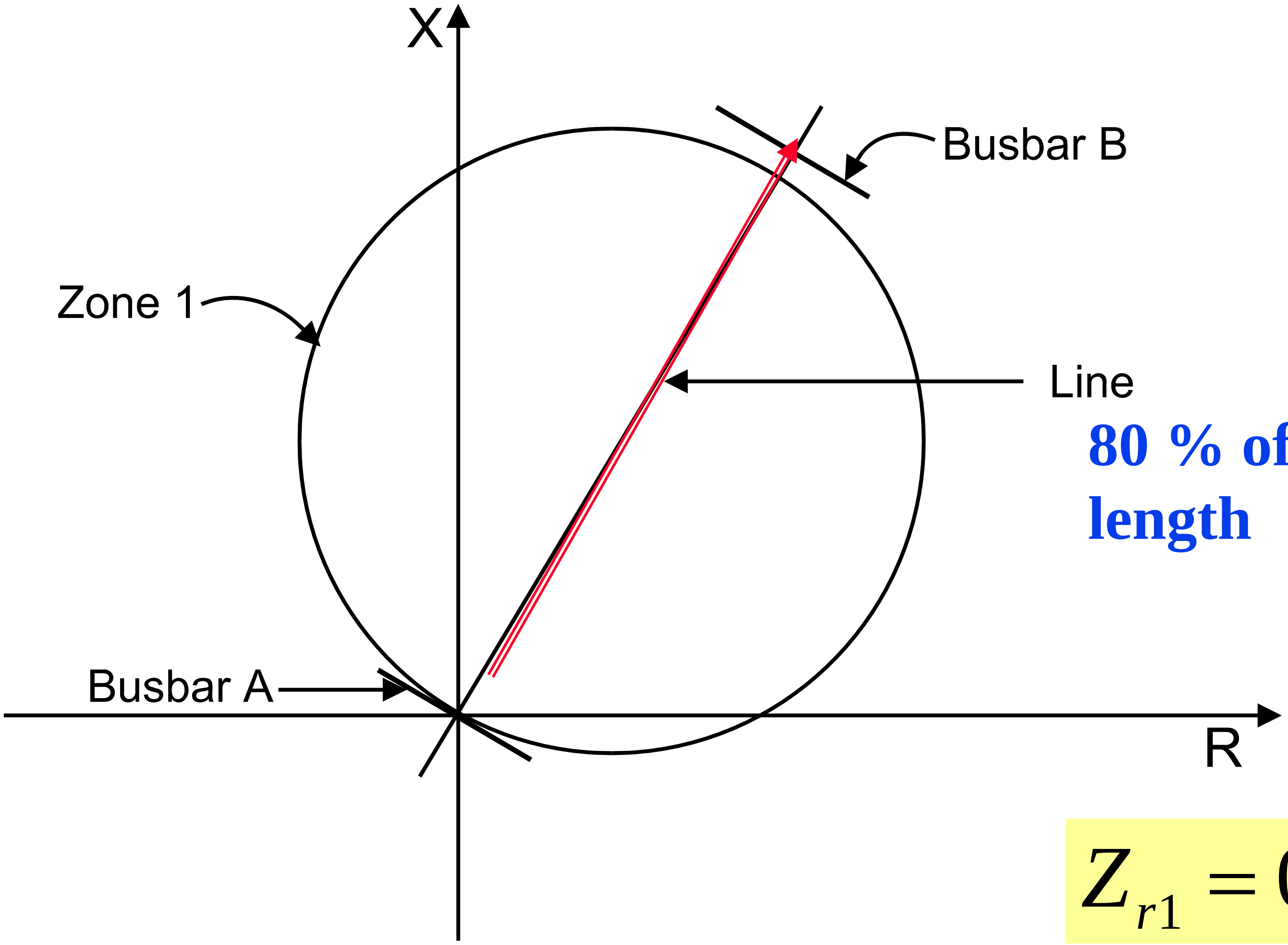


Zone 1


Local
Z1
BUS
BUS
Z1
Remote


Relay
Reach
21
Fault
Behind
Relay
Fault
Inside
Relay
Reach
Fault at
Balance
Point
Forward
Fault
Outside of
Relay Reach

Zone 1 - under-reach

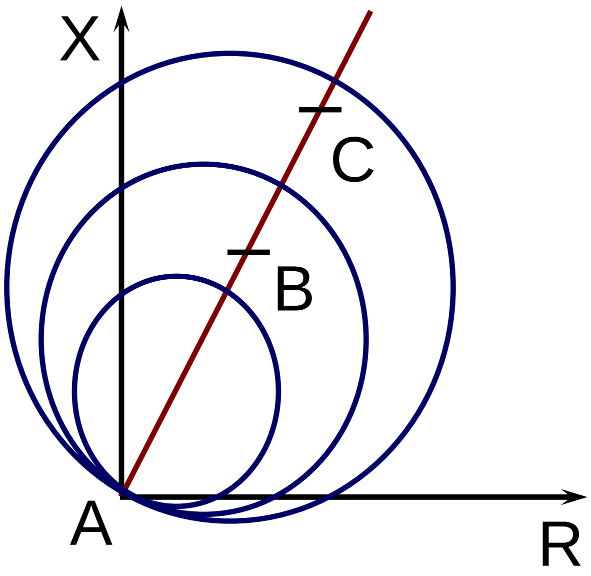


80 % of line length

$$Z_{r1} = 0.8 Z_L'$$

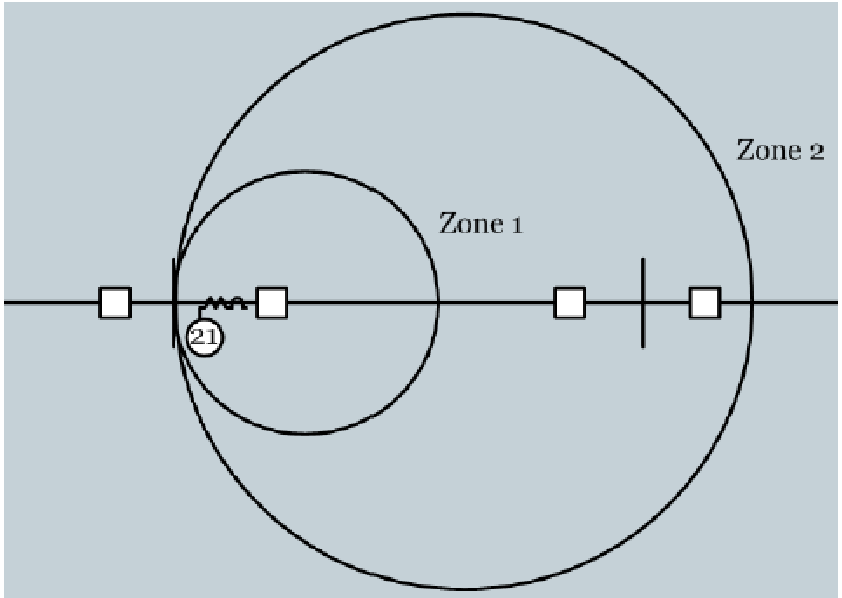
Reasons for Under-reach setting

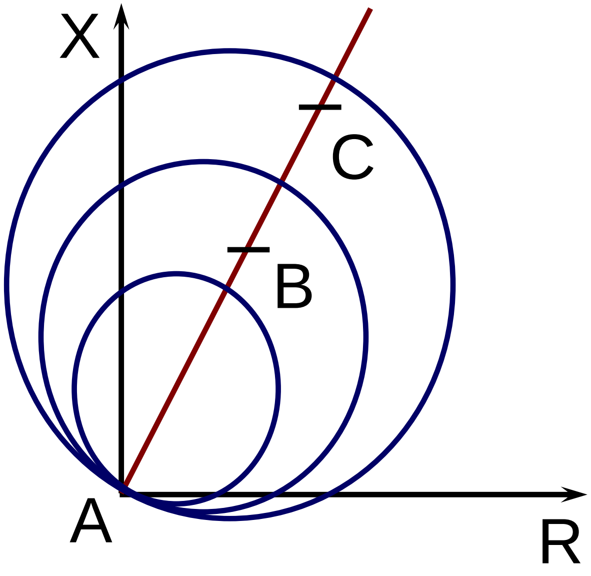
- Not practical to measure the impedance of transmission line 100% accurately.
- Errors are present in voltage and current transformers.
- Manufacturing tolerances on the relay's ability to measure accurately.
- Known as Zone 1 of the distance relay. **Instantaneous operation**



Zone 2

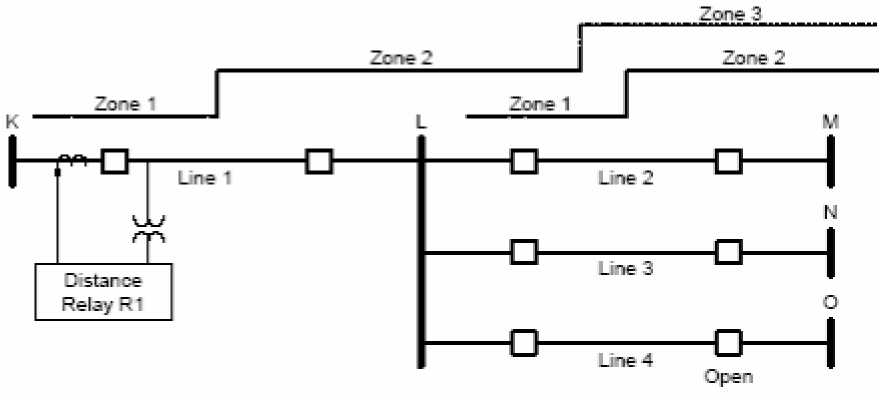
- A **second measuring element** is fitted to cover the **remaining 20% of the line length**. (normally set to measure 120% from source bus)
- Time delayed by 0.5 secs to provide the necessary co-ordination with the downstream relay (as this Zone actually **over-reaches** the next Breaker and provides **back-up**)
- Measuring element called Zone 2.





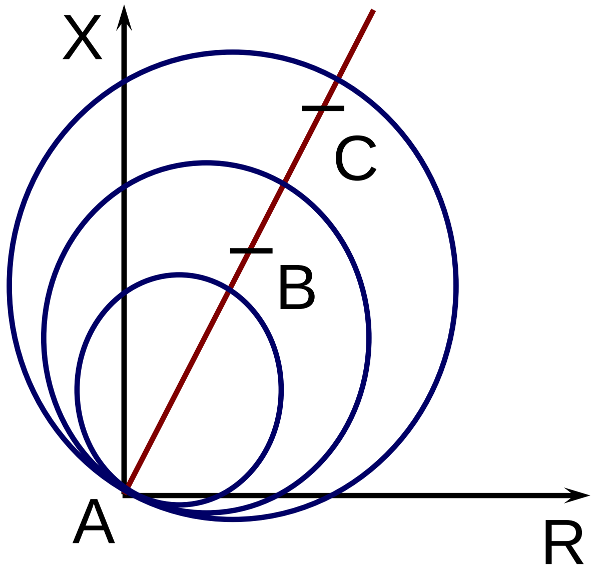
Zone 3- MHO Characteristic

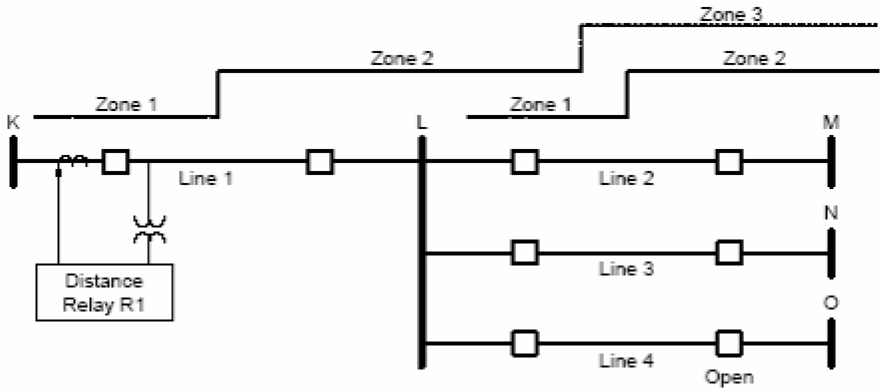
- Zone 3 element in a step-distance relay scheme provides time-delayed remote backup protection in case of failure of the primary protection at the remote station.
- Zone 3 is applied to prevent damage to the equipment and personnel.
- Zone 3 is set to cover Line 2 completely.
- Zones 1 and 2 should never overreach the end of Line 2, and Zone 3 should never underreach.
- Zones 1 and 2 are set using the actual impedance of Line 1, ignoring current infeed at Bus L, while Zone 3 reach is typically set to 120% of the impedance presented to the distance relay for a fault at the remote end of the second line section.



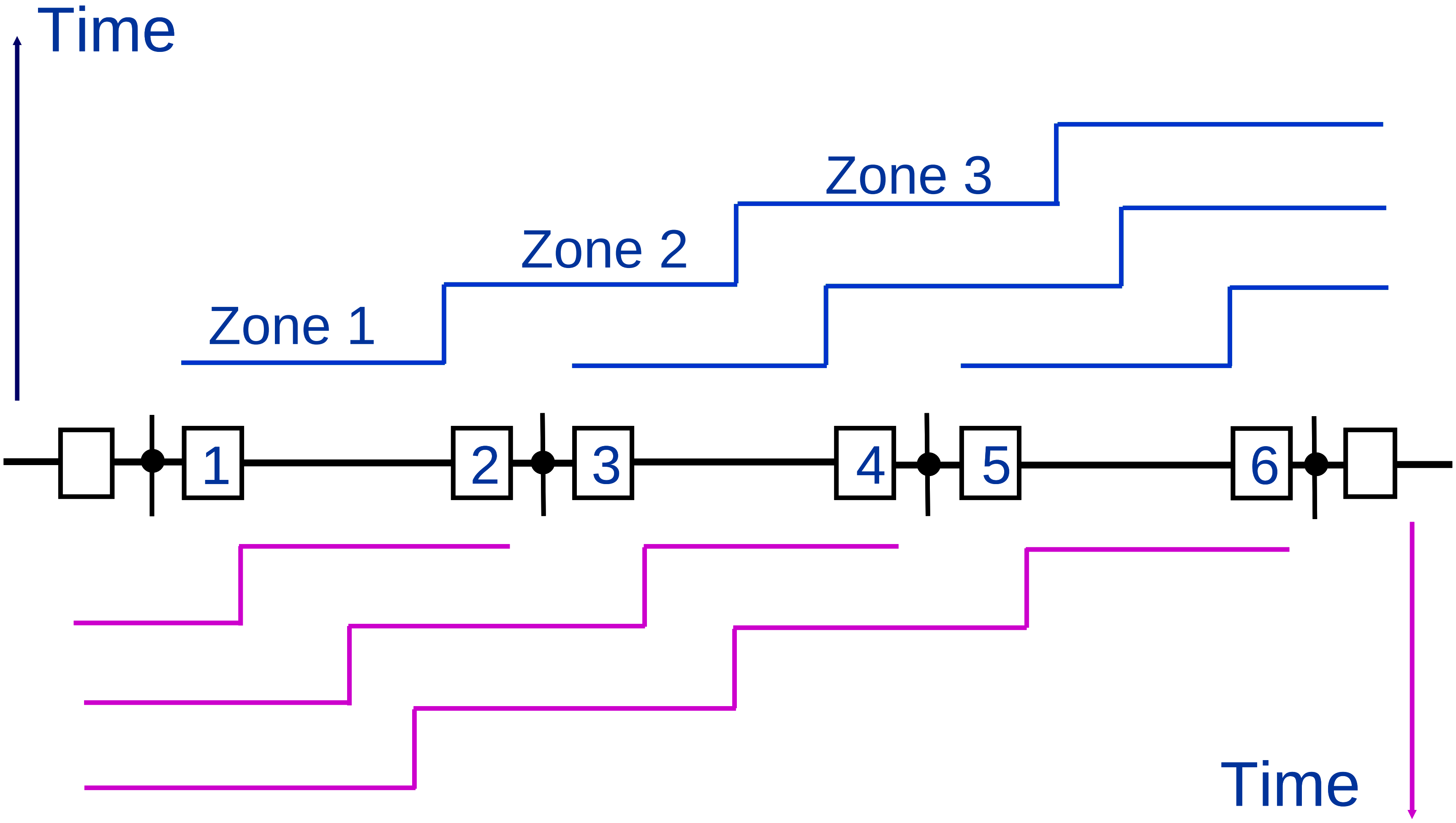
Zone 3- MHO Characteristic

- Alternatively, Zone 3 is set at 120% of the highest apparent impedance for a remote station line-end fault with the remote terminal breaker open as shown.
- The Zone 3 time delay is typically twice that of Zone 2 (i.e., 40–60 cycles) to achieve time coordination.
- The Zone 3 distance element is seldom called on to operate; however, it must not operate during extreme loading conditions, stressed power system conditions, or slow power swings.





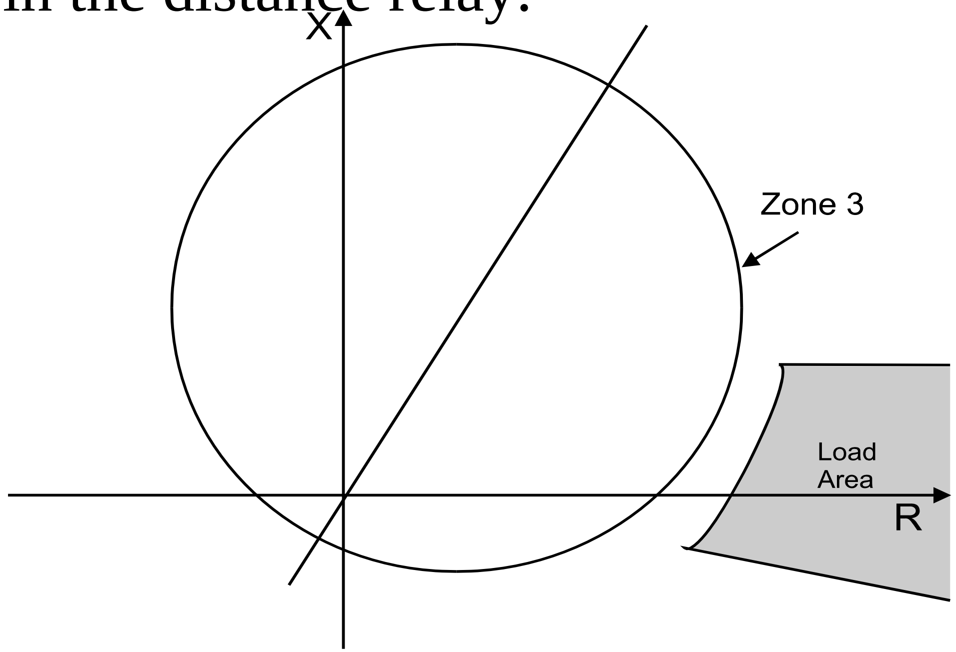
Three-Zone Distance Protection



Zone 1 Is Instantaneous

Effect of Load Current

- Overreaching and undesirable Zone 3 tripping during stressed system conditions has often contributed to cascading outages.
- Adequate measures must be taken to prevent Zone 3 operation for such conditions, using properly shaped distance characteristics or a load encroachment feature and a power-swing blocking feature in the distance relay.



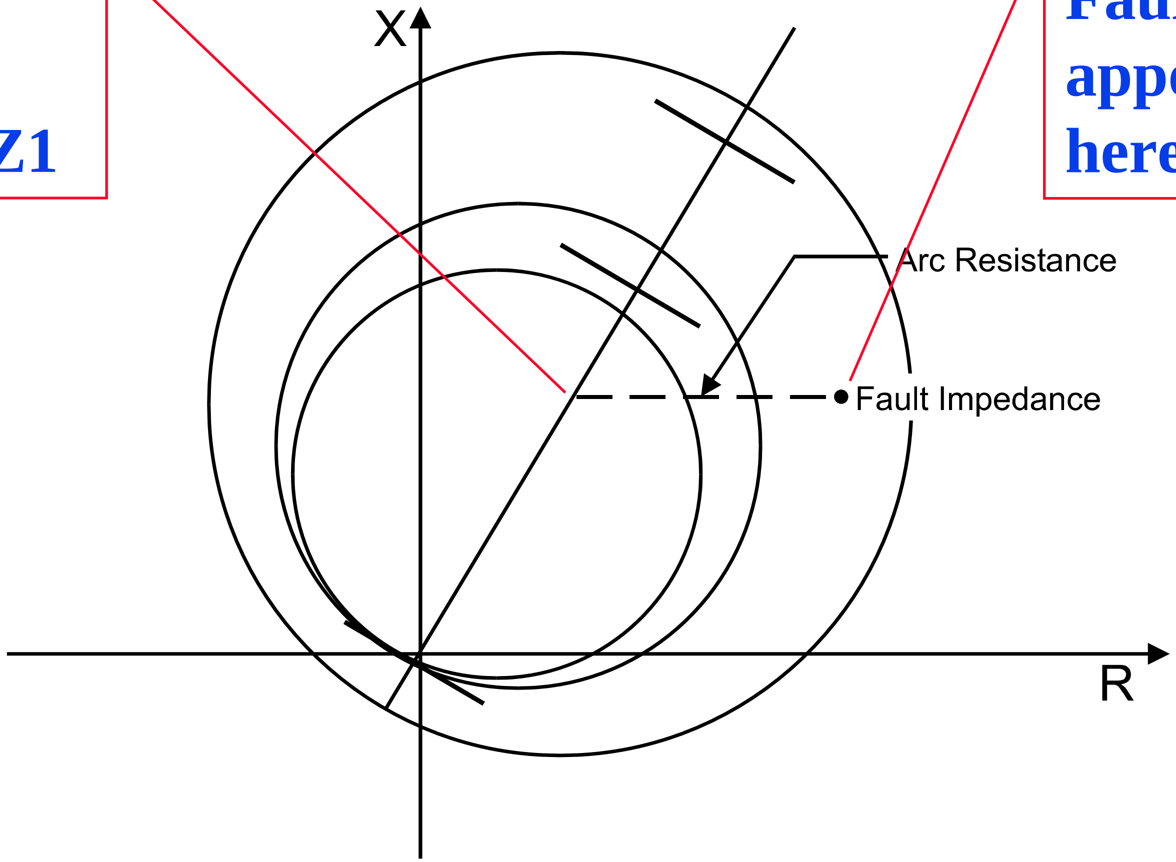
Effect of Arc Resistance

- Resistance of the fault arc can also have an impact on performance of a distance relay.
- ***R*** of fault arc takes the fault impedance outside the relay’s tripping characteristic.
- Effect of arc resistance is most significant on short lines where the reach of the relay setting is small.
- Can be a problem for faults at end of the reach.

Arc resistance can cause “Under-reaching”

Fault is actually here in Z1

Fault appears here



Effect of Arc Resistance

- High Fault Arc resistances tend to occur during mid-span flashovers to ground during a bush fire

OR

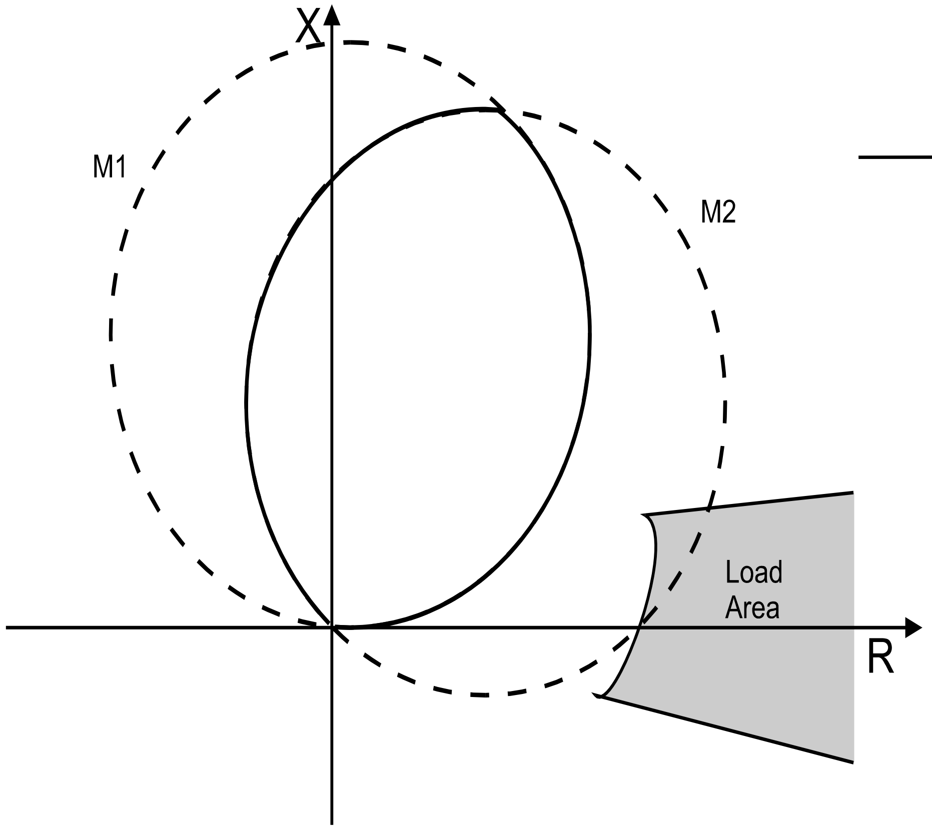
- On transmission lines carried on wood poles without earth wires.

- Overcome these problems as discussed next using different characteristic relays...

Different Shaped Characteristics

- To overcome the problems of **load encroachment** and **arc resistance**…
- Distance relays have been developed:
  - **Circular**
  - **Lenticular**
  - **Figure of Eight**
  - **Trapezoidal**
- **Digital Relays** help to **get any characteristics** as required.

Lenticular



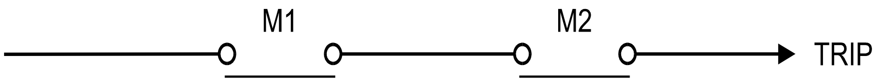
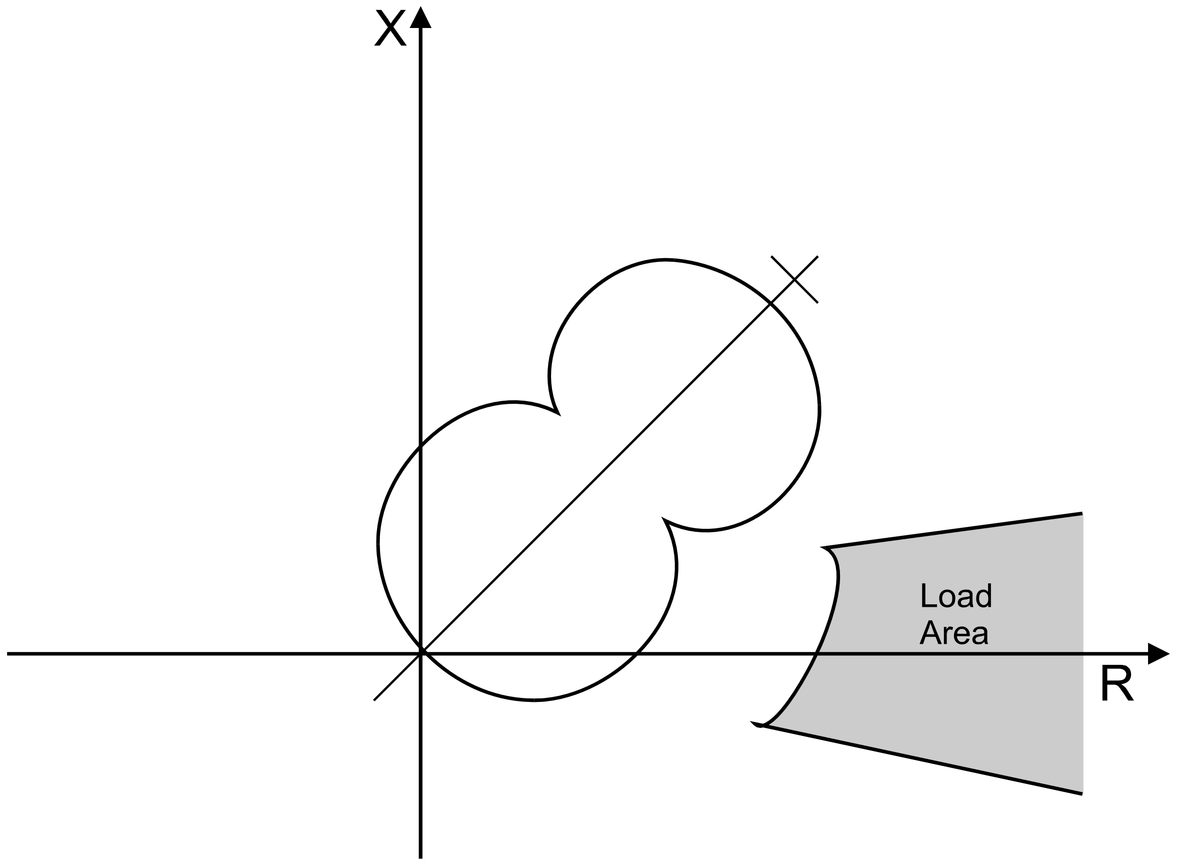
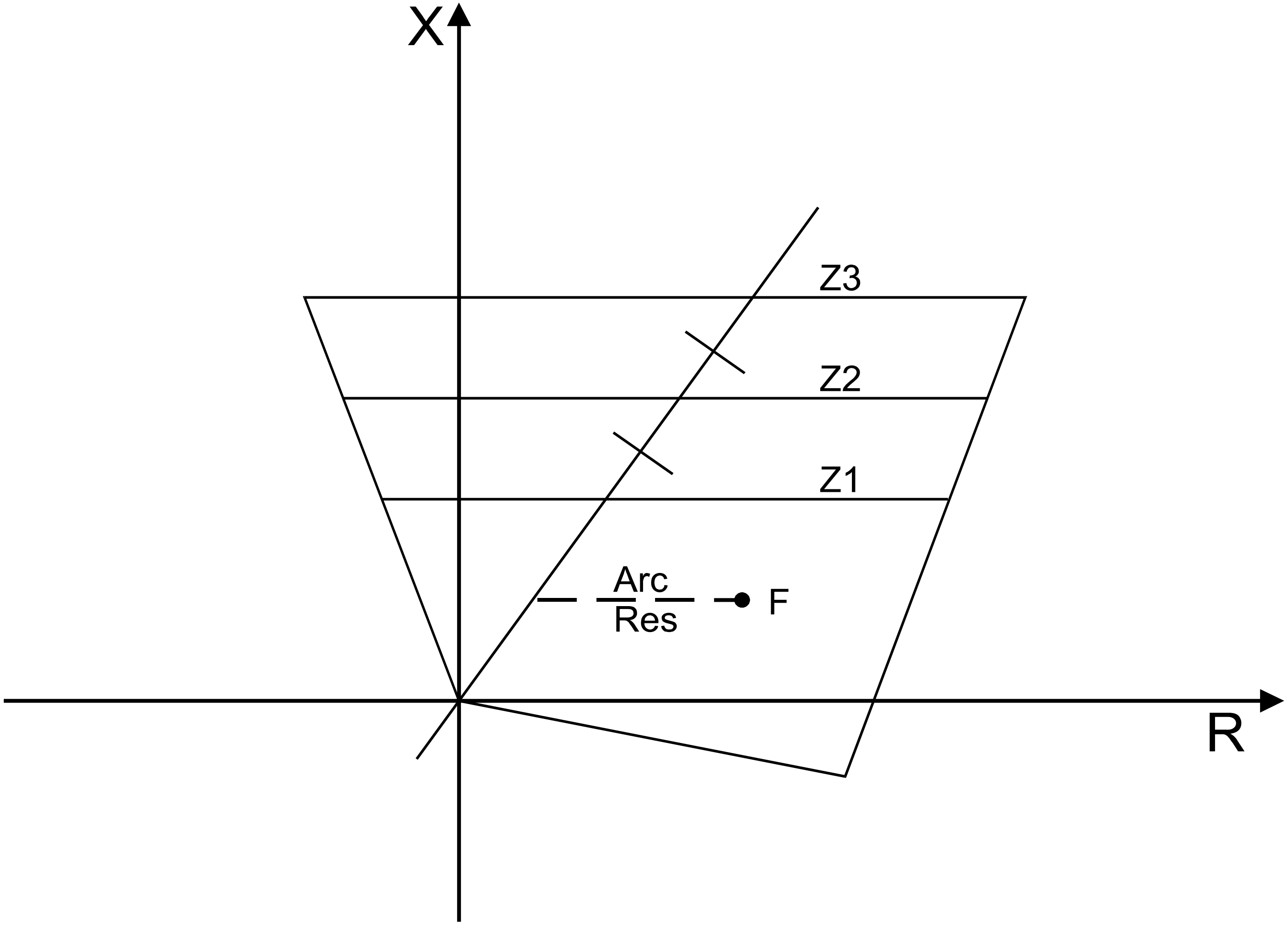


Figure of Eight



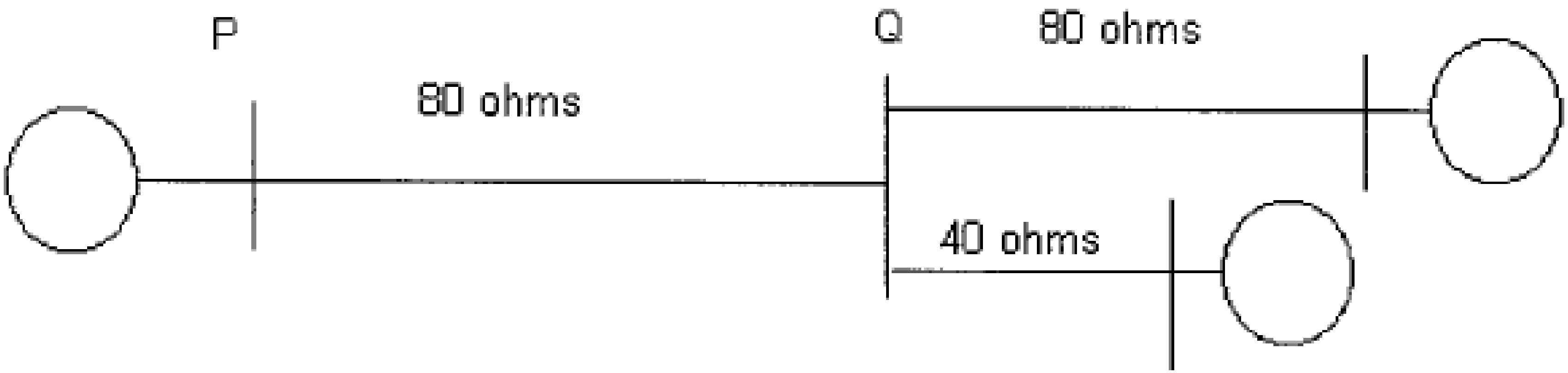
Trapezoidal

(increases arc resistance coverage)



Example

Consider the settings for line **PQ** at bus **P**.The impedance angle for all lines is **75°.** The line length is **80 Ω** . The distance relay at bus **P** is fed by current transformers rated at 2000 A:5 A and voltage transformers rated at 345 kV/200 kV Y:120 V/69 V Y. Set Zone 1 for 85% of this value (85%–90%) settings are typical for phase distance, slightly lower for ground distance):



Example

Zone 1 setting = 0.85 .80 Ω= 68 Ω ,primary ohm setting
CT ratio = 2000/5 = 400
VT ratio = 200,000/69 = 2900
Relay setting = primary setting (Ω) .CT ratio/VT ratio
= 68 .(400)/(2900)
= 9.38 relay ohms

Zone 2 setting = length×115% (minimum)
=line length+0.5 × length of shortest next adjacent line (preferred)

Example

The two next adjacent lines are 40 Ω and 80 Ω ,respectively. The shortest of these is 40 Ω. Half of that is 20 Ω. The setting 80+20Ω =100 Ω is greater than the minimum setting of 92 Ω (which guarantees seeing the entire line).

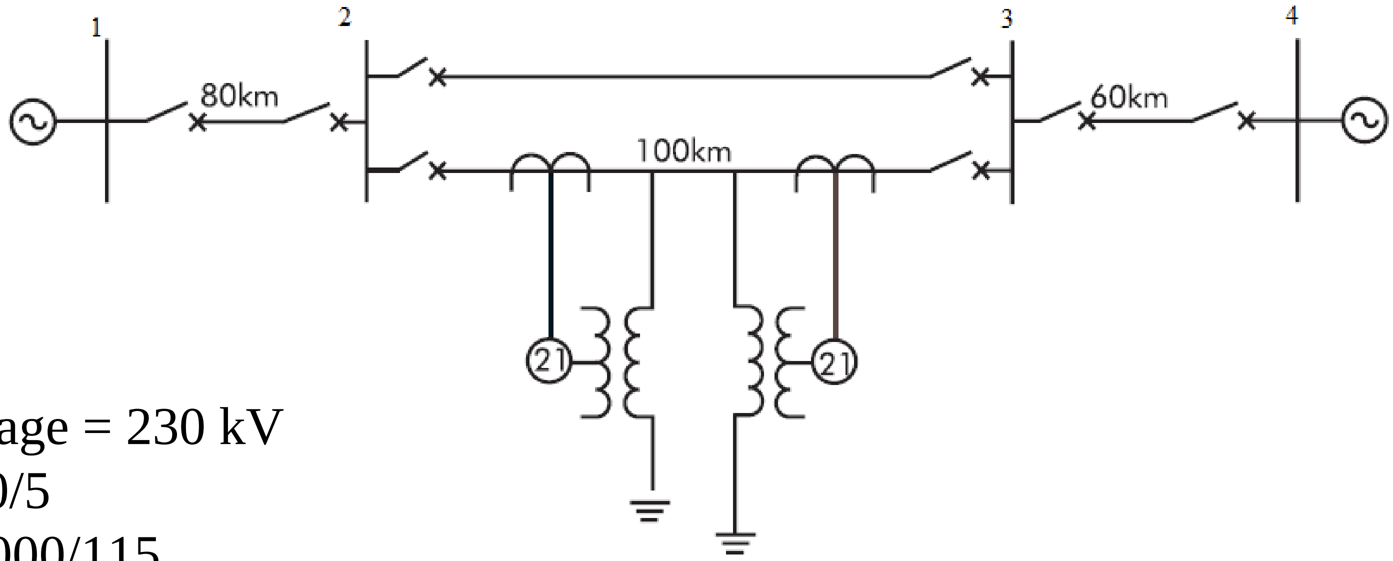
The relay setting is then

Zone 2 setting = 100 Ω (primary)

= 100400/2900 = 13.8 relay ohms

Distance Protection Setting
Exercise

- Design a 3-step (3-zone) distance protection to protect the 100km double circuit line between buses 2 and 3.



System Data

- System voltage = 230 kV
- CTR = 1200/5
- VTR = 230000/115
- $Z_l = 0.089 + j0.476\ \Omega/\text{km}$
- Line leength = 100 km

Question # 1:

Consider the system of Fig. Q1 where the values given are impedance in per-unit. Draw the per-phase equivalent circuit and find the impedance as seen by the impedance relay looking into the circuit for the following cases:

a. normal load conditions, Z_n
b. 3-phase fault at F_1, Z_{F1}
c. 3-phase fault at F_2, Z_{F2}

Plot the impedance as seen by the impedance relay on the R-X diagram.

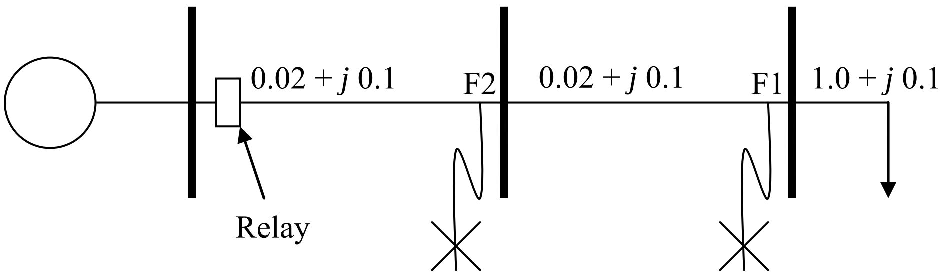
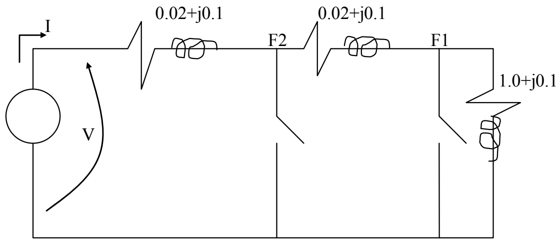


Fig. Q1

Solution:

The per-phase circuit is shown.

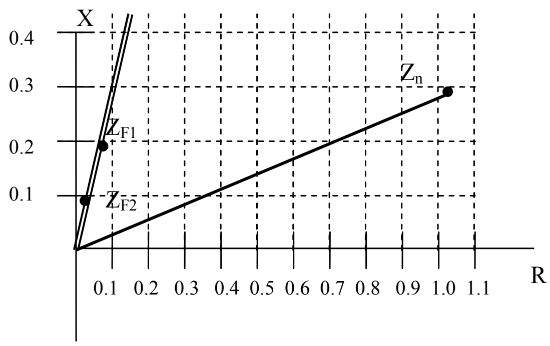


The desired impedance is $Z = \dfrac{V}{I}$

(a) Under normal load, $Z_n = \dfrac{V}{I} = 1.04 + j0.3\ pu$

(b) For a fault at F_1, $Z_{F1} = \dfrac{V}{I} = 0.04 + j0.2\ pu$

(c) For a fault at F_2, $Z_{F2} = \dfrac{V}{I} = 0.02 + j0.1\ pu$



Question # 2:

Consider a 132 kV transmission system as shown in Fig. Q2. The positive sequence impedances of the lines 1-2 and 2-3 are Z_{12} = 3 + j 40 Ω and Z_{23} = 7 + j 30 Ω respectively. The maximum peak load supplied by the line 1-2 is 110 MVA with a lagging power factor of 0.8. Assume a L-L fault of I_f=500 A occurs midway of the line 1-2 and line spacing of 3.5 m is equal to arc length. Design a distance protection system using Mho relays by determining the following:

a. Maximum load current
b. Suitable CT ratio. Secondary standard 5 A.
c. Suitable VT ratio. Secondary standard 67 V.
d. Line impedance measured by the relay.
e. Load impedance measured by the relay.
f. Zones 1, 2, and 3 setting of relay R12.
g. Value of arc resistance at fault point in Ω.
h. Show graphically, whether or not relay will clear the fault instantaneously.

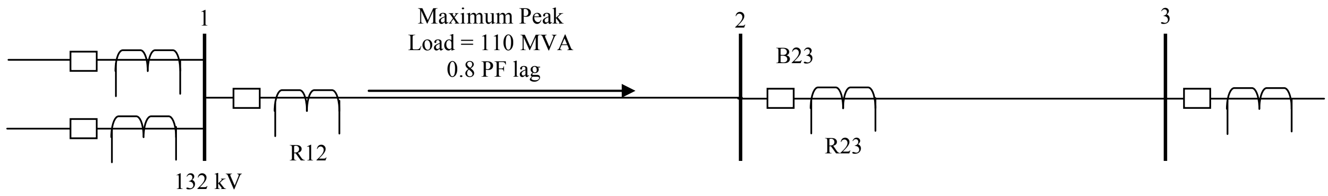


Fig. Q2

Solution:

a. Maximum load current

$$I_{L\max} = \frac{S_L}{\sqrt{3}VLL} = \frac{110\times10^6}{\sqrt{3}132\times10^3} = 481.13\ A$$

b. CT ratio

Choose $CTR = \dfrac{500}{5} = 100:1$

c. VT ratio

$$V_{ph} = \frac{V_{LL}}{\sqrt{3}} = \frac{132\times10^3}{\sqrt{3}} = 76210.2\,V = 76.21\,kV$$

Choose $VTR = \dfrac{76210}{67} = 1137.46:1$

d. Line impedance measured by the relay

$$Z_{line-\sec ondary} = \frac{V_p}{I_p}\times\frac{CTR}{VTR} = Z_{line}\times\frac{CTR}{VTR}$$

$$Z_{line-\sec ondary} = Z_{line}\times\frac{100}{1137.46} = Z_{line}\times0.0879$$

Thus the impedances of the two lines as seen by the relay R12 are approximately

Line 1-2 $\quad Z_{12} = (3+j40)\times0.0879 = 0.26+j3.52\ \Omega$

Line 2-3 $\quad Z_{23} = (7+j30)\times0.0879 = 0.615+j2.64\ \Omega$

e. Load impedance seen by the relay.

The maximum load impedance with 0.9 power factor lagging is

$$Z_{load} = \frac{V_{ph}}{I_{L\max}} \angle\cos^{-1}(0.8) = \frac{76.21\times 10^3}{481.13}\angle 36.9° = \frac{76.21\times 10^3}{481.13}(0.8 + j0.6)$$

$$= 158.4\angle 36.9°\ \Omega = 126.7 + j95.1\ \Omega$$

$$Z'_{load} = Z_{load-primary}\times\frac{CTR}{VTR} = Z_{load-primary}\times 0.0879$$

$$Z'_{load} = 158.4\angle 36.9°\ \Omega\times 0.0879 = 13.9\angle 36.9°\ \Omega$$

$$= 126.7 + j95.1\ \Omega\times\frac{40}{1189.1} = 11.1 + j8.4\ \Omega$$

$$= 13.9\angle 37.1°\ \Omega$$

f. Zones 1, 2, and 3 setting of relay R12.

The zone 1 setting of the relay R12 must **under reach** the line 1-2, so that the setting should be

$$Z_{r1} = 0.8\times Z'_{12} = 0.8\times(0.26 + j3.52)\ \Omega = 0.21 + j2.82\ \Omega = 2.83\angle 85.7°\ \Omega$$

The zone 2 setting should reach past terminal 2 of the line 1-2. Zone 2 is usually set at about 1.2×the length of the line being protected.

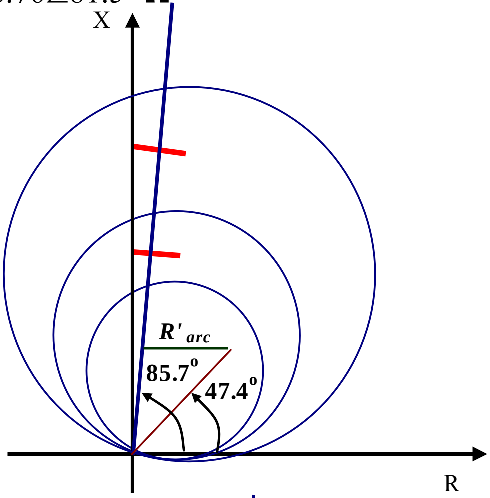
Zone 2 for R12 is therefore set at

$$Z_{r2} = 1.2\times Z'_{12} = 1.2\times(0.26 + j3.52) = 0.31 + j4.22\ \Omega = 4.2\angle 85.8°\ \Omega$$

The zone 3 setting should reach beyond the longest line connected to bus 2. Thus the zone-3 setting must be

$$Z_{r3} = Z'_{12} + 1.2\times Z'_{23}$$

$$= (0.26 + j3.52) + 1.2\times(0.615 + j2.64) = 1.0 + j6.69\ \Omega = 6.76\angle 81.5°\ \Omega$$



g. Value of arc resistance at fault point in Ω.

The empirical fault arc resistance is given by:

$$R_{arc} = \frac{2.9\times 10^4 L}{I^{1.4}}$$

Where

L is the length of arc (m) in still air

I is the fault current in A.

$$R_{arc} = \frac{2.9\times 10^4\times 3.5}{500^{1.4}} = \frac{101500}{6005.6} = 16.9\ \Omega$$

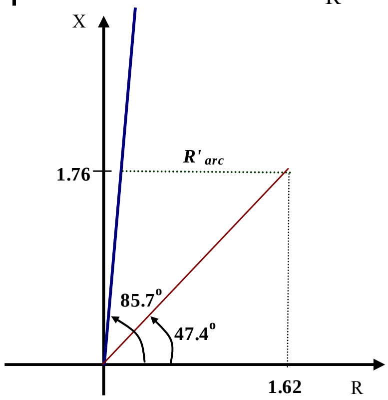
$$R'_{arc} = 16.9\times 0.0879 = 1.486\ \Omega$$

Show graphically, whether or not relay will clear the fault instantaneously.

The total impedance seen by the relay up to the fault point is Z'_f

$$Z'_f = 0.5\times Z'_{12} + R'_{arc} = 0.5\times(0.26 + j3.52) + 1.486$$

$$Z'_f = 0.13 + 1.486 + j1.76 = 1.62 + j1.76\ \Omega = 2.39\angle 47.4°\ \Omega$$

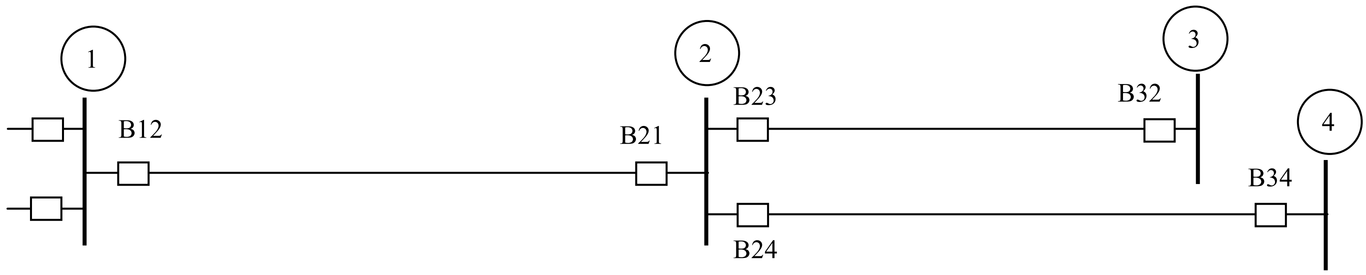


The fault lays lies in the zone 1, so it will be cleare

Question # 3:

Consider the portion of a 138 kV transmission system shown below. Lines 1-2, 2-3 and 2-4 are respectively 64, 64, and 96 km long. The positive sequence impedance of the transmission lines is (0.05 + *j* 0.5) Ω/km. The maximum load carried by line 1-2 under emergency condition is 50 MVA.

Design a 3-zone step distance relaying system to the extent of determining for R12 the zone setting which are the impedance values in terms of CT and VT secondary quantities. The zone settings give points on the R-X plane through which the zone circles of the relay characteristics must pass.



Solution:

The positive sequence impedances of the three lines are:

Line 1-2	Z12 = 3.2 + *j* 32.0 Ω
Line 2-3	Z23 = 3.2 + *j* 32.0 Ω
Line 2-4	Z24 = 4.8 + *j* 48.0 Ω

Since distance relays depend on the ratio of voltage to current (*Z=V/I*), both a CT and VT are needed for each phase. The maximum load current is

$$I_{L\max} = \frac{50\times 10^6}{\sqrt{3}\times 138\times 10^3} = 209.2\ A$$

Then, select a CT ratio of CTR= $CTR = \frac{200}{5} = 40$ which will produce about 5 A in the secondary winding under maximum loading conditions (209.2/200/5=5.23 A).

The system voltage to neutral is:

$$V_{ph} = \frac{138\times 10^3}{\sqrt{3}} = 79.67\ kV$$

The industry standard for VT secondary voltage is 67 V for line-to-neutral voltages. Consequently, select a VT ratio (VTR) of

$$VT = \frac{79.67\times 10^3}{67} = \frac{1189.1}{1}.$$

Denoting primary voltage of VT at bus 1 as V_p and the primary current of the CT as I_p, then the impedance measured by the relay is given by

$$Z_{line-secondary} = \frac{V_p}{I_p}\times\frac{CTR}{VTR} = Z_{line}\times\frac{CTR}{VTR}$$

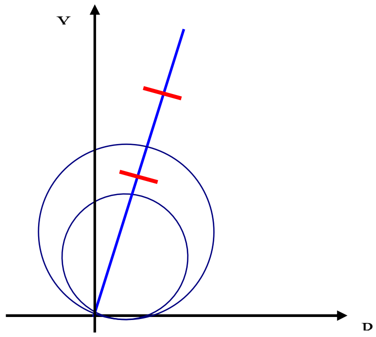
$$Z_{line-secondary} = \frac{V_p/1189.1}{I_p/40} = \frac{V_p}{I_p}\times\frac{40}{1189.1} = Z_{line}\times 0.0336$$

Thus the impedances of the three lines as seen by the relay R12 are approximately

Line 1-2	$Z12 = 0.11 + j1.1\ \Omega$
Line 2-3	$Z23 = 0.11 + j\ 1.1\ \Omega$
Line 2-4	$Z24 = 0.16 + j\ 1.6\ \Omega$

The maximum load impedance assuming a power factor of 0.8 lagging is

$$Z_{load} = \frac{V_{ph}}{I_{L\max}}\angle\cos^{-1}(0.8) = \frac{79.67\times10^3}{209.2}\angle 36.9° = \frac{79.67\times10^3}{209.2}(0.8+j0.6)$$
$$= 380.83\angle 36.9°\ \Omega = 304.6 + j228.5\ \Omega$$



The load impedance seen by the relay is

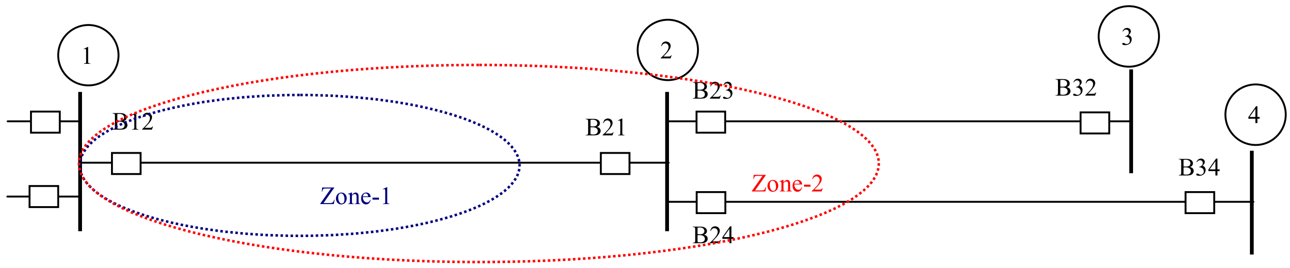
$$Z_{load-secondary} = Z_{load-primary}\times\frac{CTR}{VTR}$$

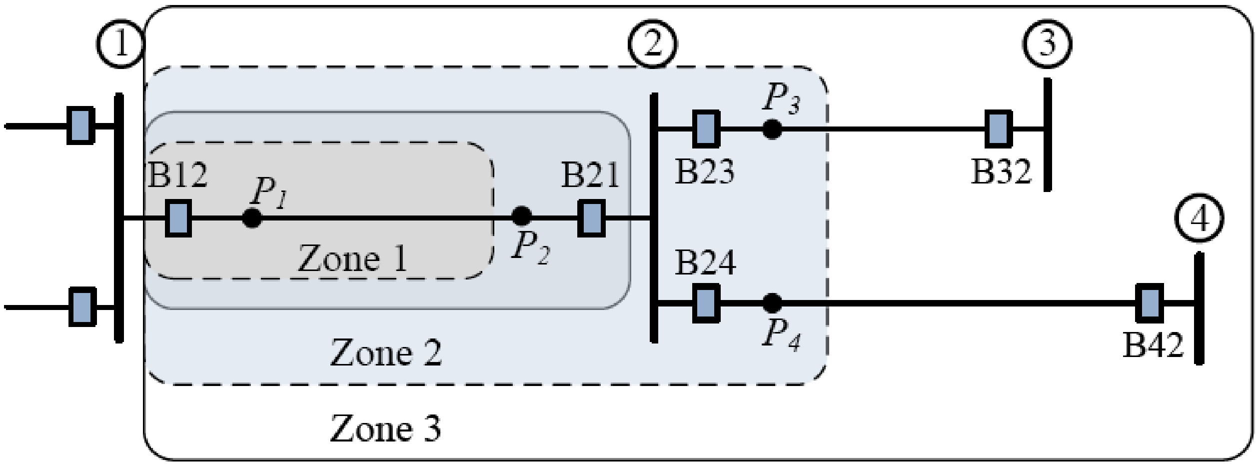
$$Z_{load-secondary} = (304.6+j228.5)\times\frac{40}{1189.1} = 10.2 + j7.7\ \Omega$$

The zone 1 setting of the relay R12 must **under reach** the line 1-2, so that the setting should be
$$Z_{r-setting-zone1} = 0.8\times Z_{line-secondary} = 0.8\times(0.11+j1.1) = 0.088+j0.88\ \Omega$$

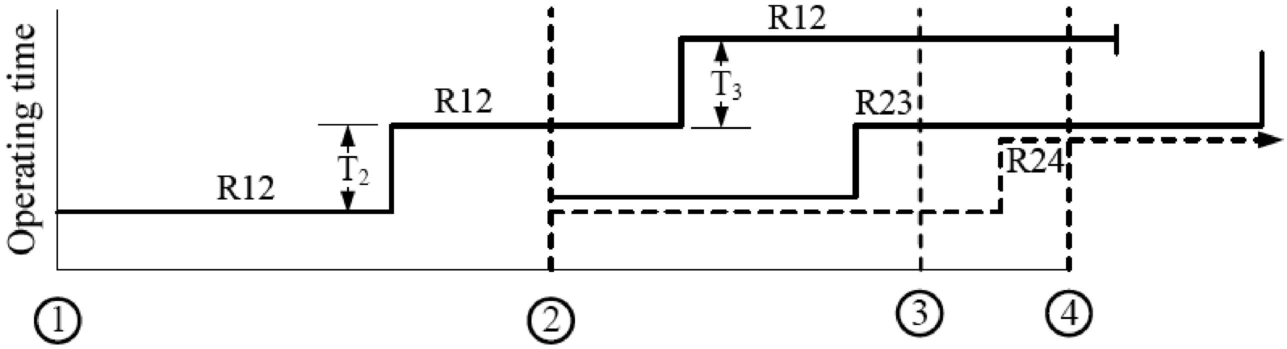
The zone 2 setting should reach past terminal 2 of the line 1-2. Zone 2 is usually set at about 1.2×the length of the line being protected.
Zone 2 for R12 is therefore set at
$$Z_{r-setting-zone2} = 1.2\times Z_{line-secondary} = 1.2\times(0.11+j1.1) = 0.13+j1.32\ \Omega$$

The zone 3 setting should reach beyond the longest line connected to bus 2. Thus the zone-3 setting must be
$$Z_{r-setting-zone3} = Z'_{12} + 1.2\times Z_{longest-line-secondary}$$
$$= (0.11+j1.1)+1.2\times(0.16+j1.6) = 0.302+j3.02\ \Omega$$

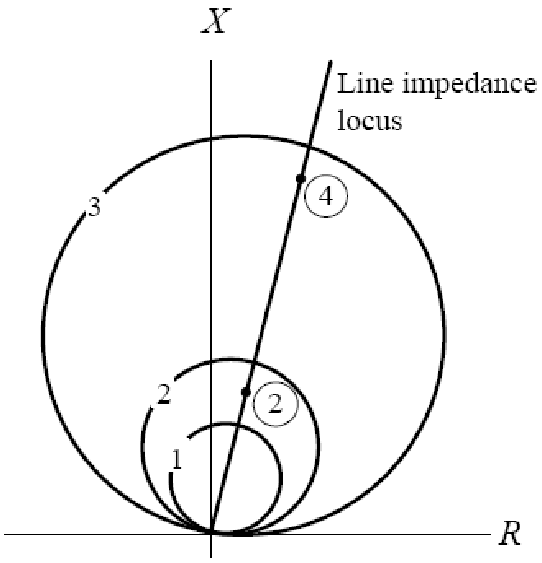




(a) Zone of protection of protection of Distance (or impedance) relays.



(b) Time delay and operating time for R12, R23 and R24



Question # 4

A 345 kV transmission network is protected by distance protection with Mho characteristics as shown in Fig. Q4. The CT and VT ratios of B12 are *CTR*=1500:5 and *VTR*=3000:1, respectively. The line data for the system are given below. The maximum load carried by line 1-2 under emergency condition is 1500 A at 0.95 PF lagging.

Line	Positive Sequence Impedance Zij	Line	Positive Sequence Impedance
1-2	$8 + j50\ \Omega$	2-4	$5.3 + j33\ \Omega$
2-3	$8 + j50\ \Omega$	3-1	$3 + j27\ \Omega$

Design a a 3-zone step distance protection system using Mho relays to the extent of determining for B12 the zone setting which are the impedance values by determining the following:

a.	impedance measured by the relay Z'_{ij} for lines 1-2, 2-3, 2-4 and 3-1	$\mathbf{Z'_{12}} = 0.8 + j5\ \Omega$ $\mathbf{Z'_{23}} = 0.8 + j5\ \Omega$ $\mathbf{Z'_{24}} = 0.53 + j3.3\ \Omega$ $\mathbf{Z'_{31}} = 0.3 + j5\ \Omega$
b.	impedance settings of B12 for Zones 1, 2, and 3, Z_{r1}, Z_{r2} and Z_{r3}	$\mathbf{Z_{r1}} = 0.64 + j4 = 4.05\angle 80.9°\ \Omega$ $\mathbf{Z_{r2}} = 0.96 + j6 = 6.08\angle 80.9°\ \Omega$ $\mathbf{Z_{r3}} = 1.55 + j8.96 = 9.07\angle 80.9°\ \Omega$
c.	the load equivalent impedance in Ohm	$\mathbf{Z_L} = 132.8\angle 18.2°\ \Omega$
d.	the load equivalent impedance as seen by the distance relay	$\mathbf{Z_{L\text{-}relay}} = 13.28\angle 18.2°\ \Omega$
e.	Will any of the relays trip during this condition?	**Yes** / **NO**

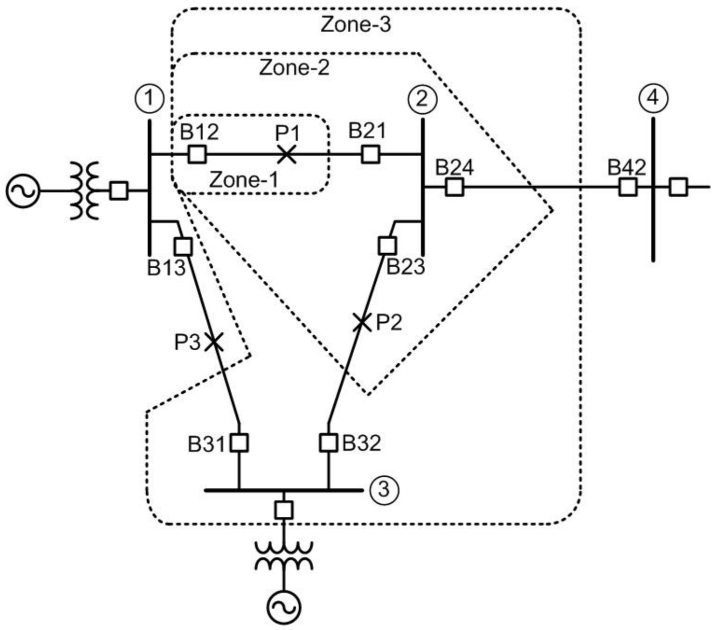


Fig.Q4a. Protection of a loop system using distance protection.

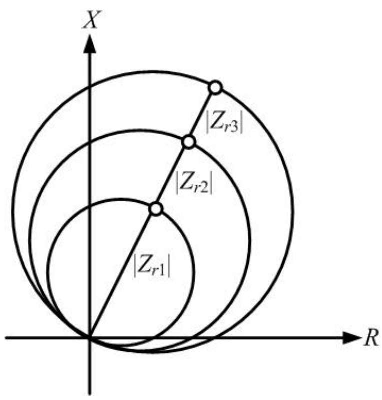


Fig.Q4b. Mho relay characteristics for the three zones

Solution:

Using the CT ratio of 1500:5 and PT ratio of 3000:1 at B12, the impedance seen by B12 is:

$$Z = \frac{V_{1(L-N)}}{I_{12}}\ \Omega$$

Using the CT and PT ratios mentioned above we have

$$Z' = \frac{V_{1(L-N)} \Big/ \left(\dfrac{3000}{1}\right)}{I_{12} \Big/ \left(\dfrac{1500}{5}\right)} = \frac{Z}{10}\ \Omega$$

Now we set Zone-1 of B12 relay for 80% reach, i.e., 80% of line 1-2 (secondary) impedance. Therefore

$$Z_{r1} = 0.80 \times \frac{8 + j50}{10} = 0.64 + j4 = 4.05\angle 80.9^\circ\ \Omega$$

The setting for Zone-2 for B12 relay, with a reach of 120%, is

$$Z_{r2} = 1.2 \times \frac{8 + j50}{10} = 0.96 + j6 = 6.08\angle 80.9^\circ\ \Omega$$

From Table Q2, we see that line 2-4 has a larger impedance than line 2-3. Therefore we set B12 for Zone-3 as 100% of line 1-2 and 120% of line 2-4. Therefore

$$Z_{r3} = 1 \times \frac{8 + j50}{10} + 1.2 \times \frac{8 + j50}{10} = 1.55 + j8.96 = 9.07\angle 80.9^\circ\ \ \Omega$$

Suppose now the bus voltage at Bus-1 is 345 kV and the maximum current for an emergency loading condition is 1500 A. Then we have

$$Z' = \frac{Z}{10} = \frac{1}{10} \times \frac{345 \times 10^3 \big/ \sqrt{3}}{1500\angle -18.2^\circ} = 13.28\angle 18.2^\circ\ \ \Omega$$

Since this impedance exceeds the Zone-3 trip setting, the impedance during the emergency loading condition is outside the trip settings of any of the zones. Therefore none of the relays will trip. Moreover, the impedance during normal loading condition will be even less and hence it will be further away from the trip regions.

EE482-Power System Analysis II

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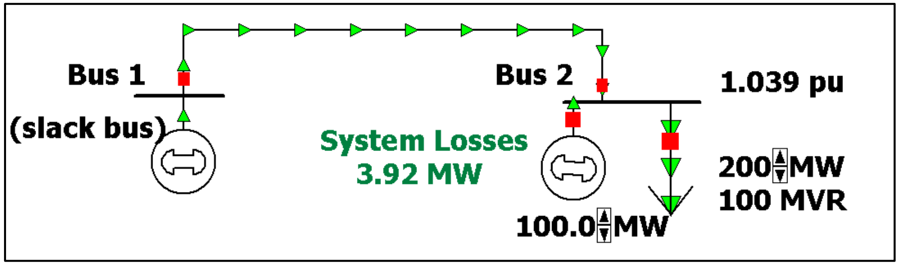
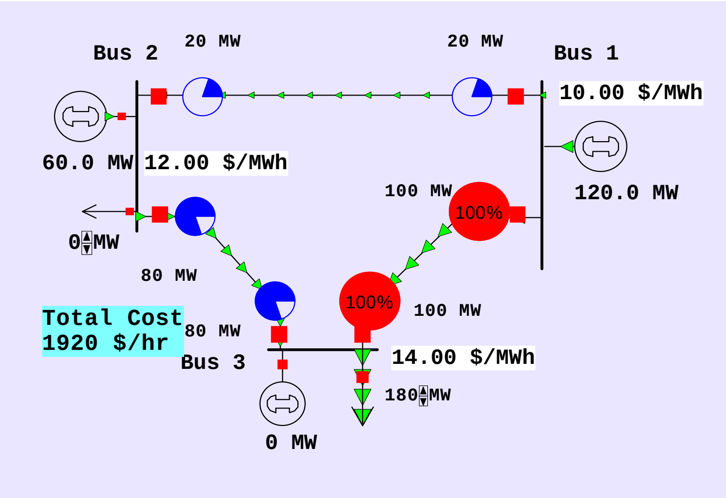


Topic 2: Economic Dispatch

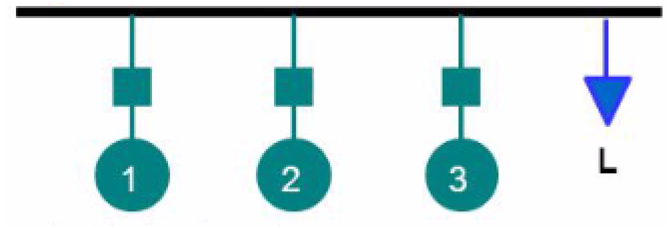
Power System Control Center

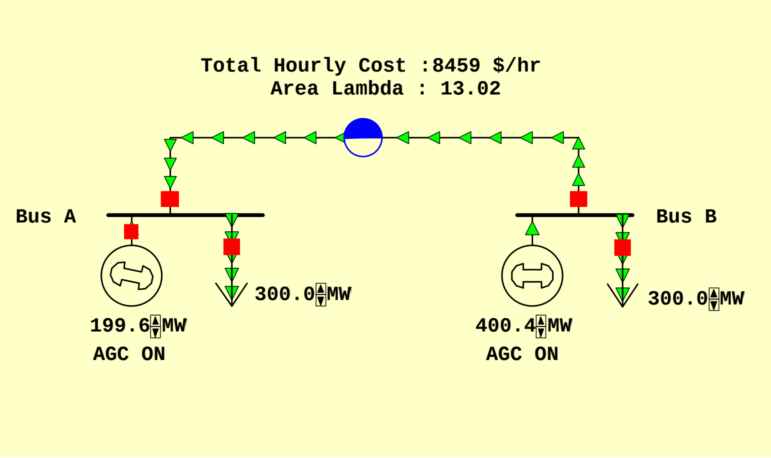


Topic 2: Economic Dispatch

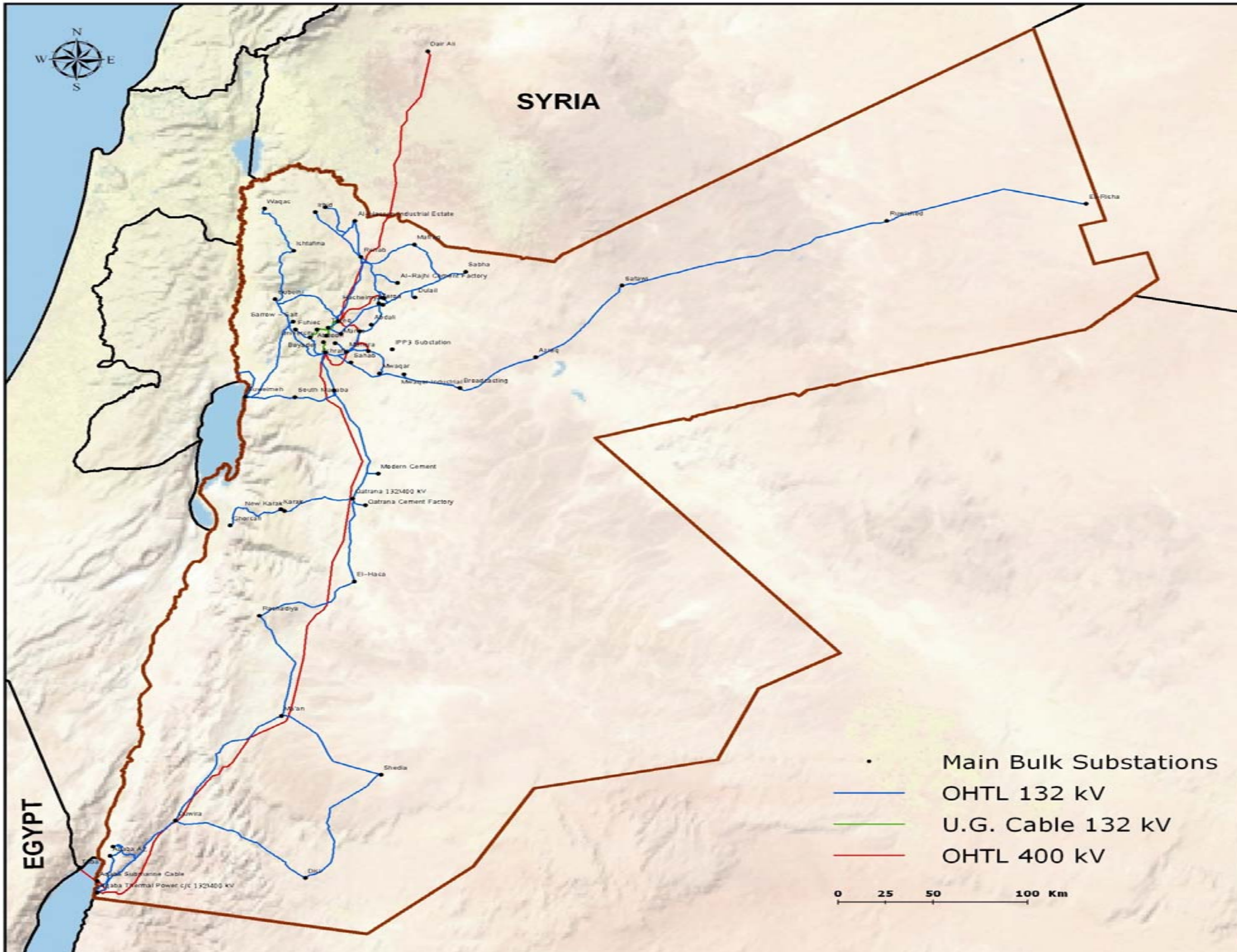
- **Economic Dispatch:** is the determination of the optimal output of a number of electricity generation facilities, to meet the system load, at the lowest possible cost, subject to transmission and operational constraints.

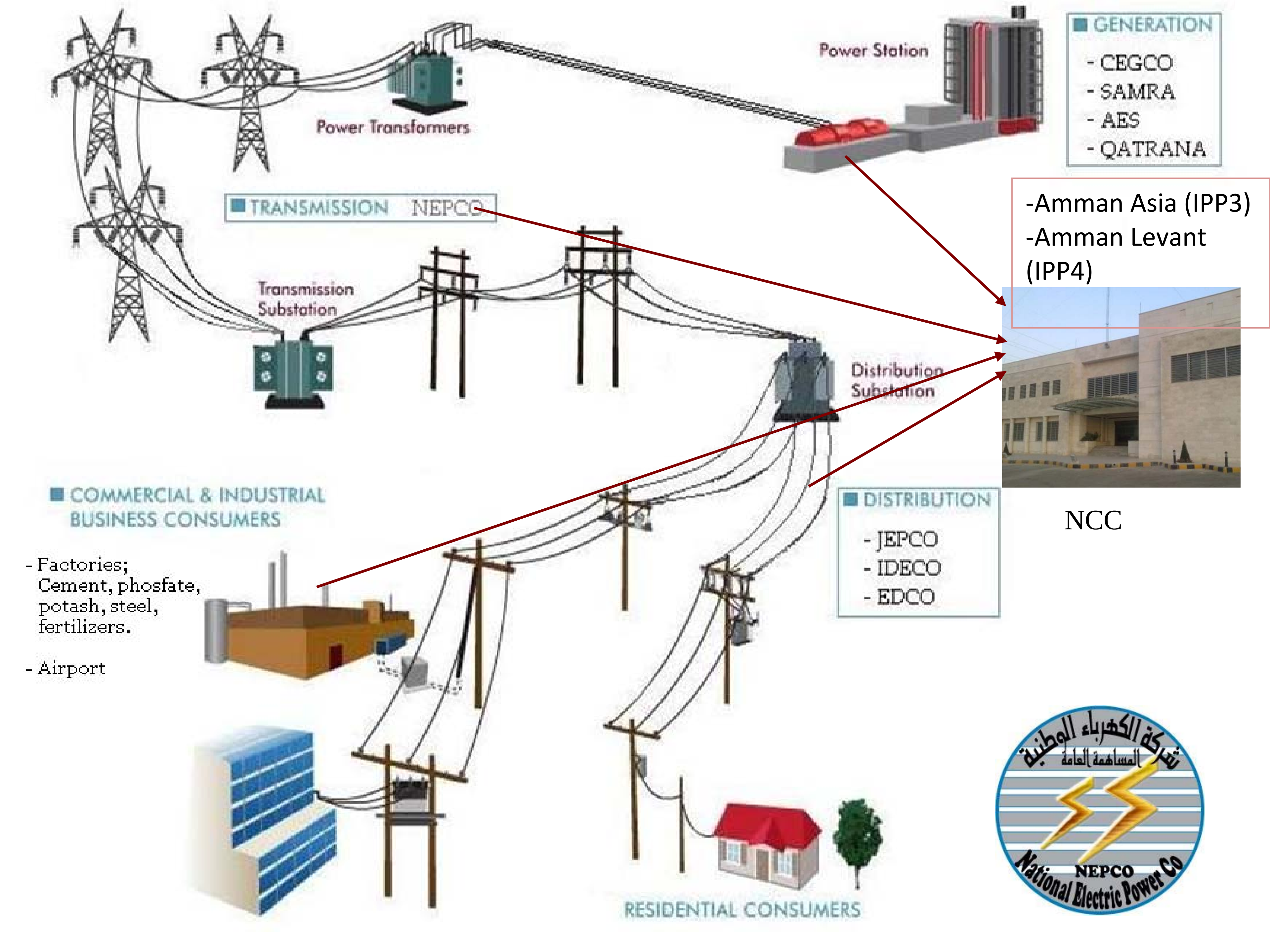
- Economic dispatch is used to determine the least cost means of using existing generating plants to meet electric demand



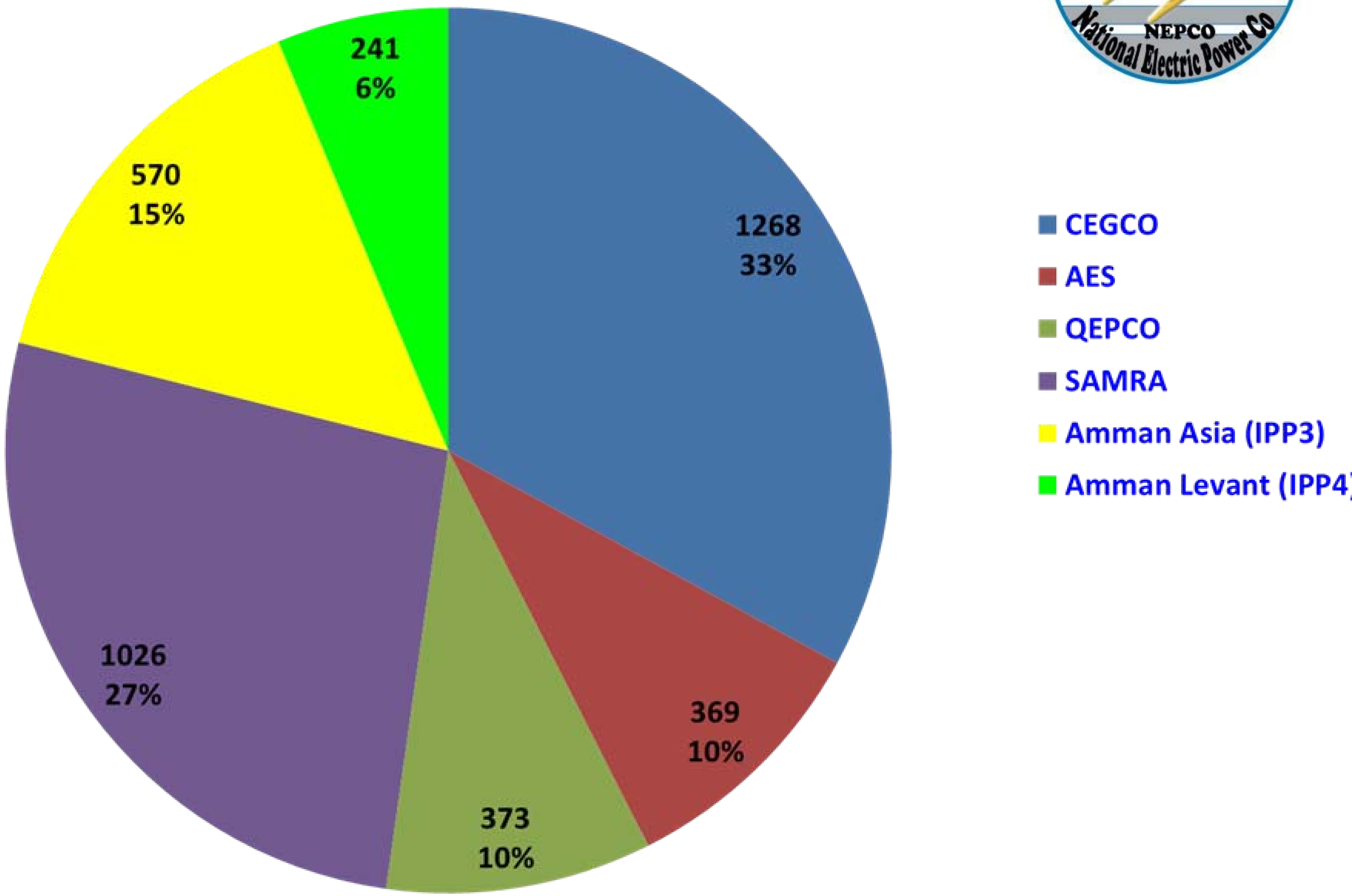


National Transmission Grid

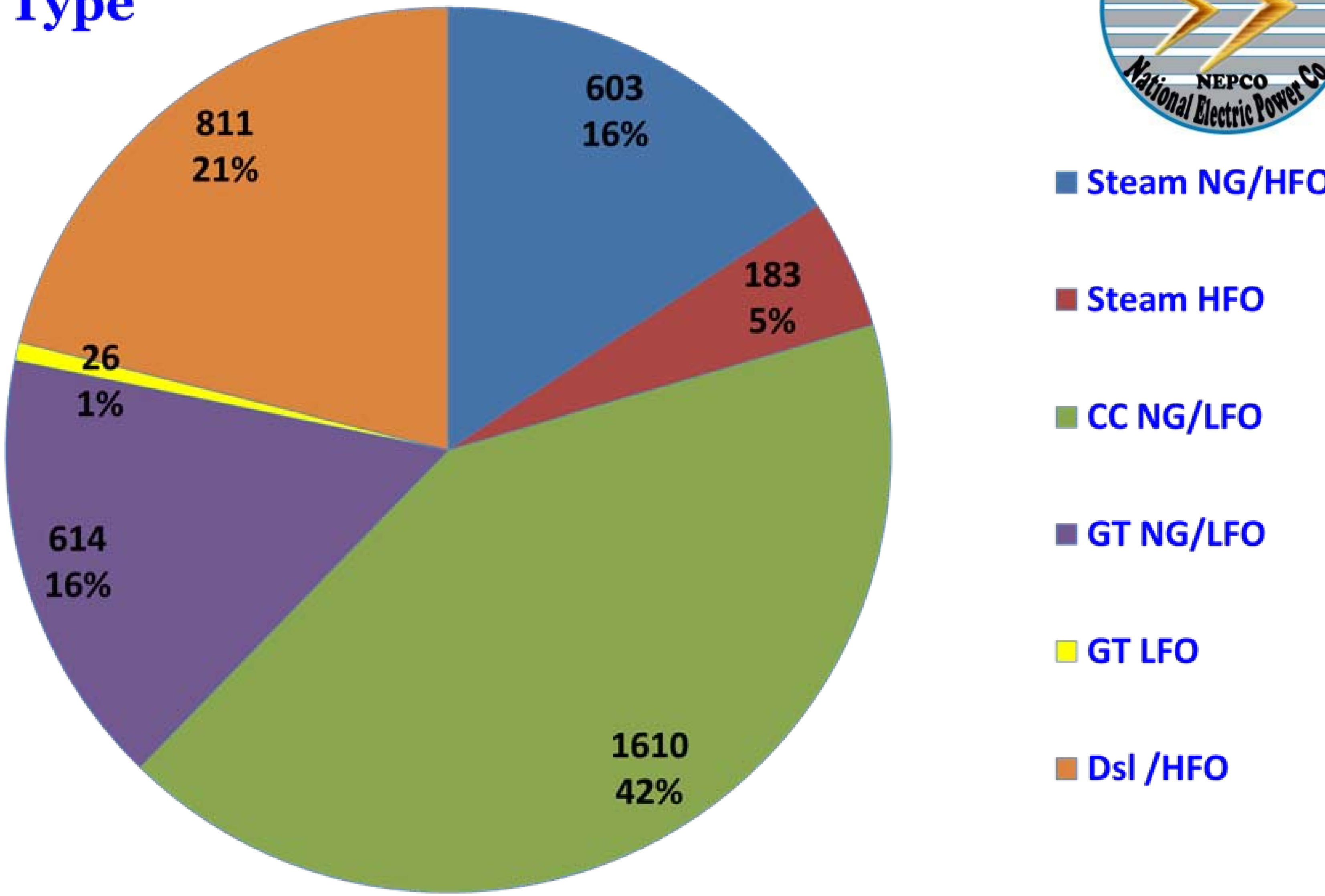


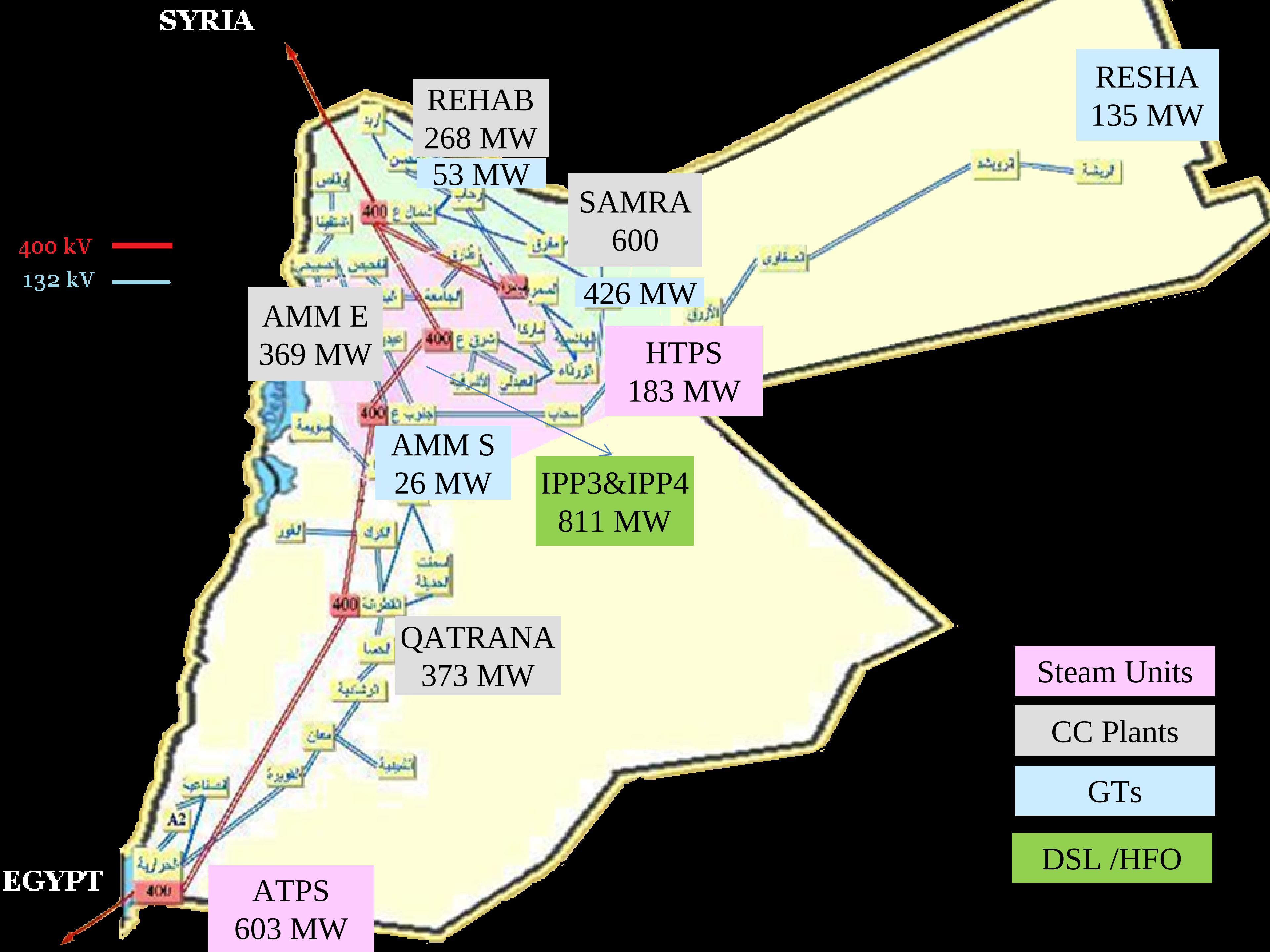

GENERATION
- CEGCO
- SAMRA
- AES
- QATRANA
Power Station
Power Transformers
-Amman Asia (IPP3)
-Amman Levant (IPP4)
TRANSMISSION NEPCO
Transmission Substation
Distribution Substation
NCC
COMMERCIAL & INDUSTRIAL BUSINESS CONSUMERS
- Factories; Cement, phosfate, potash, steel, fertilizers.
- Airport
DISTRIBUTION
- JEPCO
- IDECO
- EDCO
RESIDENTIAL CONSUMERS
شركة الكهرباء الوطنية
المساهمة العامة
NEPCO
National Electric Power Co

Generation
By Company

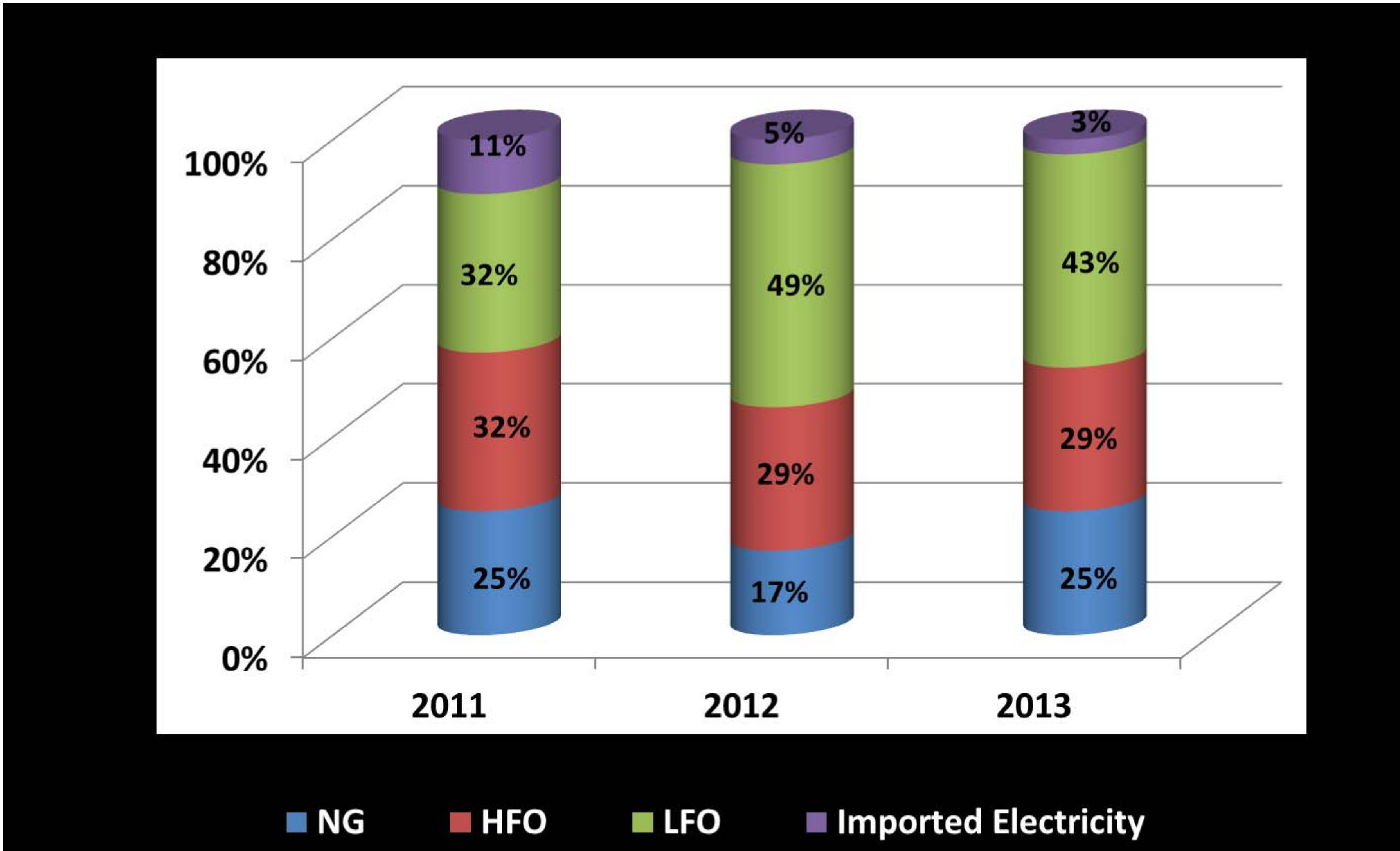


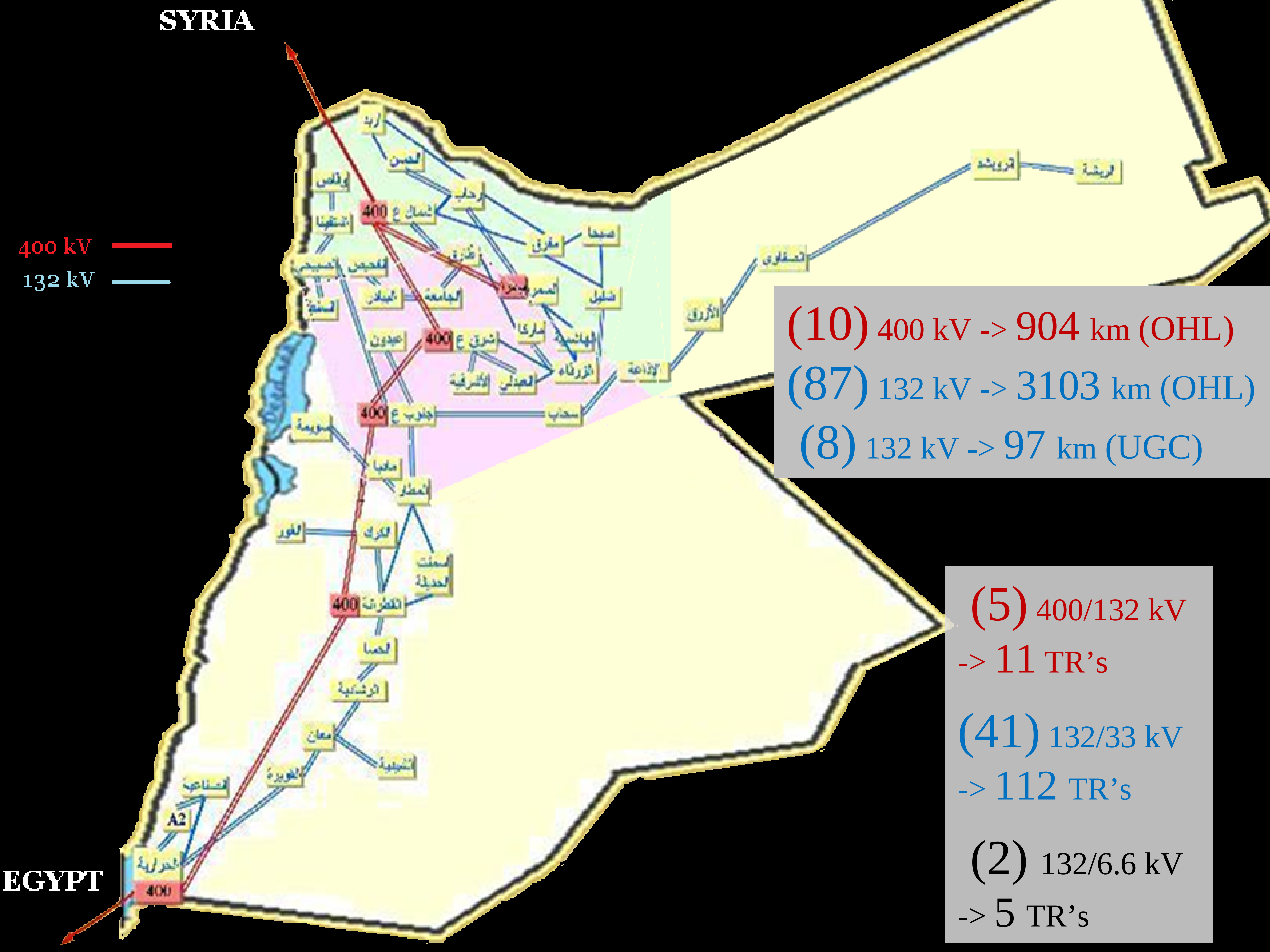
Generation
By Type




SYRIA
EGYPT
400 kV
132 kV
REHAB
268 MW
53 MW
SAMRA
600
426 MW
RESHA
135 MW
AMM E
369 MW
HTPS
183 MW
AMM S
26 MW
IPP3&IPP4
811 MW
QATRANA
373 MW
ATPS
603 MW
Steam Units
CC Plants
GTs
DSL /HFO

Energy generated by fuel type:





(10) 400 kV -> 904 km (OHL)
(87) 132 kV -> 3103 km (OHL)
(8) 132 kV -> 97 km (UGC)

(5) 400/132 kV
-> 11 TR's

(41) 132/33 kV
-> 112 TR's

(2) 132/6.6 kV
-> 5 TR's

Economic Dispatch

- Operators load the available generating units to maintain the equation:

  Generation = Load + Losses

- Based on “Daily Dispatch Schedule” which is an hourly based schedule prepared ahead of time, showing the forecasted system loads and suggested generation loading to meet such loads.

Economic Dispatch

- **In practice and in power flow analysis, there are many choices for setting the operating points of generators**
  - in the power flow analysis, generator buses are specified by P and |V|
  - generation capacity is more than load demand - generators can produce more than the customers can consume
    - there are many solution combinations for scheduling generation
  - in practice, power plants are not located at the same distance from the load centers
  - power plants use different types of fuel, which vary in cost from time to time
- **For interconnected systems, the objective is to find the real and reactive power scheduling so as to minimize some operating cost or cost function**

Optimization

- **General cost function:** $f\left(x_1, x_2, \cdots, x_n\right) = C$
- **Unconstrained parameter optimization, from calculus:**
  - the first derivative of *f* vanishes at a local extrema

$$\frac{d}{dx} f\left(x\right) = 0$$

  - for *f* to be a local minimum, the second derivative must be positive at the point of the local extrema

$$\frac{d^2}{dx^2} f\left(x\right) > 0$$

  - for a set of parameters, the gradient of f vanishes at a local extrema and to be a local minimum, the Hessian must be a positive definite matrix (i.e. positive eigenvalues)

$$\frac{\partial f}{\partial x_i} = 0 \quad i = 1, \cdots, n \quad \text{or} \quad \nabla f = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \cdots, \frac{\partial f}{\partial x_n} \right) = 0$$

Example

- **Find the minimum of**

$$f(x_1, x_2, x_3) = x_1^{\ 2} + 2x_2^{\ 2} + 3x_3^{\ 2} + x_1 x_2 + x_2 x_3 - 8x_1 - 16x_2 - 32x_3 + 110$$

  - evaluating the first derivatives to zero results in

$$\begin{aligned} \frac{\partial f}{\partial x_1} &= 2x_1 + x_2 - 8 = 0 \\ \frac{\partial f}{\partial x_2} &= x_1 + 4x_2 + x_3 - 16 = 0 \quad \textit{or} \quad \begin{bmatrix} 2 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 6 \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \hat{x}_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 16 \\ 32 \end{bmatrix} \qquad \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \hat{x}_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix} \\ \frac{\partial f}{\partial x_3} &= x_2 + 6x_3 - 32 = 0 \end{aligned}$$

Equality Constraints in Optimization

- **This type of problem arises when there are functional dependencies among the parameters to be found**
- **The problem**
  - minimize the cost function

    $$f\left(\hat{x}_1 \cdots \hat{x}_j \cdots \hat{x}_n\right)$$

  - subject to the equality constraints

    $$g_i\left(\hat{x}_1 \cdots \hat{x}_j \cdots \hat{x}_n\right) = 0 \quad i = 1, \cdots, k$$

- **Such problems may be solved by the *Lagrange muliplier* method**

Equality Constraints in Optimization

- **Lagrange Multiplier method**
  - introduce k-dimensional vector λ for the undetermined quantities

$$\mathrm{L} = f + \sum_{i=1}^{k} \lambda_i \, g_i$$

New cost function

  - The necessary conditions for finding the local minimum

$$\frac{\partial \mathrm{L}}{\partial x_i} = \frac{\partial f}{\partial x_i} + \sum_{i=1}^{k} \lambda_i \, \frac{g_i}{\partial x_i} = 0$$

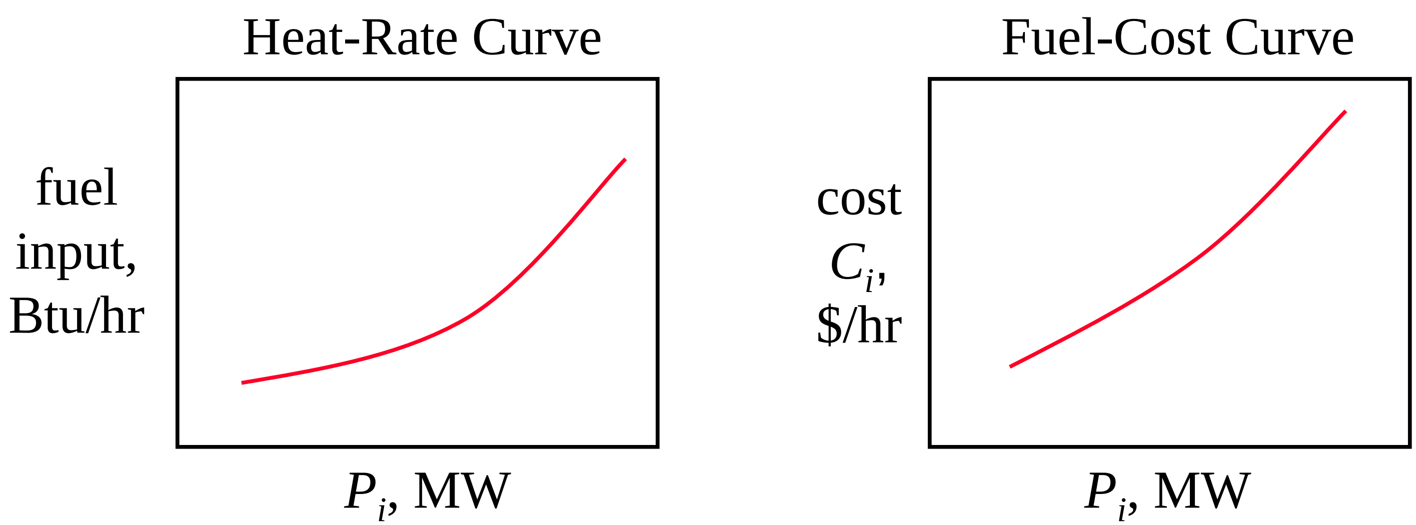
$$\frac{\partial \mathrm{L}}{\partial \lambda_i} = g_i = 0$$

Operating Costs

- **Factors influencing the minimum cost of power generation**
  - operating efficiency of prime mover and generator
  - fuel costs
  - transmission losses
- **The most efficient generator in the system does not guarantee minimum costs**
  - may be located in an area with high fuel costs
  - may be located far from the load centers and transmission losses are high
- **The problem is to determine generation at different plants to minimize the total operating costs**

Operating Costs

- **Generator heat rate curves lead to the fuel cost curves**



  - The fuel cost is commonly express as a quadratic function

$$C_i = \alpha_i + \beta_i P_i + \gamma_i P_i^2$$

  - The derivative is known as the incremental fuel cost

$$dC_i / dP_i = \beta_i + 2\gamma_i P_i$$

Economic Dispatch

- **The simplest problem is when system losses and generator limits are neglected**
  - minimize the objective or cost function over all plants
  - a quadratic cost function is used for each plant

$$C_{total} = \sum_{i=1}^{n_{gen}} C_i = \sum_{i=1}^{n_{gen}} \alpha_i + \beta_i P_i + \gamma_i P_i^2$$

  - the total demand is equal to the sum of the generators' output; the equality constrant

$$\sum_{i=1}^{n_{gen}} P_i = P_{Demand}$$

Economic Dispatch

- **A typical approach using the Lagrange multipliers**

$$L = C_{total} + \lambda\left(P_{Demand} - \sum_{i=1}^{n_{gen}} P_i\right)$$

$$\frac{\partial L}{\partial P_i} = \frac{\partial C_{total}}{\partial P_i} + \lambda(0-1) = 0 \quad \rightarrow \quad \frac{\partial C_{total}}{\partial P_i} = \lambda$$

$$C_{total} = \sum_{i=1}^{n_{gen}} C_i \quad \rightarrow \quad \frac{\partial C_{total}}{\partial P_i} = \frac{dC_i}{dP_i} = \lambda \quad \forall i = 1,\ldots,n_g$$

$$\lambda = \frac{dC_i}{dP_i} = \beta_i + 2\gamma_i P_i$$

Economic Dispatch

- the second condition for optimal dispatch

$$\frac{dL}{d\lambda} = \left(P_{Demand} - \sum_{i=1}^{n_{gen}} P_i \right) = 0 \quad \rightarrow \quad \sum_{i=1}^{n_{gen}} P_i = P_{Demand}$$

- rearranging and combining the equations to solve for λ

$$P_i = \frac{\lambda - \beta_i}{2\gamma_i}$$

$$\sum_{i=1}^{n_{gen}} \frac{\lambda - \beta_i}{2\gamma_i} = P_{Demand} \qquad \lambda = \frac{P_{Demand} + \sum_{i=1}^{n_{gen}} \frac{\beta_i}{2\gamma_i}}{\sum_{i=1}^{n_{gen}} \frac{1}{2\gamma_i}}$$

Example

- **Neglecting system losses and generator limits, find the optimal dispatch and the total cost in $/hr for the three generators and the given load demand**

$$C_1 = 500 + 5.3P_1 + 0.004P_1^2 \; [\$ / MWhr]$$

$$C_2 = 400 + 5.5P_2 + 0.006P_2^2$$

$$C_3 = 200 + 5.8P_3 + 0.009P_3^2$$

$$P_{Demand} = 800MW$$

Example

$$\lambda = \frac{P_{Demand} + \sum_{i=1}^{n_{gen}} \frac{\beta_i}{2\gamma_i}}{\sum_{i=1}^{n_{gen}} \frac{1}{2\gamma_i}} = \frac{800 + \frac{5.3}{0.008} + \frac{5.5}{0.012} + \frac{5.8}{0.018}}{\frac{1}{0.008} + \frac{1}{0.012} + \frac{1}{0.018}} = \$8.5 / MWhr$$

$$P_1 = \frac{8.5 - 5.3}{2(0.004)} = 400 MW$$

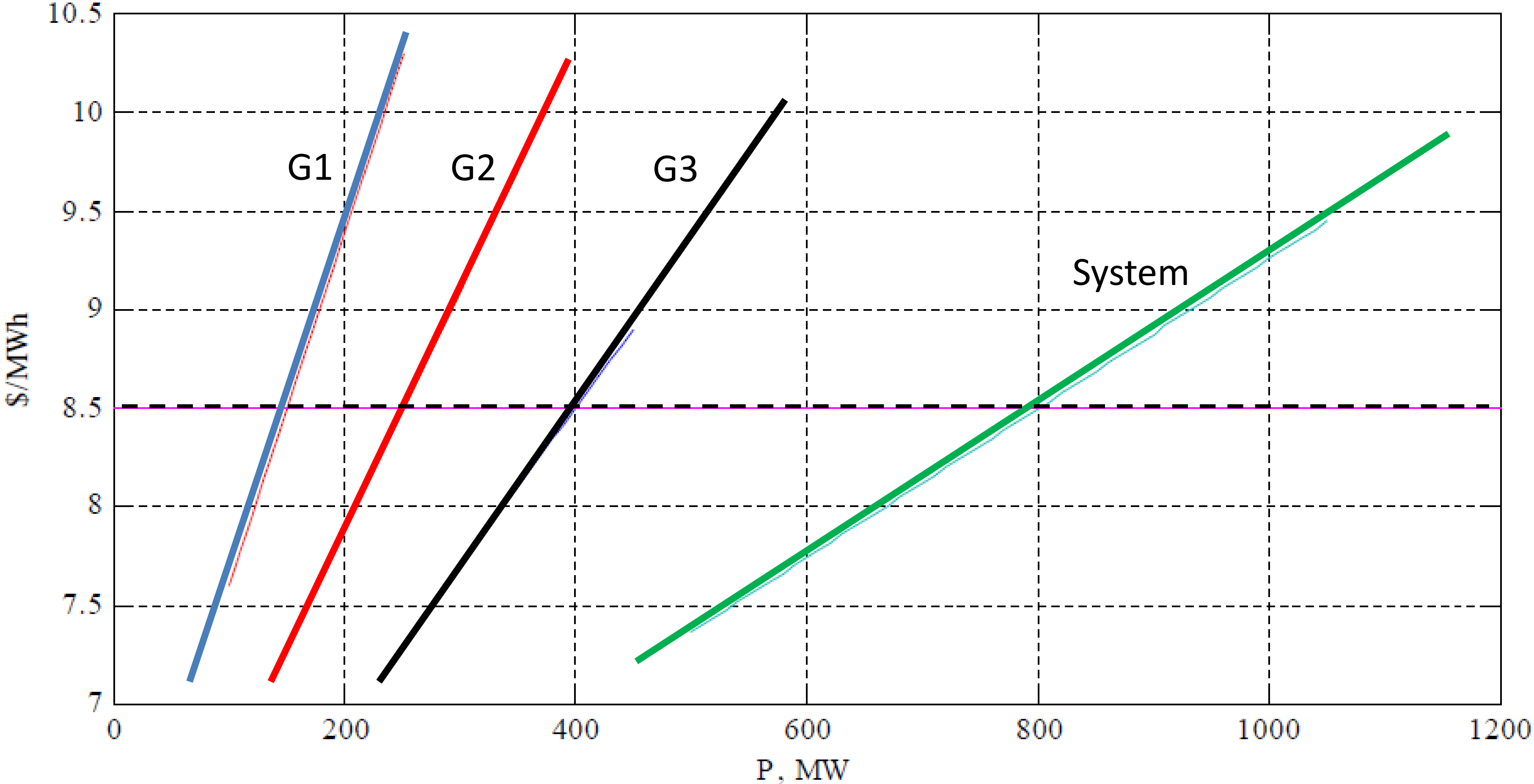
$$P_i = \frac{\lambda - \beta_i}{2\gamma_i} \qquad P_2 = \frac{8.5 - 5.5_i}{2(0.006)} = 250 MW$$

$$P_3 = \frac{8.5 - 5.8}{2(0.009)} = 150 MW$$

$$P_{Demand} = 800 = 400 + 250 + 150$$

Example

Incremental cost curves



Discussion

- Key results for Economic Dispatch?
  - Incremental cost of all generating units is equal
  - This incremental cost is the Lagrangean multiplier, λ
  - 'λ' is called the 'System λ' and is the system-wide cost of generating electricity
    - This is the price charged to customers

Economic Dispatch with Generator Limits

- **The power output of any generator should not exceed its rating nor be below the value for stable boiler operation**
  - Generators have a minimum and maximum real power output limits
- **The problem is to find the real power generation for each plant such that cost are minimized, subject to:**
  - Meeting load demand - equality constraints
  - Constrained by the generator limits - inequality constraints
- **The Kuhn-Tucker conditions**

$$dC_i / dP_i = \lambda \quad \leftarrow \quad P_{i(\min)} < P_i < P_{i(\max)}$$

$$dC_i / dP_i \leq \lambda \quad \leftarrow \quad P_i = P_{i(\max)}$$

$$dC_i / dP_i \geq \lambda \quad \leftarrow \quad P_i = P_{i(\min)}$$

Example

- **Neglecting system losses, find the optimal dispatch and the total cost in \$/hr for the three generators and the given load demand and generation limits**

$$C_1 = 500 + 5.3P_1 + 0.004P_1^2 \quad [\$/\mathrm{MWhr}]$$

$$C_2 = 400 + 5.5P_2 + 0.006P_2^2$$

$$C_3 = 200 + 5.8P_3 + 0.009P_3^2$$

$$200 \le P_1 \le 450$$

$$150 \le P_2 \le 350$$

$$100 \le P_3 \le 225$$

$$P_{Demand} = 975\ \mathrm{MW}$$

Example

$$\lambda = \frac{P_{Demand} + \sum_{i=1}^{n_{gen}} \frac{\beta_i}{2\gamma_i}}{\sum_{i=1}^{n_{gen}} \frac{1}{2\gamma_i}} = \frac{975 + \frac{5.3}{0.008} + \frac{5.5}{0.012} + \frac{5.8}{0.018}}{\frac{1}{0.008} + \frac{1}{0.012} + \frac{1}{0.018}} = \$9.163/\text{MWh}$$

$$P_i = \frac{\lambda - \beta_i}{2\gamma_i}$$

$$P_1 = \frac{9.16 - 5.3}{2(0.004)} = 483 \text{ MW}$$

$$P_2 = \frac{9.16 - 5.5_i}{2(0.006)} = 305 \text{ MW}$$

$$P_3 = \frac{9.16 - 5.8}{2(0.009)} = 187 \text{ MW}$$

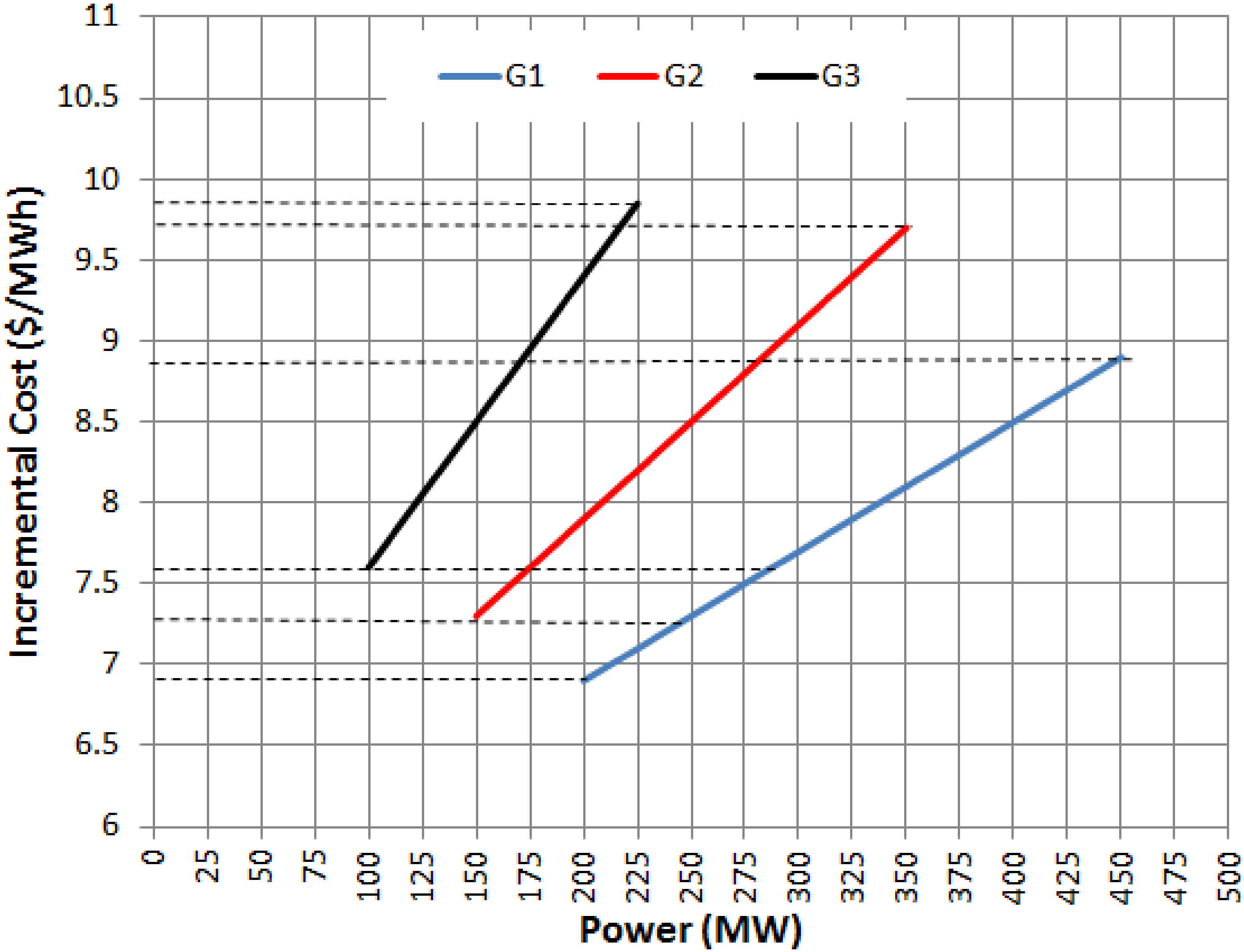
$$P_{Demand} = 975 = 450 + 315 + 210$$

Upper limit violated:
→ P1= 450 MW

→ solve the dispatch problem with two generators:
P2 + P3 = 525 MW

→ λ = \$9.4/MWh
→ P2 = 315 MW
→ P3 = 210 MW

Example



G1
G2
G3
Incremental Cost ($/MWh)
11
10.5
10
9.5
9
8.5
8
7.5
7
6.5
6
0
25
50
75
100
125
150
175
200
225
250
275
300
325
350
375
400
425
450
475
500
Power (MW)

Economic Dispatch including Losses

- **For large interconnected system where power is transmitted over long distances with low load density areas**
  - transmission line losses are a major factor
  - losses affect the optimum dispatch of generation
- **One common practice for including the effect of transmission losses is to express the total transmission loss as a quadratic function of the generator power outputs**
  - simplest form: $$P_L = \sum_{i=1}^{n_{gen}} \sum_{j=1}^{n_{gen}} P_i B_{ij} P_j$$
  - Kron's loss formula: $$P_L = \sum_{i=1}^{n_{gen}} \sum_{j=1}^{n_{gen}} P_i B_{ij} P_j + \sum_{j=1}^{n_{gen}} B_{0j} P_j + B_{00}$$

Economic Dispatch including Losses

- **B_{ij} are called the loss coefficients**
  - they are assumed to be constant
  - reasonable accuracy is expected when actual operating conditions are close to the base case conditions used to compute the coefficients
- **The economic dispatch problem is to minimize the overall generation cost, C, which is a function of plant output**
- **Constraints:**
  - the generation equals the total load demand plus transmission losses
  - each plant output is within the upper and lower generation limits - inequality constraints

Economic Dispatch including Losses

$$f: \quad C_{total} = \sum_{i=1}^{n_{gen}} C_i = \sum_{i=1}^{n_{gen}} \alpha_i + \beta_i P_i + \gamma_i P_i^2$$

$$g: \quad \sum_{i=1}^{n_{gen}} P_i = P_{demand} + P_{losses}$$

$$u: \quad P_{i(\min)} \le P_i \le P_{i(\max)} \quad i = 1, \cdots, n_{gen}$$

The resulting optimization equation

$$L = C_{total} + \lambda \left(P_{demand} + P_{losses} - \sum_{i=1}^{n_{gen}} P_i \right) + \sum_{i=1}^{n_{gen}} \mu_{i(\max)} \left(P_{i(\max)} - P_i \right)$$

$$+ \sum_{i=1}^{n_{gen}} \mu_{i(\min)} \left(P_i - P_{i(\min)} \right)$$

$$P_i < P_{i(\max)}: \quad \mu_{i(\max)} = 0 \qquad\qquad P_i > P_{i(\min)}: \quad \mu_{i(\min)} = 0$$

Economic Dispatch including Losses

- **When generator limits are not violated:**

$$\frac{\partial L}{\partial P_i} = 0 = \frac{\partial C_{total}}{\partial P_i} + \lambda\left(0 + \frac{\partial P_L}{\partial P_i} - 1\right)$$

$$\frac{\partial C_{total}}{\partial P_i} = \frac{\partial}{\partial P_i}\left(C_1 + C_2 + \cdots + C_{n_{gen}}\right) = \frac{dC_i}{dP_i}$$

$$\therefore \quad \lambda = \frac{dC_i}{dP_i} + \lambda \frac{\partial P_L}{\partial P_i} = \left(\frac{1}{1 - \partial P_L / \partial P_i}\right)\frac{dC_i}{dP_i} = L_i \frac{dC_i}{dP_i}$$

$$\frac{\partial L}{\partial \lambda} = 0 = P_D + P_L - \sum_{i=1}^{n_{gen}} P_i \qquad \therefore \sum_{i=1}^{n_{gen}} P_i = P_D + P_L$$

The Penalty Factor

- **The incremental transmission loss equation becomes the penalty factor**

$$L_i = \frac{1}{1 - \dfrac{\partial P_L}{\partial P_i}}$$

  - The effect of transmission losses introduces a penalty factor that depends on the location of the plant
  - The minimum cost is obtained when the incremental cost of each plant multiplied by its penalty factor is the same for all plants

Example

- **Find the optimal dispatch and the total cost in $/hr**
  - fuel costs and plant output limits

$$C_1 = 200 + 7.0P_1 + 0.008P_1^2 \ [\$/\mathrm{hr}] \quad 10 \le P_1 \le 85 \ \mathrm{MW}$$

$$C_2 = 180 + 6.3P_2 + 0.009P_2^2 \quad 10 \le P_2 \le 80$$

$$C_3 = 140 + 6.8P_3 + 0.007P_3^2 \quad 10 \le P_3 \le 70$$

  - real power loss and total load demand

$$P_{loss} = 0.000218\, P_1^2 + 0.000228\, P_2^2 + 0.000179\, P_3^2$$

$$P_{Demand} = 150 \ \mathrm{MW}$$

Example

$$\left(\frac{1}{1-.000436P_1}\right)(7+.016P_1) = \left(\frac{1}{1-.000456P_2}\right)(6.3+.018P_2)$$

$$= \left(\frac{1}{1-.000358P_3}\right)(6.8+.014P_3),$$

$$P_1 + P_2 + P_3 - 150 = P_{loss}$$

Results (obtained numerically):

- P_1 = 35.1 MW
- P_2 = 64.1 MW
- P_3 = 52.5 MW
- P_{loss} = 1.7 MW
- P_{demand} = 150 MW

EE482-Power System Analysis II

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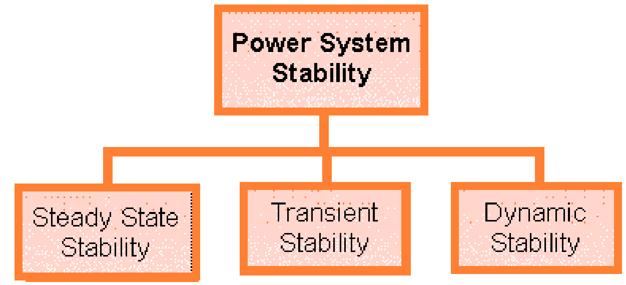
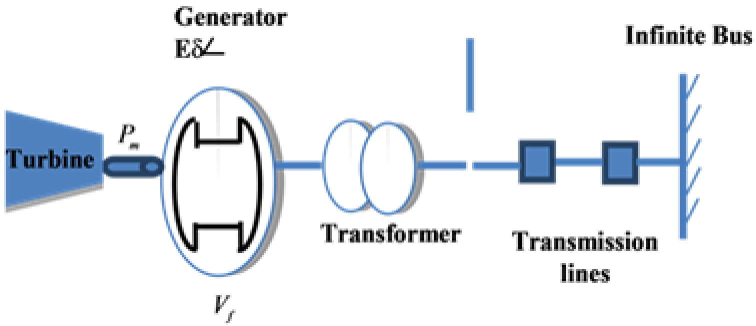
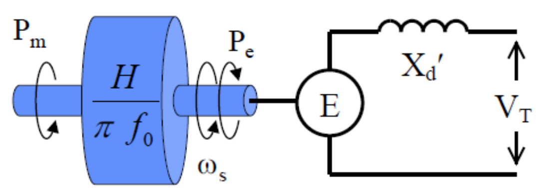
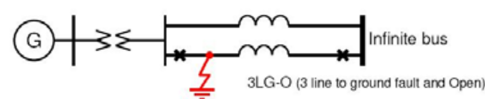
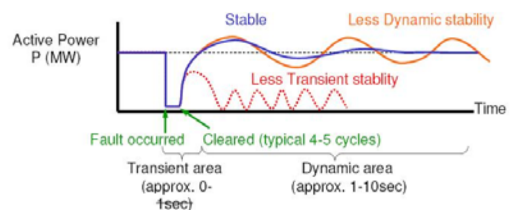


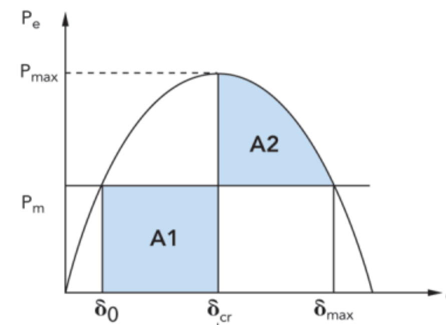
Topic 3 : Power System Stability









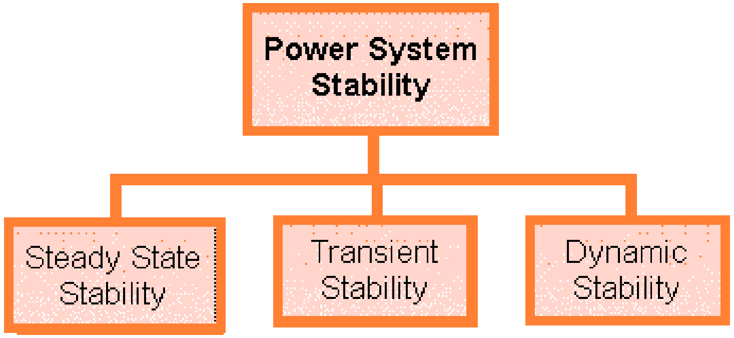


Definition of Stability

- **Stability:** The ability of the power system to remain in synchronism and maintain the state of equilibrium following a disturbing force

- The synchronous stability of a power system can be of several types depending upon the nature of disturbance, and for the purpose of successful analysis it can be classified into the following 3 types as shown below:

1. Steady state stability
2. Transient stability.
3. Dynamic stability.



Introduction

- Increase in load is a kind of disturbance. If increase in loading takes place gradually and in small steps and the system withstands this change and performs satisfactorily, then the system is said to be in **STADY STATE STABILITY**. Thus the study of steady state stability is basically concerned with the determination of upper limit of machine's loading before losing synchronism, provided the loading is increased gradually at a slow rate.
- In practice, load change may not be gradual. Further, there may be sudden disturbances due to

i. Sudden change of load

ii. Switching operation

iii. Loss of generation iv) Fault

Introduction

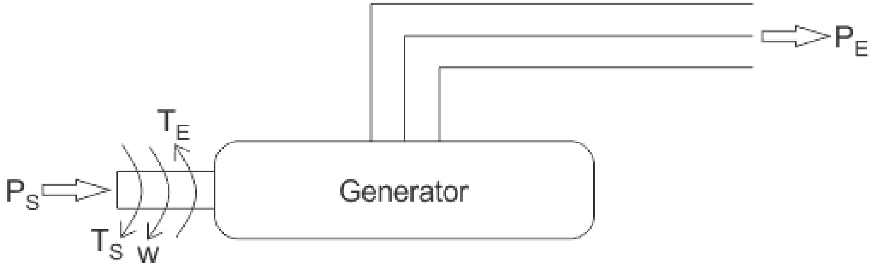
- Following such sudden disturbances in the power system, rotor angular differences, rotor speeds, and power transfer undergo fast changes whose magnitudes are dependent upon the severity of disturbances. For a large disturbance, changes in angular differences may be so large as to cause the machine to fall out of step. This type of instability is known as **TRANSIENT INSTABILITY.** Transient stability is a fast phenomenon, usually occurring within one second for a generator close to the cause of disturbance.
- Short circuit is a severe type of disturbance. During a fault, electrical powers from the nearby generators are reduced drastically, while powers from remote generators are scarily affected.
- In some cases, the system may be stable even with sustained fault; whereas in other cases system will be stable only if the fault is cleared with sufficient rapidity.
- Transient stability limit is almost always lower than the steady state limit and hence it is much important. Transient stability limit depends on the type of disturbance, location and magnitude of disturbance.

Definition of Stability

1. Steady State Stability of a Power System

- refers to the stability of a power system subject to small and gradual changes in load, and the system remains stable with conventional excitation and governor controls.

- In case the power flow through the circuit exceeds the **maximum power** permissible, then there are chances that a particular machine or a group of machines will cease to operate in synchronism, and result in yet more disturbances. In such a situation, the steady state limit of the system is said to have reached. Or in other words the **steady state stability limit** of a system refers to the **maximum amount of power** that is permissible through the system without loss of its steady state stability.



Definition of Stability

2. Transient Stability of a Power System

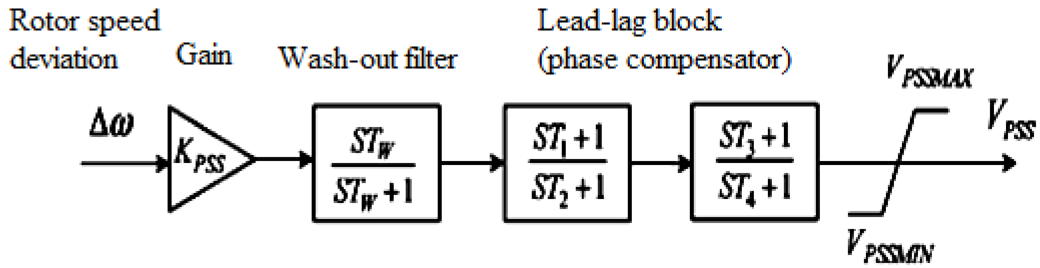
- refers to the stability of a power system subject to a sudden and severe disturbance beyond the capability of the linear and continuous supplementary stability control, and the system may lose its stability at the first swing unless a more effective countermeasure is taken, usually of the discrete type, such as dynamic resistance braking or fast valving for the electric energy surplus area, or load shedding for the electric energy deficient area. For transient stability analysis and control design, the power system must be described by nonlinear differential equations

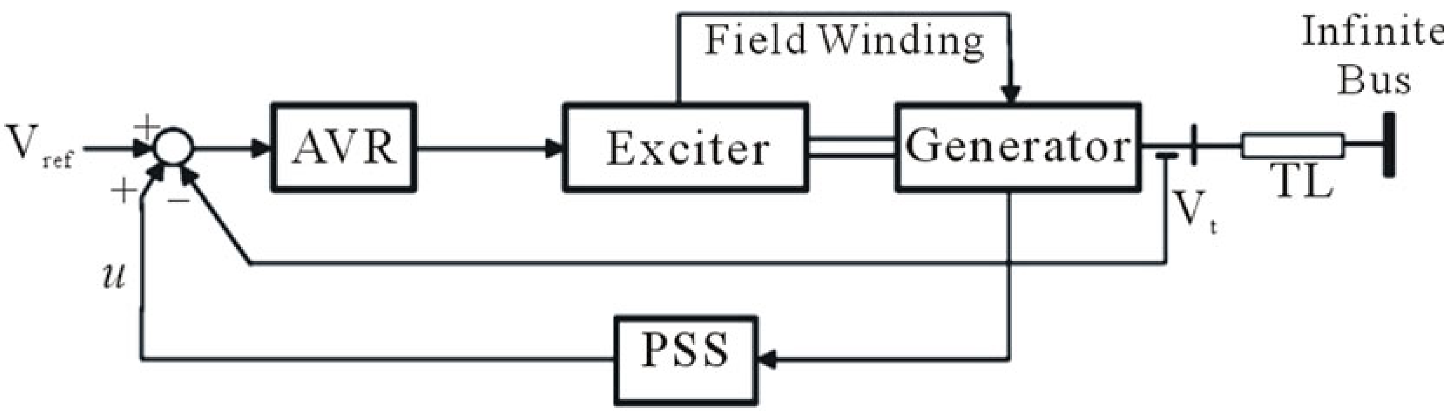
- The **maximum power** that is permissible to flow through the network without loss of stability following a sustained period of disturbance is referred to as the **transient stability limit** of the system. Going beyond that maximum permissible value for power flow, the system would temporarily be rendered as unstable.

Definition of Stability

3. Dynamic Stability of a Power System

- refers to the stability of a power system subject to a relatively small and sudden disturbance, the system can be described by linear differential equations, and the system can be stabilized by a linear and continuous supplementary stability control.





Review of Mechanics

- Transient stability analysis involves some mechanical properties of the machines in the system. After every disturbance, the machines must adjust the relative angles of their rotors to meet the condition of the power transfer involved. The problem is mechanical as well as electrical.

- The kinetic energy of an electric machine is given by

$$K.E. = \frac{1}{2} J\,\omega^2 \quad \text{Mega Joules} \qquad (1)$$

where J is the **Moment of Inertia** in Mega Joules sec^2/elec. $deg.^2$

ω is the angular velocity in elec. deg./sec.

- **Angular Momentum** M = Jω; Then from eqn. (1), K.E. can be written as

$$K.E. = \frac{1}{2} M\,\omega \quad \text{Mega Joules} \qquad (2)$$

Review of Mechanics

Relation between M and H

- The **angular momentum M** depends on the size of the machine as well as on its type.
- The **Inertia constant H** is defined as the Mega Joules of stored energy of the machine at synchronous speed per MVA of the machine. When so defined, the relation between the Angular Momentum M and the Inertia constant H can be derived as follows.

$$H = \frac{\text{Stored Energy in MJ}}{\text{Machine's Rating in MVA}}$$

Let G be the rating of the machine in MVA. Then

$$\text{Stored energy} = \text{G.H} \quad \text{MJ} \qquad (3)$$

Further

$$K.E. = \frac{1}{2} M\omega = \frac{1}{2} M\left(2\pi f\right) = M\pi f \quad \text{MJ} \qquad (4)$$

Review of Mechanics

Relation between M and H

From Eqs (3) and (4), we get $\mathrm{G.H} = M\pi f$

$$M = \frac{\mathrm{G.H}}{\pi . f} \quad \text{MJ sec./elec.rad.} \qquad (5)$$

If the power is expressed in per unit, then G = 1.0 per unit and hence

$$M = \frac{H}{\pi f} \qquad (6)$$

While the angular momentum M depend on the size of the machine as well as on its type, inertia constant H does not vary very much with the size of the machine, The quantity H has a relatively narrow range of values for each class of machine.

Review of Mechanics

Example 1:

A 50Hz, 4 pole turbo alternator rated 150 MVA, 11 kV has an inertia constant of 9 MJ / MVA. Find the stored energy at synchronous speed the rotor acceleration if the input mechanical power is raised to 100 MW when the electrical load is 75 MW, the speed at the end of 10 cycles if acceleration is assumed constant at the initial value.

Solution:

(a) Stored energy = GH = 150 × **9 = 1350 MJ**

(b) Pa = Pm – Pe = 100 – 75 = 25 MW

$$M = \frac{GH}{180f} = \frac{1350}{180 \times 50} = 0.15\ \text{MJ} - \text{s}/^{\circ}\text{e}$$

$$0.15\,\frac{d^2\delta}{dt^2} = 25$$

Example 1, Cont'd

$$\text{Acceleration } \alpha = \frac{d^2\delta}{dt^2} = \frac{25}{0.15} = 166.6 \ \ ^{o}\text{e/s}^2$$

$$= 166.6 \times \frac{2}{P} \ \ ^{o}\text{m/s}^2$$

$$= 166.6 \times \frac{2}{P} \times \frac{1}{360} \ rps\,/\text{s}$$

$$= 166.6 \times \frac{2}{P} \times \frac{1}{360} \times 60 \text{ rpm/s}$$

$$= 13.88 \text{ rpm/s}$$

* Note ^{o}e = electrical degree; ^{o}m = mechanical degree; P=number of poles.

$$\text{(c) 10 cycles} = \frac{10}{50} = 0.2 \text{ s}$$

$$N_S = \text{Synchronous speed} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

$$\text{Rotor speed at end of 10 cycles} = N_S + \quad \times 0.2$$

$$= 1500 + 13.88 \times 0.2 = 1502.776 \text{ rpm}$$

Review of Mechanics

Example 2:

Two 50 Hz generating units operate in parallel within the same plant, with the following ratings:

Unit 1: 500 MVA, 0.8 pf, 13.2 kV, 3600 rpm: H = 4 MJ/MVA

Unit 2: 1000 MVA, 0.9 pf, 13.8 kV, 1800 rpm: H = 5 MJ/MVA

Calculate the equivalent H constant on a base of 100 MVA.

Solution:

$$H_{1system} = H_{1mach} \times \frac{G_{1mach}}{G_{system}} = 4 \times \frac{500}{200} = 20\ MJ/MVA$$

$$H_{2system} = H_{2mach} \times \frac{G_{2mach}}{G_{system}} = 5 \times \frac{1000}{100} = 50\ MJ/MVA$$

$$H_{eq} = H_1 + H_2 = 20 + 50 = 70\ MJ/MVA$$

This is the equivalent inertia constant on a base of 100 MVA and can be used when the two machines swing coherently

Swing Equation

- The differential equation that relates the angular momentum M, the acceleration power P_a and the rotor angle δ is known as **SWING EQUATION**.
- Solution of swing equation will show how the rotor angle changes with respect to time following a disturbance. The plot of δ versus t is called the **SWING CURVE**. Once the swing curve is known, the stability of the system can be assessed.
- The flow of mechanical and electrical power in a generator and motor are shown in Fig. 1.

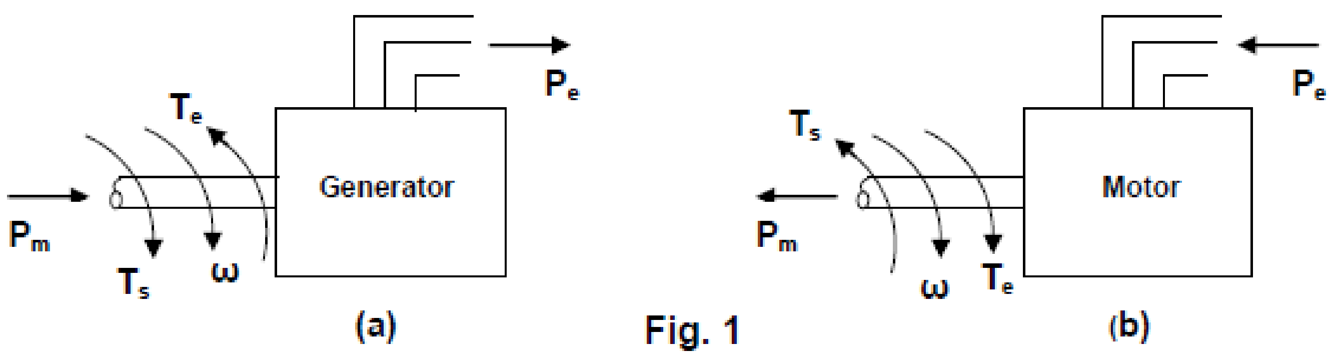


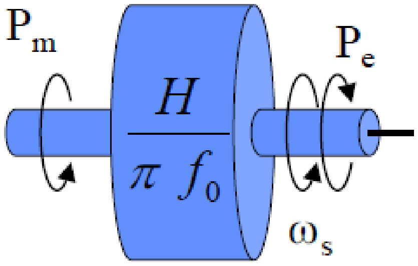
Fig. 1

Swing Equation

- Consider the generator shown in Fig. 1(a). It receives mechanical power P_m at the shaft torque T_s and the angular speed ω via. shaft from the prime-mover. It delivers electrical power P_e to the power system network via. the bus bars. The generator develops electromechanical torque T_e in opposition to the shaft torque T_s. At steady state, $T_s = T_e$.
- Assuming that the windage and the friction torque are negligible, in a generator, accelerating torque acting on the rotor is given by

$$T_a = T_s - T_e \qquad (7)$$

Multiplying by ω on both sides, we get



$$P_a = P_s - P_e \qquad (8)$$

In case of motor

$$T_a = T_e - T_s \qquad (9)$$

$$P_a = P_e - P_s \qquad (10)$$

Swing Equation

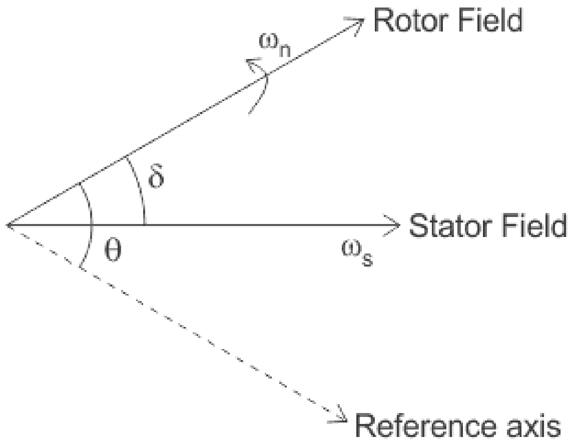
In general, the accelerating power is given by

$$P_a = \text{Input Power} - \text{Output Power} \qquad (11)$$

$$P_a = T_a\omega = J\alpha\omega = M\alpha = M\frac{d^2\theta}{d^2t}$$

Thus

$$M\frac{d^2\theta}{d^2t} = P_a$$



Angular position of rotor with respect to reference axis

Here θ = angular displacement (radians)

$$\omega = \frac{d\theta}{dt} = \text{angular velocity (rad/sec)}$$

Now we can see how the angular displacement θ can be related to rotor angle δ.

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{d^2t} = \text{angular acceleration}$$

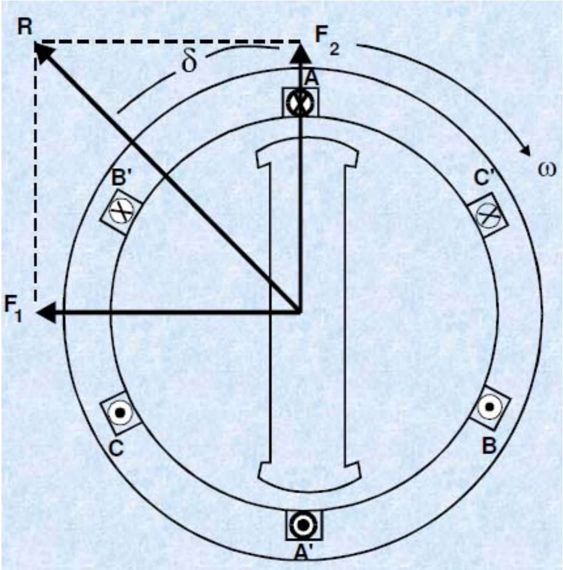
Swing Equation

- The angular displacement θ, at any time t is given by

$$\theta = \omega_s t + \delta \qquad (13)$$

where δ is the angular displacement of the rotor with respect to rotating reference axis which rotates at synchronous speed ω_s.

- The angle δ is also called as **LOAD ANGLE** or **TORQUE ANGLE**. In view of eqn.(13)



$$\frac{d\theta}{dt} = \omega_s + \frac{d\delta}{dt} \qquad (14)$$

$$\frac{d^2\theta}{d^2t} = \frac{d^2\delta}{d^2t} \qquad (15)$$

The above equation is known as

SWING EQUATION

$$M\frac{d^2\delta}{d^2t} = P_a \qquad (16)$$

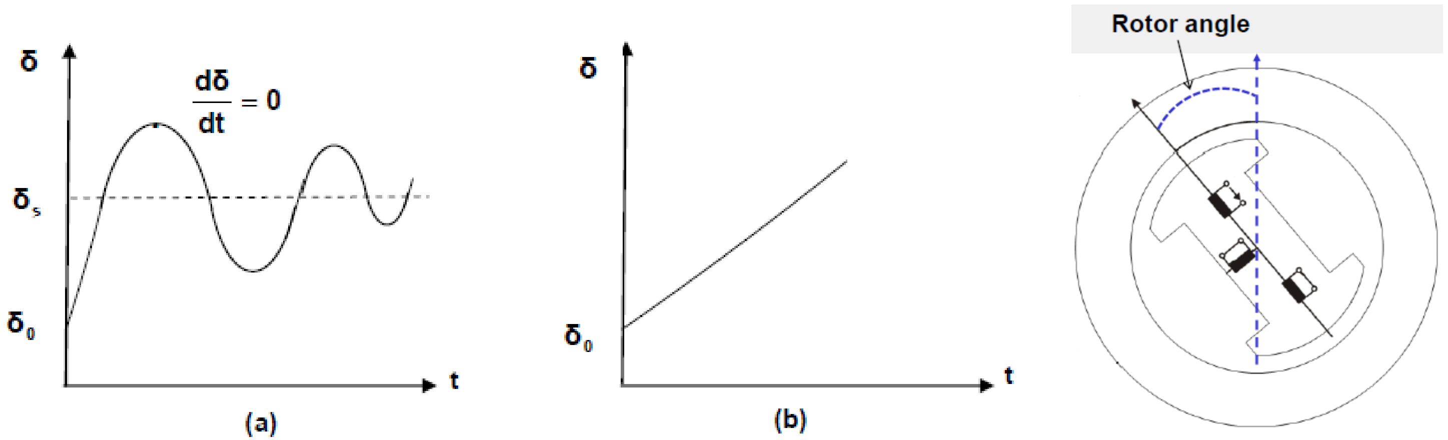
Swing Equation

- In case damping power is to be included, then eqn.(16) gets modified as

$$M\frac{d^2\delta}{d^2t} + D\frac{d\delta}{dt} = P_a \qquad (17)$$

Swing curve, which is the plot of torque angle δ vs time t, can be obtained by solving the swing equation. Two typical swing curves are shown in Fig. 2.

Fig. 2

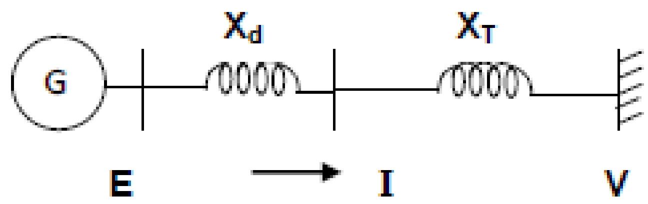


- Swing curves are used to determine the stability of the system. If the rotor angle δ reaches a maximum and then decreases, then it shows that the system has transient stability.
- On the other hand if the rotor angle δ increases indefinitely, then it shows that the system is unstable.

Stability of a Generator connected to Infinite Bus

- Consider a generator connected to infinite bus. The complex power is given by

P + *j* Q = V I *

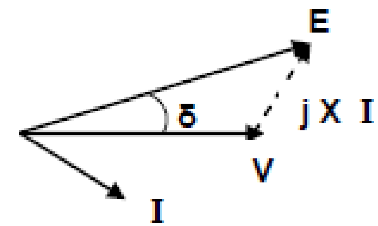


where

V is the voltage at infinite bus.

E is internal voltage of generator.

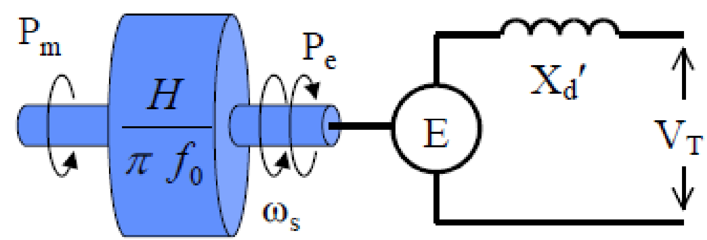
X is the total reactance



Taking $V = V\angle 0°$ as a reference phasor diagram can be obtained as

E =V + j X I

Internal voltage ***E*** leads ***V*** by angle δ. Thus

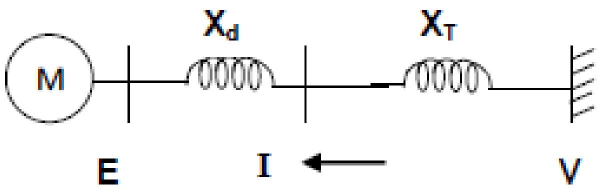


Taking $E = |E|\angle\delta$

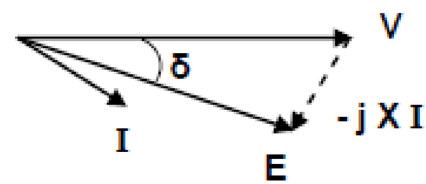
Electric output power $P_e = \frac{|E|.|V|}{X}\sin\delta = P_{max}\sin\delta$

Stability of a Synchronous Motor connected to Infinite Bus

- Consider a synchronous motor drawing power from infinite bus.



$$V - jXI = E$$



Internal Voltage E lags the terminal voltage V by angle δ.

Thus $E = |E| \angle -\delta$ Current $I = \dfrac{1}{jX}\left[\,|V| - (|E|\cos\delta - j|E|\sin\delta)\,\right]$

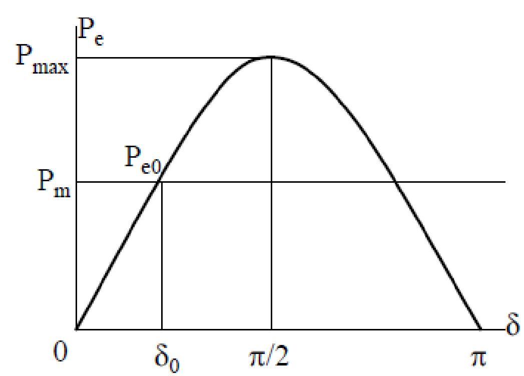
Electric input power $P_e = \mathrm{Re}\left[\,|V|\,I\,\right] = \dfrac{|E||V|}{X}\sin\delta = P_{max}\ \sin\delta$

Thus Swing equation for alternator is

$$M\,\frac{d^2\delta}{dt^2} = P_m - P_{max}\sin\delta$$

Swing equation for motor is

$$M\,\frac{d^2\delta}{dt^2} = P_{max}\sin\delta - P_m$$



power angle curve

Notice that the swing equation is second order nonlinear differential equation

Equal area criterion

- The accelerating power in swing equation will have sine term. Therefore the swing equation is non-linear differential equation and obtaining its solution is not simple.
- For **two machine system and one machine connected to infinite bus** bar, it is possible to say whether a system has transient stability or not, without solving the swing equation.
- Such criteria which decides the stability, makes use of equal area in power angle diagram and hence it is known as **EQUAL AREA CRITERION**.
- Thus the principle by which stability under transient conditions is determined without solving the swing equation, but makes use of areas in power angle diagram, is called the **EQUAL AREA CRITERION**.
- **Quickly predicts the stability after a major disturbance**
  - graphical interpretation of the energy stored in the rotating masses
- **Method provides physical insight to the dynamic behavior of machines**
  - relates the power angle with the acceleration power

Equal area criterion

- From the Fig. 2, it is clear that if the rotor angle δ oscillates, then the system is stable. For δ to oscillate, it should reach a maximum value and then should decrease.
- At that point $d\delta/dt = 0$. Because of damping inherently present in the system, subsequence oscillations will be smaller and smaller. Thus while δ changes, if at one instant of time, $d\delta/dt = 0$, then the stability is ensured.
- Let us find the condition for $d\delta/dt$ to become zero. The swing equation for the alternator connected to the infinite bus bars is

$$M\frac{d^2\delta}{d^2t} = P_a = P_s - P_e \qquad (18)$$

- Multiplying both sides by $d\delta/dt$, we get

$$M\frac{d^2\delta}{d^2t}\frac{d\delta}{dt} = (P_s - P_e)\frac{d\delta}{dt} = \frac{1}{2}M\frac{d}{dt}\left(\frac{d\delta}{dt}\right)^2 = (P_s - P_e)\frac{d\delta}{dt} \qquad (19)$$

Equal area criterion

Thus

$$\frac{d}{dt}\left(\frac{d\delta}{dt}\right)^2 \frac{dt}{d\delta} = \frac{2(P_s - P_e)}{M} \; ; \qquad \text{i.e.} \quad \frac{d}{d\delta}\left(\frac{d\delta}{dt}\right)^2 = \frac{2(P_s - P_e)}{M} \qquad \text{On integration}$$

$$\left(\frac{d\delta}{dt}\right)^2 = \int_{\delta_0}^{\delta} \frac{2(P_s - P_e)d\delta}{M} \qquad \text{i.e.} \qquad \frac{d\delta}{dt} = \sqrt{\int_{\delta_0}^{\delta} \frac{2(P_s - P_e)\,d\delta}{M}} \qquad (20)$$

Before the disturbance occurs, δ_o was the torque angle. At that time $d\delta/dt = 0$. As soon as the disturbance occurs, $d\delta/dt$ is no longer zero and δ starts changing.

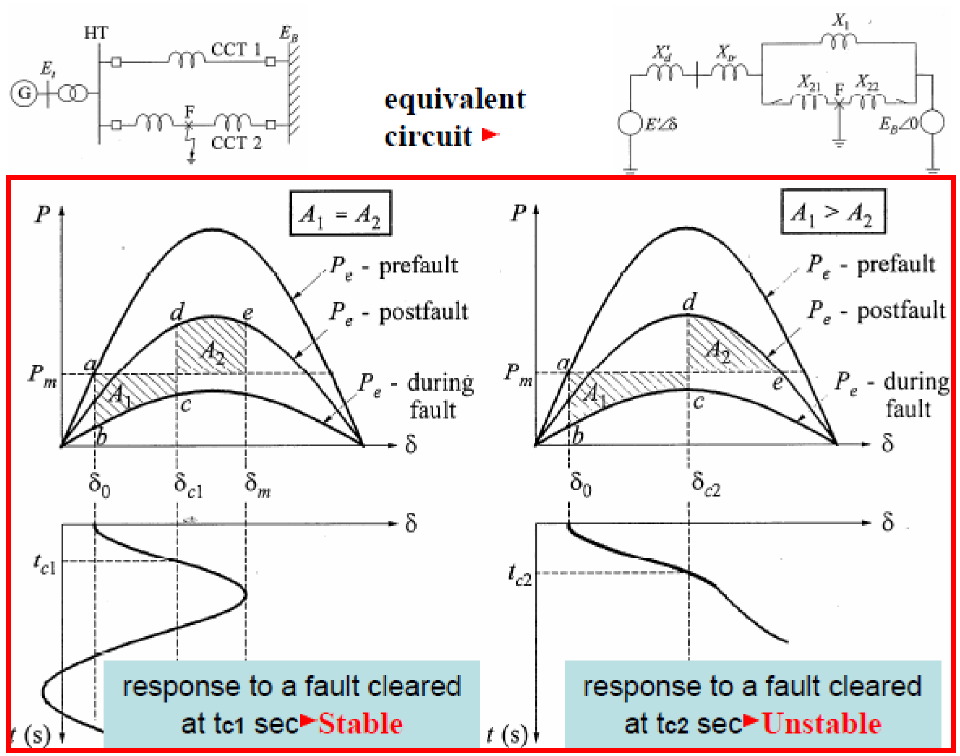
Torque angle δ will cease to change and the machine will again be operating at synchronous speed after a disturbance, when $d\delta/dt = 0$ or when

$$\int_{\delta_0}^{\delta} \frac{2(P_s - P_e)}{M}\, d\delta = 0 \qquad \int_{\delta_0}^{\delta} (P_s - P_e)\, d\delta = 0 \qquad (21)$$

Equal area criterion

- If there exist a torque angle δ for which the above is satisfied, then the machine will attain a new operating point and hence it has transient stability. The machine will not remain at rest with respect to infinite bus at the first time when $d\delta/dt = 0$. But due to damping present in the system, during subsequent oscillation, maximum value of δ keeps on decreasing.
- Therefore, the fact that δ has momentarily stopped changing may be taken to indicate stability.

  - ***Three-phase fault at F(cleared by opening circuit breakers)***



1. Sudden increase in the mechanical power input

- $P_{m1} > P_{e0}$; the acceleration power is positive
- excess energy is stored in the rotor and the power frequency increases, driving the relative power angle larger over time

$$\int_{\delta_0}^{\delta_1} (P_{m1} - P_e) d\delta > 0$$

$$\frac{d\delta}{dt} = \omega = \sqrt{\frac{2\pi f_0}{H} \int_{\delta_0}^{\delta} (P_m - P_e) d\delta} > 0$$

1. Sudden increase in the mechanical power input

- $P_{m1} > P_{e0}$; the acceleration power is positive
- excess energy is stored in the rotor and the power frequency increases, driving the relative power angle larger over time

$$\int_{\delta_0}^{\delta_1} (P_{m1} - P_e) d\delta > 0$$

$$\frac{d\delta}{dt} = \omega = \sqrt{\frac{2\pi f_0}{H} \int_{\delta_0}^{\delta} (P_m - P_e) d\delta} > 0$$

- with increase in the power angle, δ, the electrical power increases

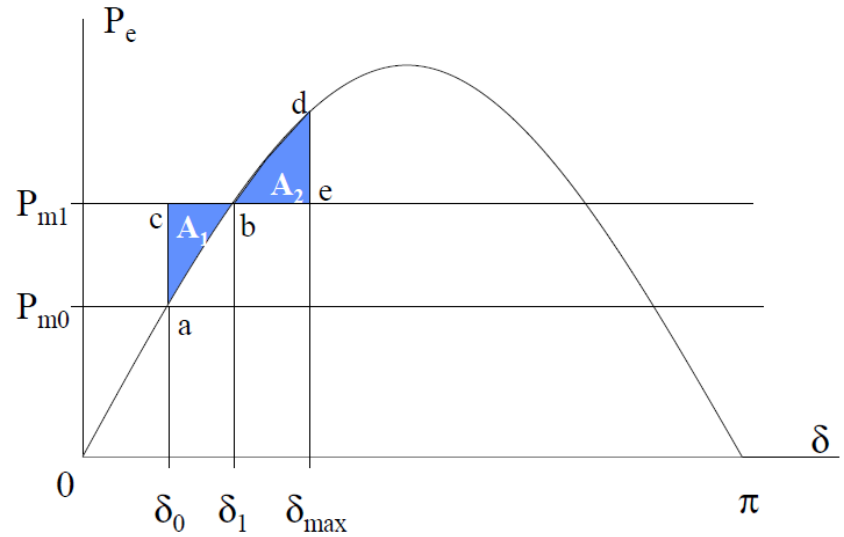
$$P_e = P_{\max} \sin\delta$$

when $\delta = \delta_1$, the electrical power equals the mechanical power, P_{m1}

1. Sudden increase in the mechanical power input

- acceleration power is zero, but the rotor is running above synchronous speed, hence the power angle, d, continues to increase
- now $P_{m1} < P_e$; the acceleration power is negative (deceleration), causing the rotor to decelerate to synchronous speed at $\delta = \delta_{max}$
- an equal amount of energy must be given up by the rotating masses

$$\int_{\delta_0}^{\delta_1} (P_{m1} - P_e) d\delta - \int_{\delta_1}^{\delta_{max}} (P_{m1} - P_e) d\delta = 0$$

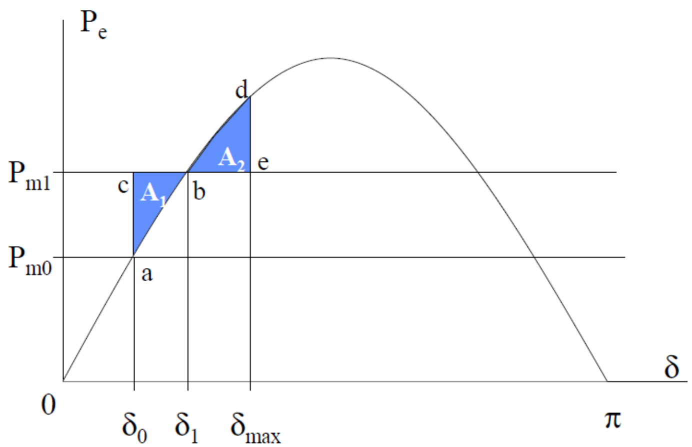


1. Sudden increase in the mechanical power input

- **The result is that the rotor swings to a maximum angle**
  - at which point the acceleration energy area and the deceleration energy area are equal

$$\int_{\delta_0}^{\delta_1} (P_{m1} - P_e)\, d\delta = \textbf{area}\ abc = \textbf{area}\ A_1$$

$$\int_{\delta_1}^{\delta_{max}} (P_{m1} - P_e)\, d\delta = \textbf{area}\ bde = \textbf{area}\ A_2$$



$$\left|\textbf{area}\ A_1\right| = \left|\textbf{area}\ A_2\right|$$ **(equal area criterion)**

- the rotor angle will oscillate back and forth between δ and δ_{max} at its natural frequency

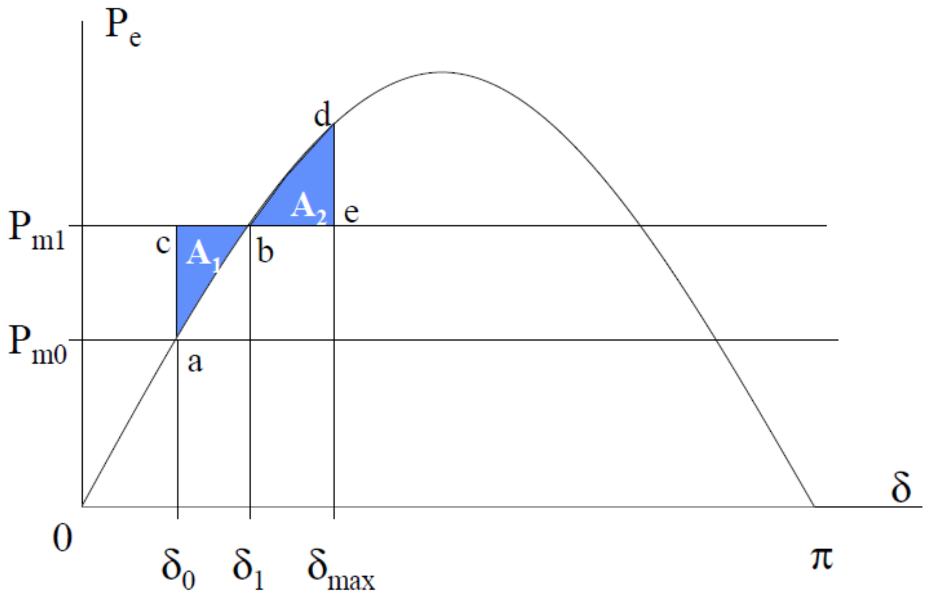
1. Sudden increase in the mechanical power input

$$P_{m1}(\delta_1 - \delta_0) - \int_{\delta_0}^{\delta_1} P_{\max} \sin\delta \, d\delta = \int_{\delta_1}^{\delta_{\max}} P_{\max} \sin\delta \, d\delta - P_{m1}(\delta_{\max} - \delta_1)$$

$$P_{m1}(\delta_{\max} - \delta_0) = P_{\max}(\cos\delta_0 - \cos\delta_{\max})$$

$$P_{m1} = P_{\max} \sin\delta_{\max} \qquad \rightarrow P_{m1} = P_{\max} \sin\delta_1$$

$$(\delta_{\max} - \delta_0)\sin\delta_{\max} = \cos\delta_0 - \cos\delta_{\max}$$



Function is nonlinear in δ_{max}
Solve using Newton-Raphson

1. Sudden increase in the mechanical power input

Example # 3:

A transmission line is acting as an interconnector between two constant voltage networks as shown in Fig. E1. Determine graphically or otherwise the maximum additional load which can be suddenly applied to this interconnector already carrying 50 MW if the power angle equation is $P_e = 100\sin\delta$.

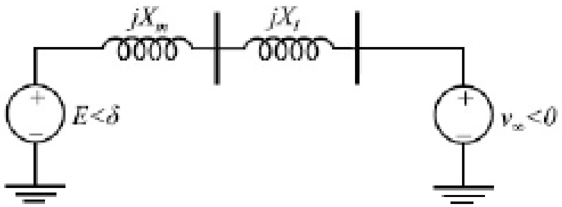
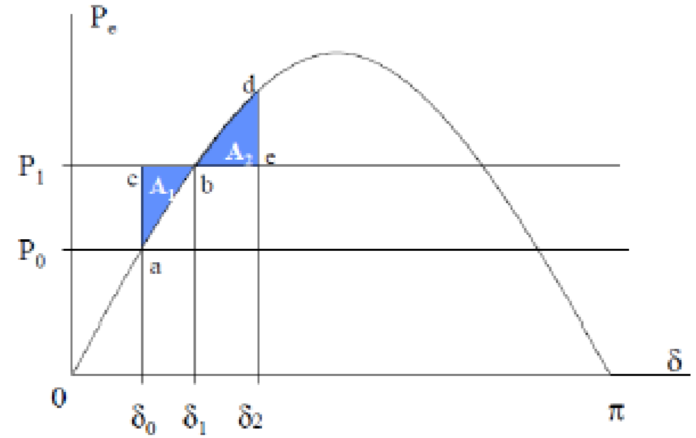


Fig. E1



Solution:

$P_o = 100\sin\delta_o = 50 \Rightarrow \delta_o = 30° \quad 0.5236$ rad

Accelerating Area is A_1 and decelerating area is A_2.

$$A_1 = \int_{\delta_o}^{\delta_1} (P_1 - 100\sin\delta) d\delta = P_1(\delta_1 - \delta_o) + 100(\cos\delta_1 - \cos\delta_o)$$

$$A_2 = \int_{\delta_1}^{\delta_2} (100\sin\delta - P_1) d\delta = -100(\cos\delta_2 - \cos\delta_1) - P_1(\delta_2 - \delta_1)$$

For limiting case $\delta_2 = \delta_{max} = \pi - \delta_1$.

Moreover, $P_1 = 100\sin\delta_1$.

Equating areas A_1 and A_2

and substituting these values we get the equation

Example 3, Cont'd

$$A_1 = A_2$$

$$P_1(\delta_1 - \delta_o) + 100(\cos\delta_1 - \cos\delta_o) = -100(\cos(\pi - \delta_1) - \cos\delta_1) - P_1(\pi - \delta_1 - \delta_1)$$

$$P_1\delta_1 - P_1\delta_o + 100\cos\delta_1 - 100\cos\delta_o = -100(\cos(\pi).\cos(\delta_1) + \sin(\pi).\sin(\delta_1)) + 100\cos(\delta_1) - P_1(\pi - \delta_1) + P_1\delta_1$$

$$-P_1\delta_o + 100\cos\delta_1 - 100\cos\delta_o = 100\cos(\delta_1) + 100\cos(\delta_1) - P_1(\pi - \delta_1)$$

$$-100\sin(\delta_1)\delta_o - 100\cos\delta_o = 100\cos(\delta_1) - 100\sin(\delta_1)(\pi - \delta_1)$$

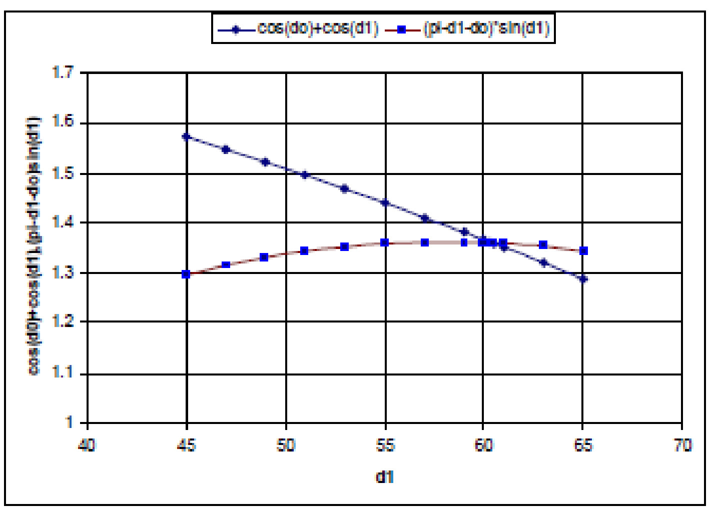
$$-\sin(\delta_1)\delta_{o1} - \cos\delta_o = \cos(\delta_1) - \sin(\delta_1)(\pi - \delta_1)$$

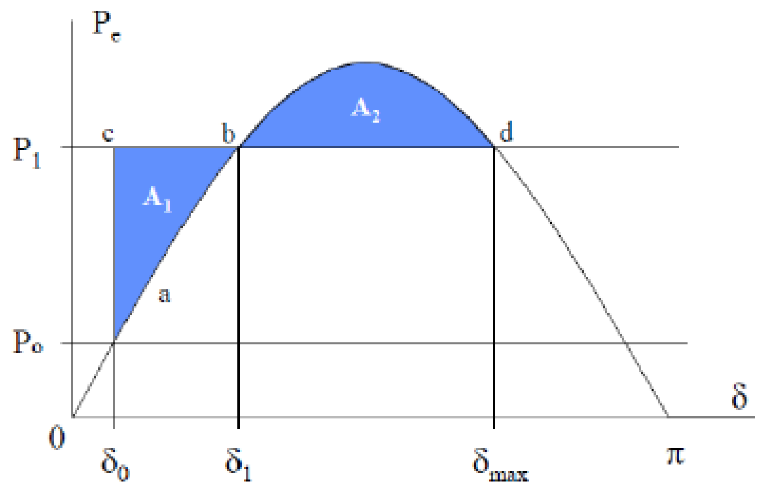
$$\sin(\delta_1)(\pi - \delta_1) - \sin(\delta_1)\delta_o + \cos\delta_1 = \cos(\delta_1) + \cos\delta_o$$

$$\cos\delta_o + \cos\delta_1 = (\pi - \delta_1 - \delta_o)\sin\delta_1$$

The angles δ_1 and δ_0 in this equation are in radians.
The equation can be solved by hit and trail. The results is $\delta_1 = 1.054179 = 60.4°$.

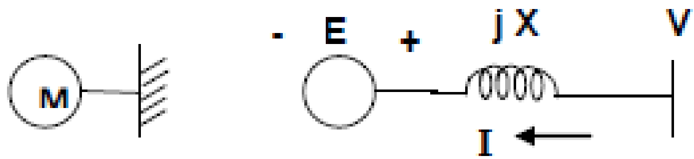
$$\cos\delta_o + \cos\delta_1 = (\pi - \delta_1 - \delta_o)\sin\delta_1 = (\delta_{max} - \delta_o)\sin\delta_1$$



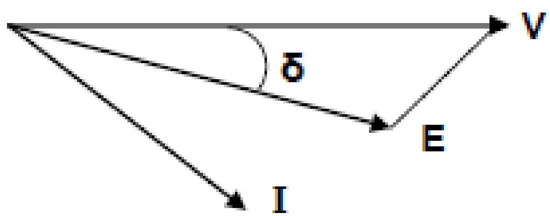


2. Sudden load increase on Synchronous motor

- Let us consider a synchronous motor connected to an infinite bus bars.



$$E = V - I\,(jX)$$



$$P_e = \frac{|E|\,|V|}{X}\sin\delta = P_{max}\sin\delta$$

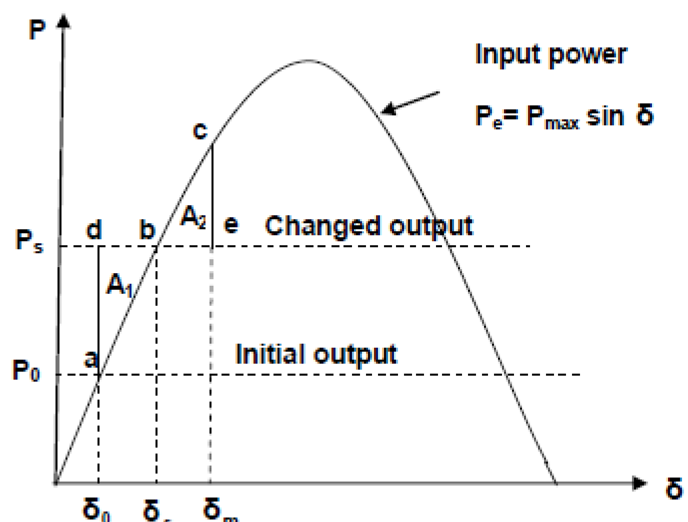


Fig. 3

2. Sudden load increase on Synchronous motor

The following changes occur when the load is increased suddenly.

Point a Initial condition; Input = output = P_0; $\omega = \omega_s$; $\delta = \delta_0$

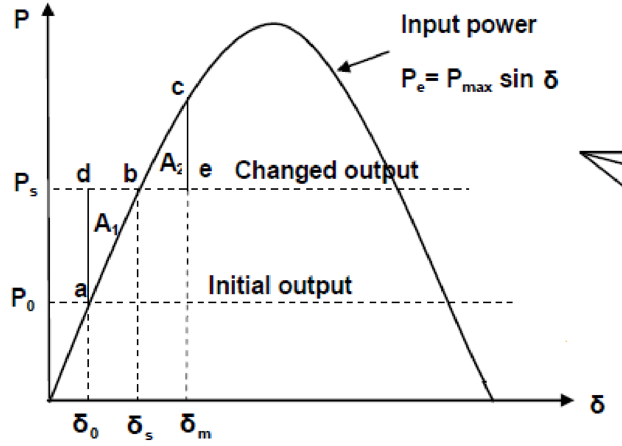
Due to sudden loading, output = P_s; output > Input;

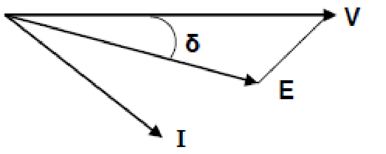
ω decreases from ω_s; δ increases from δ_0.

Between a-b Output > Input; Rotating mass starts loosing energy resulting deceleration; ω decreases; δ increases.

Point b Output = Input; $\omega = \omega_{min}$ which is less than ω_s; $\delta = \delta_s$

Since ω is less than ω_s, δ continues to increase.





2. Sudden load increase on Synchronous motor

<u>Between b-c</u> Input > output; Rotating masses start gaining energy; Acceleration; ω starts increasing from minimum value but still less than ω_s; δ continues to increase.

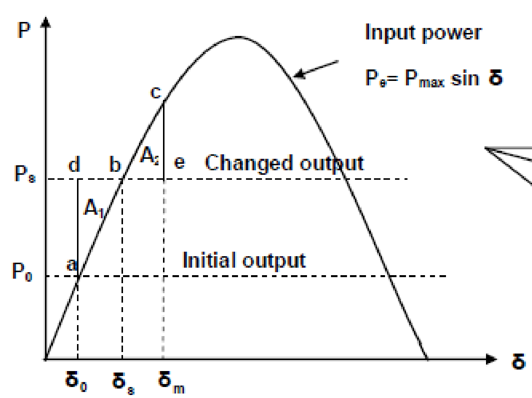
<u>Point c</u> Input > output; $\omega = \omega_s$; $\delta = \delta_m$; There is acceleration; ω is going to increase from ω_s; hence δ is going to decrease from δ_m.

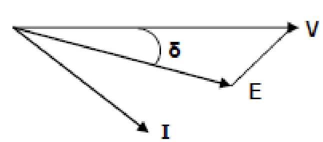
<u>Between c-b</u> Input > output; Acceleration; ω increases and δ decreases.

<u>Point b</u> Input = output; $\omega = \omega_{max}$; $\delta = \delta_s$. Since ω is greater than ω_s, δ continues to decrease.

<u>Between b-a</u> Output > input; Deceleration; ω starts decreasing from ω_{max} ; but still greater than ω_s; δ continues to decrease.

<u>Point a</u> $\omega = \omega_s$; $\delta = \delta_0$; Output > Input; The cycle repeats.





2. Sudden load increase on Synchronous motor

Because of damping present in the system, subsequent oscillations become smaller and smaller and finally b will be the steady state operating point.

<u>Interpretation of equal area</u>

As discussed earlier (eqn. 21), the condition for stability is

$$\int_{\delta_0}^{\delta} (P_s - P_e)\, d\delta = 0 \quad \text{i.e.} \quad \int_{\delta_0}^{\delta} P_e\, d\delta = \int_{\delta_0}^{\delta} P_s\, d\delta$$

From Fig. 4, $\int_{\delta_0}^{\delta_m} P_e\, d\delta$ = area δ_0 a b c δ_m

and $\int_{\delta_0}^{\delta_m} P_s\, d\delta$ = area δ_0 a d e δ_m

Thus for stability,

area δ_0 a b c δ_m = area δ_0 a d e δ_m

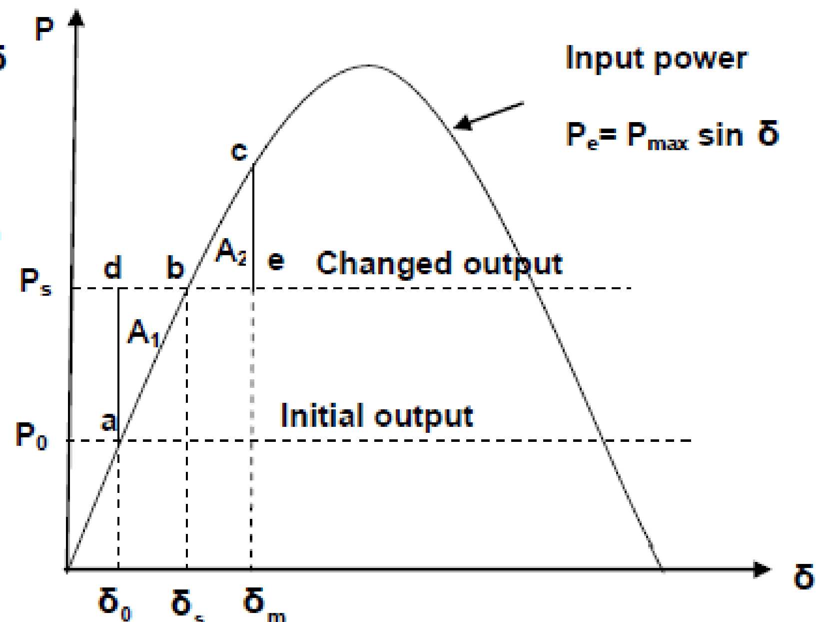


Fig. 4

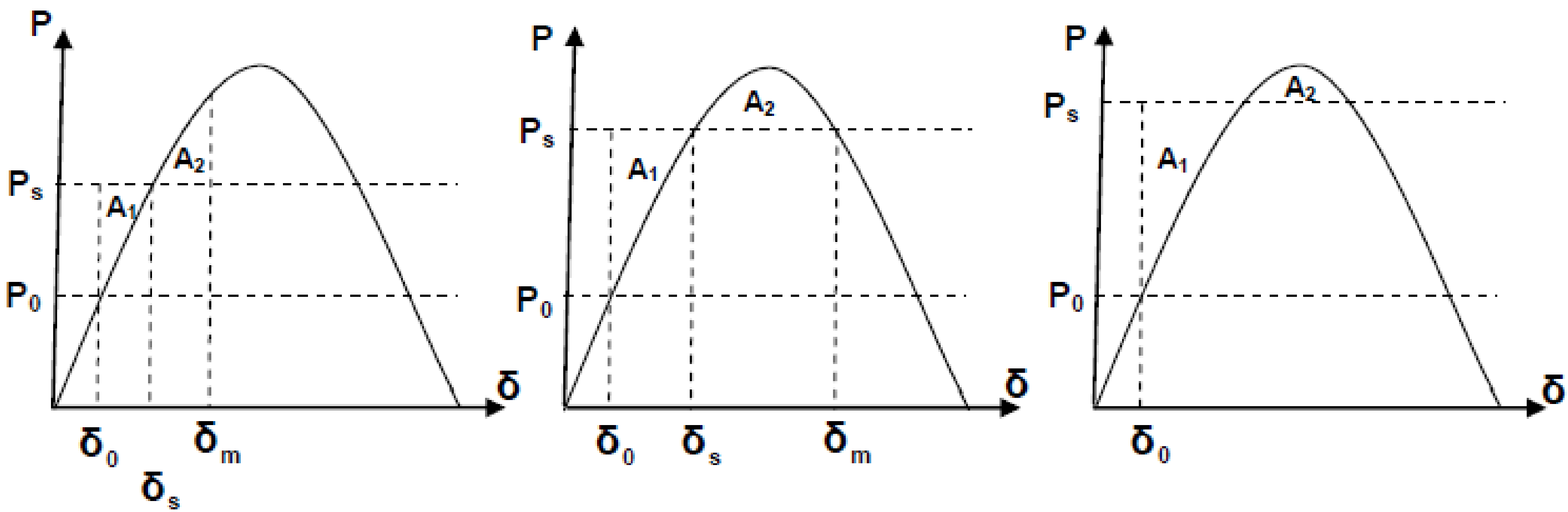
Subtracting area δ_0 a b e δ_m from both sides of above equation, we get $A_2 = A_1$.

Thus for stability,

$$A_2 = A_1 \qquad (22)$$

2. Sudden load increase on Synchronous motor

Fig. 5 shows three different cases: The one shown in case a is **STABLE**. Case b indicates **CRITICALLY STABLE** while case c falls under **UNSTABLE**.



Case a   Case b   Case c

Fig. 5

Note that the areas A_1 and A_2 are obtained by finding the difference between INPUT and OUTPUT.

2. Sudden load increase on Synchronous motor

Example 1

A synchronous motor having a steady state stability limit of 200 MW is receiving 50 MW from the infinite bus bars. Find the maximum additional load that can be applied suddenly without causing instability.

Solution

Referring to Fig. 6,

for critical stability

$A_2 = A_1$

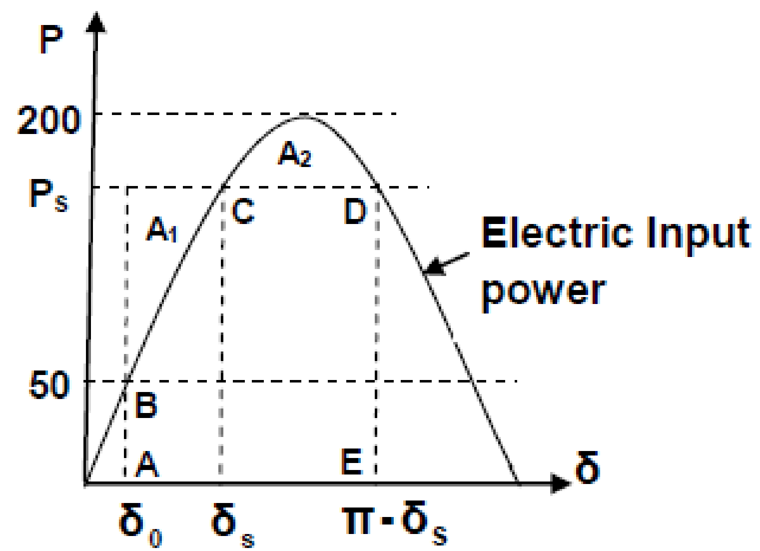


Fig. 6

$200 \sin \delta_0 = 50$ i.e.

$$\delta_0 = \sin^{-1} \frac{50}{200} = 0.25268 \text{ rad}$$

Further $200 \sin \delta_S = P_S$

Adding area ABCDEA to both A_1 and A_2 and equating the resulting areas

2. Sudden load increase on Synchronous motor

$$\cancel{200} \sin \delta_S \quad (\pi - \delta_S - \delta_0) \;=\; \int_{\delta_0}^{\pi-\delta_s} \cancel{200} \sin \delta \, d\delta \quad \text{i.e.}$$

$$(\pi - \delta_S - \delta_0) \;\sin \delta_S \;=\; \cos \delta_0 - \cos(\pi - \delta_S) = \cos \delta_0 + \cos \delta_S \quad \text{i.e.}$$

$$(\pi - \delta_S - 0.25268) \sin \delta_S \;-\; \cos \delta_S \;=\; 0.9682458$$

The above equation can be solved by trial and error method.

δ_S	0.85	0.9	0.95
RHS	0.8718	0.9363	0.9954

Using linear interpolation between second and third points we get δ_S = 0.927 rad.

0.927 rad. = 53.11 deg.

Thus $P_S = 200 \sin 53.11^0 = 159.96$ MW

Maximum additional load possible = 159.96 – 50 = 109.96 MW

3. Opening of one of the parallel lines

- When a generator is supplying power to an infinite bus over two parallel transmission lines, the opening of one of the lines will result in increase in the equivalent reactance and hence decrease in the maximum power transferred.
- Because of this, depending upon the initial operating power, the generator may loose synchronism even though the load could be supplied over the remaining line under steady state condition.

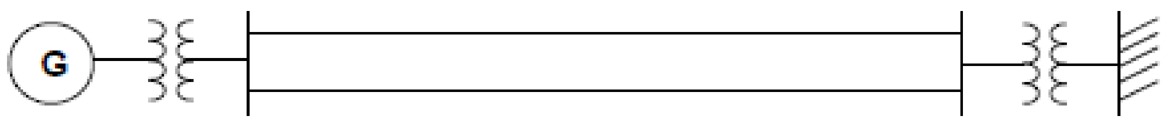


Fig. 7

Consider the system shown in Fig. 7. The power angle diagrams corresponding to stable and unstable conditions are shown in Fig. 8.

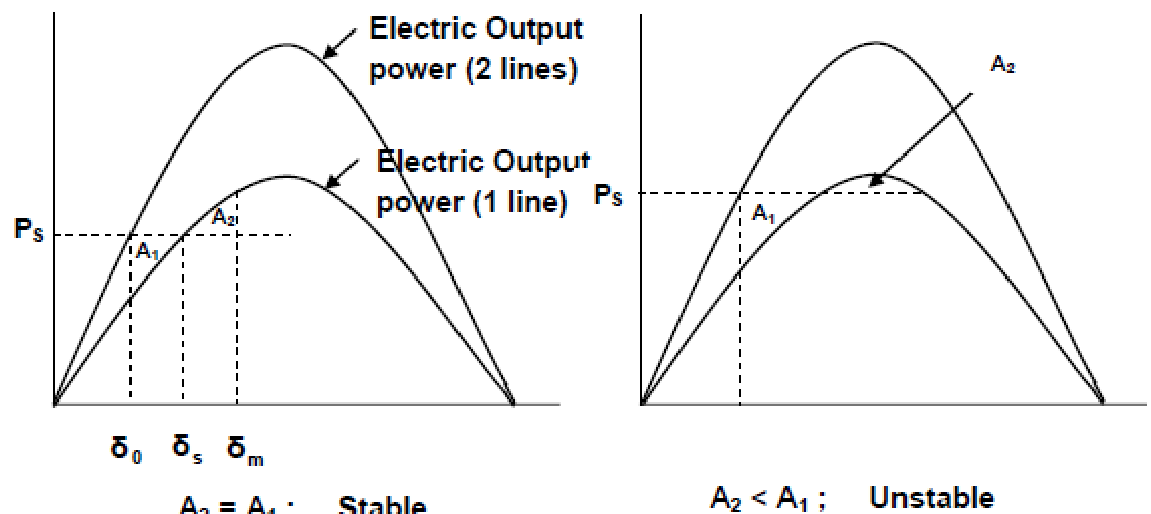


Fig. 8

Example # 4:

For the system shown in Fig. E2, the per unit value of the system are:
$|E| = 1.2$ pu, $|V_\infty| = 1.0$ pu, $X_d' = 0.2$ pu, $X_1 = X_2 = 0.4$ pu. The system is operating in equilibrium with $P_o = P_{eo} = 1.5$ pu when one of the lines is suddenly switched out. Predict whether the system will be stable or not. If the system is stable find the maximum value of δ attains.

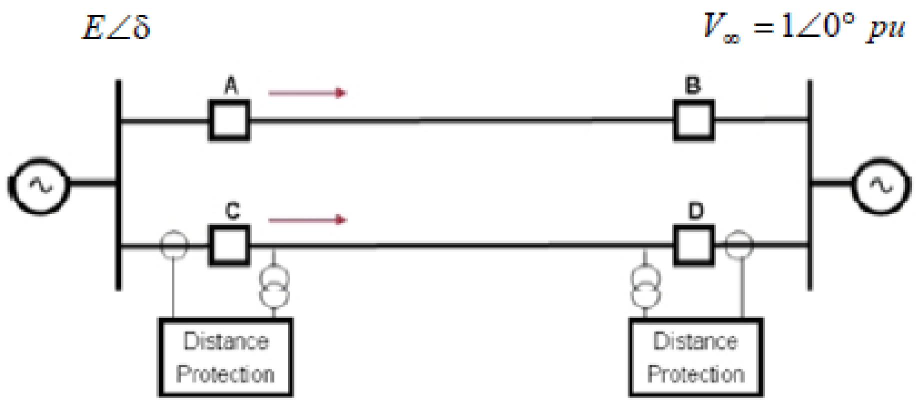


Fig. E2

Solution:

$$P_e = \frac{EV_\infty}{X_{eq}}\sin\delta = \frac{EV_\infty}{X_d' + \dfrac{X_1 \times X_2}{X_1 + X_2}}\sin\delta = \frac{1.2\times 1}{0.2 + \dfrac{0.4\times 0.4}{0.4+0.4}}\sin\delta = \frac{1.2}{0.4}\sin\delta = 3\sin\delta$$

$P_e(\delta_o) = 3\sin\delta_o = 1.5 \Rightarrow \delta_o = 30°\ (0.524\ radians)$

when one line is switched out

$$P_e' = \frac{EV_\infty}{X_{eq}}\sin\delta = \frac{EV_\infty}{X_d' + X_1}\sin\delta = \frac{1.2\times 1}{0.2+0.4}\sin\delta = \frac{1.2}{0.6}\sin\delta = 2\sin\delta$$

$P_e = P_e'(\delta_s) = 2\sin\delta_s = 1.5 \Rightarrow \delta_s = 48.6°\ (0.848\ radians)$

$P_e = P_e'(\delta_m) = 2\sin\delta_m = 1.5 \Rightarrow \delta_m = 131.4°\ (0.2.293\ radians)$

Example # 4, Cont'd

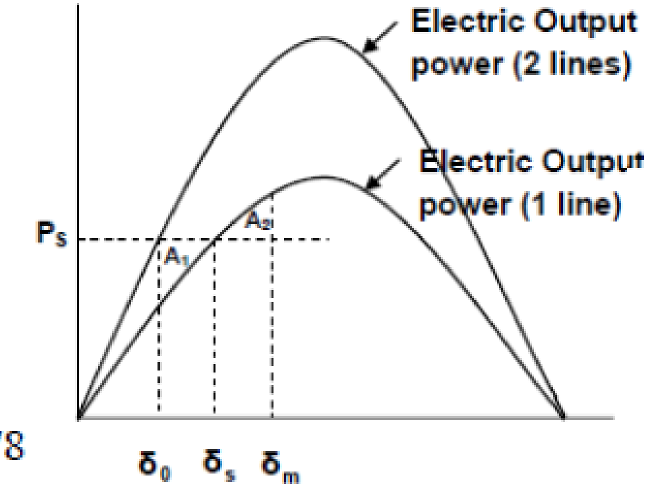
$$A_1 = \int_{\delta_o}^{\delta_s} (P_s - 2\sin\delta)d\delta = P_s(\delta_s - \delta_o) + 2(\cos\delta_s - \cos\delta_o)$$

$$= 1.5(0.848 - 0.524) + 2(\cos(48.6^\circ) - \cos(30^\circ) = 0.0773$$

$$A_{2\max} = \int_{\delta_s}^{\delta_{\max}} (2\sin\delta - P_s)d\delta = -2(\cos\delta_{\max} - \cos\delta_s) - P_s(\delta_{\max} - \delta_s)$$

$$= -2(\cos(131.4^\circ) - \cos(48.6^\circ)) - 1.5(2.293 - 0.848) = 0.0478$$

since A_{2max} (0.478) > A_1 (0.0773), the system is stable.



$$A_2 = \int_{\delta_s}^{\delta_2} (2\sin\delta - P_s)d\delta = -2(\cos\delta_m - \cos\delta_s) - P_s(\delta_m - \delta_s) = -2(\cos(\delta_m) - \cos(\delta_s)) - 1.5(\delta_m - \delta_s)$$

$$= -2(\cos(\delta_m) - \cos(48.6^\circ)) - 1.5(\delta_m - 0.848)$$

$$A_2 = \int_{\delta_s}^{\delta_2} (2\sin\delta - P_s)d\delta = -2(\cos\delta_m - \cos\delta_s) - P_s(\delta_m - \delta_s)$$

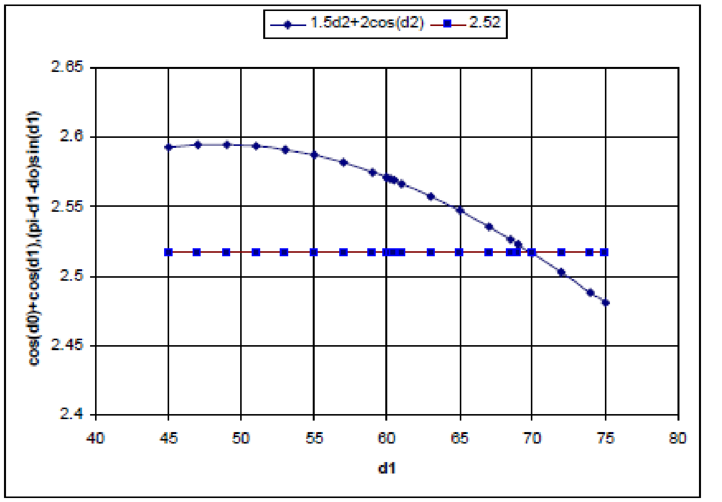
$$= -2(\cos(\delta_m) - \cos(\delta_s)) - 1.5(\delta_m - \delta_s)$$

$$= -2(\cos(\delta_m) - \cos(48.6^\circ)) - 1.5(\delta_m - 0.848)$$

$$-2(\cos(\delta_m) - \cos(48.6^\circ)) - 1.5(\delta_m - 0.848) = 0.0773$$

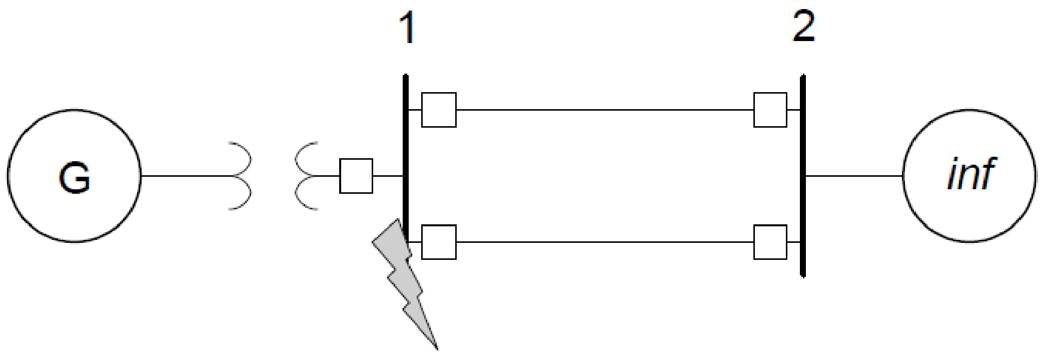
$$1.5\delta_m + 2\cos(\delta_m) = 1.5 \times 0.848 + 2\cos(48.6^\circ) - 0.0773$$

$$1.5\delta_m + 2\cos(\delta_m) = 2.52$$

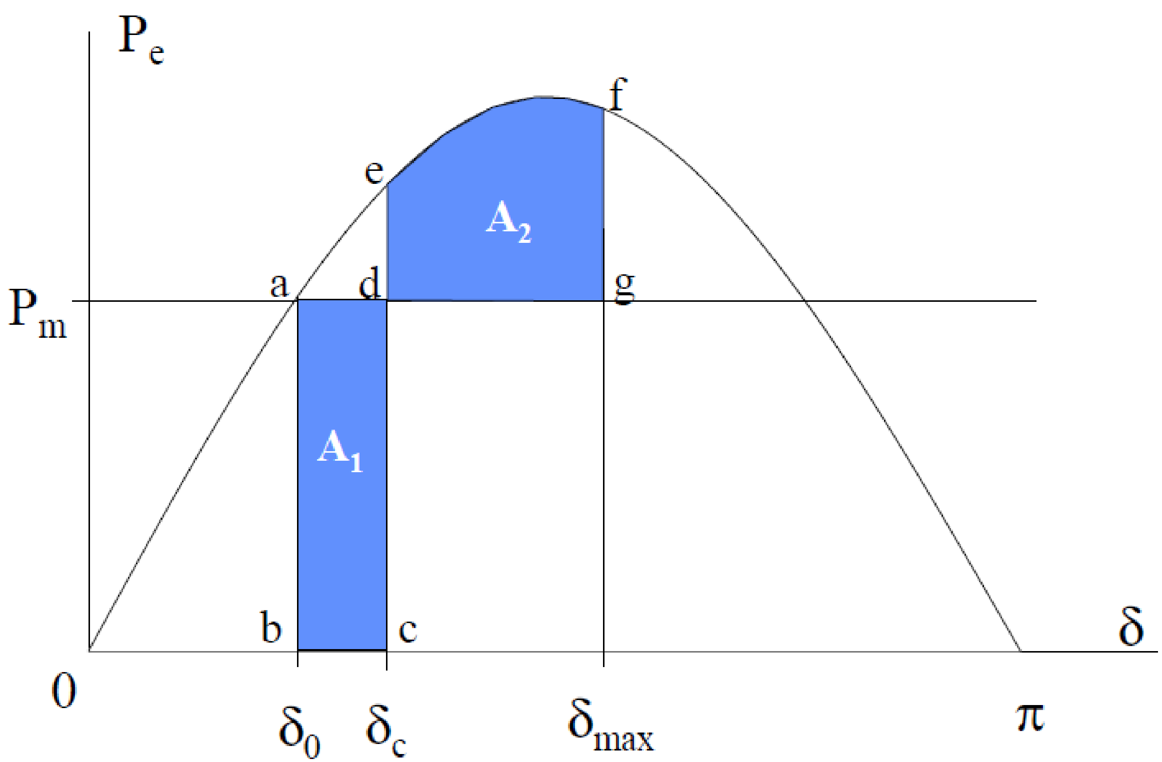


4. Short circuit occurring in the system

- **3-Phase Fault**

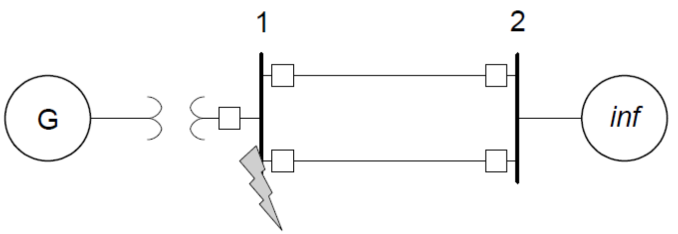


Equal Area Criterion - 3 phase fault



4. Short circuit occurring in the system

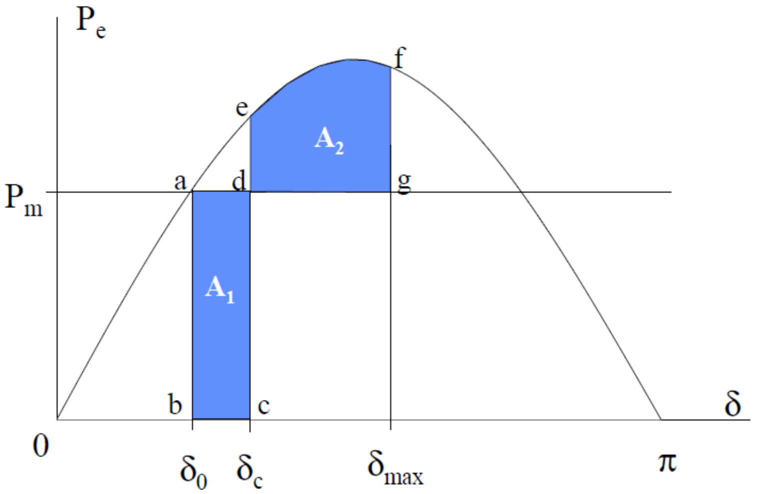
Equal Area Criterion - 3 phase fault



$$\int_{\delta_0}^{\delta_c} P_m \, d\delta = \int_{\delta_c}^{\delta_{\max}} \left(P_{\max} \sin\delta - P_m\right) d\delta$$

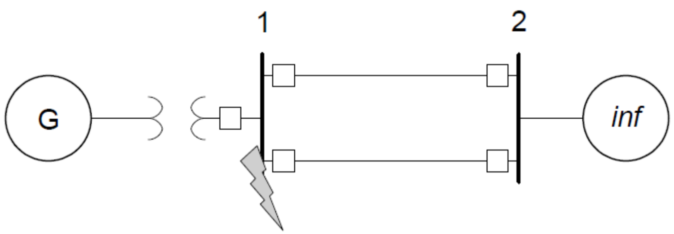
$$P_m\left(\delta_c - \delta_0\right) = P_{\max}\left(\cos\delta_c - \cos\delta_{\max}\right) - P_m\left(\delta_{\max} - \delta_c\right)$$

$$\cos\delta_c = \frac{P_m}{P_{\max}}\left(\delta_{\max} - \delta_c\right) + \cos\delta_{\max}$$



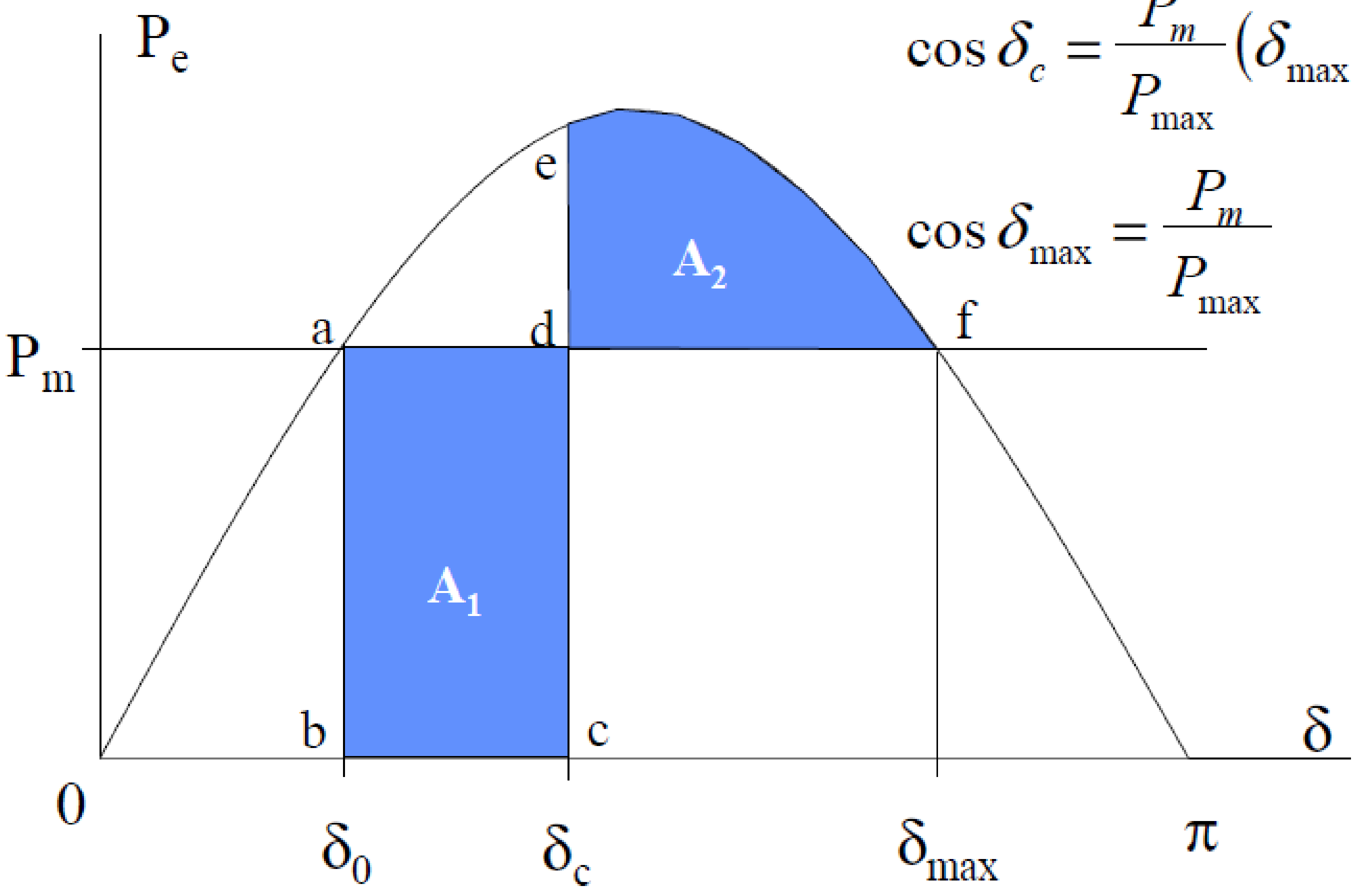
4. Short circuit occurring in the system

Critical Clearing Time



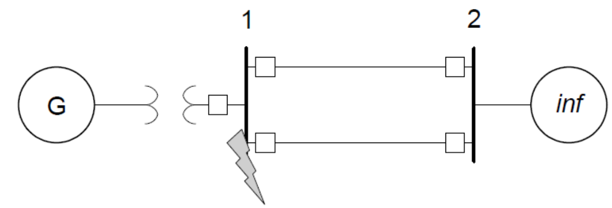
$$\cos\delta_c = \frac{P_m}{P_{max}}(\delta_{max} - \delta_c) + \cos\delta_{max}$$

$$\cos\delta_{max} = \frac{P_m}{P_{max}}$$



4. Short circuit occurring in the system

Critical Clearing Time



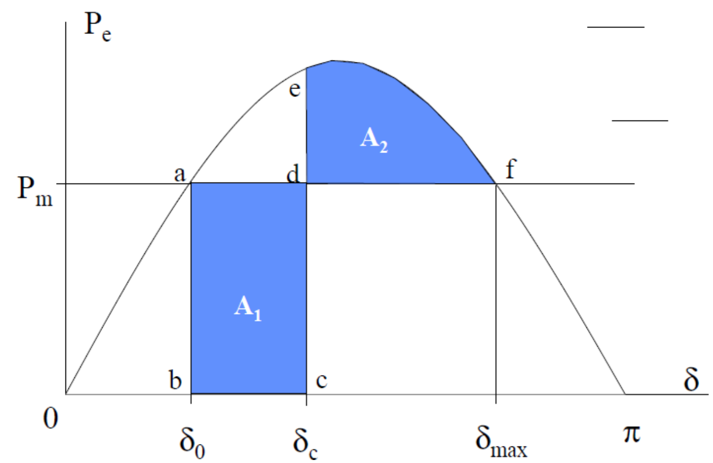
$$\frac{H}{\pi f_0}\frac{d^2\delta}{dt^2} = P_m - P_e = P_m \neg\ P_e = 0$$

$$\frac{d^2\delta}{dt^2} = \frac{\pi f_0}{H} P_m$$

$$\frac{d\delta}{dt} = \frac{\pi f_0}{H} P_m \int_0^t dt = \frac{\pi f_0}{H} P_m t$$

$$\delta = \frac{\pi f_0}{2H} P_m t^2 + \delta_0$$

$$t_c = \sqrt{\frac{2H(\delta_c - \delta_0)}{\pi f_0 P_m}}$$



Example 5

A generator is transferring power to a load through a short line as shown in Fig. E3. The power angle equation is $P_e = P_{\max} \sin\delta$. The initial power is P_m pu when a 3-phase fault occurs at the terminals of generator.

a. Use equal are criterion to find equation for critical clearing angle and the critical clearing time.

b. Find the critical clearing time angle δ_{cc} if $P_{max} = 2$ and $P_m = 1.0$ pu. H=6 MJ/MVA and f_o=50Hz.

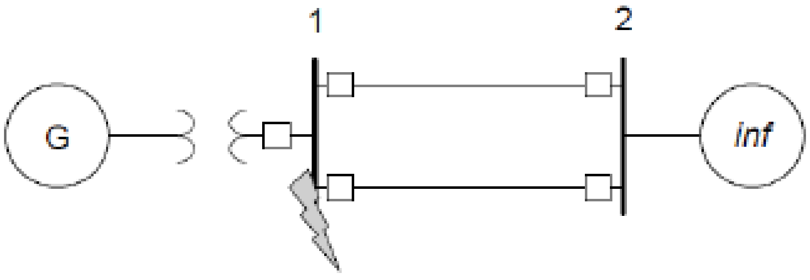


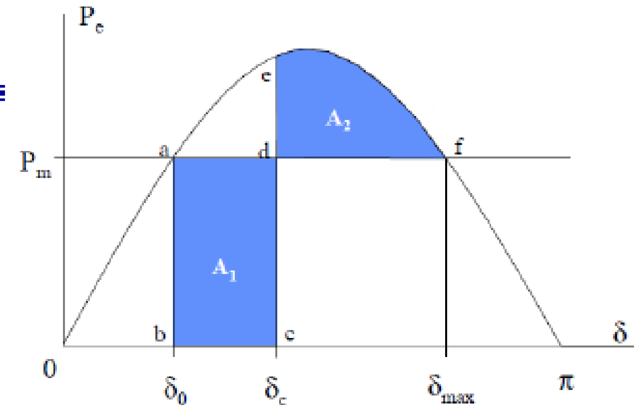
Fig. E3

Solution:

$$A_1 = \int_{\delta_o}^{\delta_c} P_m d\delta = P_m(\delta_c - \delta_o)$$

$$A_2 = \int_{\delta_c}^{\delta_{\max}} (P_{mac} \sin\delta - P_m) d\delta = -P_{\max}(\cos\delta_{\max} - \cos\delta_c) - P_m(\delta_{\max} - \delta_c)$$

Example 5, Cont'd



For stability $A_1 = A_2$.

$$P_m(\delta_c - \delta_o) = -P_{max}(\cos\delta_{max} - \cos\delta_c) - P_m(\delta_{max} - \delta_c)$$

but $\delta_{max} = \pi - \delta_o$ and $P_m = P_{max}\sin\delta_o$

$$P_m(\delta_c - \delta_o) = P_{max}\cos\delta_c - P_{max}\cos\delta_{max} - P_m(\pi - \delta_o - \delta_c)$$

$$P_{max}(\delta_c - \delta_o)\sin\delta_o = P_{max}\cos\delta_c - P_{max}\cos\delta_{max} - P_{max}(\pi - \delta_o - \delta_c)\sin\delta_o$$

$$P_{max}(\delta_c - \delta_o)\sin\delta_o = P_{max}\cos\delta_c + P_{max}\cos\delta_o - P_{max}(\pi - \delta_o - \delta_c)\sin\delta_o$$

$$(\delta_c - \delta_o)\sin\delta_o = \cos\delta_c + \cos\delta_o - (\pi - \delta_o - \delta_c)\sin\delta_o$$

$$\cos\delta_c = (\pi - 2\delta_o)\sin\delta_o - \cos\delta_o$$

$$\delta_c = \cos^{-1}[(\pi - 2\delta_o)\sin\delta_o - \cos\delta_o]$$

During fault, power transfer is zero.

$$M\frac{d^2\delta}{dt^2} = P_m - 0 = P_{max}\sin\delta_o$$

$$\int_{\omega_s}^{\omega} d\omega = \int_0^t \frac{P_m}{M}dt$$

$$\frac{d^2\delta}{dt^2} = \frac{P_m - 0}{M} = \frac{P_{max}\sin\delta_o}{M}$$

$$\omega - \omega_s = \omega_r = \frac{d\delta}{dt} = \frac{P_m}{M}t \Rightarrow d\delta = \frac{P_m}{M}tdt$$

$$\frac{d}{dt}\left(\frac{d\delta}{dt}\right) = \frac{P_m}{M} \Rightarrow d\left(\frac{d\delta}{dt}\right) = \frac{P_m}{M}dt$$

$$\int_{\delta_o}^{\delta} d\delta = \int_0^t \frac{P_m}{M}tdt$$

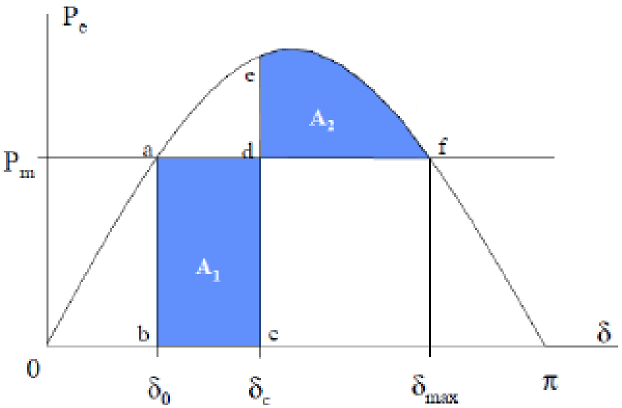
$$\delta(t) = \frac{P_m}{2M}t^2 + \delta_o$$

$$(d\omega) = \frac{P_m}{M}dt$$

Example 5, Cont'd

$$at\ t = t_c \Rightarrow \delta = \delta_c \Rightarrow \delta_c = \frac{P_m}{2M}t_c^2 + \delta_o$$

$$t_c = \sqrt{\frac{2M}{P_m}(\delta_c - \delta_o)} = \sqrt{\frac{2H}{\pi f_o P_m}(\delta_c - \delta_o)}$$



b. P_{max} = 2 and P_m =1.0 pu. H=6 MJ/MVA and f_o=50Hz.

$$\Rightarrow M = \frac{H}{\pi f_o} = \frac{6}{\pi \times 50}, \quad P_m = P_{\max}\sin\delta_o \Rightarrow 1.0 = 2\sin\delta_o \Rightarrow \delta_o = 30° \ (0.5236\ rad)$$

$$\delta_c = \cos^{-1}[(\pi - 2\delta_o)\sin\delta_o - \cos\delta_o] = \cos^{-1}\left[(\pi - 2\times 0.5236)\sin 30° - \cos 30°\right]$$

$$\delta_c = 79.6°\ 1.389\ rad$$

$$t_c = \sqrt{\frac{2H}{\pi f_o P_m}(\delta_c - \delta_o)} = \sqrt{\frac{2\times 6}{\pi\times 50\times 1}(1.389 - 0.52336)}$$

$$t_c = 0.257\ s$$

$$t_c = 12.85\ cycles\ of\ 50\ Hz$$

4. Short circuit occurring in the system

- Short circuit occurring in the system often causes loss of stability even though the fault may be removed by isolating it from the rest of the system in a relatively short time. A three phase fault at one end of a double circuit line is shown in Fig.9(a) which can be reduced as shown in Fig. 9(b).

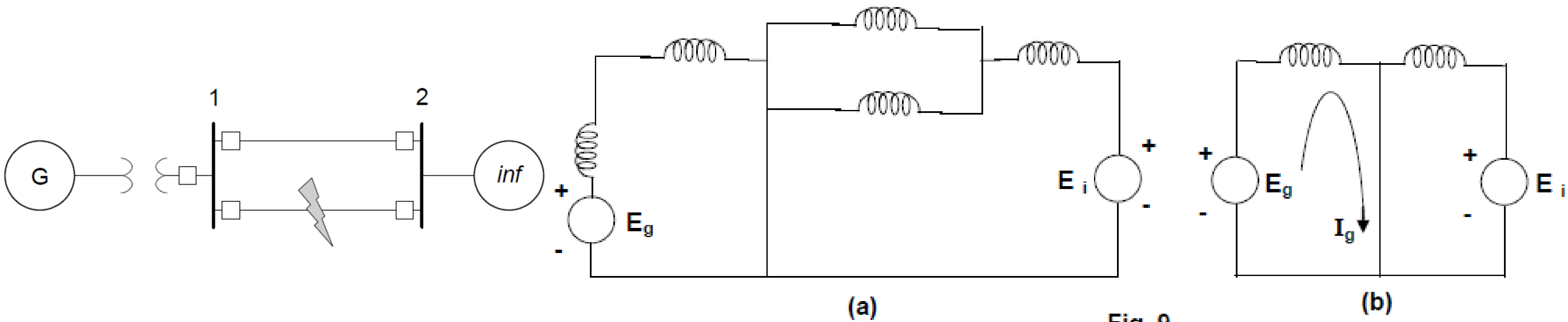


Fig. 9

- It is to be noted that all the current from the generator flows through the fault and this current I_g lags the generator voltage by 90°. Thus the real power output of the generator is zero. Normally the input power to the generator remains unaltered. Therefore, if the fault is sustained, the load angle δ will increase indefinitely because entire the input power will be used for acceleration. This may result in unstable condition.

4. Short circuit occurring in the system

- When the three phase fault occurring at one end of a double circuit line is disconnected by opening the circuit breakers at both ends of the faulted line, power is again transmitted. If the fault is cleared before the rotor angle reaches a particular value, the system will remain stable; otherwise it will loose stability as shown in Fig. 10.

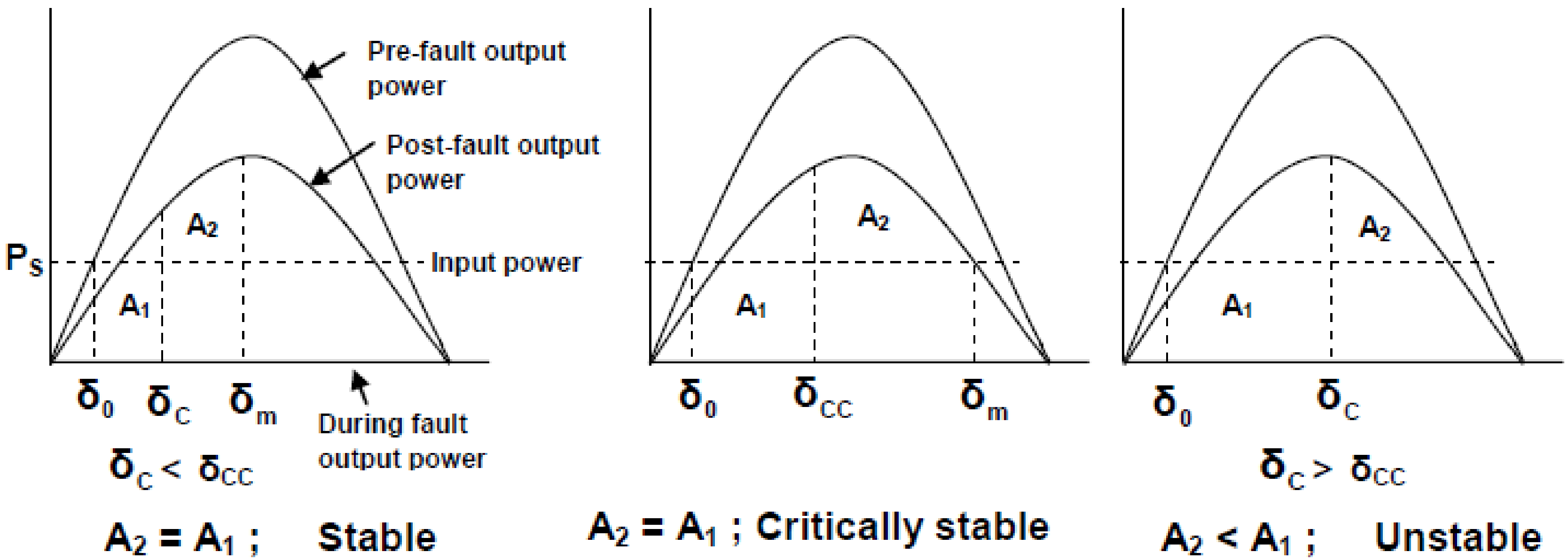


Fig. 10

- Note that the areas A1 and A2 are obtained by finding difference between INPUT and OUTPUT.

4. Short circuit occurring in the system

- When a three phase fault occurs at some point on a double circuit line, other than on the extreme ends, as shown in Fig. 11(a), there is some finite impedance between the paralleling buses and the fault. Therefore, some power is transmitted during the fault and it may be calculated after reducing the network to a delta connected circuit between the internal voltage of the generator and the infinite bus as shown in Fig. 11(b).

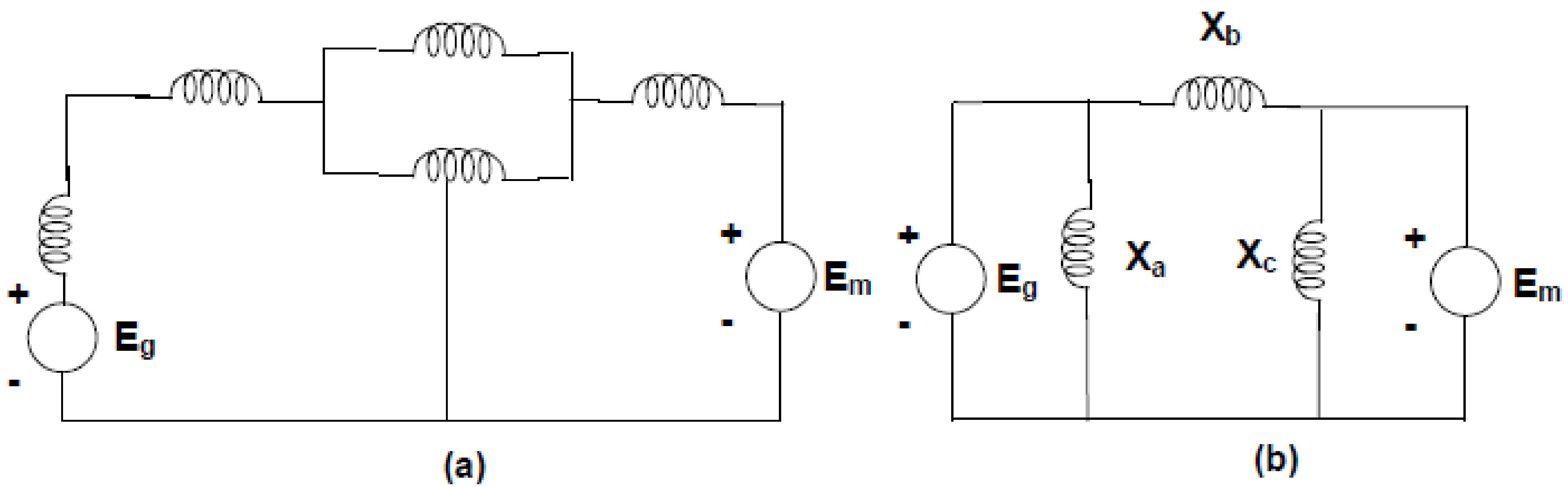
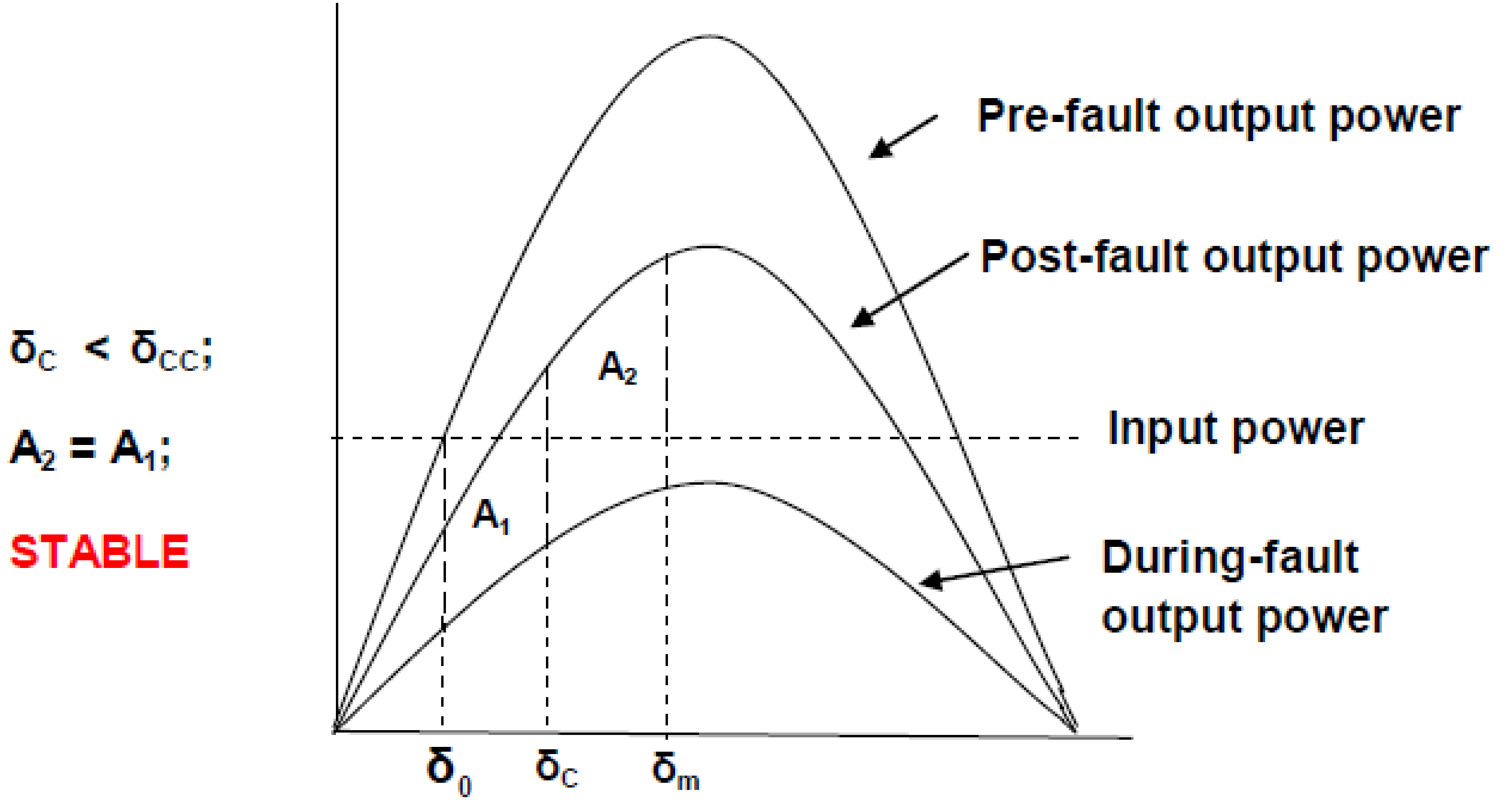


Fig. 11

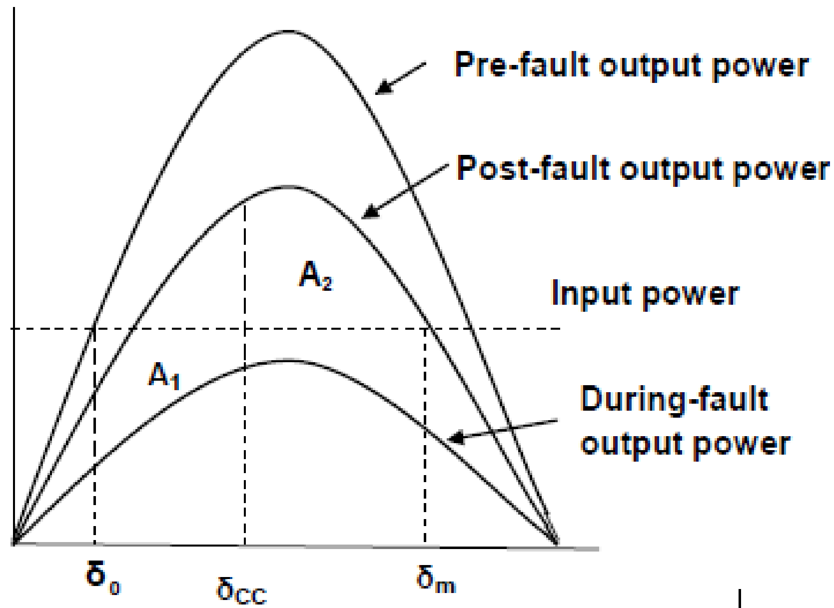
$$\text{Power transmitted during the fault} = \frac{|E_g||E_m|}{X_b}\sin\delta \qquad (23)$$

4. Short circuit occurring in the system

- Stable, critically stable and unstable conditions of such systems are shown:



4. Short circuit occurring in the system



$\delta_C = \delta_{CC}$;

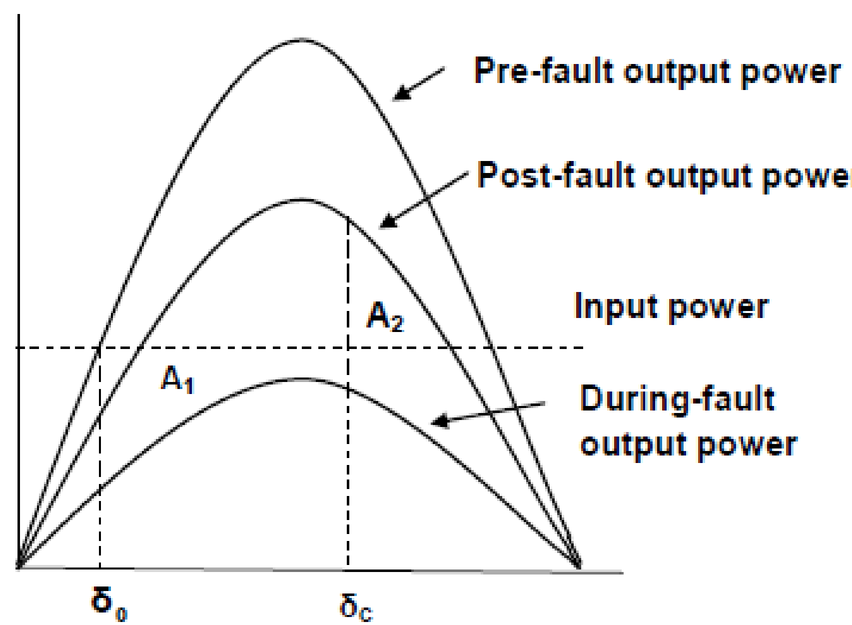
$A_2 = A_1$;

CRITICALLY STABLE

$\delta_C > \delta_{CC}$;

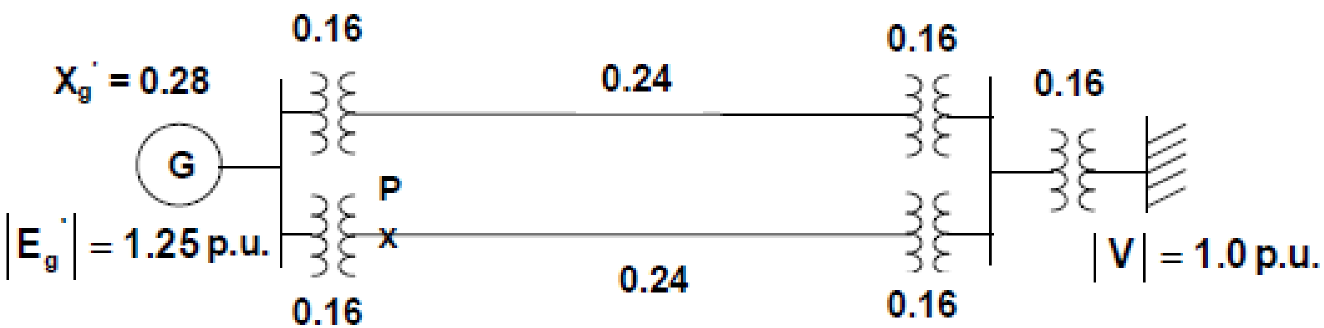
$A_2 < A_1$;

UNSTABLE



Example 6

In the power system shown in Fig. 12, three phase fault occurs at P and the faulty line was opened a little later. Find the power output equations for the pre-fault, during fault and post-fault conditions.

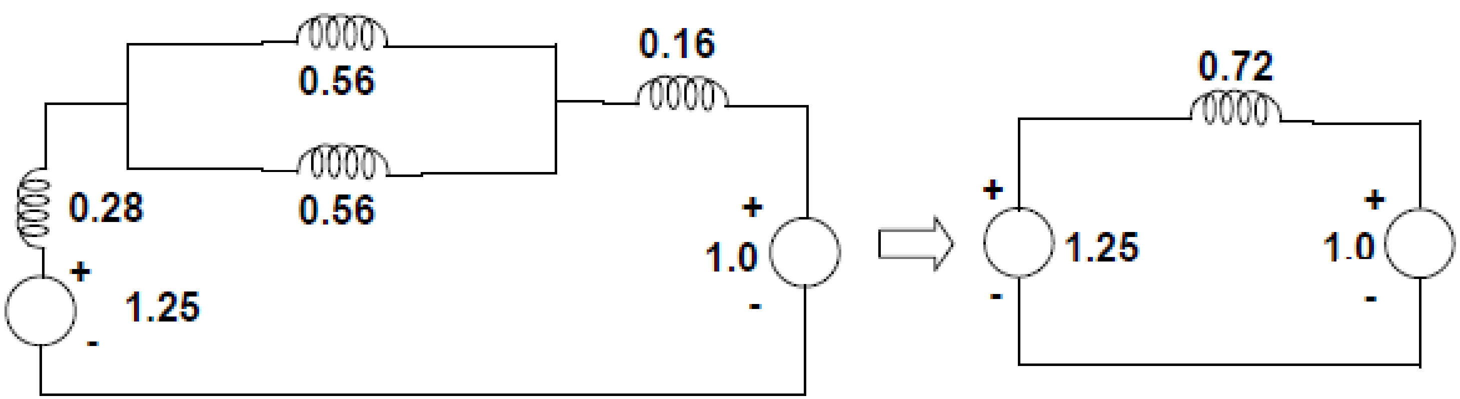


Values marked are p.u. reactances

Fig. 12

<u>Solution</u>

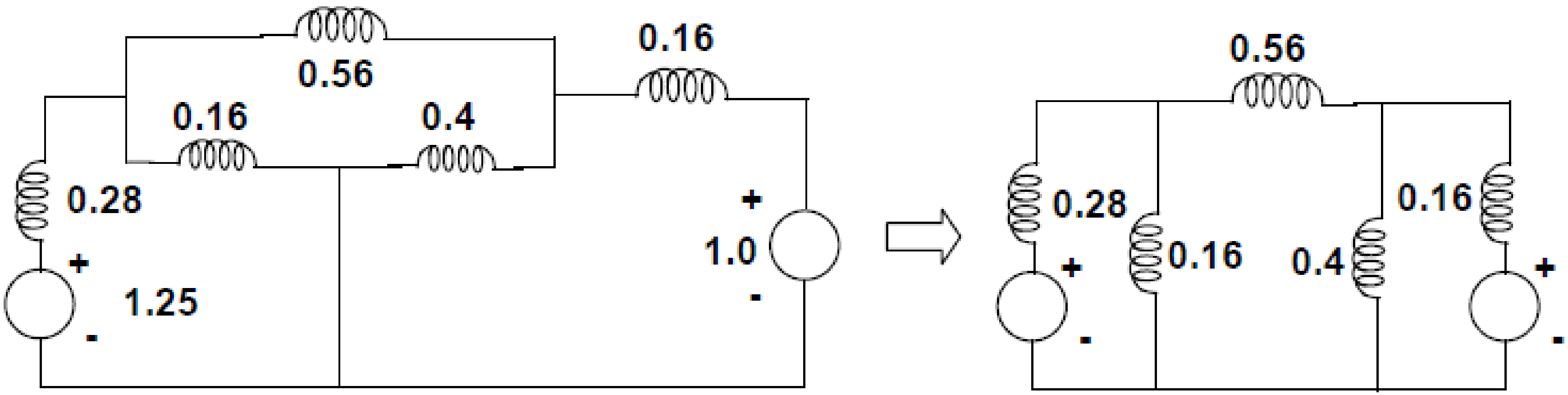
Pre-fault condition

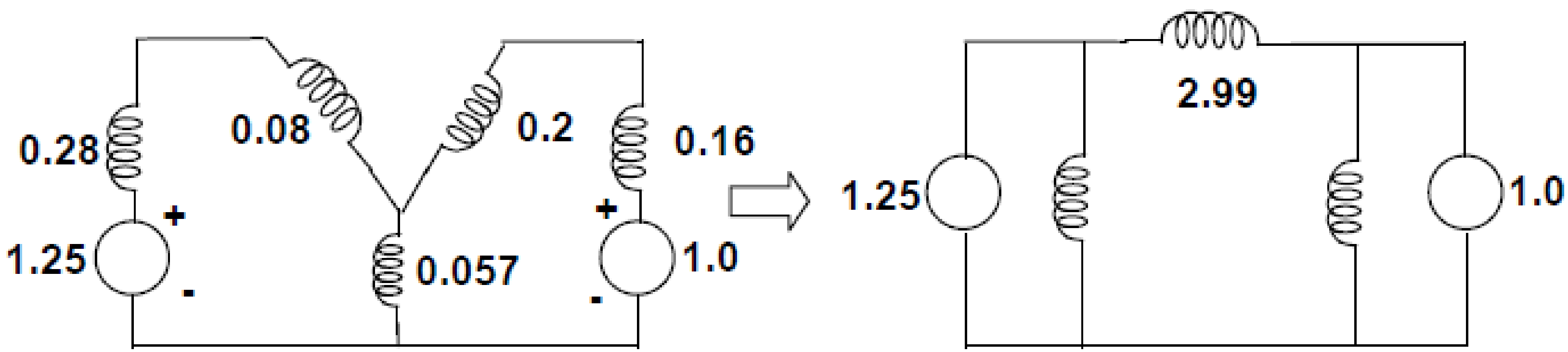


$$\text{Power output } P_e = \frac{1.25 \times 1.0}{0.72} \sin\delta = 1.736 \sin\delta$$

Example 6

During fault condition:



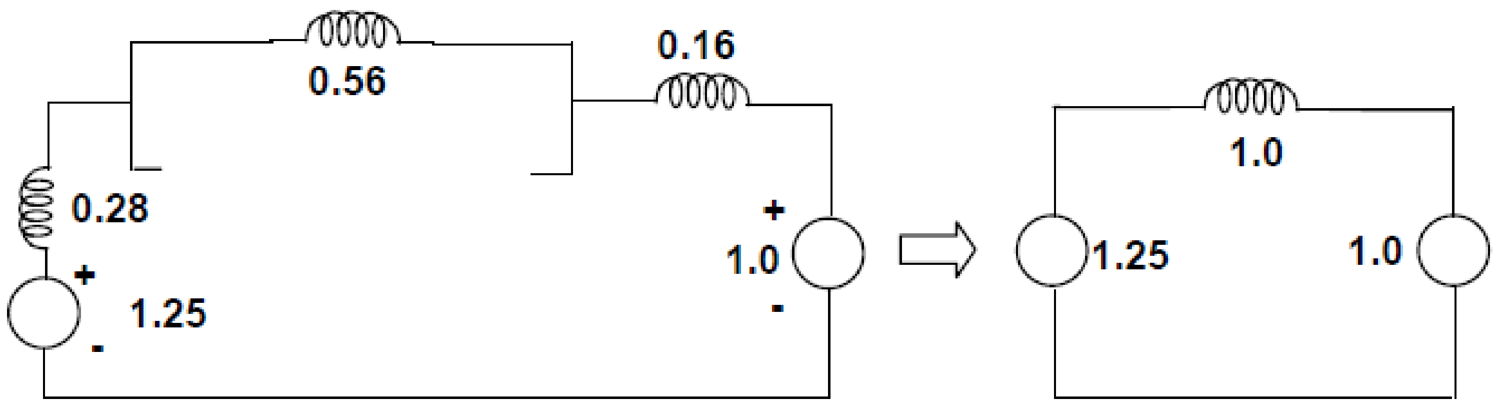


(0.36x0.36 + 0.36x0.057 + 0.057x0.36) / 0.057 = 2.99

Power output $P_e = \dfrac{1.25 \times 1.0}{2.99} \sin\delta = 0.418 \sin\delta$

Example 6

Post-fault condition:



Power output $P_e = \dfrac{1.25 \times 1.0}{1.0} \sin\delta = 1.25 \sin\delta$

Thus power output equations are:

Pre-fault	$P_e = P_{m1} \sin\delta = 1.736 \sin\delta$
During fault	$P_e = P_{m2} \sin\delta = 0.418 \sin\delta$
Post fault	$P_e = P_{m3} \sin\delta = 1.25 \sin\delta$

Here

$P_{m1} = 1.736$; $P_{m2} = 0.418$; $P_{m3} = 1.25$;

Expression for critical clearing angle δ_{cc}

Consider the power angle diagrams shown in Fig. 13

$$P_{m1} \sin \delta_0 = P_s$$

$$P_{m3} \sin \delta_s = P_s$$

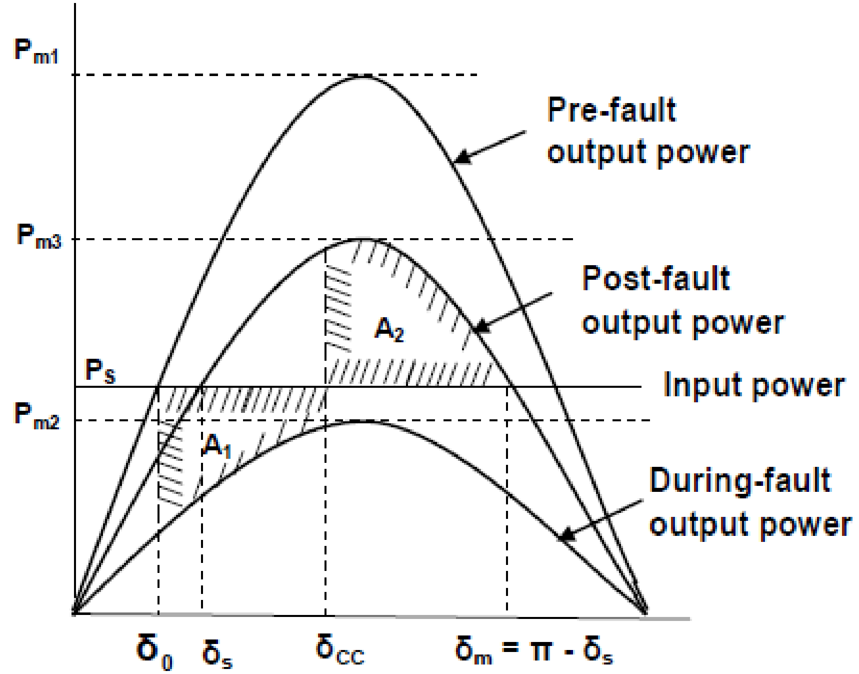


Fig. 13

$$\text{Area } A_1 = P_s(\delta_{CC} - \delta_0) - \int_{\delta_0}^{\delta_{CC}} P_{m2} \sin\delta \, d\delta$$

$$= P_s \delta_{CC} - P_s \delta_0 + P_{m2} \cos \delta_{CC} - P_{m2} \cos \delta_0 \qquad (24)$$

$$\text{Area } A_2 = \int_{\delta_{CC}}^{\delta_m} P_{m3} \sin\delta \, d\delta - P_s(\delta_m - \delta_{CC})$$

$$= P_{m3} \cos \delta_{CC} - P_{m3} \cos \delta_m - P_s \delta_m + P_s \delta_{CC} \qquad (25)$$

Expression for critical clearing angle δ_{cc}

$$A_1 = P_s\,\delta_{CC} - P_s\,\delta_0 + P_{m2}\cos\delta_{CC} - P_{m2}\cos\delta_0 \qquad (24)$$

$$A_2 = P_{m3}\cos\delta_{CC} - P_{m3}\cos\delta_m - P_s\,\delta_m + P_s\,\delta_{CC} \qquad (25)$$

Area A_2 = Area A_1

$$P_{m3}\cos\delta_{CC} - P_{m3}\cos\delta_m - P_s\,\delta_m + \cancel{P_s\,\delta_{CC}} = \cancel{P_s\,\delta_{CC}} - P_s\delta_0 + P_{m2}\cos\delta_{CC} - P_{m2}\cos\delta_0$$

$$(P_{m3} - P_{m2})\cos\delta_{CC} = P_s(\delta_m - \delta_0) + P_{m3}\cos\delta_m - P_{m2}\cos\delta_0$$

$$\cos\delta_{CC} = \frac{P_s(\delta_m - \delta_0) + P_{m3}\cos\delta_m - P_{m2}\cos\delta_0}{P_{m3} - P_{m2}}$$

Thus CRITICAL CLEARING ANGLE is given by

$$\delta_{CC} = \cos^{-1}\left[\frac{P_s(\delta_m - \delta_0) + P_{m3}\cos\delta_m - P_{m2}\cos\delta_0}{P_{m3} - P_{m2}}\right] \qquad (26)$$

Here the angles are in radian. Further, since

$P_{m1}\sin\delta_0 = P_s$, $P_{m3}\sin\delta_s = P_s$ and $\delta_m = \pi - \delta_s$ angles δ_0 and δ_m are given by

$$\delta_0 = \sin^{-1}\left(\frac{P_s}{P_{m1}}\right) \qquad \delta_m = \pi - \sin^{-1}\left(\frac{P_s}{P_{m3}}\right) \qquad (27)$$

Example 6

In the power system described in the previous example, if the generator was delivering 1.0 p.u. just before the fault occurs, calculate δ_{cc} .

<u>Solution</u>

$P_{m1} = 1.736;$ $P_{m2} = 0.418;$ $P_{m3} = 1.25;$ $P_s = 1.0$

$1.736 \sin\delta_0 = 1.0;$ $\sin\delta_0 = 0.576;$ $\delta_0 = 0.6139$ rad.

$1.25 \sin \delta_s = 1.0;$ $\sin \delta_s = 0.8;$ $\delta_s = 0.9273$ rad.; $\delta_m = \pi - \delta_s = 2.2143$ rad.

$$\cos \delta_{cc} = \frac{P_s(\delta_m - \delta_0) + P_{m3} \cos \delta_m - P_{m2} \cos\delta_0}{P_{m3} - P_{m2}}$$

$$= \frac{1.0(2.2143 - 0.6139) + 1.25 \cos 2.2143 - 0.418 \cos 0.6139}{1.25 - 0.418} = 0.6114$$

Critical clearing angle $\delta_{cc} = 52.31^0$

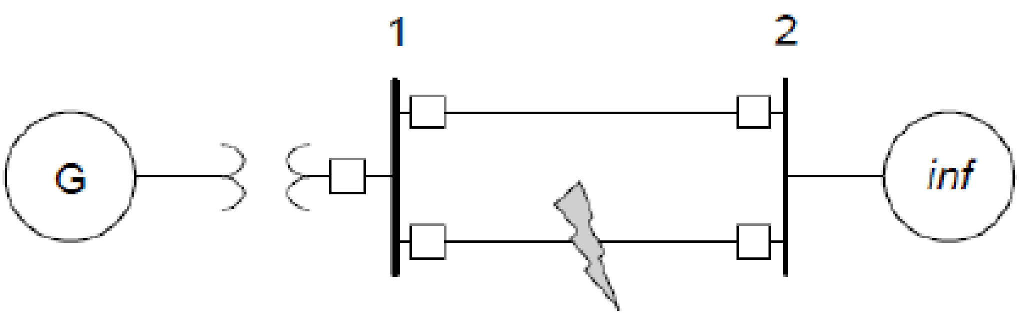
Example 7

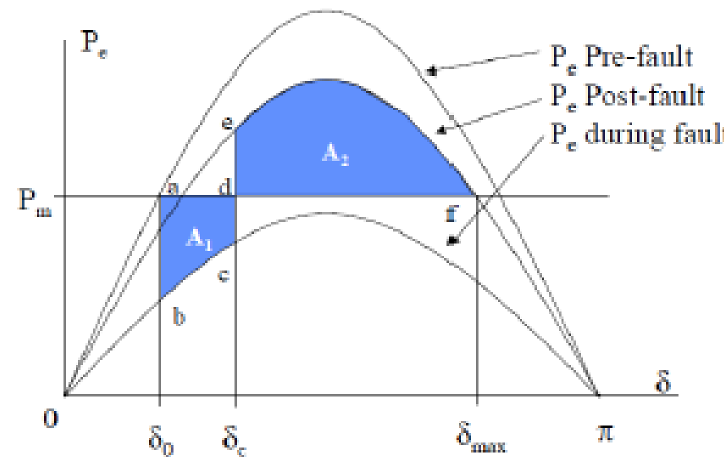
A balanced 3-phase fault occurs at middle point of line 2 when the power transfer is 1.5 pu in the system. The system data are $|E|$ = 1.2 pu, $|V_\infty|$ = 1.0 pu, X_d'= 0.2 pu, X_1= X_2= 0.4 pu.

a. Determine whether the system is stable for a sustained fault.

b. The fault is cleared at $\delta = 60^o$. Is the system stable? If so find the maximum rotor swing.

c. Find the critical clearing angle





Solution:

a. Pre-fault condition

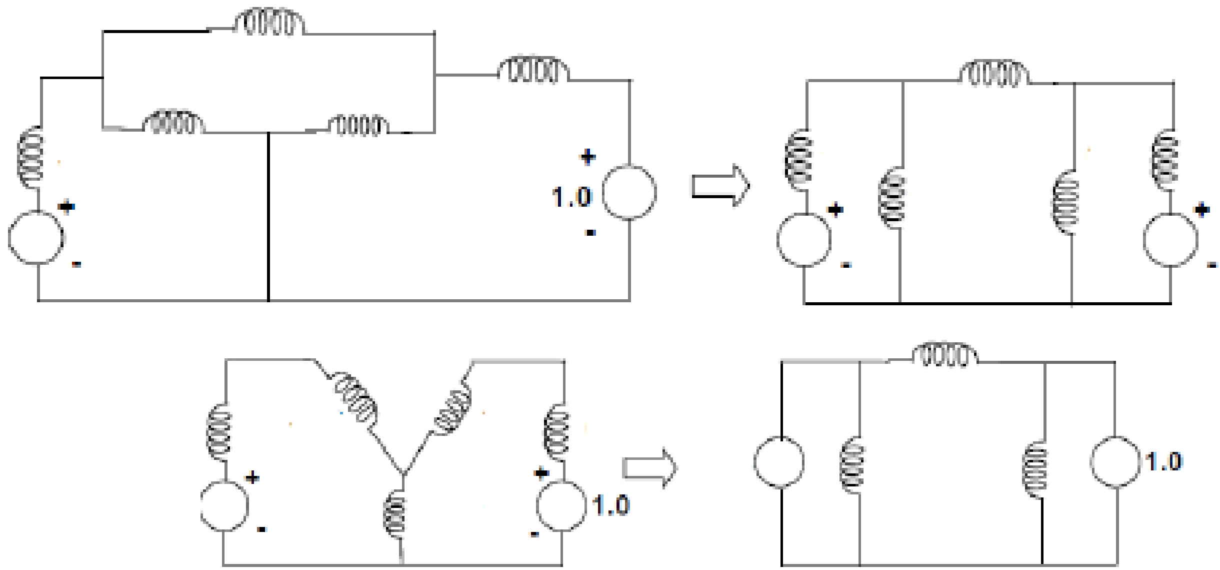
Transfer reactance X_{eq}= 0.2+0.4//0.4=0.4 pu

$$P_e = \frac{EV_\infty}{X_{eq}}\sin\delta = \frac{EV_\infty}{X_d' + \dfrac{X_1 \times X_2}{X_1 + X_2}}\sin\delta = \frac{1.2\times 1}{0.2 + \dfrac{0.4\times 0.4}{0.4+0.4}}\sin\delta = \frac{1.2}{0.4}\sin\delta = 3\sin\delta$$

Example 7, Cont’d

b. During-fault condition

$$P_e' = \frac{1.2\times 1}{1.0}\sin\delta = 1.2\sin\delta$$

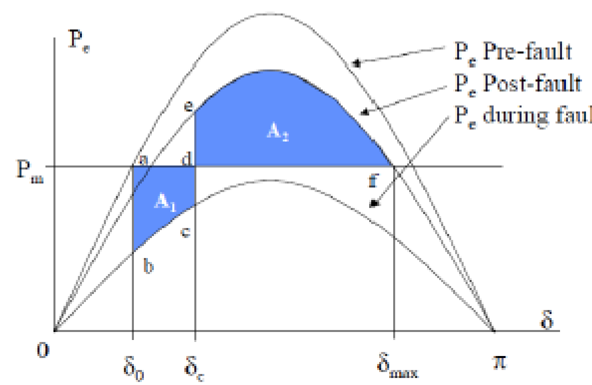


Since the initial load is 1.5 pu , and the maximum possible value of power transfer during faut is condition is 1,2 pu, therefore stability is impossible for a sustained fault.

c. Post-fault condition

$$P_e'' = \frac{1.2\times 1}{0.2+0.4}\sin\delta = \frac{1.2}{0.6}\sin\delta = 2\sin\delta$$

$$1.5\delta_2 + 2\cos(\delta_2) = 2.225 \Rightarrow \delta_2 = 1.848\ rad\ (105.9°)$$



$$\cos\delta_{cc} = \frac{P_m(\delta_{max}-\delta_o) + P_{3max}\cos\delta_{max} - P_{2max}\cos\delta_o}{P_{3max}-P_{2max}}$$

$$= \frac{1.5(2.293-0.524) + 2\cos(2.293) - 1.2\cos(0.524)}{2-1.2}$$

$$\delta_{cc} = 1.196\,rad\ (68.6°)$$

Steady State Stability

- **The ability of the power system to remain in synchronism when subject to small disturbances**
- **Stability is assured if the system returns to its original operating state (voltage magnitude and angle profile)**
- **The behavior can be determined with a linear system model**

  - **Assumption:**
    - the automatic controls are not active
    - the power shift is not large
    - the voltage angles changes are small

Steady State Stability

- **Simplification of the swing equation**

$$\frac{H}{\pi f_0}\frac{d^2\delta_0}{dt^2}+\frac{H}{\pi f_0}\frac{d^2\Delta\delta}{dt^2}=P_m-P_{max}\left[\sin\delta_0\cos\Delta\delta+\cos\delta_0\sin\Delta\delta\right]$$

Substitute the following approximations

$$\Delta\delta << \delta \qquad \cos\Delta\delta\approx 1 \qquad \sin\Delta\delta\approx\Delta\delta$$

$$\frac{H}{\pi f_0}\frac{d^2\delta_0}{dt^2}+\frac{H}{\pi f_0}\frac{d^2\Delta\delta}{dt^2}=P_m-P_{max}\sin\delta_0-P_{max}\cos\delta_0\cdot\Delta\delta$$

Group steady state and transient terms

$$\frac{H}{\pi f_0}\frac{d^2\delta_0}{dt^2}-P_m+P_{max}\sin\delta_0=-\frac{H}{\pi f_0}\frac{d^2\Delta\delta}{dt^2}-P_{max}\cos\delta_0\cdot\Delta\delta$$

Steady State Stability

- **Simplification of the swing equation**

$$\frac{H}{\pi f_0}\frac{d^2\delta_0}{dt^2} - P_m + P_{max}\sin\delta_0 = -\frac{H}{\pi f_0}\frac{d^2\Delta\delta}{dt^2} - P_{max}\cos\delta_0 \cdot \Delta\delta$$

$$0 = \frac{H}{\pi f_0}\frac{d^2\Delta\delta}{dt^2} + P_{max}\cos\delta_0 \cdot \Delta\delta$$

Steady state term is equal to zero

$$\left.\frac{dP_e}{d\delta}\right|_{\delta_0} = \left.\frac{d}{d\delta}P_{max}\sin\delta\right|_{\delta_0} = P_{max}\cos\delta_0 = P_s$$

$$\frac{H}{\pi f_0}\frac{d^2\Delta\delta}{dt^2} + P_s \cdot \Delta\delta = 0$$

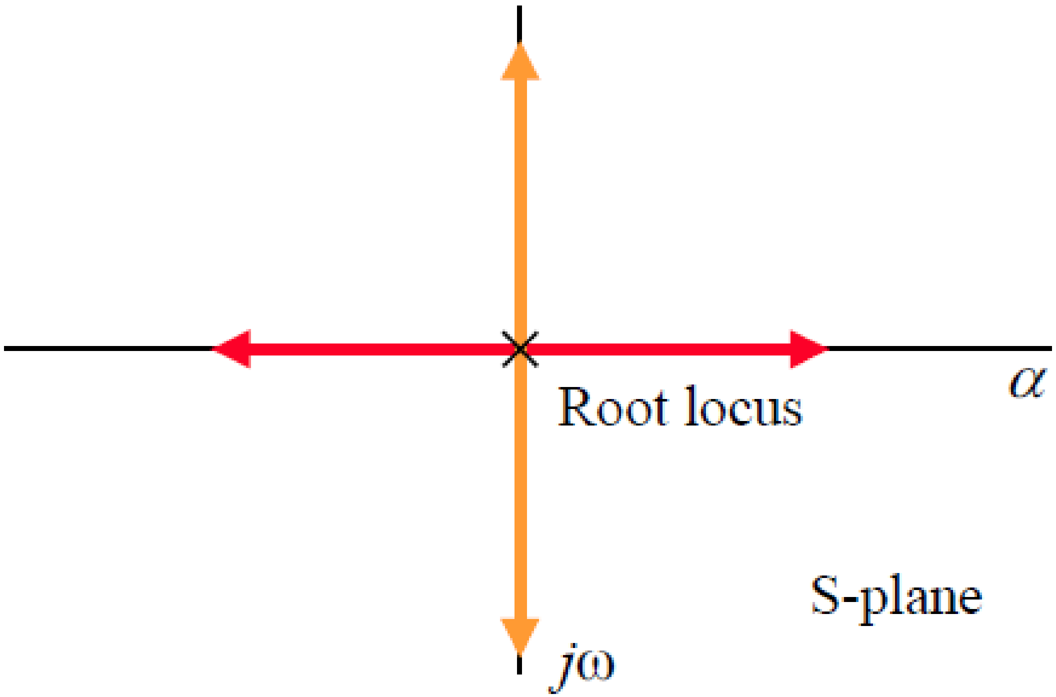
Second order equation.
The solution depends on the roots of the characteristic equation

Steady State Stability

- **Stability Assessment**
  - When P_s is negative, one root is in the right-half s-plane, and the response is exponentially increasing and stability is lost
  - When P_s is positive, both roots are on the $j\omega$ axis, and the motion is oscillatory and undamped, the natural frequency is:

$$s^2 = -\frac{\pi f_0}{H} P_S$$

$$\omega_n = \sqrt{\frac{\pi f_0}{H} P_S}$$



Damping Torque

$$P_D = D\frac{d\delta}{dt}$$

Damping force is due to air-gap interaction

$$\frac{H}{\pi\ f_0}\frac{d^2\Delta\delta}{dt^2} + D\frac{d\Delta\delta}{dt} + P_S\Delta\delta = 0$$

$$\frac{d^2\Delta\delta}{dt^2} + \frac{\pi\ f_0}{H}D\frac{d\Delta\delta}{dt} + \frac{\pi\ f_0}{H}P_S\Delta\delta = 0$$

$$\frac{d^2\Delta\delta}{dt^2} + 2\zeta\,\omega_n\frac{d\Delta\delta}{dt} + \omega_n^2\Delta\delta = 0$$

$$\zeta = \frac{D}{2}\sqrt{\frac{\pi\ f_0}{H\,P_S}}$$

Characteristic Equation

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

$$\zeta = \frac{D}{2}\sqrt{\frac{\pi f_0}{H P_S}} < 1 \qquad \text{for normal operation conditions}$$

$$s_1, s_2 = -\zeta\omega_n \pm j\,\omega_n\sqrt{1-\zeta^2} \qquad \text{complex roots}$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2} \qquad \text{the damped frequency of oscillation}$$

Laplace Transform Analysis

$$x_1 = \Delta\delta, \quad x_2 = \frac{d\Delta\delta}{dt}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$$

$$\mathcal{L}\{\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}\} \rightarrow s\mathbf{X}(s) - \mathbf{x}(0) = \mathbf{A}\mathbf{X}(s)$$

$$\mathbf{X}(s) = (s\mathbf{I} - \mathbf{A})^{-1}\mathbf{x}(0)$$

$$(s\mathbf{I} - \mathbf{A}) = \begin{bmatrix} s & -1 \\ \omega_n^2 & s + 2\zeta\omega_n \end{bmatrix}$$

$$\mathbf{X}(s) = \frac{\begin{bmatrix} s + 2\zeta\omega_n & 1 \\ -\omega_n^2 & s \end{bmatrix}}{s^2 + 2\zeta\omega_n s + \omega_n^2}\mathbf{x}(0)$$

Laplace Transform Analysis

$$\Delta\delta(s) = \frac{(s + 2\zeta\omega_n)\Delta\delta_0}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\Delta\omega(s) = \frac{\omega_n^2 \Delta\delta_0}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

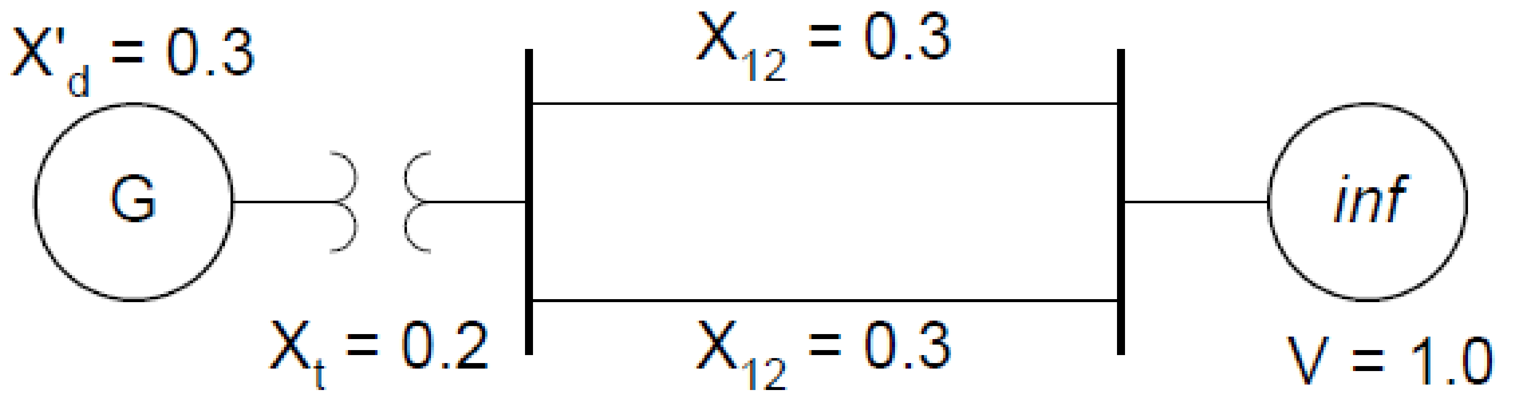
$$\Delta\delta(t) = \frac{\Delta\delta_0}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_d t + \theta), \quad \theta = \cos^{-1}\zeta$$

$$\Delta\omega(t) = -\frac{\omega_n \Delta\delta_0}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_d t)$$

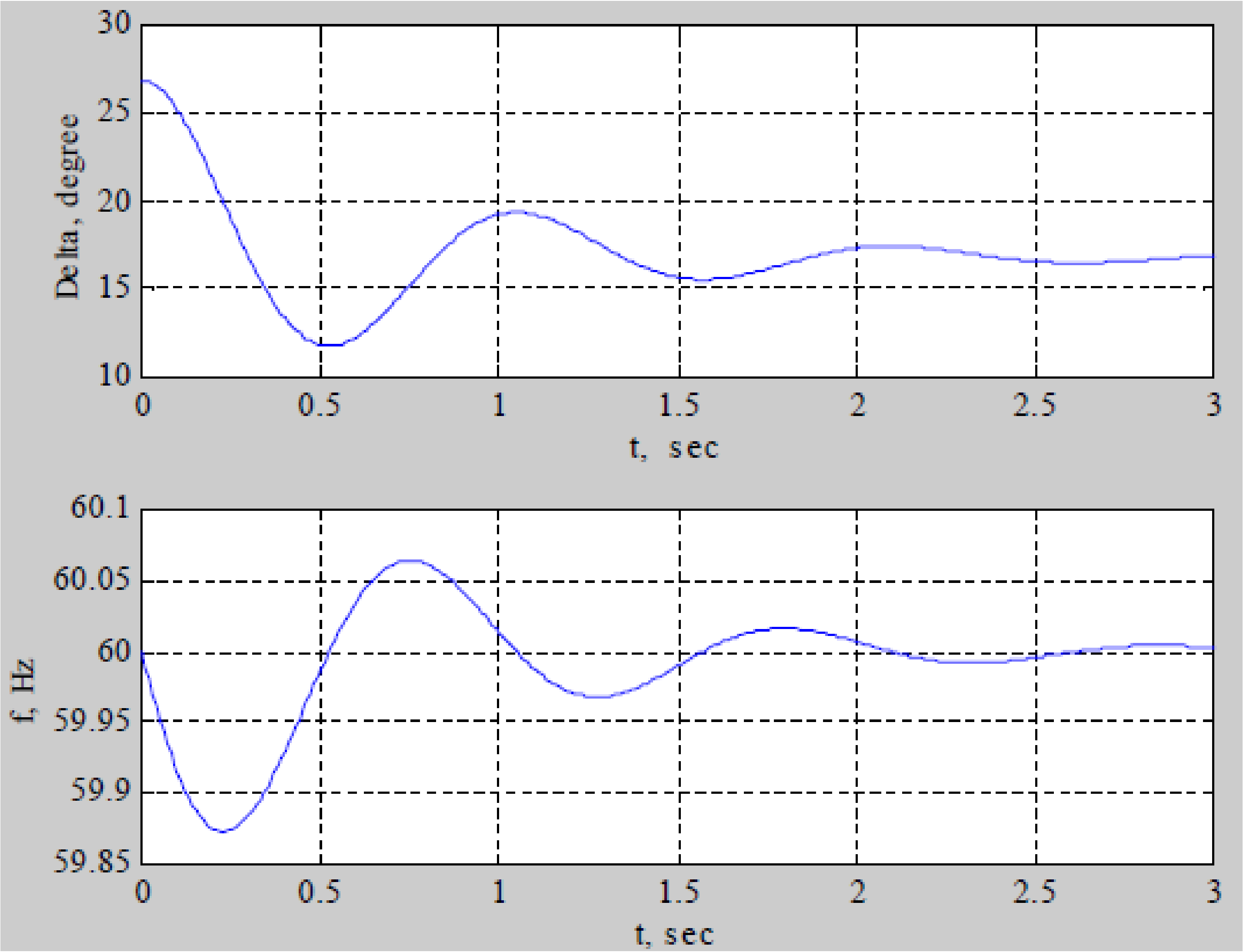
$$\delta(t) = \delta_0 + \Delta\delta(t), \quad \omega(t) = \omega_0 + \Delta\omega(t)$$

Example

A 60 Hz synchronous generator having inertia constant H = 9.94 MJ/MVA and a transient reactance X_d = 0.3 pu is connected to an infinite bus through the following network. The generator is delivering 0.6 *pu* real power at 0.8 power factor lagging to the infinite bus at a voltage of 1 *pu*. Assume the damping power coefficient is $D = 0.138$ *pu*. Consider a small disturbance of 10^o or 0.1745 radians. Obtain equations of rotor angle and generator frequency motion.



Example


30
25
20
15
10
Delta, degree
0
0.5
1
1.5
2
2.5
3
t, sec
60.1
60.05
60
59.95
59.9
59.85
f, Hz
0
0.5
1
1.5
2
2.5
3
t, sec

Question # 1:

A four-pole, 60-Hz synchronous generator has a rating of 200 MVA, 0.8 power factor lagging. The moment of inertia of the rotor is 45,100 kgm^2. Determine *M* and *H*.

Solution:

$$n_s = \frac{120 f_0}{P} = 1800 \ \text{rpm}$$
$$\omega_{sm} = \frac{2\pi n_s}{60} = 188.495 \ \text{rad/sec}$$

$$W_k = \frac{1}{2} J\omega_{sm}^2 = \frac{1}{2}(45100)(188.495)^2 = 801.2 \ \text{MJ}$$

$$M = \frac{2W_k}{\omega_{sm}} = \frac{2(801.2)}{188.495} = 8.5\text{MJ sec}$$
$$H = \frac{W_k}{S_B} = \frac{801.2}{200} = 4 \ \text{MJ/MVA}$$

Question # 2:

A two-pole, 60-Hz synchronous generator has a rating of 250 MVA, 0.8 power factor lagging. The kinetic energy of the machine at synchronous speed is 1080 MJ. The machine is running steadily at synchronous speed and delivering 60 MW to a load at a power angle of 8 electrical degrees. The load is suddenly removed. Determine the acceleration of the rotor. If the acceleration computed for the generator is constant for a period of 12 cycles, determine the value of the power angle and the rpm at the end of this time.

Solution:

$$H = \frac{W_k}{S_B} = \frac{1080}{250} = 4.32 \ \text{MJ/MVA}$$, $$P_m = \frac{60}{250} = 0.24 \ \text{pu}$$, $$12 \text{ cycle} = \frac{12}{60} = 0.2 \ \text{sec}$$

$$\frac{H}{180 f}\frac{d^2\delta}{dt^2} = 0.24 - 0$$, or
$$\frac{d^2\delta}{dt^2} = 600 \ \text{degree/sec}^2$$
$$= \frac{60}{360}(600) = 100 \ \text{rpm/sec}$$

Integrating the above expression, we have

$$\frac{d\delta}{dt} = 600t$$
$$\delta = \frac{1}{2}(600)t^2 + \delta_0$$
$$= \frac{1}{2}(600)(0.2)^2 + 8 = 20^\circ$$, $$n_s = \frac{120 f_0}{P} = \frac{(120)(60)}{2} = 3600 \ \text{rpm}$$

Speed at the end of 0.2 second is

$$n = 3600 + 0.2(100) = 3620 \ \text{rpm}$$

Question # 3:

Determine the kinetic energy stored by a 250-MVA, 60-Hz, two-pole synchronous generator with an inertia constant *H* of 5.4 MJ/MVA. Assume the machine is running steadily at synchronous speed with a shaft input of 331,100 hp. The electrical power developed suddenly changes from its normal value to a value of 200 MW. Determine the acceleration or deceleration of the rotor. If the acceleration computed for the generator is constant for a period of 9 cycles, determine the change in the power angle in that period and the rpm at the end of 9 cycles.

Solution:

$$P_m = (331100)(746)(10^{-6}) = 247 \text{ MW}$$
$$= \frac{247}{250} = 0.988 \text{ pu}$$
$$P_e = \frac{200}{250} = 0.8 \text{ pu}$$
$$9 \text{ cycle} = \frac{9}{60} = 0.15 \text{ sec}$$
$$\frac{5.4}{(180(60)} \frac{d^2\delta}{dt^2} = 0.988 - 0.8$$

or

$$\frac{d^2\delta}{dt^2} = 376 \text{ degree/sec}^2$$
$$= \frac{60}{360}(736) = 62.6667 \text{ rpm/sec}$$

Integrating the above expression, we have

$$\frac{d\delta}{dt} = 376t$$
$$\delta = \frac{1}{2}(376)t^2 + \delta_0$$
$$= \frac{1}{2}(376)(0.15)^2 + \delta_0$$

or

$$\Delta\delta = \delta - \delta_0 == 4.23^\circ$$

$$n_s = \frac{120 f_0}{P} = \frac{(120)(60)}{2} = 3600 \text{ rpm}$$

Speed at the end of 0.15 second is

$$n = 3600 + 0.15(62.6667) = 3609.4 \text{ rpm}$$

Question # 4:

The swing equations of two interconnected synchronous machines are written as

Solution:

Denote the relative power angle between the two machines by $\delta = \delta_1 - \delta_2$. Obtain a swing equation equivalent to that of a single machine in terms of δ, and show that

$$\frac{H_1}{\pi f_0}\frac{d^2\delta_1}{dt^2} = P_{m1} - P_{e1}$$
$$\frac{H_2}{\pi f_0}\frac{d^2\delta_2}{dt^2} = P_{m2} - P_{e2}$$
$$\frac{H}{\pi f_0}\frac{d^2\delta}{dt^2} = P_m - P_e$$

where

$$H = \frac{H_1 H_2}{H_1 + H_2}$$

$$P_m = \frac{H_2 P_{m1} - H_1 P_{m2}}{H_1 + H_2} \quad \text{and} \quad P_e = \frac{H_2 P_{e1} - H_1 P_{e2}}{H_1 + H_2}$$

Subtracting the second swing equation from the first, we obtain

$$\frac{1}{\pi f_0}\left(\frac{d^2\delta_1}{dt^2} - \frac{d^2\delta_2}{dt^2}\right) = \left(\frac{P_{m1}}{H_1} - \frac{P_{m2}}{H_2}\right) - \left(\frac{P_{e1}}{H_1} - \frac{P_{e2}}{H_2}\right)$$

or

$$\frac{1}{\pi f_0}\left(\frac{d^2(\delta_1 - \delta_2)}{dt^2}\right) = \left(\frac{H_2 P_{m1} - H_1 P_{m2}}{H_1 H_2}\right) - \left(\frac{H_2 P_{e1} - H_1 P_{e2}}{H_1 H_2}\right)$$

Multiplying both side by $\frac{H_1 H_2}{H_1 + H_2}$, results in

$$\frac{1}{\pi f_0}\frac{H_1 H_2}{H_1 + H_2}\left(\frac{d^2(\delta_1 - \delta_2)}{dt^2}\right) = \left(\frac{H_2 P_{m1} - H_1 P_{m2}}{H_1 + H_2}\right) - \left(\frac{H_2 P_{e1} - H_1 P_{e2}}{H_1 + H_2}\right)$$

or

$$\frac{H}{\pi f_0}\frac{d^2\delta}{dt^2} = P_m - P_e$$

where H, δ, P_m, and P_e are defined above.

Question # 5:

Two synchronous generators represented by a constant voltage behind transient reactance are connected by a pure reactance $X = 0.3$ per unit, as shown in Figure Q5. The generator inertia constants are $H1 = 4.0$ MJ/MVA and $H2 = 6$ MJ/MVA, and the transient reactances are $X_1' = 0.16$ and $X_2' = 0.20$ per unit. The system is operating in the steady state with $E_1' = 1.2$, $P_{m1} = 1.5$ and $E_2' = 1.1$, $P_{m2} = 1.0$ per unit. Denote the relative power angle between the two machines by $\delta = \delta_1 - \delta_2$. Referring to Question 4, reduce the two-machine system to an equivalent one-machine against an infinite bus. Find the inertia constant of the equivalent machine, the mechanical input power, and the amplitude of its power angle curve, and obtain the equivalent swing equation in terms of δ.

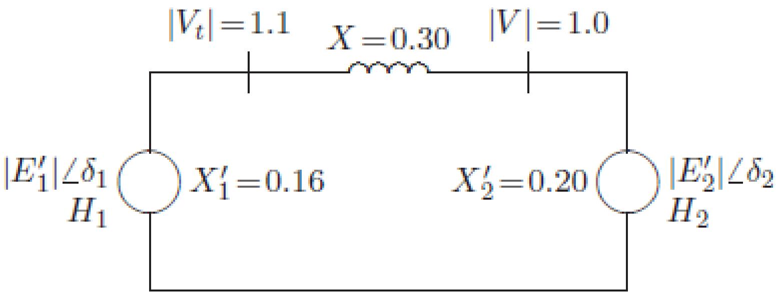


Fig. Q5

Solution:

Referring to Question.4, the equivalent parameters are

$$H = \frac{(4)(6)}{4+6} = 2.4 \ \text{ MJ/MVA}$$

$$P_m = \frac{(6)(1.50) - (4)(1)}{4+6} = 0.5 \ \text{ pu}$$

$$P_{e1} = \frac{|E_1||E_2|}{X}\sin(\delta_1 - \delta_2) = \frac{(1.2)(1.1)}{0.66}\sin\delta = 2\sin\delta$$

Since $P_{e2} = -P_{e1}$, we have

$$P_e = \frac{(6)(2\sin\delta) + (4)(2\sin\delta)}{4+6} = 2\sin\delta$$

Therefore, the equivalent swing equation is

$$\frac{2.4}{(180)(60)}\frac{d^2\delta}{dt^2} = 0.5 - 2\sin\delta$$

or

$$\frac{d^2\delta}{dt^2} = 4500(0.5 - 2\sin\delta) \quad \text{where } \delta \text{ is in degrees}$$

Question # 6:

A 60-Hz synchronous generator has a transient reactance of 0.2 per unit and an inertia constant of 5.66 MJ/MVA. The generator is connected to an infinite bus through a transformer and a double circuit transmission line, as shown in Fig. Q6. Resistances are neglected and reactances are expressed on a common MVA base and are marked on the diagram. The generator is delivering a real power of 0.77 per unit to bus bar 1. Voltage magnitude at bus 1 is 1.1. The infinite bus voltage $V = 1.0\angle 0°$ per unit. Determine the generator excitation voltage and obtain the swing equation.

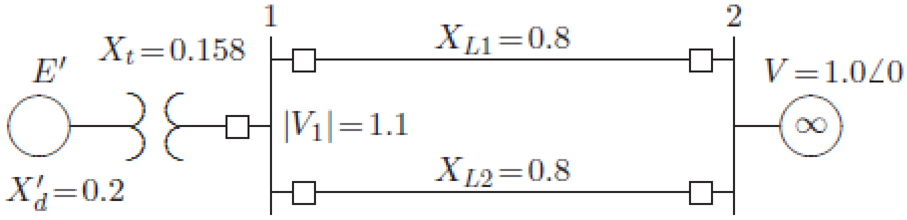


Fig. Q6

Solution:

$$P = \frac{|V_1||V_2|}{X_L}\sin\delta_1$$
$$0.77 = \frac{(1.1)(1.0)}{0.4}\sin\delta_1$$

or

$$\delta_1 = 16.26°$$

$$I = \frac{V_1 - V_2}{jX_L} = \frac{1.1\angle 16.26° - 1.0\angle 0°}{j0.4} = 0.77 - j0.14$$
$$= 0.7826\angle -10.305° \quad \text{pu}$$

The total reactance is X = 0.2+0.158+0.4 = 0.758, and the generator excitation voltage is

$$E' = 1.0 + j0.758(0.77 - j0.14) = 1.25\angle 27.819°$$

the swing equation with ± in radians is

$$\frac{5.66}{60\pi}\frac{d^2\delta}{dt^2} = 0.77 - \frac{(1.25)(1)}{0.758}\sin\delta$$
$$0.03\frac{d^2\delta}{dt^2} = 0.77 - 1.65\sin\delta$$

Question # 7:

A three-phase fault occurs on the system of Question 6 at the sending end of the transmission lines. The fault occurs through an impedance of 0.082 per unit. Assume the generator excitation voltage remains constant at $E' = 1.25$ per unit. Obtain the swing equation during the fault. The impedance network with fault at bus 1, and with with $Z_f = j0.082$ is shown in Fig. Q7.

.

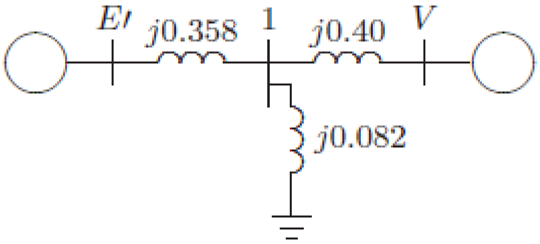


Fig. Q7

Solution:

Transforming the Y-connected circuit in FigQ7 into an equivalent Δ, the transfer reactance between *E*' and *V* is

$$X = \frac{(0.358)(0.082) + (0.358)(0.4) + (0.4)(0.082)}{0.082} = 2.5 \text{ pu}$$

$$P_{2max} = \frac{(1.25)(1)}{2.5} = 0.5$$

Therefore, the swing equation during fault with ± in radians is

$$0.03\frac{d^2\delta}{dt^2} = 0.77 - 0.5\sin\delta$$

Question # 8:

The machine in the power system of Question 6 has a per unit damping coefficient of D = 0.15. The generator excitation voltage is E' = 1.25 per unit and the generator is delivering a real power of 0.77 per unit to the infinite bus at a voltage of V = 1.0 per unit. Write the linearized swing equation for this power system. Find the equations describing the motion of the rotor angle and the generator frequency for a small disturbance of $\Delta\delta = 15^o$.

Solution:

The initial operating angle is given by

$$0.77 = \frac{(1.25)(1)}{0.758}\sin\delta \qquad \text{or} \qquad \delta = 27.835^\circ = 0.4858 \ \text{ radian}$$

The synchronizing power coefficient given by

$$P_s = P_{max}\cos\delta_0 \ = \ \frac{(1.25)(1)}{0.758}\cos 27.835^\circ = 1.4583$$

The undamped angular frequency of oscillation and damping ratio are

$$\omega_n = \sqrt{\frac{\pi f_0}{H}P_s} = \sqrt{\frac{(\pi)(60)}{5.66}1.4583} = 6.969 \ \text{ rad/sec}$$

$$\zeta = \frac{D}{2}\sqrt{\frac{\pi f_0}{HP_s}} = \frac{0.15}{2}\sqrt{\frac{(\pi)(60)}{(5.66)(1.4583)}} = 0.3584$$

The linearized force-free equation which determines the mode of oscillation given by with δ in radian is

$$\frac{d^2\Delta\delta}{dt^2} + 5\frac{d\Delta\delta}{dt} + 48.565\Delta\delta = 0$$

The damped angular frequency of oscillation is

$$\omega_d = \omega_n\sqrt{1-\zeta^2} = 6.969\sqrt{1-(0.3584)^2} = 6.506 \ \text{ rad/sec}$$

corresponding to a damped oscillation frequency of

$$f_d = \frac{6.506}{2\pi} = 1.035 \ \text{ Hz}$$

$$\theta = \cos^{-1} 0.3584 = 69^\circ$$

the motion of rotor relative to the synchronously revolving field in electrical degrees and the frequency excursion in Hz are given by equations

$$\delta = 27.835^\circ + 16.0675e^{-2.4977t}\sin(6.506t + 69^\circ)$$

$$f = 60 - 0.311e^{-2.4977t}\sin 6.506t$$

Question # 9:

Write the linearized swing equation of Question 8 in state variable form.

$$\dot{X} = AX + BU$$
$$Y = CX + DU$$
.

Solution:

$$A = \begin{bmatrix} 0 & 1 \\ -48.5649 & -4.9955 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Question # 10:

The generator of Question 8 is operating in the steady state at $\delta o = 27.835^{o}$ when the input power is increased by a small amount ΔP = 0.15 per unit. The generator excitation and the infinite bus voltage are the same as before. Find the equations describing the motion of the rotor angle and the generator frequency for a small disturbance of ΔP = 0.15 per unit.

Solution:

Substituting for H, δ_o, ς and ω_n evaluated in Question 9 and expressing the power angle in degree, we get

$$\delta = 27.835^{\circ} + \frac{(180)(60)(0.15)}{(5.66)(48.565)}[1 - \frac{1}{\sqrt{1-(0.3548)^2}}e^{-2.4977t}\sin(6.506t + 69^{\circ})]$$

or

$$\delta = 27.835^{\circ} + 5.8935[1 - 1.0712e^{-2.4977t}\sin(6.506t + 69^{\circ})]$$

Also,

$$f = 60 + \frac{(60)(0.15)}{2(5.66)(6.506)\sqrt{1-(0.3584)^2}}e^{-2.4977t}\sin 6.506t$$

or

$$f = 60 + 0.1222e^{-2.4977t}\sin 6.506t$$

Question # 1:

The fuel-cost functions in $/h for two 800 MW thermal plants are given by

$$C_1 = 400 + 6.0P_1 + 0.004P_1^2$$
$$C_2 = 500 + \beta P_2 + \gamma P_2^2$$

where P_1 and P_2 are in MW.

a. The incremental cost of power λ is $8/MWh when the total power demand is 550 MW. Neglecting losses, determine the optimal generation of each plant.

b. The incremental cost of power λ is $10/MWh when the total power demand is 1300 MW. Neglecting losses, determine the optimal generation of each plant.

c. From the results of (a) and (b) find the fuel-cost coefficients β and γ of the second plant.

Solution:

$$\frac{dC_1}{dP_1} = 6 + 0.008P_1 = \lambda$$
$$\frac{dC_2}{dP_2} = \beta + 2\gamma P_2 = \lambda$$

(a) For $\lambda = 8$, and $P_D = 550$ MW, we have

$$P_1 = \frac{8-6}{0.008} = 250 \text{ MW}$$
$$P_2 = P_D - P_1 = 500 - 250 = 300 \text{ MW}$$

(b) For $\lambda = 10$, and $P_D = 1300$ MW, we have

$$P_1 = \frac{10-6}{0.008} = 500 \text{ MW}$$
$$P_2 = P_D - P_1 = 1300 - 500 = 800 \text{ MW}$$

(c) The incremental cost of power for plant 2 are given by

$$\beta + 2\gamma(300) = 8$$
$$\beta + 2\gamma(800) = 10$$

Solving the above equations, we find $\beta = 6.8$, and $\gamma = 0.002$

Question # 2:

The fuel-cost functions in \$/h for three thermal plants are given by

$$\begin{aligned} C_1 &= 350 + 7.20P_1 + 0.0040P_1^2 \\ C_2 &= 500 + 7.30P_2 + 0.0025P_2^2 \\ C_3 &= 600 + 6.74P_3 + 0.0030P_3^2 \end{aligned}$$

where P_1, P_2, and P_3 are in MW. The governors are set such that generators share the load equally. Neglecting line losses and generator limits, find the total cost in \$/h when the total load is

(i) PD = 450 MW
(ii) PD = 745 MW
(iii) PD = 1335 MW

Solution:

(i) For $P_D = 450$ MW, $P_1 = P_2 = P_3 = \frac{450}{3} = 150$ MW. The total fuel cost is

$$\begin{aligned} C_t = {} & 350 + 7.20(150) + 0.004(150)^2 + 500 + 7.3(150) + 0.0025(150)^2 + \\ & 600 + 6.74(150) + 0.003(150)^2 = 4,849.75 \ \$/\mathrm{h} \end{aligned}$$

(ii) For $P_D = 745$ MW, $P_1 = P_2 = P_3 = \frac{745}{3}$ MW. The total fuel cost is

$$\begin{aligned} C_t = {} & 350 + 7.20\left(\frac{745}{3}\right) + 0.004\left(\frac{745}{3}\right)^2 + 500 + 7.3\left(\frac{745}{3}\right) + 0.0025\left(\frac{745}{3}\right)^2 \\ & +600 + 6.74\left(\frac{745}{3}\right) + 0.003\left(\frac{745}{3}\right)^2 = 7,310.46 \ \$/\mathrm{h} \end{aligned}$$

(iii) For $P_D = 1335$ MW, $P_1 = P_2 = P_3 = 445$ MW. The total fuel cost is

$$\begin{aligned} C_t = {} & 350 + 7.20(445) + 0.004(445)^2 + 500 + 7.3(445) + 0.0025(445)^2 + \\ & 600 + 6.74(445) + 0.003(445)^2 = 12,783.04 \ \$/\mathrm{h} \end{aligned}$$

Question # 3:

Neglecting line losses and generator limits, determine the optimal scheduling of generation for each loading condition in Question # 2

a. by analytical technique.
b. find the savings in \$/h for each case compared to the costs in Question # 2 when the generators shared load equally.

Solution:

(a) (i) For $P_D = 450$ MW, from (7.33), λ is found to be

$$\lambda = \frac{450 + \frac{7.2}{0.008} + \frac{7.3}{0.005} + \frac{6.74}{0.006}}{\frac{1}{0.008} + \frac{1}{0.005} + \frac{1}{0.006}}$$
$$= \frac{450 + 3483.333}{491.666} = 8.0 \quad \$/\text{MWh}$$

Substituting for λ in the coordination equation, the optimal dispatch is

$$P_1 = \frac{8.0 - 7.2}{2(0.004)} = 100$$
$$P_2 = \frac{8.0 - 7.3}{2(0.0025)} = 140$$
$$P_3 = \frac{8.0 - 6.74}{2(0.003)} = 210$$

(a) (ii) For $P_D = 745$ MW, from (7.33), λ is found to be

$$\lambda = \frac{745 + 3483.333}{491.666} = 8.6 \quad \$/\text{MWh}$$

Substituting for λ in the coordination equation, the optimal dispatch is

$$P_1 = \frac{8.6 - 7.2}{2(0.004)} = 175$$
$$P_2 = \frac{8.6 - 7.3}{2(0.0025)} = 260$$
$$P_3 = \frac{8.6 - 6.74}{2(0.003)} = 310$$

(a) (iii) For $P_D = 1335$ MW, from (7.33), λ is found to be

$$\lambda = \frac{1335 + 3483.333}{491.666} = 9.8 \quad \$/\text{MWh}$$

Substituting for λ in the coordination equation, the optimal dispatch is

$$P_1 = \frac{9.8 - 7.2}{2(0.004)} = 325$$
$$P_2 = \frac{9.8 - 7.3}{2(0.0025)} = 500$$
$$P_3 = \frac{9.8 - 6.74}{2(0.003)} = 510$$

(c)(i) For $P_1 = 100$ MW, $P_2 = 140$ MW, and $P_3 = 210$ MW, the total fuel cost is

$$C_t = 350 + 7.20(100) + 0.004(100)^2 + 500 + 7.3(140) + 0.0025(140)^2 + 600 + 6.74(210) + 0.003(210)^2 = 4,828.70 \ \ \$/\mathrm{h}$$

Compared to Question #2 (i), when the generators shared load equally,
the saving is 4,849.75 - 4,828.70 = 21.05 $/h.

(c)(ii) For $P_1 = 175$ MW, $P_2 = 260$ MW, and $P_3 = 310$ MW, the total fuel cost is

$$C_t = 350 + 7.20(175) + 0.004(175)^2 + 500 + 7.3(260) + 0.0025(260)^2 + 600 + 6.74(310) + 0.003(310)^2 = 7,277.20 \ \ \$/\mathrm{h}$$

Compared to Question #2 (ii), when the generators shared load equally,
the saving is 7,310.46 - 7,277.20 = 33.25 $/h.

(c)(iii) For $P_1 = 325$ MW, $P_2 = 500$ MW, and $P_3 = 510$ MW, the total fuel cost is

$$C_t = 350 + 7.20(325) + 0.004(325)^2 + 500 + 7.3(500) + 0.0025(500)^2 + 600 + 6.74(510) + 0.003(510)^2 = 12,705.20 \ \ \$/\mathrm{h}$$

Compared to Question #2 (iii), when the generators shared load equally,
the saving is 12,783.04 - 12,705.20 = 77.84 $/h.

Question # 4:

Repeat Question # 3 (a), but this time consider the following generator limits (in MW)

$$122 \le P_1 \le 400$$
$$260 \le P_2 \le 600$$
$$50 \le P_3 \le 445$$

Solution:

In Question # 3, in part (a) (i), the optimal dispatch are P_1 = 100 MW, P_2 = 140 MW, and P_3 = 210 MW. Since P_1 and P_2 are less that their lower limit, these plants are pegged at their lower limits. That is, P_1 = 122, and P_2 = 260 MW. Therefore, P_3 = 450 - (122 + 260) = 68 MW.

In Question # 3, in part (a) (ii), the optimal dispatch are P_1 = 175 MW, P_2 = 260 MW, and P_3 = 310 MW, which are within the plants generation limits.

In Question # 3, in part (a) (iii), the optimal dispatch are P_1 = 325 MW, P_2 = 500 MW, and P_3 = 510 MW. Since P_3 exceed its upper limit, this plant is pegged at P_2 = 445. Therefore, a load of 1335 - 445 = 890 MW must be shared between plants 1 and 2, with equal incremental fuel cost give by

$$\lambda = \frac{890 + \frac{7.2}{0.008} + \frac{7.3}{0.005}}{+\frac{1}{0.008} + \frac{1}{0.005}}$$
$$= \frac{890 + 2360}{325} = 10 \ \$/\mathrm{MWh}$$

Substituting for λ in the coordination equation, the optimal dispatch is

$$P_1 = \frac{10 - 7.2}{2(0.004)} = 350$$
$$P_2 = \frac{10 - 7.3}{2(0.0025)} = 540$$

Since P_1 and P_2 are within their limits the above result is the optimal dispatch.