Power I

NoteBook

Dr. Hisham Hamdan

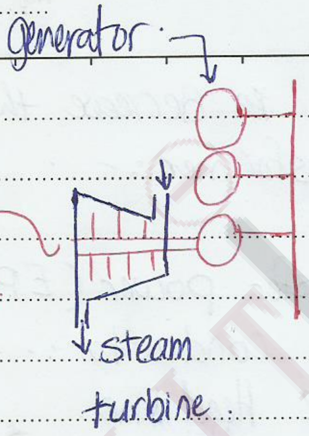
By:Luma & Asma' Hakouz

POWER UNIT
ELECTRICAL ENGINEERING

Power

No.

18/2/14.



⊛ Generation system:-

⊛ To change any process (concerness):
1. Technical
2. economical
3. enviRoment.

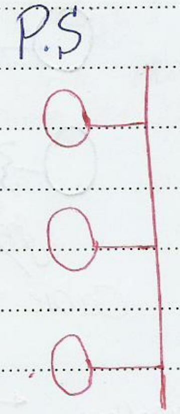
water
heat.
1. (Fuel oiL)
2. (gas)
3. (coal)
4. (nuclear reactor)

U^{235} → fissionable 0.7%

U^{238} → non fissionable → 99.3% → [illegible] → U^{239} → fissionable.

5. Solar cell → photo voltic cell (PV cell)

6. Wind Energy → (20 km/hr)



⊛ if we want to have more "S":-
- more I_L → more copper.
- more V_{LL} → more Insulation.

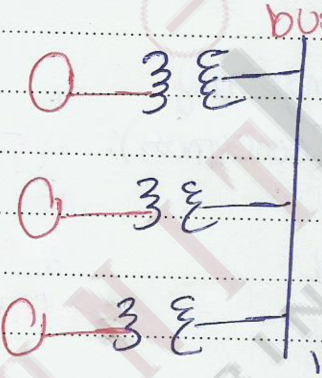
⊛ where to put the generators:-
- near Load centre.
- near water & fuel supplies

(11-24) kV

$S = \sqrt{3} V_L I_L$.

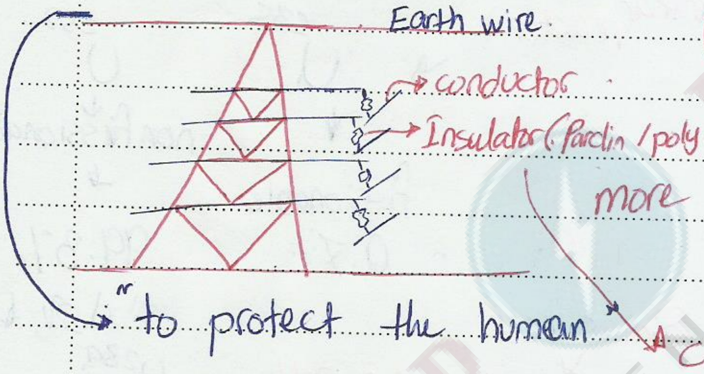
note:- → cheaper to transfer electrical power than to transfer water & fuel

* to decrease the Voltage drop we use Transformer :- $(P \propto V^2)$



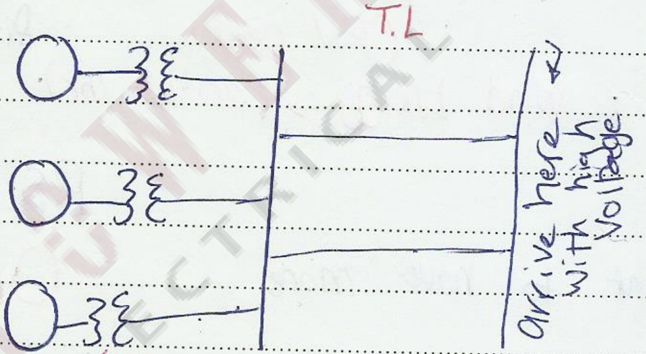
⊛ to transfer power (E.P) we use a conductor :-

1. OVer Head Line



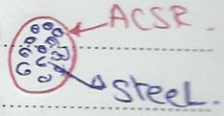
more voltage more power ←

132 KV
220 (230) KV
(275) (300) (345) KV
400 KV
500 KV
765 KV
1000 KV
1500 KV

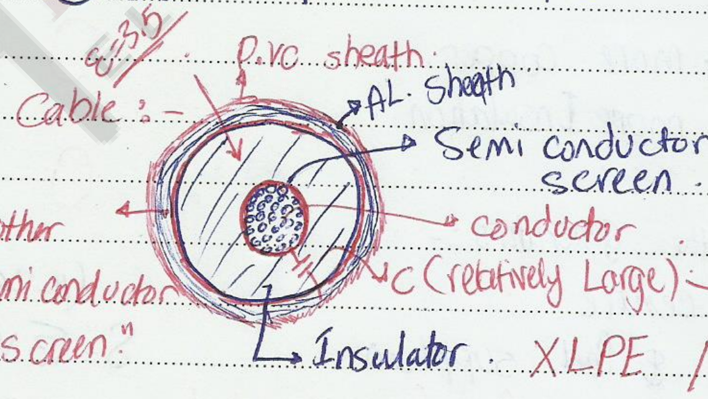


note Insulators made from polymers or parclain.

ACSR (Aluminuem conductor steel Reinforce).



Cable :-



C (relatively Large) → I large → $I = j\omega C V$

Insulator . XLPE / paper.

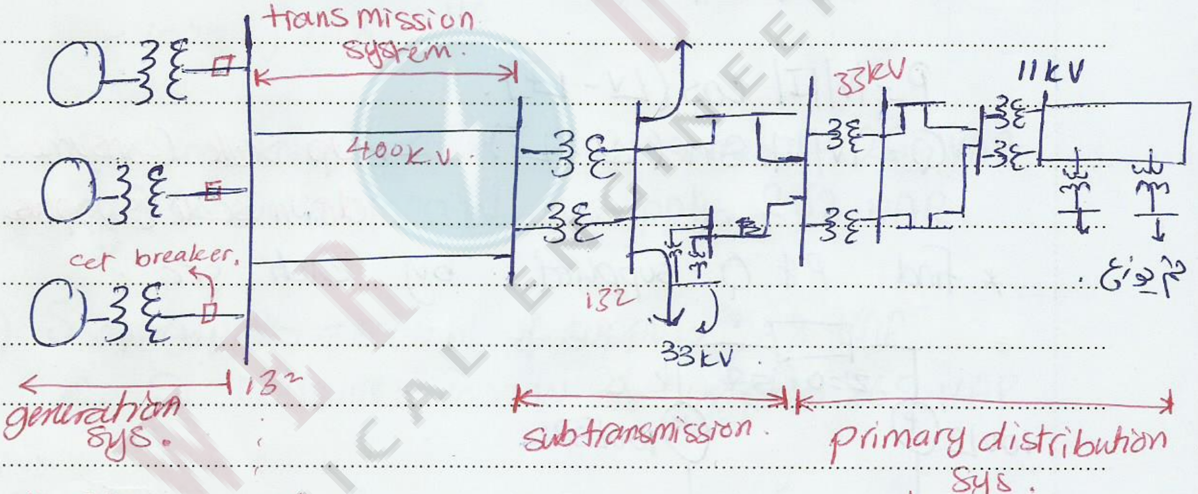
* OHL → over head Line .

No.

⊛ Cost of cable / Cost OHL

Cost of cable / Cost OHL	Voltage
18	400 kV
13	275 kV
8	132 kV
2-3	L.V

Lecture "2" (2/20/[illegible]14)



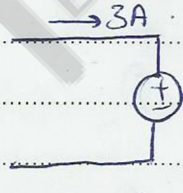
generation sys.

11-24 1500'

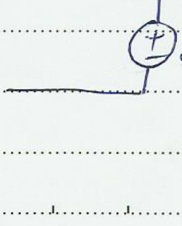
sub transmission.

primary distribution sys.
↓
for a circle that has radius of 500 m.

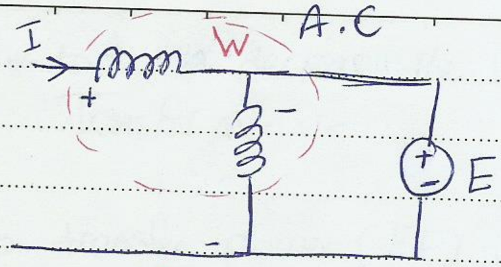
* CH.1 → 1.7 "Direction of power flow" :-



→ src consumes power.



→ supplies power.



$W = V.I$

$= VI \cos\theta$

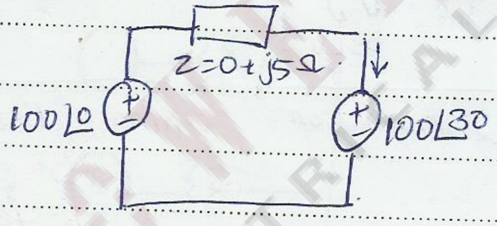
→ if the wattmeter reads up scale, src is consuming power.

→ if the wattmeter reads down scale, src is supplying power.

$P = |V||I| \cos(\angle V - \angle I)$.

$Q = |V||I| \sin(\angle V - \angle I)$ → Up scale (consumes Q)
down scale (supplies Q)

* find P & Q supplied by each src.



$$I = \frac{100\angle 0 - 100\angle 30}{j5} = -10 - j2.68 \text{ A}.$$

$$= 10.35\angle 195^\circ \text{ A}.$$

$S_2 = V_2 . I^*$ " Consumed).

$= 100\angle 30 * 10.35\angle -195$

$= 1035\angle -165$

$= -1000 - j268$

Src 2 consumes -1000 W.

Src 2 consumes -268 VAR

OR src 2 supplies 1000 W & supplies 268 VAR.

Src 1 → A) Consumed.

$S_1 = V_1 \cdot (-I^*)$

$\leftarrow$ $S_1 = 100\angle 0 * (-10.35\angle -195)$

$= 100\,(10 - j2.68)$

$= 1000 - j268$ VA

$(-I = 10 + j2.68)$

B) Supplied

$S_1 = V_1 \cdot I^*$

$= 100\,(-10 + j2.68)$

$= -1000 + j268$ VA

For (A) S_1 → consumes 1000 W & consumes − 268 VAR
OR S_1 consumes 1000 W & supplies 268 VAR

For (B) S_1 → supplies −1000 W & supplies 268 VAR
OR S_1 consumes 1000 W & supplies 268 VAR

"Active power → supplies = consumes (no resistance).
Reactive power → ~~the~~ both src are suppling so the reactance must consumes the sum of Q from S_1 & S_2.

So →

$Q_2 = I^2 X = (10.35)^2 (5) = 536$ VAR "consumes"

So

$Q_{supplies} = Q_{consumes}$ #."

No.

"who leads gives power"

$100\angle 30 \rightarrow$ lead $100\angle 0$ So $100\angle 30$ gives power that's ~~consum~~ being consumed by $100\angle 0$.

Q → depends on the magnitude.

P → depends on who leads who.

1.10 Per Unit System:—

V , I , S , Z

$$V_{P.U} = \frac{\text{Actual value}}{\text{Base value}}$$

* Lecture "3" * Continue to PU quantities:-

V, I, S, Z

* Base $KVA_{1\phi}$, Base KV_{LN} :—

① Base current (A) = $\frac{\text{Base } KVA_{1\phi}}{\text{Base } KV_{LN}}$ #

② Base Impedance (Ω) = $\frac{\text{Base } KV_{LN} * 1000}{\text{Base current}}$ → Just for Units always Proud

Smile up

No.

& to find the Base value of Z :—

→ $$\text{Base}(Z) = \frac{(\text{Base value of kV})^2 * 1000}{\text{Base kVA}}$$

$$= \frac{(\text{base kV}_{LN})^2}{\text{base MVA}_{1\phi}}$$

// Just take two base values of V, S, I or Z.

→ for Power :—

→ $S = P + jQ$ * if we divide S by S base, then P & Q must be divided by S_{base} too *.

→ $$\frac{S}{S_{base}} = \frac{P + jQ}{S_{base}}$$

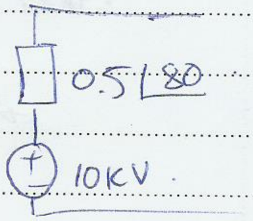
* Base value of P & Q is the same value of S *.

* Take a 1φ Gen. where :-

$V_{actual} = 10.000$ V, → Line to Neutral.

$S_{actual} = 10$ MVA → 1φ S

$Z = 0.5 \angle 80$ Ω



→ Base $kV_{LN} = 10$

Base $MVA_{1\phi} = 10$

→

* $V_{PU} = \frac{\text{actual value}}{\text{Base Value}} = \frac{10}{10} = \boxed{1\ PU = V_{PU}}$ — ⊛

* $S_{PU} = \frac{\text{Actual}}{\text{Base}} = \frac{10}{10} = \boxed{1\ PU = S}$

* $Z_{base} = \frac{(\text{Base}\ \&\ KV_{LN})^2}{\text{Base}\ MVA_{1\phi}} = \frac{(10)^2}{10} = 10\ \Omega$

* $Z_{PU} = \frac{0.5\angle 80}{10} = 0.05\angle 80\ PU$

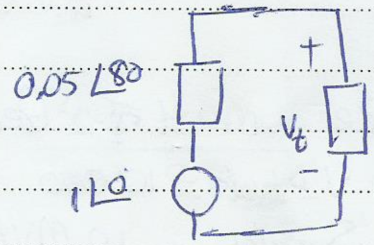
* $I_{sc(p.u)} = \frac{1\angle 0}{0.05\angle 80} = 20\angle -80$

* if we put a load → have voltage V_t

$V_t = 1\angle 0 - IZ$

$= 1\angle 0 - |I|\angle \phi\ |Z|\angle 80$

[illegible].



make the Gen. take it's full load @ $I = 1PU$

$V_t = 1 - 0.05 = 0.95$

$I = 1\angle -80$

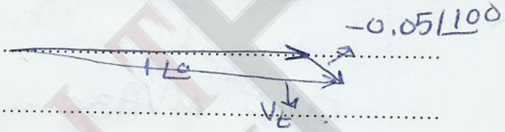
No.

if $I = 1\angle 20$

$V_t = 1\angle 0 - 1\angle 20 * 0.05\angle 180$

$= 1\angle 0 - 0.05\angle 100$

Capacitive load.



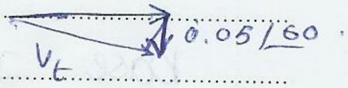
* See the current lag or lead to know Z_L inductive or capacitive load respectively *.

* $V_t \to$ min when $\angle\theta_I = -\angle\theta_Z$ *

$V_t = 1\angle 0 - \underset{1\angle -20}{\underset{\downarrow}{I}} Z$

Inductive Load.

$= 1\angle 0 - 0.05\angle 60$



* when I (leads) → gives the max value of V_t & the oppisite when it lags.

* $\frac{1}{PU(Z)}$ = max value of I_{SC}

* ~~For 3ϕ values :~~

Actual values →

3φ system (Y)	equivalent 1-φ sys.
$MVA_{3\phi} = 18$ MVA	$MVA_{1\phi} = 18/3 = 6$ MVA
$KV_{LL} = 108$ KV	$KV_{LN} = 18/\sqrt{3}$
Base $MVA_{3\phi} = 30$ MVA →	base $MVA_{1\phi} = 10$ MVA
Base $KV_{LL-3\phi} = 120$ KV	base $KV_{LN} = 120/\sqrt{3}$

PU values must be the Same :-

PU(S) = 0.6	= 0.6
PU(V) = 0.9	= 0.9

Lecture "4"

(*) base current = $\frac{\text{base } KVA_{3\phi}}{\sqrt{3} * \text{base } KV_{LL}}$

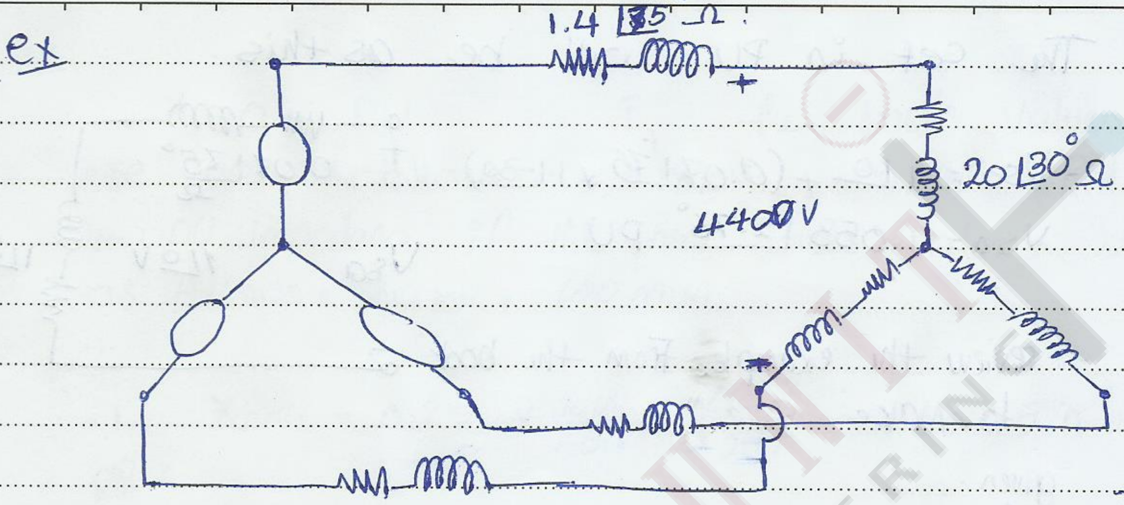
base Impedance = $\frac{\text{base } KV_{LL}/\sqrt{3}}{\text{base current (A)}} * 1000$ → to balance Units.

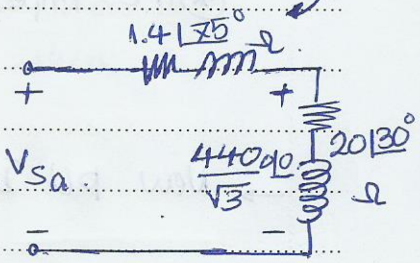
→ Base Impedance (Ω) = $\frac{(\text{base } KV_{LL}/\sqrt{3})(\sqrt{3} \text{ base } KV_{LL}) * 1000}{\text{base } KVA_{3\phi}}$

$= \frac{(\text{base } KV_{LL})^2}{\text{base } MVA_{3\phi}}$

[in 3φ → V_{LL} , $S_{3\phi}$
in 1φ → V_{LN} , $S_{1\phi}$]

ex





$$V_{sa} = \frac{4400}{\sqrt{3}}\angle 0 + I\left(1.4\angle 75^\circ\right)$$

∴ to Find I :

$$I = \frac{\frac{4400\angle 0}{\sqrt{3}}}{20\angle 30}$$

$$\boxed{I = 127\angle -30^\circ A} - **$$

then : $V_{sa} = 2670\angle 2.7^\circ$

$V_{LL} = 2670\sqrt{3}$ (as a magnitude if he doesn't Give me the sequence).

* Solve previous example using Per Unit if base values as follows :

$V_{base} = 4400V$, $I_{base} = 127A$

base MVA = $V_{base} * I_{base} * \sqrt{3}$ → L to L voltage.

$= 4.4 * \sqrt{3} * 0.127$

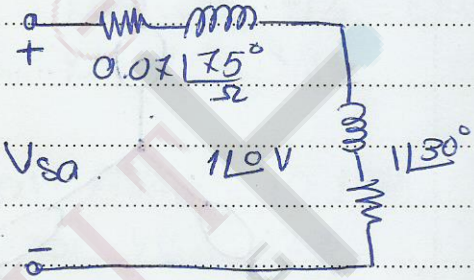
$= 0.96787$ MVA.

base Impedance $= \frac{(4.4)^2}{0.96787} = 20\ \Omega$

The cct in P.U will be as this:

$V_{sa} = 1\angle 0 + (0.07\angle 75 * 1\angle -30)$

$V_{sa} = 1.053\angle 2.76^\circ$ PU



"review the example From the book to make sure".

given:

$$\text{PU impedance} = \text{actual imp.} * \frac{\text{Given base MVA}}{(\text{given base KV})^2} \quad — ①$$

$$\rightarrow \text{New PU Imp.} = \text{actual imp} * \frac{\text{new base MVA}}{(\text{new base KV})^2} \quad — ②$$

:- divide 2 by 1

$$\frac{\text{new PU Imp.}}{\text{Given PU imp.}} = \frac{\text{new base MVA}}{\text{Given base MVA}} * \left(\frac{\text{Given base KV}}{\text{new base KV}}\right)^2$$

thus:-

$$\text{new PU Imp} = \text{Given PU imp} * \frac{\text{new base MVA}}{\text{given base MVA}} * \left(\frac{\text{Given base KV}}{\text{new base KV}}\right)^2$$

→

ex

$X = 0.25$ PU & the base's values are as follows: 18KV, 500 MVA find the new pu impedance if the new base's values become as follows: 20 KV, 100 MVA.

-1- $X_{new} = 0.25 * \frac{100}{500} \times \left(\frac{18}{20}\right)^2 = 0.45\ \Omega$

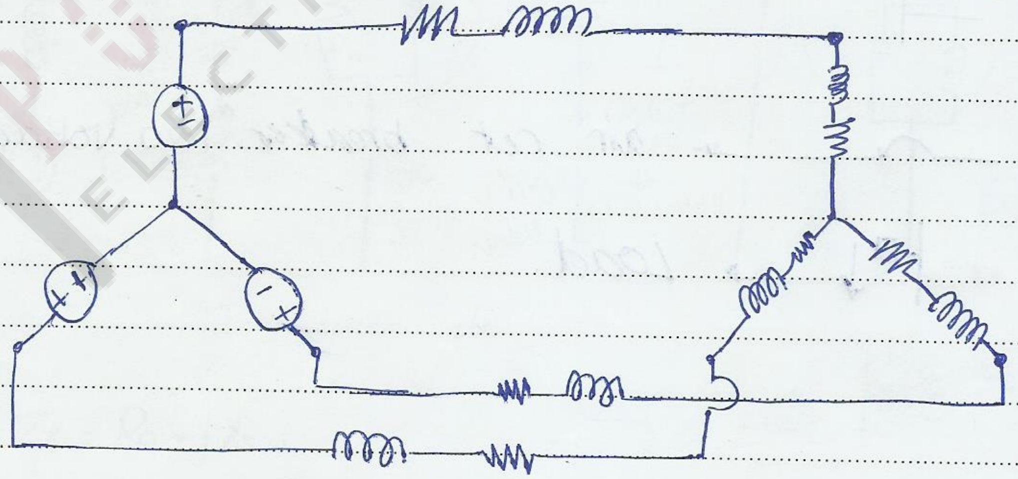
OR:

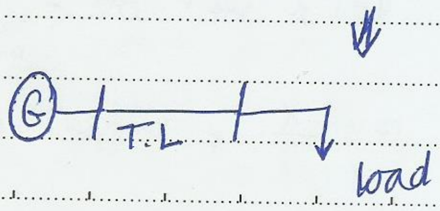
we can find the actual value of X_{old} :-

$X_{old} = \frac{0.25 * (18)^2}{500}$ & then

$X_{pu} = \frac{0.25 * (18)^2/500}{(20)^2/100}$.

* One Line Diagram * "Single line Diagram"

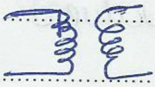


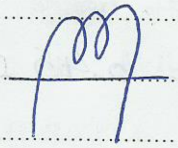


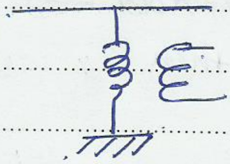
No.

Some Symbols :-

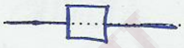

→ synchronous Generator (S.G)


→ Power Transformer.


→ Current Transformer.


→ voltage Transformer.

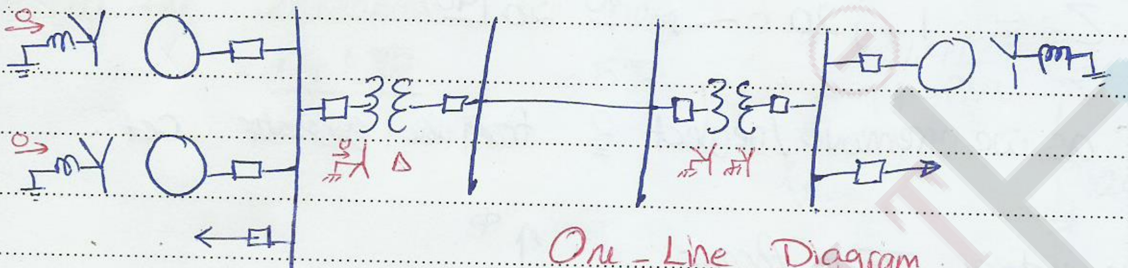

→ Transmithion Line.


→ cct breaker "high voltage"


→ bus bar.


→ air cct breaker "low voltage"


→ Load.



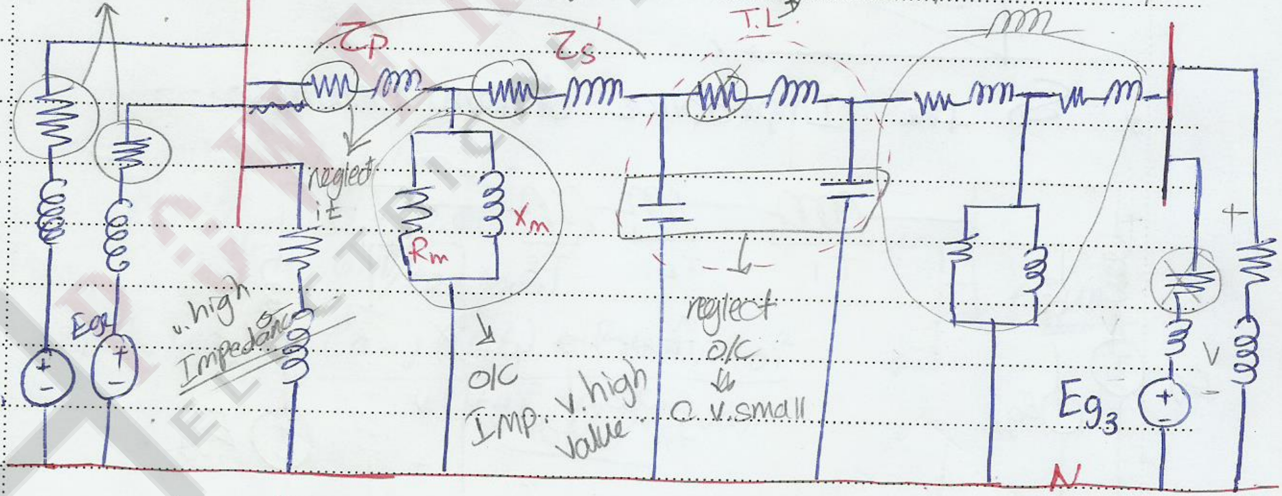
One_Line Diagram

* generator → cct breaker → step up transformer → T.L → step down Trans. → load
└→ motor or any other type of machine.

Lecture "5" * Thr. 27/2/2014.

In Balance $\sum I$ in Y Connection $= 0$. #

as we remove this $Z_{eq_1} = R_{eq_1} + jX_{eq_1}$ H.V.TL : $\frac{X}{R} = 3-7$ (neglect R)



"Impedance diagram"

$Z_s = R_a + jX_s$

$\frac{X_s}{R_a} > 20$ "For a well-designed generator"

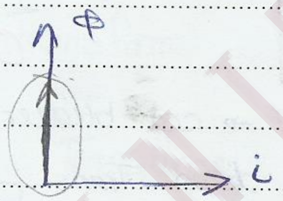
for the Transformer → $\frac{X}{R} \simeq 10$

* $Z_L >> Z_{Line}$.

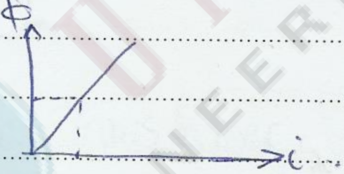
e.g. $Z_S = 1 + j20 \rightarrow \simeq 20\angle 90^\circ$

as we remove / neglect R from the generator cct.

* ideal Trans former.

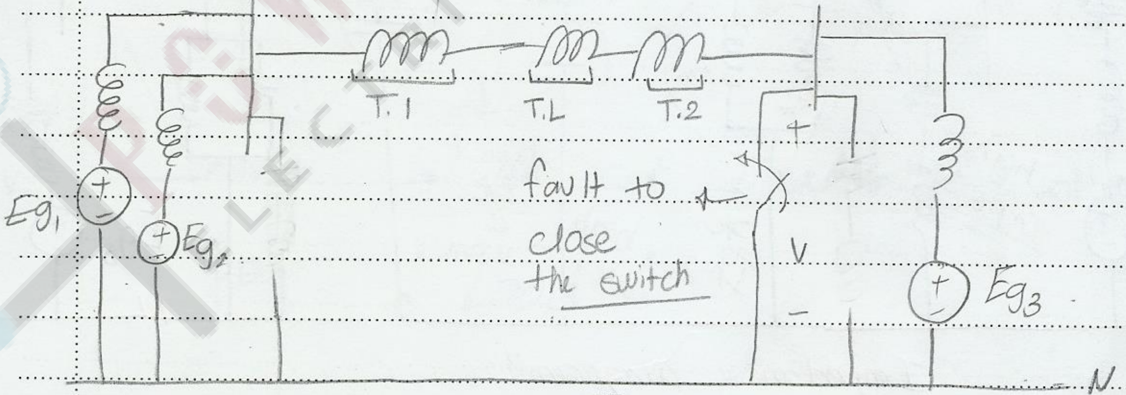


* for practical Transformer.

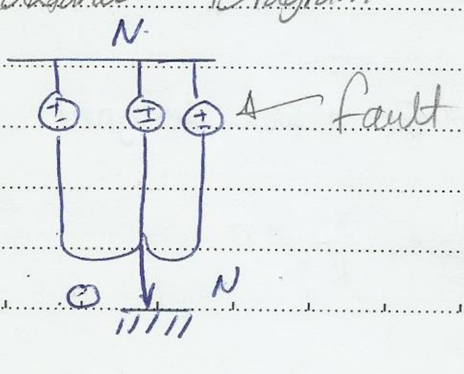


* Linear Transformer.

* So the Impedance digram becomes:--

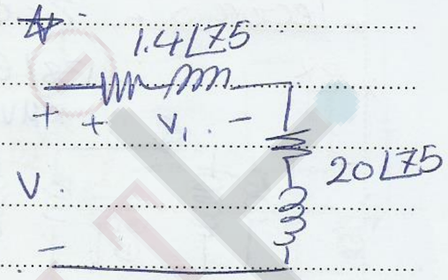


Reactance Diagram

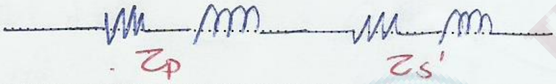


* all the resistances were negected

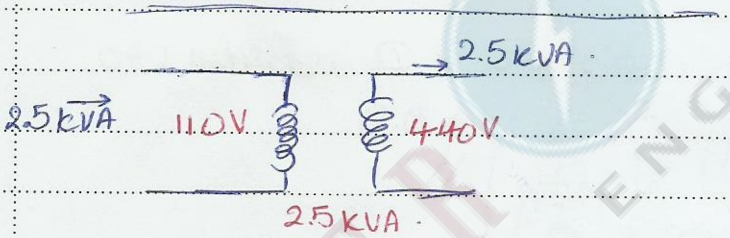
$$V_1 = \frac{1.4\angle 75}{1.4\angle 75 + 20\angle 75} = 5\%$$



"* PU Calculations in Transformer circuits"



referred to primary



base values:
110V, 2.5 kVA.

$Z_{1eq} = 0 + j\,0.06\ \Omega$

→ pU calculations :-

$$P.U\ Z_{eq_1} = \frac{0 + j0.06}{0.11^2/(2.5/1000)}$$

$$PU\ Z_{eq_2} = \frac{(0 + j\,0.06) * (440)^2/(110)^2}{(0.44)^2/(2.5/1000)}$$

→ equal

-1- V_1 → given V_2 → taken as it's good with turns ratio

-2- $KVA_{in} = KVA_{out}$

* Lecture "6" * $Z = 0 + j0.8$

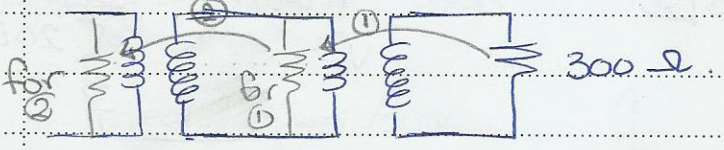
* ex 138/69 KV

10 MVA

base values :-

69 kV

10 MVA



13.8/138 KV

10 MVA.

$Z = 0 + j0.1$ PU.

10 MVA.

$$PU.R = \frac{300}{(69)^2/10} = 0.63 \text{ PU}$$ ① and then to

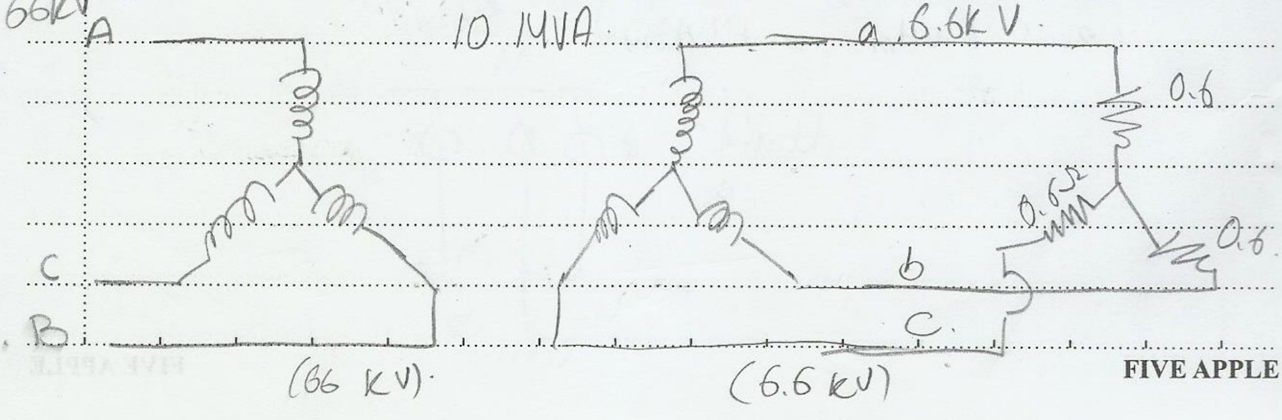
reflect it →

$$= \frac{\frac{300 * (138)^2}{(69)^2}}{(138)^2/10} = 0.63 \text{ PU}$$ &

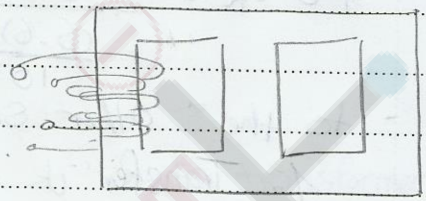
② then to reflect it again:-

$$= \frac{300 \times \frac{(138)^2}{(69)^2} * \frac{(13.8)^2}{(138)^2}}{(13.8)^2/10} = 0.63 \text{ PU}$$

(*) for 3-ph connections :- (Y-Y) connection



for 3-phase transformer :-



——— **

Continue :

Base MVA for all system is 10 MVA.

to find the PU ~~Base~~ of R $= \frac{0.6}{\left(\frac{6.6^2}{10}\right)}$ PU

equal

to reflect R to the other side :-

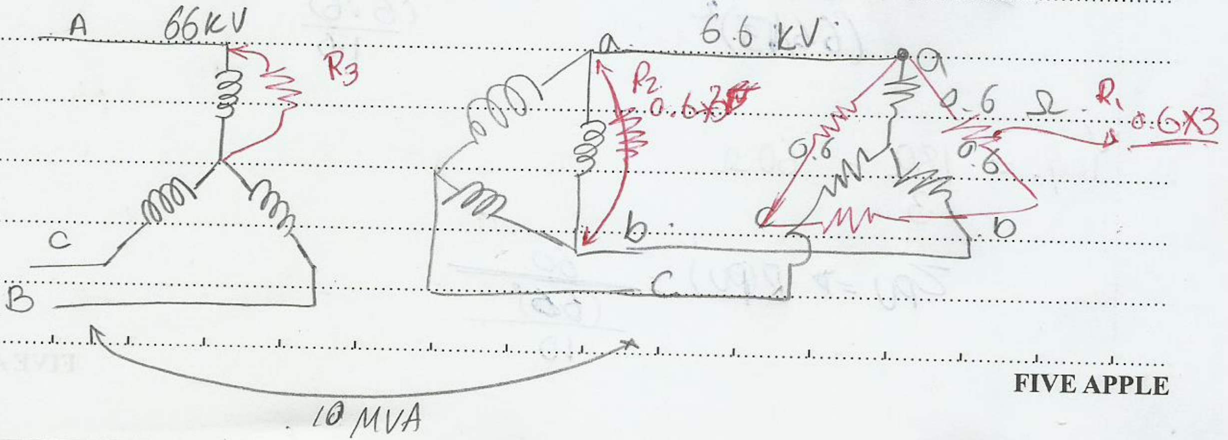
$$0.6 \times \left(\frac{\frac{66}{\sqrt{3}}}{\frac{6.6}{\sqrt{3}}}\right)^2 \Big/ \frac{66^2}{10}$$

phase voltage

$$= \boxed{} \text{ PU}$$

$= 0.6 \times \frac{66^2}{6.6^2}$ if we multiply it by (squere of turns ratio OR transformation ratio) is the same

For Y-Δ connection :-



* load is Y & we want to reflected to

No. .... Y- load.

$$PU.R = \frac{0.6}{\frac{(6.6)^2}{10}}$$ & then to reflect it to the other side:-

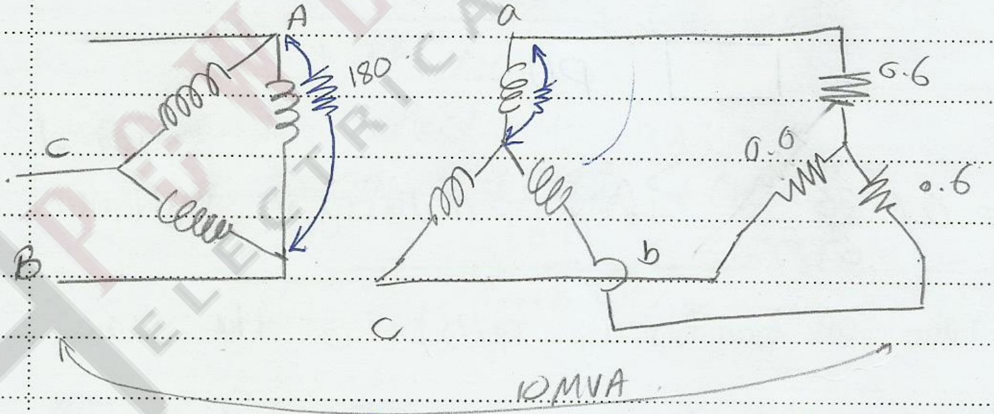
1. Transfer it to Δ → $R_1 * 3$.

$$R_3 = 0.6 \times 3 \times \left(\frac{66/\sqrt{3}}{6.6}\right)^2$$

$$= 0.6 * \left(\frac{66}{6.6}\right)^2 \ \Omega$$

$$PU.R = \frac{0.6\left(\frac{66}{6.6}\right)^2}{66^2/10}$$

* For Δ-Y connection:-



$$0.6 \times \frac{66^2}{(6.6/\sqrt{3})^2} \qquad \frac{0.6}{\frac{(6.6)^2}{10}}$$

$$Y_{eq.} = \frac{180}{3} = 60\,\Omega$$

$$Z_{PU} \Rightarrow R(PU) = \frac{60}{\frac{(6.6)^2}{10}}$$

(*) multiply it by the (transformation ratio)^2 irrespective to the transformer connection

ex 3 single phase transformer ⟶ 3-ϕ transformer

25 MVA

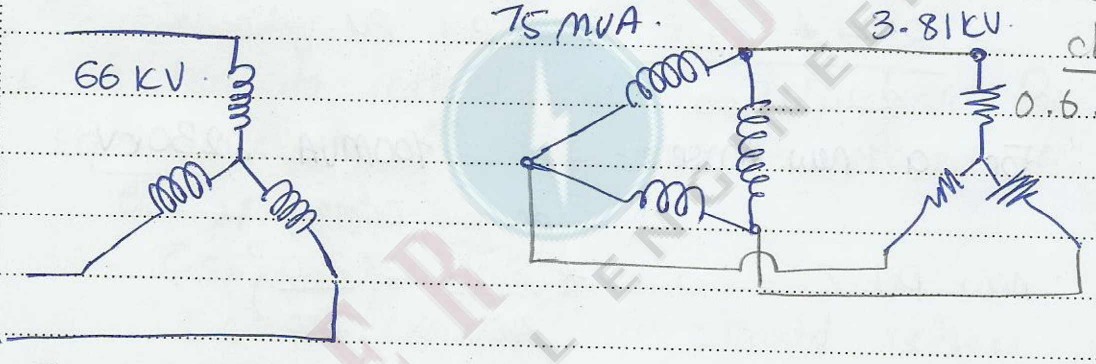
$\underbrace{38.1}_{LN} \text{ KV} / 3.81 \text{ KV}$

λ / Δ

75 MVA

$\underbrace{66}_{LL} / \underbrace{3.81}_{} \text{ KV}$

Δ doesn't change



R In PU ⟶ $= \dfrac{0.6}{(3.81)^2/75} = 3.1 \text{ PU}$

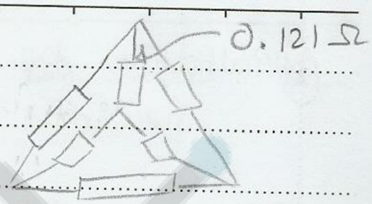
reflect it in PU ⟹

$$= \frac{\boxed{0.6 \times \dfrac{66^2}{(3.81)^2}}}{(66)^2/75} = 3.1 \text{ PU}$$

In Ω

No. ....

ex 3-ϕ transformer
400 MVA
220Y/22Δ kV



In PU → $\dfrac{0.121}{(22^2/400)}$ = 0.1 PU

& then to reflect it :-

$$= \frac{0.121 * \frac{220^2}{22^2}}{220^2/400} = 0.1 \text{ PU}$$

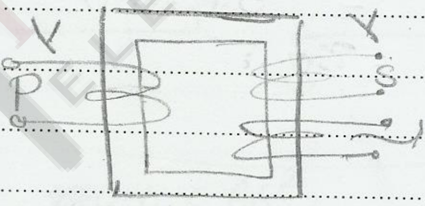
For a new base :- 100MVA , 230kV
on the primary

$$\text{new PU } Z = 0.1 * \frac{100}{400} * \left(\frac{220}{230}\right)^2$$

$$= 0.0228 \text{ PU}$$

Lecture "7" :-

3-winding 3-phase transformer :-



Same MVA

tertiary (tΔ) "because of the 3rd harmonic"

* and if Δ-Y we put (t) to take power as a low voltage put it on it (step down the voltage)

* if we make (S) s/c & (t) o/s then P won't be affected by t so we will calculate Z in (P) Called Z_{ps}

$Z_{ps} = Z_p + Z_s' (\Omega)$.

* & if we make (S) o/c & t s/c then we will calculate $Z_{pt} = Z_p + Z_t'$ in (P) (Ω).

* or Z_{st} as will $\rightarrow Z_{st} = Z_s + Z_t' (\Omega)$ in Secondary

* we had to reflect all the impedance for the Same side so if we must referred it to the primary

$Z_{st}' = Z_s' + Z_t'$

"In all examples we should reflect all the impedances to the primary"

* $Z_{ps} + Z_{pt} - Z_{st} = 2Z_p$ thus :

$Z_p = \frac{1}{2} (Z_{ps} + Z_{pt} - Z_{st})$ So:

$Z_s = \frac{1}{2} (Z_{ps} + Z_{st} - Z_{pt})$ &

$Z_t = \frac{1}{2} (Z_{st} + Z_{pt} - Z_{ps})$

* $\underline{S_s + S_t = S_p}$

No.

ex

3-Φ rating of a 3-winding transformer

P: Y- connected 66 KV , 15 MVA

S: Y- connected 13.2 KV , 10 MVA

t: Δ- connected 2.3 KV , 5 MVA

neglect R

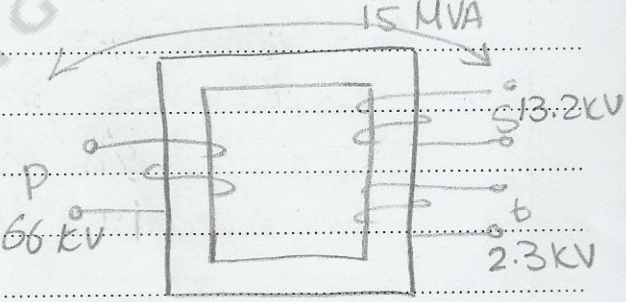
$Z_{ps} = 0.07$ PU , with base 15 MVA & 66 KV

$Z_{pt} = 0.09$ PU , " " 15 MVA & 66 KV

$Z_{st} = 0.08$ PU , " " 10 MVA & 13.2 KV

find Z with 15 MVA , 66 KV In p ?

sol



$Z_{st} = 0.08 \times \frac{15}{10}$ PU

base 15 MVA , 13.2 KV

$Z_{st}(\Omega) =$ PU * base

$= 0.08 \times \frac{(13.2)^2}{10}$ in secondary

$= 0.08 \times \frac{(13.2)^2}{10} \times \left(\frac{66}{13.2}\right)^2$ in primary

In PU : to 15 MVA base , 66 KV

$= 0.08 * \frac{(13.2)^2}{10} * \left(\frac{66}{13.2}\right)^2 \Big/ \frac{(66)^2}{15}$

Z_{st} (PU) $= 0.12$ PU in primary.

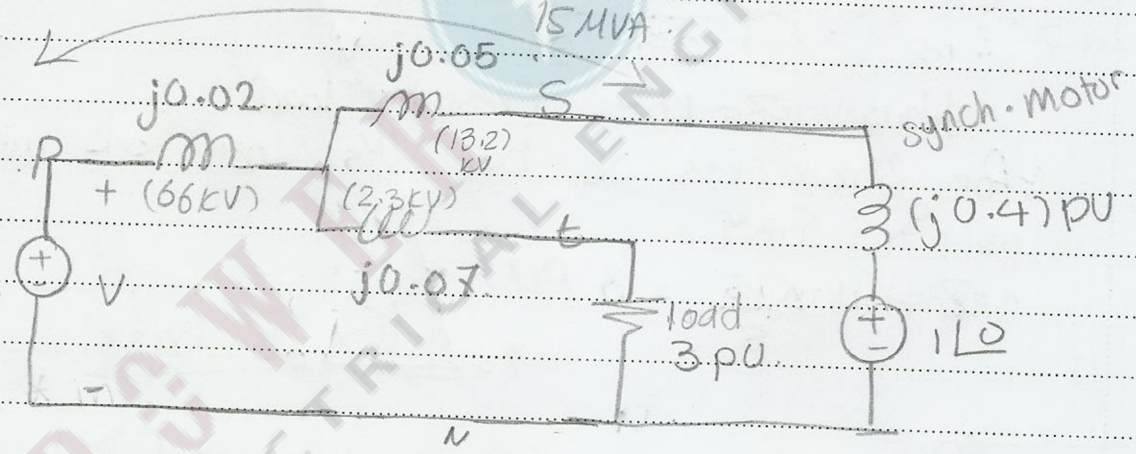
$Z_P = \frac{1}{2}(0.07 + 0.09 - 0.12)\, j$

$Z_p = 0.02\, j$ PU

↳ neglecting R.

$Z_S = \frac{1}{2}(0.07 + 0.12 - 0.09) = j\, 0.05$ PU

$Z_t = j\, 0.07$



* infinite bus:

V constant (infinite number of generators)

f. constant

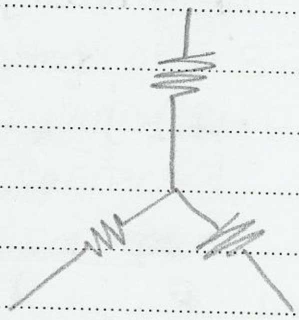
$Z_S = 0$

* the resistive load:-

5MW, 2.3 KV

to a base of 2.3 kV, 5 MVA

Z = 1 PU (prove it) →



$Z = \frac{V_{LN}}{I}$

$I = \frac{S}{3V_{LN}}$

$Z = \frac{3V_{LN}^2}{S} = \frac{V_{LL}^2}{S} = \frac{(2.3)^2}{5} (\Omega)$

$\text{In } PU = \frac{(2.3)^2/5}{(2.3)^2/5} = 1$

thus $Z = 1\angle 0$ (resistive load).

for 23KV it's ok but it's not ok with S, so..

$Z = 1 \times \frac{15}{5} = 3 \text{ PU}$

* for synchronous motor :-

7.5 MVA

13.2 KV

$X = 0.2j \text{ PU}$.

$X(PU)_{new} = 0.2j * \frac{15}{7.5} = j0.4$ (in S)

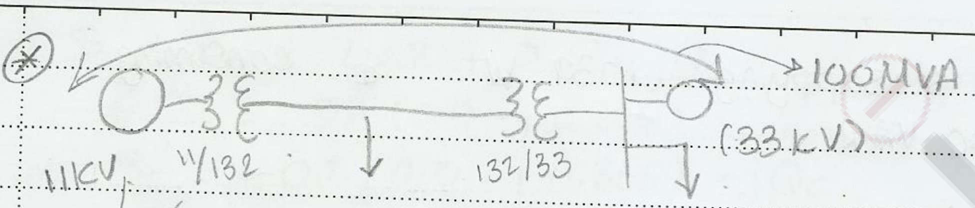
↓

Base values: 15 MVA

13.2 KV

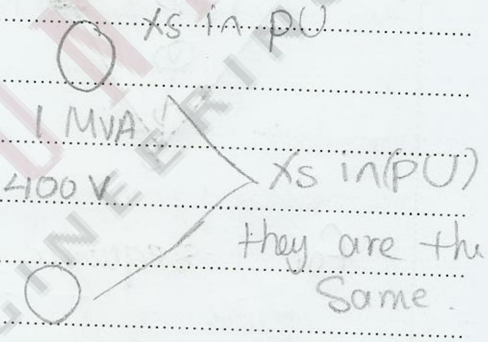
"experience makes you perfect" Dr. Hisham Hamdan

No. ....



base kV = 132 kV
base MVA = 100 MVA

Z rating Z rating.



they are the Same.

100MVA
13.2 kV

In (Ω) they are different.

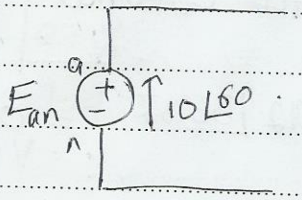
*end of CH.2 *

Solving problems of CH.1

1.10 $E_{an} = -120\angle 210$ V

$I_{na} = 10\angle 60$ A

$E_{an} = 120\angle 30$ add 180°



source is supplying.

$S = EI^* = 120\angle 30 * 10\angle -60$

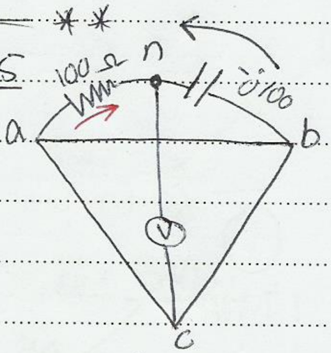
$= 1200\angle -30$

$= 1039 - j600$ VA →

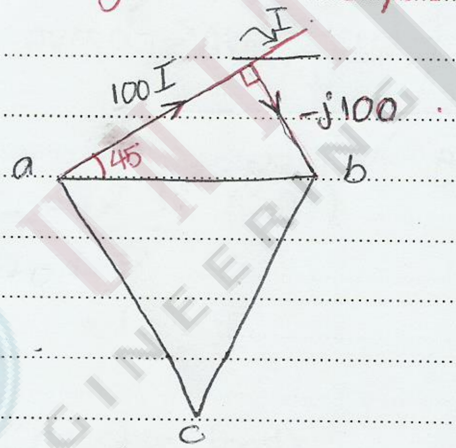
Source is supplying 1039 W & consuming 600 VAR.

—— * *

1.15

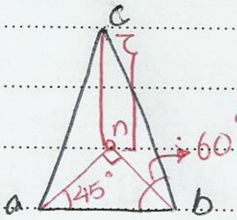


Leading Iab → Capacitor



for sequence abc.

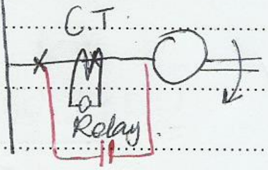
for sequence c b a



—— * * *

1.21

V= constant



* motor consumes

$S_1 = 0.92 \angle 36.87 = 0.92 (0.8 + j 0.6)$ MVA

* S_2 = generator supplies $0.1 \angle -60$ = $0.1 (0.5 - j 0.866)$
leading PF

For successful operation : $|I_1| = |I_2|$
Condenser consumes $-jQ_c$

$S_1' = 0.92\ (0.8 + j0.6) - jQ_c$

→ $S_2 = -0.1\ (0.5 - j\ 0.866)$

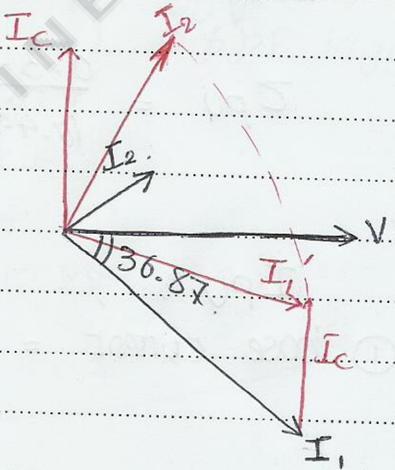
$S_2' = -0.1\ (0.5 - j\ 0.866) - jQ_c$

$|S| = |V||I^*|$ & $|I_1'| = |I_2'|$

$|S_1'| = |S_2'|$

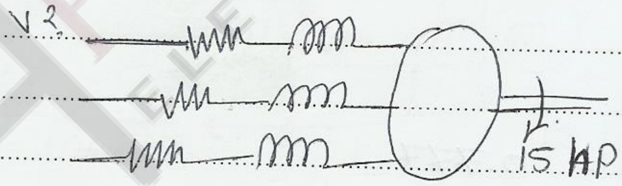
Solve it to find $\underline{\underline{Q_c}}$

Using phasor Diagram :-



* *

1.16 3-ϕ motor



15 HP, $\eta = 90\%$

pF = 0.8 lag.

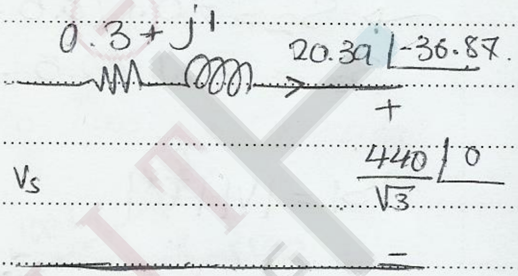
V = 440V.

$$P_{in} = \frac{15 \times 746}{0.9}$$

$$S_{in} = \frac{15 \times 746}{0.9 \times 0.8} \angle \pm 36.87 = 15514.67 \angle 36.87 \text{ VA}$$

$S = \sqrt{3} * V * I \angle 36.87$

$\longrightarrow I = 20.34 \angle -36.87$.



$V_s = \frac{440}{\sqrt{3}} + (0.3+j1)(20.34 \angle -36.87)$.

$V_s = 271.45 \angle 2.67°$

$*$ Using PU :-

base : 20 kVA , 440 V.

$$Z_{pu} = \frac{0.3+j1.0}{(0.44)^2/\left(\frac{20}{1000}\right)}$$

I_{PU} :

① base current $= \frac{20 \times 10^3}{\sqrt{3} * 440} = 26.24$ A

$$I_{PU} = \frac{20.39}{26.24} \angle -36.87 = 0.777$$

OR

② $S_{p.u} = \frac{15541.67}{20} = 0.777$.

$S = V I^*$

$0.777 \angle 36.87 = 1 \angle 0 \; I^*$

$\rightarrow I^* = 0.777 \angle -36.87$.

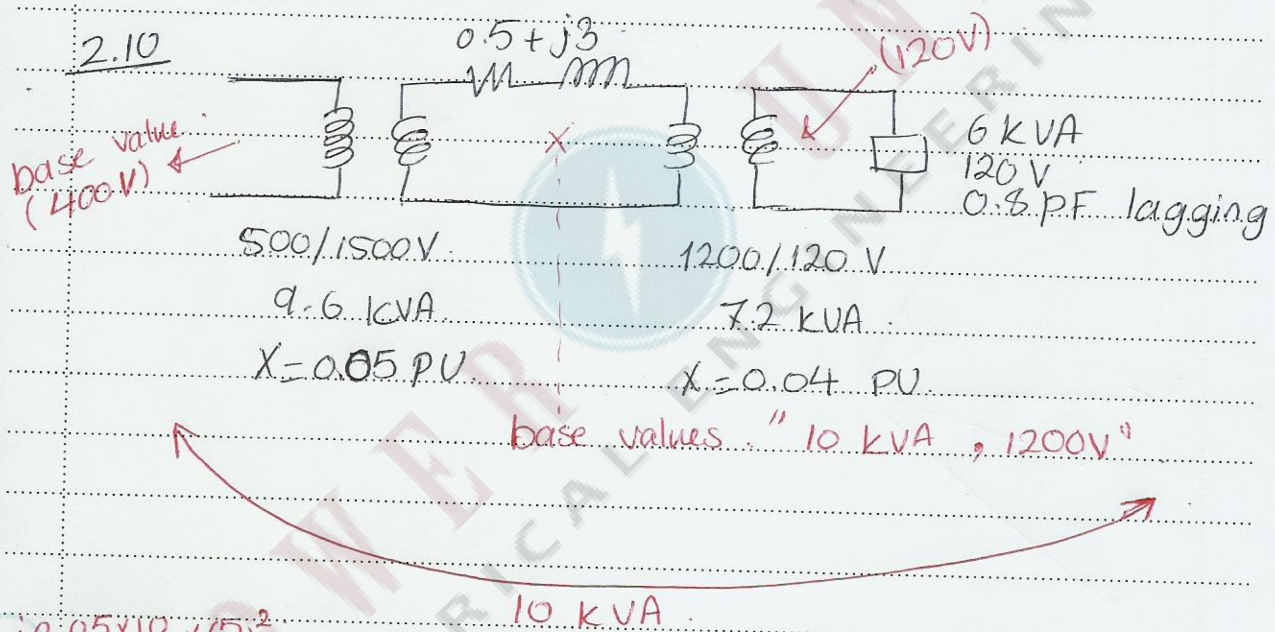
$V_S = 1\angle 0 + (0.777\angle{-36.87})(0.0301 + j\,0.301)$

$$V_R = \frac{271.45 - (440/\sqrt{3})}{(440/\sqrt{3})}$$

———— **

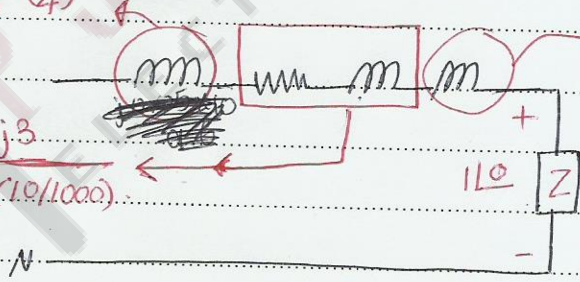
Problems of CH.2

2.10



$j\,0.05 \times \frac{10}{9.6} \times \left(\frac{5}{4}\right)^2$

$j\,0.04 \times \frac{10}{7.2}$



$\frac{0.5 + j3}{(1.2)^2/(10/1000)}$

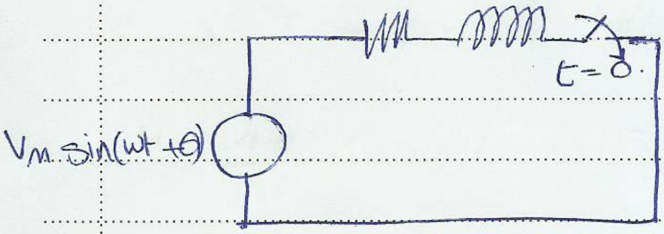
$1\angle 0$

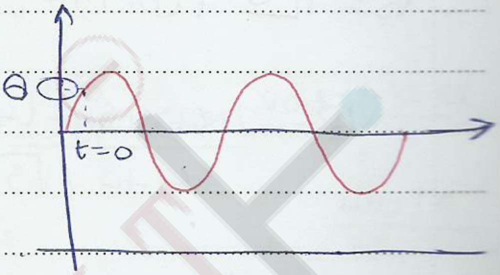
$Z \rightarrow 1 \times \frac{10}{6}\angle{+36.87^\circ}$

$I\,(PU) = \frac{1\angle 0}{10/6\angle 36.87} = 0.6\angle{-36.87^\circ}$

No.

Lecture "9" CH.3





$i_{tot} = i_{ss} + i_{tr}$

$$i_{tot} = \frac{V_{max}}{Z} \sin(\omega t + \theta - \phi) + A e^{-\frac{R}{L} t}$$

$\phi = \tan^{-1} \frac{\omega L}{R}$

θ : point on wave at which the switch closes.

at $t=0 \rightarrow$

$$i(t) = 0 = \frac{V_{max}}{Z} \sin(\theta - \phi) + A$$

So: $A = -\frac{V_{max}}{Z} \sin(\theta - \phi)$

$$i(t) = \frac{V_{max}}{Z} \left(\sin(\omega t + \theta - \phi) - \sin(\theta - \phi) e^{-\frac{R}{L} t}\right)$$

for $\theta = 0$ & $\phi = 90 \rightarrow R \simeq 0$.

AC component of $i \rightarrow$ fig(1)

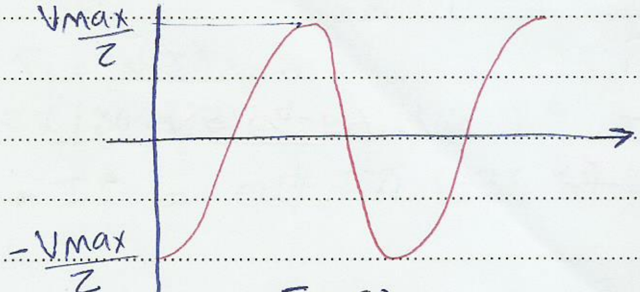


Fig (1)

for θ & $\Phi = 90$ (fig 2)

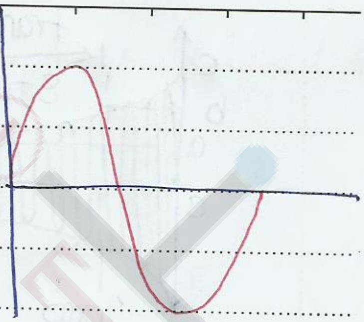
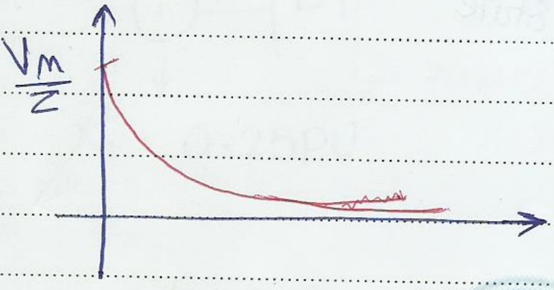
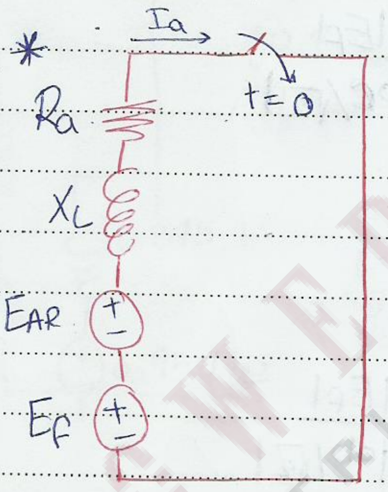


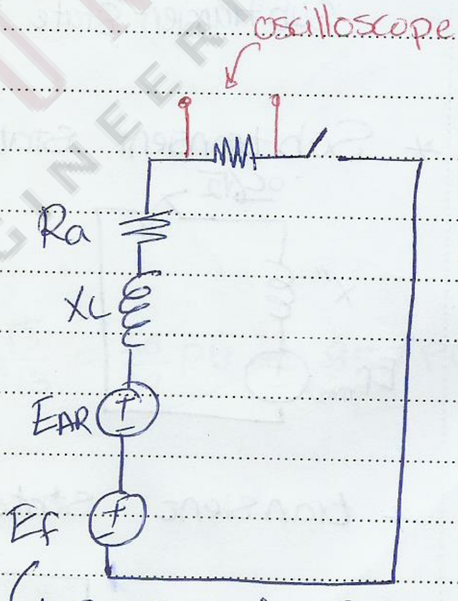
fig.(2)

& for the DC component fig (3):





$\Longrightarrow$ put a shunt resistor



$\hookrightarrow E_f = 4.44\, \Phi\, N f\, K_d\, K_p$

Φ = flux, f = freq → speed, $K_d\,K_p$ = constant of the gen.

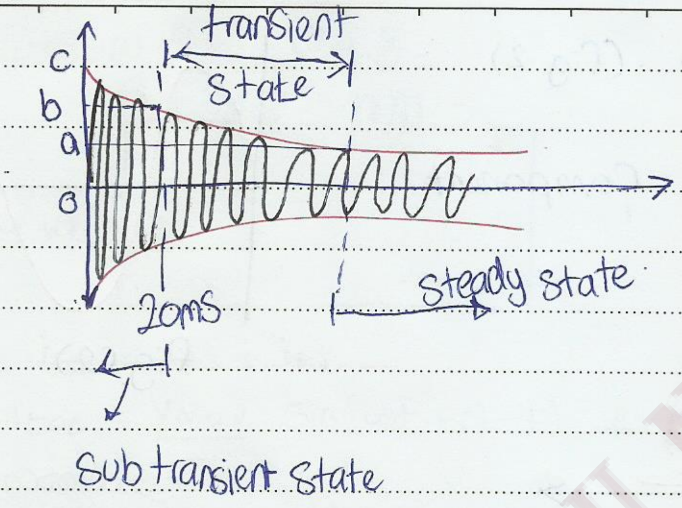
$E_{AR} = -j K I_a$

$-E_f - \underbrace{E_{AR}}_{-(jKI_a)} + j I_a X_L + I_a R_a = 0$

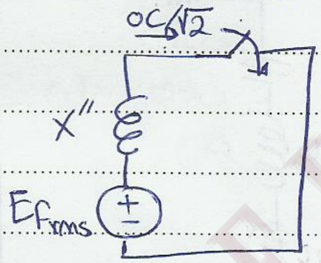
$-E_f + jK I_a + j I_a X_L + I_a R_a = 0$

$-E_f + j I_a \underbrace{(K + X_L)}_{X_S} + I_a R_a = 0$

X_S → at steady state

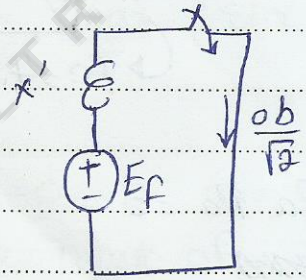


* Sub transient state → $X_d'' = \frac{|E_f|}{|oc/\sqrt{2}|}$

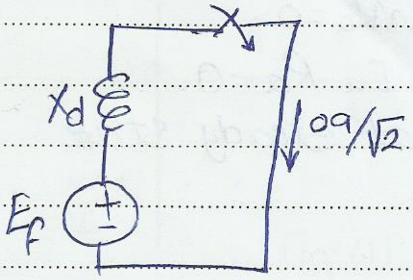


* transient state → $X_d' = \frac{|E_f|}{|ob/\sqrt{2}|}$

direct axis

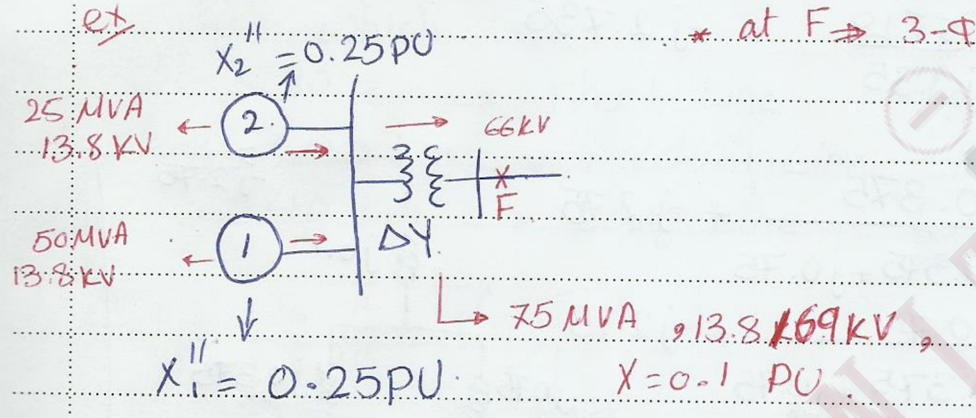


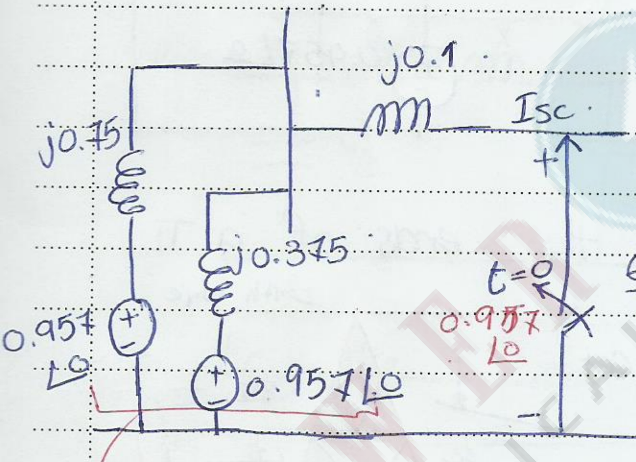
* Steady state → $X_d = \frac{|E_f|}{|oa/\sqrt{2}|}$



ex

* at F → 3-Φ s.c balanced fault.



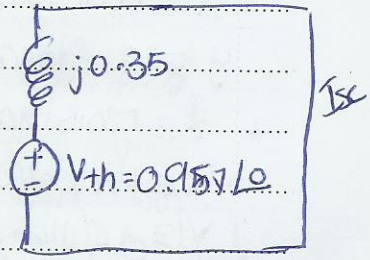


$$\frac{66}{\sqrt{3}}V \rightarrow \frac{66\sqrt{3}}{69\sqrt{3}} = \frac{66}{69}PU = 0.957PU \angle 0$$

$$j0.75 = 0.25 * \frac{75}{25}$$

$$j0.375 = 0.25 * \frac{75}{50}$$

there's no circulating current.



when the Switch is open.

$V_{th} = 0.957\angle 0$

$$Z_{th} = j0.1 + \frac{j0.75 * j0.375}{j.75 + j0.375} = j0.35PU.$$

→

No.

$I_{sc} = \frac{0.957\angle 0}{j0.35} = -j2.735$.

$I_{g_2} = \frac{j0.375}{j0.375 + j0.75} * -j2.75$.

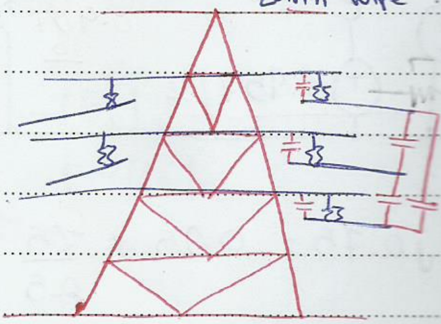
$I_{g_1} = \frac{j0.75}{j0.375 + j0.75} * -j2.75$.

* Solve problems 12 & 13 *

Lecture "10" * CH.3 *

V, I, S Relations at the ends of a TL

Double cct line



* $r \equiv$ resistance per unit Length (Ω/Km).

$x \equiv$ reactance per unit length.

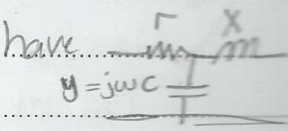
$y \equiv$ admittance per unit length.

* $R \equiv$ resistance of a total line.

$X \equiv$ reactance of a total Line.

$Y \equiv$ admittance of the total line.

each line have



$r + jx = z$

$R + jX = Z$

* Line which is:

< 80 Km → Short Line "fig(1)"

80 - 240 Km → medium Line "fig(2)"

> 240 km → Long Line "Fig (3)"

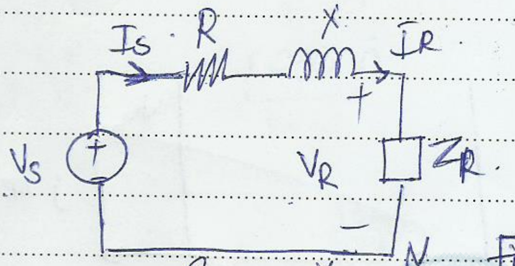
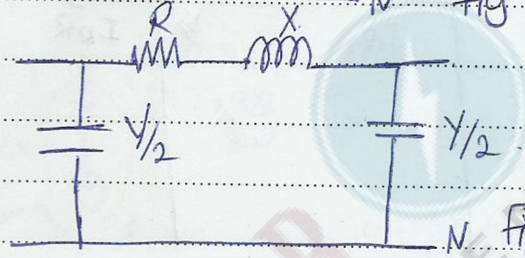


Fig (1)



fig(2)

Long Line : exact solution (only mathmatical)

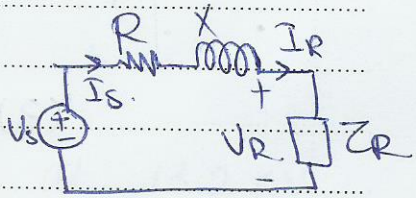
fig(3)

* The relation[illegible] between Vs & VR

$I_S = I_R$ ——①

$V_S = V_R + I_R Z_R$ ——②

Using two ports network:



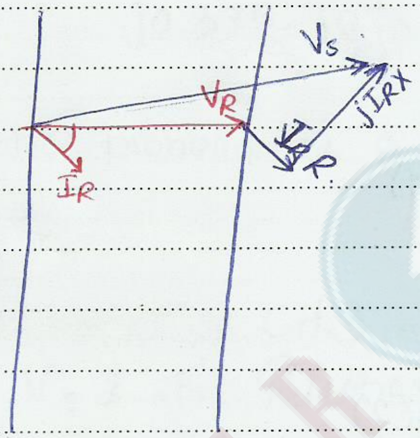
$V_S = \underset{A}{\boxed{t_{11}}} V_R \underset{B}{\boxed{-t_{12}}} I_R$

$I_S = \underset{C}{\boxed{t_{21}}} V_R + \underset{D}{\boxed{-t_{22}}} I_R$

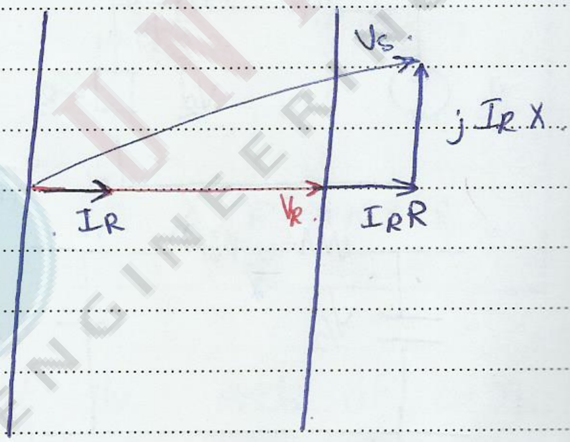
→

* $A = 1$
$B = Z$
$C = 0$
$D = 1$ } "according to the equations" ① & ②

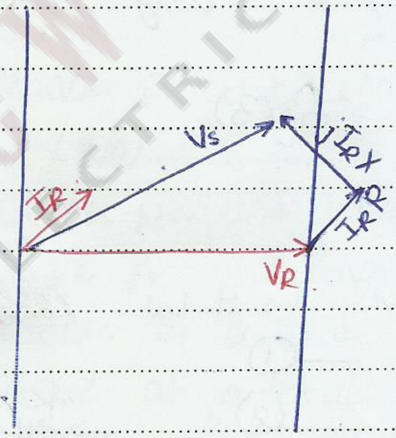
*



(1) Lagging PF



(2) Unity PF.

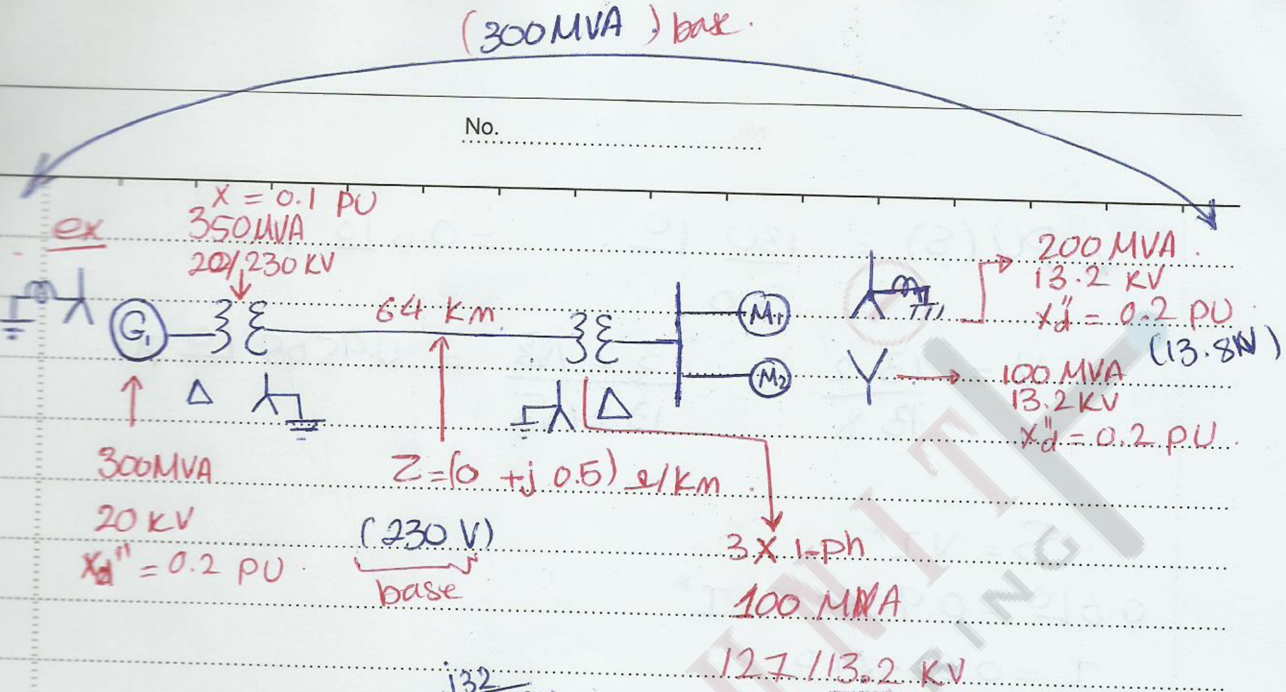


(3) Leading PF.

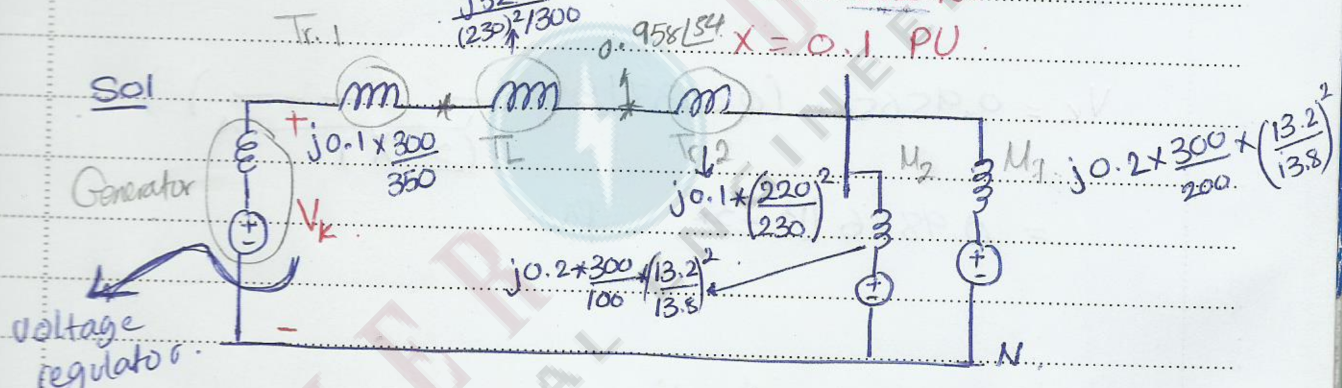
$$VR = \frac{|V_{NL}| - |V_{FL}|}{|V_{FL}|}$$ then :-

$$VR = \frac{|V_s| - |V_R|}{|V_R|}$$

ex



Sol



3-ph rating :

300 MVA.

$127\sqrt{3}$ / 13.2 → 220 / 13.2

$\boxed{220}$

$\frac{230}{220} \times 13.2 = 13.8$

M1 → 120 MW at U.PF at 13.2 KV

M2 → 60 MW at U.PF at 13.2 KV

M1 + M2 = 180 MW

S = 180∠0° MVA.

$* \; PU(S) = \frac{180}{300} \angle 0 \quad = 0.6 \angle 0 \; pU$

$* \; V = \frac{13.2}{13.8} = \frac{13.2/\sqrt{3}}{13.8/\sqrt{3}} = 0.9565 \angle 0 \; pU.$

$S = V I^*$

$0.6\angle 0 = 0.9565 * I^*$

$I = 0.6237\angle 0 .$

$V_K = 0.9565 + (0.6237\angle 0 * (\underbrace{- + - + -}_{(\Sigma X)}))$

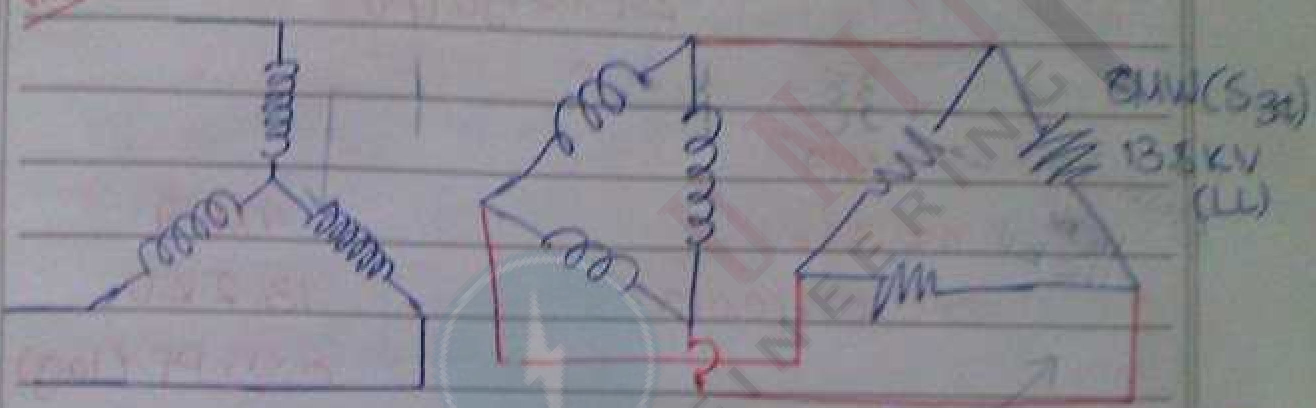
$= 0.9826 \angle 13.237^\circ \; pU.$

VR across the Line :-

$VR = \frac{0.9826 - 0.9582}{0.9582} .$

Continue Solving problem For ch.2

n.2



coz it's $\Delta \Rightarrow \frac{(13.8)^2}{R} = \frac{8}{3}$

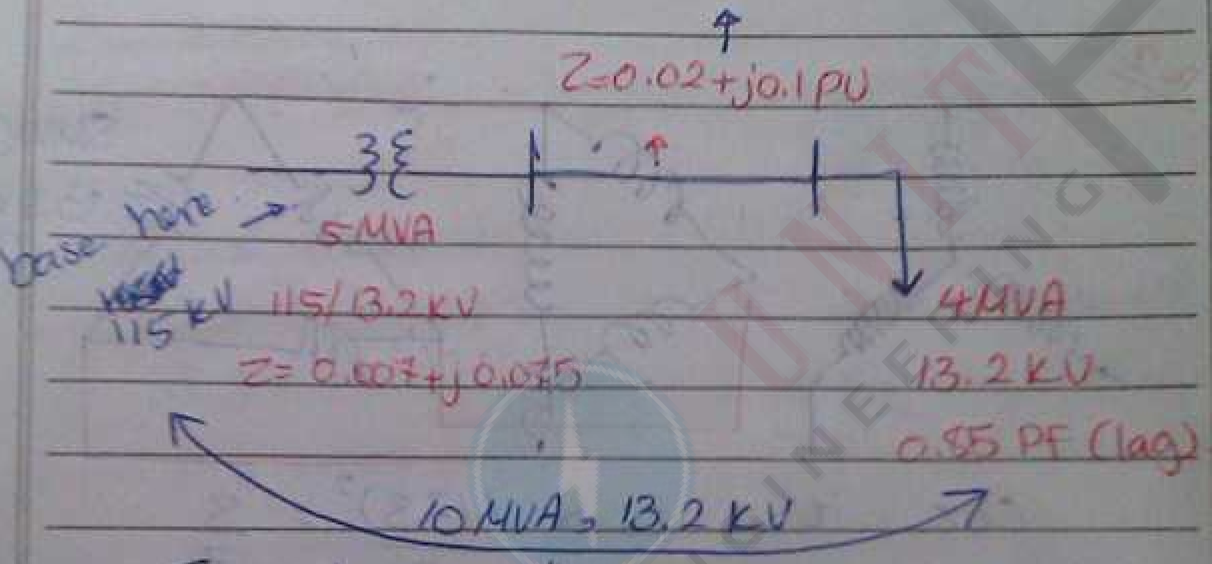
$\Rightarrow R = 71.415\ \Omega$

Note If it was Y-connected to Find $R \Rightarrow \frac{(13.8)^2}{R} = 8 \rightarrow R = \frac{71.415}{3}$

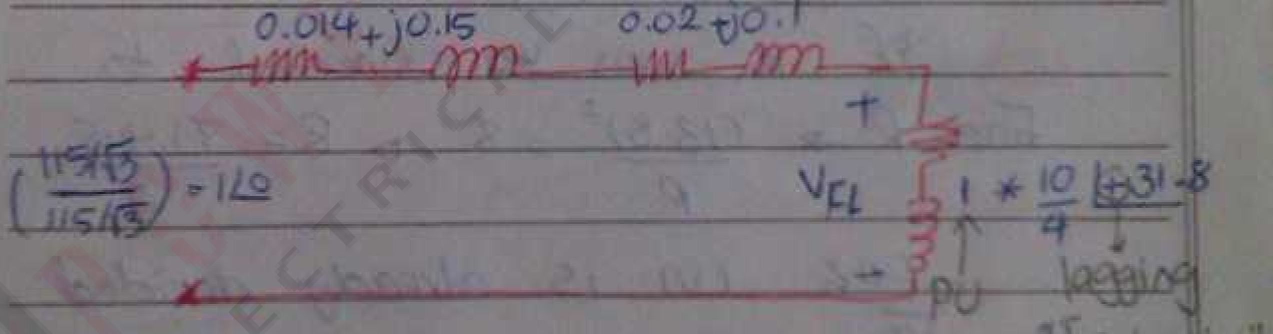
coz (V) is already devided by $\sqrt{3}$.

Ex. 3

base values "10MVA, 13.2 KV"



Impedance diagram:



$$\left(\frac{115/\sqrt{3}}{115/\sqrt{3}}\right) = 1\angle 0$$

$\cancel{V_{FL}} =$

$$V_{FL} = \frac{2.5\angle 31.8}{2.5\angle 31.8 + 0.14 + j0.15 + 0.02 + j0.1} * 1\angle 0$$

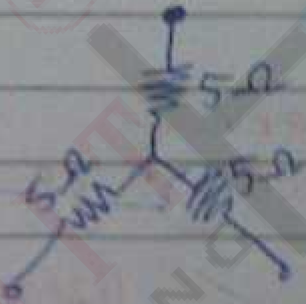
$$VR = \frac{\overset{1\angle 0}{V_{NL}} - V_{FL}}{V_{FL}}$$

16.2 3x 1-ph

1.2/0.12 KV

7.2 KVA

X=0.05PU.

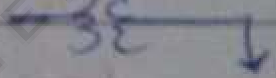


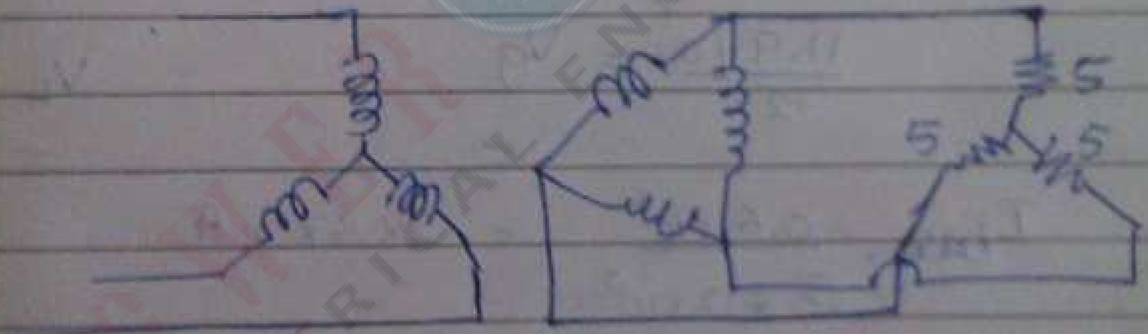
If we make it Y-Δ

(1.2 * √3 /0.12) KV

$7.2 * 3 = 21.6$ KVA

$X = 0.05$ PU





$$5 * \left(\frac{(1.2\sqrt{3})^2}{(21.6/1000)} \right) = j10\ \Omega$$

We see the Impedance as

$(1500 + j10)\ \Omega$

In PU → $\frac{1500+j10}{(1.2/\sqrt{3})^2/(21.6/1000)}$

17.2

pF = 0.8 lagging

(12/0.6kV

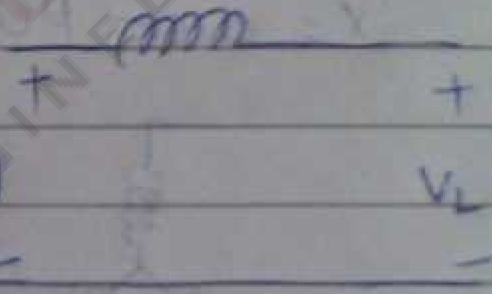
base ⇒ 12 kV

Δ λ

0.6 MVA

$X = 0.1$ PU

↳ base values: 0.6 kV , 0.6 MVA



$$\frac{11.9}{12}\angle 0 = V_g$$

$$I_{base} = \frac{0.6 \times 10^6}{\sqrt{3} * 12 \times 10^3} = 28.875 \text{ A}$$

$$I(\text{in PU}) = \frac{20}{28.875} = 0.693 \angle -36.87$$

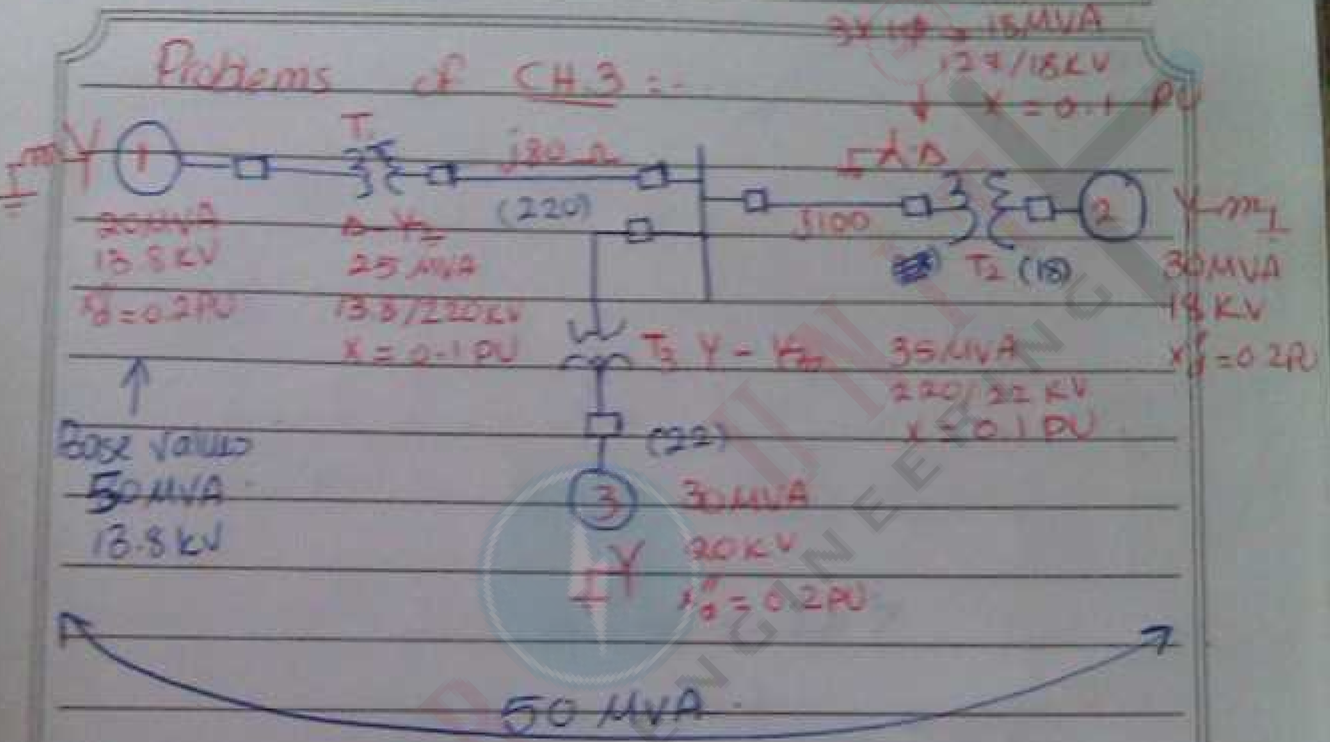
$$V_L = \frac{11.9}{12}\angle 0 - j0.1 * 0.693 \angle -36.87$$

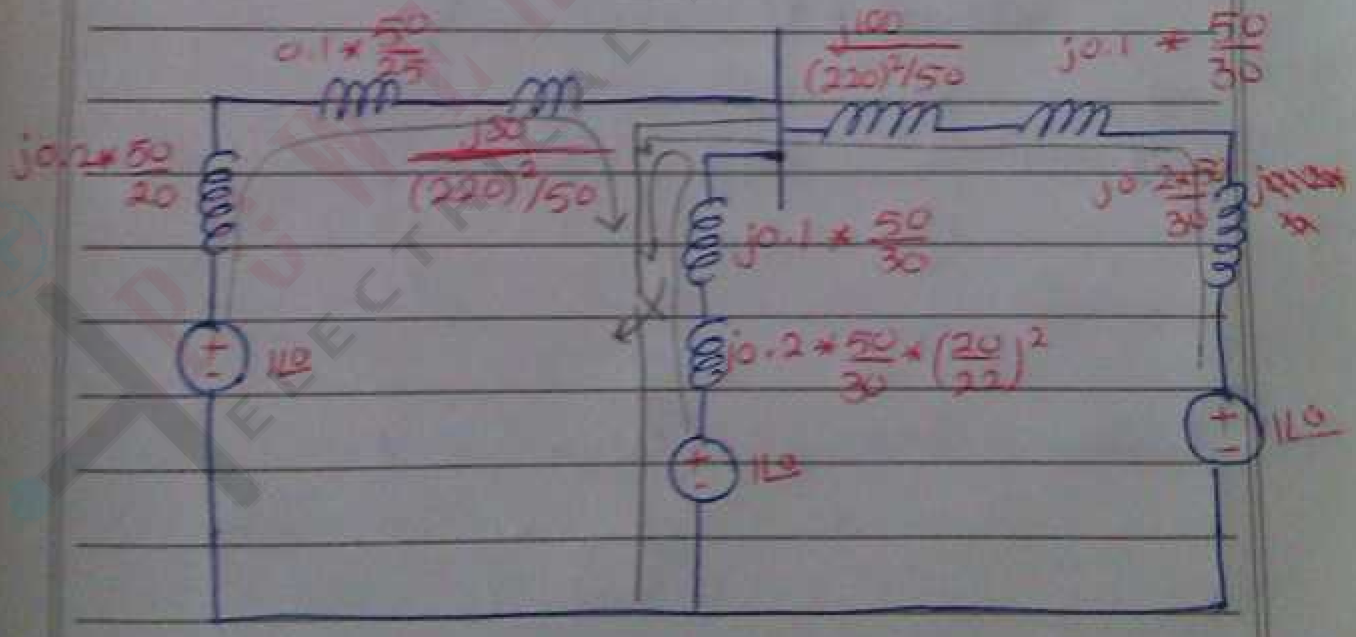
$$V_L = 0.95174 \angle -3.33$$

$$V_L = 0.95174 * \frac{600}{\sqrt{3}} = V \boxed{} V_{ph}$$

$$V_{L(LL)} = 0.95174 * 600 = \boxed{} V_{LL}$$

Problems of CH.3 ::

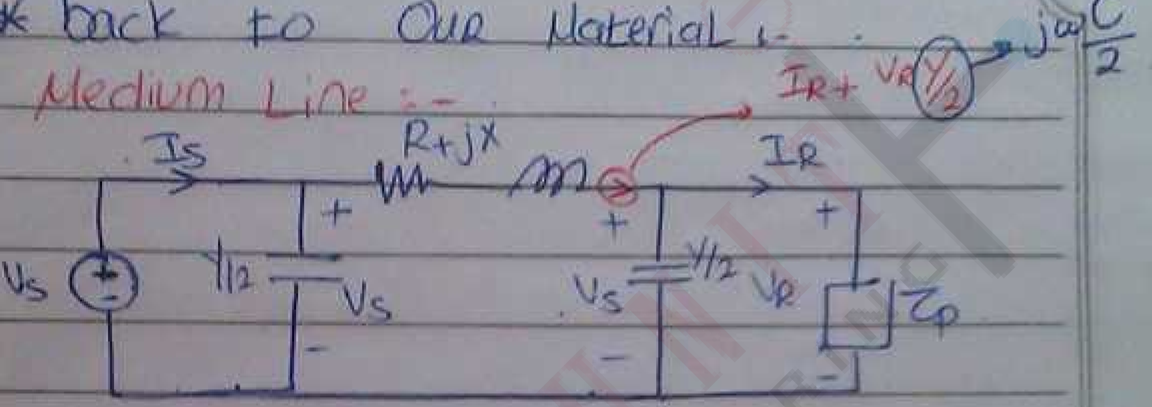




all the three current moves to the s/c → $S = VI^*$

* back to Our Material :-

Medium Line :-



$$* \quad V_S = V_R + (I_R + V_R \cdot Y/2) Z$$

$$V_S = (1 + ZY/2) V_R + I_R Z$$

$$* \quad I_S = (V_S * Y/2) + (V_R * Y/2) + I_R$$

$$= \left[\left(\left(1 + \frac{ZY}{2}\right) V_R + I_R Z\right) Y/2\right] + (V_R * Y/2) + I_R$$

$$I_S = Y\left(1 + \frac{YZ}{4}\right) V_R + \left(1 + \frac{YZ}{2}\right) I_R$$

$A = 1 + \frac{YZ}{2}$ (dimenssionless)

B

$B = Z$ (Ω)

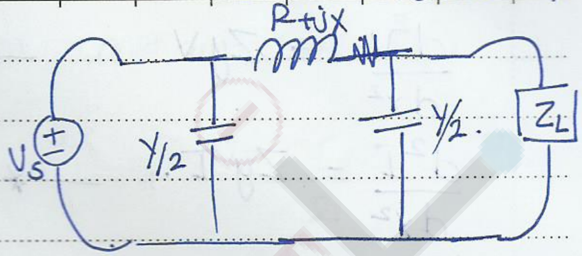
$C = Y\left(1 + \frac{YZ}{4}\right)$ (℧)

$D = \left(1 + \frac{YZ}{2}\right)$ (dimenssionless).

No. power.

Sun (16/3/2014)

* medium Line :-



$$V_S = \left(1+\frac{\overset{jwC}{Y}\,\overset{jwL}{Z}}{2}\right)V_R + Z\,I_R$$

$$I_S = Y\left(1+\frac{YZ}{4}\right)V_R + \left(1+\frac{YZ}{2}\right)I_R$$

$$VR = \frac{|V_{NL}| - |V_{FL}|}{|V_{FL}|} \rightarrow VR = \frac{\frac{|V_S|}{|A|} - V_{FL}}{V_{FL}}$$

$$V_S = A\,V_R + B\times 0$$

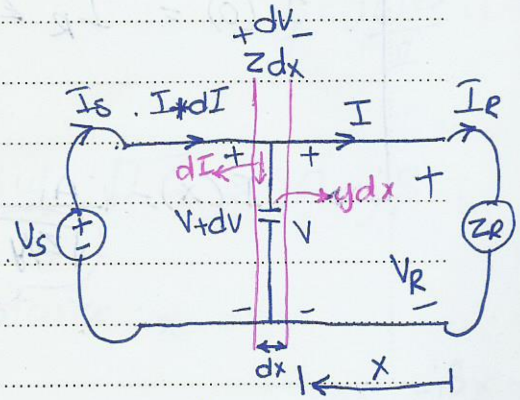
$$|V_{RNL}| = \frac{|V_S|}{|A|}$$

* For $A \leq 1$

$|A| = 0.85 - 1$

$\angle A = \alpha = 0 - 3^\circ$

* Long Line *



$$dV = \frac{I + (\overset{\approx 0}{dI} + I)}{2}\cdot Z\,dx$$

$$= Z I\,dx$$

$$\frac{dV}{dx} = Z I$$

where : z = impedance / unit length
y = admittance / unit length

$$dI = \overset{V}{\cancel{\frac{V+dV+V}{2}}}\cdot y\,dx$$

$$\frac{dI}{dx} = yV$$

$$\frac{d^2V}{dx^2} = ZyV \quad —*— ①$$

$$\frac{d^2I}{dx^2} = Zy\,I \quad —*— ②$$

* FOR ① *

$$* \quad \frac{d^2V}{dx^2} - yZV = 0$$

$$(s^2 - yZ)\,V(s) = 0$$

$$* \quad V(x) = A_1\, e^{\sqrt{yZ}\,x} + A_2\, e^{-\sqrt{yZ}\,x} \quad —— **$$

$$\& \quad I(x) = \left(\frac{dV}{dx}\right) / Z = \frac{A_1\sqrt{yZ}\, e^{\sqrt{yZ}\,x} - A_2\sqrt{yZ}\, e^{-\sqrt{yZ}\,x}}{Z}$$

$$V(0) = V_R = A_1 + A_2 \quad (\text{for } x=0) \quad — ③$$

$$I(0) = I_R Z_c = A_1 - A_2 \quad — ④$$

$$\rightarrow I(x) = \frac{A_1}{\sqrt{Z/y}}\, e^{\sqrt{yZ}\,x} - \frac{A_2}{\sqrt{Z/y}}\, e^{-\sqrt{yZ}\,x}$$

$\sqrt{Z/y}$ → charactaristic impedance Z_c

Solving for ③ & ④ :

$$A_1 = \frac{V_R + I_R Z_c}{2}$$

$$A_2 = \frac{V_R - I_R Z_c}{2}$$

For $\sqrt{yz}$ = propagation constant (γ)

then :-

$$V(x) = \frac{V_R + I_R Z_c}{2} e^{\gamma x} + \frac{V_R - I_R Z_c}{2} e^{-\gamma x}$$

& $$I(x) = \frac{\frac{V_R}{Z_c} + I_R}{2} e^{\gamma x} - \frac{\frac{V_R}{Z_c} - I_R}{2} e^{-\gamma x}$$

For $\gamma = \alpha + j\beta$ → phase constant (rad/unit length)

γ ↓ propagation constant

α ↓ attenuation constant (nepper / unit length).

$\gamma = \sqrt{Zy}$; for a lossless Line : $Z = (j\omega L)/l$

$y = j\omega \frac{C}{l}$ → total Line

thus .

$$\gamma = \sqrt{j\omega \frac{L}{l} \cdot j\omega \frac{C}{l}} = \frac{j\omega}{l}\sqrt{LC} = 0 + j\beta .$$

* $\alpha = 0$ → no resistance . *

$$V(x) = \underbrace{\frac{V_R + I_R Z_c}{2} e^{\alpha x} e^{j\beta x}}_{\text{incident wave}} + \underbrace{\frac{V_R - I_R Z_c}{2} e^{-\alpha x} e^{-j\beta x}}_{\text{reflected wave}}$$

incident wave

(مصدره عند src

كل ما اقتربنا زاد) .

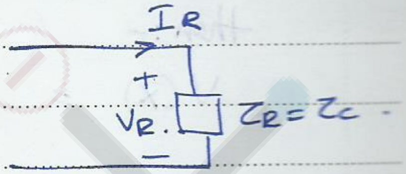
reflected wave

⇓

مصدره عند end pnt.

if $(Z_R = Z_C)$: no reflected wave .

$$\frac{V_R - I_R Z_C}{2} e^{-\alpha x} e^{-j\beta x} = 0$$



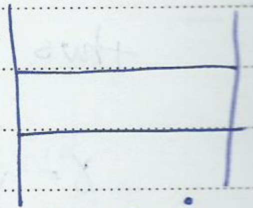
→ So the Line is called flate line , OR infinite Long Line OR infinite Length.

* $Z_C = 400 \underline{\angle 0-(-15)^\circ}$ — * ¢ .

$$Z_C = \sqrt{\frac{Z}{y}} = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{|Z| \angle 60-90}{|Y| \angle 90}} = \sqrt{\frac{|Z|}{|Y|}} \angle 0-(-15)^\circ$$

* For lossless Line → $Z_C = \sqrt{\frac{j\omega L}{j\omega C}}$ → phase = 0

* For two parallel Lines :



$$Z_C = 200 \angle 0-(-15)$$

$$= \sqrt{\frac{\frac{1}{2}|Z| \angle 60-90}{2|Y| \angle 90}} = \underline{\underline{200}} \angle 0-(-15)^\circ .$$

↳ same phase

For $Z_C = 400 \angle 0$; Z_C is called " surge Impedance ".

(*) Surge Impedance Loading (SIL) → "3¢ value"

$$SIL_{3\phi} = \frac{V_{LL}^2}{Z_C}$$

⊛ λ = wave Length $= \frac{2\pi}{\beta}$

⊛ V = wave velocity $= f.\lambda = \frac{2\pi f}{\beta}$

For β (In a lossless Line) $= \frac{\omega}{\ell}\sqrt{LC}$

thus $V = \frac{\ell}{\sqrt{LC}}$ (m/s) $= 3\times10^{8}$ m/s.

* $\cosh\theta = (e^{\theta} + e^{-\theta})/2$.

$\sinh\theta = (e^{\theta} - e^{-\theta})/2$.

We will use them to simplify V(x) & I(x) :-

*– $V(x) = \cosh(\gamma x)\,V_R + I_R Z_C \sinh\gamma x$.

*– $I(x) = \sinh(\gamma x)\frac{V_R}{Z_C} + I_R\cosh(\gamma x)$

all of them are (1×)

$$\left[\begin{array}{l}\text{For } (x=\ell).\\ V(\ell) = V_s = \overbrace{V_R\cosh(\gamma\ell)}^{A} + \overbrace{I_R Z_C \sinh(\gamma\ell)}^{B}\\ I(\ell) = I_s = \underbrace{\frac{\sinh(\gamma\ell)}{Z_C}}_{C} V_R + I_R \underbrace{\cosh(\gamma\ell)}_{D}\end{array}\right.$$

→ A = D.

→ AD − BC = 1

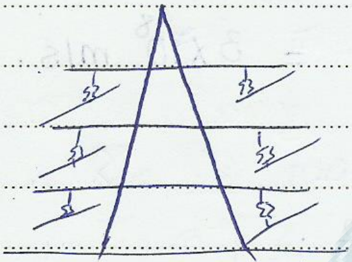
$$\cosh\gamma L = \frac{e^{\gamma L} + e^{-\gamma L}}{2} = \frac{e^{\alpha L}\angle\beta L + e^{-\alpha L}\angle-\beta L}{2}$$

$$\sinh\gamma L = \frac{e^{\alpha L}\angle\beta L - e^{-\alpha L}\angle-\beta L}{2}$$

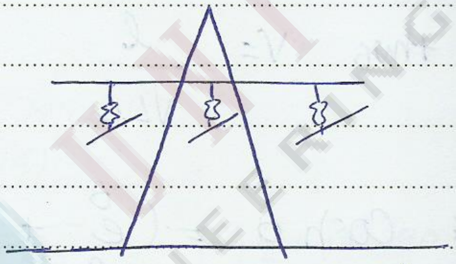
(*) $\cosh \gamma L = 1 + \frac{(\gamma l)^2}{2!} \Big| + \frac{(\gamma L)^4}{4!}$ → في العادة نقف هنا (تقريب)

$\sinh \gamma L = \gamma l + \frac{(\gamma l)^3}{3!} \Big| + \frac{(\gamma l)^5}{5!}$

→ نقف هنا (تقريب)



double circuit



Single circuit.

ex Single circuit.

230 mi

$Z = 0.8431 \angle 79.04^\circ$ Ω/mi

$y = 5.105 \times 10^{-6} \angle 90$ S/mi

receiving end

$S_R = 125 MVA$

215 kV

at u.pF.

Find: V_S, I_S, S_S?

sol

$$\gamma l = \sqrt{0.8431 \angle 79.04 * 5.105 * 10^{-6} \angle 90} * 230 = \sqrt{yz}\, l$$

$$= 0.4772 \angle 84.52$$

$$= 0.0456 + j\, 0.475$$

$$Z_c = \sqrt{\frac{z}{y}} = \sqrt{\frac{0.8431 \angle 79.04}{5.105 \times 10^{-6} \angle 90}} = 406.4 \angle -5.48^\circ$$

No.

$$\cosh \gamma L = \frac{e^{0.0456} \angle 0.475^{rad} + e^{-0.0456} \angle -0.475 \text{ rad}}{2}$$

$$= 0.8904 \angle 1.34^\circ$$

$$\sinh \gamma L = 0.4597 \angle 84.93^\circ$$

$$V_S = \left(0.8904 \angle 1.34 * \frac{215000 \angle 0}{\underset{\uparrow \text{ per phase}}{\sqrt{3}}}\right) + \left(406.4 \angle -5.48 * 0.4597 \angle 84.93 * 335 \angle 0\right)$$

$$= 137860 \angle 27.77^\circ$$

$$I_S = \left(\frac{0.4597 \angle 84.93}{406.4 \angle -5.48} * \frac{215000 \angle 0}{\sqrt{3}}\right) + \left(0.8904 \angle 1.34 * 335.7 \angle 0\right)$$

$$= 332.31 \angle 26.33^\circ$$

$$S_S = 3 V_S I_S^*$$

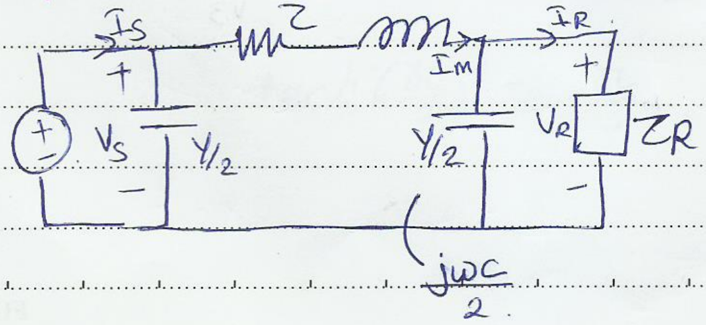
$$= 3|V_S||I_S| \angle 27.77^\circ - 26.33^\circ = 3|V_S||I_S| \angle 1.44^\circ$$

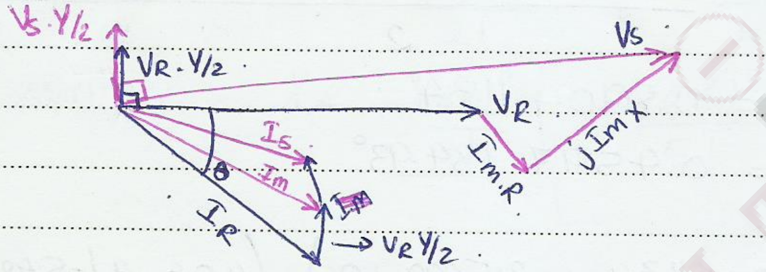
* before Finding Vs & Is *

$$I_R = \frac{125 * 10^6}{\sqrt{3} * 215 * 10^3}$$

$$= 335.7 \angle 0$$

* phasor Diagram For medium Line:-





Continue Solving the example:

$$VR = \frac{\frac{|Vs|}{|A|} - |V_R|}{|V_R|}$$

$$= \frac{\frac{137860}{0.8904} - \frac{215000}{\sqrt{3}}}{215000/\sqrt{3}} = 24.7\ \%$$

* Solving the same example in PU:-

Base values: 125 MVA

215 KV

base $Z = (215)^2/125$

$S_{pu} = 1\angle 0 = 1\angle 0 * I_R^*$

$I_R = 1\angle 0$

$$V_S = A V_R + B I_R$$

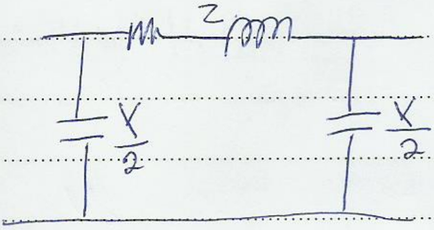
$$= (0.8904\angle 1.34 * 1\angle 0) + \underbrace{370}_{Z_C} \sinh \gamma L * 1\angle 0$$

cuz(A) is dimenssionless

$$= 1.102\angle 27.75^{\circ} \quad PU \Rightarrow * \frac{215000}{\sqrt{3}} = 137860\angle 27.75$$

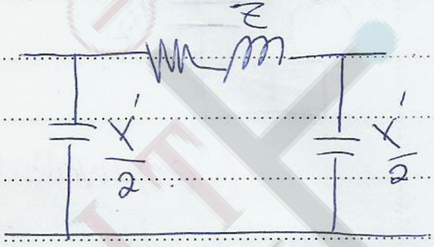
* ~~equi~~ π equivalent circuit for a long Line :-

for medium Line -



→

for long Line



$$V_S = \left(1 + \frac{YZ}{2}\right) V_R + Z I_R$$

(nominal π)

$$V_S = \boxed{\cosh \gamma l}\, V_R + \boxed{Z_c \sinh \gamma l}\, I_R$$

$$V_S = \boxed{\left(1 + \frac{Y'Z'}{2}\right)} V_R + \boxed{Z'} I_R$$

proof :→

$$\cosh \gamma L = \left(1 + \frac{Y'Z'}{2}\right)$$

$$Z_c \sinh \gamma L = Z'$$

⊛ $Z' = Z_c \sinh \gamma L$

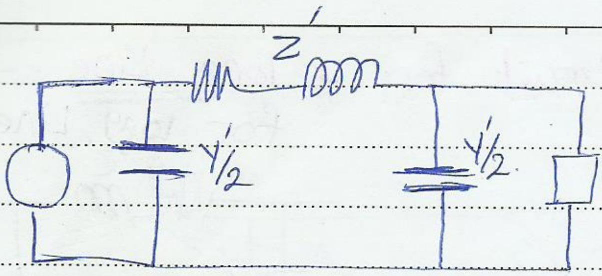
$$= \frac{\sqrt{Z}}{\sqrt{y}} \cdot \frac{\sqrt{Z}}{\sqrt{Z}} \cdot \frac{l}{l} \sinh \gamma L$$

$$Z' = Z \cdot \frac{\sinh \gamma L}{\gamma L} \rightarrow \sinh \gamma L \cong \gamma L$$

⊛ $1 + \frac{Y'Z'}{2} = \cosh \gamma L$

$$\frac{Y'}{2} = \frac{\cosh \gamma L - 1}{Z_c \sinh \gamma L} = \frac{Y}{2} \frac{\tanh\left(\frac{\gamma L}{2}\right)}{\left(\frac{\gamma L}{2}\right)}$$

$$\tanh\left(\frac{\gamma L}{2}\right) \cong \frac{\gamma L}{2}$$



(equivalent π)

ex

$Z = 230 \times 0.8431 \angle 79.04$

$= 193.9 \angle 79.04$

$Z' = Z_c \sinh \gamma L = Z \dfrac{\sinh \gamma L}{\gamma L}$

$= 193.9 \angle 79.04 * \dfrac{0.4597 \angle 84.93}{0.4772 \angle 84.52}$

$= 186.82 \angle 79.45$

$\dfrac{Z'}{Z} = 96.2\%$ → error $= -3.8\%$

$\dfrac{Y'}{2} = \dfrac{0.8904 \angle 1.34^\circ - 1}{186.82 \angle 79.45} = 0.000599 \angle 89.82$

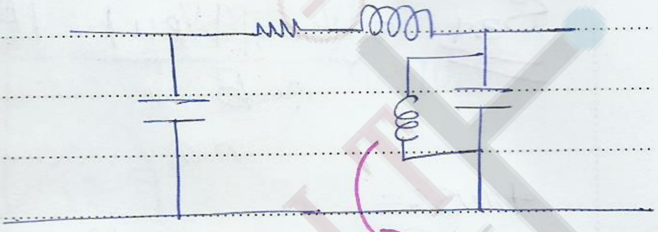
$Y/2 = \dfrac{5.105 \times 10^{-6} \times 230}{2} = 0.000587 \angle 90$

$\dfrac{Y'/2}{Y/2} = 1.02$

↳ error $= +2\%$

$V_S = A\ V_R \longrightarrow V_R = \frac{V_S}{A}$ (at no load)

$(|V_S| = |V_R|)$



at resonance.
$I = 0 \rightarrow$ $V_S = V_R$

Circle Diagram:- (recieving end)

$V_S = AV_R + B\,I_R$

$I_R = \frac{V_S - AV_R}{B}$; $I_R = |I_R| \angle \theta$

$V_R = |V_R| \angle 0^\circ$

$V_S = |V_S| \angle \delta \longrightarrow$ Power Angle.

$A = |A| \angle \alpha$, $B = |B| \angle \beta$

$$I_R = \frac{|V_S| \angle \delta - |A|\,|V_R| \angle \alpha}{|B| \angle \beta}$$

$$= \frac{|V_S|}{|B|} \angle \delta - \beta - \frac{|A|\,|V_R|}{|B|} \angle \alpha - \beta$$

$$I_R^* = \frac{|V_S|}{|B|} \angle \beta - \delta - \frac{|A|\,|V_R|}{|B|} \angle \beta - \alpha$$

multiply by V_R

$$V_R\, I_R^* = \frac{|V_S|\,|V_R|}{B} \angle \beta - \delta - \frac{|A|(|V_R|^2)}{|B|} \angle \beta - \alpha$$

single phase $\swarrow$

$\hookrightarrow = S_{R\,1\phi}$

No. ....

* V_S & V_R are both L-N voltages.

→ * multiply by 3 → V_S & V_R by $\sqrt{3}$ * ←

$$\underline{\underline{S_{3\phi}}} = \frac{|V_{LLS}||V_{RLL}|}{B} \angle \beta-\delta - \frac{|A||V_{RLL}|^2}{|B|} \angle \beta-\alpha$$

thus:

$$S_R = \frac{|V_S||V_R|}{|B|} \angle \beta-\delta - \frac{|A||V_R|^2}{|B|} \angle \beta-\alpha .$$

$$= P + jQ \longrightarrow$$

constant

$$① \quad P = \frac{|V_S||V_R|}{|B|} \cos(\beta-\delta) - \frac{|A||V_R|^2}{|B|} \cos(\beta-\alpha)$$

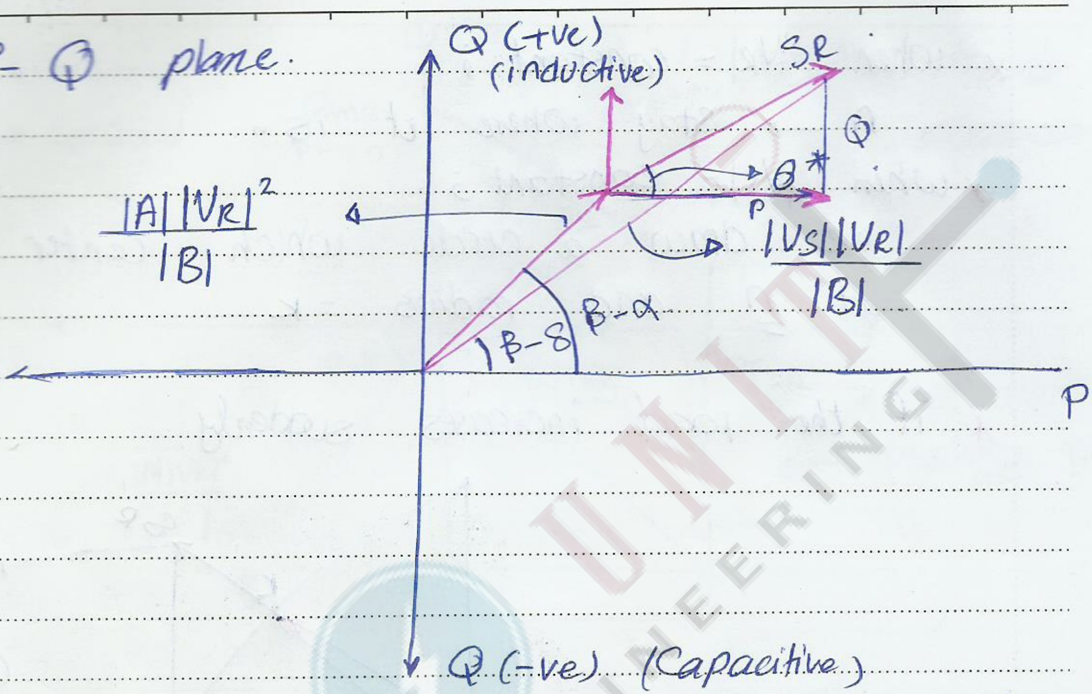
to make P maximum :- $(\beta = \delta)$.

exact term ← $$P_{max} = \frac{|V_S||V_R|}{|B|} - \frac{|A||V_R|^2}{|B|} \cos(\beta-\alpha) .$$

Approximate term ← $$P_{max} = \frac{|V_S||V_R|}{|B|} \quad \text{when } \beta \cong 90^\circ \text{ \& } \alpha \cong 0^\circ .$$

$$② \quad Q = \frac{|V_S||V_R|}{|B|} \sin(\beta-\delta) - \frac{|A||V_R|^2}{|B|} \sin(\beta-\alpha) .$$

(*) P- Q plane.



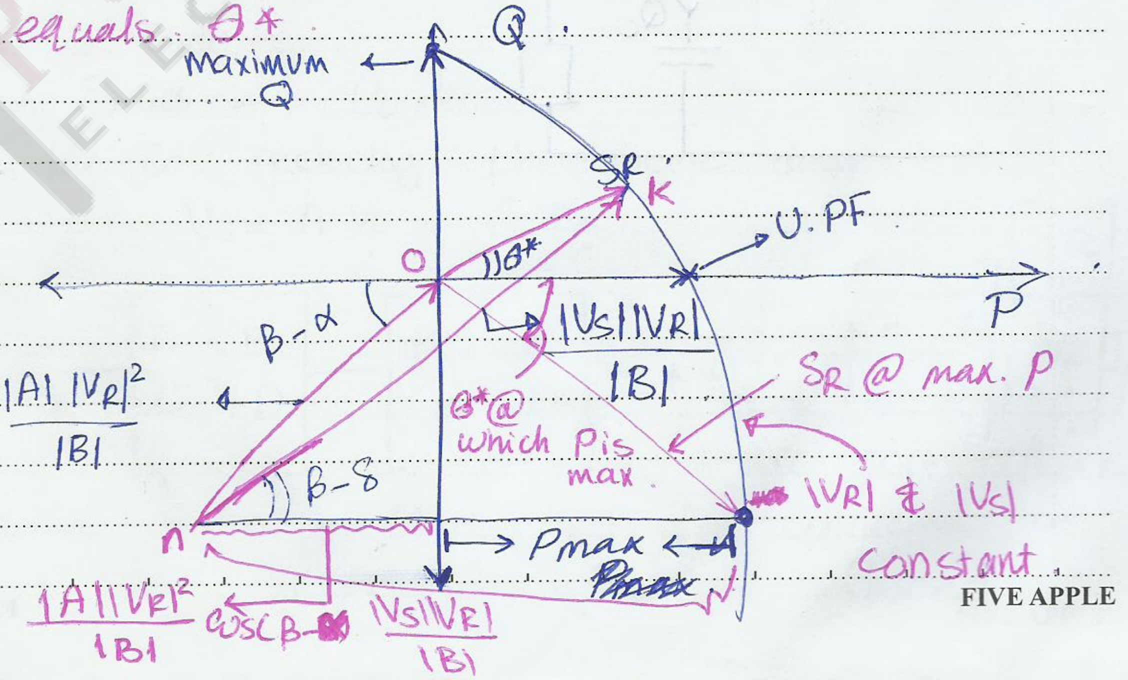
(*) Prove that (δ) at no load equals to (α).

$$V_S = A\,V_R + B\,\overset{0}{\cancel{I_R}} \quad \text{at no load.}$$

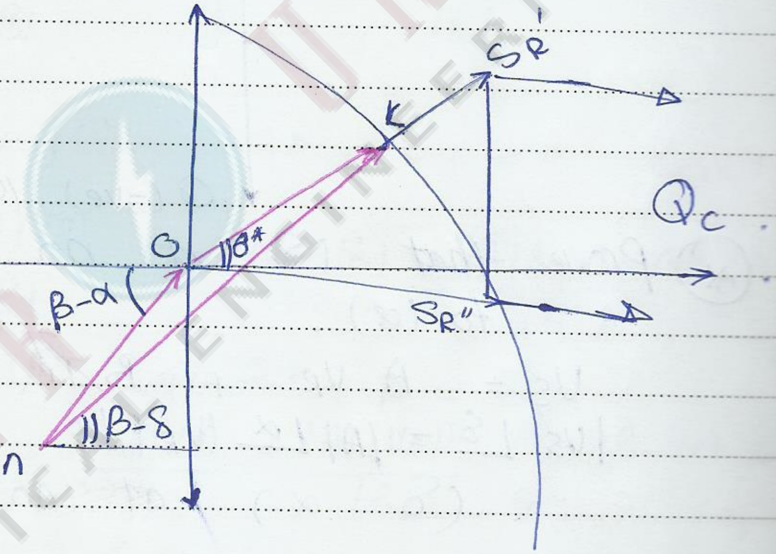
$$|V_S|\angle\delta = |A|\angle\alpha\,|V_R|\angle 0 \quad \text{thus}$$

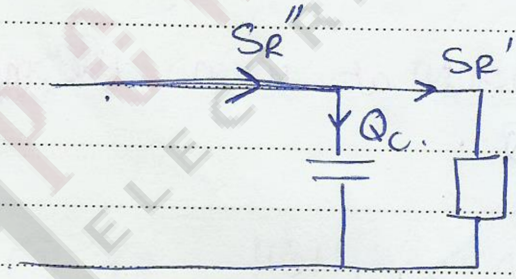
$$(\delta = \alpha) \quad \text{at no load.}$$

move the axis to point at when the angle equals θ*



- * when $|V_R|$ = constant ;
  - n → stay where it is.
- * when $|V_S|$ = constant ;
  - we draw a circle which center is at n and radius = K
- * if the load increases suddenly

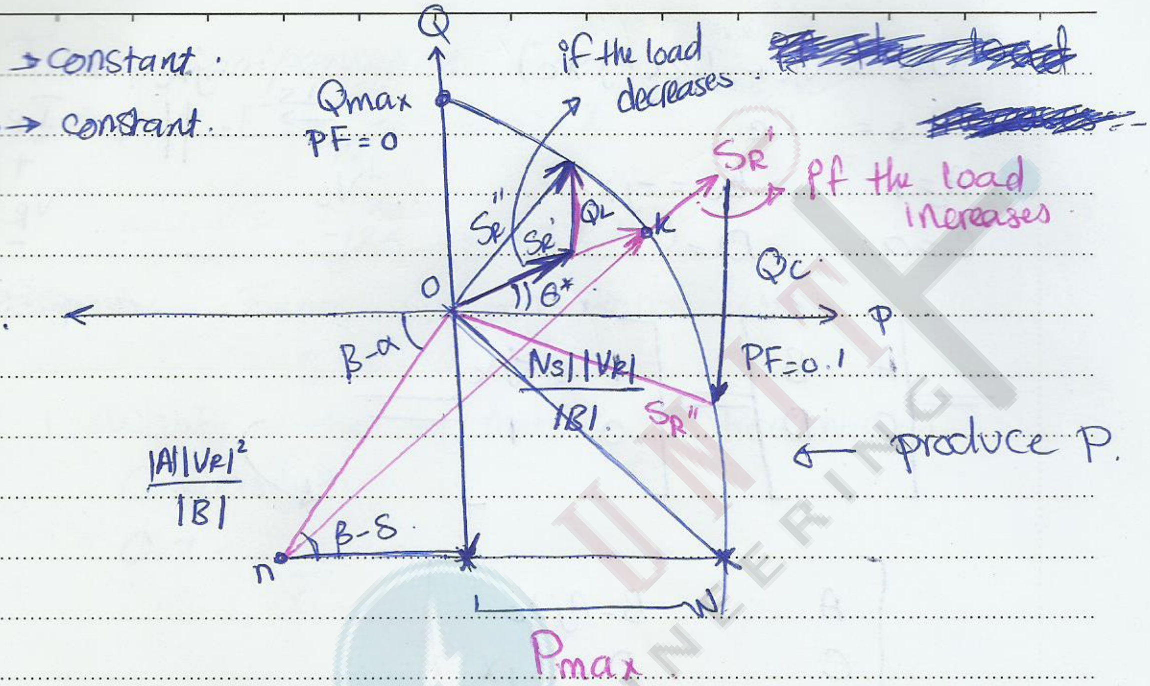


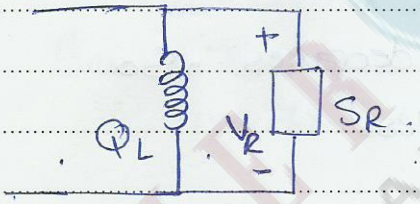


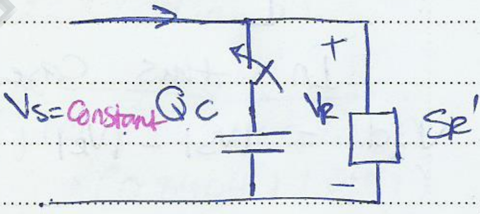
1cm = ---- MVA

$V_R \rightarrow$ constant.

$V_s \rightarrow$ constant.







$S_R'' = S_R' + Q_L$ "decreases"

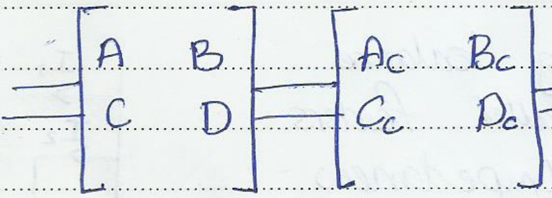
$S_R'' = S_R' + Q_c$ "increases"

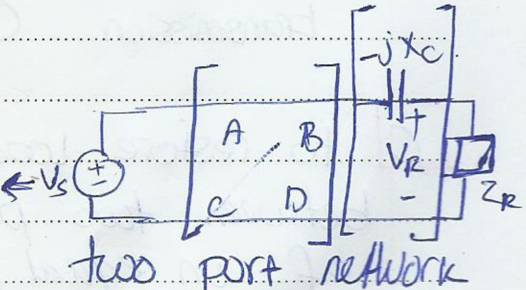
* Compensation:-

1. Series $\rightarrow$ Condenser:

$\boxed{A}$ For reducing the voltage drop.

$V_s = A\,V_R + B\,I_R$





two port network Connected in Cascade.

* For a Long Line

$V_S = V_R + I_R(-jX_C)$

$I_S = I_R$

$A_C = 1$ $B_C = -jX_C$

$C = 0$ $D = 1$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & -jX_C \\ 0 & 1 \end{bmatrix}$$

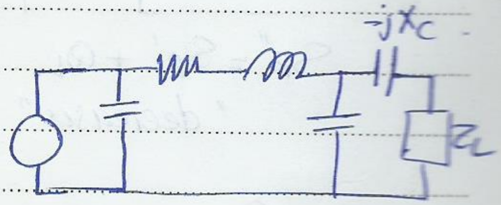
$$\begin{bmatrix} A & B - AjX_C \\ C & D - CjX_C \end{bmatrix}$$

In this case → B decreases , So

V.d = $|V_S| - |V_R|$ is less.

▲note▲ ~~For Medium Line :-~~

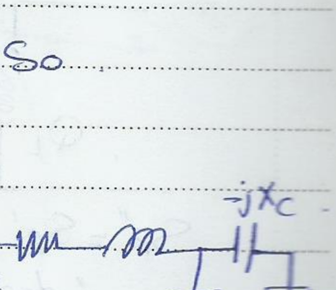
~~we~~ In medium Line we put C in that way :-



---*

B $P_{max} \simeq \frac{|V_S||V_R|}{|B|}$, So to increase the ~~tzo~~ transmission Capacity of the Line

C to restore Loading balance between two parallel feeders of un equal Impedances.



* There is no series Inductor.

* Series Compansation Factor (Series C.F):

Series C.F = $\frac{X_C}{X_L}$ = 50%.

ex

$B = 186.78 \angle 77.48^\circ$.

~~2. Shunt~~: Series Compensation with

C.F = 70%.

1. What is the increase in transmission Capability?

$0.7 = \frac{X_C}{X_L} \rightarrow$

$X_L = 230 \times 0.8277$

$X_C = 0.7 * 230 * 0.8277$ (in Ω)

$B_{new} = B_{old} - A j X_C \quad \sim \overline{-jX_C}$

$= 186.78 \angle 77.46^\circ - \underbrace{(j(0.7*230*0.8277) * (0.8404 \angle 1.34))}_{A}$

$B_{new} \cong 75 \angle 60.5 \ \Omega$

$P_{max} \propto \frac{|V_S||V_R|}{|B|}$ for (V_R & V_S = constants)

$\rightarrow \frac{P_{max(new)}}{P_{max(old)}} = \frac{B_{old}}{B_{new}} = \frac{186.78}{75} = 250\%$.

lecture # 18 27/3/2014

- **Load (Power) flow analysis:-**

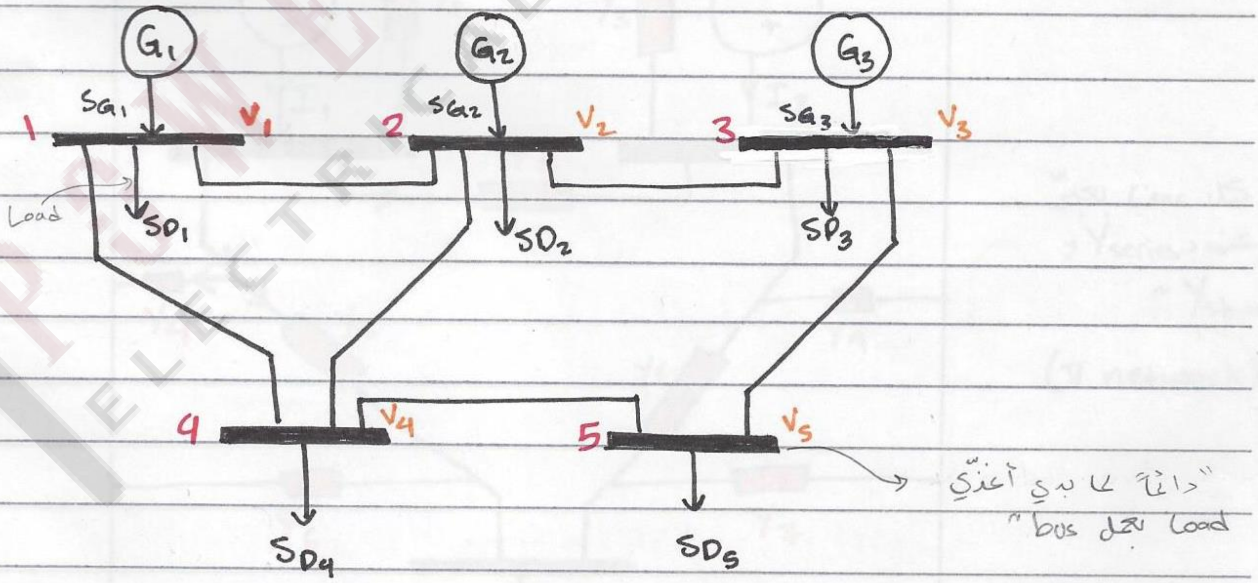
$V = ZI$

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{21} & Z_{22} & Z_{23} \\ Z_{31} & Z_{32} & Z_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} \longrightarrow$$ mesh analysis (linear equations).

$I = yV$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} & y_{13} \\ y_{21} & y_{22} & y_{23} \\ y_{31} & y_{32} & y_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \longrightarrow$$ Nodal analysis (linear equ.)

What about $S \propto V^2$ (non-linear)? this is solve using numerical methods (i.e newton-raphson, gauss seidel, ..).



"دائماً لا بدي أغذّي load بكل bus"

S_{G_1}: complex power generated by gen. 1.

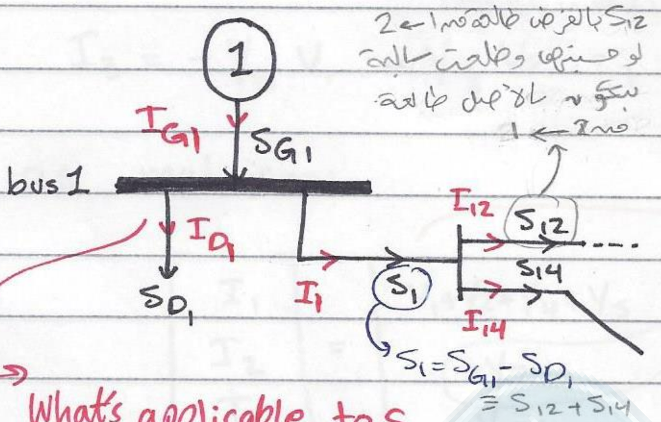
S_{D_1}: ~ ~ delivered (drawn). by load at bus 1.

V_i: voltage at bus i. $= |V_i| \angle \theta_i$

at every bus (i) there are S_{G_i}, S_{D_i} (might be zero).

$\rightarrow S_i = S_{G_i} - S_{D_i}$

$S_i \equiv$ complex power injected from bus (i) to the system.

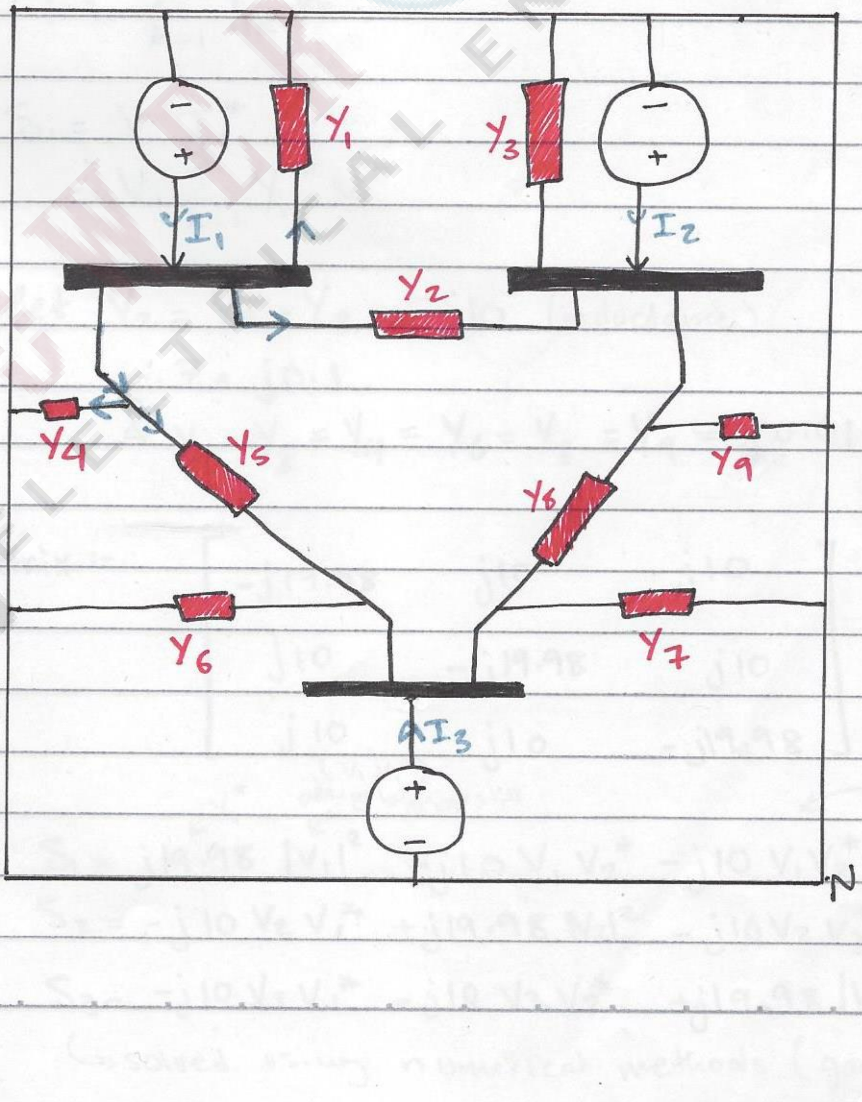


S_{12} بالفرض طالعة من 1 ← 2
لو حسبتها وطلعت سالبة
بيكون بالأصل طالعة
من 2 ← 1

What's applicable to S, is applicable to I. ($I = S/V$)

* I have to supply power at all times with acceptable V & frequency limits, with minimum cost.

** في حالة حدوث fault بفصل الـ faulty section كاملاً.
و بستخدم الـ Load flow Program حتى
أفحص الـ I والـ V على الـ lines وأعالج أي
مشكلة أو تخطي القيم اللي بيتحملها الـ Line



"كل Line بعبر عنه بـ Y_{series} و Y_{shunt}"
(π network).

$$I_1 = Y_1 V_1 + Y_2 (V_1 - V_2) + Y_4 V_1 + Y_5 (V_1 - V_3)$$

$$= (Y_1 + Y_2 + Y_4 + Y_5) V_1 - \boxed{Y_2} V_2 - \boxed{Y_5} V_3$$

مجموع المنتهية على bus1 ← ، بين ① & ② ، بين ① & ③

$$I_2 = -Y_2 V_1 + (Y_2 + Y_3 + Y_8 + Y_9) V_2 - Y_8 V_3$$

مجموع المنتهية على bus2

$$I_3 = -Y_5 V_1 - Y_8 V_2 + (Y_5 + Y_6 + Y_7 + Y_8) V_3$$

as matrices:-

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \overbrace{\begin{bmatrix} Y_1+Y_2+Y_4+Y_5 & -Y_2 & -Y_5 \\ -Y_2 & Y_2+Y_3+Y_8+Y_9 & -Y_8 \\ -Y_5 & -Y_8 & Y_5+Y_6+Y_7+Y_8 \end{bmatrix}}^{Y\text{-matrix}} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

$(Y_{ij} = Y_{ji})$

$$I_i = \sum_{k=1}^{n} Y_{ik} V_k$$

$$S_i = V_i I_i^*$$
$$= V_i \sum_{k=1}^{n} Y_{ik}^* V_k^*$$

ex:- let $Y_2 = Y_5 = Y_8 = \ominus j10$ (inductance)

$\therefore Z = j0.1$

& $Y_1 = Y_3 = Y_4 = Y_6 = Y_7 = Y_9 = \oplus j0.01$ (capacitance).

Ymatrix :-

$$\begin{bmatrix} -j19.98 & j10 & j10 \\ j10 & -j19.98 & j10 \\ j10 & j10 & -j19.98 \end{bmatrix}$$

equations:- $= V_1 \sum_{k=1}^{n} Y_{ik}^* V_k^*$

Y_{11}^* ، الزاويتين بروحوا مع بعض $(V_1 V_1^*)$

$$S_1 = j19.98 |V_1|^2 - j10 V_1 V_2^* - j10 V_1 V_3^* \quad \dots ①$$
$$S_2 = -j10 V_2 V_1^* + j19.98 |V_2|^2 - j10 V_2 V_3^* \quad \dots ②$$
$$S_3 = -j10 V_3 V_1^* - j10 V_3 V_2^* + j19.98 |V_3|^2 \quad \dots ③$$

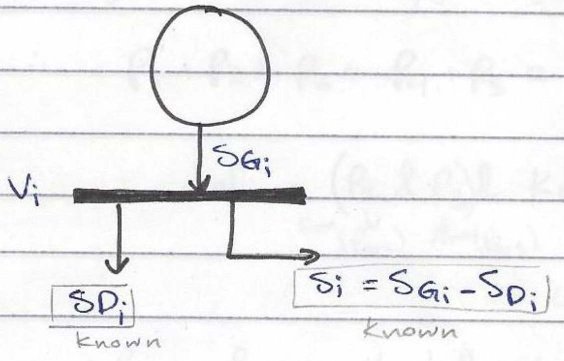
} non-linear equations.

↳ solved using numerical methods (gauss, gauss seidel, newton-raphson ...).

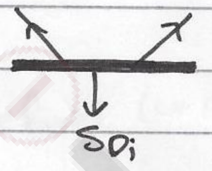
Lecture #20

30/3/2014

bus bar types:-



$\boxed{S_{D_i}}$ known $\boxed{S_i = S_{G_i} - S_{D_i}}$ known



$S_i = \underset{\overset{\uparrow}{S_{G_i}}}{\underline{\underline{0}}} - S_{D_i}$ لانه ما في طالع من الـ bus

Generator bus. **Load bus.**

- in generator bus, we can control ① $\underline{\underline{P_{G_i}}}$ (by controling steam input to the generator). "بتوقف الـ angle" (Using steam value.)

  ② we can control $|V_i|$ by controling I_f (field current).

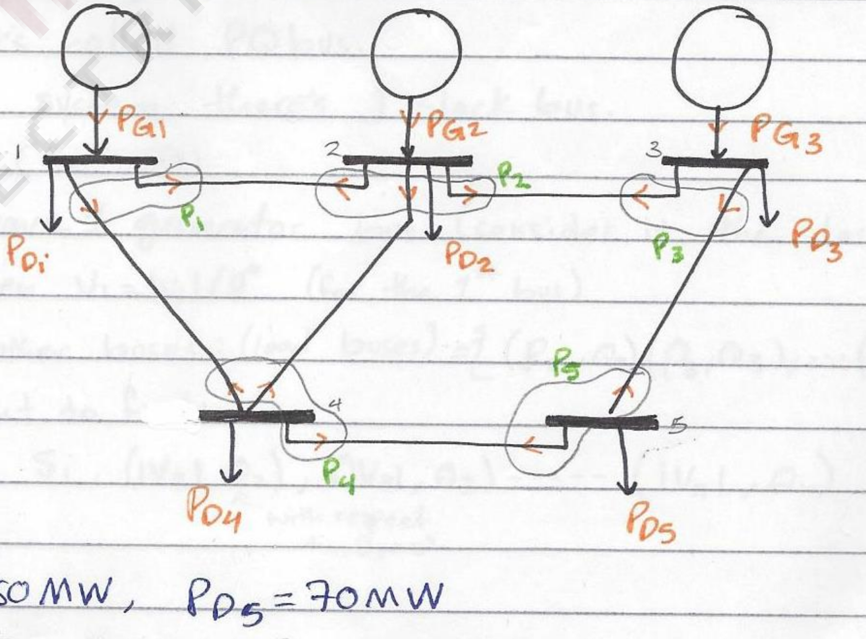
  ∴ at generator bus we know:-

  ① $|V_i|$, ② $P_i = P_{G_i} - P_{D_i}$ (P_{G_i} ← انا بحدده. , P_{D_i} ← known)   while $\overset{\times}{Q_i} = \underline{\underline{Q_{G_i}}} - \underline{Q_{D_i}}$ (I don't know this.)

- @ load bus, we can't control V_i, but we know S_i

  ∴ ① $P_i = -P_{D_i}$

  ② $Q_i = -Q_{D_i}$



given $P_{D4} = 80\,MW$, $P_{D5} = 70\,MW$

∴ $P_4 = -80\,MW$, $P_5 = -70\,MW$

but $P_1 + P_2 + P_3 + P_4 + P_5 \neq 0$ (there are losses in T.L.s that must be considered).

$\cancel{70} \quad 30 \quad 50 \quad -70 \quad -80 \neq 0$

$\therefore P_1 + P_2 + P_3 + P_4 + P_5 = P_{Loss}$

$\therefore$ define $(\underset{from (P_{G2})}{P_2} \,\&\, \underset{from (P_{G3})}{P_3})$ & keep $\textcircled{P_1}$ undefined (to compensate for losses).

e.g (70 + losses).

عادةً بختارها عند أكبر generator عشان هو اللي رح يعوض الـ(losses).

$\therefore P_{G1}$, P_1 aren't defined, but $|V_1|$ is defined.

* let this bus be your refference

$V_1 = |V_1| \angle 0°$ ← "سحبنا منها امتياز (P undifined) بس نعطيها امتياز آخر (بجعلها refference)"

- busses types (based on the info. given at the bus):

1. V_i, $(\theta_i = \angle V_i = 0°)$, if it wasn't bus 1, rename your buses to make it 1.

   it's called "Slack bus" (الفائض) / "Swing bus" / "Voltage reference bus".

   هاد رح نعتمده

   why swing? لأنه متغيّر لأنه يغطّي الـlosses

   "هاد الاسم الناس بيهربوا منه لأنه بيعمل confusion مع Neutral"

2. P_i, $|V_i|$ → generator bus

   it's called $P,|V|$ bus / Voltage control bus.

3. P_i, Q_i → load bus

   it's called PQ bus.

* in each system there's 1 slack bus.

<u>Case 1</u>:- (الأكثر شيوعاً)

given:
- we have 1 generator bus (consider it the slack bus).
- $\therefore$ given $V_1 = |V_1| \angle 0°$ (for the 1st bus)
- for other boses: (load buses) $= \{(P_2, Q_2), (P_3, Q_3), \cdots (P_n, Q_n)\}$

we want to find:-

$$S_1, \; (|V_2|, \underset{\text{with respect to } \theta_1 = 0°}{\theta_2}), \; (|V_3|, \theta_3) \text{ ----- } (|V_n|, \theta_n)$$

Case 2 :- (الأكثر شيوعاً)

(m)generator buses -1 ; Load buses

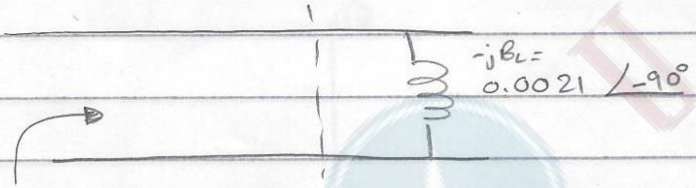
given (known) : $\underbrace{|V_1|\angle 0^\circ}_{\text{Slack bus.}}, \underbrace{(|V_2|, P_2), \dots (|V_m|, P_m)}$, $\underbrace{(P_{m+1}, Q_{m+1}) \dots (P_n, Q_n)}$

---

* Chapter 6 problems:-

6.25 250 MVAR
345 kV
Shunt reactor.

$Y = \frac{1}{j\omega L} = jB_L = 0.021\angle -90^\circ$ is added.



$A = 0.818\angle 1.3^\circ = D$
$B = 172.2\angle 84.2^\circ$
$C = 0.001933\angle 90.4^\circ$

$A_{new} = A_{old} - jB_L B$
$= 0.818\angle 1.3^\circ - j(0.0021)(172.2\angle 84.2^\circ) = 1.178\angle -0.88^\circ$

$C_{new} = C_{old} - jB_L D$ ---

D & B are the same.

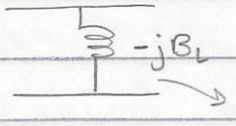
$S = \frac{V^2}{Z \leftarrow j\omega L}$

$Z = \frac{V^2}{S}$, $|Y| = \frac{|\overset{Q}{S}|}{|V|^2} = \frac{250}{(345)^2} = 0.0021$!

6.24: $y_c = 0 + j6.89 \times 10^{-6}$ S/mi

$C.F = 0.6 = \frac{B_L}{\boxed{B_c}}$ $\times 300$ mi

$\therefore B_L = 0.001237$ S



$$\begin{bmatrix} 1 & 0 \\ -j0.001237 & 1 \end{bmatrix}$$

calculate A, B, C, D ...

6.16:- 3-phase, long line

$Z = 35 + j140 \ \Omega$

$Y = 930 \times 10^{-6} \angle 90^\circ \ S$

$\gamma l = \sqrt{yz} \ l$

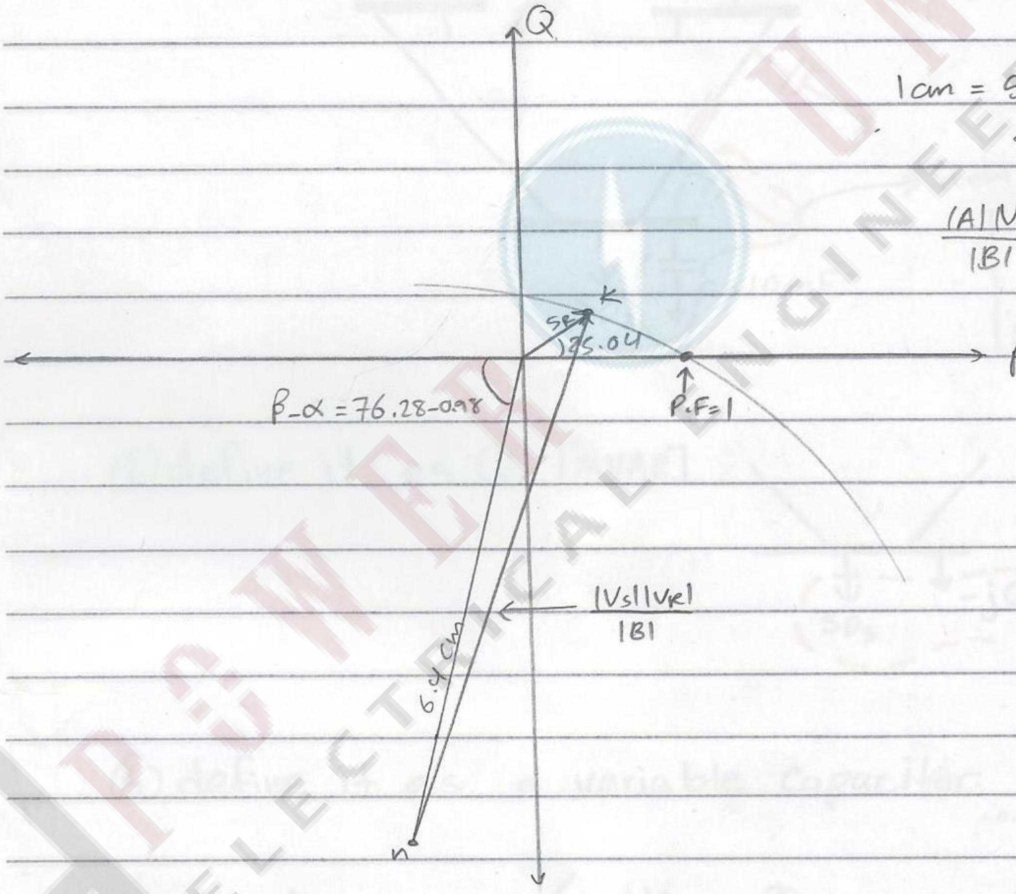
$= \sqrt{yz l^2} = \sqrt{YZ} = 0.3663 \angle 83^\circ$

$A = \cosh(\gamma l) = 0.9355 \angle 0.98^\circ$

$Z_c = \sqrt{\frac{Z}{Y}} = 394 \angle -7.02$

$B = Z_c \sinh(\gamma l) = 141.428 \angle 76.28$

$C = \sinh(\gamma l) / Z_c$ ---



1cm = 50MVA

$S_R = \frac{40}{0.9} \angle{+25.04} = 44 \angle 25.0$ MVAR

$\frac{|A||V_R|^2}{|B|} = 320$ MVAR

Lecture #21

1/4/2014

from previous lec.:-

- **Case 1:-**

  Known:- $V_1, S_2, S_3, \dots S_n$

  find :- $S_1, V_2, V_3 \dots V_n$

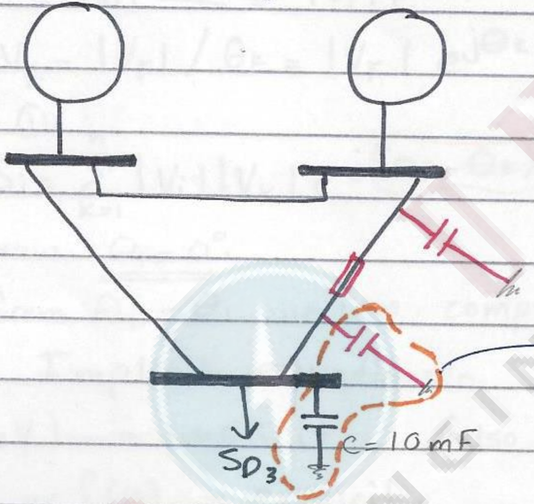
- **Case 2:-**

  Known:- $V_1, \underbrace{(P_2,V_2),(P_3,V_3)\dots(P_m,V_m)}_{\text{PV buses.}}, \underbrace{S_{m+1}, S_{m+2} \dots S_n}_{\text{PQ buses}}$

→ if I want to add a capacitor at a PQ bus (load bus):

(ممكن يكون في ٣ تعريفات مختلفة حسب كيف عرّفت C)

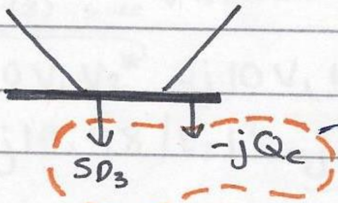
① define it as (C)



→ add it to the Y-matrix

$$\begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & \boxed{Y_{33} + j\omega \times 10 \times 10^{-3}} \end{bmatrix}$$

② define it as Q_c [MVAR]

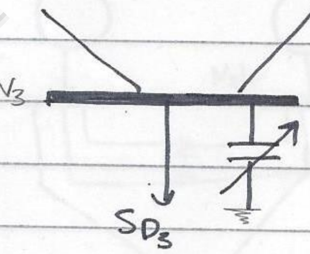


→ add it to S_{D_3}

$$S_{D_3}' = S_{D_3} + (-jQ_c)$$
$$= P_{D_3} + j(Q_{D_3} - Q_c)$$

③ define it as a variable capacitor: "عشان يحافظ على V_3 ثابتة عند قيمة معينة"

(e.g $V_3 = 1.02$ p.u.)



it's a PQ bus, but we'll define it as a PV bus (after adding ⇗‖). since $|V_3|$ is now Known. & Q_3 variable.

$P_3 = P_{G3} - P_{D_3} = 0 - P_{D_3}$

↖ capacitor doesn't generate active Power

*Notes:-

* أنواع الـbuses الي حكم عَر عليك نوعين (Load bus, gen. bus)

واحد منهم Slack ↖

* على كل bus في ٢ معروفين و ٢ مجهولين من (P, V, Q, θ)

* وين الزاوية بتظهر بالمعادلة ؟

$$S_i = V_i \sum_{k=1}^{n} Y_{ik}^{*} V_k^{*} \quad \ldots ①$$

$$V_i = |V_i| \angle \theta_i = |V_i| e^{j\theta_i}$$

$$\therefore V_k = |V_k| \angle \theta_k = |V_k| e^{j\theta_k}$$

Plug in ①

$$S_i = \sum_{k=1}^{n} |V_i||V_k| e^{j\boxed{(\theta_i - \theta_k)}} Y_{ik}^{*}$$

"angle difference" تظهر

we know $\underline{\theta_1 = 0^\circ}$

$\therefore$ from $\theta_i - \theta_1$ we can compute θ_i ....

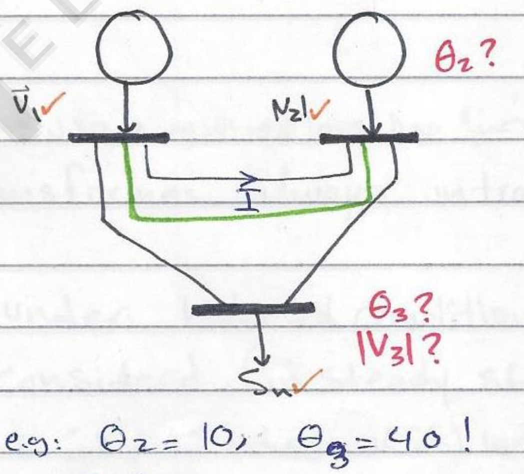
- Explicit & Implicit equations:-
  - $f(x,y) \longrightarrow$ implicit (also $f(x^2)$ is implict)
  - $f(x)$ or $f(y) \longrightarrow$ explicit.

“أنا بتهمنيش” بس مكتوبة بكتابك..

for P.system equ.:

$$\text{1 exlicit}\ \begin{cases} S_1 = j19.98|V_1|^2 \overbrace{- j10 V_1 V_2^{*}}^{f(\overset{V_2}{x}),\ \text{since } V_1 \text{ is known}} \underbrace{- j10 V_1 V_3^{*}}_{f(y)} & \leftarrow \text{explicit.} \\ S_2 = -j10 V_2 V_1^{*} + j19.98|V_2|^2 \underbrace{- j10 V_2 V_3^{*}}_{\substack{f(x,y) \\ \text{2 variables}}} & \leftarrow \text{implicit} \\ \vdots \end{cases}\ \overset{\text{الباقي}}{\text{implicit}}$$

- if the current is exceeded?



if we find $V_1, V_2 --- V_n$, it's solved.

$$I = \frac{V_2 - V_1}{(\text{T.L}_{12}\,\text{imp.})} \quad ----- \quad I_n$$

↳ this current must not exceed the thermal capability of the line.

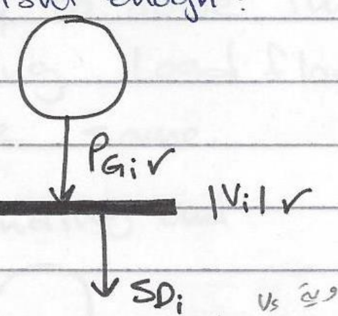
Prob. 1 * if the current is ↑ (exceeded) : Suggested Solution: add parall line.

Prob 2:- e.g: $\theta_2 = 10$, $\theta_3 = 40$!

الفرق المفروض يكون بين 5°-4°

to maintain stability.

- if Q_i isn't enogh?



Q_i?
θ_i?

I might put a constraint to Q_i

$$Q_i^{min} \leq Q_i \leq Q_i^{max}$$

eg $[40 \leq Q_i \leq 70]$ MVAR

$P_i = P_{Gi} - P_{Di}$ ✓

زاوية Vs (power angle)

"بدّك تودّي Q بدّك ترفع الVoltage، Q مرتبطة بـ V , و P مرتبطة بـ θ"

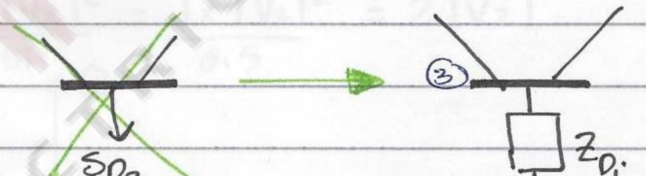
if $Q_i = 50$ MVAR ✓

$Q_i = 30$ MVAR ✗ not within the range!

↳ solution:- $Q_i' = 40$ MVAR
the lower bound (أقرب قيمة مقبولة لـ 30MVAR)

P_i ✓ known & $Q_i = Q_i'$ ✓ known
& $|V_i|$ is no longer known
& this bus is considered as a PQ bus since the $|V_i|$ it gives does not satisfy my constraints, thus $|V_i|$ is changed.

- if the load was defined as an impedence instead of S_{Di}?



Sol:- add it to the Y-matrix. (+ to Y_{ii})

$$\begin{bmatrix} - & - & - \\ - & - & - \\ - & - & -Y_{33} + \frac{1}{Z_{D_3}} \end{bmatrix}$$

& $S_{D_3}' = 0$

↳ بلغي الـbus لأنه صار داخل ضمنياً مع الـloss على اللاين من خلال تعديل الـ Y-matrix

- Transformes always introduce a phase shift! ↳ (0°, 30°, 180°, 60°, ...).

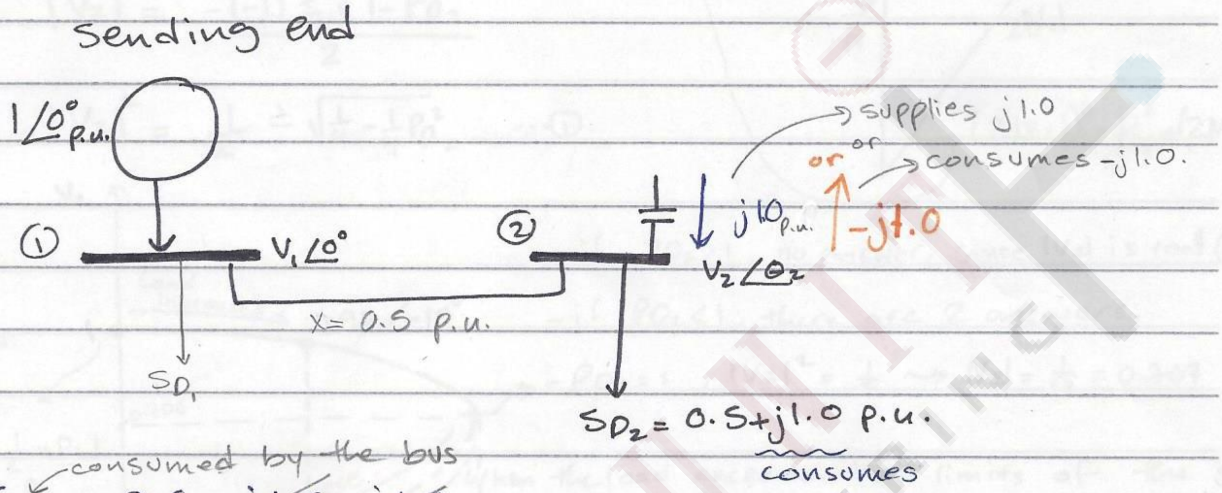
we'll see it (ch#11). "vector groups."

↳ under balanced conditions this phase shift isn't considerd (in steady state solution). لأنه الـShift للتيار و الـV نفسه فالزاوية بتضل ثابتة.

"رح نتذوقها أكثر لما نروح على تشابتر الـ ١١"

ex:- try to solve this using ch#6 method then using Load flow, & see that answers will be the same.

Sending end



consumed by the bus

$S_{D_2} = 0.5 + \cancel{j1.0} \cancel{-j1.0}$

$S_2 = -(0.5 + j0) = -0.5$

supplied from the bus.

Lecture#21
3/4/2014

→ Chapter 6 method:- (T.L analysis & circle diagram).

$S_R = P_{D_2} + j1.0 - j1.0 = P_{D_2} = 0.5$

for the Line:- (short line ; since $Y=0$)

$A = 1\angle 0°$

$B = 0.5\angle 90°$ Ω

$$\frac{|A||V_R|^2}{|B|} = \frac{1 \times |V_2|^2}{0.5} = 2|V_2|^2$$

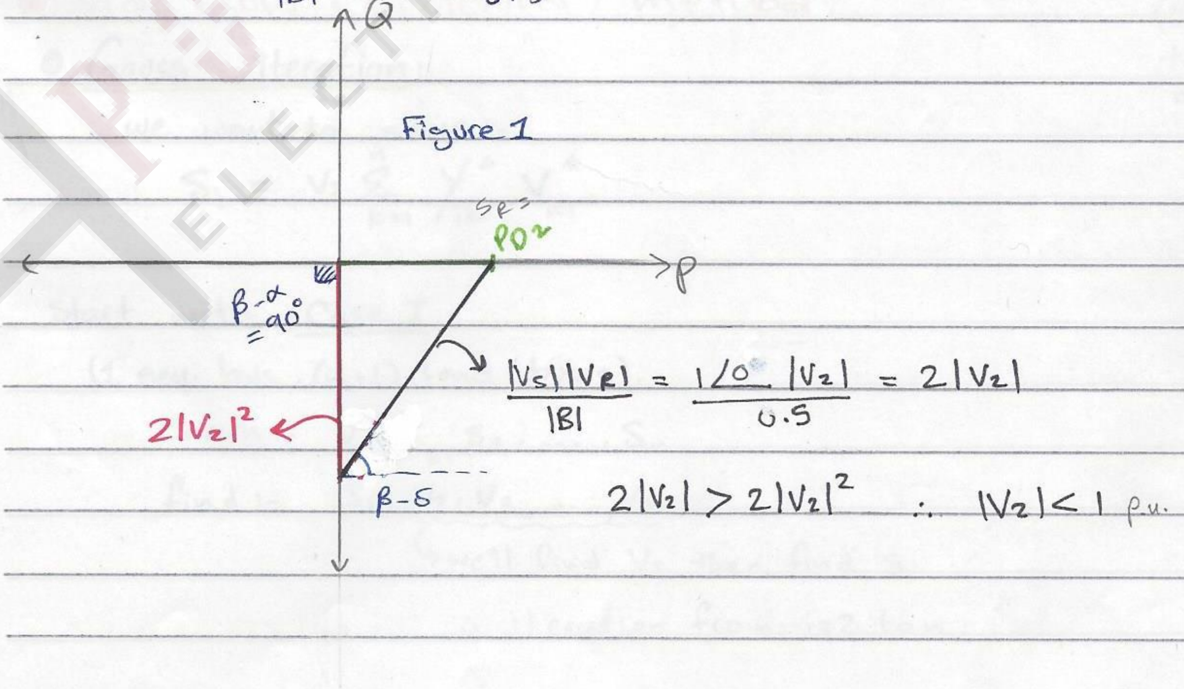


Figure 1

$$\frac{|V_S||V_R|}{|B|} = \frac{1\angle 0° \cdot |V_2|}{0.5} = 2|V_2|$$

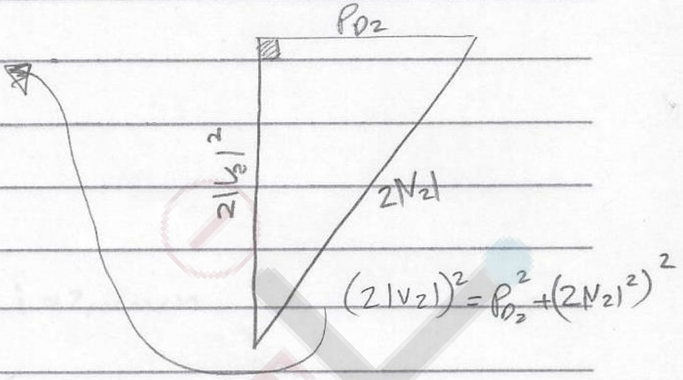
$2|V_2| > 2|V_2|^2 \quad \therefore \ |V_2| < 1$ p.u.

lecture # 22

3/4/2014

ex (cont.)

$$|V_2|^4 - |V_2|^2 + \frac{1}{4} P_{D_2}^2 = 0$$



$(2|V_2|)^2 = P_{D_2}^2 + (2|V_2|^2)^2$

$$|V_2|^2 = \frac{-(-1) \pm \sqrt{1 - P_{D_2}^2}}{2}$$

$$|V_2|^2 = \frac{1}{2} \pm \sqrt{\frac{1}{4} - \frac{1}{4} P_{D_2}^2} \quad \cdots ①$$

@ no-load:
$P_{D_2} = 0$
$|V_2|^2 = \frac{1}{2} \pm \frac{1}{2} = 0, 1$

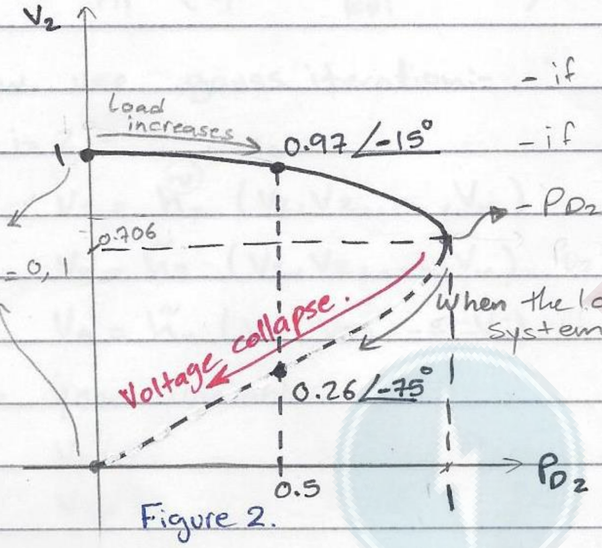


Figure 2.

- if $P_{D_2} > 1$, no answer, since $|V_2|$ is real (not imag.)
- if $P_{D_2} < 1$, there are 2 answers.
- $P_{D_2} = 1$, $|V_2|^2 = \frac{1}{2} \rightsquigarrow |V_2| = \frac{1}{\sqrt{2}} = 0.707$

When the load exceeds the limits of the gen. system collapses.
"Voltage Stability Problem".

- we find $|V_2|$ from equatin ① (figure 2), & $\angle V_2$ from (figure 1)
  - * $\angle V_2 = -\delta \rightarrow$ since V_s was my refference $= 1\angle 0°$!
- at $P_{D_2} = 0.5$ there are two solutions, $0.97\angle -15$ ✓, $0.26 \angle -75°$

this is not Acceptable (~~this voltage~~ is too weak to drive the load"

Load flow (numerical) method:

- **Gauss iteration:**

we want to solve:-

$$S_i = V_i \sum_{k=1}^{n} Y_{ik}^* V_k^*$$

start with Case I:

(1 gen. bus, (n-1) load buses).

Known:- $V_1, S_2, S_3, \ldots, S_n$

find:- S_1, $\underbrace{V_2, V_3, \ldots, V_n}$

↳ we'll find V_s then find S_1.

∴ iteration from i=2 to n.

$$S_i^* = V_i^* \sum_{k=1}^{n} Y_{ik} V_k$$

$$\frac{S_i^*}{V_i^*} = \underbrace{Y_{ii} V_i}_{} + \sum_{\substack{k=1 \\ \boxed{k \neq i}}}^{n} Y_{ik} V_k$$

ضفناها عال Σ

$$V_i = \frac{1}{Y_{ii}} \left(\frac{S_i^*}{V_i^*} - \sum_{\substack{k=1 \\ k \neq i}}^{n} Y_{ik} V_k \right), \quad i = 2, \ldots, n$$

now, use gauss iteration:-

$i = 2$:-

tilde

$V_2 = \tilde{h}_2 (V_2, V_3, \ldots, V_n)$

$V_3 = \tilde{h}_3 (V_2, V_3, \ldots, V_n)$

$\vdots$

$V_n = \tilde{h}_n (V_2, V_3, \ldots, V_n)$

rename your variables:-

$V_2 = X_1$

$V_3 = X_2$

$\vdots$

$V_n = V_{n-1} \equiv V_N \quad , \quad N = n-1$

now,

$X_1^{v+1} = h_1 (X_1^v, X_2^v, \ldots, X_N^v)$

$X_2^{v+1} = h_2 (X_1^v, X_2^v, \ldots, X_N^v)$

$X_N^{v+1} = h_N (X_1^v, X_2^v, \ldots, X_N^v)$

in general,

vectors.

$$X^{v+1} = h^v(X)$$

1st step, we guse $\overset{v}{V_2} \xleftarrow{} X_1 , \overset{v}{V_3} \xleftarrow{} X_2 . \ldots$ "كلهم قريبات عال 1 لأنّا بنشتغل p.u."

* we stop the iteration when

$X^{v+1} - X^v \leq \epsilon$ (for both phase & magnitude"

* In gauss iteration we complete the whole iteration based on the same initial values.

↳ we use X^v to calculate X^{v+1} for all $i \in [2, n]$ ($X_1 \rightarrow X_N$)

then we use X^{v+1} to calculate X^{v+2}, ... and so on.

* In gauss seidel we update the values of X step by step:-

G.S :-

$$x_1^{v+1} = h_1(x_1^v, x_2^v, \dots x_N^v) \quad \dots ①$$

$$x_2^{v+1} = h_2(\underline{x_1^{\boxed{v+1}}}, x_2^v, \dots x_N^v)$$

↳ calculated from 1.

$$x_3^{v+1} = h_3(x_1^{v+1}, x_2^{v+1}, \dots x_N^v)$$

$$\vdots$$

$$x_N = h_N(x_1^{v+1}, x_2^{v+1}, x_3^{v+1}, \dots, x_N^v$$

for our example both methods are the same because I have 1 variable only. (V_2)

↳ given :- $S_2 = -S_{D2} = -0.5$

$V_1 = 1\angle 0^\circ$

Find :- $S_1, \underline{\underline{V_2}}$?

↳ من هون ببدا

$$V_2^{v+1} = \frac{1}{Y_{22}} \left(\frac{S_2^*}{V_2^{v*}} - \sum_{\substack{k=1 \\ k\neq 2}}^{n} Y_{ik} V_k^v \right) \dots ①$$

Y-matrix (2×2)

$$\frac{1}{j0.5} \rightarrow \begin{bmatrix} -j2 & j2 \\ j2 & -j2 \end{bmatrix}$$

back to ①

$$V_2^{v+1} = \frac{1}{-j2} \left(\frac{-0.5}{(V_2^v)^*} - (j2 \times \overset{k=1}{1}) \right)$$

$$V_2^{v+1} = \frac{-j0.25}{(V_2^v)^*} + 1$$

بالعادة بنبدأ Start
عن فرض $V_2 = V_1 = 1\angle 0^\circ$
حتى نوحّد الكل.

Iteration	V_2	V_2 (using another guess)
0	$1\angle 0^\circ$ ← initial guess	$0.1\angle 0^\circ$
1	$1.030776\angle -14.03624^\circ$	$1.092982\angle -68.7$
2	$0.970143\angle -14.036$	⋮
⋮	⋮	⋮
6	$0.965948\angle -14.990572^\circ$	iteration#8:- $0.965918\angle -14.995^\circ$

رجعنا لنفس الحل بس بعدد iter. أكبر
لأنه ال guess بعيد.

"ما امشش على 0.26 لأنه unstable point"

6/4/2014
lecture#23

Case II

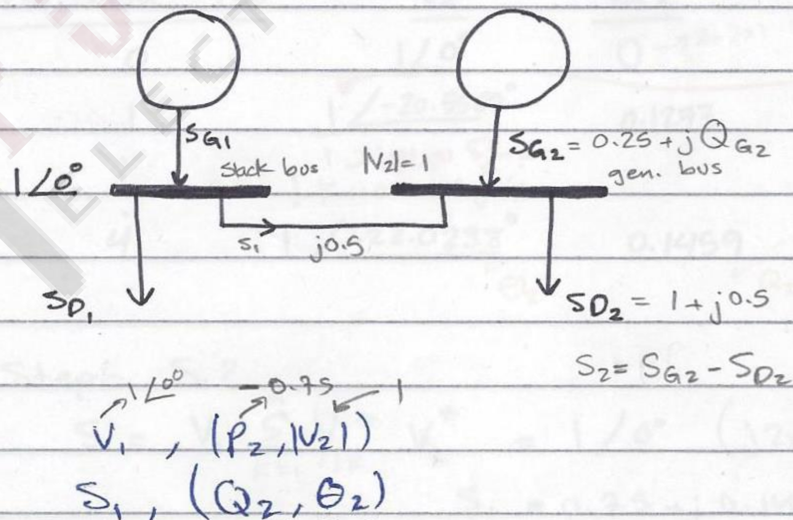
$$\begin{array}{lllll} \text{known:-} & \underbrace{\underline{V_1}}_{\text{Slack bus}}, & \overbrace{(|V_2|, P_2), \ldots (|V_m|, P_m)}^{\text{gen. buses}}, & \overbrace{(S_{m+1}) \ldots (S_n)}^{\text{load buses}} \\ \text{find:-} & S_1, & (Q_2, \theta_2), \ldots (Q_m, \theta_m), & V_{m+1}, \ldots V_n \end{array}$$

$$V_i^{v+1} = \frac{1}{Y_{ii}} \left(\frac{\overset{?\ Q_i\ \text{unknown.}}{S_i^*}}{V_i^{v*}} - \sum_{\substack{k=1 \\ k \neq i}}^{n} Y_{ik} V_k \right) \quad ①$$

but for PV buses we don't know Q_i!

$$Q_i^v = I_m \{S_i^v\} = I_m \{ V_i^v \sum_{k=1}^{n} Y_{ik}^* V_k^{v*} \} \quad ②$$

ex:



$$S_2 = S_{G2} - S_{D2} = \underbrace{-0.75}_{P_2} + j(Q_{G2} - 0.5)$$

given $\overset{1\angle 0^\circ}{V_1}, (\overset{-0.75}{P_2}, \overset{1}{|V_2|})$

find $\underbrace{S_1}_{2^{nd}\ \text{step}}, \underbrace{(Q_2, \theta_2)}_{1^{st}\ \text{step}}$

$$V_2^{v+1} = \frac{1}{Y_{22}}\left(\frac{S_1^*}{V_2^{v*}} - \sum_{\substack{k=1 \\ k\neq 2}}^{n} Y_{2k} V_k\right) \quad \dots①$$

$$Y\text{-matrix} = \begin{bmatrix} -j2 & j2 \\ j2 & -j2 \end{bmatrix}$$

$$V_2^{v+1} = \frac{1}{-j2}\left(\frac{-0.75 - jQ_2^v}{V_2^{v*}} - j2\times 1\right)$$

$$\boxed{V_2^{v+1} = 1 + \frac{-0.75 - jQ_2^v}{V_2^{v*}}} \quad —①$$

$$Q_2^v = I_m\left\{V_2^v \sum_{k=1}^{n} Y_{2k}^* V_k^*\right\}$$

$$= I_m\left\{V_2^v\left(-j2\times \overset{1\angle 0^\circ}{\cancel{V_1^*}} + j2 V_2^{v*}\right)\right\}$$

$$= I_m\{-j2V_2^v + j2\times 1\} = I_m\{j(2-2V_2^v)\}$$

$$\boxed{Q_2^v = 2 - 2Re\{V_2^v\}} \quad —②$$

هوالي رح يطلعني من Im لأنضربه مع (j)

we start with initial guess for V_2, $V_2 = \overset{known}{1}\,\overset{guess.}{\angle 0^\circ} = V_1$ (flat start).

iteration No	V_2	Q_2	V_2^{v+1}
0	$1\angle 0^\circ$	0 $\rightarrow$ 2-2x1	$1.068\angle -20.5560^\circ$
1	$1\angle -20.5560^\circ$ مباشرة نعدلها لـ 1 لأنها known !	0.1273	$1.003\angle -21.9229^\circ$
⋮			
4	$1\angle -22.0238^\circ$ ✓ θ_2	0.1459 ✓ Q_2	

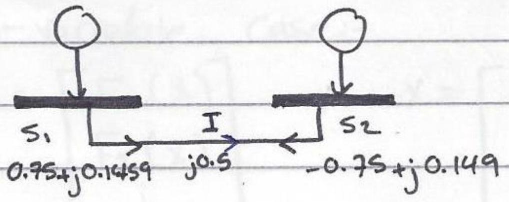
2^{nd} Step:- S_1?

$$S_1 = V_1 \sum_{k=1}^{n} Y_{1k}^* V_k^* = 1\angle 0^\circ\left(j2\times 1\angle 0^\circ + -j2\times 1\angle{+22.0238^\circ}\right)$$

$$S_1 = 0.75 + j\,0.1459$$

"بنشط لما أشوفها هيك :)"

* ① 0.75 is supplied by G_1 & consumed by G_2 since no (R) through line. ② both G_1, G_2 supplies the same amount of reactive power since they have the same $|V_i|$ → لأنه اللي بيقرر مين بغذي الـ Q هو |V| و اللي بقرر مين بغذي P هو الزاوية $\angle V$. (Leading source supplies P)



$$I = \frac{1\angle 0^\circ - 1\angle -22.0238^\circ}{j0.5} = 0.764\angle -11.011^\circ$$

$$I^2 x = (0.764)^2 * 0.5 = 0.292 = \underline{\underline{2 \times 0.1459}}$$

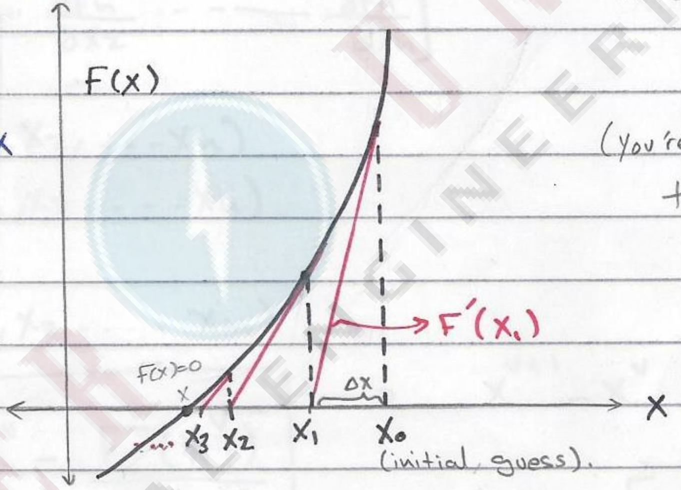
$$Q_x = Q_1 + Q_2$$

Newton-Raphson Method N-R

(better ; faster convergent more stable).

let F(X) = 0

then solve for X



(you're linearizing the problem).

$$F'(X_0) = \frac{F(X_0)}{\Delta X}$$

$$\rightarrow \Delta X = X_0 - X_1 = \frac{F(X_0)}{F'(X_0)}$$

$$X_1 = X_0 - \frac{F(X_0)}{F'(X_0)}$$

in general:- $$X^{v+1} = X^{v} - \frac{F(X^{v})}{F'(X^{v})}$$

simple example:- find $\sqrt[3]{64}$?

$$X = \sqrt[3]{64}$$

$$X^3 = 64 \longrightarrow X^3 - 64 = 0$$

start with initial guess $X_0 = 5$

$$X_1 = 5 - \frac{125-64}{3 \times 25} = 4.1867$$

$$X_2 = 4.1867 - \frac{(4.1867)^3 - 64}{3(4.1867)^2} = 4.0082 \approx 4$$ from 2 iterations only!

For multi-variable case:-

$$F(x)=\begin{bmatrix}F_1(x)\\F_2(x)\\\vdots\\F_n(x)\end{bmatrix},\quad x=\begin{bmatrix}x_1\\x_2\\\vdots\\x_n\end{bmatrix}$$

$$F'(x)=\begin{bmatrix}\frac{\partial F_1}{\partial x_1} & \frac{\partial F_1}{\partial x_2} & \cdots & \frac{\partial F_1}{\partial x_n}\\ \frac{\partial F_2}{\partial x_1} & \frac{\partial F_2}{\partial x_2} & \cdots & \frac{\partial F_2}{\partial x_n}\\ \vdots & & & \\ \frac{\partial F_n}{\partial x_1} & \frac{\partial F_n}{\partial x_2} & \cdots & \frac{\partial F_n}{\partial x_n}\end{bmatrix}\equiv \text{jacobian } J(x)$$

$F_1(x)=F_1(x_1,x_2,\dots x_n)$

$F_2(x)=F_2(x_1,x_2\dots x_n)$

$\vdots$

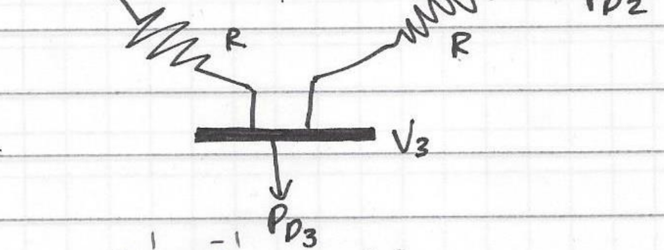
$F_n(x)=F_n(x_1,x_2\dots x_n)$

$$\boxed{X^{v+1}=X^{v}-\frac{F(X^{v})}{J(X^{v})}}\longrightarrow X^{v+1}-X^{v}=\Delta X^{v}=\frac{-F(X^{v})}{J(X^{v})}$$

$$=-\left[J(X^{v})\right]^{\textcircled{-1}}F(X^{v})$$

it migh cause problems for very large J matrix

ex:- DC load flow "عشان نقلل المتغيرات بنهمل الزوايا وQ = 0".



R = 0.01 p.u.

$V_1 = 1.0\ V$

V_2, V_3 ?

$P_{D2} = 1$ p.u.

$P_{D3} = 0.5$ p.u.

$P_{D1} = 0.5$ p.u.

$\therefore P_2 = -1.0$ p.u.

$P_3 = -0.5$ p.u.

Known:- $\underline{\overset{1}{V_1}}, \underline{\overset{-1}{P_2}}, \underline{\overset{-0.5}{P_3}}$

find:- $\underset{\nwarrow P_1}{\delta_1}, V_2, V_3$

y-matrix: $\begin{bmatrix} 200 & -100 & -100 \\ -100 & 200 & -100 \\ -100 & -100 & 200 \end{bmatrix}$

$F(x) = ?$

$$S_i = V_i \sum_{k=1}^{n} y_{ik}^* V_k^*$$

for D.C:

$$P_i = V_i \sum_{k=1}^{n} y_{ik} V_k$$

$$P_1 = 200 V_1^2 - 100 V_1 V_2 - 100 V_1 V_3 \quad \ldots ①$$

$$F_1(x) \rightarrow P_2 = -100 V_2 V_1 + 200 V_2^2 - 100 V_2 V_3 \quad \ldots ②$$

$$F_2(x) \rightarrow P_3 = -100 V_3 V_1 - 100 V_2 V_3 + 200 V_3^2 \quad \ldots ③$$

$$X = \begin{bmatrix} V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

S_1 "في P_1 (D.C.)" بنكتبها على جنب وبنقلها
انتي بالأخير بنحلك ؟

8/4/2014
lecture #24 →

$$X^{v+1} = X^{v} - [J(X^{v})]^{-1} F(X^{v}) \quad \ldots$$

$$X^{1} = \underset{\nwarrow \text{guess} \Rightarrow \text{flat start } \begin{bmatrix}1\\1\end{bmatrix}}{X^{0}} - [J(X^{0})]^{-1} F(X^{0})$$

$$\begin{bmatrix} V_2^1 \\ V_3^1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} - [J(X^0)]^{-1} F(X^0) \quad \ldots Ⓐ$$

1^{st} we have to find $F(X^0)$ then calculate the jacobian.

from ② & ③

$$P_2(x) = -100 V_1 V_2 + 200 V_2^2 - 100 V_2 V_3 = \overset{P_2}{-1}$$

$$\hookrightarrow F_1(x) = -100 V_1 V_2 + 200 V_2^2 - 100 V_2 V_3 + 1 = 0$$

$$P_3(x) = -100 V_1 V_3 - 100 V_2 V_3 + 200 V_3^2 = -0.5$$

$$\hookrightarrow F_2(x) = -100 V_1 V_3 - 100 V_2 V_3 + 200 V_3^2 + 0.5 = 0$$

8/4/2014

$$J(x) \longrightarrow \begin{bmatrix} \overset{\frac{\partial F_1}{\partial V_2}}{\left(-100V_1+400V_2-100V_3\right)} & \overset{\frac{\partial F_1}{\partial V_3}}{\left(-100V_2\right)} \\ \underset{\frac{\partial F_2}{\partial V_2}}{\left(-100V_3\right)} & \underset{\frac{\partial F_2}{\partial V_3}}{\left(-100V_1-100V_2+400V_3\right)} \end{bmatrix}$$

plug $x = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{matrix} \rightarrow V_2' \\ \leftarrow V_3' \end{matrix}$, $\overset{\text{given}}{V_1 = 1}$

$$J(x^0) = \begin{bmatrix} 200 & -100 \\ -100 & 200 \end{bmatrix}$$

iteration 0:-

Plug in Ⓐ $$\begin{bmatrix} V_2^1 \\ V_3^1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 200 & -100 \\ -100 & 200 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}$$

$$= \begin{bmatrix} 0.991667 \\ 0.993333 \end{bmatrix}$$

iteration 1:-

$$\underset{x^{v+1}}{\begin{bmatrix} V_2^2 \\ V_3^2 \end{bmatrix}} = \underset{x^{v}}{\begin{bmatrix} 0.991667 \\ 0.993333 \end{bmatrix}} - \left(\begin{bmatrix} 197.333 & -99.1667 \\ -99.333 & 198.1667 \end{bmatrix}^{-1} \overset{\text{mismatch}}{\begin{bmatrix} 0.00843 \\ 0.00322 \end{bmatrix}} \right)$$

$$= \begin{bmatrix} 0.991599 \\ 0.993283 \end{bmatrix}$$

this term is approaching zero

thus the difference between x^{v+1} & x^{v} is approaching zero.

now,

for actual P.System: ← not only D.C.

$$S_i = V_i \sum_{k=1}^{n} Y_{ik}^{*} V_k^{*} \quad i = 2 \ldots n$$, solve for $V_2, V_3, \ldots V_n$ then find S_1.

$V_i = |V_i| e^{j\theta_i}$

$V_k = |V_k| e^{j\theta_k}$

$Y_{ik} = (\underset{\text{conductance}}{g_{ik}} + j \overset{\text{susceptance.}}{b_{ik}})$

$$S_i = \sum_{k=1}^{n} |V_i| e^{j\theta_i} |V_k| e^{\overset{\text{conjugate}}{-j\theta_K}} Y_{ik}^{*}$$

$$= \sum_{k=1}^{n} |V_i||V_k| e^{j(\theta_i-\theta_K)} Y_{ik}^{*}$$

$$S_i = \sum_{k=1}^{n} |V_i||V_k| \left[\cos(\theta_i-\theta_K) + j\sin(\underbrace{\theta_i-\theta_K}_{\equiv\,\theta_{ik}})\right]\left(g_{ik} \overset{\text{conjugate}}{-} j\, b_{ik}\right)$$

also, $S_i = P_i + jQ_i$

$$\therefore\ P_i = \sum_{k=1}^{n} |V_i||V_k| \left[\overset{\text{real x real}}{g_{ik}\cos(\theta_i-\theta_K)} + \overset{\text{Imaginary * Imag.}}{b_{ik}\sin(\theta_i-\theta_K)}\right] \quad \text{--- ①}$$

$$Q_i = \sum_{k=1}^{n} |V_i||V_k| \left[g_{ik}\sin(\theta_i-\theta_K) - b_{ik}\cos(\theta_i-\theta_K)\right] \quad \text{--- ②}$$

our variables:-

$$X = \begin{bmatrix} \theta_2 \\ \theta_3 \\ \vdots \\ \theta_n \\ \hline |V_2| \\ |V_3| \\ \vdots \\ |V_n| \end{bmatrix} \equiv \begin{bmatrix} \theta \\ \hline |V| \end{bmatrix}$$

"ترتيبهم ما بغيّر اشي بس بختار هذا الترتيب لأنه رح اقدر من خلاله أبسّط اشي قدام"

What about $F(x)$?

P_i, Q_i given. from ① & ②:-

$$F_1(x) = P_i(x) - P_i = 0$$

$$F_2(x) = Q_i(x) - Q_i = 0$$

دائماً P مرتبطة مع θ
و Q مرتبطة مع $|V|$

$$\therefore\ F(x) = \begin{bmatrix} P_2(x) - P_2 \\ P_3(x) - P_3 \\ \vdots \\ P_n(x) - P_n \\ \hline Q_2(x) - Q_2 \\ \vdots \\ Q_n(x) - Q_n \end{bmatrix} \begin{matrix} \left.\vphantom{\begin{matrix}a\\a\\a\\a\end{matrix}}\right\} P \\ \left.\vphantom{\begin{matrix}a\\a\\a\end{matrix}}\right\} Q \end{matrix}$$

we won't write it for simplicity

$$J(x) = \left[\begin{array}{cccc|cccc}
\frac{\partial P_2(x)}{\partial\theta_2} & \frac{\partial P_2}{\partial\theta_3} & \cdots & \frac{\partial P_2}{\partial\theta_n} & \frac{\partial P_2}{\partial|V_2|} & \frac{\partial P_2}{\partial|V_3|} & \cdots & \frac{\partial P_2}{\partial|V_n|} \\
\frac{\partial P_3}{\partial\theta_2} & \frac{\partial P_3}{\partial\theta_3} & \cdots & \frac{\partial P_3}{\partial\theta_n} & \frac{\partial P_3}{\partial|V_2|} & \frac{\partial P_3}{\partial|V_3|} & \cdots & \frac{\partial P_3}{\partial|V_n|} \\
\vdots & \vdots & \boxed{J_{11}} & \vdots & \vdots & \vdots & \boxed{J_{12}} & \vdots \\
\frac{\partial P_n}{\partial\theta_2} & \frac{\partial P_n}{\partial\theta_3} & \cdots & \frac{\partial P_n}{\partial\theta_n} & \frac{\partial P_n}{\partial|V_2|} & \frac{\partial P_n}{\partial|V_3|} & \cdots & \frac{\partial P_n}{\partial|V_n|} \\
\hline
\frac{\partial Q_2}{\partial\theta_2} & \frac{\partial Q_2}{\partial\theta_3} & \cdots & \frac{\partial Q_2}{\partial\theta_n} & \frac{\partial Q_2}{\partial|V_2|} & \frac{\partial Q_2}{\partial|V_3|} & \cdots & \frac{\partial Q_2}{\partial|V_n|} \\
\frac{\partial Q_3}{\partial\theta_2} & \frac{\partial Q_3}{\partial\theta_3} & \cdots & \frac{\partial Q_3}{\partial\theta_n} & \frac{\partial Q_3}{\partial|V_2|} & \frac{\partial Q_3}{\partial|V_3|} & \cdots & \frac{\partial Q_3}{\partial|V_n|} \\
\vdots & \vdots & \boxed{J_{21}} & \vdots & \vdots & \vdots & \boxed{J_{22}} & \vdots \\
\frac{\partial Q_n}{\partial\theta_2} & \frac{\partial Q_n}{\partial\theta_3} & \cdots & \frac{\partial Q_n}{\partial\theta_n} & \frac{\partial Q_n}{\partial|V_2|} & \frac{\partial Q_n}{\partial|V_3|} & \cdots & \frac{\partial Q_n}{\partial|V_n|}
\end{array}\right]$$

$$\left[\begin{array}{c|c}
\overset{J_{11}}{\frac{\partial P}{\partial\theta}} & \overset{J_{12}}{\frac{\partial P}{\partial|V|}} \\
\hline
\underset{J_{21}}{\frac{\partial Q}{\partial\theta}} & \underset{J_{22}}{\frac{\partial Q}{\partial|V|}}
\end{array}\right]$$

→ اعتماد P على $|V|$ و Q على θ محدود ≈ 0

$\therefore J_{12}$ & $J_{21} \approx 0$ (we'll justify this later)

→ effective terms

$P \propto \theta$ & $Q \propto |V|$

$$X^{v+1} - X^{v} = \Delta X^{v} = -j(X)^{-1}\, F(X)$$

$$P_2(X) = \sum_{k=1}^{n} |V_2||V_k|\left[g_{2k}\cos(\theta_2-\theta_k) + b_{2k}\sin(\theta_2-\theta_k)\right]$$

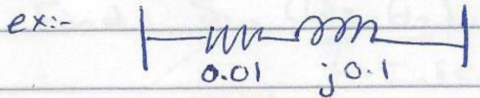
$$\frac{\partial P_2}{\partial \theta_2} = \sum_{\substack{k=1\\ \boxed{k\neq 2}}}^{n} |V_2||V_k|\left[g_{2k}(-\sin(\theta_2-\theta_k)) + b_{2k}\cos(\theta_2-\theta_k)\right] \ldots ①$$

* الفرق بين الزوايا صغير ← إذا $\theta_{ik} \approx 2\text{-}4^\circ \Leftarrow \sin(\theta_{ik}) \approx 0$ & $\cos(\theta_{ik}) \approx 1$

* "في هذه الحالة ضربنا b (الكبيرة) بـ 1 و g (الصغيرة) بـ 0 ← وبالتالي $\frac{\partial P}{\partial \theta}$ كبيرة (إلها حجم)."

**** Note:** in power systems usually $b > g$

ex:-



$$y = \left[\frac{1}{0.01+j0.1}\right] \times \frac{0.01-j0.1}{0.01-j0.1} = \underset{\sim}{1} - j\underset{\sim}{10}$$

$$\underset{\sim}{g} < \underset{\sim}{b}$$

θ_3 only shows up at $k=3$.

$$\frac{\partial P_2(X)}{\partial \theta_3} = |V_2||V_3|\left[g_{23}\sin(\theta_2-\theta_3) - b_{23}\cos(\theta_2-\theta_3)\right]$$

$$\frac{\partial P_2(X)}{\partial |V_2|} = \sum_{k=1}^{n} |V_k|\left[g_{2k}\cos(\theta_2-\theta_k) + b_{2k}\sin(\theta_2-\theta_k)\right] + |V_2|g_{22}$$

at $k=2$ this term $= |V_2|g_{22}$

$\oplus$

$2|V_2|g_{22}$ since at $k=2$ it $\frac{\partial(|V_2|^2 g_{22})}{\partial |V_2|}$

10/4/2014

$$\frac{\partial P_2}{\partial |V_3|} = |V_2|\left(g_{23}\cos(\theta_2-\theta_3) + b_{23}\sin(\theta_2-\theta_3)\right)$$

this is a small quantity

"بتتفق مع مفهومنا: تغيير |V| لا يؤثر على الـP"

$$Q_2(X) = \sum_{k=1}^{n} |V_2||V_k|\left(g_{2k}\sin(\theta_2-\theta_k) - b_{2k}\cos(\theta_2-\theta_k)\right)$$

$$\frac{\partial Q_2}{\partial \theta_2} = \sum_{\substack{k=1\\ k\neq 2}}^{n} |V_2||V_k|\left[g_{2k}\cos(\theta_2-\theta_k) + b_{2k}\sin(\theta_2-\theta_k)\right]$$

$$\frac{\partial Q_2}{\partial \theta_3} = |V_2||V_3|\left[-g_{23}\cos(\theta_2-\theta_3) - b_{23}\sin(\theta_2-\theta_3)\right]$$

$$\frac{\partial Q_2}{\partial |V_2|} = \sum_{k=1}^{n}\left(g_{2k}\sin(\theta_2-\theta_k) - b_{2k}\cos(\theta_2-\theta_k)\right) - b_{22}|V_2|$$

to adjust the value of the sum at $k=2$.

$$\frac{\partial Q_2}{\partial |V_3|} = |V_2|\left(g_{22}\sin(\theta_2-\theta_3) - b_{23}\cos(\theta_2-\theta_3)\right)$$

$$x^{v+1} = x^v - \frac{F(x^v)}{J(x^v)}$$

$$-F(x^v) = (x^{v+1} - x_v)\, J(x^v)$$

$$= \Delta(x^v)\, J(x^v)$$

Case II :-

given: V_1, $(P_2, |V_2|)$, ---- $(P_m, |V_m|)$, S_{m+1} --- , S_n

Find: $\underline{\underline{S_1}}$, $(\underline{\underline{Q_2}}, \underline{\theta_2})$, --- $(Q_m, \underline{\theta_m})$, $\underline{V_{m+1}}$, ..., $\underline{V_n}$

↳ $Im\{S_2\}$

"بالأول بنجيب ال θ_s وال $|V|_s$"

↳ بنحسبها بالآخر

$$S_1 = V_1 \sum_{k=1}^{n} V_k^{*} y_{1k}^{*}$$

we'll derive the matrices for case II from those of case 1.

$$J(x^v)\,(\Delta x^v) = -F(x^v)$$

if bus2 was a PV bus (case II) (more than 1 gen).

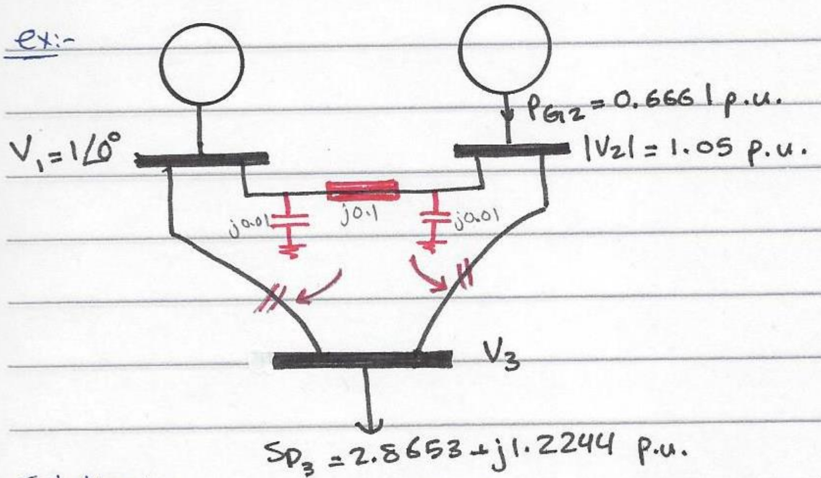
$$
\left[\begin{array}{cccc|cccc}
\frac{\partial P_2}{\partial\theta_2} & \frac{\partial P_2}{\partial\theta_3} & \cdots & \frac{\partial P_2}{\partial\theta_n} & \cancel{\frac{\partial P_2}{\partial|V_2|}} & \frac{\partial P_2}{\partial|V_3|} & \cdots & \frac{\partial P_2}{\partial|V_n|}\\
\frac{\partial P_3}{\partial\theta_2} & \frac{\partial P_3}{\partial\theta_3} & \cdots & \frac{\partial P_3}{\partial\theta_n} & \cancel{\frac{\partial P_3}{\partial|V_2|}} & \frac{\partial P_3}{\partial|V_3|} & \cdots & \frac{\partial P_3}{\partial|V_n|}\\
\vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots\\
\frac{\partial P_n}{\partial\theta_2} & \frac{\partial P_n}{\partial\theta_3} & \cdots & \frac{\partial P_n}{\partial\theta_n} & \cancel{\frac{\partial P_n}{\partial|V_2|}} & \frac{\partial P_n}{\partial|V_3|} & \cdots & \frac{\partial P_n}{\partial|V_n|}\\
\hline
\cancel{\frac{\partial Q_2}{\partial\theta_2}} & \cancel{\frac{\partial Q_2}{\partial\theta_3}} & \cdots & \cancel{\frac{\partial Q_2}{\partial\theta_n}} & \cancel{\frac{\partial Q_2}{\partial|V_2|}} & \cancel{\frac{\partial Q_2}{\partial|V_3|}} & \cdots & \cancel{\frac{\partial Q_2}{\partial|V_n|}}\\
\frac{\partial Q_3}{\partial\theta_2} & \frac{\partial Q_3}{\partial\theta_3} & \cdots & \frac{\partial Q_3}{\partial\theta_n} & \cancel{\frac{\partial Q_3}{\partial|V_2|}} & \frac{\partial Q_3}{\partial|V_3|} & \cdots & \frac{\partial Q_3}{\partial|V_n|}\\
\vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots\\
\frac{\partial Q_n}{\partial\theta_2} & \frac{\partial Q_n}{\partial\theta_3} & \cdots & \frac{\partial Q_n}{\partial\theta_n} & \cancel{\frac{\partial Q_n}{\partial|V_2|}} & \frac{\partial Q_n}{\partial|V_3|} & \cdots & \frac{\partial Q_n}{\partial|V_n|}
\end{array}\right]
\left[\begin{array}{c}
\theta_2^{v+1} - \theta_2^{v}\\
\theta_3^{v+1} - \theta_3^{v}\\
\vdots\\
\theta_n^{v+1} - \theta_n\\
\hline
\cancel{|V_2|^{v+1} - |V_2|}\\
|V_3|^{v+1} - |V_3|\\
\vdots\\
|V_n|^{v+1} - |V_n|^{v}
\end{array}\right]
= -
\left[\begin{array}{c}
P_2(X) - P_2\\
P_3(X) - P_3\\
\vdots\\
P_n(X) - P_n\\
\hline
\cancel{Q_2(X) - Q_2}\\
Q_3(X) - Q_3\\
\vdots\\
Q_n(X) - Q_n
\end{array}\right]
$$

→ = 0 because $|V_2|$ is known

↳ على حسب عدد ال PV buses بنشطب ال $|V|$ مع (1col & 1row)

* in newton-raphson: increasing the # of generator buses (PV buses) decreases the dimensionality (complexity) of the problem. while in gauss it complecates the problem. (adds extra steps to it).

ex:-



, apply N-R method to solve for variables:

Solution:-

Case II:-

$$\text{Given:- } \overset{1\angle 0^\circ}{V_1},\ (\overset{P_{G2}-P_{D2}=P_{G2}=0.6661}{P_2}, \overset{1.05\,p.u.}{|V_2|}),\ S_3 \leftarrow S_3 = -S_{D_3} = -2.8653 - j1.2244$$

$$\text{find:- } S_1,\ (Q_2, \underset{1}{\underline{\theta_2}}),\ V_3 \rightsquigarrow (\underset{3}{\underline{|V_3|}}, \underset{2}{\underline{\theta_3}})$$

$$X = \begin{bmatrix} \theta_2 \\ \theta_3 \\ |V_3| \end{bmatrix} \begin{matrix} \rightarrow \propto P_2 \\ \rightarrow \propto P_3 \\ \rightarrow \propto Q_3 \end{matrix} \qquad Y_{matrix} = \begin{bmatrix} -j19.98 & j10 & j10 \\ j10 & -j19.98 & j10 \\ j10 & j10 & -j19.98 \end{bmatrix}$$

$$P_2(x) = \sum_{k=1}^{n} |V_2||V_k| \left[\cancel{g_{2k} \cos(\theta_2 - \theta_k)} + b_{2k} \sin(\theta_2 - \theta_k) \right]$$

g_{2k} =0, since Y doesn't have real part.

$$= |V_2||V_1| \times 10 \sin(\theta_2 - \theta_1) + \cancel{|V_2|^2 \times zero} + |V_2||V_3| \times 10 \sin(\theta_2 - \theta_3)$$

10 → be careful it's not j10 (b= real value equals the imaginar
Part of Y (without "j")).

$$\therefore\ P_2(x) = 10.5 \sin(\theta_2) + 10.5 |V_3| \sin(\theta_2 - \theta_3) \quad \cdots ①$$

$$P_3(x) = |V_3||V_1| \times 10 \sin(\theta_3 - \theta_1) + |V_3||V_2| \times 10 \sin(\theta_3 - \theta_2) + 0$$

$$P_3(x) = 10 |V_3| \sin(\theta_3) + 10.5 |V_3| \sin(\theta_3 - \theta_2) \quad \cdots ②$$

$$Q_3(x) = \sum_{k=1}^{n} |V_3||V_k| \left[\cancel{\overset{=0}{g_{3k}} \overset{0}{\sin(\theta_3 - \theta_k)}} - b_{3k} \cos(\theta_3 - \theta_k) \right]$$

$$= -|V_3||V_1| \left(10 \cos(\theta_3)\right) - |V_3||V_2| \times 10 \cos(\theta_3 - \theta_2) + |V_3|^2 \times 19.98 \times 1$$

$$Q_3(x) = -10 |V_3| \cos(\theta_3) - 10.5 |V_3| \cos(\theta_3 - \theta_2) + 19.98 |V_3|^2 \quad \cdots ③$$

13/4/2014 lecture #26

cont. ⇒

$$X = \begin{bmatrix} \theta_2 \\ \theta_3 \\ \hline |V_3| \end{bmatrix} \qquad J(X)?$$

$$\rightarrow \frac{\partial P_2(X)}{\partial \theta_2} = 10.5\cos(\theta_2) + 10.5|V_3|\cos(\theta_2-\theta_3) \quad \ldots(11)$$

$$\frac{\partial P_2(X)}{\partial \theta_3} = \boxed{-}10.5|V_3|\cos(\theta_2-\theta_3) \quad \ldots(12)$$

"إذا ما طلعتش بالسالب لازم أشك
لأنه لما أزيد θ_3 بدها تزيد P_3 و بالتالي P_2 بدها تقل"

$$\frac{\partial P_2(X)}{\partial |V_3|} = 10.5\sin(\theta_2-\theta_3) \quad \ldots(13)$$

$$\rightarrow \frac{\partial P_3(X)}{\partial \theta_2} = -10.5|V_3|\cos(\theta_3-\theta_2) \quad \ldots(21)$$

$$\frac{\partial P_3(X)}{\partial \theta_3} = 10|V_3|\cos\theta_3 + 10.5|V_3|\cos(\theta_3-\theta_2) \quad \ldots(22)$$

$$\frac{\partial P_3(X)}{\partial |V_3|} = 10\sin(\theta_3) + 10.5\sin(\theta_3-\theta_2) \quad \ldots(23)$$

$$\rightarrow \frac{\partial Q_3}{\partial \theta_2} = -10.5|V_3|\sin(\theta_3-\theta_2) \quad \ldots(31)$$

$$\frac{\partial Q_3}{\partial \theta_3} = 10|V_3|\sin\theta_3 + 10.5|V_3|\sin(\theta_3-\theta_2) \quad \ldots(32)$$

$$\frac{\partial Q_3}{\partial |V_3|} = -10\cos(\theta_3) - 10.5\cos(\theta_3-\theta_2) + 2\times19.98|V_3| \quad \ldots 33$$

$$\rightarrow F_1(X) = \overset{0 \leftarrow P_2(X^0)}{\cancel{P_2(X)}} - P_2 \leftarrow 0.6661 \text{ p.u}$$

$$F_2(X) = \overset{0}{\cancel{P_3(X)}} - P_3 \leftarrow -2.8653 \text{ p.u.}$$

$$F_3(X) = \underset{-0.52}{\cancel{Q_3(X)}} - Q_3 \leftarrow -1.2244 \text{ p.u}$$

$$X^0 = \begin{bmatrix} 0 \\ 0 \\ \hline 1 \end{bmatrix} \quad \text{flat start}$$

→ depends on the value of Stack ~~bus~~ voltage $|V_1|$ here = 1 p.u.

$$F(X^0) = \begin{bmatrix} -0.6661 \\ 2.8653 \\ 0.7044 \end{bmatrix}$$

$$J(x^0) = \begin{bmatrix} 21 & -10.5 & 0 \\ -10.5 & 20.5 & 0 \\ 0 & 0 & 19.96 \end{bmatrix}$$ → the inverse of this matrix can be found by inverting each individual sub matrix

$$\Delta(x^0) = J(x^0)^{-1} F(x^0)$$

$$\begin{bmatrix} \theta_2^1 - \cancelto{0}{\theta_2^0} \\ \theta_3^1 - \cancelto{0}{\theta_3^0} \\ |V_3|^1 - \underset{1}{\cancel{|V_3|^0}} \end{bmatrix} = - \begin{bmatrix} 21 & -10.5 & 0 \\ -10.5 & 20.5 & 0 \\ 0 & 0 & 19.96 \end{bmatrix}^{-1} \begin{bmatrix} -0.6661 \\ 2.8653 \\ 0.7044 \end{bmatrix}$$

$$= \begin{bmatrix} -0.0513 \ \underline{rad} \\ -0.166 \ \underline{rad} \\ -0.0169 \ p.u. \end{bmatrix} = \begin{bmatrix} -3.9396^\circ \\ -9.5139^\circ \\ -0.0619 \ p.u. \end{bmatrix}$$

$\therefore \theta_2^1 = -3.9396^\circ$

$\theta_3^1 = -9.5139^\circ$

$|V_3|^1 = 1 - 0.0619 = 0.9389$

then calculate $S_1^1 = V_1^1 \Sigma Y_{1k}^* V_k^*$ & $Q_2^1 = Im\{S_2^1\}$ ...

$V_1^1 = 1\angle 0^\circ$, $V_2^1 = 1.05\angle -3.9396^\circ$

$V_3^1 = 0.9389 \angle -9.5139^\circ$

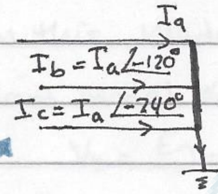
CHAPTER #10: Balanced 3-phase faults:

- what is a fault?

→ Any failure or condition in the system that results in the generated current (or part of it) not reaching the load (exept the charging current).

- Types of fault:-

1. Balanced 3-phase fault.



"أقسى أنواع الـ fault"

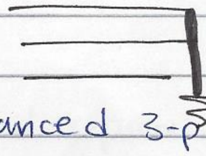
2. 1-phase to earth fault (1-P-E)

"الأكثر شيوعاً"

(not balanced).

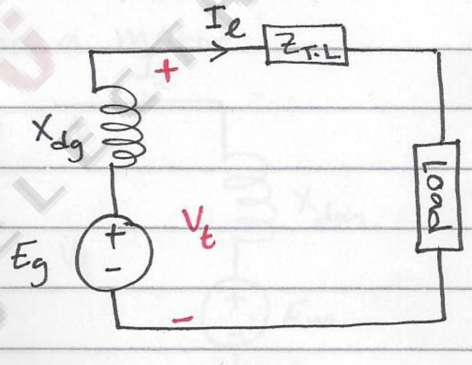
3. Phase to phase (P-P) fault.

4. Phase to phase to Earth fault (P-P-E)



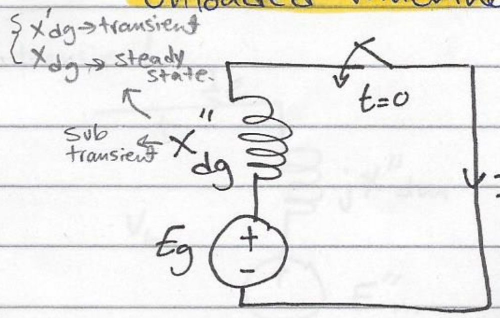
in this chapter we'll study the balanced 3-ph fault...

model#1: for calcualtions of Load current (I_L):



$V_t = E_g - jX_{dg} I_L$

model#2: for calculation of Short CKT current of Unloaded machine.

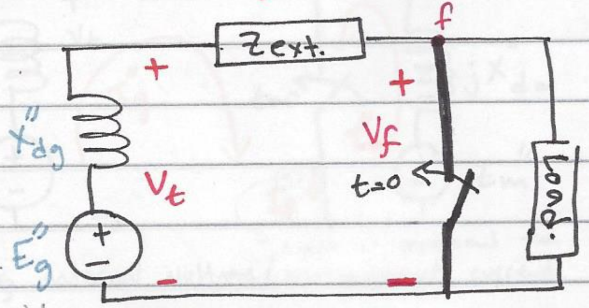


↳ before the fault there was no Armature reaction ($I_L=0$) "No-Load".

* model #1 is used to find I_L only, model #2 for I_f only but what about a loaded machine that has suddenly encountered a fault? (we'll use model #3) ...

model #3: for calculations of both I_L & I_f.

↳ Internal Voltages Method.



before the fault I want to use X_{dg}, but when $t=0$ (fault happens, I must use X''_{dg} (sinee armature reaction will change)

↳ $E_{AR} = -jIX_{ar}$

this will consider the armature reaction

In order to be able to use this model to calculate I_L V_t in this model must equal V_t in model #1.

from model #1:

$$V_t = E_g - I_L \times jX_{dg} \longrightarrow E_g = V_t + jI_L X_{dg} \quad \text{---①}$$

from model #3:

$$V_t = E''_g - j\underbrace{I_L}_{\text{(before fault)}} X''_{dg} \longrightarrow E''_g = V_t + jI_L X''_{dg} \quad \text{---②}$$

① - ②

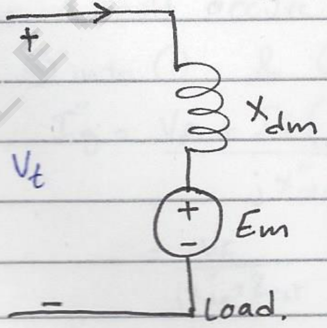
↳ $E_g - E''_g = jI_L \underbrace{(X_{dg} - X''_{dg})}_{X_{ar} \text{ at } I_L}$

→ E''_g ≡ internal voltage. (mathmatical representation, can't be measured practically).

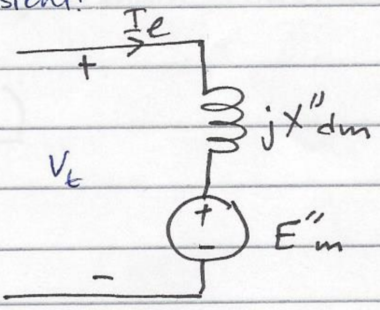
⇒ for transient calculation use X'_{dg}, E'_g. "just replace (″) with (′)"

→ for a motor:

Steady state:



Sub-transient:



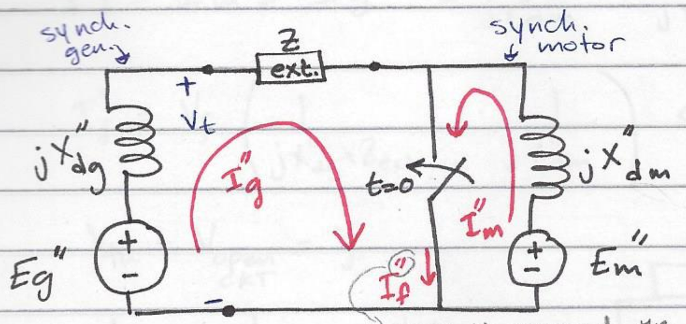
$E_m = V_t - I_L jX_{dm}$

$E''_m = V_t - jIX''_{dm}$

for transient: $E'_m = V_t - jIX'_{dm}$

15/4/2014

lecture #27



since it represent the subtransient current.

before the fault happens we can find I_L, $\tilde{E}''_g$, E''_m

After the fault we can compute I''_f.

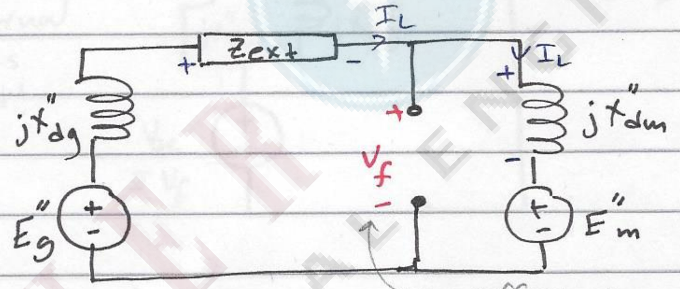
Solve using internal voltage/ method:

mesh analysis:-

$$I''_g = \frac{E''_g}{jX''_{dg} + Z_{ext}} \quad ---①$$

$$I''_m = \frac{E''_m}{jX''_{dm}} \quad ---②$$

E''_m ? E''_g ? use the CKT before the fault:



my refference (Vt بدل)

$$E''_g = V_f + I_L(Z_{ext} + jX''_{dg}) \quad ---ⓐ$$

$$E''_m = V_f - I_L(jX''_{dm}) \quad ---ⓑ$$

When a fault occurs E''_m, E''_g, V_f remain constant.

Plug ⓐ into ① & ⓑ into ②

$$I''_g = \frac{V_f + I_L(jX''_{dg} + Z_{ext})}{jX''_{dg} + Z_{ext}}$$

$$= \frac{V_f}{jX''_{dg} + Z_{ext}} + I_L$$

$$I''_m = \frac{V_f - jI_L X''_{dm}}{jX''_{dm}}$$

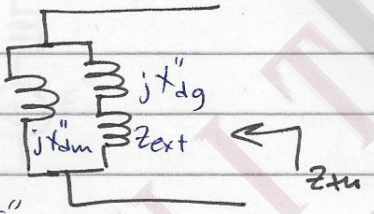
$$= \frac{V_f}{jX''_{dm}} - I_L$$

$$I''_f = I''_m + I''_g = \frac{V_f}{jX''_{dm}} + \frac{V_f}{jX''_{dg} + Z_{ext}}$$

$$I''_f = V_f \left(\frac{1}{jX''_{dg} + Z_{ext}} + \frac{1}{jX''_{dm}} \right) \Leftarrow \text{thevinin !}$$

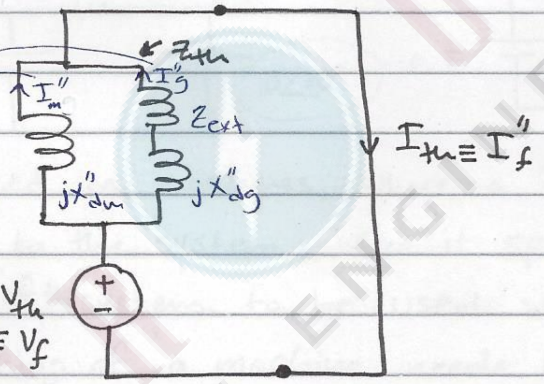
$$V_{th} = V_{\underset{CKT}{open}} = V_f$$

$$\frac{1}{Z_{th}} = \frac{1}{jX''_{dg} + Z_{ext}} + \frac{1}{jX''_{dm}}$$



$$I_{S/C} = V_f \times \frac{1}{Z_{th}} = I_{th.} = I''_f$$

≠ I''_m & I''_g calculated in the internal Voltages method!



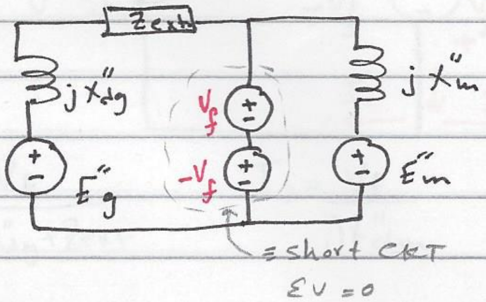
$$I''_g = \frac{V_f}{Z_{ext} + jX''_{dg}}$$

$$I''_m = \frac{V_f}{jX''_{dm}}$$

(we can use thevinin method for the external ckt (Load) only, we can't use it for the internal CKT values).

∴ thwinin is a very fast method to find I''_f. but what about I''_m & I''_g ?

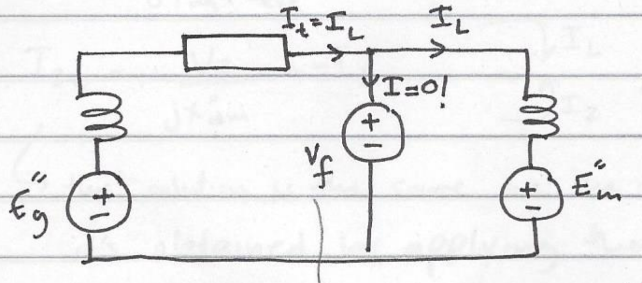
let's do some CKT modifications:



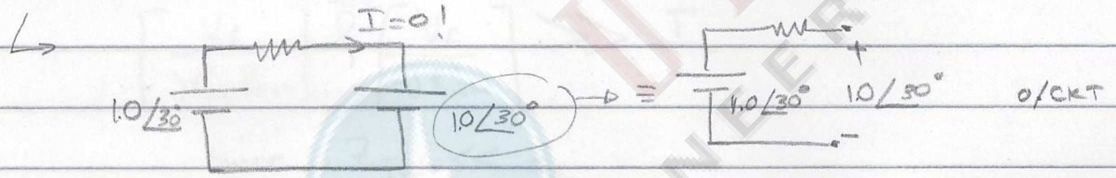
now, by super position :-

↳ solution #1 :-

let $\left[E_g'' , E_m'' \;\; V_f\right]^{\text{be}}_{\uparrow} \text{ON}$ & $[-V_f]$ off (short CKT).



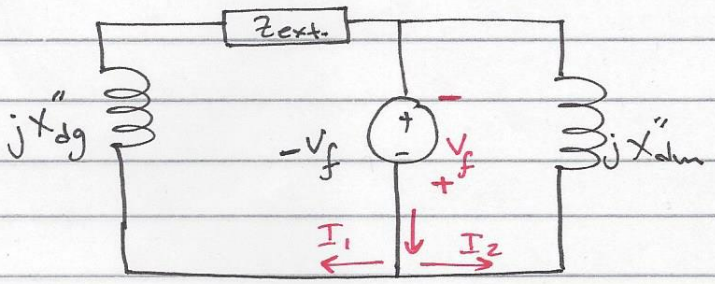
"حطيت Voltage له نفس القيمة ونفس الزاوية لـ V_f ⇐ ما رح ينزل فيه تيار"
قيمة V قبل ما أضيف الـ source



↳ this idea is used in machines, where a "spinning reserved" machine is connected to the system, where it spins but doesn't contribute to the total (I) of the system. to be used when an error occurs. (since starting of a machine needs long time due to its huge inertia). $V_t = V_{\text{before inserting it.}}$

↳ solution #2:

let $(-V_f)^{\text{be}}\text{ON}$, (E_g'', E_m'', V_f) off

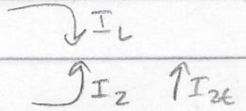


$I_1 = \dfrac{V_f}{jX''_{dg} + Z_{ext}}$ $\qquad I_2 = \dfrac{V_f}{jX''_{dm}}$

↳ the total solution (add sol.# ① & ②).

$$I_{1t} = \frac{V_f}{jX''_{dg} + Z_{ext}} + I_L$$

$$I_{2t} = \frac{V_f}{jX''_{dm}} + -I_L$$



↳ this solution is the same as the internal voltages method's sol. it's obtained by applying thevinin then add Loading condition.

$$\left[\frac{V_f}{jX''_{dg} + Z_{ext}}\right] \& \left[\frac{V_f}{jX''_{dm}}\right] >> I_L$$

Since $Z_{T.L} << Z_L$

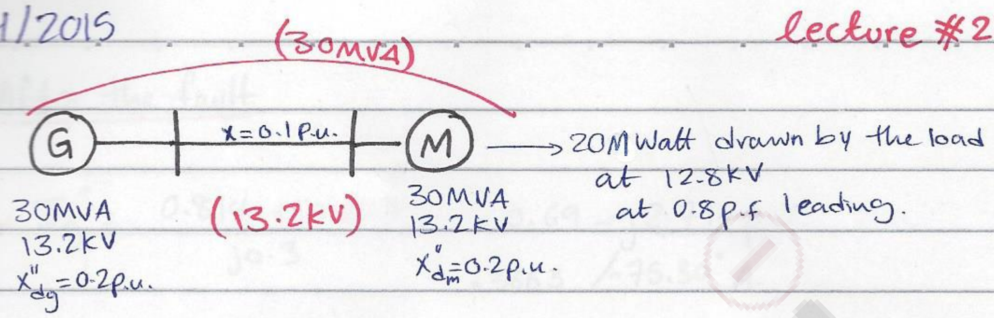
"مشان هيك بقلك حل" الكل الثاني وما في داعي للأول (I_L)"

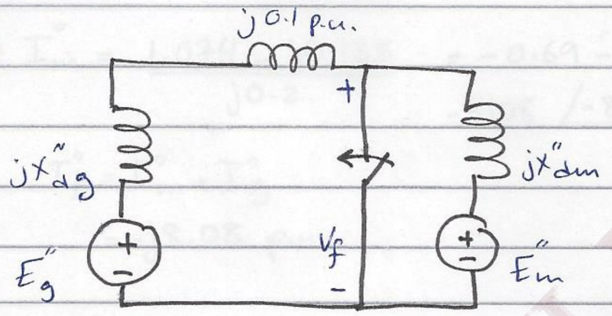
بالامتحان إذا ما حدد طريقة الحل ، حل عال thevinin واهمل I_L.

17/4/2015

lecture #28

ex:





1. Internal Voltages Method:

- before fault:

$$V_f = \frac{12.8}{13.2} = 0.97\angle 0^\circ \text{ p.u.}$$

$$E''_g = 0.97\angle 0^\circ + I\,(jx''_{dg} + jX_{T.L}) \quad \ldots ①$$

@the terminals of the motor:- $V = 0.97\angle 0^\circ$ p.u.

$$S = \frac{20/0.8}{30} \angle -36.87 \text{ p.u.}$$

$$I = \frac{S^*}{V^*} = \frac{\frac{20/0.8}{30}\angle +36.87^\circ}{0.97\angle 0^\circ} = 0.86\angle 36.87 \text{ p.u.}$$

$$= 0.69 + j0.52 \text{ p.u.}$$

back to ①:

$$* \; E''_g = 0.97\angle 0^\circ + 0.86\angle 36.87^\circ\,(j0.3)$$

$$= 0.819 + j0.207$$

$$* \; E''_m = V_f - j\,x''_{dm}\,I_L$$

$$= 0.97\angle 0^\circ - j\,0.2 \times 0.86\angle 36.87^\circ$$

$$= 1.075 - j0.138$$

$$\text{base current} = \frac{30 \times 10^6}{\sqrt{3} \times 13.2 \times 10^3} = 1312\text{ A}$$

• After the fault

* $I_g'' = \dfrac{0.814 + j0.207}{j0.3} = 0.69 - j2.71$ p.u.
$= 3663 \angle -75.30^\circ$ A

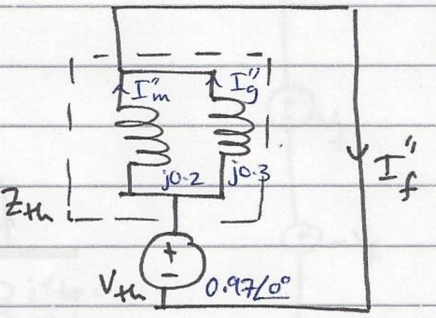
* $I_m'' = \dfrac{1.074 - j0.138}{j0.2} = -0.69 - j5.37$ p.u.
$= 7108 \angle -87.3^\circ$ A

$I_f'' = I_m'' + I_g''$
$= -j8.08$ p.u.

2. Thevinin Method:

$V_{th} = V_f = 0.97\angle 0^\circ$ p.u.
$Z_{th} = j0.3 \,//\, j0.2 = j0.12$ p.u.

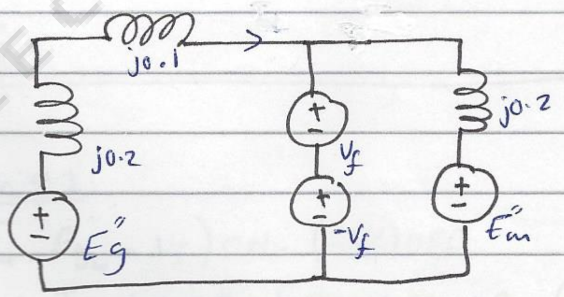
$I_f'' = \dfrac{0.97\angle 0^\circ}{j0.12} = -j8.08$ p.u.



$I_m'' = -j3.23 \neq I_m''$ in the previous meth.
$I_g'' = -j4.84 \neq I_g''$ " " "

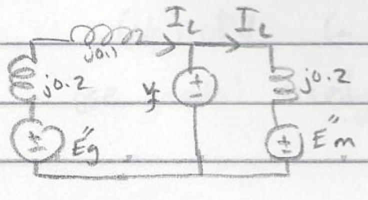
solution? use superposition:

3. Super Position:

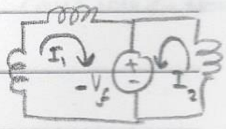


① $-V_f$ off, others ON

$I_L = 0.86 \angle 36.87^\circ$ p.u.



② $-V_f$ ON, others OFF



$I_1 = \dfrac{0.97\angle 0^\circ}{j0.3} = -j3.23$

$I_2 = \dfrac{0.97\angle 0^\circ}{j0.2} = -j4.85$ p.u.

③ total solution:-

$I''_g = -j3.23 + 0.69 + j0.52 = 0.69 - j2.71$ p.u. $= 3660\angle{-75.3^\circ}$ A

$I''_m = -j4.85 - 0.69 - j0.52 = -0.69 - j5.37$ p.u. $= 7108\angle{-87.3^\circ}$ A

$I''_f = -j10600$A (base current = 1312A)

in thevinin's method:-

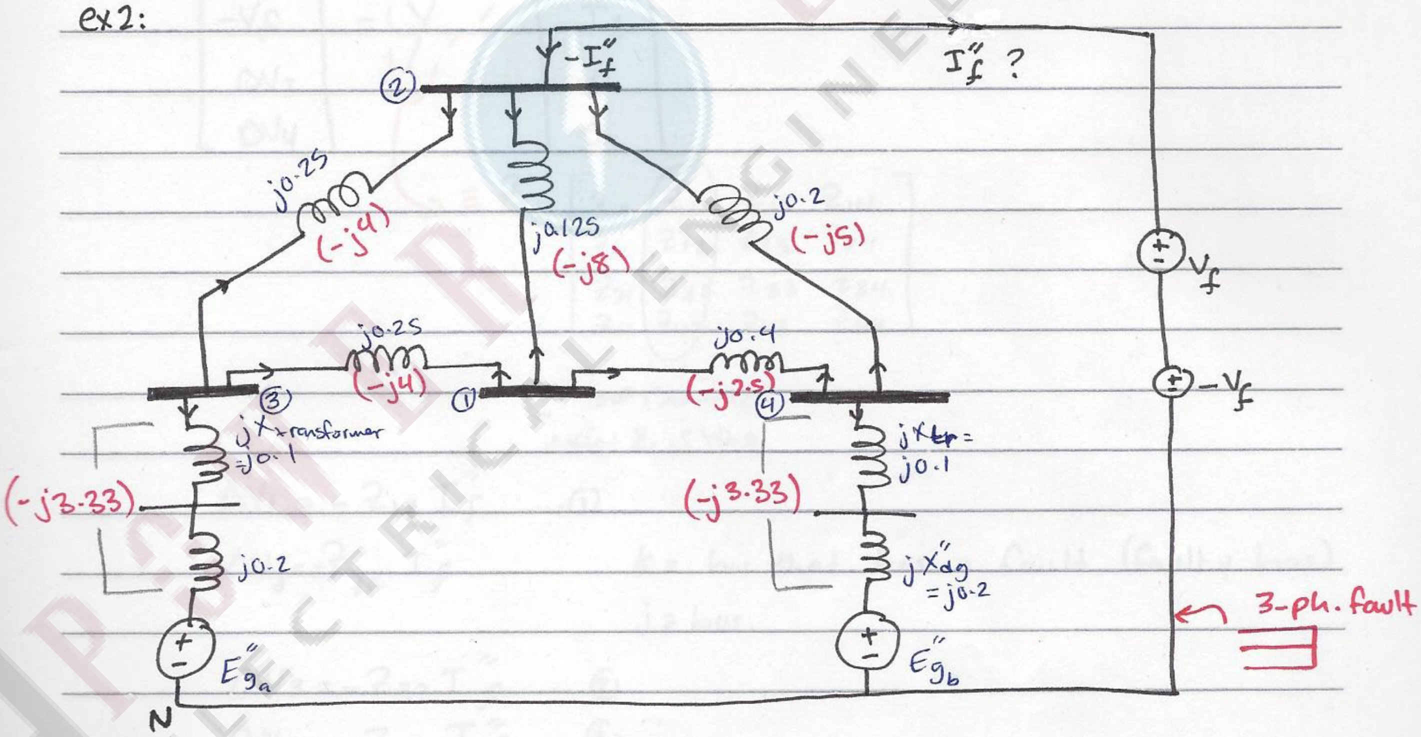
$I''_m = -j\,6360$ A

$I''_g = -j\,4240$A $4240 - 3660 \approx 800$A

"هون متواضع ال fault current لكنه بالعادة أكثر بكثير فالفرق بين نتيجة ال Thevinin والطريقة الثانية بكون صغير جداً عكس هالحالة"

now, we'll focus on the 2nd sol. ($I_L = 0$)

ex2:



Solution #1:

$(E''_{g_a}, E''_{g_b}, V_f)$ ON, $(-V_f)$ OFF

assume I_L in all branches $= 0$ (don't take it into consideration).

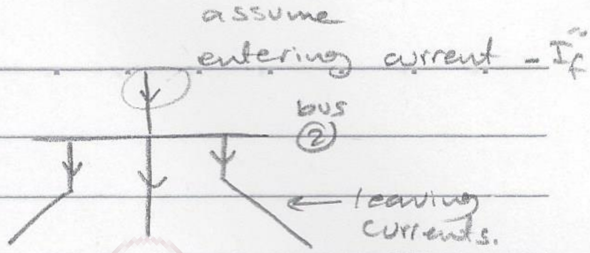
$\therefore V_1 = V_2 = V_3 = V_4 = E''_{g_a} = E''_{g_b} = V_f$

$\therefore$ go to Solution #2:

$(-V_f)$ ON, $(+V_f, E''_{g_a}, E''_{g_b})$ OFF

use nodal analysis. in p.sys. → bus ≡ node entering I

$$I = (YV)$$ leaving I



لكل row المجموع = 0 إذا ما كان في اشي بينه والـ bus و N

$$\begin{bmatrix} 0 \\ -I_f'' \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -j14.5 & j8 & j4 & j2.5 \\ j8 & -j17 & j4 & j5 \\ j4 & j4 & -j11.33 & 0 \\ j2.5 & j5 & 0 & -j10.83 \end{bmatrix} \begin{bmatrix} \Delta V_1 \\ \Delta V_2 = -V_f \\ \Delta V_3 \\ \Delta V_4 \end{bmatrix}$$

حسب اللي فرضته بالـ superposition

$\Sigma \neq 0$

لأنه في gen بينهم وبين الـ neutral

$$\begin{bmatrix} \Delta V_1 \\ -V_f \\ \Delta V_3 \\ \Delta V_4 \end{bmatrix} = Y^{-1} \begin{bmatrix} 0 \\ -I_f'' \\ 0 \\ 0 \end{bmatrix}$$

$$\equiv Z = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{14} \\ Z_{21} & Z_{22} & Z_{23} & Z_{24} \\ Z_{31} & Z_{32} & Z_{33} & Z_{34} \\ Z_{41} & Z_{42} & Z_{43} & Z_{44} \end{bmatrix}$$

إحنا بنستخدم هاد
والباقي Z بتصفر

$\Delta V_1 = -Z_{12} I_f''$ ...①

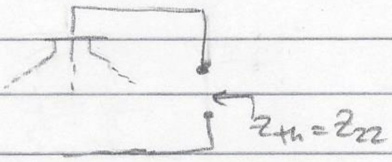
$\Delta V_j = -Z_{jk} I_f''$ k ≡ bus that has a fault. (faulty bus)
j ≡ bus.

$\Delta V_3 = -Z_{32} I_f''$ ...②

$\Delta V_4 = -Z_{42} I_f''$ ...③

$-V_f = -I_f'' Z_{22}$ ...④

$\hookrightarrow I_f'' = \dfrac{V_f}{Z_{22}}$ $Z_{22} \equiv Z_{th}$



Plug I_f'' in ② & ③

$$\Delta V_3 = -Z_{32} \frac{V_f}{Z_{22}} \quad , \quad \Delta V_4 = -Z_{42} \frac{V_f}{Z_{22}}$$

$$\Delta V_j = V_f - \frac{Z_{jk}}{Z_{kk}} V_f$$

2014/2014 **lecture #29**

cont. → now, add the 2 solutions:

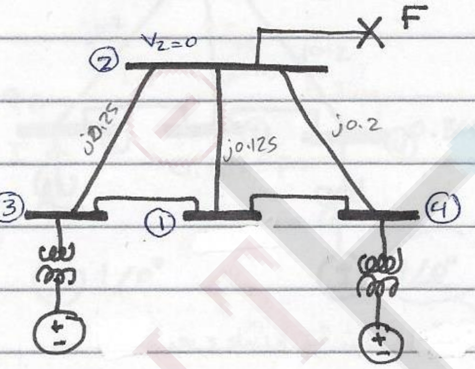
$$V_1 = V_f - \frac{Z_{12}}{Z_{22}} V_f$$

$$V_2 = V_f - \overset{1}{\frac{\cancel{Z_{22}}}{\cancel{Z_{22}}}} V_f = 0$$

$$V_3 = V_f - \frac{Z_{32}}{Z_{22}} V_f$$

$$V_4 = V_f - \frac{Z_{42}}{Z_{22}} V_f$$

one-line diagram



in general, $V_j = \underset{1^{st}\ sol.}{\boxed{V_f}} - \underset{2^{nd}\ Sol.}{\boxed{\frac{Z_{jK}}{Z_{KK}} V_f}}$

given $V_f = 1.0\angle 0°$ p.u.

"من جدول [illegible]" (col. 2)

$$Z_{bus} = \begin{bmatrix} - & j0.1938 & - & - \\ - & j0.2295 & - & - \\ - & j0.1494 & - & - \\ - & j0.1506 & - & - \end{bmatrix} \rightarrow$$

in two-port networks:

$$Z_{ij} = \frac{V_i}{I_j} \Bigg|_{\substack{V_k = 0 \\ k \neq i}} \neq 0$$

"لأنه معناها إنه bus i ضارب ب bus j (S/CKT) وهاد مستحيل يصير (bus ضارب ب bus)!"

$$I_f'' = \frac{V_f}{Z_{22}} = \frac{1.0\angle 0°}{j0.2295} = -j\,4.3573 \text{ p.u.}$$

$$V_1 = 1.0\angle 0° - \frac{j0.1938}{j0.2295} 1.0\angle 0° = 0.1556 \text{ p.u.}$$

$$V_2 = 0$$

$$V_3 = 1\angle 0° - \frac{j0.1494}{j0.2295} 1\angle 0° = 0.349 \text{ p.u.}$$

$$V_4 = 1.0\angle 0° - \frac{j0.1506}{j0.2295} 1\angle 0° = 0.3438 \text{ p.u.}$$

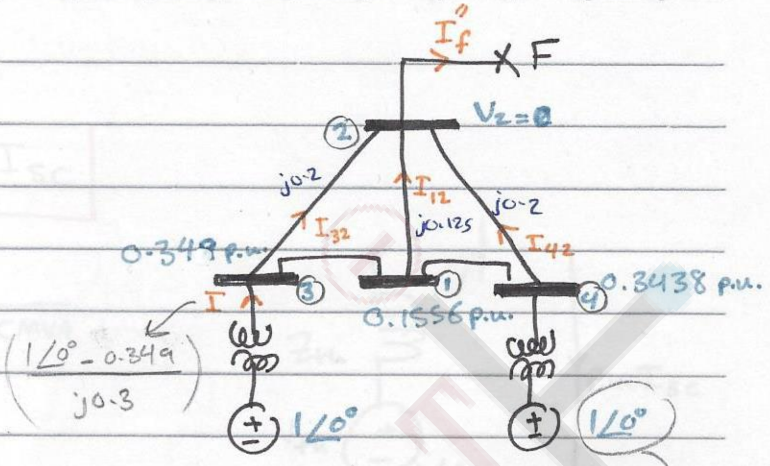
fault at bus 2 → $V_2 = 0$ → @other buses $|V|$ will decrease, , the most affected bus (i) is the one thas has the smallest impedence between its terminal & the faulty bus's ≡ Z_{i2}, in this example it's bus 1.

$$I_{12} = \frac{0.1556 - 0}{j0.125}$$

$$I_{32} = \frac{0.349 - 0}{j0.25}$$

$$I_{42} = \frac{0.3438 - 0}{j0.2}$$

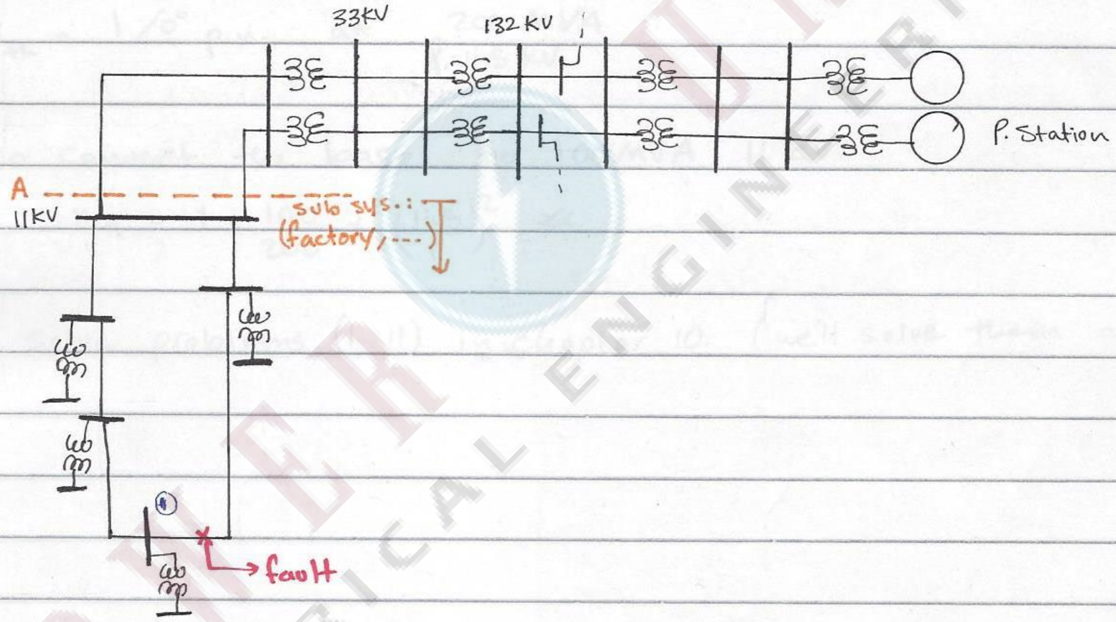
$$I_f'' = I_{32} + I_{12} + I_{42}$$



بضل ثابت قبل واثناء وبعد الـ fault

$I_f = \text{const.}$ ، $I_L = 0$

لا يتغير



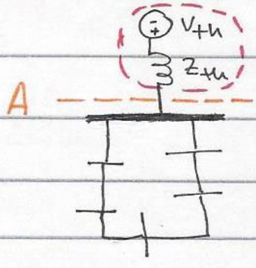
for the sub-system calculations & analysis we just need to know V_{th} & Z_{th} at A, we don't need other details about the remaining P. system.

how to find thevinin equivalent?

usually, you'll be given (3-ph) Short CKT MVA (SCMVA) at A.

↳ at nominal voltage

القيمة المترتبة/ القيمة الاسمية



$$\text{SCMVA} = \sqrt{3} \times \text{Nominal Voltage(kV)} \times I_{sc}\,(A) \times 10^{-3} \quad \text{---} ①$$

✓given ✓ $I_{sc} \equiv I_f''$

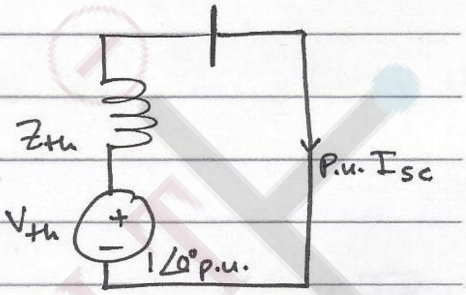
$$\text{Base MVA} = \sqrt{3} \times \underbrace{\text{base Voltage kV}}_{} \times I_{base}\,(A) \times 10^{-3} \quad \text{---} ②$$

↳ let it equal nominal Voltage.

divide ① by ②:

$$\boxed{SCMVA_{(p.u)} = p.u.\ I_{sc}}$$

$$X_{th\,(p.u.)} = \frac{1}{p.u.\ I_{sc}} = \frac{1}{p.u.\ SCMVA}$$



another method:

consider a fault at bus 1

SCMVA = 200 MVA

KV = 11.5 KV

let

$\rightarrow X_{th} = 1\angle 0°$ p.u. at base: 200 MVA & 11.5 KV

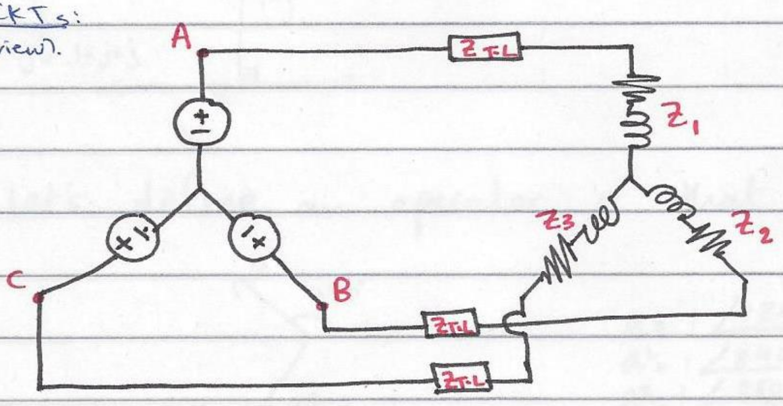
to convert the base to 100 MVA, 11 KV

$$X'_{th} = 1 \times \frac{100}{200} \times \left(\frac{11.5}{11}\right)^2 \quad \divideontimes$$

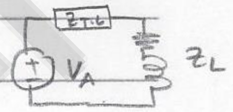
solve problems (1-11) in chapter 10. (we'll solve them on sunday).

CHAPTER #11: Symmetrical Components:

- in CKTs: (review).



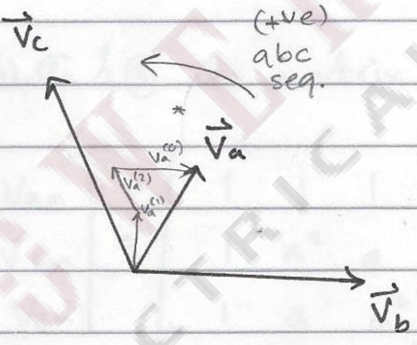
if $Z_1 = Z_2 = Z_3$ (balanced): we use perphase CKT



if $Z_1 \neq Z_2 \neq Z_3 \rightarrow$ we use mesh analysis.

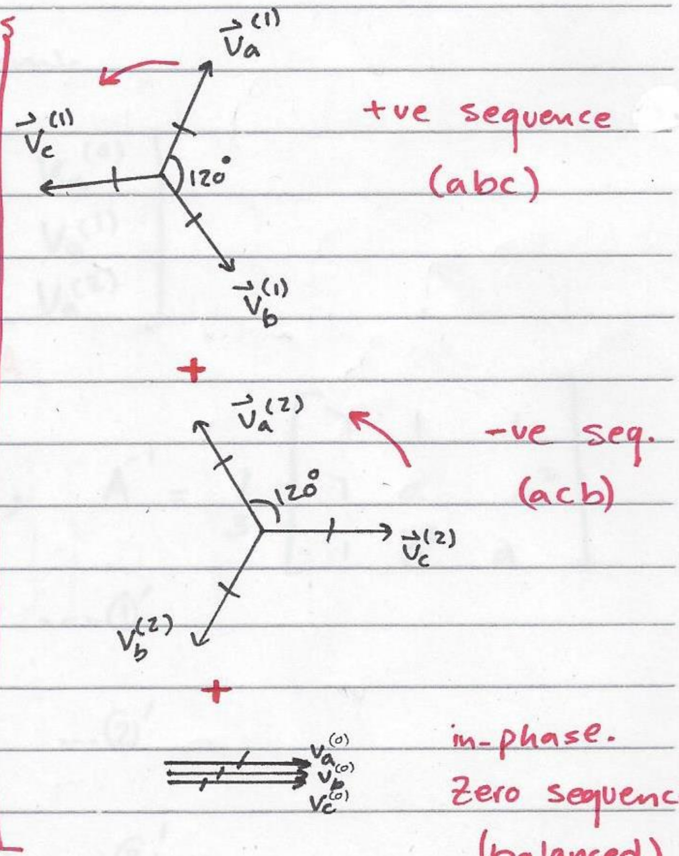
- An unbalanced system of (n) related phasors can be resolved into (n) balanced systems (we're here concerned about n=3). each containing (n=3 ←in our case.) phasors equal in magnitude & equally displaced. (Fortescue's Thm.)

24/4 → lecture #30



unbalanced System!

$\equiv$

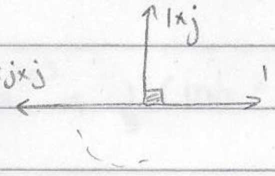


$V_a = V_a^{(0)} + V_a^{(1)} + V_a^{(2)}$

$V_b = V_b^{(0)} + V_b^{(1)} + V_b^{(2)}$

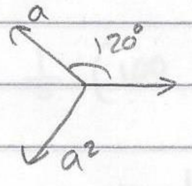
$V_c = V_c^{(0)} + V_c^{(1)} + V_c^{(2)}$

$j = 1\angle 90^\circ$ (j) is used to shift any phasor by 90°



$j^2 = 1\times j\times j$

let's define an operator a that shifts the phasor 120°



$a = 1\angle 120$
$a^2 = 1\angle 240^\circ$
$a^3 = 1\angle 360^\circ$

also: $a = -0.5 + j0.866$

back to the equations:-

$$V_{an} = V_a^{(0)} + V_a^{(1)} + V_a^{(2)} \quad \text{---} ①$$

$$V_{bn} = \overset{V_a^{(0)}}{\cancel{V_b^{(0)}}} + \overset{a^2 V_a^{(1)}}{\cancel{V_b^{(1)}}} + \overset{a V_a^{(2)}}{\cancel{V_b^{(2)}}} \quad \text{---} ②$$

$$V_{cn} = \overset{V_a^{(0)}}{\cancel{V_c^{(0)}}} + \overset{a V_a^{(1)}}{\cancel{V_c^{(1)}}} + \overset{a^2 V_a^{(2)}}{\cancel{V_c^{(2)}}} \quad \text{---} ③$$

$* \; V_b^{(1)} = V_a^{(1)} \angle \overset{+240^\circ}{\text{or } -120^\circ} = a^2 V_a^{(1)}$

$V_c^{(1)} = V_a^{(1)} \angle \overset{-240^\circ}{\text{or } +120^\circ} = a\, V_a^{(1)}$

$* \; V_b^{(2)} = a V_a^{(2)}$

$V_c^{(2)} = a^2 V_a^{(2)}$

put ①, ② & ③ in matrix form:-

$$\begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}}_{\text{matrix A}} \begin{bmatrix} V_a^{(0)} \\ V_a^{(1)} \\ V_a^{(2)} \end{bmatrix}$$

$$\begin{bmatrix} V_a^{(0)} \\ V_a^{(1)} \\ V_a^{(2)} \end{bmatrix} = A^{-1} \begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix} \quad , \quad A^{-1} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}$$

$$\therefore \; V_a^{(0)} = \frac{1}{3}(V_a + V_b + V_c) \quad \text{---} ①'$$

$$V_a^{(1)} = \frac{1}{3}(V_a + a V_b + a^2 V_c) \quad \text{---} ②'$$

$$V_a^{(2)} = \frac{1}{3}(V_a + a^2 V_b + a V_c) \quad \text{---} ③'$$

in a case of a balanced system with (+ve) seq. : $|V_a| = |V_b| = |V_c| = 100$.

$$V_a^{(0)} = \frac{1}{3}(100\angle 0^\circ + 100\angle -120^\circ + 100\angle -240) = 0$$

$$V_a^{(1)} = \frac{1}{3}(100\angle 0^\circ + 100\angle 0^\circ + 100\angle 0^\circ) = 100\angle 0^\circ$$

$$V_a^{(2)} = \frac{1}{3}(100\angle 0^\circ + 100\angle 120^\circ + 100\angle -120^\circ) = 0$$

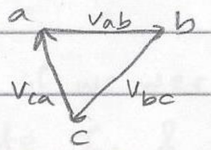
but is it applicable to 3 line to line Voltages? (V_{ab}, V_{bc}, V_{ca})? yes!

$$\begin{bmatrix} V_{ab}^{(0)} \\ V_{ab}^{(1)} \\ V_{ab}^{(2)} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix}$$

= 0 (closed loop a→b→c).

$$V_{ab}^{(0)} = \frac{1}{3}(V_{ab} + V_{bc} + V_{ca})$$

also, $= V_{an}^{(0)} - V_{bn}^{(0)} = 0$



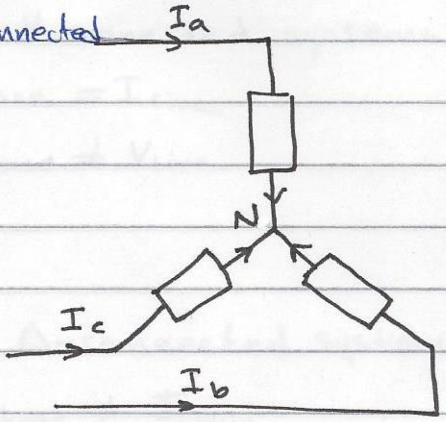
It's also applicable to currents, ... (any 3 phasors).

$$\begin{bmatrix} I_a^{(0)} \\ I_a^{(1)} \\ I_a^{(2)} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

for line currents:-

$$\begin{bmatrix} I_{ab}^{(0)} \\ I_{ab}^{(1)} \\ I_{ab}^{(2)} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_{ab} \\ I_{bc} \\ I_{ca} \end{bmatrix}$$

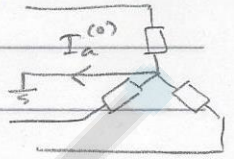
- in a Y-connected load:



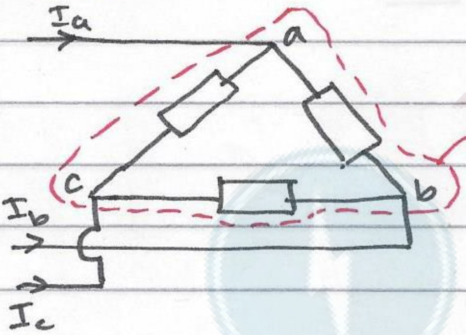
if the neutral (N) was isolated

$I_a^{(0)}=0$

else: $I_a^{(0)}$ might not be zero.



- in a Δ-connected load:

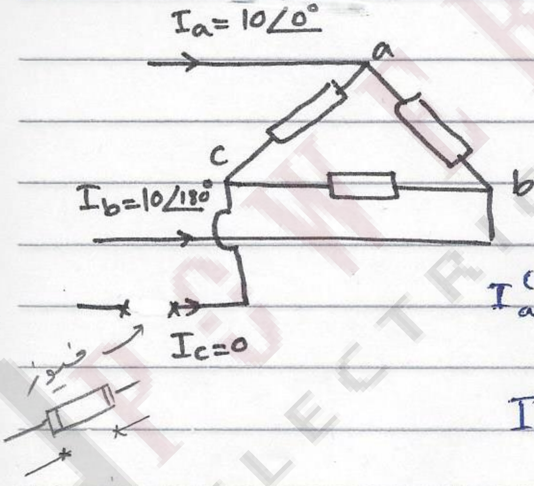


$(I_a+I_b+I_c)$ most equal zero.

$\Sigma I=0$

consider Δ as a super node. $\Sigma I = I_{\text{leaving the node}} = 0.$

ex:-



$\Sigma I=0$ ∴ if we were give I_a we can calculate I_b & vice versa.

find $I_a^{(0)}, I_a^{(1)}, I_a^{(2)}, \ldots\ldots I_c^{(2)}$:

$$I_a^{(0)} = \frac{1}{3}(I_a+I_b+I_c) = \frac{1}{3}\Sigma I = 0.$$

$$I_a^{(1)} = \frac{1}{3}(10\angle 0^\circ + 10\angle 300^\circ + 0) = 5.78\ \angle -30^\circ\ A$$

$$I_a^{(2)} = \frac{1}{3}(10\angle 0^\circ + 10(\overset{\equiv 60^\circ}{\angle 180^\circ \angle 240^\circ}) + 0)$$

$$= 5.78\ \angle +30^\circ\ A$$

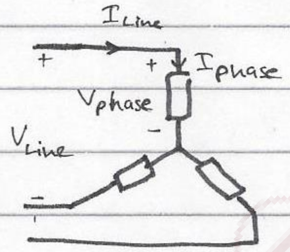
$I_b^{(1)} = a^2 I_a^{(1)}$, $I_c^{(1)} = a I_a^{(1)}$

$I_b^{(2)} = a I_a^{(2)}$, $I_c^{(2)} = a^2 I_a^{(2)}$

in a Y-connected system:-

$I_{phase} = I_{Line}$

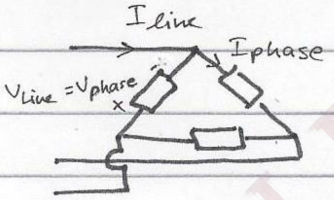
$V_{phase} \neq V_{Line}$



in a Δ-connected system:-

$I_{phase} \neq I_{line}$

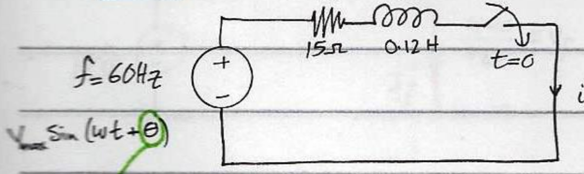
$V_{phase} = V_{Line}$.



27/4/2014 lecture#31

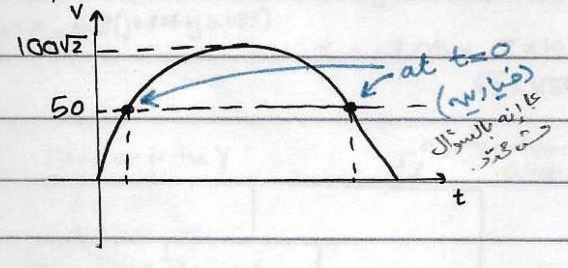
CHAPTER 10 problems:-

10.1



→ θ: Point on the waveform at which the switch closes

a)



$50 = 100\sqrt{2}\,\sin(\theta)$

$\theta = 20.7^\circ$ or 159.3°

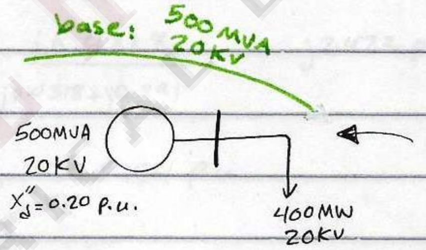
$$i(t) = \underbrace{\frac{V_{max}}{Z}\sin(\omega t+\theta-\phi)}_{\text{steady state component.}} - \underbrace{\frac{V_{max}}{Z}\sin(\theta-\phi)\,e^{-\frac{R}{L}t}}_{\text{transient component.}}$$

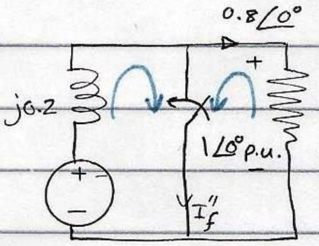
$Z = 15 + j2\pi \times 60 \times 0.12 = 47.66\angle 71.66^\circ\ \Omega$

* max. value at t=0.

⋮

10.4





$S_{Load} = \frac{400}{500}\angle 0^\circ$

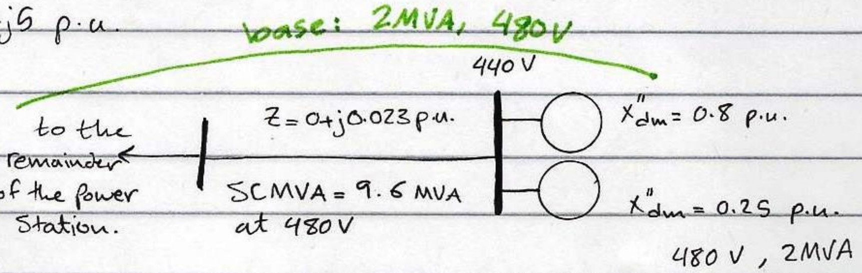
$= 0.8$ p.u.

$I_{Load} = \frac{S^*_{Load}}{V^*_{Load}} = \frac{0.8}{1} = 0.8$ p.u.

$E_g'' = 1\angle 0^\circ + 0.8\angle 0^\circ \times j0.2 = 1 + j0.16$ p.u.

$I_g'' = \frac{1+j0.16}{j0.2} = 0.8 - j5$ p.u.

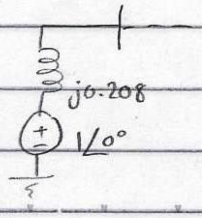
10-6

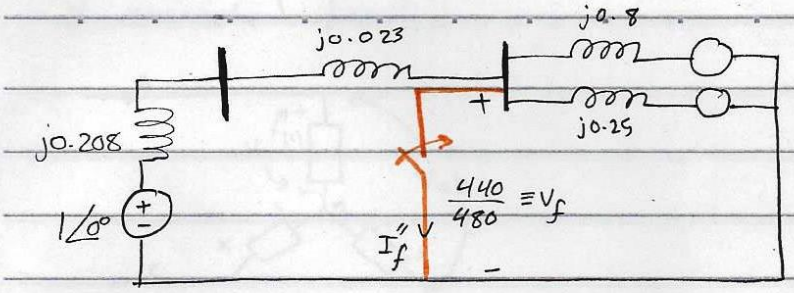


$V_{th} = 480\text{V} \equiv 1\angle 0^\circ$ p.u.

$X_{th} = \frac{1}{SCMVA_{p.u.}} = \frac{1}{\left(\frac{9.6}{2}\right)} = j0.208$ p.u.

base

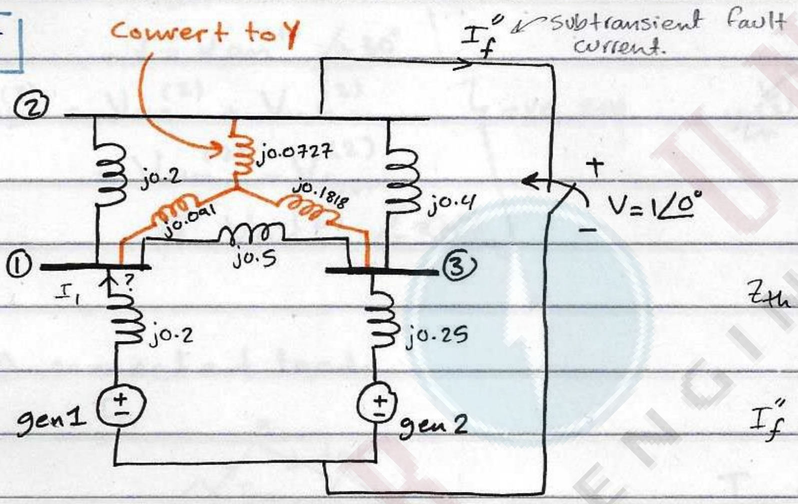




$$I_f'' = \frac{V_f}{(j0.8 // j0.25) // (j0.208 + j0.023)} = -j7.05 \text{ p.u.}$$

$$\equiv -j7.05 \times \frac{2\times10^6}{\sqrt{3}\times 480} \quad \text{A}$$

$\boxed{10.8}$



Use thevinin method (since Load isn't mentioned)...

$Z_{th} = j0.0727 + (j0.1818 + j0.25) // (j0.2 + j0.2)$

$= j0.2465$ p.u.

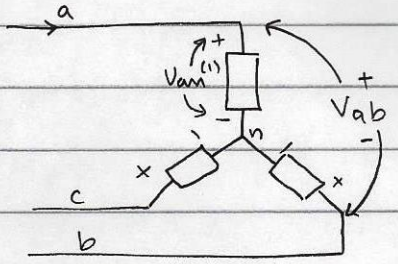
$I_f'' = \frac{V_f}{Z_{th}} = \frac{1\angle 0^\circ}{j0.2405} = -j4.056$ p.u.

$I_1 = -j4.056 \times \frac{j0.4318}{j0.4318 + j0.291} = -j2.423$ p.u.

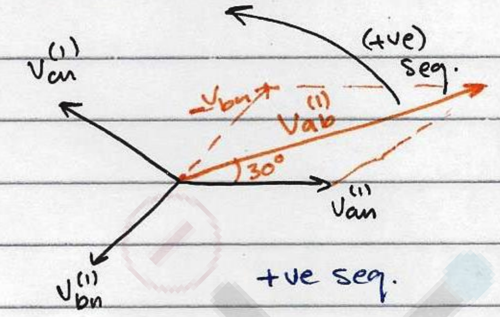
$V_1 = 1\angle 0^\circ - j0.2 I_1 = 0.5154$ p.u.

$I_{12} = \frac{0.5154 - \overset{V_2}{0}}{j0.2}$

* Y-connected load

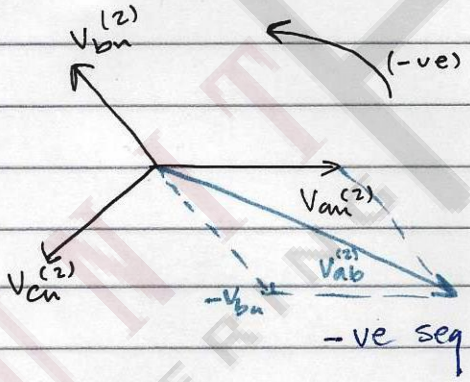


(+ve) sequence.

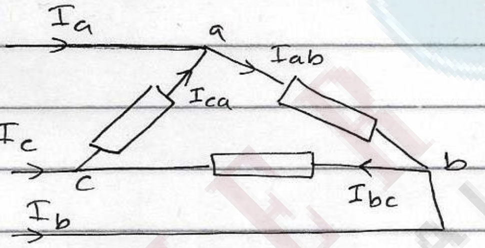


$$V_{ab}^{(1)} = V_{an}^{(1)} + V_{nb}^{(1)}$$
$$= V_{an}^{(1)} - V_{bn}^{(1)}$$
$$= \sqrt{3}\, V_{an}^{(1)} \angle +30^\circ$$
} +ve seq.

$$V_{ab}^{(2)} = V_{an}^{(2)} + V_{nb}^{(2)}$$
$$= V_{an}^{(2)} - V_{bn}^{(2)}$$
$$= \sqrt{3}\, V_{an}^{(2)} \angle -30^\circ$$
} -ve seq.



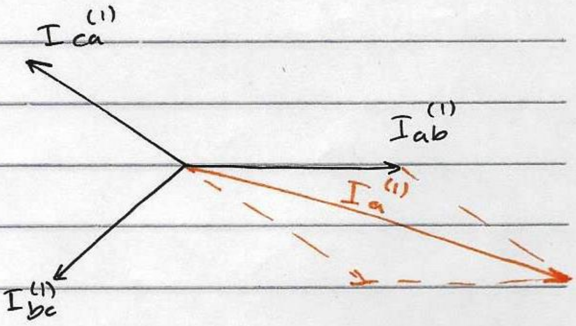
* Δ-connected load:-



$$I_a + I_{ca} = I_{ab}$$
$$\rightarrow I_a = I_{ab} - I_{ca}$$

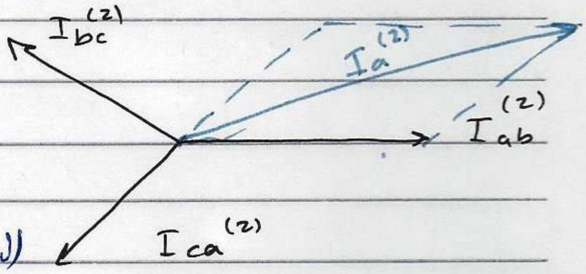
for (+ve) seq.:

$$I_a^{(1)} = I_{ab}^{(1)} - I_{ca}^{(1)}$$
$$I_a^{(1)} = \sqrt{3}\, I_{ab}^{(1)} \angle -30^\circ$$



for (-ve) seq:

$$I_a^{(2)} = \sqrt{3}\, I_{ab}^{(2)} \angle +30^\circ$$



** $S = 3 V_a I_a^*$ (for a balanced sys.)

$= V_a I_a^* + V_b I_b^* + V_c I_c^*$ (in general)

eg:-

$V_a = 100 \angle 0^\circ$

$V_b = 100 \angle -120^\circ$

$V_c = 100 \angle -240^\circ$

$I_a = 10 \angle 60^\circ$

$I_b = 10 \angle -60^\circ$

$I_c = 10 \angle 180^\circ$

$S = 1000\angle -60^\circ + 1000\angle -60^\circ + 1000\angle -60^\circ$

$= 3 \times (1000 \angle -60)$

- 3-phase complex power:

→ for a balanced system:

$$S_{3\phi} = 3V_a I_a^*$$

→ for an unbalanced system:

$$S_{3\phi} = V_a I_a^* + V_b I_b^* + V_c I_c^*$$

in matrix form:

$$S_{3\phi} = \begin{bmatrix} V_a & V_b & V_c \end{bmatrix} \begin{bmatrix} I_a^* \\ I_b^* \\ I_c^* \end{bmatrix} \equiv \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}^{\overset{\text{transpose}}{\boxed{T}}} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}^* \ldots ①$$

remember:

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_a^{(0)} \\ V_a^{(1)} \\ V_a^{(2)} \end{bmatrix}$$

$$\equiv [A][V_{012}]$$

plug in ①

$$S_{3\phi} = \Big(\big[[A][V_{012}] \big]^T \Big) \big[[A][I_{012}] \big]^*$$

$$= \Big([V_{012}]^T [A]^T \Big) [A]^* [I_{012}]^* \ldots ②$$

$$\big[[A][B] \big]^T \equiv [B]^T [A]^T$$

$$\rightarrow [A]^T = [A]$$

$$\rightarrow [A]^* = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \longrightarrow A^T A^* = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}}_{3A^{-1}}$$

$$= \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} \leftarrow 3AA^{-1} \equiv 3I \leftarrow \text{Identity matrix}$$

plug in ②:

$$S_{3\phi} = 3 \left[V_a^{(0)} \; V_a^{(1)} \; V_a^{(2)} \right] \begin{bmatrix} I_a^{*(0)} \\ I_a^{*(1)} \\ I_a^{*(2)} \end{bmatrix}$$

$$= \underbrace{3 V_a^{(0)} I_a^{*(0)}} + 3 V_a^{(1)} I_a^{*(1)} + 3 V_a^{(2)} I_a^{*(2)} \qquad \overset{\text{unit}}{[VA]} \quad \ldots ③$$

each one of them is balanced, but the total system is unbalanced.

this term equals (0) when:-

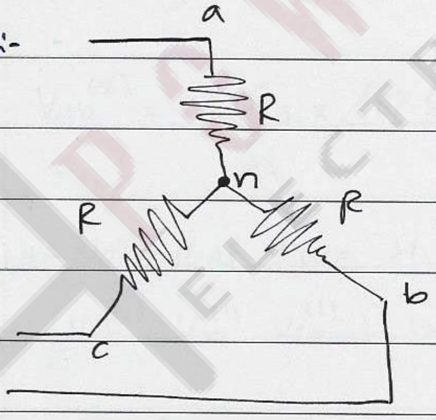
1. it's a delta connected load.
2. it's a Y-connected load with an isolated neutral (not connected to earth).

base: $3 V_a I_a$

$\therefore$ in ③; p.u $S_{3\phi}$:

$$S_{3\phi} = V_a^{(0)} I_a^{*(0)} + V_a^{(1)} I_a^{*(1)} + V_a^{(2)} I_a^{*(2)} \qquad [p.u.]$$

ex:-



given: $|V_{ab}| = 1860V$ (+ve) sequence

$|V_{bc}| = 2780V$

$|V_{ca}| = 2300V$

find: $V_{an}^{(0)}$, $V_{an}^{(1)}$, $V_{an}^{(2)}$

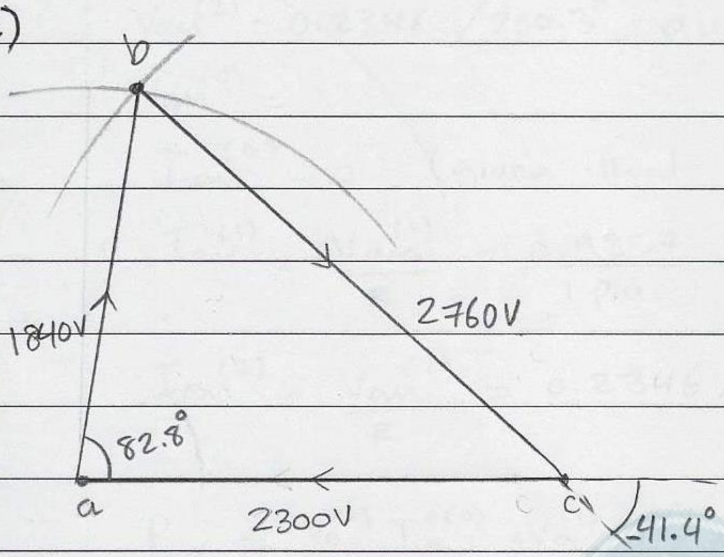
Solution: $V_{ab}^{(0)} = 0$ (always) since $(V_{ab} + V_{bc} + V_{ca})$ forms a closed loop.

$$V_{ab}^{(1)} = \frac{1}{3}(V_{ab} + a V_{bc} + a^2 V_{ca})$$

I only have $|V|$, what to do?

1) first take a reference, let's say Vca $= 2300\angle 180°$.

2)



$V_{ab} = 1840 \angle 82.8°$ V

$V_{bc} = 2760 \angle -41.4°$ V

$V_{ca} = 2300 \angle 180°$ V

$\boxed{400V : 1cm}$

→ choose base: 2300V, 500kVA

→ given that the ratings of the 3-ph load are: 2300V, 500kVA

$\therefore\ R = 1.0\ p.u. \equiv \frac{(2300)^2}{500 \times 10^3} = 10.58\ \Omega$

$$V_{ab}^{(1)} = \frac{1}{3}\left(0.8\angle 82.8° + 1.2\angle -41.4° + 120° + 1\angle 180° + 240°\right) = 0.9857\angle 73.6°$$

$$V_{ab}^{(2)} = \frac{1}{3}\left(0.8\angle 82.8° + 1.2\angle -41.4° + 240° + 1\angle 180° + 120\right) = 0.2346\angle 220.3°$$

** Note that since the system is (+ve) seq., the (+ve) seq component ($V_{ab}^{(1)}, V_{bc}^{(1)}, V_{ca}^{(1)}$) is stronger.

$$\left.\begin{aligned} V_{ab}^{(1)} &= \sqrt{3}\, V_{an}^{(1)} \angle +30° \\ \therefore\ V_{an}^{(1)} &= \frac{V_{ab}^{(1)}}{\sqrt{3}} \angle -30° \end{aligned}\right\} \text{in Volts.}$$

$V_{an}^{(1)} = V_{ab}^{(1)} \angle -30°$ in p.u.

$\hookrightarrow$ base: 2300

$\hookrightarrow$ base: $\frac{2300}{\sqrt{3}}$

$\therefore V_{an}^{(1)} = 0.9857 \underline{/43.6^\circ}$ p.u. $\left(\text{base: } \frac{2300}{\sqrt{3}}\right)$

$V_{an}^{(2)} = 0.2346 \underline{/250.3^\circ}$ p.u. $=$

$V_{an}^{(0)} = 0$

$I_{an}^{(0)} = 0$ (given that neutral is isolated).

$I_{an}^{(1)} = \frac{V_{an}^{(1)}}{R} = \frac{0.9857}{1\,\text{p.u.}} \angle 43.6^\circ$ p.u.

$I_{an}^{(2)} = \frac{V_{an}^{(2)}}{R} = 0.2346 \underline{/250.3^\circ}$ p.u.

$P_{3\phi} = V_a^{(0)} I_a^{*(0)} + V_a^{(1)} I_a^{*(1)} + V_a^{(2)} I_a^{*(2)}$ [p.u.]

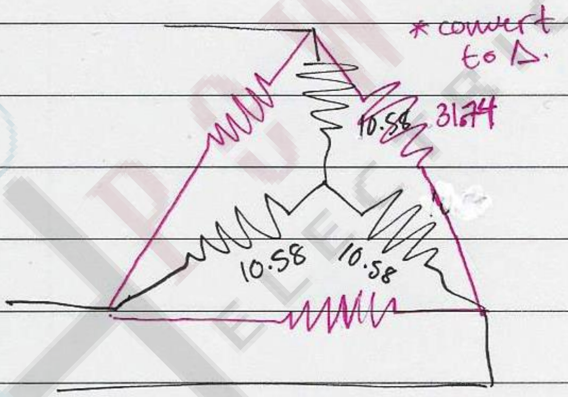
↳ since it's a resistive load

$= 0 + (0.9857)^2 + (0.2346)^2$

$= 1.02664$ p.u.

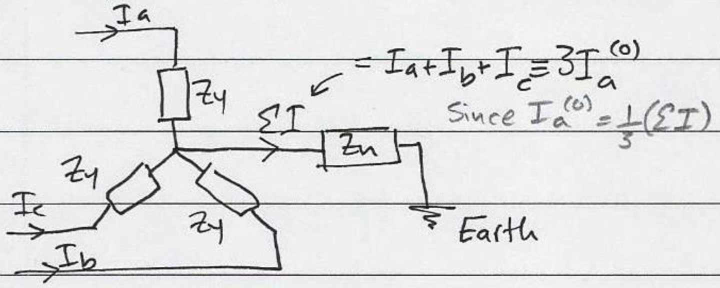
$\equiv 1.02664 \times 500\,\text{KVA} = 513.32\,\text{KW}$

we can solve this question using CKT I method!



$P_{3\phi} = \frac{1840^2}{31.74} + \frac{2760^2}{31.74} + \frac{2300^2}{31.74} = 513.32\,\text{KW}$

- now, consider a non-isolated neutral:



$= I_a + I_b + I_c \equiv 3I_a^{(0)}$

Since $I_a^{(0)} = \frac{1}{3}(\Sigma I)$

* the current flowing through (n) is a zero sequence current.

$$\begin{array}{rcccccc}
 & & & & 0 & & 0 \\
\Sigma I = & I_a^{(0)} & + & I_a^{(1)} & + & I_a^{(2)} & \\
 & + I_b^{(0)} & + & I_b^{(1)} & + & I_b^{(2)} & \\
 & + I_c^{(0)} & + & I_c^{(1)} & + & I_c^{(2)} & \\
\hline
 & = 3I_a^{(0)} & + & 0 & + & 0 &
\end{array}$$

* from now on $V_a \equiv V_{aE}$

"now E is my reference instead of n, where V_{nE} might not equa zero".

$$V_{nE} = 3I_a^{(0)} Z_n$$

$$\therefore V_{aE} = I_a Z_Y + (3I_a^{(0)} + Z_n)$$
$$V_{bE} = I_b Z_Y + (3I_a^{(0)} + Z_n)$$
$$V_{cE} = I_c Z_Y + (3I_a^{(0)} + Z_n)$$

even if it's not mentioned, it's there by default ($V_c \equiv V_{cE}$)

Subject lecture #33 Date 3/5/2014 No.

- for Y-connection:

$$[A]^{-1}\times \underbrace{\begin{bmatrix}V_a\\V_b\\V_c\end{bmatrix}}_{[A][V_{012}]} = Z_y \underbrace{\begin{bmatrix}I_a\\I_b\\I_c\end{bmatrix}}_{[A][I_{012}]} + 3I_a^{(0)} Z_n \begin{bmatrix}1\\1\\1\end{bmatrix} \times [A]^{-1}$$

$$\overset{I}{\cancel{[A]^{-1}[A]}}\begin{bmatrix}V_a^{(0)}\\V_a^{(1)}\\V_a^{(2)}\end{bmatrix} = Z_y \overset{I}{\cancel{[A^{-1}][A]}}\begin{bmatrix}I_a^{(0)}\\I_a^{(1)}\\I_a^{(2)}\end{bmatrix} + \cancel{3}I_a^{(0)} Z_n \times \frac{1}{\cancel{3}} \underbrace{\begin{bmatrix}1&1&1\\1&a&a^2\\1&a^2&a\end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix}}_{\begin{bmatrix}3\\0\\0\end{bmatrix}}$$

$$\begin{bmatrix}V_a^{(0)}\\V_a^{(1)}\\V_a^{(2)}\end{bmatrix} = Z_y \begin{bmatrix}I_a^{(0)}\\I_a^{(1)}\\I_a^{(2)}\end{bmatrix} + 3I_a^{(0)} Z_n \begin{bmatrix}1\\0\\0\end{bmatrix}$$

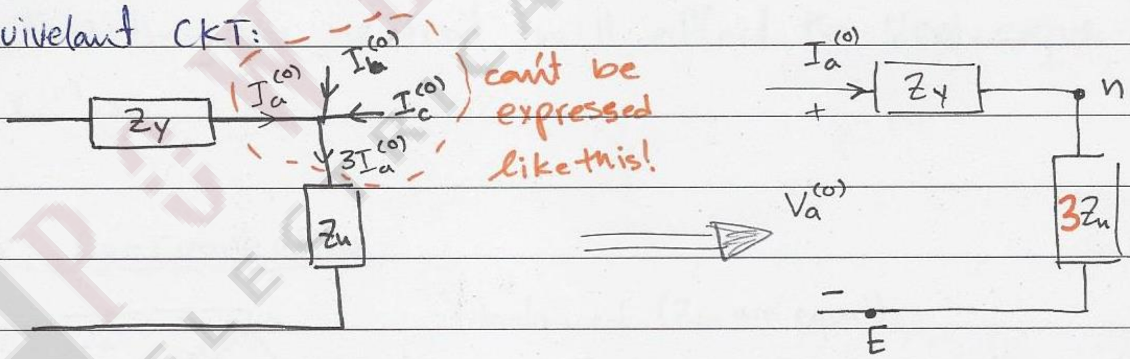
$$V_a^{(0)} = I_a^{(0)}\,[Z_y + 3Z_n]$$

$$V_a^{(1)} = I_a^{(1)}\,Z_y$$

$$V_a^{(2)} = I_a^{(2)}\,Z_y$$

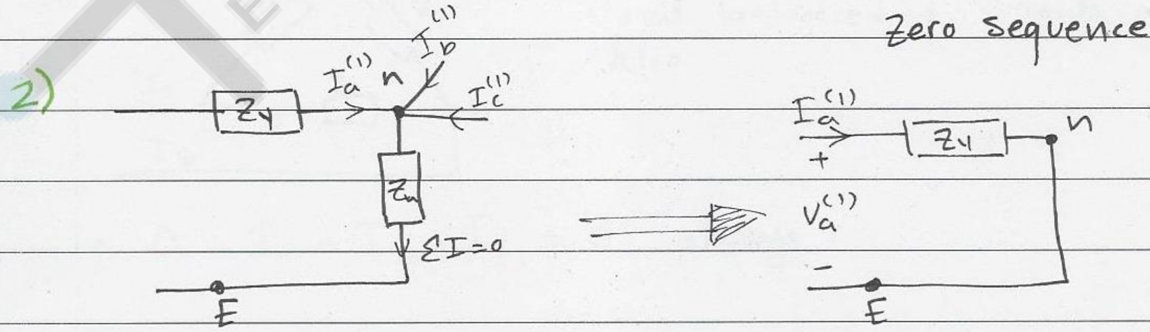
equivelant CKT:

1)



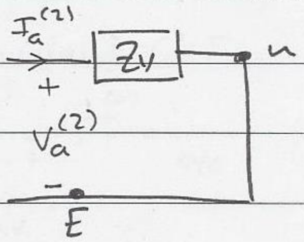
Zero sequence Network.

2)



Positive sequence network.

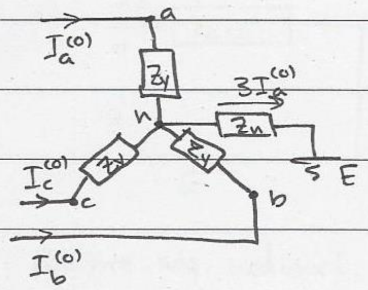
3)

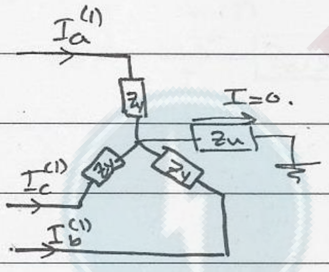


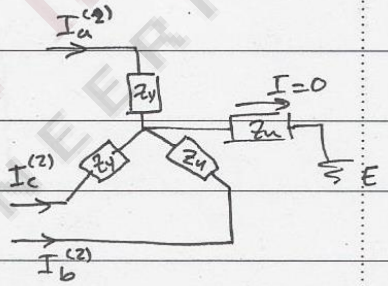
negative Sequence network.

* the same is applied for phases B & C.

3-phase equivelant:







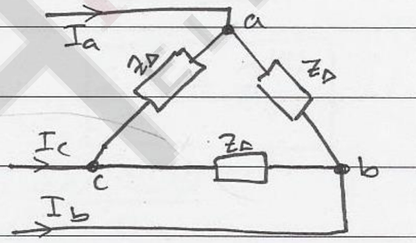
$V_a^{(0)} = I_a^{(0)} [Z_y + 3Z_n]$ $V_a^{(1)} = I_a^{(1)} Z_y$ $V_a^{(2)} = I_a^{(2)} Z_y$

zero-seq. +ve-seq. -ve-seq.

** Isolating the neutral will affect the zero-sequence only

$I_a^{(0)} = 0$.

- for Δ-connection:

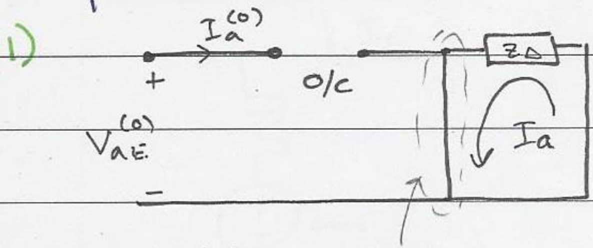


balanced (Z_Δs are equal).

V aren't balanced → ∴ currents aren't balanced also

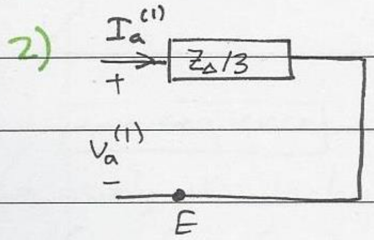
in Δ, $I_a + I_b + I_c = 0$ always

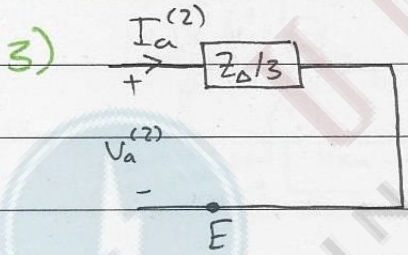
equivelant CKT:

1) 

zero-seq. network.

"بعض الكتب بتحطها!
لو كان في mutual inductance, transformer, ...
بيقدر يمشي فيه بس ما بيمشي بالعاديّة"

2) 

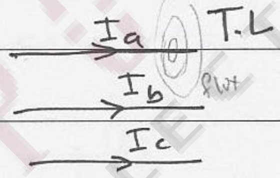
3) 

Positive-seq. network.

negative-seq. network.

$V_a^{(1)}, V_b^{(1)}, V_c^{(1)}$ → are star connected
$V_a^{(2)}, V_b^{(2)}, V_c^{(2)}$ ↗

- for transmission line:



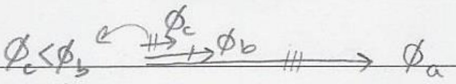
$$L_a^{(i)} = \frac{\overset{\text{flux Linkage}}{\Psi_a^{(i)}}}{I_a^{(i)}}$$

$$\boxed{Z_{T.L}^{(1)} = Z_{T.L}^{(2)}}$$

$$\boxed{\begin{gathered} X_1 = X_2 \\ X_0 = (\underbrace{2 - 3.5}_{\text{range.}}) X_1 \end{gathered}}$$

$\underline{\Psi_a = \Sigma \phi}$:

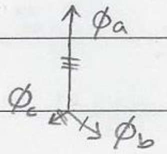
for zero seq. :



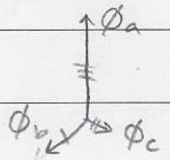
$\phi_c < \phi_b$

ابعد عن a

+ve seq.

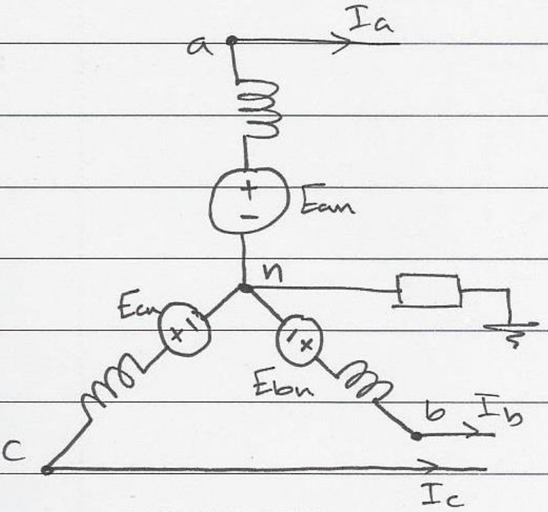


-ve seq.



Ψ is higher (they add up)
∴ l is higher

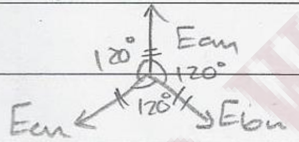
• for generator :



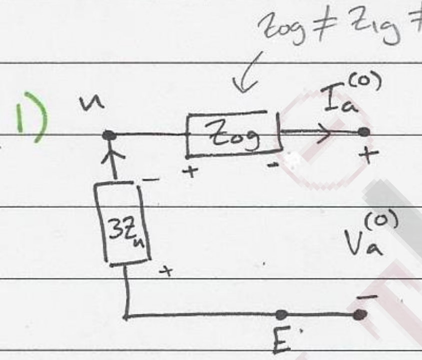
+ve sequence

usualy it's balanced ($E_{an} = E_{bn} = E_{cn}$)

* a perfectly designed generator generates only +ve seq. (V)s with $\Delta\theta = 120°$ between adjacent phases.



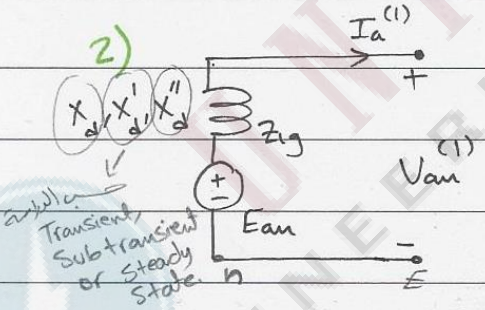
1)



$$\boxed{V_a^{(0)} = -(Z_{0g}+3Z_n)I_a^{(0)}}$$

zero-seq. network.

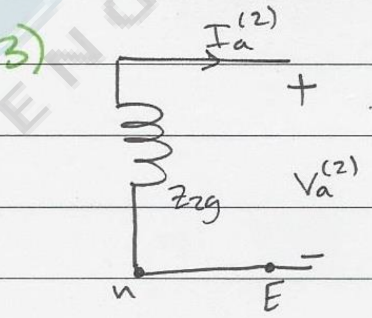
2)



$$\boxed{V_a^{(1)} = E_{an} - I_a^{(1)} Z_{1g}}$$

+ve-seq. network

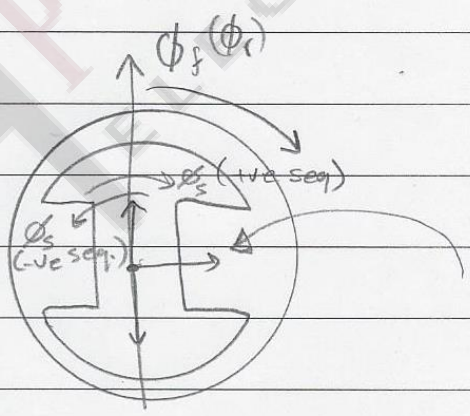
3)

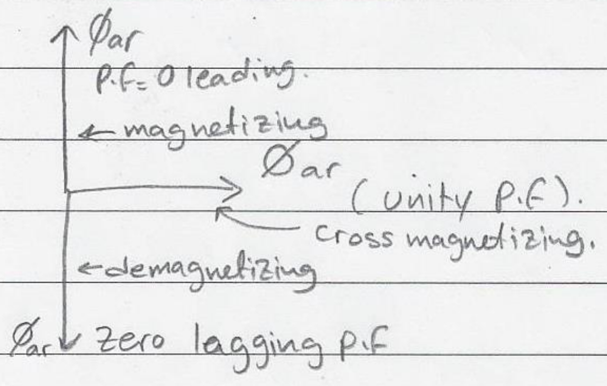


$$\boxed{V_a^{(2)} = -I_a^{(2)} Z_{2g}}$$

-ve-seq. network.

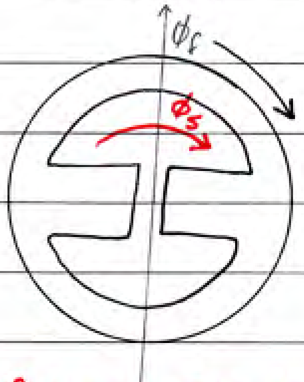
* the same CKTs apply for phases B & C.



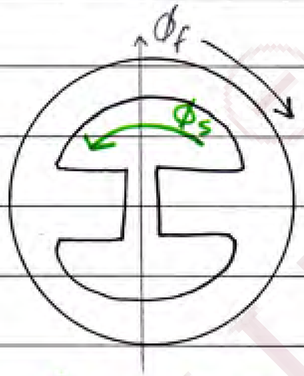


lecture #34

Subject 6/5/2014 Date No.



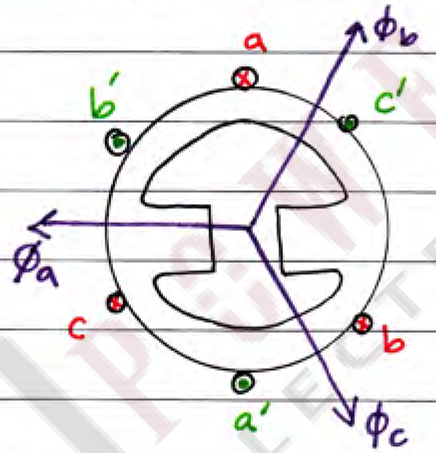
for +ve seq. current



for -ve seq. current
(ϕ_s will flip).

- for +ve seq: ϕ_s is in the same direction of rotor's rotation ("بشرفش الـ rotor")
- for -ve seq: ϕ_s is in the opposite direction of rotor's rotation. ∴ it's relative velocity is doubled. which will induce current in the field winding, damper winding, & rotor body. thus, rotor will heat up.

- Armature reaction is weak for (-ve) seq. current, thus $Z_{2g} \approx X''_d \approx X_L$ ← leakage reactance



$|\phi_a| = |\phi_b| = |\phi_c|$ since $I_a^{(0)} = I_b^{(0)} = I_c^{(0)}$

$\therefore \phi_a + \phi_b + \phi_c = 0$

thus, zero-seq. current doesn't introduce any armature reaction.

*شرط (spacing between adjacent phases = 120°)

for zero-seq. current

$X_{2og} \cong X''_d$ (direct axis subtransient reactance).

$X_{1og} \rightarrow X''_d \rightarrow X'_d \rightarrow X_d$

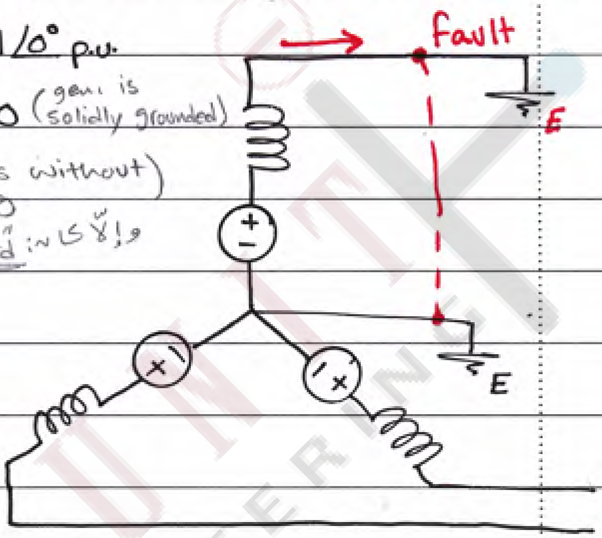
$X_{og} = X_\ell$ (exactly). (no armature reaction).

*if we assumed no damper windings X_{2og} will increase.

** induced voltages E_{an}, E_{bn}, E_{cn} appears only in one sequence. (+ve or -ve)

↳ (حسب اصطلاحي في الـ System الأصلي)

ex: 3-phase generator, without damper windings, Rated 20MVA,
13.8KV, $X''_d = 0.25$ p.u., $E_{an} = 1\angle 0°$ p.u.
$X_{0g} = 0.1$ p.u., $Z_n = 0$ (gen. is solidly grounded)
$X_{2g} = 0.35$ p.u. (since it's without damping
$X_{2og} \approx \underline{X''_d}$ وإلا بكون)



initialy unloaded.

after fault:-

$V_{aE} = 0$

$V_{bE} = 1.013 \angle -102.25°$

$V_{cE} = 1.013 \angle 102.25°$

$I_a = ?$

Solution:

$I_b = 0, \quad I_c = 0$

$$\begin{bmatrix} V_a^{(0)} \\ V_a^{(1)} \\ V_a^{(2)} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} \overset{0}{\cancel{V_a}} \\ V_b \\ V_c \end{bmatrix} \begin{matrix} \\ \leftarrow 1.013\angle -102.25° \\ \leftarrow 1.013\angle 102.25° \end{matrix}$$

$$= \begin{bmatrix} -0.143 \\ 0.643 \\ -0.5 \end{bmatrix} \leftarrow V_a^{(0)} + V_a^{(1)} + V_a^{(2)} = 0 \equiv V_a \quad \#$$

$V_a^{(0)} = -I_a^{(0)} * j0.1$

$\hookrightarrow I_a^{(0)} = \boxed{-j1.43 \text{ p.u.}}$

$V_a^{(1)} = E_{an} - I_a^{(1)} Z_1$

$0.643 = 1\angle 0° - I_a^{(1)} * j0.25$

$I_a^{(1)} = \boxed{-j1.43 \text{ p.u.}}$

$V_a^{(2)} = -I_a^{(2)} Z_2$

$I_a^{(2)} = \boxed{-j1.43 \text{ p.u.}}$

$* \; I_a^{(0)} = I_a^{(1)} = I_a^{(2)}$

this always hold in a single phase to earth fault (we'll prove it later).

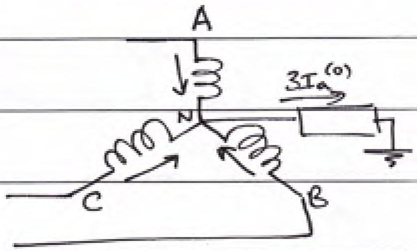
$I_a = I_a^{(0)} + I_a^{(1)} + I_a^{(2)} = -j1.43\times3 = -j4.29$ p.u.

base current $= \dfrac{20\times10^6}{\sqrt{3}\times13.8\times10^3}$ A

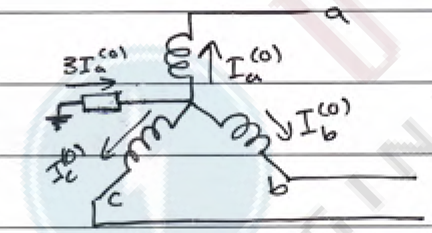
$V_{ab} = V_{ac} - V_{bc} = 0 - 1.013\angle{-102.25^\circ} = 1.013\angle{77.75^\circ}$ p.u.

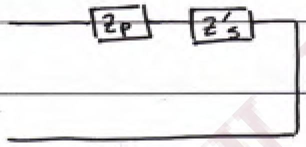
Sequence CKTs of transformer:-

Y-Y transformer:-

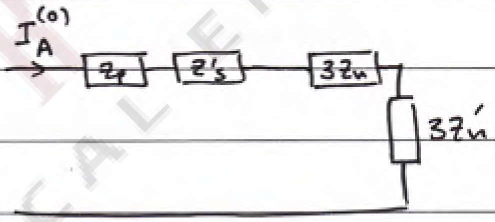


Primary.





equivelent CKT for +ve & -ve seq.



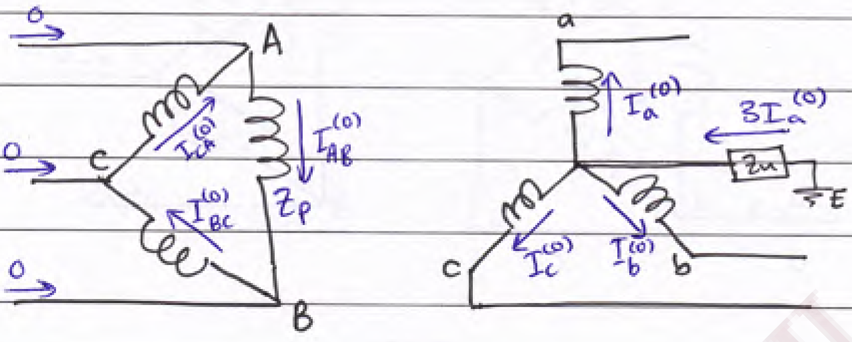
equ. ckT for zero. sequence.

- if neutral (N) is opened : $I_A^{(0)}=0 \longrightarrow \therefore I_a^{(0)}=0$.
- if ~ (n) ~ ~ : $I_a^{(0)}=0 \longrightarrow \therefore I_A^{(0)}=0$.

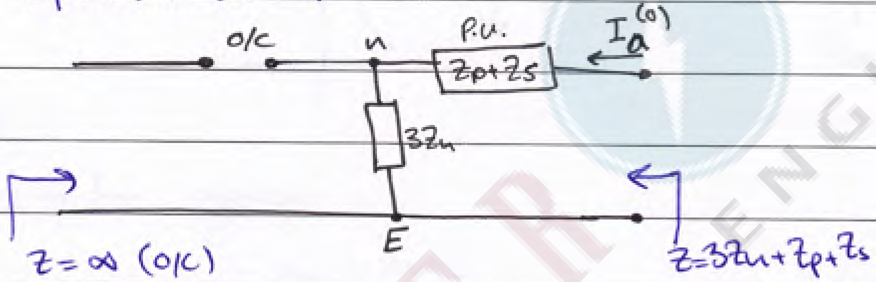
bject lecture #35 Date 8/5/2014 No.

ΔY transformer sequence ckTs:

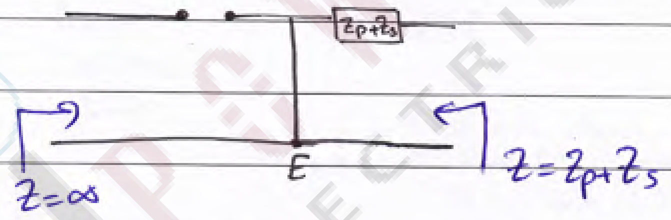
* بشكل عام "ال ±ve seq. مثل ما تعودنا قبل ما في خلاف عليهم ، الخلاف عال zero-seq. "



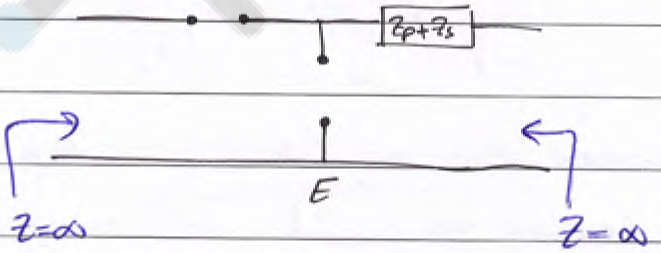
* zero-seq equivelant ckT:



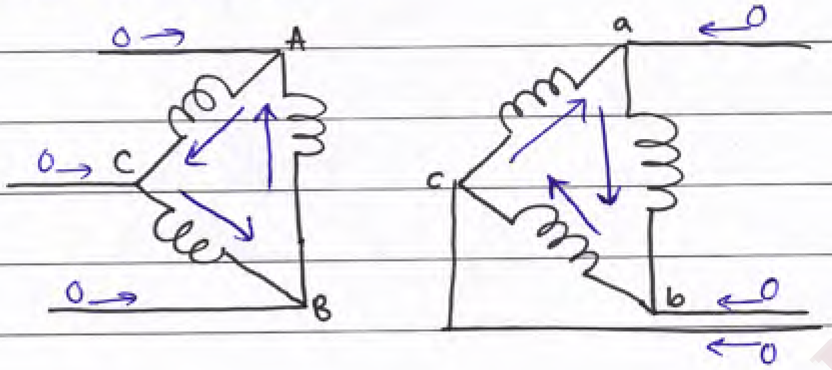
→ if (n) was short-circuited:

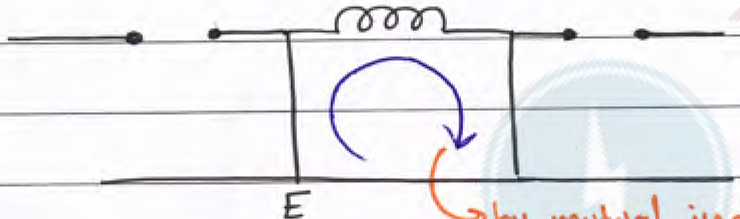


→ if (n) was isolated (o/c):



Δ-Δ transformer:-





equ. CKT
for zero seq.

↳ by mutual inductance.

• Vector group

Dy1

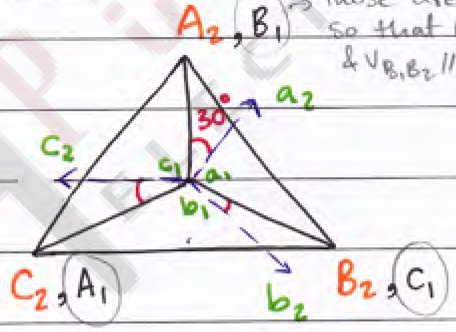
D → high Voltage side

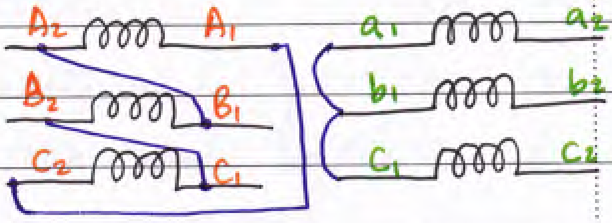
y → Low Voltage Side

1 → clock angle $1 \equiv \angle 30°$ → this angle is between the equivelant Y for H.V side & the L.V side.
(considering +ve sequence)

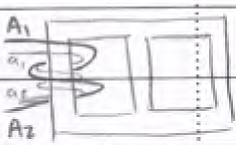
لأنه التسمية مأخوذة
بالـ balanced ولا +ve seq.

A_2, B_1 → those are chosen so that ($V_{A_1A_2} \parallel V_{a_1a_2}$ & $V_{B_1B_2} \parallel V_{b_1b_2}$, ...)





- $(a_1 a_2)$ & $(A_1 A_2)$ are both wounded on the same limb
thus, $\overbrace{(V_{a_1a_2} \text{ \& } V_{A_1A_2})}^{\text{induced voltage}}$ are in-phase.



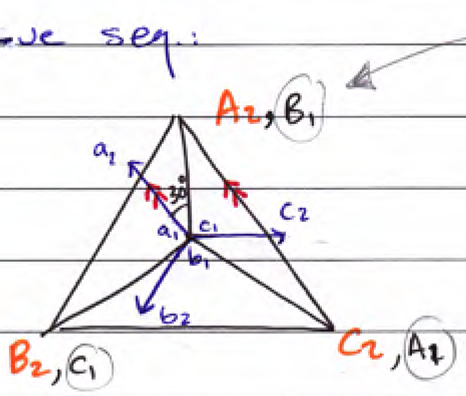
" نلف الـ L.V جوّا لأنه أسهل
لي أعزلها عن الـ E وحتى
أقلل الـ leakage flux "

$$\underbrace{V_{AN}^{(1)} = V_{an}^{(1)}}_{\text{magnitude is the same in p.u}} \angle +30° \text{ p.u.}$$ (for +ve seq.)

but they have different bases.

for -ve seq:

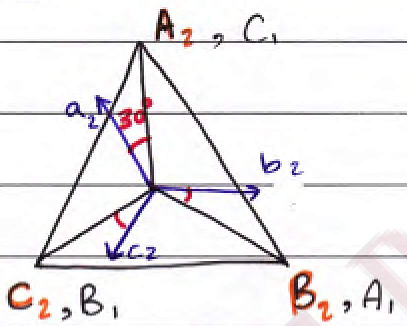
"التركيبة نفسها بقدرش أغيرها"

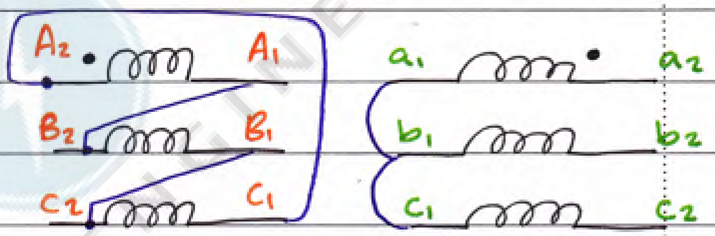


$V_{AN}^{(2)} = V_{an}^{(2)} \angle -30°$ p.u.

Dy11

"لـ يكونوا in-parallel لازم كلهم نفس النوع ، بصيرش Dy11 // Dy1"

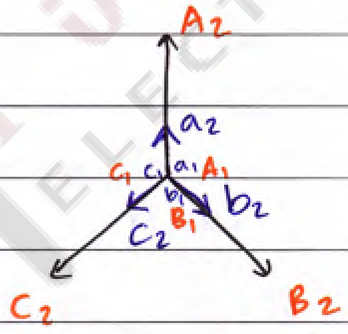


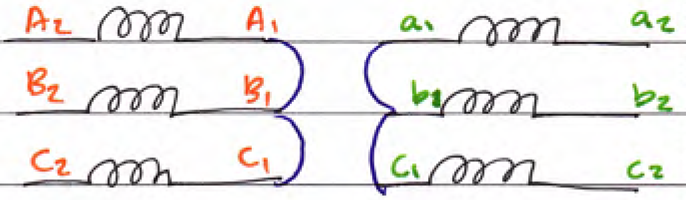


$V_{AN}^{(1)} = V_{an}^{(1)} \angle -30°$ p.u. (for +ve)

$V_{AN}^{(2)} = V_{an}^{(2)} \angle +30°$ p.u. (for -ve)

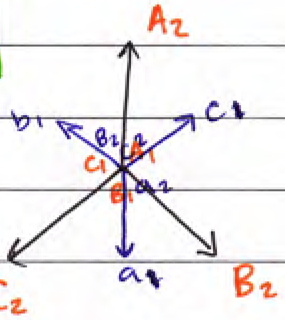
Yy0

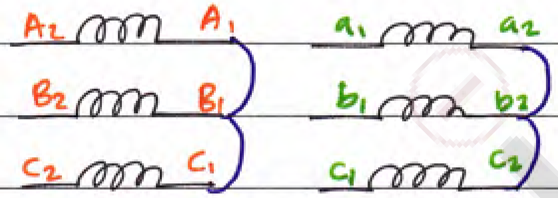




$V_{AN}^{(1)} = V_{an}^{(1)}$ p.u.

Yy6





$$V_{AN}^{(1)} = V_{an}^{(1)} \angle 180^\circ \text{ p.u.}$$

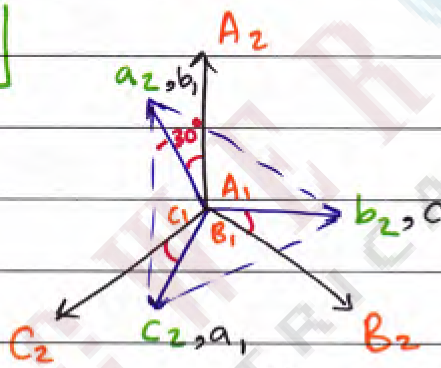
$$V_{AN}^{(2)} = V_{an}^{(2)} \angle 180^\circ \text{ p.u.}$$

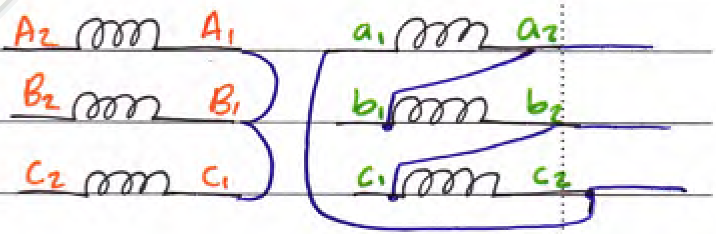
Yy9 → can't be made.

Dy5 → the same as Dy11 (just take the output from a₁, b₁, c₁ in secondary instead of a₂, b₂, c₂).

the same for Dy7 & Dy1

Yd 11

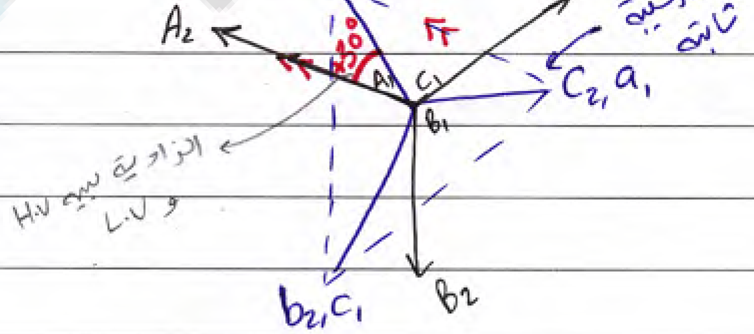




$$V_{AN}^{(1)} = V_{an}^{(1)} \angle -30^\circ \text{ p.u.}$$

$$V_{AN}^{(2)} = V_{an}^{(2)} \angle +30^\circ \text{ p.u.}$$

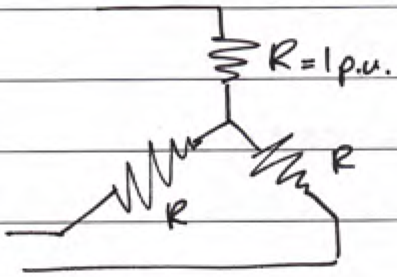
for -ve seq:



* الاشارات مطلوب
بالامتحان!
مع ال Connection
diagram.

Subject lecture #36 Date 11/5/2014 No.

Previous ex:



$V_{ab} = 0.8 \angle 82.8°$ p.u.

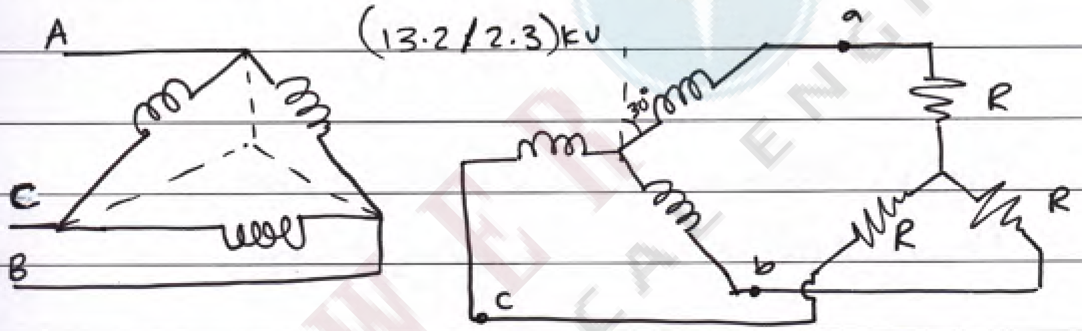
$V_{bc} = 1.2 \angle -41.4°$ p.u.

$V_{ca} = 1 \angle 180°$ p.u.

$$I_{an}^{(1)} = \frac{V_{an}^{(1)}}{1 p.u.} = 0.9856 \angle 43.6°$$

$$\frac{V_{an}^{(2)}}{1 p.u.} = I_{an}^{(2)} = 0.2346 \angle 250.3°$$

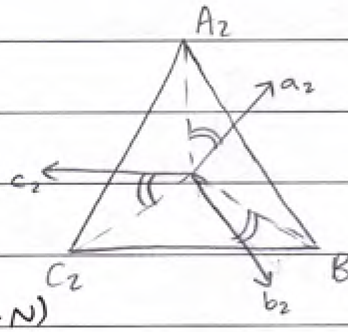
if we add a Dy1 transformer



Dy1 : $V_{AN}^{(2)} = V_{an}^{(2)} \angle -30°$ p.u.

$V_{AN}^{(1)} = V_{an}^{(1)} \angle +30°$ p.u.

↳ reference $\left(\frac{2300}{\sqrt{3}}\right)$



$V_{AN}^{(1)} = 0.9856 \angle 43.6° + 30°$ p.u.

↳ it's base is $\frac{13200}{\sqrt{3}}$ (base L-N)

$V_{AN}^{(2)} = 0.2346 \angle 250.3 - 30°$ (Base L-N)

↳ base = $\frac{13200}{\sqrt{3}}$

$V_{AN}^{(0)} = 0$

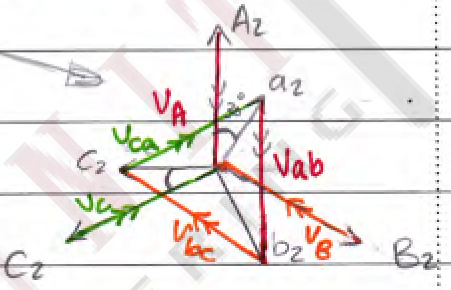
$V_{AN} = V_{AN}^{(0)} + V_{AN}^{(1)} + V_{AN}^{(2)} \equiv V_{AE}$ since in this ex. $V_{NE}=0$.

$= \boxed{0.8 \angle 82.8^\circ} = V_{ab}$! (p.u)

$V_B = V_B^{(1)} + V_B^{(2)} + V_B^{(0)}$

$= \left(\overset{V_A^{(1)}}{0.9857 \angle 73.6}\right) a^2 + \left(\overset{V_A^{(2)}}{0.2346 \angle 220.3}\right) a + 0$

$= \boxed{1.2 \angle -41.4^\circ} = V_{bc}$ (p.u).



$V_{AB} = V_{AE} - V_{BE}$

$= 0.8 \angle 82.8 - 1.2 \angle -41.4$

$= 1.78 \angle 116.8 \quad \left(L\text{-}N,\ \frac{13200}{\sqrt{3}}\right)$

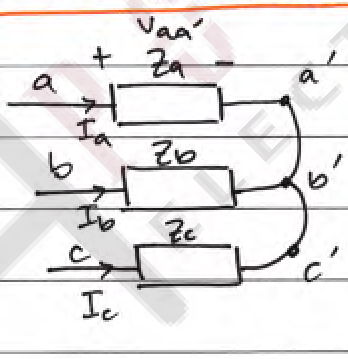
also

$\equiv \frac{1.78}{\sqrt{3}} \angle 116.8^\circ \quad (L\text{-}L,\ 13200)$

$V_{BC} = V_{BE} - V_{CE} = \ldots = 2.09 \angle -27.7^\circ \quad (L\text{-}N)$

$V_{CA} = V_{CE} - V_{AE} = \ldots = 1.356 \angle 215.8^\circ \quad (L\text{-}N)$

- for an unbalanced load $(Z_a \neq Z_b \neq Z_c)$ (general case).



$$\left(\begin{bmatrix} V_{aa'} \\ V_{bb'} \\ V_{cc'} \end{bmatrix} = \begin{bmatrix} Z_a & 0 & 0 \\ 0 & Z_b & 0 \\ 0 & 0 & Z_c \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \right) \quad \text{pre-multiply by } * [A]^{-1}$$

$$\overset{\text{unity matrix}}{\cancel{[A]^{-1}[A]}} \begin{bmatrix} V_{aa'}^{(0)} \\ V_{aa'}^{(1)} \\ V_{aa'}^{(2)} \end{bmatrix} = [A]^{-1} \begin{bmatrix} Z_a & 0 & 0 \\ 0 & Z_b & 0 \\ 0 & 0 & Z_c \end{bmatrix} [A] \begin{bmatrix} I_a^{(0)} \\ I_a^{(1)} \\ I_a^{(2)} \end{bmatrix}$$

$$\begin{bmatrix} V_{aa'}^{(0)} \\ V_{aa'}^{(1)} \\ V_{aa'}^{(2)} \end{bmatrix} = A^{-1} \begin{bmatrix} Z_a & 0 & 0 \\ 0 & Z_b & 0 \\ 0 & 0 & Z_c \end{bmatrix} [A] \begin{bmatrix} I_a^{(0)} \\ I_a^{(1)} \\ I_a^{(2)} \end{bmatrix}$$

$$V_{aa'}^{(0)} = \frac{1}{3} I_a^{(0)} (Z_a + Z_b + Z_c) + \frac{1}{3} I_a^{(1)} (Z_a + a^2 Z_b + a Z_c) + \frac{1}{3} I_a^{(2)} (Z_a + a Z_b + a^2 Z_c)$$

$$V_{aa'}^{(1)} = \frac{1}{3} I_a^{(0)} (Z_a + a Z_b + a^2 Z_c) + \frac{1}{3} I_a^{(1)} (Z_a + Z_b + Z_c) + \frac{1}{3} I_a^{(2)} (Z_a + a^2 Z_b + a Z_c)$$

$$V_{aa'}^{(2)} = \frac{1}{3} I_a^{(0)} (Z_a + a^2 Z_b + a Z_c) + \frac{1}{3} I_a^{(1)} (Z_a + a Z_b + a^2 Z_c) + \frac{1}{3} I_a^{(2)} (Z_a + Z_b + Z_c).$$

for a balanced load: $Z_a = Z_b = Z_c = Z_y$. $\rightarrow$ $V_{aa'}^{(0)} = I_a^{(0)} Z_a$, $V_{aa'}^{(1)} = I_a^{(1)} Z_a$, $V_{aa'}^{(2)} = I_a^{(2)} Z_a$

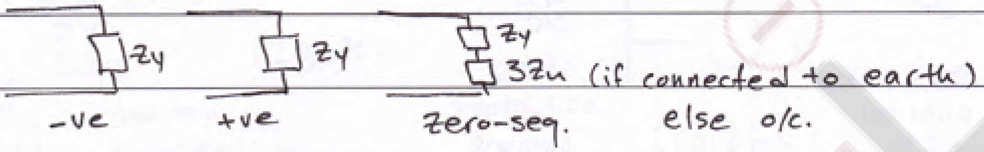
→ in this case each voltag (+ve, -ve, zero) creates 3 seq. current (+ve, -ve, zero)! complicated!

∴ if $Z_a \approx Z_b \approx Z_c$ I neglect the difference & consider them equal.

Summary:

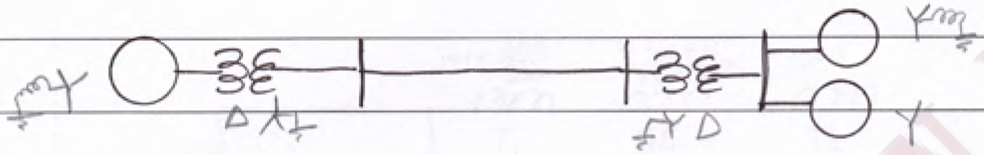
- for generater: x_{1g} : x''_d or x'_d or x_d
  $x_{2g} \approx x''_d$ (if no damper winding exists → $x_{2og} > x''_d$)
  x_{og} is different.
- for T.L: $Z_1 = Z_2$
  Z_0 differs due to flux linkage difference $= (2 \sim 3.5) X_1$

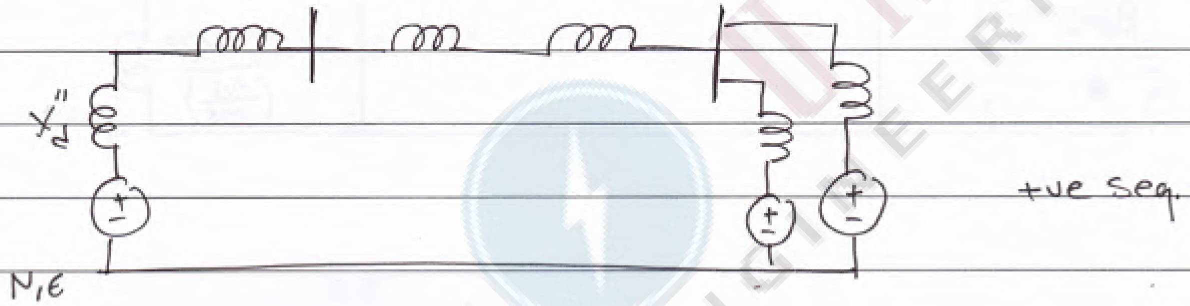
- for load : assume $Z_a = Z_b = Z_c = Z_y$

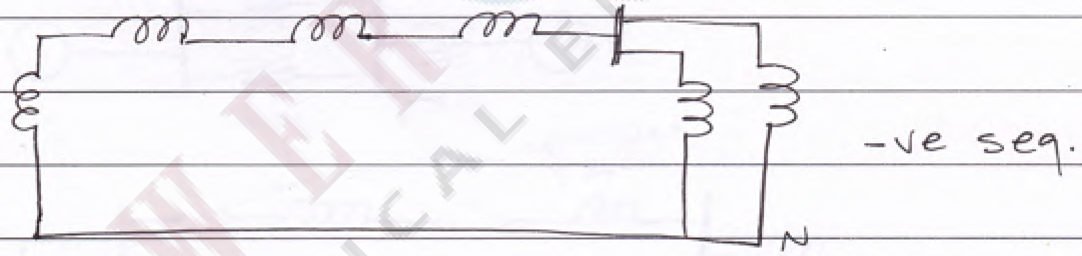


(if connected to earth) else o/c.

ex:

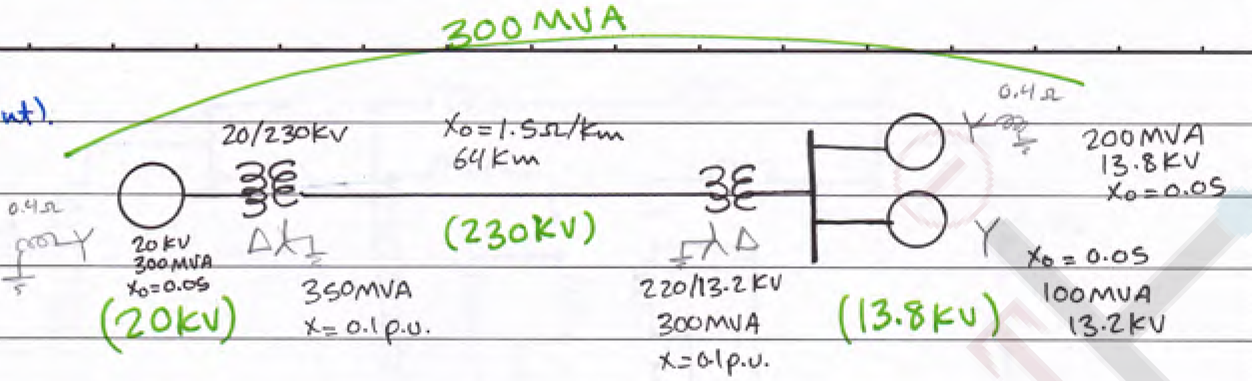


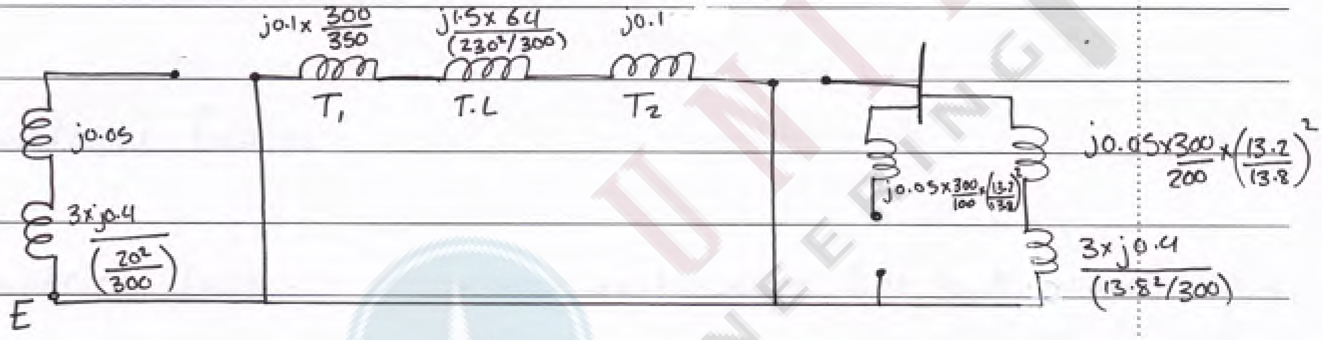




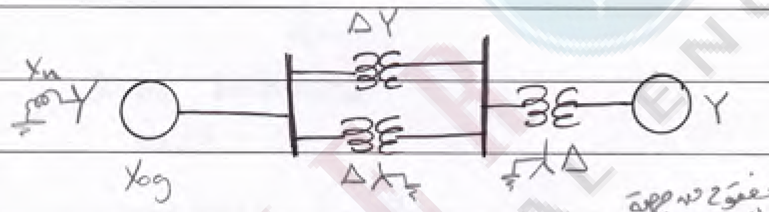
Subject lecture #37 Date 13/5/2014 No.

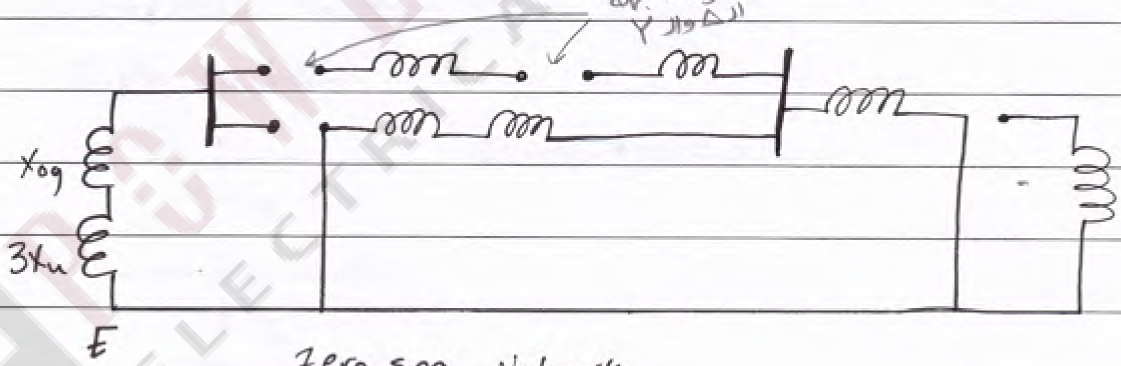
ex: (cont).





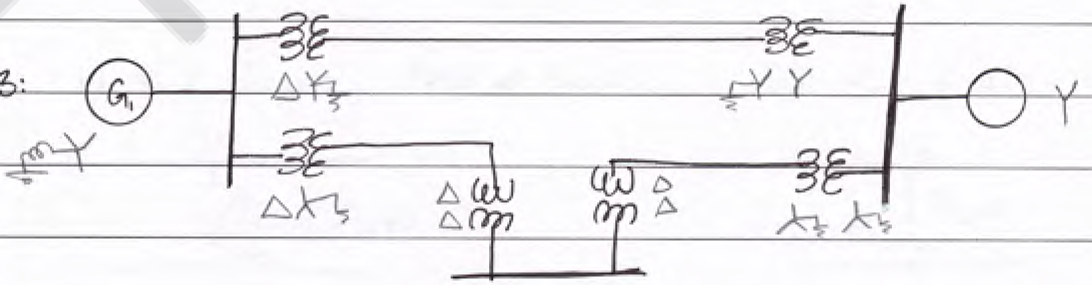
ex 2:

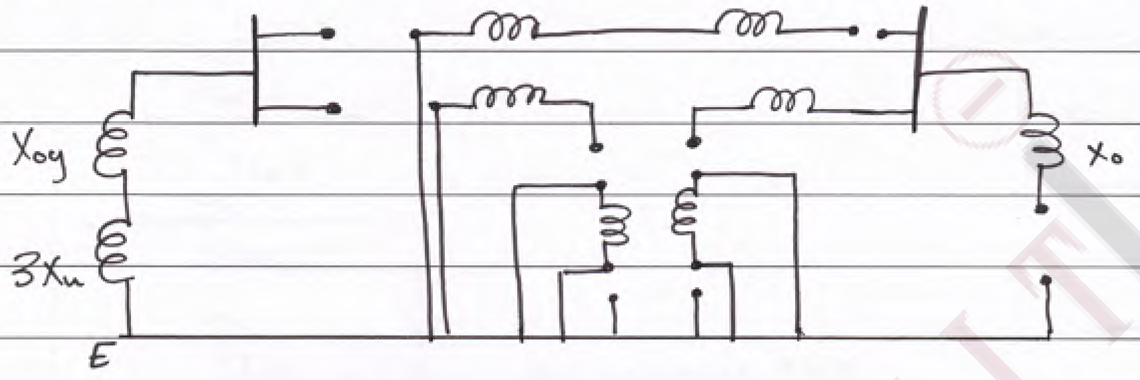




Zero seq. Network

ex3:





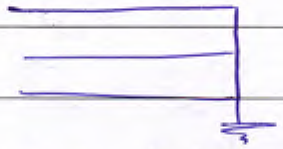
• UnSymmetrical faults:-

→ Symmetrical fault:-

* the most severe fault that might happen (worst case scenario).

"إذا تحمّل الـSystem أكيد رح يتحمل غيره"

* نادر الحدوث

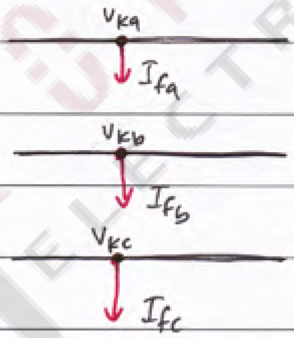


3-ph balanced fault

→ Unsymmetrical faults:-

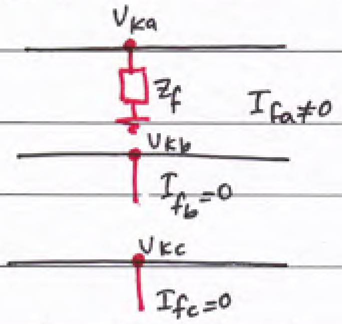
3-ph system:

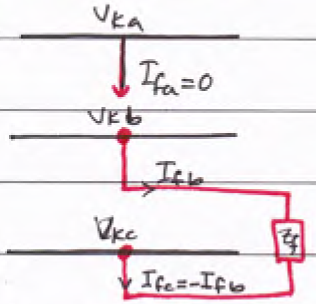
"عند الـfault بنعمل تَفْرِيعَة Gx"



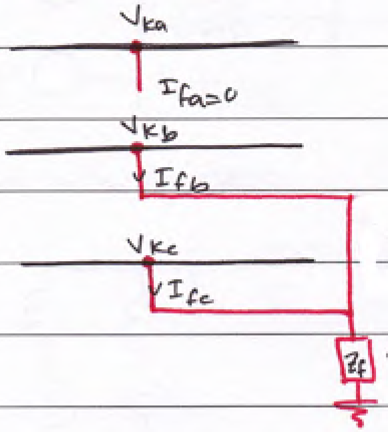
1) P-E fault: (85% of faults).

2) P-P fault



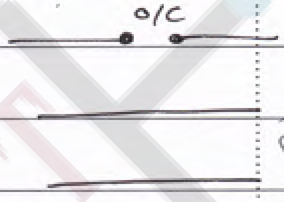


3) P-P-E fault:



this impedence limits the current, so sometimes we assume it zero to consider the worst case scenario.

4) damaged jumper or fuse (open CKT).



"مش كثير بهم تحليله لأنه ما بينتج عنه تيارات عالية"

"لاحظ إنه دائماً افترضنا phase a يكون هو المختلف"

$$I_{fa} = I_{fa}^{(0)} + I_{fa}^{(1)} + I_{fa}^{(2)}$$

$$I_{fb} = I_{fb}^{(0)} + I_{fb}^{(1)} + I_{fb}^{(2)}$$

$$I_{fc} = I_{fc}^{(0)} + I_{fc}^{(1)} + I_{fc}^{(2)}$$

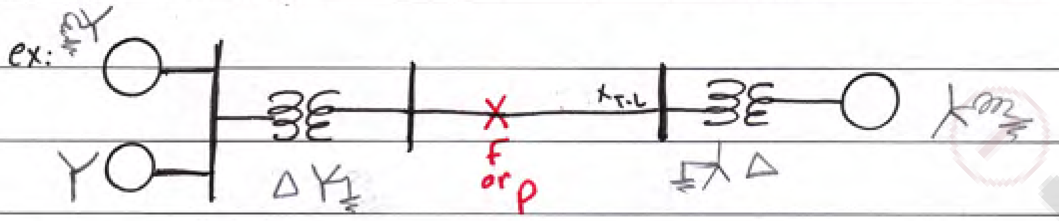
in symm. fault $\quad V_j = V_f - \frac{Z_{jk}}{Z_{kk}} V_f$

now: $\quad V_{ja} = V_{ja}^{(0)} + V_{ja}^{(1)} + V_{ja}^{(2)}$

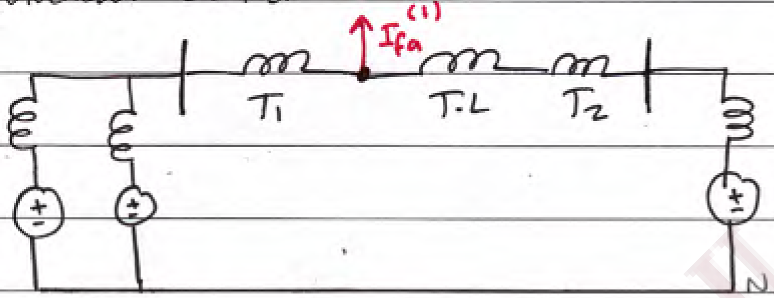
$$V_{jb} = V_{jb}^{(0)} + V_{jb}^{(1)} + V_{jb}^{(2)}$$

$$V_{jc} = V_{jc}^{(0)} + V_{jc}^{(1)} + V_{jc}^{(2)}$$

ex:



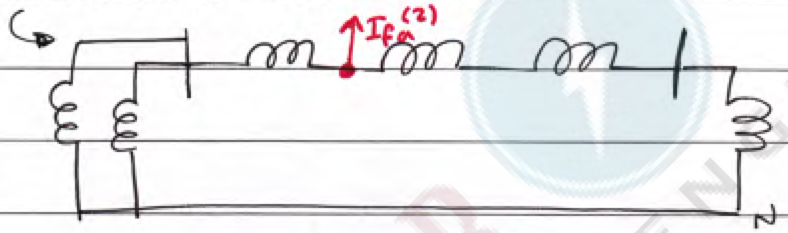
3 equivalent CKTs:



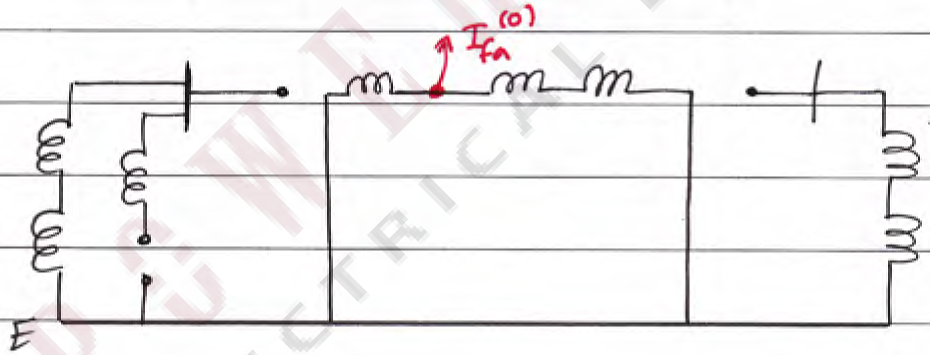
+ve seq.

"ما إلبش دعوة كيف ال earth ! connected"

Sources are short-circuited



-ve seq.



Zero-seq.

We'll use Zbus method. (3(Zbus)s) $Z_{bus}^{(0)}, Z_{bus}^{(1)}, Z_{bus}^{(2)}$

$$Z_{bus}^{(1)} = \begin{bmatrix} Z_{11}^{(1)} & Z_{12}^{(1)} & \cdots & Z_{1K}^{(1)} & \cdots & Z_{1N}^{(1)} \\ Z_{21}^{(1)} & Z_{22}^{(1)} & \cdots & Z_{2K}^{(1)} & \cdots & Z_{2N}^{(1)} \\ \vdots & \vdots & & \vdots & & \vdots \\ Z_{K1}^{(1)} & Z_{K2}^{(1)} & \cdots & Z_{KK}^{(1)} & \cdots & Z_{KN}^{(1)} \\ \vdots & \vdots & & \vdots & & \vdots \\ Z_{N1}^{(1)} & Z_{N2}^{(1)} & \cdots & Z_{NK}^{(1)} & \cdots & Z_{NN}^{(1)} \end{bmatrix}$$

from +ve seq. network.

$$Z_{bus}^{(2)} = \begin{bmatrix} Z_{11}^{(2)} & Z_{12}^{(2)} & \cdots & Z_{1K}^{(2)} & \cdots & Z_{1N}^{(2)} \\ Z_{21}^{(2)} & Z_{22}^{(2)} & \cdots & Z_{2K}^{(2)} & \cdots & Z_{2N}^{(2)} \\ \vdots & & & & & \\ Z_{K1}^{(2)} & Z_{K2}^{(2)} & \cdots & Z_{KK}^{(2)} & \cdots & Z_{KN}^{(2)} \\ \vdots & & & & & \\ Z_{N1}^{(2)} & Z_{N2}^{(2)} & \cdots & Z_{NK}^{(2)} & \cdots & Z_{NN}^{(2)} \end{bmatrix}$$

from -ve seq. network.

$$Z_{bus}^{(0)} = \begin{bmatrix} Z_{11}^{(0)} & Z_{12}^{(0)} & \cdots Z_{1k}^{(0)} & \cdots Z_{1N}^{(0)} \\ Z_{21}^{(0)} & Z_{22}^{(0)} & \cdots Z_{2k}^{(0)} & \cdots Z_{2N}^{(0)} \\ \vdots & & & \\ Z_{k1}^{(0)} & Z_{k2}^{(0)} & \cdots Z_{kk}^{(0)} & \cdots Z_{kN}^{(0)} \\ \vdots & & & \\ Z_{N1}^{(0)} & Z_{N2}^{(0)} & \cdots Z_{Nk}^{(0)} & \cdots Z_{NN}^{(0)} \end{bmatrix}$$

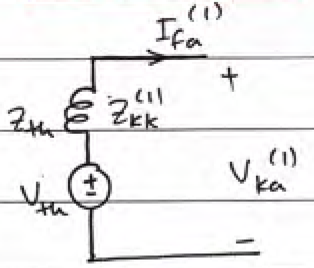
V_{th} → only in +ve seq.

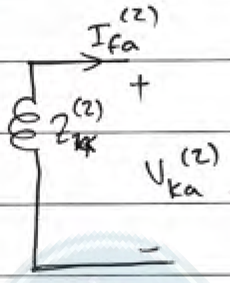
∴ first solution:-

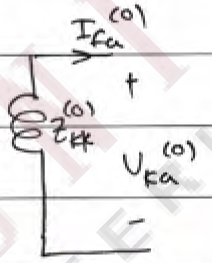
$V_{ja}^{(1)} = V_f$, $V_{ja}^{(2)} = 0$, $V_{ja}^{(0)} = 0$

lecture #38
15/5/2014

Solution #1 :







+ve seq. −ve seq. Zero-seq.

$$\begin{bmatrix} V_{1a}^{(1)} \\ V_{2a}^{(1)} \\ \vdots \\ V_{ka}^{(1)} \\ \vdots \\ V_{Na}^{(1)} \end{bmatrix} = \begin{bmatrix} V_f \\ V_f \\ \vdots \\ V_f \\ \vdots \\ V_f \end{bmatrix}$$

← here we didn't take I_L into consideration

$$\begin{bmatrix} V_{1a}^{(2)} \\ V_{2a}^{(2)} \\ \vdots \\ V_{ka}^{(2)} \\ \vdots \\ V_{Na}^{(2)} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \end{bmatrix} \qquad \begin{bmatrix} V_{1a}^{(0)} \\ V_{2a}^{(0)} \\ \vdots \\ V_{ka}^{(0)} \\ \vdots \\ V_{Na}^{(0)} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$\boxed{V_{ja}^{(1)} = V_f}$ ...① $\boxed{V_{ja}^{(2)} = 0}$ ...② $\boxed{V_{ja}^{(0)} = 0}$ ...③

Solution #2:

$$\begin{bmatrix} \Delta V_{1a}^{(1)} \\ \Delta V_{2a}^{(1)} \\ \vdots \\ \Delta V_{ka}^{(1)} \\ \vdots \\ \Delta V_{Na}^{(1)} \end{bmatrix} = \begin{bmatrix} Z_{11}^{(1)} & Z_{12}^{(1)} & \cdots & Z_{1k}^{(1)} & \cdots & Z_{1N}^{(1)} \\ Z_{21}^{(1)} & Z_{22}^{(1)} & \cdots & Z_{2k}^{(1)} & \cdots & Z_{2N}^{(1)} \\ \vdots & & & & & \\ Z_{k1}^{(1)} & Z_{k2}^{(1)} & \cdots & Z_{kk}^{(1)} & \cdots & Z_{kN}^{(1)} \\ \vdots & & & & & \\ Z_{N1}^{(1)} & Z_{N2}^{(1)} & \cdots & Z_{Nk}^{(1)} & \cdots & Z_{NN}^{(1)} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ -I_{fa}^{(1)} \\ \vdots \\ 0 \end{bmatrix}$$

$\boxed{\Delta V_{ja}^{(1)} = -Z_{jk}^{(1)} I_{fa}^{(1)}}$ ...①′

$$\begin{bmatrix} \Delta V_{1a}^{(2)} \\ \Delta V_{2a}^{(2)} \\ \vdots \\ \Delta V_{ka}^{(2)} \\ \vdots \\ \Delta V_{Na}^{(2)} \end{bmatrix} = \begin{bmatrix} Z_{11}^{(2)} & Z_{12}^{(2)} & \cdots & Z_{1k}^{(2)} & \cdots & Z_{1N}^{(2)} \\ Z_{21}^{(2)} & Z_{22}^{(2)} & \cdots & Z_{2k}^{(2)} & \cdots & Z_{2N}^{(2)} \\ \vdots \\ Z_{k1}^{(2)} & Z_{k2}^{(2)} & \cdots & Z_{kk}^{(2)} & \cdots & Z_{kN}^{(2)} \\ \vdots \\ Z_{N1}^{(2)} & Z_{N2}^{(2)} & \cdots & Z_{Nk}^{(2)} & \cdots & Z_{NN}^{(2)} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ -I_{fa}^{(2)} \\ \vdots \\ 0 \end{bmatrix}$$

$$\boxed{\Delta V_{ja}^{(2)} = -Z_{jk}^{(2)} I_{fa}^{(2)}} \cdots ②'$$

$$\begin{bmatrix} \Delta V_{1a}^{(0)} \\ \Delta V_{2a}^{(0)} \\ \vdots \\ \Delta V_{ka}^{(0)} \\ \vdots \\ \Delta V_{Na}^{(0)} \end{bmatrix} = \begin{bmatrix} Z_{11}^{(0)} & Z_{12}^{(0)} & \cdots & Z_{1k}^{(0)} & \cdots & Z_{1N}^{(0)} \\ Z_{21}^{(0)} & Z_{22}^{(0)} & \cdots & Z_{2k}^{(0)} & \cdots & Z_{2N}^{(0)} \\ \vdots \\ Z_{k1}^{(0)} & Z_{k2}^{(0)} & \cdots & Z_{kk}^{(0)} & \cdots & Z_{kN}^{(0)} \\ \vdots \\ Z_{N1}^{(0)} & Z_{N2}^{(0)} & \cdots & Z_{Nk}^{(0)} & \cdots & Z_{NN}^{(0)} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ -I_{fa}^{(0)} \\ \vdots \\ 0 \end{bmatrix}$$

$$\boxed{\Delta V_{ja}^{(0)} = -Z_{jk}^{(0)} I_{fa}^{(0)}} \cdots ③'$$

total solution: ① + ①', ...

$$V_{ja}^{(1)} = V_f - Z_{jk}^{(1)} I_{fa}^{(1)}$$

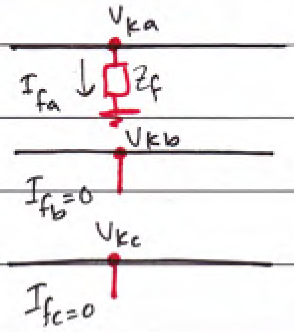
$\hookrightarrow V_{ka}^{(1)} = V_f - Z_{kk}^{(1)} I_{fa}^{(1)}$ V at the faulty bus not zero as before

$$V_{ja}^{(2)} = -Z_{jk}^{(2)} I_{fa}^{(2)}$$

$$V_{ja}^{(0)} = -Z_{jk}^{(0)} I_{fa}^{(0)}$$

$I_{fa}^{(1)} = ? \rightarrow$ (حسب نوع ال fault)

P-E fault:



$$I_{fb} = I_{fc} = 0$$

$$Z_{ka} = I_{fa} Z_f$$

$$\begin{bmatrix} I_{fa}^{(0)} \\ I_{fa}^{(1)} \\ I_{fa}^{(2)} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_{fa} \\ \cancel{I_{fb}} \xrightarrow{} 0 \\ \cancel{I_{fc}} \xrightarrow{} 0 \end{bmatrix}$$

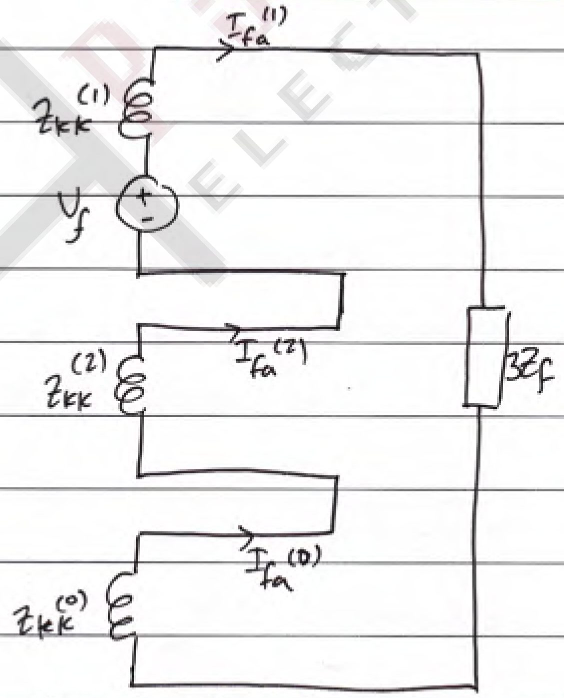
$$\therefore \boxed{I_{fa}^{(0)} = I_{fa}^{(1)} = I_{fa}^{(2)} = \frac{1}{3} I_{fa}}$$

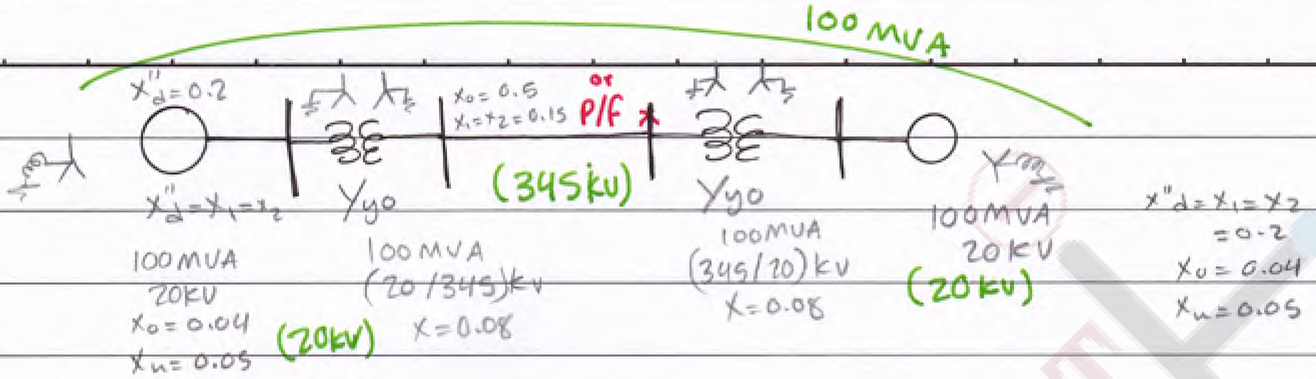
$$V_{ka} = (V_{ka}^{(1)}) + V_{ka}^{(2)} + V_{ka}^{(0)} = I_{fa} Z_f$$

$$(V_f - I_{fa}^{(1)} Z_{kk}^{(1)}) - Z_{kk}^{(2)} \underset{= I_{fa}^{(1)}}{\cancel{I_{fa}^{(2)}}} - Z_{kk}^{(0)} \underset{= I_{fa}^{(1)}}{\cancel{I_{fa}^{(0)}}} = 3 I_{fa}^{(1)} Z_f$$

$$\therefore V_f = I_{fa}^{(1)} \left[Z_{kk}^{(1)} + Z_{kk}^{(2)} + Z_{kk}^{(0)} + 3Z_f \right)$$

$$\boxed{I_{fa}^{(1)} = \frac{V_f}{Z_{kk}^{(1)} + Z_{kk}^{(2)} + Z_{kk}^{(0)} + 3Z_f}}$$

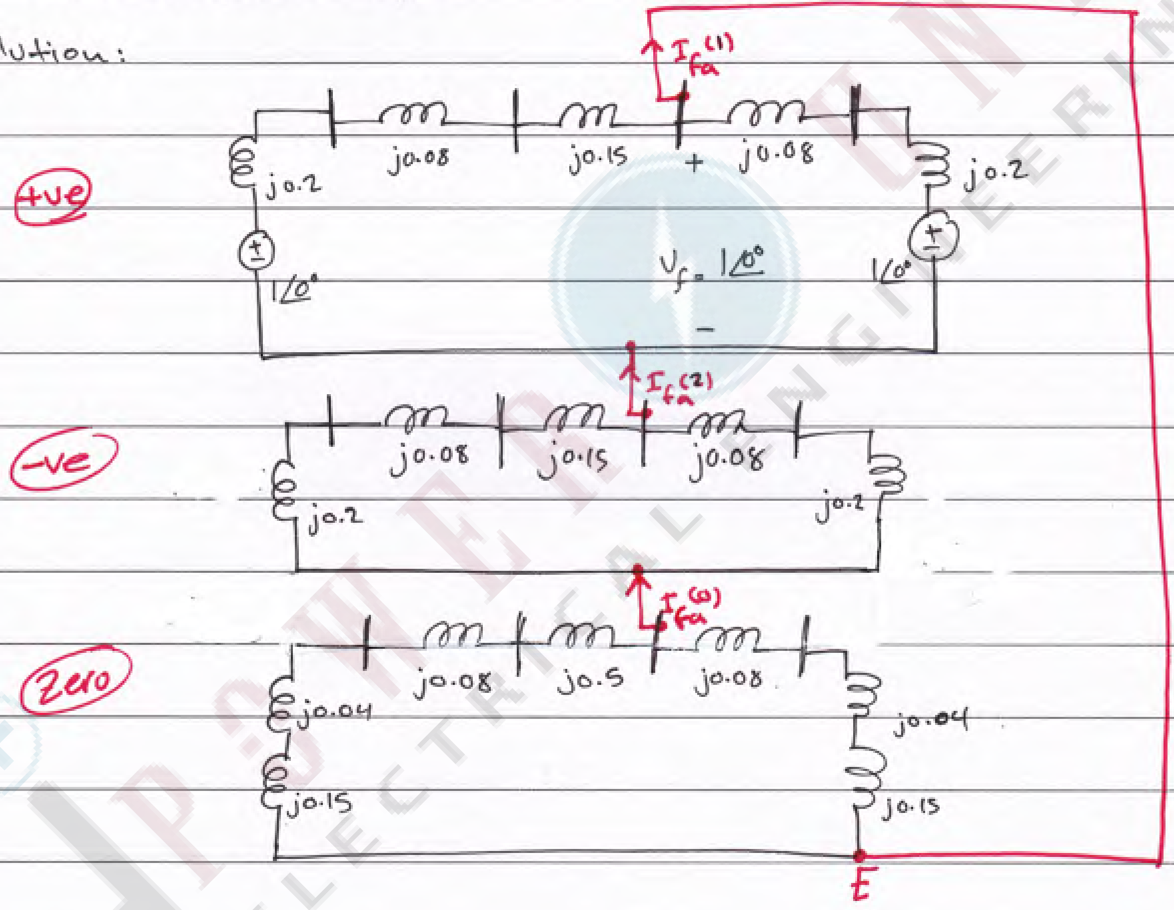




→ $I_L = 0$ before fault

* bolted fault ≡ $Z_f = 0$

solution:



$Z_{kk}^{(1)} = j0.28 // j0.48$

$Z_{kk}^{(2)} = Z_{kk}^{(1)}$ from symmetry

$Z_{kk}^{(0)} = j0.24 // j0.32$

$$I_{f\textcircled{A}}^{(1)} = \frac{1 \angle 0^\circ}{j0.1696 + j0.1696 + j0.1999} = -j1.8549.$$

as fault [illegible] H.V [illegible] side

base current.

$$I_{f\textcircled{A}} = 3I_{fA}^{(0)} = -j1.8549 \times 3 \text{ p.u.} \quad \times \left(\frac{100 \times 10^6}{\sqrt{3} \times 345 \times 10^3}\right)$$

ex continued.

$I_{fA}^{(1)} = I_{fA}^{(2)} = I_{fA}^{(0)} = -j1.8549 = \frac{(I_{fA})}{3}$

$$Z_{bus}^{(1)} = \underset{\text{by symmetry}}{Z_{bus}^{(2)}} = \begin{bmatrix} - & - & j0.0789 & - \\ - & - & j0.1104 & - \\ - & - & j0.1894 & - \\ - & - & j0.1211 & - \end{bmatrix}$$

$$Z_{bus}^{(0)} = \begin{bmatrix} - & - & j0.443 & - \\ - & - & j0.0701 & - \\ - & - & j0.1999 & - \\ - & - & j0.1407 & - \end{bmatrix}$$

$I_{fA} = 3I_{fA}^{(0)} = -j5.5647$

find V_{4a}, V_{4b}, V_{4c}?

Zbus method:

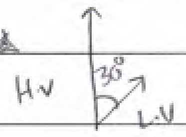
$V_{4a}^{(0)} = -Z_{43}^{(0)} I_{fA}^{(0)} = -j0.1407 \times -j1.8549 = -0.261$ p.u.

$V_{4a}^{(1)} = 1 - Z_{43}^{(1)} I_{fA}^{(1)} = 1 - j0.1211 \times -j0.18549 = 0.7754$ p.u.

$V_{4a}^{(2)} = -Z_{43}^{(2)} I_{fA}^{(2)} = -j0.1211 \times -j0.18549 = -0.2246$ p.u.

$$\begin{bmatrix} V_{4a} \\ V_{4b} \\ V_{4c} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{4a}^{(0)} \\ V_{4a}^{(1)} \\ V_{4a}^{(2)} \end{bmatrix} \quad \text{---} ①$$

if the transformer was Yd1 :-



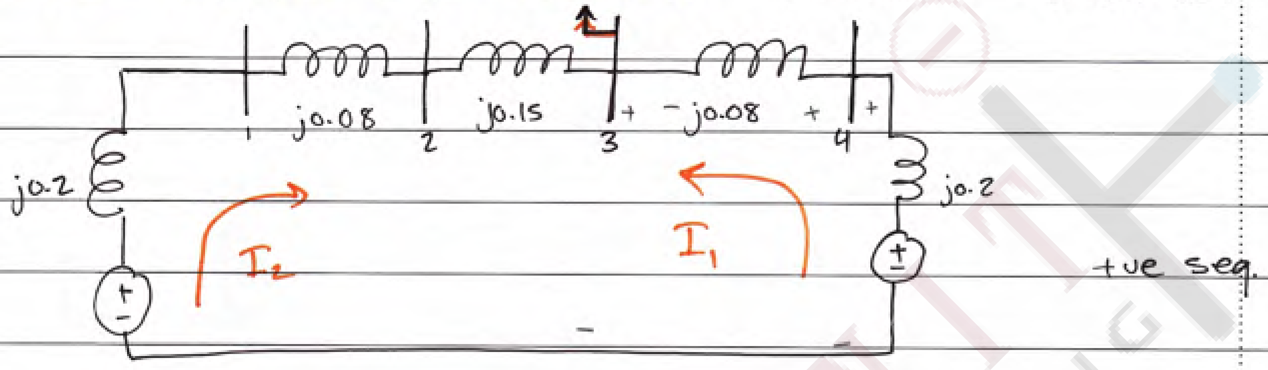
→ for +ve seq $\angle -30°$

" -ve " $\angle +30°$

Since V_{4a} is in the L.V side.

** shift due to transformers only appears in ±ve sequences.

- another method (CKT method):- (to calculate V_{4a}, V_{4b}, V_{4c})



$I_1 = -j1.8549 \times \frac{j0.43}{j0.71}$ current division.

$= -j1.3733$

& it's the same for the -ve seq.

in the zero sequence:

$I_1 = -j1.8549 \times \frac{0.27}{1.04} = -j1.12339$

since the system was unloaded.

$V_{4a}^{(1)} = 1\angle 0^\circ - j0.2(-j1.3733) = 0.7753$ p.u.

$V_{4a}^{(2)} = 0 - j0.2(-j1.3733) = -0.22546$ p.u.

$V_{4a}^{(0)} = 0 - j0.19(-j1.1233) = -0.261$

- 3^{rd} method:

$V_{3A}^{(0)} = -Z_{kk}^{(0)} I_{fa}^{(0)} = -0.37079$ p.u.

$V_{3A}^{(1)} = 1 - Z_{kk}^{(1)} I_{fa}^{(1)} = 0.6854$ p.u.

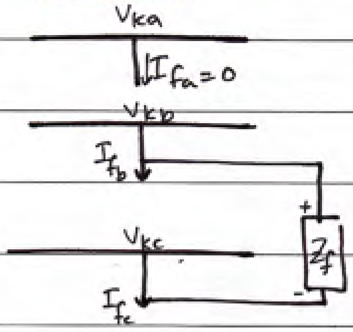
$V_{3A}^{(2)} = -Z_{kk}^{(2)} I_{fa}^{(2)} = -0.3146$ p.u. $\boxed{V_{3A} = 0}$

then $V_{4a}^{(1)} = V_{3A}^{(1)} + I^{(1)} \times j0.08$

$V_{4a}^{(0)} = V_{3A}^{(0)} + (I^{(0)} \times j0.08)$

⋮

$\boxed{P-P}$ fault



$$I_{fa} = 0$$

$$I_{fb} = -I_{fc}$$

$$V_{kb} - V_{kc} = I_{fb} Z_f$$

$$\begin{bmatrix} I_{fa}^{(0)} \\ I_{fa}^{(1)} \\ I_{fa}^{(2)} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} \overset{0}{\cancel{I_{fa}}} \\ I_{fb} \\ \underset{-I_{fb}}{\cancel{I_{fc}}} \end{bmatrix}$$

$$\boxed{I_{fa}^{(0)} = 0} \quad ..①$$

$$I_{fa}^{(1)} = \frac{1}{3} a I_{fa} - \frac{1}{3} a^2 I_{fb} = -I_{fa}^{(2)} \quad ..② \qquad \boxed{I_{fa}^{(1)} = -I_{fa}^{(2)}}$$

zero seq. doesn't exist

$$V_{kb} = a^2 V_{ka}^{(1)} + a V_{ka}^{(2)} + 0$$

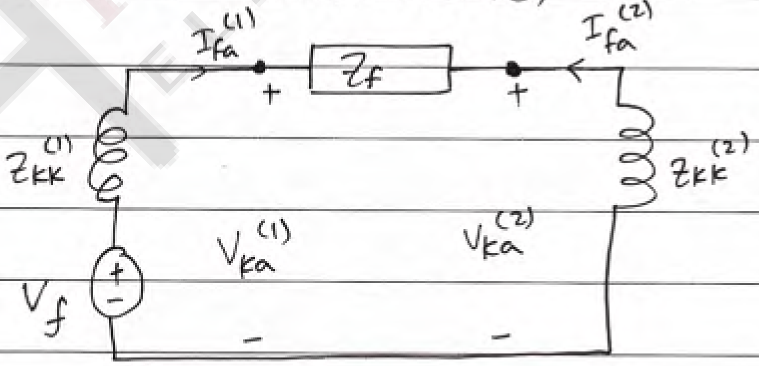
$$V_{kb} - V_{kc} = \left(a^2 V_{ka}^{(1)} + a V_{ka}^{(2)}\right) - \left(a V_{ka}^{(1)} + a^2 V_{ka}^{(2)}\right) = I_{fb} Z_f$$

$$\hookrightarrow \left(a^2 I_{fa}^{(1)} + a \overset{-I_{fa}^{(1)}}{\cancel{I_{fa}^{(2)}}}\right) Z_f$$

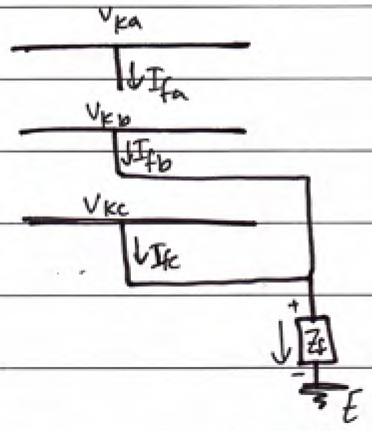
$$= \cancel{(a^2-a)} V_{ka}^{(1)} - \cancel{(a^2-a)} V_{ka}^{(2)} = \cancel{(a^2-a)} I_{fa}^{(1)} Z_f$$

$$\boxed{V_{ka}^{(1)} - V_{ka}^{(2)} = I_{fa}^{(1)} Z_f} \quad ..③$$

- equivelant CKT:- (satisfies ①②&③)



$\boxed{\text{P-P-E}}$ fault:



$\rightarrow I_{fa} = 0$

$\rightarrow V_{kb} = V_{kc} = (I_{fb} + I_{fc}) Z_f$

$$\begin{bmatrix} I_{fa}^{(0)} \\ I_{fa}^{(1)} \\ I_{fa}^{(2)} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} \overset{0}{\cancel{I_{fa}}} \\ I_{fb} \\ I_{fc} \end{bmatrix}$$

$$I_{fa}^{(0)} = \frac{1}{3}(I_{fb} + I_{fc}) \Rightarrow I_{fb} + I_{fc} = 3I_{fa}^{(0)}$$

$$\begin{bmatrix} V_{ka}^{(0)} \\ V_{ka}^{(1)} \\ V_{ka}^{(2)} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_{ka} \\ V_{kb} \\ \underset{V_{kb}}{\cancel{V_{kc}}} \end{bmatrix}$$

$$V_{ka}^{(1)} = \frac{1}{3}(V_{ka} + aV_{kb} + a^2 V_{kb})$$

$$V_{ka}^{(2)} = \frac{1}{3}(V_{ka} + a^2 V_{kb} + aV_{kb})$$

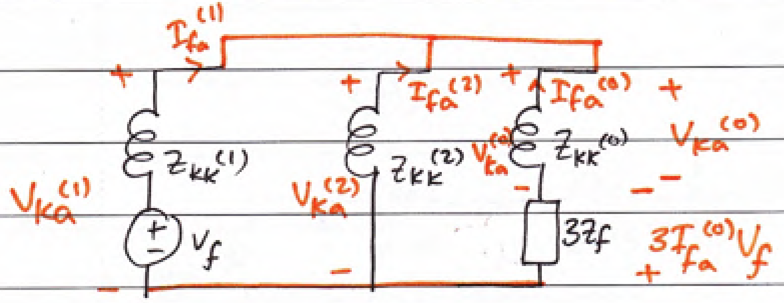
$$\boxed{V_{ka}^{(1)} = V_{ka}^{(2)}}$$

$$3V_{ka}^{(0)} = V_{ka} + 2V_{kb}$$

$$\overset{2}{\cancel{3}} V_{ka}^{(0)} = (\cancel{V_{ka}^{(0)}} + \underbrace{\cancel{V_{ka}^{(1)} + V_{ka}^{(2)}}}_{\cancel{2}V_{ka}^{(1)}}) + \cancel{2} \times (\underline{\overset{(I_{fb}+I_{fc})}{3I_{fa}^{(0)}}} Z_f)$$

$$\boxed{V_{ka}^{(0)} = V_{ka}^{(1)} + 3I_{fa}^{(0)} Z_f}$$

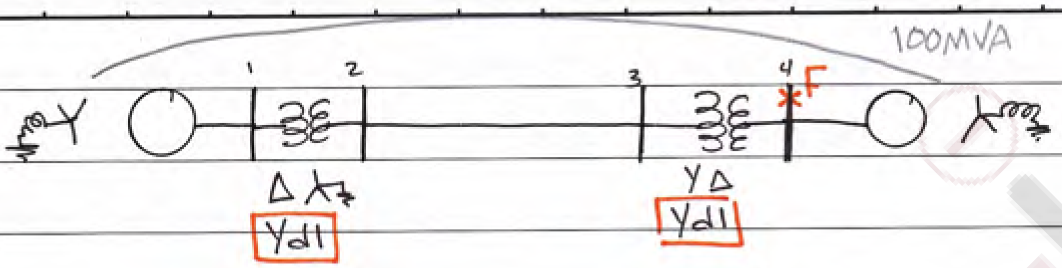
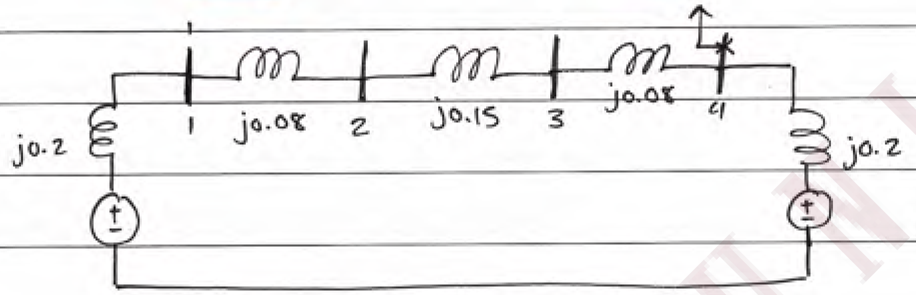
equivelent CKT:



$$V_{ka}^{(1)} = V_{ka}^{(0)} - 3I_{fa}^{(0)} \; V_f$$

$$I_{fa}^{(2)} = -I_{fa}^{(1)}\left(\frac{Z_{kk}^{(0)} + 3Z_f}{Z_{kk}^{(2)} + Z_{kk}^{(0)} + 3Z_f}\right)$$

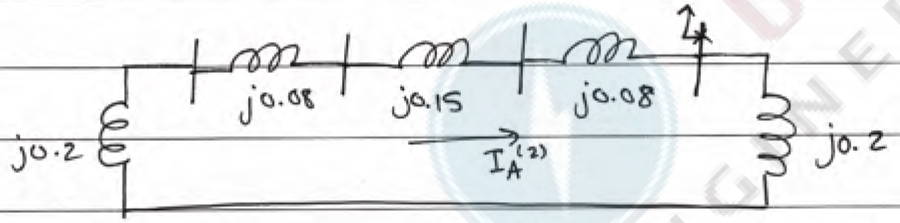
$$I_{fa}^{(0)} = -I_{fa}^{(1)}\left(\frac{Z_{kk}^{(2)}}{Z_{kk}^{(2)} + Z_{kk}^{(0)} + 3Z_f}\right)$$

ex:



+ve seq.

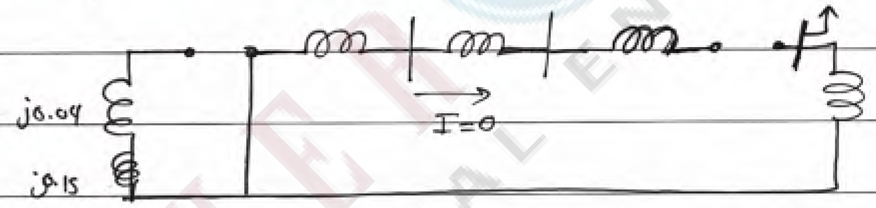
$Z_{44}^{(1)} = j0.2 // (j0.51)$

$= j0.1437$



-ve seq.

$Z_{KK}^{(2)} = j0.1437$

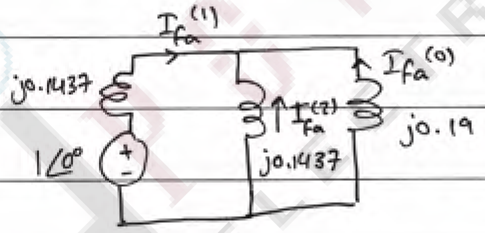


zero-seq.

$Z_{44}^{(0)} = j0.19$

$= j0.15 + j0.04$

L-L-E fault:



$$I_{fa}^{(1)} = \frac{1\angle 0^\circ}{j0.1437 + (j0.19 // j0.1437)} = -j4.4342 \text{ p.u.}$$

$$I_{fa}^{(2)} = -(-j4.4342) \times \frac{0.19}{0.19 + 0.1437} = j2.5247 \text{ p.u.}$$

$$I_{fa}^{(0)} = -(-j4.4342)\left(\frac{0.1437}{0.1437+0.19}\right) = j1.9095 \text{ p.u.}$$

$$\begin{bmatrix} I_{fa} \\ I_{fb} \\ I_{fc} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} j1.9095 \\ -j4.4342 \\ j2.5247 \end{bmatrix} \rightarrow I_{fa} = 0$$

$I_{fb} = 6.6726 \angle 154.6^\circ$ p.u.

$I_{fc} = 6.6726 \angle 25.4^\circ$ p.u.

base current $= \frac{100 \times 10^6}{\sqrt{3} \times 20 \times 10^3}$

$V_{4a}^{(1)} = V_{4a}^{(2)} = V_{4a}^{(0)}$ in parallel (no Z_f is mentioned).

$V_{4a}^{(1)} = 1 - j0.1437 \times (-j4.4342) = 0.3628$ p.u.

$\hookrightarrow V_{4a}^{(2)} = V_{4a}^{(1)} = V_{4a}^{(0)}$

$\therefore V_{4a} = 3V_{4a}^{(1)} = 3 \times 0.3628$ p.u.

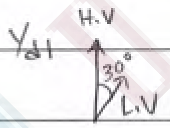
$V_{4b} = 0, \quad V_{4c} = 0$ (Z_f isn't considered).

now find V_{2a} ?

① $I_A^{(2)} = j2.547 \times \frac{0.2}{0.71} = j0.7118$

+ phase shift

$I_A^{(2)} = j0.7118 \angle \boxed{-}30^\circ$ ← since it's a(-ve) seq.



$\angle +30^\circ$ من L.V لل H.V

$I_A^{(1)} = -j4.434 \times \frac{0.2}{0.71} \angle +30^\circ = -j1.249 \angle 30^\circ$

$I_A^{(0)} = 0$

$I_A = I_A^{(1)} + I_A^{(2)}$

V at bus 2:

$V_{2a}^{(1)} = 1\angle 30^\circ - j0.28 \times (-j1.249 \angle 30^\circ)$ ← due to phase shift

$= 0.65028 \angle 30^\circ$

" نقدر اعمل الحسابات من دون ما اعتبر الـ Phase shift وبالآخير اعملها عشان ما اغلطش! "

$V_{2a}^{(2)} = (0 - j0.28 \times j0.7118) \angle -30^\circ$

$= 0.19913 \angle -30^\circ$

$V_{2a}^{(0)} = 0$

base $= \frac{345 \text{ kV}}{\boxed{\sqrt{3}}}$

② using Z_{bus} method:

$V_{2a}^{(0)} = 0,$

$$Z_{bus}^{(1)} = \begin{bmatrix} - & - & - & j0.0563 \\ - & - & - & j0.0784 \\ - & - & - & j0.1211 \\ - & - & - & j0.1437 \end{bmatrix}$$

$V_{2a}^{(1)} = 1 - Z_{24}^{(1)} I_{fa}^{(1)}$

$= 1\angle 0^\circ - j0.0784 \times -j4.4342$

$= (0.65236) \angle +30$ ← add it in

$V_{2a}^{(0)} = 0$ what does it mean?

$V_{2a}^{(0)} = -Z_{24}^{(0)} \, \underset{\nwarrow \neq 0}{I_{fa}^{(0)}}$

$\therefore Z_{24}^{(0)} = 0$; ∴ no connection between 2 & 4.

$Z_{bus}^{(0)}$ $\begin{bmatrix} \cdots 0 \\ \cdots 0 \\ \cdots 0 \\ \cdots j0.19 \end{bmatrix}$

remember that $Z_{24} \neq Z$ between bus 2 & 4

at bus 1: 2 shifts will occur +30 then −30

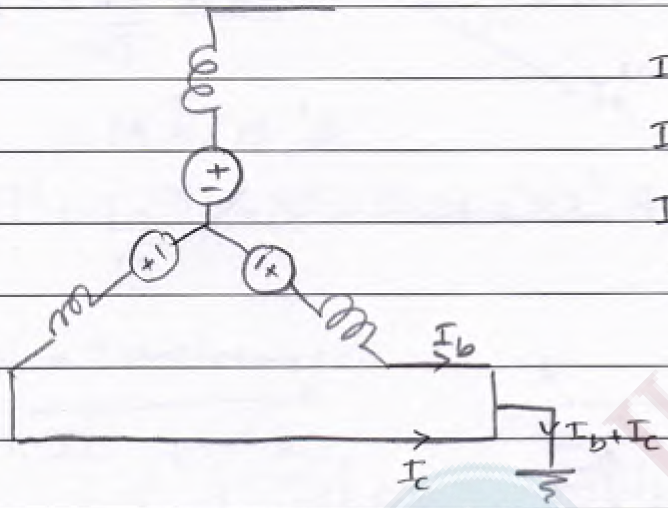
"عشان هيك بأول المثال لاعتبرنا كأنه أنا بدي الـ voltage معكوس

المعكوس عن الـ LV side الثاني transformed لجهة اصبح V_{2a} يعني لا صارت قيمتها $1\angle 0^{\circ}$

بالنسبة لل bus الي عليها fault و $1\angle 30^{\circ}$ بالنسبة لل bus2 "

Chapter # 11 Problems:-

11.2:



$I_a^{(1)} = 600\angle{-90^\circ}$ A

$I_a^{(2)} = 250\angle{90^\circ}$ A

$I_a^{(0)} = 350\angle{90^\circ}$ A

$I_a = 0$

$I_b^{(1)} = 600\angle{-90+240^\circ}$

$I_b^{(2)} = 250\angle{90+120^\circ}$

$I_b^{(0)} = I_a^{(0)} = 350\angle{90^\circ}$

$* \ I_b = 904.1\angle{144.5^\circ}$

$I_c^{(1)} = 600\angle{-90+120}$

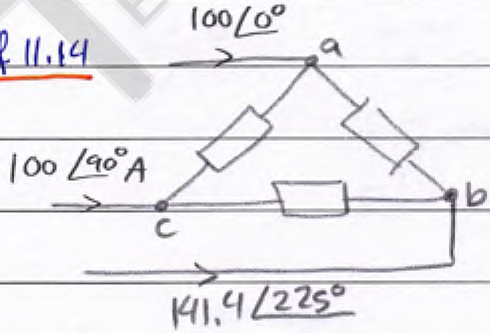
$I_c^{(2)} = 250\angle{90+240^\circ}$

$I_c^{(0)} = 350\angle{90^\circ}$

$* \ I_c = 904.1\angle{35.5^\circ}$

$I_b + I_c = j1050$ A $= \boxed{3I_a^{(0)}}$ you can solve it directly.

11.4 & 11.14



$I_a^{(0)} = 0$ (Δ-load)

$I_a^{(1)} = \frac{1}{3}\left(100\angle{0^\circ} + 141.4\angle{225+120} + 100\angle{90+240}\right)$

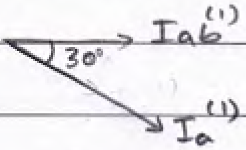
$= 111.5\angle{-15^\circ}$

$I_a^{(2)} = \frac{1}{3}\left(100\angle{0^\circ} + 141.4\angle{225+240^\circ} + 100\angle{90+120}\right)$

$= 29.9\angle{105^\circ}$

→ then solve for $I_b^{(1)}, I_b^{(2)}, \ldots I_c^{(1)}, I_c^{(2)}, \ldots$

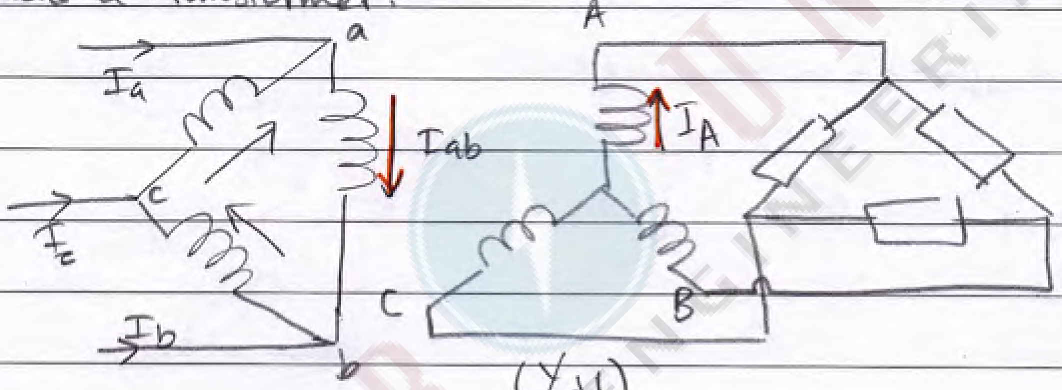
$$I_{ab}^{(1)} = \frac{I_a^{(1)}}{\sqrt{3}} \angle +30^\circ$$



$$= 64.4 \angle 15^\circ \text{ A}$$

$$I_{ab}^{(2)} = \frac{I_a^{(2)}}{\sqrt{3}} \angle -30^\circ = 17.36 \angle 75^\circ \text{ A}$$

11/4 → add a transformer:



(Yd1)
13.2/66KV
10MVA

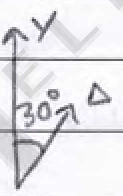
$$I_{ab} = \overset{I_A}{100\angle 0} \times \overbrace{\left(\frac{\frac{66}{\sqrt{3}}}{13.2}\right)}^{\text{turns ratio.}} \leftarrow = 2.89$$

↳(since Iab is a phase current.)

by symm. components

$$I_{ca} = 100 \angle 90^\circ \times 2.89 \qquad I_a = I_{ab} - I_{ca} = 408.48 \angle -45^\circ$$

now, since it's a Yd1 transformer



$$I_a^{(1)} = I_A^{(1)} \angle -30^\circ \text{ p.u.}$$

if I want it in Amperes

$$I_a^{(1)} = I_A^{(1)} \times \underbrace{(\text{transformation ratio})}_{} \angle -30^\circ \text{ A}$$

since I_a & I_A are line to line currents.

$$\text{in p.u.}: I_a^{(1)} = I_A^{(1)} \angle -30^\circ = \frac{111.5 \angle -15}{\left(\frac{10 \times 10^6}{\sqrt{3} \times 66 \times 10^3}\right)} \angle -30^\circ = 1.274 \angle -45 \text{ p.u.}$$

base: $\frac{10 \times 10^6}{\sqrt{3} \times \underline{13.2} \times 10^3}$
L-LKV

Subject Date No.

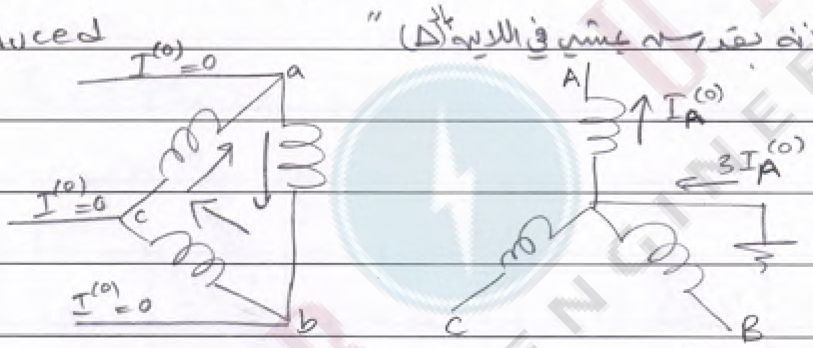
$$I_a^{(2)} = \frac{29.9\angle 105^\circ}{\text{base current}} \quad \angle 30 \leftarrow \text{-ve seq.}$$

$$= \underline{0.342\angle 135^\circ} \text{ p.u.}$$

$$I_a = I_a^{(1)} + I_a^{(2)} = 0.932\angle -45 \text{ p.u.}$$

$$= 408\angle -45^\circ \text{ A} \quad = I_b = I_c$$

if the Y side was grounded a zero seq. current will be introduced "بجيبه بال turns ratio لانه بقدر يمشي في اللوب (Δ)"

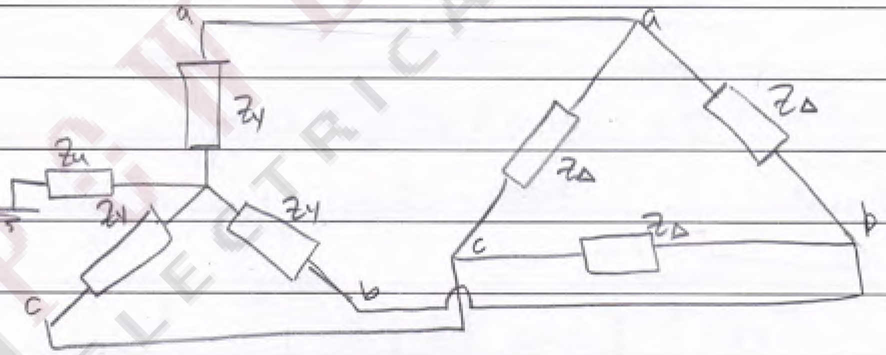


$I_{ab}^{(2)}$ from $I_A^{(2)}$

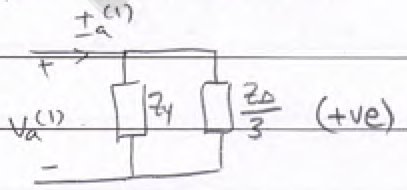
$I_{ab}^{(1)}$ from $I_A^{(1)}$

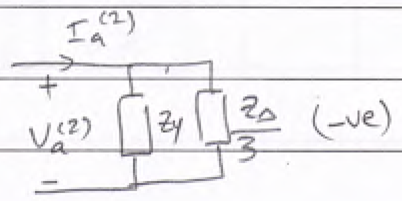
$I_{ab}^{(0)}$ from $I_A^{(0)}$ (بال turns ratio)

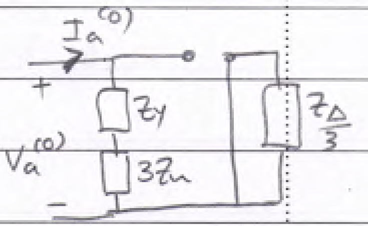
11.9



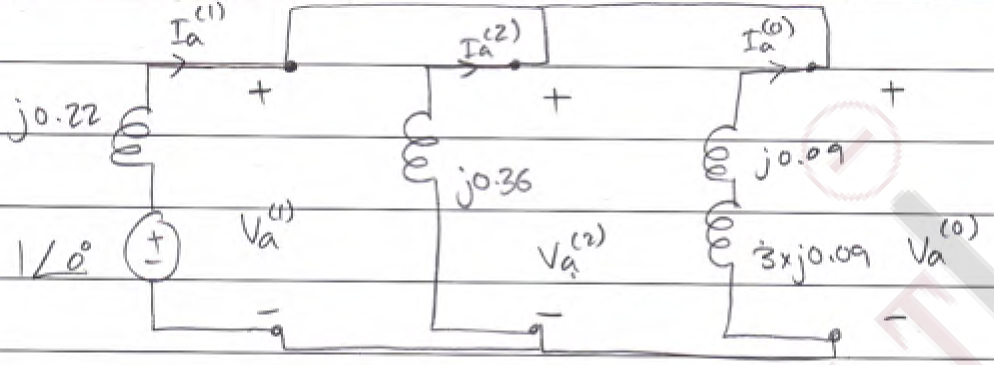
Sequence networks:







11.12



rated terminal voltage $= 1\angle 0°$

fault currents:-

$I_a = 0$ $\quad I_c = 3.75\angle 30°$ p.u.

$I_b = 3.75\angle 150°$ p.u.

V_a, V_b, V_c ?

$$\begin{bmatrix} I_a^{(0)} \\ I_a^{(1)} \\ I_a^{(2)} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} 0 \\ 3.75\angle 150° \\ 3.75\angle 30° \end{bmatrix} = \begin{bmatrix} j1.25 \\ -j2.5 \\ j1.25 \end{bmatrix}$$

"من صورة بتعرف
النوع ال fault
" b-c-E

$V_a^{(1)} = 1\angle 0° - j0.22 \times -j2.5$

$= 0.45$

$V_a^{(2)} = -j0.36 \times j1.25 = 0.45$

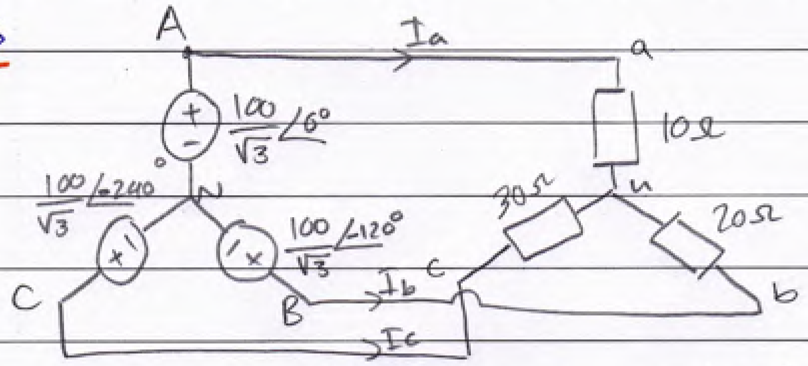
$V_a^{(0)} = -j0.36 \times j1.25 = 0.45$

→ $V_a^{(0)} = V_a^{(1)} = V_a^{(2)}$ since they're in parallel (V_f wasn't mentioned)

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}\begin{bmatrix} 0.45 \\ 0.45 \\ 0.45 \end{bmatrix} = \begin{bmatrix} 1.35 \\ 0 \\ 0 \end{bmatrix} \text{ p.u.}$$

$0, 0$ — since $V_f = 0$

11.16



* CKT method (3 meshes the solve for currents).

* Symmetrical components:-

$$\underline{V_{AN}} = \underline{V_{An}} + \underline{V_{nN}}$$

↳ zero-seq. only

+ve seq. only since it's a balanced source

most contain zero & +ve seq. only

↳ to cancel the zero seq. from V_{nN}

$\therefore V_{an}^{(2)} = 0$

$V_{an}^{(1)} = \frac{100}{\sqrt{3}}\angle 0°$

$V_{an}^{(2)} = 0$

$I_a^{(0)} = 0$

$$V_{an}^{(1)} = \frac{100}{\sqrt{3}}\angle 0° = \frac{1}{3} I_a^{(1)} (Z_a + Z_b + Z_c) + \frac{1}{3} I_a^{(2)} (Z_a + a^2 Z_b + a Z_c)$$

$$V_{an}^{(2)} = 0 = \frac{1}{3} I_a^{(1)} (Z_a + a Z_b + a^2 Z_c) + \frac{1}{3} I_a^{(2)} (Z_a + Z_b + Z_c)$$

$I_a^{(0)} = 0$

$I_a^{(1)} = 3.147$

$I_a^{(2)} = 0.908\angle 30°$

$I_a = 3.959\angle 6.35°$