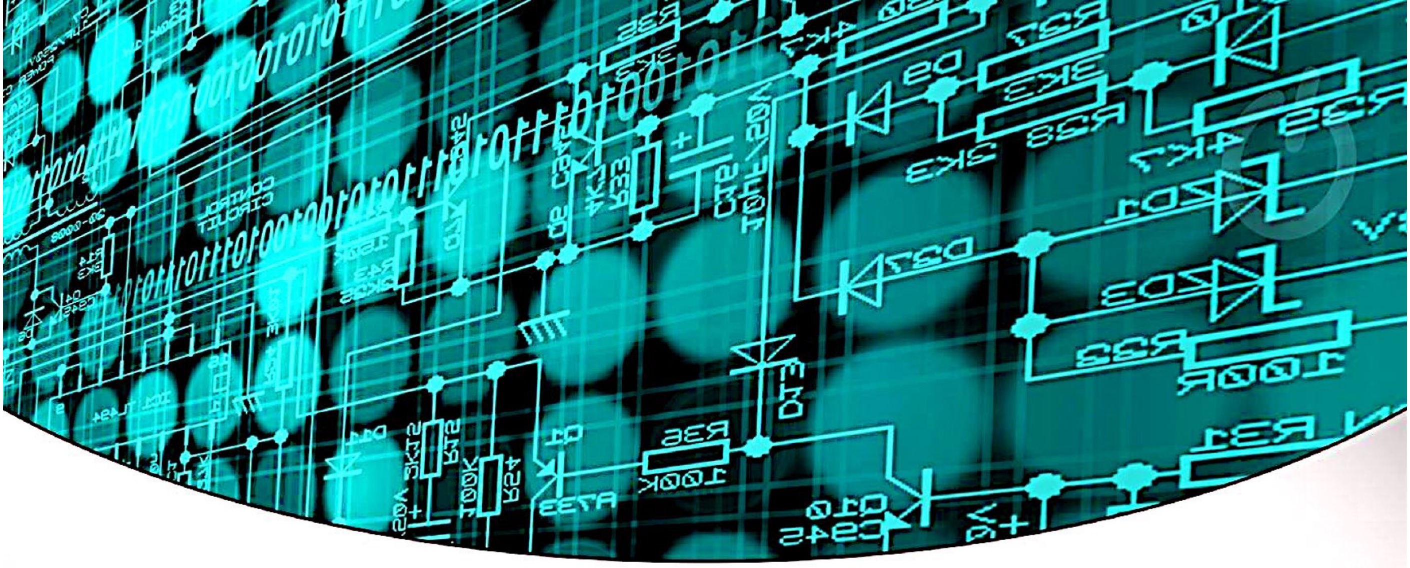

Power Electronics

Fall017

Dr.Sereen Althaher

By: Mhmd Abuhashya

www Powerunit-ju.com

Power Electronics

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Fall 2017-2018

Notebook.

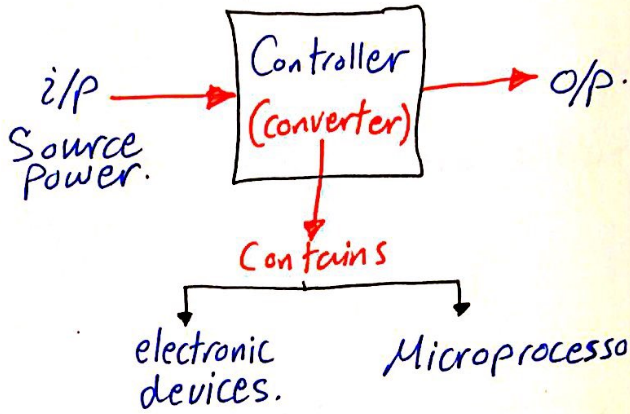
By. Mohammad Abu Hashya.

※ Power Electronics (PE):

Power → Source (V,I)

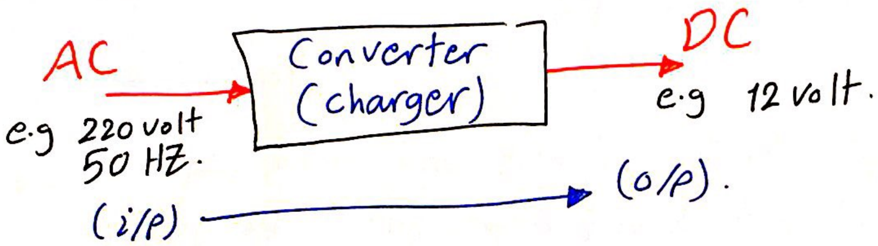
Electronics → Devices.

Devices → Source: Control



※ Application of PE:

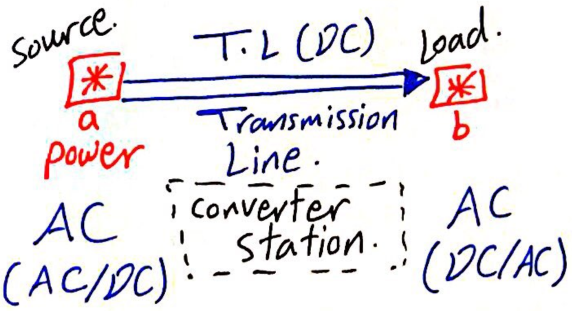
{1} Charger:



{2} High Voltage DC (HVDC):

e.g

- Convert power from one city to another. (Egypt & Jordan).



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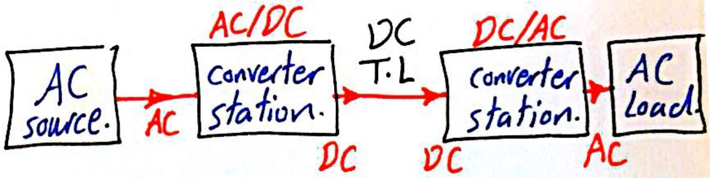
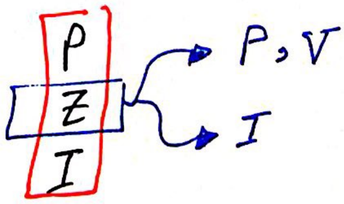


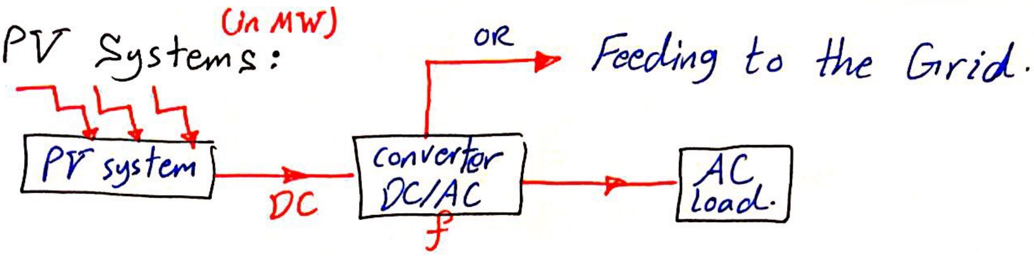
This operation could be reversed.

{3} Energy Efficiency:

- Like the inverters in the Airconditioners & Refrigerators.
- This applied using PIZ model.

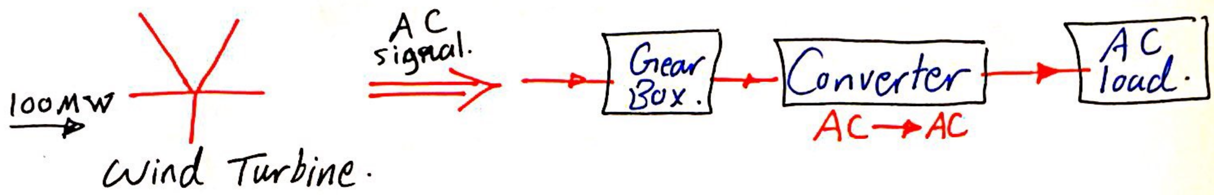


{4} PV Systems: (in MW)



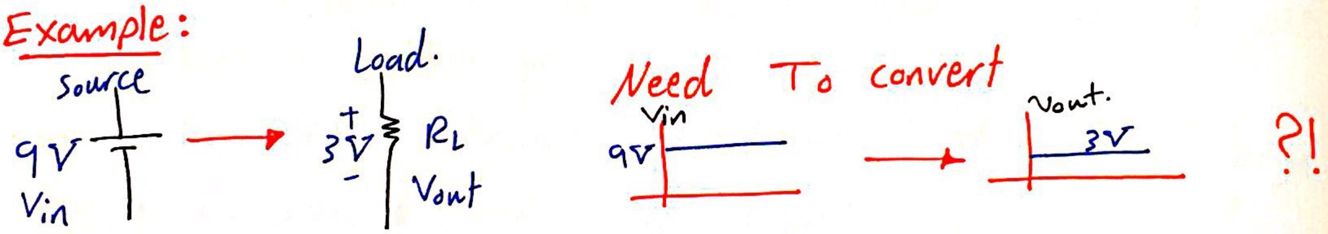
⇒ Here the inverter face a problem with synchronization with the exist frequancy.

{5} Wind Generation Systems :



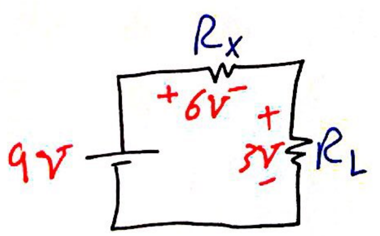
- The input signal has freq. & Amplitude changing because the speed of the wind, and the AC Load need fixed RMS & fixed freq , and this is solved By a converter (AC/AC).

Example:



Solutions:

- Simple solution by using Resistance R_x To drop the voltage.



⇒ $\boxed{R_x = 2R_L}$ → Problem due to Losses.

$$P_{in} = VI = 9I = \frac{27}{R_L}$$

$$P_{out} = \frac{V^2}{R_L} = \frac{9}{R_L}$$

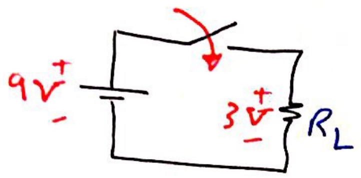
$\frac{18}{R_L}$ Watt lost @ R_x.

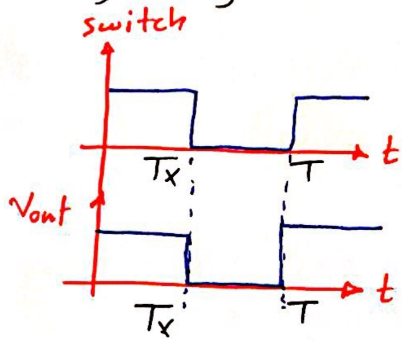
$$\eta = \frac{P_{out}}{P_{in}} = \frac{9/R_L}{27/R_L} = 33\% \text{ (very bad).}$$

→ Continue.

⇒

- Another solution by using a switch.



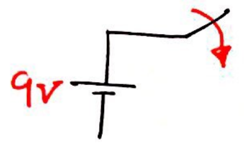


$$DC = Avg. = \frac{1}{T}\int_T v(t)\,dt$$

$$= \frac{1}{T}\int_0^{T_x} 9\,dt = 3$$

$$\Rightarrow \frac{9\,T_x}{T} = 3$$

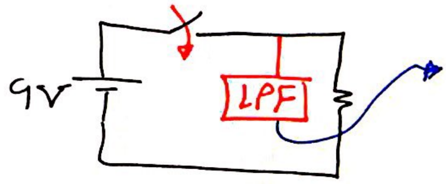
$$\Rightarrow \boxed{T_x = \frac{T}{3}}$$



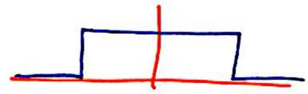
close ⇒ S.C ⇒ V = 0 so P = 0

open ⇒ O.C ⇒ I = 0 so P = 0

$\eta = 100\%$



Low-Pass-Filter.



- The converter already include the filter & the switch.

* For an Organization like our university, which is better to use PV system or Wind system?

PV system better, because Wind system Need very large area & make a very Large Noise.

* Types of Converters:

1. DC to AC (inverter) → used in PV systems.
2. AC to DC (Rectifier) → used in HVDC.
3. AC to AC ⇒ called AC chopper.

used for.
- Induction motor.
- RMS fixed to variable RMS.
- Variable RMS to fixed RMS.

(4) DC to DC ⇒ called DC chopper.

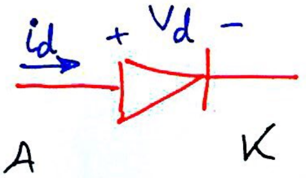
(5) Cyclo Converter (Not included in our Material).

↳ Control on the frequancy.

* Electronic Devices: → Diode. → Thy ristor. → Transistor.

1 Diode:

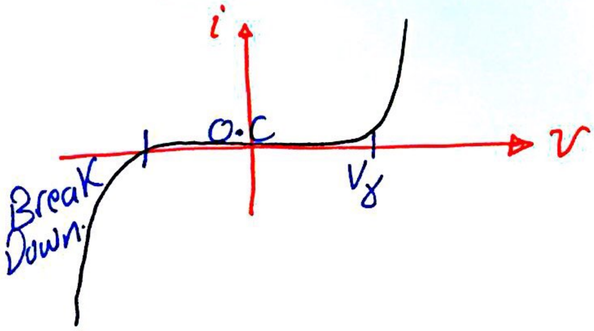
- It is Uncontrolable device (Because it depend on the connected circuit).



ideal iv-char:

work almost on any voltage > 0.
(we will deal with ideal diode in this course)

iv-char. of diode:

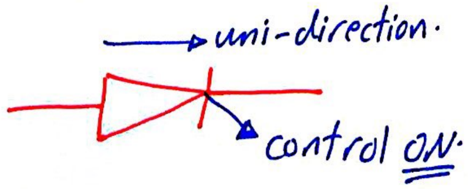


2 Thyristor:

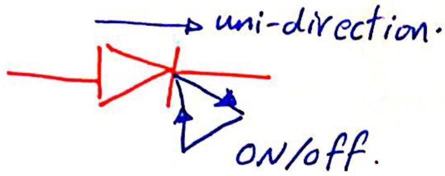
- Controlable device, But Not all types.
  → Control (ON)
  → Control (ON/off) → Need two circuits to do ON/off.

* Types:

1) • SCR ⇒ Silicon Control Rectifier.

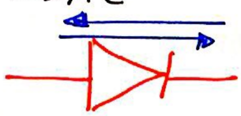


2) • GTO ⇒ Gate Turn Off.



"Comutation Circuit"

3) • TRIAC



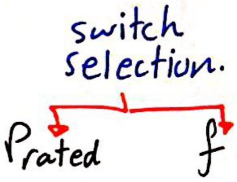
Bi-direction.

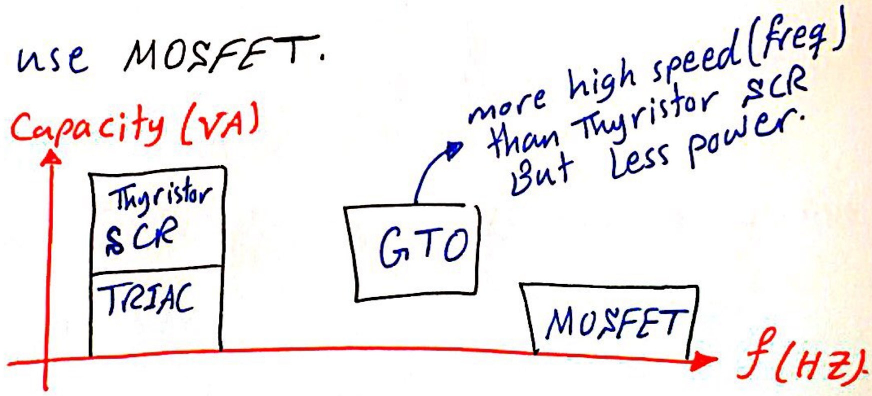
3 Transistor:

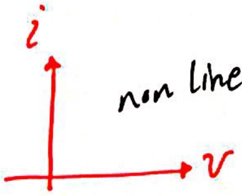
- Need just one circuit to do ON/off.
- Types: ① IGBT ② BJT ③ MOSFET

* All previous types will be used as a swith (ON/off) in PE.

* Electrical vehicles use MOSFET.





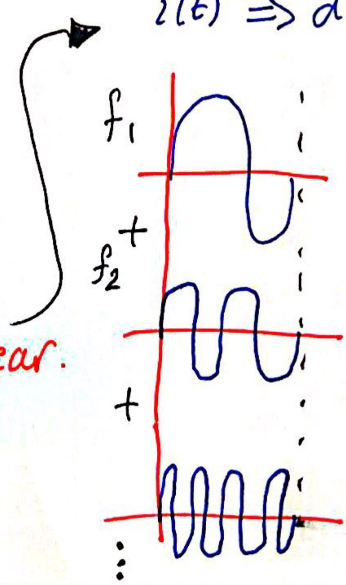


non linear loads.

$v(t) = A\cos\omega t$
$i(t) \Rightarrow$ sinusoidal } Linear load.

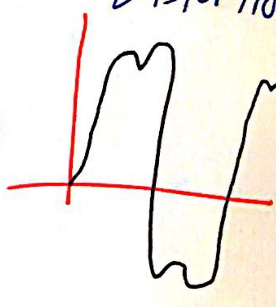
$i(t) \Rightarrow$ non sinusoidal } Non-linear.

$i(t) \Rightarrow$ distortion.



⇒

Distortion.

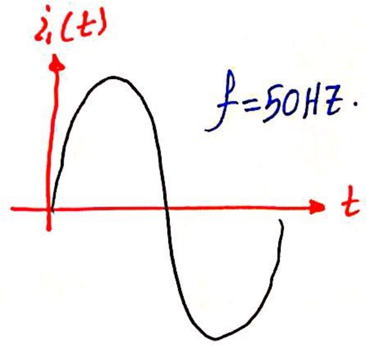


※ Harmonics:

* **Harmonics**: are non sinusoidal current, voltage. that are exist in sinusoidal network.

→ fundemental freq. , other freq.

50HZ
100HZ
150HZ
⋮



$f = 50HZ$. + other currents multiple freq.

if the current increased ↑:

1) Over loaded Network.
2) Over heating transformer.

} This added extra cost on the network.

3) 3^{rd} Harmonics – in nutral losses.
4) Machines (reverse in the Rotation).
5) Cause a decrease in pf. ↓

* Analysis of Harmonics:

The Best way by applying "Fourier Series".

- Fourier Series: for any non sinusoidal/periodic signal $f(t)$ could be written as a sum of cosines & sines.

$$f(t) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos(n\omega_0 t) + b_n \sin(n\underbrace{\omega_0}_{\text{Fundemental Frequancy}} t) \right)$$

a_0 = Avg. value = DC.

$$\Rightarrow a_0 = \frac{1}{T} \int_{-\frac{T}{2}}^{T/2} f(t)\,dt.$$

$$a_n = \frac{2}{T} \int_{-\frac{T}{2}}^{T/2} f(t) \cos(n\omega_0 t)\,dt$$

$$b_n = \frac{2}{T} \int_{-\frac{T}{2}}^{T/2} f(t) \sin(n\omega_0 t)\,dt.$$

* In Compact Form:

$$f(t) = a_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega t + \theta_n)$$

$$C_n = \sqrt{a_n^2 + b_n^2}$$

$$\theta_n = \tan^{-1}\left(\frac{-b_n}{a_n}\right)$$

or we could write it as: $f(t) = a_0 + \sum_{n=1}^{\infty} C_n \sin(n\omega t + \theta_n)$

$$\Rightarrow \quad \theta_n = \tan^{-1} \frac{a_n}{b_n}$$

* Evaluating RMS for $f(t)$ & the Average Power:

- RMS: $$F_{rms} = \sqrt{a_0^2 + \sum_{n=1}^{\infty} \left(\frac{C_n}{\sqrt{2}}\right)^2}$$

- Average Power: $$v(t) = \overset{DC}{V_0} + \sum_{n=1}^{\infty} V_n \cos(n\omega t + \theta_n)$$
$$i(t) = \overset{DC}{I_0} + \sum_{n=1}^{\infty} I_n \cos(n\omega t + \phi_n)$$

$$P_{avg} = \frac{1}{T} \int_T v(t)\, i(t)\, dt$$

$$\rightarrow \quad P_{avg} = V_0 I_0 + \sum_{n=1}^{\infty} V_{rms_n} I_{rms_n} \cos(\theta_n - \phi_n)$$

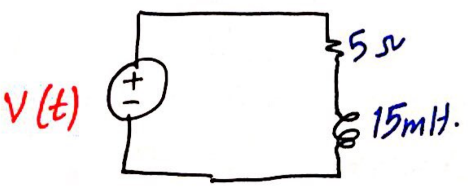
OR

$$P_{avg} = V_0 I_0 + \sum_{n=1}^{\infty} \frac{V_{max,n} I_{max,n}}{2} \cos(\theta_n - \phi_n)$$

Example: Non-sinusoidal voltage has the following fourier series form:

$$v(t) = 10 + 20\cos(2\pi * 60\, t - 25) + 30\cos(2(2\pi * 60)t + 20)$$

Connected cct as follows:



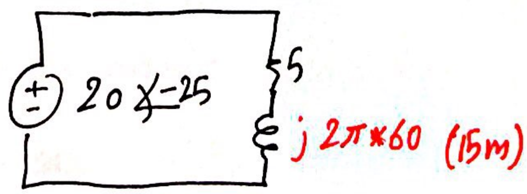
Find power absorbed by the load ?

solution: using $P_{av} = V_0 I_0 + \sum V_{rms,n} I_{rms,n} \cos(\theta_n - \phi_n)$

$I_0 = \frac{10}{5} = \boxed{2}\,A$. (remember @ DC, inductor s.c). $\Rightarrow P_0 = V_0 I_0$

$\boxed{P_0 = 20W}$

I_1 @ $f = 60HZ$.



$\Longrightarrow$ Contine.

$\Rightarrow I_1 = \dfrac{20\angle -25}{5 + j(2\pi * 60 * 0.015)} = \boxed{2.65\angle -73.5^\circ}\ A.$

$\hookrightarrow P_1 = \dfrac{20}{\sqrt{2}} * \dfrac{2.65}{\sqrt{2}} \cos(-25 + 73.5)$

$\Rightarrow \boxed{P_1 = 17.4\ W.}$

I_2 @ 120Hz: $I_2 = \dfrac{30\angle 20}{5 + j2\pi * 120 * (0.015)}$

$\boxed{I_2 = 2.43\angle -46.2}\ A.$

$\hookrightarrow P_2 = \dfrac{30}{\sqrt{2}} * \dfrac{2.43}{\sqrt{2}} \cos(20 + 46.2) \Rightarrow \boxed{P_2 = 14.8\ W.}$

$P_{Total} = 20 + 17.4 + 14.8 \Rightarrow P_T = 52.2\ W.$ #

※ Sinusoidal Source + Non Linear Load:

$v_s(t) = V_s \cos(\omega t + \theta_s).$

$I(t) = I_0 + \sum_{n=1}^{\infty} I_n \cos(n\omega t + \theta_n)$

$\Rightarrow P_{avg} = \frac{1}{T}\int v_s(t)\, I(t)\, dt.$

$= V_0 I_0 + \sum V_{rms_n} I_{rms_n} \cos(\theta_n - \phi_n).$

$n = 1$ for $v_s(t)$.

$\Rightarrow \underline{\underline{V_0 = 0}} \Rightarrow P_{avg} = V_{s,rms}\, I_{rms,1} \cos(\theta_s - \phi_1)$, $\boxed{pf = \frac{P}{S}}$

$\Rightarrow S = V_{rms} I_{rms} = V_{s,rms}\left[\sqrt{I_0^2 + \sum_{n=1}^{\infty}\left(\frac{I_n}{\sqrt{2}}\right)^2}\right]$

$\Rightarrow pf = \dfrac{\cancel{V_s}_{rms}\, I_{1,rms} \cos(\theta_s - \phi_1)}{\cancel{V_{s,rms}}\, I_{rms}.} \Rightarrow pf = \dfrac{I_{1,rms}}{I_{rms}} \cos(\theta_s - \phi_1)$

Distortion Factor ≡ DF.

Total

Cause Harmonics. pf↓

$$\boxed{DF = \frac{I_{1,rms}}{I_{rms}}}$$

* Total Harmonic Distortion: (THD)

$THD = \sqrt{\dfrac{I_{rms}^2 - I_{1,rms}^2}{I_{1,rms}^2}}$ OR $THD = \sqrt{\dfrac{\sum_{n\neq 1}^{\infty} I_{n,rms}^2}{I_{1,rms}^2}}$

I_{rms} ≡ Total wich include Harmonics.

$I_{1,rms}$ ≡ Fundemental RMS current.

$DF = \sqrt{\dfrac{1}{1 + (THD)^2}}$

** for S : $S = V_{rms} I_{rms} = V_{rms,s}\sqrt{I_o^2 + \sum_{n=1}^{\infty}\left(\frac{I_n}{\sqrt{2}}\right)^2} = \sqrt{P^2+Q^2}$

$P = V_{rms} \; I_{1,rms} \cos(\theta_s - \phi_1)$

$Q = V_{rms,s} \sum I_{rms} \sin(\theta_s - \phi_n)$ .... $S = \sqrt{P^2 + Q^2 + D^2}$

P → fund. Q → fund.

$\Rightarrow D = V_{1,rms}\sqrt{\sum_{n\neq 1}^{\infty} I_{n,rms}^2}$

$D^2 = V_{1,rms}^2\left(\sqrt{\sum_{n\neq 1}^{\infty} I_{n,rms}^2}\right)^2$

Example: $v(t) = 100\cos(377t)$

$i_L(t) = 8 + 15\cos(377t+30^\circ) + 6\cos(2*377t+45^\circ) + 2\cos(3*377+60^\circ)$

Find the following:

a) Power Absorbed by the load.
b) Pf of the load.
c) DF load current.
d) THD load current.

Solution:

a) $P_{av} = V_{s,rms} \; I_{1,rms} \cos(\theta_s - \phi_1) = \frac{100}{\sqrt{2}} * \frac{15}{\sqrt{2}} \cos(0-30) = \boxed{650\,W.}$

b) $Pf = \frac{I_{1,rms}}{I_{rms}} \cos(\theta_s - \phi_1)$

$I_{1,rms} = \frac{15}{\sqrt{2}}A$, $I_{rms} = \sqrt{(8)^2 + \left(\frac{15}{\sqrt{2}}\right)^2 + \left(\frac{6}{\sqrt{2}}\right)^2 + \left(\frac{2}{\sqrt{2}}\right)^2} = 14A.$

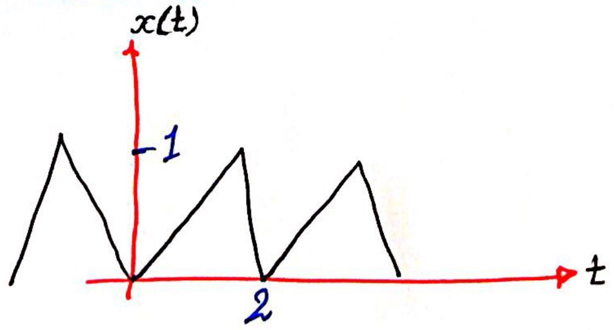
$\Rightarrow \boxed{Pf = 0.66}$

c) $DF = \frac{I_{1,rms}}{I_{rms}} = \boxed{0.76}$

d) $\boxed{THD = 0.86.}$

→ $THD = \sqrt{\frac{14^2 - \frac{225}{2}}{\frac{225}{2}}} = \underline{0.86}$

Example: Find THD:



a_0, a_n, b_n

$\Rightarrow$ RMS → THD ✓

⟹ solution.

Solution: Assume we need THD for first 5 Harmonics.

it is even signal so $\boxed{b_n = 0}$

$$a_0 = \frac{1}{T}\int_T x(t)dt = \frac{1}{2} * Area = \frac{1}{2}\left(\frac{1}{2}*2*1\right) \Rightarrow \boxed{a_0 = \frac{1}{2}}$$

$$a_n \Rightarrow x(t) = \begin{cases} t & , t>0 \\ -t & , t<0 \end{cases} \qquad a_n = \frac{2}{T}\int_{-1}^{1} x(t)\cos(n\omega t)\,dt$$

$$\Rightarrow a_n = 2\int_0^1 t\cos(n\omega t)\,dt \quad \text{(By parts)}$$

$$= 2\left[\frac{t}{n\omega}\sin(n\omega t) + \frac{\cos(n\omega t)}{(n\omega)^2}\right]_0^1 \qquad \text{Note: } \omega = \frac{2\pi}{T} = \pi$$

$$a_n = \frac{-2}{(n\pi)^2} + \frac{2\cos(n\pi)}{(n\pi)^2} = \frac{-2+2(-1)^n}{(n\pi)^2}$$

- n even $a_n = 0$
- n odd $a_n = \dfrac{-4}{(n\pi)^2}$

$$** \quad x(t) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{-4}{\pi^2(2n-1)^2}\cos((2n-1)\pi t)$$

$$= \frac{1}{2} + \frac{-4}{\pi^2}\cos(\pi t) + \frac{-4}{9\pi^2}\cos(3\pi t) + \frac{-4}{25\pi^2}\cos(5\pi t)$$

$$THD^2 = \frac{X_{rms}^2 - X_{rms,1}^2}{X_{rms,1}^2} \Rightarrow X_{rms}^2 = \frac{1}{4} + \left(\frac{4/\pi^2}{\sqrt{2}}\right)^2 + \left(\frac{4/9\pi^2}{\sqrt{2}}\right)^2 + \left(\frac{4/25\pi^2}{\sqrt{2}}\right)^2$$

$$= 0.333$$

$$X_{rms,1}^2 = \left(\frac{4/\pi^2}{\sqrt{2}}\right)^2 = 0.082$$

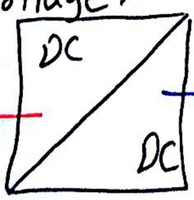
So Now:

$$THD = \sqrt{\frac{0.333-0.082}{0.082}} \Rightarrow THD = 1.75$$

※ DC-DC Converter: (DC chopper).

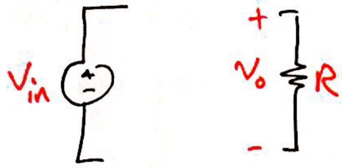
- Application: 1) DC power source. 2) Speed DC motors.
  ↳ Control the speed of the DC motor.

voltage.



constant voltage (fixed). → different voltage out.

- Idea:

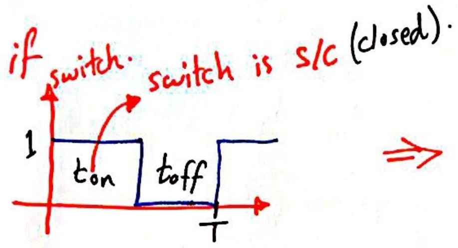


Desired $V_o < V_{in}$.

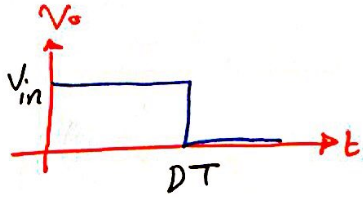
↳ solution: using ideal switch.

⇓

$V_{o_{avg}} = DC$.



⇒



$$V_{o_{avg}} = \frac{1}{T}\int_0^{DT} V_{in}\,dt$$

$$\Rightarrow \boxed{V_{o_{avg}} = DV_{in}}.$$

$D \equiv$ Duty Factor. $\Rightarrow \boxed{D = \frac{t_{on}}{T} = \frac{t_{on}}{t_{on}+t_{off}}}$

⇒ solution also include using LPF.

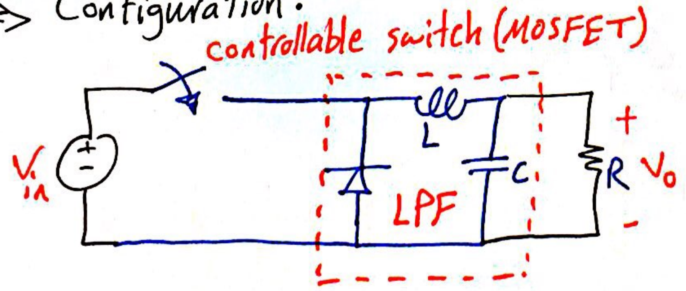
$$\boxed{f_{switching} = \frac{1}{T}}$$

* • Three Types of DC-DC converter:

1) step down (Buck). 2) step up (Boost). 3) step down/step up (Buck/Boost).

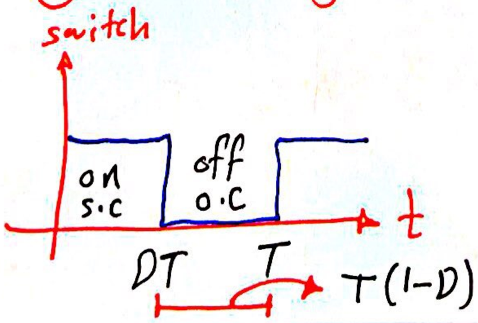
- Buck:

⇒ Configuration:



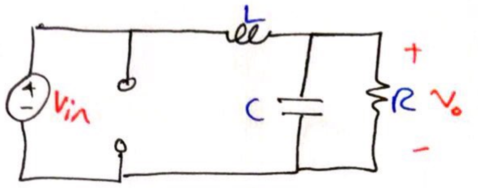
* To study this circuit: (Assumptions).

① Switch signal: assume ideal switch.



case (1):

** when switch closed:

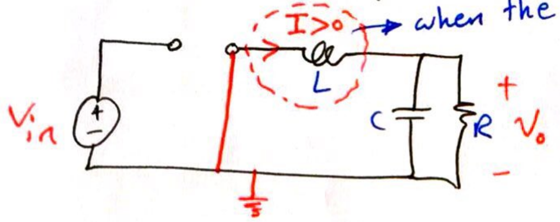


$\Rightarrow$ Diode $\rightarrow$ Reverse (o.c).

case(2):

** when switch opened:

$I>0 \rightarrow$ when the inductor in continuous mode.



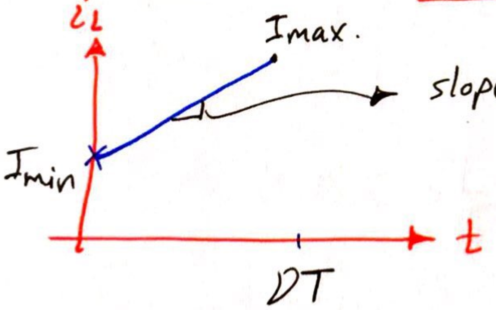
$\Rightarrow$ Diode $\rightarrow$ Forward (S.C).

Analysis:

for Case(1): @ steady state (S.S) $\left[\begin{array}{l} I_{av,C} = 0 \ (o.c) \\ V_{avg,L} = 0 \ (S.C). \end{array}\right.$

(C.C.M) continuous Current Mode. $I_L > 0$.

$$L\frac{di_L}{dt} = V_L \Rightarrow \frac{di_L}{dt} = \frac{1}{L}(V_{in} - V_{out}) \rightarrow \text{since C is o.c \& } V_L = V_{in} - V_{out}.$$

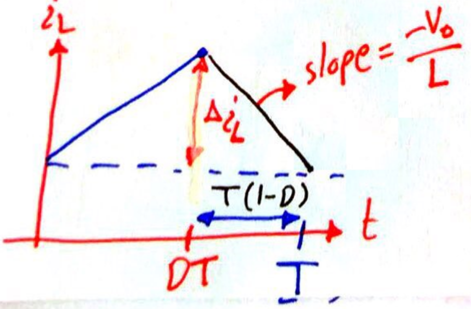


$$\text{slope} = \frac{V_{in} - V_o}{L} \equiv \frac{\Delta i_L}{\Delta t} = \frac{I_{max} - I_{min}}{DT}$$

$$\Rightarrow \boxed{(\Delta i_L)_{close} = \frac{DT}{L}(V_{in} - V_o)}$$

for Case(2):

$$L\frac{di_L}{dt} = V_L = 0 - V_o \Rightarrow \frac{di}{dt} = \frac{-V_o}{L}$$



for S.S, C.C.M:

$$\frac{(\Delta i)_{open}}{T(1-D)} = \frac{-V_o}{L} \Rightarrow \boxed{(\Delta i)_{open} = \frac{-V_o}{L}(1-D)T}$$

Given S.S, C.C.M :

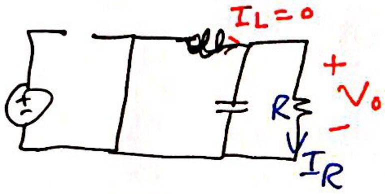
$(\Delta i)_{close} + (\Delta i)_{open} = 0 \Rightarrow \frac{DT}{L}(V_o - V_{in}) + \left(\frac{-V_o}{L}\right)(1-D)T = 0$

$\Rightarrow DV_{in} - DV_o = V_o - DV_o \Rightarrow V_o = DV_{in}$

- We want To design Buck converter (LPF), so find C, L.

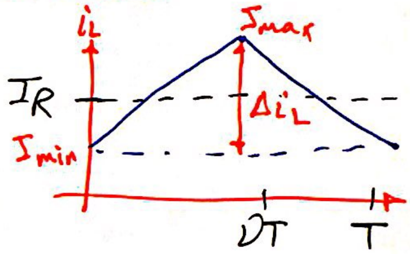
* To find I_{min} & I_{max}: C.C.M

$I_{min} > 0$



S.S: $I_{av,C} = 0$

$I_{L,avg} = I_R = \frac{V_o}{R}$



$$\boxed{\begin{aligned} I_{max} &= I_R + \frac{\Delta i_L}{2} \\ I_{min} &= I_R - \frac{\Delta i_L}{2} \end{aligned}}$$

for the inductor:

we choose $(\Delta i)_{open}$ since we want a relation with just V_o.

$\Rightarrow I_{min} = \frac{V_o}{R} - \frac{1}{2}\left|\left(\frac{-V_o}{L}\right)(1-D)T\right| > 0$ (C.C.M) [L_{min}]

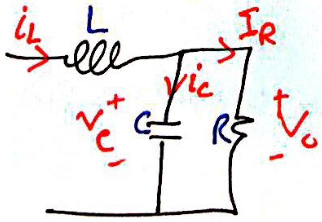
$\frac{V_o}{R} + \frac{1}{2}\left(\frac{V_o}{L}\right)(1-D)T > 0 \Rightarrow \frac{1}{2}\frac{V_o}{L}(1-D)T > \frac{-V_o}{R}$

$\Rightarrow (1-D)\frac{T}{2} > \frac{-L}{R} \Rightarrow L > \frac{1}{2}RT(1-D) \Rightarrow L_{min}$

$L_{min} = \frac{R}{2}T(1-D)$ $\Rightarrow T = \frac{1}{f}$ so $L \propto \frac{1}{f}$ (To have L_{min} use Higher freq.)

for the capacitor:

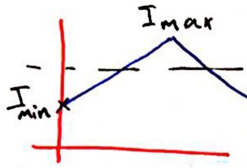
Need to minimize V_o variation. (Ripple $= \frac{\Delta V_o}{V_o}$). need it $<< 1$



$C\frac{dV_c}{dt} = i_c \Rightarrow V_c = \frac{1}{C}\left(\int i \cdot dt\right)$

$\frac{\Delta q}{\Delta t} = i \Rightarrow \Delta q = i\Delta t$ q

$i_C = i_L - I_R$



$I_{min} = \frac{V_o}{R} - \frac{1}{2}\left(\frac{V_o}{L}\right)(1-D)T$

• $I_{C_{min}} = I_{min} - I_R \quad \left(= \frac{V_o}{R}\right)$

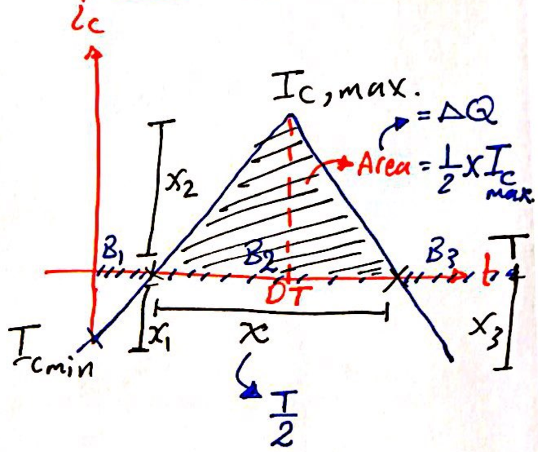
$I_{C_{min}} = -\frac{1}{2}\left(\frac{V_o}{L}\right)(1-D)T$

• $I_{C,max} = I_{L_{max}} - \frac{V_o}{R}$

$\Rightarrow \quad I_{C_{max}} = \frac{T V_o (1-D)}{2L}$

Derive I_{max}:
it will result:

$I_{max} = V_o\left(\frac{1}{R} + \frac{T(1-D)}{2L}\right)$



from $x = \frac{T}{2}$ & $I_{C_{max}}$
find C?!

$i_c = C\frac{dV_c}{dt}$

$i_c = \frac{dq}{dt} = C\frac{dV_c}{dt} \Rightarrow \boxed{\Delta q = C\,\Delta V_C}$

same ΔV_o

$$\boxed{\begin{aligned} I_{max,L} &= V_o\left(\frac{1}{R} + \frac{(1-D)}{2Lf}\right) \\ I_{min,L} &= V_o\left(\frac{1}{R} - \frac{(1-D)}{2Lf}\right) \\ L &= \frac{(V_{in}-V_o)D}{\Delta i_L f} \end{aligned}}$$

$\longrightarrow L = \frac{\left(\frac{V_o}{D} - V_o\right)D}{\Delta i_L f} \Rightarrow L = \frac{V_o(1-D)}{\Delta i_L f}$

"used when R is"
Unknown.

* Ripple $= \frac{\Delta V_o}{V_o}$, $i_c = i_L - I_R$

* Note on the graph of i_c vs. t: Distance $x_1 = x_2 = x_3$

also $B_1 + B_3 = B_2$ $\quad B_1 + B_2 + B_3 = T$ $\quad$ so $B_2 = \frac{T}{2}$

$i_c = C\frac{dV_c}{dt}$, $\Delta Q = C\Delta V$

$\Rightarrow i_C = \frac{dq}{dt} \Rightarrow \Delta q = \int i\,dt \quad (i > 0)$

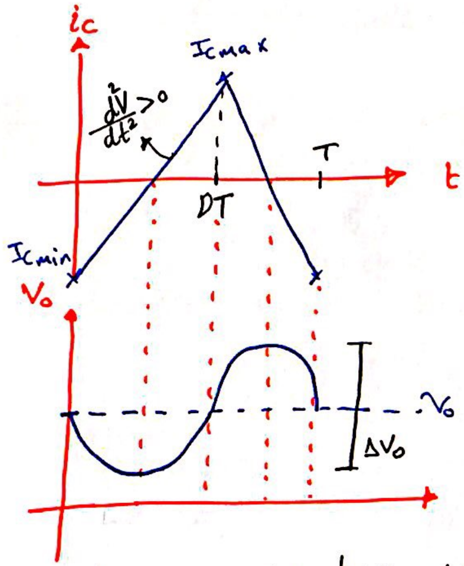
so $\Delta V = \frac{\Delta Q}{C} = \frac{\int i\,dt}{C}$

$\Delta Q =$ positive area of $I_C = \frac{1}{2} B_2 I_C = \frac{1}{2}\left(\frac{T}{2}\right)\left(\frac{V_o(1-D)}{2Lf}\right)$ $\Longrightarrow$

$$\Delta V_o = \frac{\Delta Q}{C} = \frac{V_o(1-D)}{8Lf^2C} \Rightarrow \text{Ripple} = \frac{\Delta V_o}{V_o} = \frac{(1-D)}{8Lf^2C}$$

↳ from this equation we find C.

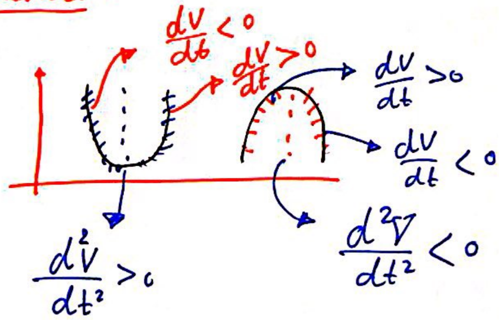
*** Note:** In practice: $L = 1.25\, L_{min}$ $\uparrow f \Rightarrow$ heat loss $\uparrow$



$$i_c = C\frac{dV}{dt}$$

$$\Rightarrow V = \frac{1}{C}\int i_c\, dt.$$

memember:



we have min & max of V_o when $i_c = 0$.

Example: Buck converter has the following:

$V_s = 50V$, $D = 0.4$, $L = 400\mu H$, $C = 100\mu F$, $f = 20KHZ$, $R = 20\Omega$

Find: a) V_o b) $I_{max,L}$, $I_{min,L}$ c) V_o, Ripple.

solution:

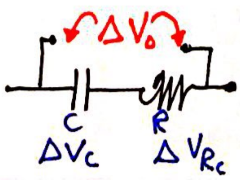
a) $V_o = DV_{in} = (0.4)(50) \Rightarrow \boxed{V_o = 20 \text{ volt.}}$

b) $I_{L,max} = V_o\left(\frac{1}{R} + \frac{(1-D)}{2Lf}\right) \Rightarrow \boxed{I_{Lmax} = 1.75A.}$

$I_{L,min} = V_o\left(\frac{1}{R} - \frac{(1-D)}{2Lf}\right) \Rightarrow \boxed{I_{Lmin} = 0.25A.}$

c) $\text{Ripple} = \frac{\Delta V_o}{V_o} = \frac{1-D}{8Lf^2C} \Rightarrow \boxed{\text{Ripple} = 0.47\%}$

*** Note:** In practive:



$\Delta V_o = \Delta V_C + \Delta V_{R_C}$ But ΔV_C very small compare to ΔV_{R_C}

$$* \Delta V_o \simeq \Delta V_{R_C} = i_c R_C *$$

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$$\Delta i_c = I_{C_{max}} - I_{C_{min}} = 2\left(\frac{V_o(1-D)}{2Lf}\right) \Rightarrow \Delta i_c = \frac{V_o(1-D)}{Lf}$$

for the example: $\boxed{\Delta i_c = 1.5A}$.

given $r_c = 0.1\Omega \Rightarrow$ Ripple $= \frac{\Delta V_o}{V_o} = \frac{(0.1)(1.5)}{20} = \underline{0.75\%}$

Homework: For the example Re-design the Buck (C_{new}, f_{new}) To make ripple $< 0.47\%$

Example: Design a buck converter to produce $V_o = 18V$. across $R = 10\Omega$ with switching freq. 40KHZ Given that:

- V_o Ripple $\leq 0.5\%$
- DC supply $= 48V$.
- Ideal component, C.C.M, S.S. ($r_c = 0$)

Determine:
a) Duty cycle.
b) L, C
c) Find the peak voltage for each device → diode, L, C
d) RMS of I_L, I_C.

Solution:

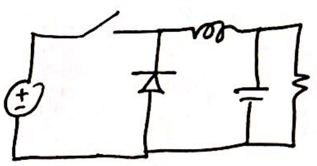
a) $D = \frac{V_o}{V_{in}} = \frac{18}{48} \Rightarrow \boxed{D = 0.375}$

b) $L_{min} = \frac{(1-D)R}{2f} \Rightarrow$ in Design: $L_{min} * 1.25 = L$

$\Rightarrow \underline{L_{min} = 78\mu H}$ $\quad \boxed{L = 97.5\mu H}$

$C = \frac{(1-D)}{8Lf^2 * \text{Ripple}} = \frac{1-0.375}{8*97.5\mu(40K)^2(0.005)}$

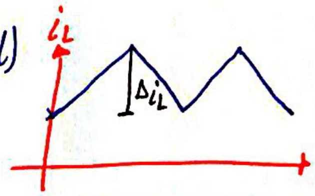
$\Rightarrow \boxed{C = 100\mu F}$.

c) 

• Diode:
switch closed $\Rightarrow V_D = V_{in} = \underline{48\,volt}$.
switch opened $\Rightarrow V_D = \underline{0}$

• L:
switch closed $\Rightarrow V_L = V_{in} - V_o = 48 - 18 = \underline{30V}$
switch opened $\Rightarrow V_L = -V_o = \underline{-18V}$

• C:
$V_c = V_o = \underline{18V}$.

d) 

$$I_{rms_L} = \sqrt{(DC)^2 + \left(\frac{\frac{\Delta i_L}{2}}{\sqrt{3}}\right)^2}$$

$\Delta i_L = DT\left(\frac{V_{in} - V_o}{L}\right) \Rightarrow \boxed{I_{L,rms} = 1.98A}$

For I_C (No DC):

$I_{C_{rms}} = \frac{\frac{\Delta i}{2}}{\sqrt{3}} \Rightarrow \boxed{I_{C_{rms}} = 0.83A.}$

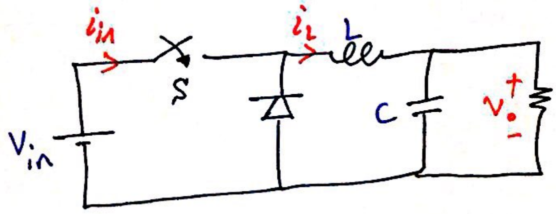
if we need $I_{L_{max}}$:

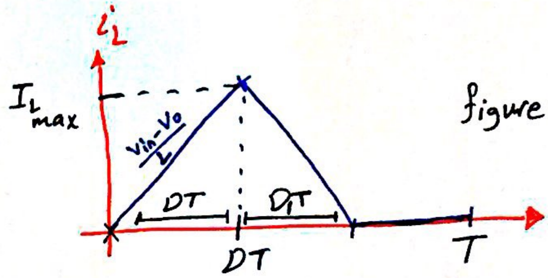
$$I_{L_{max}} = \frac{V_o}{R} + \frac{\Delta i}{2}$$

* Discontinuous Current Mode (D.C.M):

Need a relation for:

$V_o, V_{in} \Rightarrow D, D_1, R, L, C$



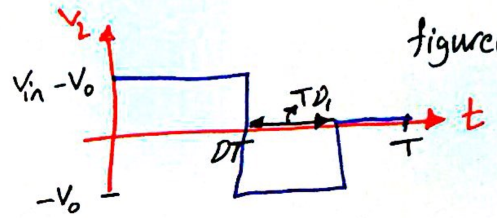


figure(1).

$V_L = L\frac{di}{dt}$

$0 \rightarrow DT \Rightarrow V_L = V_{in} - V_o$

$> DT \Rightarrow V_L = -V_o$



figure(2).

* for ideal case:

$P_{in} = P_{out} \Rightarrow V_{in} i_{in} = \frac{V_o^2}{R}$

$P_{avg,L} = 0 = \langle V_L \rangle \langle i_L \rangle$ so, $\langle V_L \rangle = 0$

@ S.S: $\langle V_c \rangle = 0$

For figure(2): $\langle V_L \rangle = 0 \Rightarrow (V_{in} - V_o) DT = V_o D_1 T$

$(V_{in} - V_o) D = D_1 V_o$

$\Rightarrow D V_{in} - V_o D = V_o D_1 \Rightarrow D V_{in} = V_o (D + D_1) \Rightarrow V_o = V_{in} \frac{D}{D + D_1}$

from figure(1):

$V_L = L\frac{di}{dt} = L\left(\frac{I_{max}}{DT}\right) = \frac{V_{in} - V_o}{L} * L \Rightarrow V_{in} - V_o = \frac{L\, I_{max}}{DT}$ (slope.)

$\Rightarrow \boxed{I_{max} = \frac{DT}{L}(V_{in} - V_o)}$

$\langle i_L \rangle = \frac{V_o}{R} = \frac{1}{T}\left(\frac{1}{2}(DT + D_1 T) I_{max}\right) = \frac{V_o}{R} \Rightarrow \frac{1}{2}(D + D_1) I_{max} = \frac{V_o}{R}$

$\Rightarrow (D_1 + D)\, DT \frac{(V_{in} - V_o)}{L} = \frac{2V_o}{R} \Rightarrow DT(D + D_1)(V_{in} - V_o) = \frac{2LV_o}{R}$

$\Rightarrow D^2 V_{in} - D^2 V_o + DD_1 V_{in} - DD_1 V_o = \frac{2L}{RT} V_o$ $\quad V_o = V_{in}\left(\frac{D}{D + D_1}\right)$

$(DD_1 + D^2) V_{in} - V_o\left(D^2 + DD_1 + \frac{2L}{RT}\right) = 0 \Rightarrow DD_1 + D^2 - \frac{D}{D + D_1}\left(D^2 + DD_1 + \frac{2L}{RT}\right) = 0$

Divide equation by D Then multiply by $(D + D_1)$:

$\Rightarrow (D_1 + D)(D_1 + D) = D^2 + DD_1 + \frac{2L}{RT} \Rightarrow D_1^2 + 2DD_1 + \cancel{D^2} = \cancel{D^2} + DD_1 + \frac{2L}{RT}$

$\Rightarrow D_1^2 + DD_1 - \frac{2L}{RT} = 0$

Contine. $\Longrightarrow$

Solving the equation: $D_1 = \dfrac{-D \pm \sqrt{D^2 - 4\left(\frac{-2L}{RT}\right)}}{2}$

negative answer is neglected.

$\Rightarrow$ So, $D_1 = \dfrac{-D + \sqrt{D^2 + \frac{8L}{RT}}}{2}$

$V_o = V_{in}\left(\dfrac{D}{D+D_1}\right)$

D.C.M. $\rightarrow D_1 < 1-D$.

Example: A Buck converter has the following parameters:

$V_{in} = 24V$, $L = 200\mu H$, $R = 20\Omega$, $f = 10KHZ$, $D = 0.4$

a) Is this Buck work in C.C.M or D.C.M ?

b) Find V_o ? c) Plot V_o as a function of D ?

Solution:

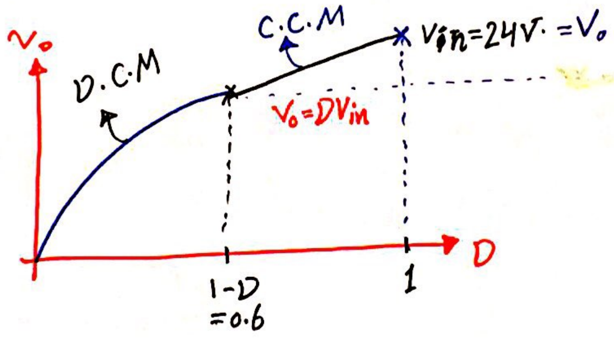
a) $D_1 = \dfrac{-0.4 + \sqrt{(0.4)^2 + \frac{8(200\mu)}{20 * \frac{1}{10K}}}}{2} = \underline{0.29}$, $1-D = 1-0.4 = 0.6$

since $D_1 < 1-D$

it is D.C.M

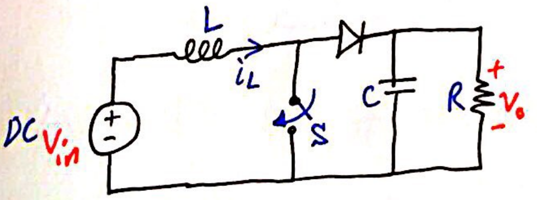
b) $V_o = 24\left(\dfrac{0.4}{0.29+0.4}\right) \Rightarrow \boxed{V_o = 13.9V}$.

c) $V_o = V_{in}\left(\dfrac{D}{D+D_1}\right)$



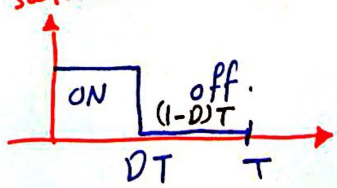
• Boost Converter:

* circuit configuration:



* Assumption:
  - ideal component.
  - steady state.
  - C.C.M ($I_L > 0$).

switch.



we need relations for:

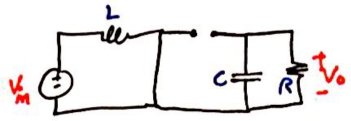
V_o, V_{in}

$i_c, V_c \Rightarrow$ Ripple $\Rightarrow C$

$I_{L_{avg}}, I_{L_{max}}, I_{L_{min}} \Rightarrow L$

case(1): switch closed. $(t \leq DT)$

- Diode Reverse (o.c).

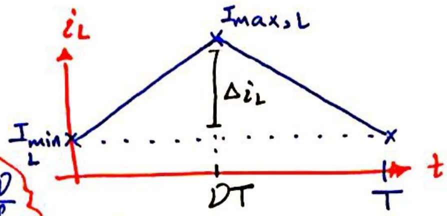


$V_L = V_{in}$

$V_L = L\frac{di_L}{dt} = V_{in} \Rightarrow \frac{di_L}{dt} = \frac{V_{in}}{L}$ = constant.

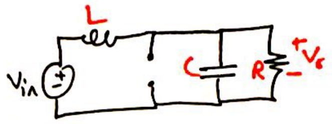
so $\frac{V_{in}}{L} = \frac{(\Delta i_L)_{closed}}{DT}$

$\Rightarrow (\Delta i_L)_{closed} = V_{in}\frac{DT}{L} = \frac{V_{in}D}{Lf}$



case(2): switch off $(t > DT)$

- Diode (S.C).



$V_L = V_{in} - V_{out} = L\frac{di_L}{dt}$

$\frac{di_L}{dt} = \frac{1}{L}(V_{in} - V_{out}) = \frac{(\Delta i_L)_{open}}{(1-D)T} \Rightarrow (\Delta i_L)_{open} = \frac{(V_{in}-V_o)(1-D)}{Lf}$

$\Delta i_{L_{open}} + \Delta i_{L_{closed}}$ = Zero $\Rightarrow \frac{V_{in}-V_o}{\cancel{Lf}}(1-D) + \frac{V_{in}D}{\cancel{Lf}} = 0 \Rightarrow \cancel{V_{in}} - \cancel{DV_{in}} - V_o + DV_o + \cancel{V_{in}D} = 0$

so $V_o = \frac{V_{in}}{1-D}$ ← Boost. $D<1$

Note that: if $D=1 \Rightarrow V_o = \infty$ (always switch is ON)

if $D=0 \Rightarrow V_o = V_{in}$ (always off)

$I_{L_{max}} = I_{L_{avg}} + \frac{\Delta i_L}{2}$

$I_{L_{min}} = I_{L_{avg}} - \frac{\Delta i_L}{2}$

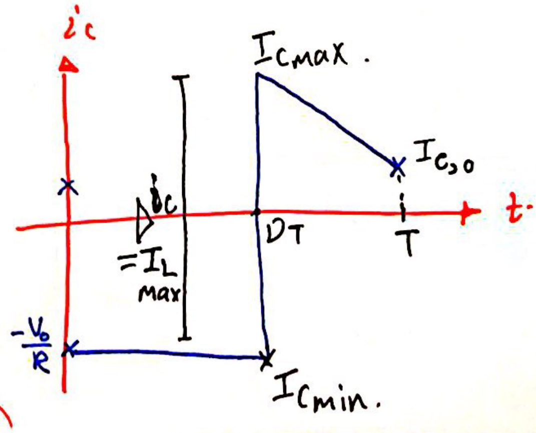
for $I_{L_{avg}}$: $P_{in} = P_{out} \Rightarrow V_{in} I_{L_{avg}} = \frac{V_o^2}{R}$

$\Rightarrow I_{L_{avg}} = \frac{V_o^2}{V_{in}R} = \left(\frac{V_{in}}{1-D}\right)^2\left(\frac{1}{V_{in}R}\right) \Rightarrow I_{L(avg)} = \frac{V_{in}}{(1-D)^2 R}$

$$I_{L_{max/min}} = \frac{V_{in}}{R(1-D)^2} \pm \frac{V_{in}D}{2Lf}$$

@ C.C.M : $I_{L_{min}} > 0$ we find L_{min}. $\frac{V_{in}}{(1-D)^2R} - \frac{V_{in}D}{2Lf} > 0 \Rightarrow L > \frac{D(1-D)^2R}{2f}$.

so $L_{min} = \frac{D(1-D)^2R}{2f}$



* for case(1): $i_c = \frac{-V_o}{R}$ (-ve current)

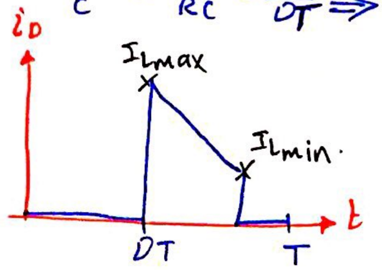
* for case(2): $i_c = I_L - \frac{V_o}{R}$

$\Delta i_c = \Delta i_L - \frac{V_o}{R}$

$= I_{L_{max}} - \frac{V_o}{R} - \left(\frac{-V_o}{R}\right)$ so $\Delta i_c = I_{L_{max}}$

⇒ Now To find C: $i_c = C\dfrac{dV_o}{dt} \Rightarrow \dfrac{\Delta V_o}{(1-D)T} = \dfrac{\Delta i_c}{C} = \dfrac{I_{Lmax}}{C}$

$\Rightarrow \dfrac{i_c}{C} = \dfrac{-V_o}{RC} = \dfrac{\Delta V_o}{DT} \Rightarrow \dfrac{\Delta V_o}{V_o} \equiv \text{Ripple} = \dfrac{DT}{RC} \Rightarrow \boxed{C_{min} = \dfrac{DT}{R * \text{Ripple}}}$.



Example: Design a Boost Converter that will have $V_o = 30V$, $V_s = 12V$, Ripple Voltage $\leq 1\%$, $R_L = 50\Omega$, $f_s = 25KHZ$. Given C.C.M.

Find: ➡ D, L_{min}, C, $I_{L\,max\,\&\,min}$?

Solution:

$V_o = \dfrac{V_{in}}{1-D}$

$30 = \dfrac{12}{1-D} \Rightarrow \boxed{D = 0.6}$

$L = \dfrac{DT(1-D)^2 R_L}{2} = 96\mu H$

In Design: $L = L_{min} * 1.25$

$\Rightarrow \boxed{L = 120\mu H}$

$C = \dfrac{D}{Rf * \text{Ripple}} \Rightarrow \boxed{C = 48\mu F}$

Now for I_L: $I_{L\,max/min} = I_{L\,avg} \pm \dfrac{\Delta i_L}{2}$

$I_{L\,avg} = \dfrac{V_{in}}{(1-D)^2 R} = 1.5A$.

$\Delta i_L = \dfrac{DT\,V_{in}}{L} = 2.4A$.

substitute:

$\boxed{\begin{array}{l} I_{L\,max} = 2.7A. \\ I_{L\,min} = 0.3A. \end{array}}$

Note $I_{L\,min} > 0$ so C.C.M.

Example: A Boost Converter has the following Requirements:

1) $V_o = 8V$, Load current $= 1A$.

2) $2.7 \leq V_{in} \leq 4.2$

3) Duty cycle is controlled to keep V_o constant, $f = 200KHZ$.

Determine: ① L s.t (such that) the variation in the inductor current $\leq 40\%\ I_{L\,avg}$.

② C s.t Ripple Voltage $\leq 2\%$. ③ Max R_c s.t Ripple voltage $\leq 2\%$

Solution:

for $V_{in} = 2.7V$

for $D \Rightarrow V_o = \dfrac{V_{in}}{1-D} \Rightarrow 8 = \dfrac{2.7}{1-D}$

$\boxed{D = 0.663}$

$L = \dfrac{DT\,V_{in}}{\Delta i_L}$; $\Delta i_L \leq 0.4\, I_{L\,avg}$

$\Rightarrow I_{L\,avg} = \dfrac{V_{in}}{(1-D)^2 R}$ ($V_{in} \to 2.7$, $D \to 0.663$, $R = \dfrac{V}{I} = \dfrac{8}{1}$) $\Rightarrow \boxed{I_{L\,avg} = 2.96A.}$

$\Delta i_L \leq 2.96 * 0.4 = 1.19A.$ so $L = 7.5\mu H$

for $V_{in} = 4.2$ volt. $\Rightarrow \boxed{\begin{array}{l} I_{L\,avg} = 1.9A. \\ L = 13.1\mu H. \end{array}}$

we select this L since it is Larger.

∗ for this design:

choose $D|_{@4.2V} = 0.475$ & $L = 13.1\mu H$.

$C = \frac{D}{Rf * ripple} = \frac{0.663}{8 * 200K * 0.02} = \boxed{20.7\mu F}$ choosing the higher C.

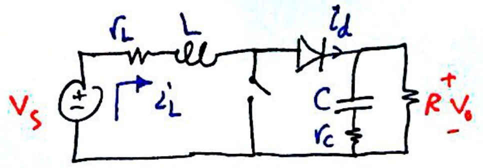
$I_{L_{max}} = I_{L_{avg}} + \frac{\Delta i_L}{2}$ $\quad \Delta i_L = \frac{DT V_s}{L}$

$I_{L_{max}} @ 2.7V = 2.96 + \frac{\Delta i_L}{2} = 3.6A.$ $\quad I_{L_{max}} @ 4.2V = I_{L_{avg}} + \frac{\Delta i_L}{2} = 2.28$

$\frac{\Delta V_o}{V_o} = 0.02 \Rightarrow \Delta V_o = 0.02 V_o \Rightarrow \Delta V_o = I_{L_{max}} r_C \Rightarrow r_C = \frac{0.02 * 8}{3.6} \Rightarrow \boxed{r_C = 44 m\Omega}$

$\hookrightarrow \Delta I_C = I_{L_{max}}.$

* * *



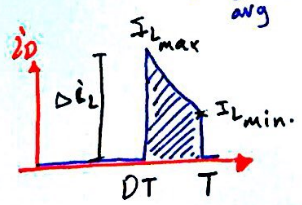
r_L in boost convertor effect on the relation between V_s & V_o unlike buck converter.

P_{loss} in Buck convertor $= 0$

Average: $P_{in} = P_{out} + P_{Loss}$

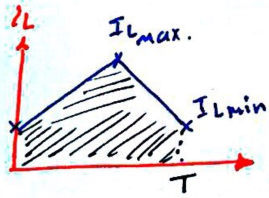
$V_s I_{L_{avg}} = V_o I_o + r_L I_{L_{avg}}^2$

$I_{o_{avg}} = I_{D_{avg}} - I_{C_{avg}}$ $\quad (I_{C_{avg}} \to 0)$



$\Rightarrow I_{D_{avg}} = \frac{1}{T}\left[(1-D)T\, I_{min} + \frac{1}{2}(1-D)T\,(I_{max} - I_{min})\right]$

$I_{o_{avg}} = \frac{(1-D)}{2}(I_{min} + I_{max})$



$I_{L_{avg}} = \frac{1}{T}\left(\frac{1}{2}T\,(I_{max} - I_{min}) + I_{min}T\right)$

$I_{L_{avg}} = \frac{1}{2}(I_{max} + I_{min})$

So, $I_{o_{avg}} = (1-D)\, I_{L_{avg}}$, $\quad P_{in} = V_s I_{L_{avg}} = V_o (1-D) I_{L_{avg}} + r_L I_{L_{avg}}^2$

$\boxed{V_o = \frac{V_s}{1-D}}$; without r_L.

$V_o = \frac{V_s}{1-D}\left[\frac{1}{1 + \frac{r_L}{R_o(1-D)^2}}\right]$; with r_L

r_L: inductor resistance.

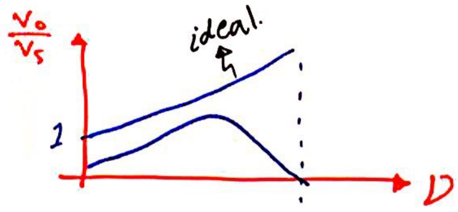
R_o: Load resistance.

$V_o \downarrow$

$P_o \downarrow = \frac{V_o^2}{R} = I^2 R$

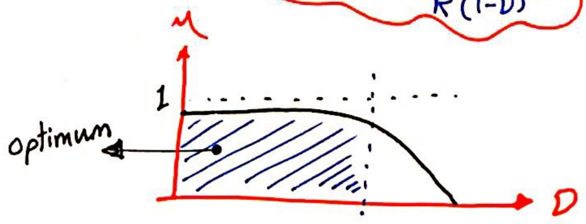
$I_o \downarrow$

$$\frac{V_o}{V_s} = \frac{1}{1-D}\left(\frac{1}{1+\frac{r_L}{R_o(1-D)^2}}\right)$$

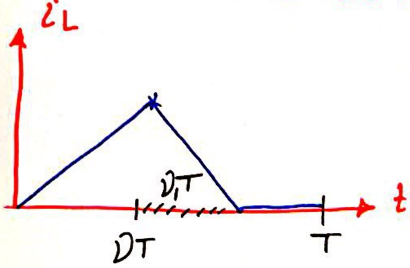


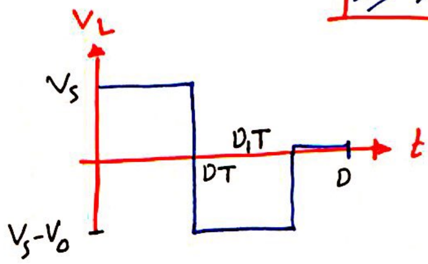
* Efficiency:

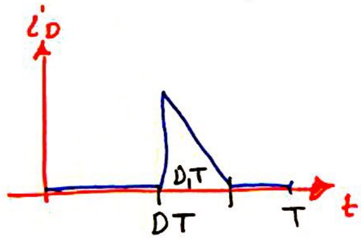
$$\eta = \frac{P_o}{P_{in}} = \frac{V_o^2/R = P_o}{P_o + \underbrace{P_{Loss}}_{\to I_L^2 r_L}} \Rightarrow \boxed{\eta = \frac{1}{1+\frac{r_L}{R(1-D)^2}}}$$



* Discontinuous Current Mode:







@ S.S: $V_{L_{avg}} = 0 \Rightarrow \frac{1}{T}\left[V_s DT + (V_s - V_o) D_1 T\right] = 0$

$\Rightarrow V_s D + V_s D_1 - V_o D_1 = 0 \Rightarrow \boxed{V_o = \frac{V_s}{D_1}(D + D_1)} \cdots ①$

$i_{D_{avg}} = I_{o_{avg}} = \frac{V_o}{R} \Rightarrow \frac{V_o}{R} = \frac{1}{T}\left[\frac{1}{2} D_1 T I_{L_{max}}\right]$

$\frac{di}{dt} = \frac{\Delta i_L}{\Delta t} = \frac{I_{max} - \overset{I_{min}}{0}}{DT} = \frac{V_s}{L} \Rightarrow \boxed{I_{max} = \frac{V_s DT}{L}}$

$\Rightarrow \boxed{D_1 = \frac{2 V_o L}{R V_s DT}}$ ②

substitute ② in ①: (solve the equation)

$\boxed{\frac{V_o}{V_s} = \frac{1}{2}\left(1 + \sqrt{\frac{2D^2RT}{L} + 1}\right)} \to$ Discontinuous.

$\boxed{\frac{V_o}{V_s} = \frac{1}{1-D}} \to$ Continuous.

Example: A Boost converter $V_s = 20V$, $C = 100\mu F$, $D = 0.6$, $L = 100\mu H$, $R = 50$, $f = 15KHZ$.

1) Verify that the inductor is in D.C.M ? 2) find V_o ? 3) find $I_{L_{max}}$?

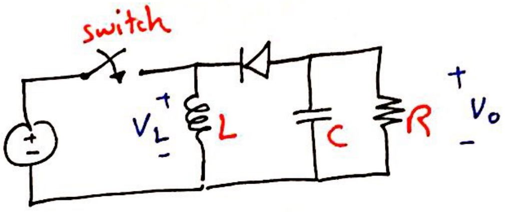
Solution:

1) $I_{L_{min}} = 0$ (→ C.C.M) OR < 0 (→ D.C.M) $\Rightarrow I_{L_{min}} = \frac{V_s}{R(1-D)^2} - \frac{V_s DT}{2L} = \underline{-1.5A}$.

2) $V_o = \underline{60 \text{ volt}}$.

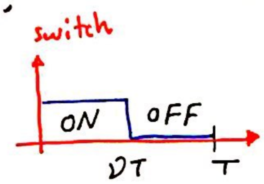
3) $I_{L_{max}} = \frac{V_s DT}{L} = \underline{8A}$.

• Buck - Boost Converter:



@ S.S, C.C.M:

$I_{L_{min}} \geq 0$



$f = \frac{1}{T}$

case(1): switch close: Diode o.c.

$V_{in} = V_L = L\frac{di}{dt} \Rightarrow \frac{\Delta i}{DT} = \frac{V_{in}}{L} \Rightarrow (\Delta i)_{close} = V_{in}\frac{DT}{L}$

case(2): switch open (off): Diode s/c.

$V_L = V_o = L\frac{di_L}{dt} \Rightarrow (\Delta i_L)_{open} = V_o\frac{(1-D)T}{L}$

DC Machines

• As we Know: $(\Delta i)_{close} + (\Delta i)_{open} = 0 \Rightarrow V_{in}\frac{D\cancel{T}}{\cancel{L}} + V_o\frac{(1-D)\cancel{T}}{\cancel{L}} = 0 \Rightarrow V_o = -\frac{V_{in} D}{1-D}$

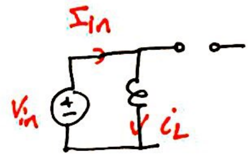
if $\frac{D}{1-D} > 1$ it will work as Boost.

$\Rightarrow D > 1-D \Rightarrow D > 0.5 \longrightarrow$ work as Boost.

$D < 0.5 \longrightarrow$ work as Buck.

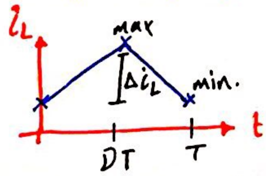
for Average Values:

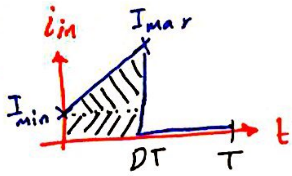
$P_{in} = P_{out} \rightarrow V_{in}(I_{in})_{av.} = \frac{V_o^2}{R}$, $V_o = V_{in}\left(\frac{D}{1-D}\right)$



$I_{in} = I_L \Rightarrow$ when $0 \rightarrow DT$.

& $I_{in} = 0$ when $DT \rightarrow T$





so $(I_{in})_{av} = \frac{1}{T}\left[DT I_{L_{min}} + \frac{1}{2}DT(I_{max} - I_{min})\right] = \frac{D}{2}\left[I_{L_{max}} + I_{L_{min}}\right]$

$(I_L)_{av} = \frac{1}{T}\left[T I_{L_{min}} + \frac{1}{2}T(I_{max} - I_{min})\right] = \frac{I_{max} + I_{min}}{2}.$

so $I_{in_{avg}} = D I_{L_{avg}}$

Now: $V_{in} I_{in_{avg}} = \frac{V_o^2}{R} \longrightarrow V_{in} D I_{L_{avg}} = \left(\frac{V_{in} D}{1-D}\right)^2 * \frac{1}{R} \longrightarrow I_{L_{avg}} = \frac{V_{in} D}{(1-D)^2 R}$

$\Rightarrow I_{L_{min}} = I_{L_{avg}} - \frac{\Delta i_L}{2} \Rightarrow \Delta i_L = \frac{DTV_{in}}{L}$

$\Rightarrow$ continue.

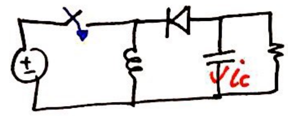
Now $I_{L_{min}}$ becomes:

$$I_{L_{min}} = \frac{V_{in} D}{(1-D)^2 R} - \frac{1}{2L} DT V_{in}$$

remember @ C.C.M $I_{L_{min}} > 0$ → we found L_{min}.

$$V_{in} D \left(\frac{1}{R(1-D)^2} - \frac{T}{2L} \right) > 0 \Rightarrow L_{min} = \frac{T}{2} R (1-D)^2$$

Now for C: from the Ripple.



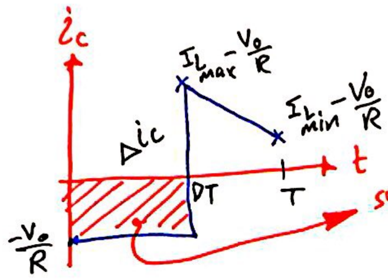
$$i_c = C \frac{dV}{dt} = C \frac{\Delta V_o}{\Delta t}$$

when switch on (Diode o/c).

$$i_c = \frac{-V_o}{R}$$

when switch off (Diode s/c).

$$i_c = I_L - \frac{V_o}{R}.$$



$\Delta i_c = I_{L_{max}}$.

switch ON Area $= DT \frac{V_o}{R} = i_c = C \frac{dV_o}{dt} = C \frac{\Delta V_o}{DT} \Rightarrow C = \frac{(DT)^2}{R * \text{Ripple}}$.

* if there is r_L There is NO effect.
* if there is r_c as we took in Buck: $\Delta V_o = \Delta i_c r_c$ & $\Delta i_c = I_{L_{max}}$.

Example: Buck-Boost Converter has the following: $V_{in} = 24V$, $D = 0.4$, $R_L = 5\Omega$, $L = 20\mu H$, $C = 80\mu F$, $f = 100 KHZ$.

Find: V_o, $I_{L_{avg}}$, $I_{L\,min\,\&\,max}$, Ripple?

Solution: since $D = 0.4$ here it is Buck.

$V_o = -V_{in} \frac{D}{1-D} \Rightarrow \boxed{V_o = -16V}$, $I_{L_{avg}} = \frac{V_{in} D}{R(1-D)^2} = \boxed{5.33A}$, $\Delta i_L = \frac{DT V_{in}}{L} = \boxed{4.8A}$

$I_{L_{min/max}} = 5.33 \mp \frac{4.8}{2}$ To be sure $I_{L_{min}}$ must be > 0.

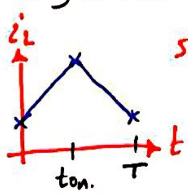
$$\text{Ripple} = \frac{\Delta V_o}{V_o} = \frac{(DT)^2}{RC}$$

Example: A Buck Converter has a switch that open & close @ 20 HZ. and remains close for 3msec. per cycle. if the avg Load current is 70A.

Find: Avg inductor current & Avg source Current?

Solution: $t_{on} = 3 msec$

$T = \frac{1}{20}$

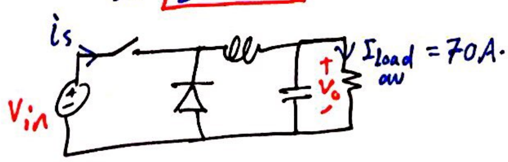


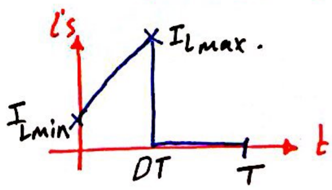
since $I_{c_{av}} = 0$ so $I_{L_{avg}} = I_{Load_{av}} = 70A$.

$\Rightarrow DT = t_{on}$

so $\boxed{D = 0.06}$

* when switch close: $i_s = i_L$
  " " open: $i_s = 0$





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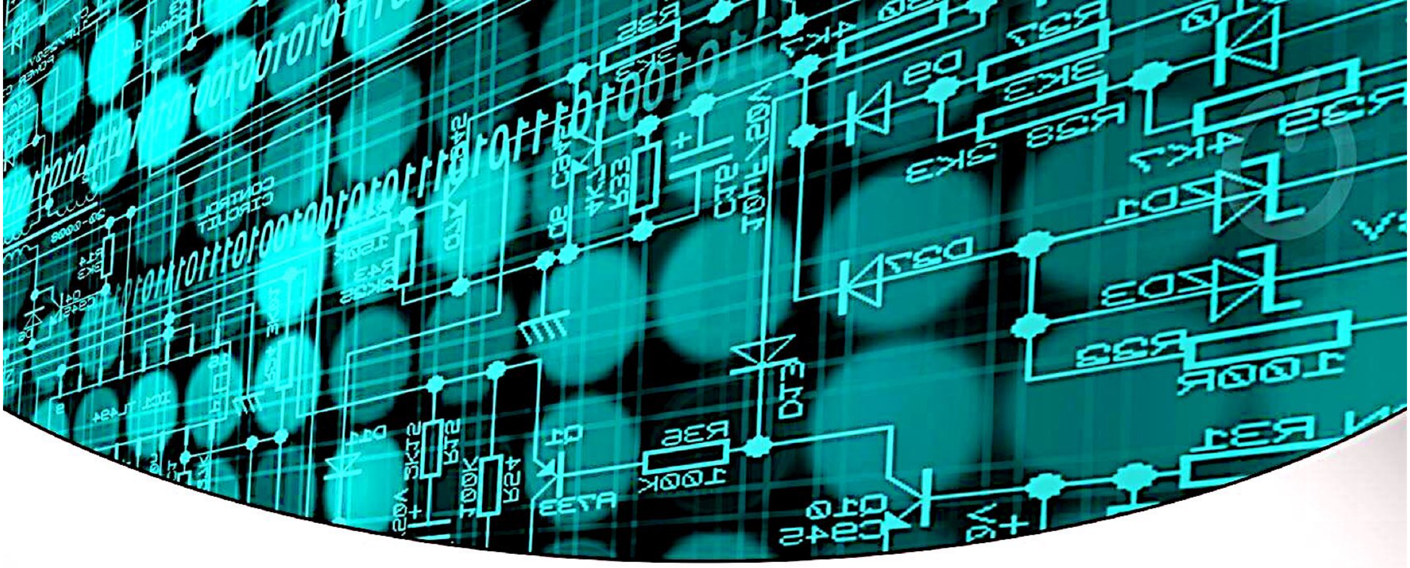
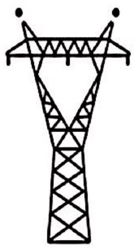


Power Electronics

Fall017

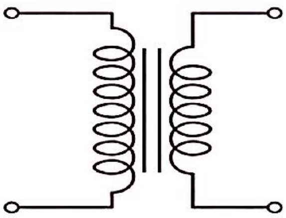




Dr.Sereen Althaher

By: Mhmd Abuhashya





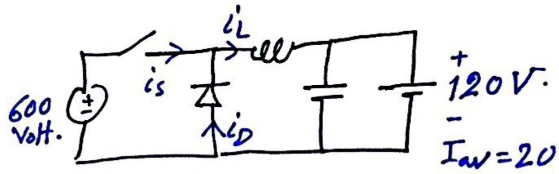
$$(i_s)_{avg} = \frac{1}{T}\left[DT\, I_{L_{min}} + \frac{1}{2} DT \left(I_{L_{max}} - I_{L_{min}}\right)\right] = \frac{D}{2}\underbrace{\left[I_{L_{max}} + I_{L_{min}}\right]}_{\to I_{L_{avg}}}$$

so $(i_s)_{avg} = D * I_{L_{avg}} = (0.06) * (70) = \underline{4.2A}$.

Example: We wish to charge a 120V battery from 600V. DC source using a DC chopper. The avg. battery current should be 20A with peak to peak ripple current 2A if the chopper freq is 200 HZ.

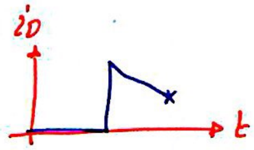
↳ $\Delta i = 2A$.

Find: 1) DC current drawn from the source.
2) DC current in the diode.
3) Duty cycle + ton
4) Inductance.



Solution: ① @ Avg. $P_{in} = P_{out} \rightarrow V_{in}(I_{in})_{avg} = V_o\, I_{o_{avg}}$

$$I_{in_{avg}} = \frac{120 * 20}{600} \Rightarrow \boxed{I_{in_{avg}} = 4A}$$



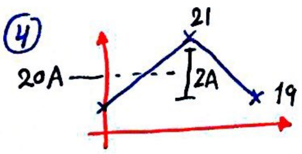
② $i_s + i_D = i_L \Rightarrow i_D = i_L - i_s$; $i_{L_{avg}} = i_{Load_{avg}}$ so $i_{D_{avg}} = 20 - 4 \Rightarrow \boxed{i_{D_{avg}} = 16A}$.

③ $$(I_D)_{avg} = \frac{1}{T}\left[(1-D)T\, I_{L_{min}} + \frac{1}{2}(1-D)\left(I_{L_{max}} - I_{L_{min}}\right)\right] = (1-D)\, I_{L_{avg}} = I_{D_{avg}}$$

so $16 = (1-D) * 20 \Rightarrow \boxed{D = 0.2}$ OR By using $(I_s)_{avg} = D\, I_{L_{avg}}$

$\Rightarrow \underline{D = 0.2}$

$\frac{t_{on}}{T} = D \Rightarrow t_{on} = 1\,msec.$

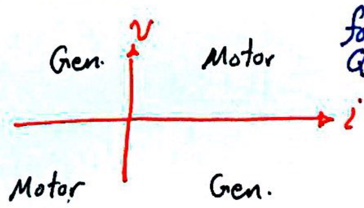
④ 

$\Delta i_L = \frac{V_o(1-D)T}{L}$ so $\boxed{L = 0.24H}$

*   *   *   end of first material

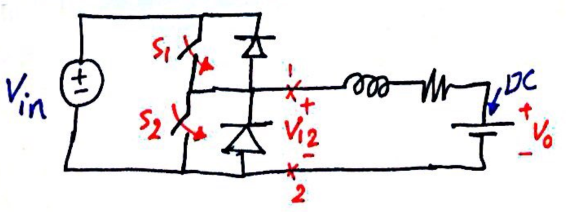
DC-DC: Buck } same polarity of voltage. } powerflow in unidirection.
Boost
Buck-Boost ⇒ change polarity (-).   source → Load.

Application: wind turbine. ⇒ { change Polarity, powerflow }

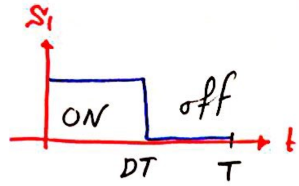


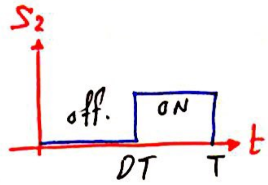
four Quadrant.

※ 2-Quadrent Converter:



* S_1 & S_2 Can't be closed @ the same time.
But Can be opened together for a short time, this called Dead Zone. ⇒ S_1 & S_2 off.





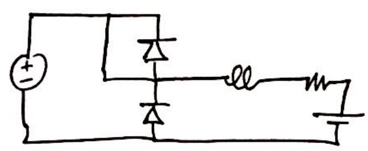
@ Av. inductor $V_L = 0$

$\Uparrow$

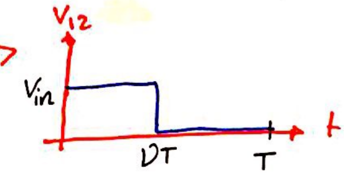
$(V_{12})_{av} = \frac{1}{T}[DT\,V_{in}]$

$V_{12} = D V_{in}$

Case(1):

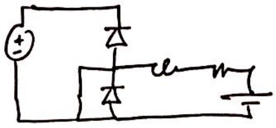


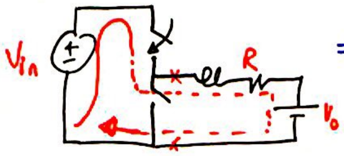
$\Rightarrow$



$\Rightarrow i_L = \frac{V_{12} - V_o}{R} \Rightarrow i_L = \frac{DV_{in} - V_o}{R}$

Case(2):





$\Rightarrow -V_{in} + Ri + V_o = 0$

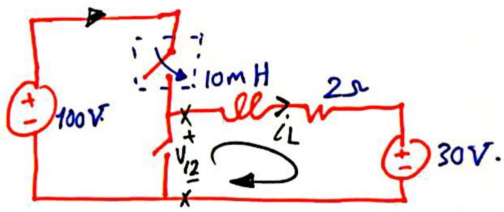
$V_o = V_{in} - Ri$

*when S_2 closed & S_1 opened $\Rightarrow$ it will make V_o feed the cct, reverse the direction of power flow.

Example: For the shown figure:

$D_{S_1} = 0.2$, $f = 20$ KHZ.

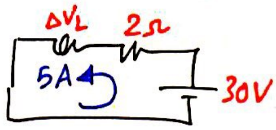
1) Find the value & the direction of DC current i_L ?
2) find Δi_L ?
3) if D increased to 0.45 repeat ① & ② ?



Solution:

① S_1 closed $\Rightarrow V_{12} = DV_{in}$ $\Rightarrow 2i_L + 30 - (0.2)(100) = 0 \Rightarrow \boxed{i_L = -5A}$

② $L\frac{di_L}{dt} = V_L \Rightarrow \frac{\Delta i_L}{\Delta t} = \frac{\Delta V_L}{L}$, when S_2 closed:



$\Rightarrow \Delta V_L = 30 - 2 * 5 = 20$ volt.

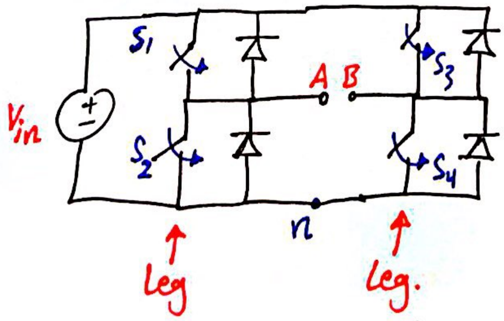
$\Delta t = (1-D)T$

$\Rightarrow \boxed{\Delta i_L = 0.08\,A.}$

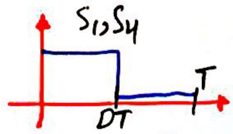
③ $V_{12} = 0.45 * 100 = 45$ volt.

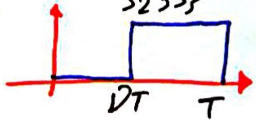
$\Rightarrow i_L = \frac{45-30}{2} = 7.5A$ (so changing D cause to reverse the direction of i_L).

4-Quadrent Converter:



S_1, S_4 ON, OR S_2, S_3 ON





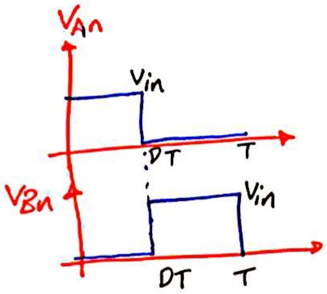
ALL S_1, S_2, S_3, S_4 off
"Dead Zone".

Case(1): S_1, S_4 closed.

$V_{An} = V_{in}$
$V_{Bn} = \text{Zero}$

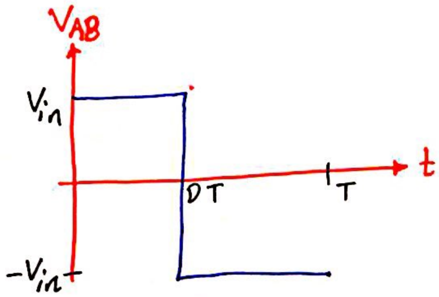
Case(2): S_2, S_3 closed.

$V_{an} = 0$
$V_{Bn} = V_{in}$



$\Rightarrow V_{An} - V_{Bn} = V_{AB}$

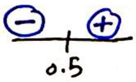
$\Rightarrow$



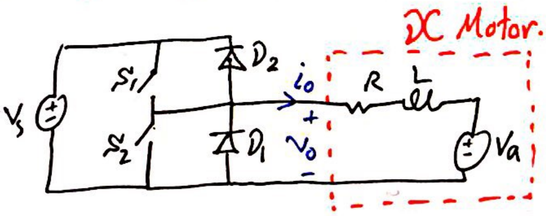
$[V_{AB}]_{av} = \frac{1}{T}\left[DTV_{in} - V_{in}T(1-D)\right] = DV_{in} - V_{in} + V_{in}D \Rightarrow \boxed{V_{AB} = V_{in}[2D-1]}$

$2D-1=0 \Rightarrow D=\frac{1}{2}$

$2D-1=1 \Rightarrow D=1$

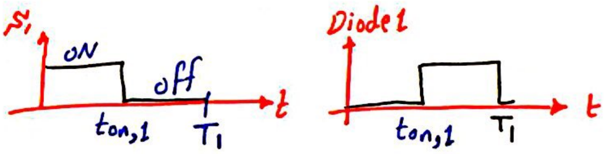


* 2-quadrant DC/DC:

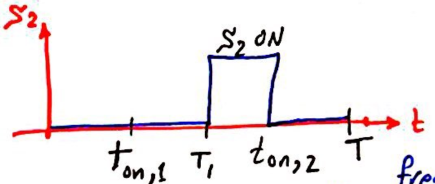


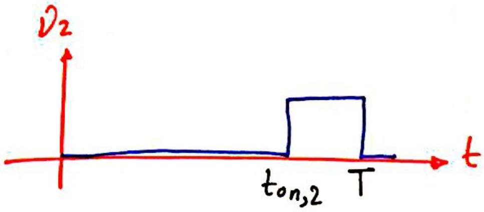
$S_1, D_1 \Rightarrow$ chopper 1.

$S_2, D_2 \Rightarrow$ chopper 2 (turned off).



* when S_2, D_2 ON:

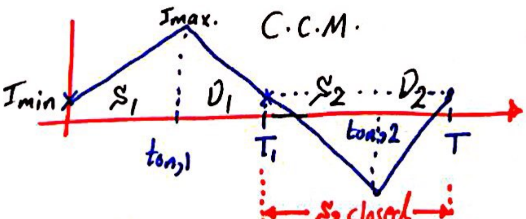




$T \equiv \frac{1}{f}$ frequancy of the chopper.

*Cases:

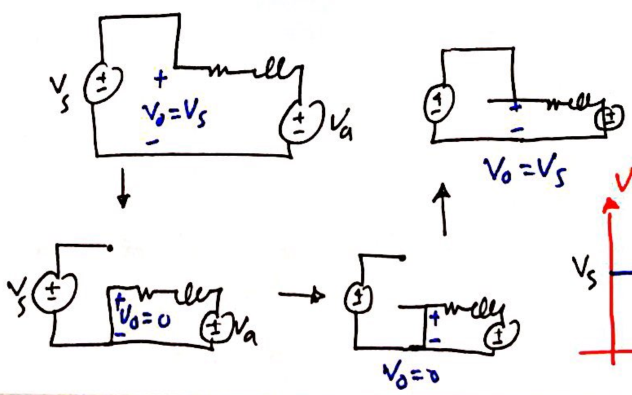
- S_1, D_1 (turned on).

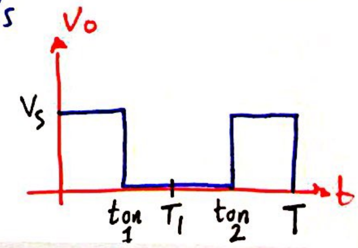


& others opened.
The D_2 F.A
& others opened.

case(1): S_1, D_1 are ON.

$\left.\begin{matrix} I_{max} > 0 \\ I_{min} > 0 \end{matrix}\right\} \rightarrow (I_o)_{avg} > 0$ so $P > 0$ (source $\rightarrow$ load) Q_1

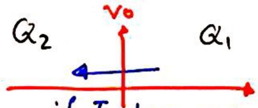




Case(2): $I_{min} < 0$, $I_{max} > 0$ $\Rightarrow$ $I_o > 0 \Rightarrow Q_1$; $I_o < 0 \Rightarrow Q_2$

case(3):

$P = V_o I_o$



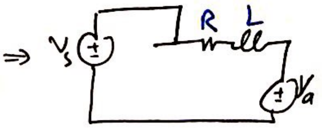
if I_o become Less than 0.

case(4):
S_1, D_1 turned off:
$t_{on,1} = 0$, $T_1 = 0$
S_2, D_2 are ON.

$I_{min} < 0$, $I_{max} < 0$ $\Rightarrow I_o < 0$

$\underline{Q_2}$

※Analysis:
Assume S_1, D_1 (ON).
∗ Take S_1 (ON), D_1 (off) $\Rightarrow$



$\Rightarrow \boxed{V_s = R i_o + L \dfrac{di_o}{dt} + V_a}$ differential Equation.

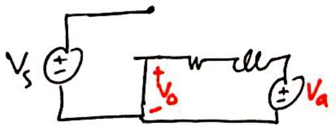
↓ if we solve for i_o:

$$i_o(t) = \frac{V_s - V_a}{R}\left(1 - e^{-t/\tau}\right) + I_{min}\, e^{-t/\tau} \quad \ldots \{1\}$$

$\Rightarrow$ for $\underline{0 < t \leq t_{on,1}}$ where: $\tau = L/R$

Note that:
Before we dealed in case $L >> 1$
(gives i_o Linear Relation).

∗ Take D_1 (ON).



$\Rightarrow \boxed{V_o = \text{Zero} = R i_o + L \dfrac{di_o}{dt} + V_a}$ $\Rightarrow$ for $t_{on,1} < t < T_1$

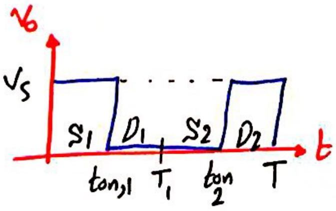
$$\Rightarrow i_o(t) = \frac{-V_a}{R}\left(1 - e^{-t/\tau}\right) + I_{max}\, e^{-t/\tau} \quad \ldots \{2\}$$

∗ from {1} & {2} you can obtain I_{min} & I_{max}:

$$I_{max} = \frac{V_s}{R}\left(\frac{1 - e^{-T_1/\tau}}{1 - e^{-T/\tau}}\right) - \frac{V_a}{R}$$

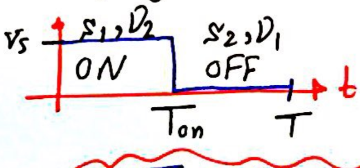
$$I_{min} = \frac{V_s}{R}\left(\frac{e^{T_1/\tau} - 1}{e^{T/\tau} - 1}\right) - \frac{V_a}{R}$$

$$\text{Ripple} = \Delta I = I_{max} - I_{min}.$$



$$\Rightarrow (I_o)_{avg} = \frac{(V_o)_{avg} - V_a}{R} \quad \ldots \boxed{3}$$

∗ for easier calculations:
Duty Cycle:



$$D = \frac{T_{on}}{T} \equiv \text{Duty Cycle}.$$

So Now find $[V_o]_{avg}$: $\bar{V_o} = \dfrac{1}{T}\displaystyle\int_0^{T_{on}} V_s\, dt$

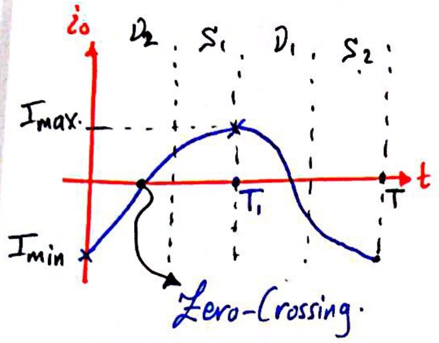
$$\Rightarrow \bar{V_o} = V_s \frac{T_{on}}{T} \Rightarrow V_{o\,avg} = D V_s \quad \ldots \boxed{4}$$

substitute $\boxed{4}$ in $\boxed{3}$:

$$I_{o\,avg} = \frac{D V_s - V_a}{R}$$

RMS for V_o: $V_{o\,rms} = \sqrt{\dfrac{1}{T}\displaystyle\int_0^{T_{on}} V_s^2\, dt} = \sqrt{\dfrac{T_o}{T} V_s^2}$

$$\Rightarrow V_{o\,rms} = \sqrt{D}\, V_s$$



* When we do Control: we avoid this Zero-crossing to occure, Because S/C happen @ the Machine. $I_o = \frac{DV_s - V_a}{R} \neq 0$

Example: 2Q-DC/DC converter Feeds an inductive load with $R = 10\Omega$, $L = 50mH$. & back emf $= 100V$ DC from a source of $340V$ DC. if the chopper is operate at $200HZ$. with 25% on-state duty cycle. Find:

1) $(V_o)_{avg}$, $(V_o)_{rms}$. 2) $(I_o)_{max,min}$, ΔI_o = Ripple. 3) $(I_o)_{avg}$. 4) $i_o(t)$.

5) Zero-crossing point of $i_o(t)$. 6) Plot V_o, i_o, I_s.

7) Find the value of D if $(I_o)_{avg}$ = Zero.

8) Find the value of V_a if $D = 0.25$, $I_o = 0$

Solution: $D = 0.25$

① $V_{o_{avg}} = DV_s \Rightarrow \boxed{V_{o_{avg}} = 85 \text{ volt}}$.

$V_{o_{rms}} = \sqrt{D}\, V_s \Rightarrow \boxed{V_{o_{rms}} = 170 \text{ volt}}$

② $I_{max} = \frac{V_s}{R}\left(\frac{1 - e^{-T_1/\tau}}{1 - e^{-T/\tau}}\right) - \frac{V_a}{R}$

$\Rightarrow \boxed{I_{min} = -4.38A \quad I_{max} = 1.9A.}$

$D = \frac{T_{on}}{T} = 0.25$

$\Rightarrow T_1 = 1.25$ msec.

$T = \frac{1}{200} = 5$ msec

$\tau = L/R$

④ $i_o = 24 - 28.38\, e^{-t/5m}$; $0 \leq t \leq 1.25m$

$i_o = -10 + 11.9\, e^{-t/5m}$, $t \leq T - T_1$

$\Rightarrow t \leq 3.75m$

③ $\overline{I_o} = \frac{\overline{V_o} - V_a}{R} = \frac{85 - 100}{10} = \boxed{-1.5A} \rightarrow \therefore Q_{II}$

$\Delta I_o = I_{max} - I_{min} = 1.9 - (-4.38) = \boxed{6.28A.}$

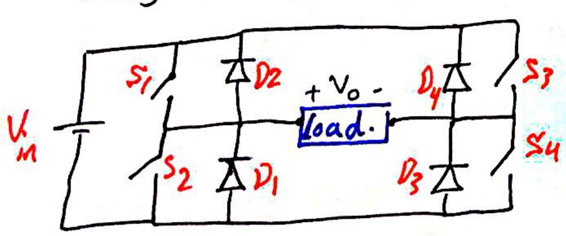
⑤ i_o = Zero $\Rightarrow 24 = 28.3\, e^{-t/5m}$

$10 = 11.9\, e^{-t/5m}$

$\Rightarrow \boxed{t = 0.83 \text{ msec.} \quad t = 0.87 \text{ msec.}}$

* Four Quadrant DC/DC Converter:

- Application: DC Motor.
- Configuration:



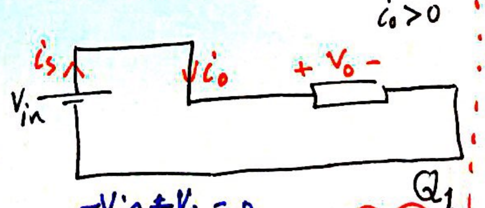
All of them work of freq. of the chopper, $f = \frac{1}{T}$.

Case (1): T_1 & T_4 (ON), D_1 & D_4 (OFF) Then D_1, D_4 (ON) T_1, T_4 (off.)

Case (2): T_2, T_3 (ON) Then D_2, D_3 (ON).

* for case(1): S_1, S_4 S.C.

$i_o > 0$



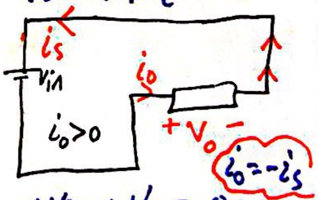
$-V_{in} + V_o = 0$

so $\boxed{V_{in} = V_o}$, $i_o = i_s$

Q_1

D_1, D_4 ON.

اتجاه i_s مع اتجاه Diode.



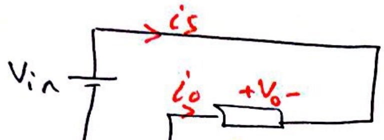
$+V_{in} + V_o = 0$

so $\boxed{V_o = -V_{in}}$

Q_4

* for Case (2):

S_2, S_3 S.C



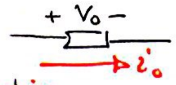
$i_o < 0$

$i_s = -i_o$

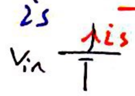
$\boxed{V_o = -V_{in}}$

Q_3

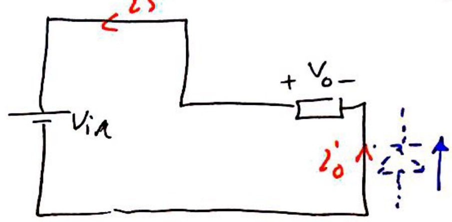
*Always By Convention direction of i_o



" of i_s



D_2, D_3 ON.



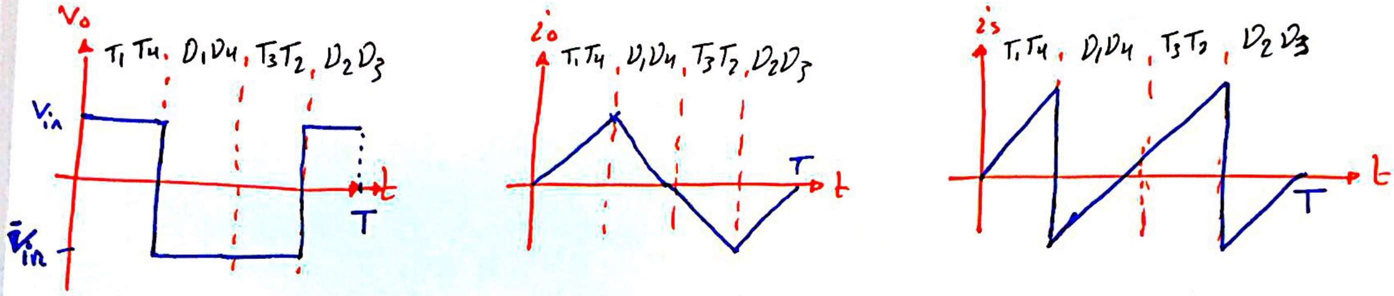
$i_o < 0$

$-i_o = -i_s$

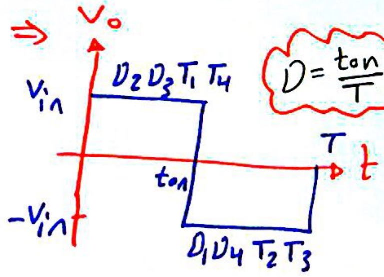
$\Rightarrow i_o = i_s$

$\boxed{V_{in} = V_o}$

Q_2



$\Rightarrow$



$D = \frac{t_{on}}{T}$

$$\Rightarrow V_{o_{av.}} = \frac{1}{T}\left[\int_0^{t_{on}} V_{in}\,dt + \int_{t_{on}}^{T} -V_{in}\,dt\right] = \frac{V_{in}}{T}\left[t_{on} - [T - t_{on}]\right]$$

$$\Rightarrow V_{o_{av.}} = \frac{V_{in}}{T}\left[2t_{on} - T\right] \Rightarrow \overline{V_o} = V_{in}[2D-1]$$

$$\overline{I_o} = \frac{\overline{V_o} - V_a}{R}$$

- when $D=0$ (T_1, T_4 off)

  $V_o = -V_{in}$

- $D = 1$ (T_1, T_4 ON)

  $V_o = V_{in}$

- Worst Case: $\overline{V_o} = 0$

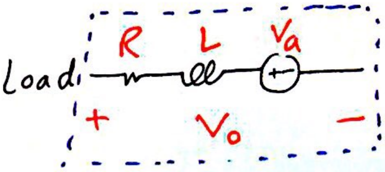
  $2D - 1 = 0$

  $D = 0.5$

$D > 0.5 \rightarrow V_o > 0$

$D < 0.5 \rightarrow V_o < 0$

Load:



$$\Rightarrow i_o R + L\frac{di_o}{dt} + V_a = V_{in}$$

* You Can Observe the following Relations:

① • $i_o(t) = \frac{V_s - V_a}{R}\left(1 - e^{-t/\tau}\right) + I_{min}e^{-t/\tau}$, $0 \le t \le t_{on}$ ($D \geq 0.5$).

② • $i_o(t) = -\left(\frac{V_s + V_a}{R}\right)\left(1 - e^{-t/\tau}\right) + I_{max}e^{-t/\tau}$, $0 \le t \le T - t_{on}$ ($D \leq 0.5$)

③ • $0 \le t \le t_{on}$ ($D < 0.5$)
$0 \le t \le T - t_{on}$ ($D \geq 0.5$)

$$i_o = \frac{-V_a}{R}\left(1 - e^{-t/\tau}\right) + I_{max}e^{-t/\tau}, \quad \begin{cases} 0 \le t \le t_{on}, & D < 0.5 \\ 0 \le t < T - t_{on}, & D \geq 0.5 \end{cases}$$

$$I_{max} = \frac{V_s}{R}\left(\frac{1 - e^{-\frac{t_{on}}{\tau}}}{1 - e^{-T/\tau}}\right) - \frac{V_a}{R}$$

$$I_{min} = \frac{V_s}{R}\left(\frac{e^{t_{on}/\tau} - 1}{e^{T/\tau} - 1}\right) - V_a/R$$

$$\tau = \frac{L}{R}$$

Example: 4-quadrant DC chopper feeds an inductive load. $R = 10\Omega$, $L = 50\,mH$, $D = 0.25$, back emf $= 55V$, $V_s = 340V$, $f_c = 200HZ$. Find:

(i) $\bar{V_o}$? (ii) $\bar{I_o}$, Determine quadrant of operation? (iii) if $\bar{I_o}$ is halved, find: a) Duty Cycle? b) $\bar{V_o}$?

Solution:

(i) $\bar{V_o} = V_s(2D-1)$
$\Rightarrow \boxed{\bar{V_o} = -170\,volt}$.

(ii) $\bar{I_o} = \dfrac{\bar{V_o} - V_a}{R}$
$\Rightarrow \boxed{\bar{I_o} = -22.5A}$.

$\Rightarrow$ since $\bar{I_o}$ & $\bar{V_o}$ are both $-ve$
$\Rightarrow$ Q_3 "Motor".

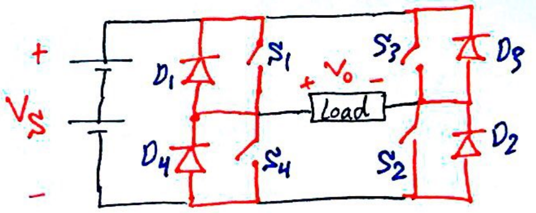
(iii) $I_o = \dfrac{-22.5}{2} = \dfrac{V_o - 55}{10} \Rightarrow \boxed{\bar{V_o} = -57.5\,volt}$; $\bar{V_o} = (2D-1)V_s \Rightarrow \boxed{D = 0.415}$

if we asked to find $i_o(t)$ $\Rightarrow$ Apply the relation with $D \leq 0.5$.

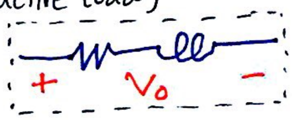
* * *

* Single Phase Inverter:

• Configuration:



* load : (inductive load)



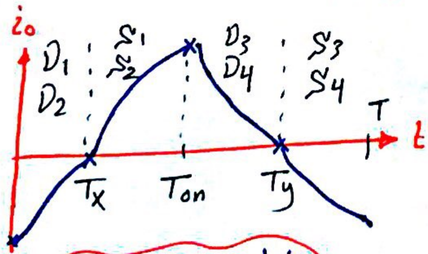
* No way that both Diode on the same leg work together (D_1 will be ON with D_2, and D_3 ON with D_4).

* here we will work on $D = 0.5$ which will give a zero DC value.

* fourier transform: $V_o(t) = \dfrac{4}{\pi} V_s \sum_{n=odd} \dfrac{1}{n} \sin n\omega_o t$, $\boxed{\omega_o = \dfrac{2\pi}{T} = 2\pi f_c}$

$\Rightarrow$ RMS value: $V_{o1_{rms}} = \dfrac{V_{o1}}{\sqrt{2}} = \dfrac{4}{\pi} \dfrac{V_s}{\sqrt{2}} \Rightarrow V_{o1_{rms}} = 0.9V_s$
($n=1$) ↳ fundemental freq.

• D_1, D_2 ($i_o < 0$)
• S_1, S_2 ($i_o > 0$)



$T_x, T_y \equiv$ time when zero crossing occurs.
To find them put $i_o(t) = 0$

• $t \leq \dfrac{T}{2}$: $i_o(t) = \dfrac{V_s}{R} - \left(\dfrac{V_s}{R} - I_{min}\right) e^{-t/\tau}$

• $t \geq \dfrac{T}{2}$: $i_o(t) = \dfrac{-V_s}{R} + \left(\dfrac{V_s}{R} + I_{max}\right) e^{\frac{-(t - T/2)}{\tau}}$

* fourier transform: $i_o(t) = \dfrac{4}{\pi} \sum_{n=odd} \dfrac{V_s}{nZ_n} \sin(n\omega_o t - \phi_n)$; $Z_n = \sqrt{R^2 + (n\omega L)^2}$

$\phi_n = \tan^{-1}\left(\dfrac{n\omega L}{R}\right)$

@ $n=1$:

$i_{o1_{rms}} = \dfrac{4}{\pi} \dfrac{V_s}{|Z_1|\sqrt{2}} \Rightarrow i_{o1_{rms}} = \dfrac{0.9V_s}{|Z_1|}$

$\boxed{P_1 = V_{rms} I_{rms}}$

Example: A single-phase inverter supplies $10\Omega + 50mH$ from $340V$ DC

$f = 50Hz$. Find:

(i) $i_o(t)$? (ii) $V_{1,rms}$? (iii) Zero-crossing points on i_o ?

Solution:

$$I_{max} = -I_{min} = \frac{V_s}{R}\left(\frac{1-e^{\frac{-T/2}{\tau}}}{1+e^{\frac{-T/2}{\tau}}}\right) \Rightarrow \text{for single phase converter.}$$

(i) $\tau = L/R$, $T = \frac{1}{f}$, $I_{max} = 25.9A$, $I_{min} = -25.9A$.

Substitute these values in both Relations of $i_o(t)$ $t \le \frac{T}{2}$ & $t \ge \frac{T}{2}$:

for $t \le \frac{T}{2}$: $i_o(t) = 34 - 59.9\, e^{-t/5m}$ A. for $t \ge \frac{T}{2}$: $i_o(t) = -34 + 59.9\, e^{-\left(\frac{t-10m}{5m}\right)}$

(ii) $V_{1rms} = 0.9 V_s \Rightarrow \boxed{V_{1rms} = 306 \text{ volt}}$.

(iii) find T_x & T_y:

$0 = 34 - 59.9\, e^{-T_x/5m}$

solving: $T_x = 2.83\,msec$

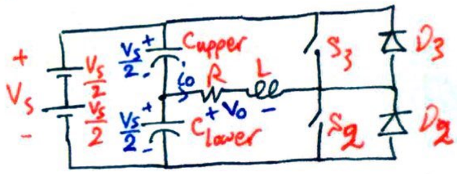
$0 = -34 + 59.9\, e^{-\left(\frac{T_y-10m}{5m}\right)}$

$T_y = 12.83\,msec$

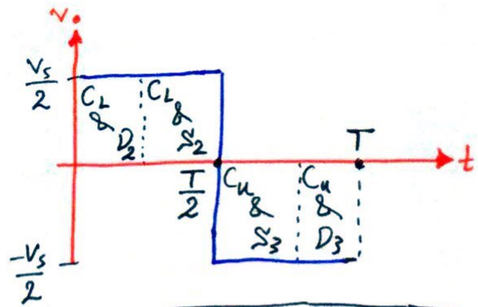
Example: A single phase inverter supplies a load with $R = 10\Omega$, $L = 50mH$ & fed from $340V$ DC source. if the bridge is operating @ $50Hz$. "Half Bridge Inverter"

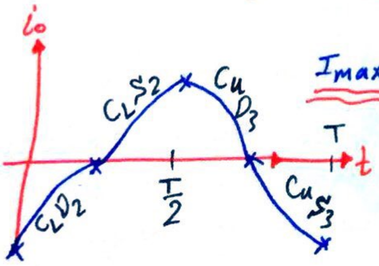
Find: a) V_o? b) $V_{o rms}$? c) $I_{o max}$? d) $i(t)$?

Solution:



- in case C_{upper}, S_3 (ON):
  $i_o < 0 \Rightarrow -V_o - \frac{V_s}{2} = 0 \Rightarrow V_o = \frac{-V_s}{2}$
- in case C_{upper}, D_3 (ON):
  $i_o > 0 \Rightarrow +\frac{V_s}{2} + V_o = 0 \Rightarrow V_o = \frac{-V_s}{2}$
- in case C_{lower}, S_2 (ON):
  $i_o > 0 \Rightarrow +V_o - \frac{V_s}{2} = 0 \Rightarrow V_o = \frac{V_s}{2}$
- in case C_{lower}, D_2 (ON):
  $i_o < 0 \Rightarrow -V_o + \frac{V_s}{2} = 0 \Rightarrow V_o = \frac{V_s}{2}$





$I_{max} = -I_{min}$.

$$V_{o rms} = \sqrt{\frac{1}{T}\int_0^{T/2}\left(\frac{V_s}{2}\right)^2 dt + \int_{T/2}^{T}\left(\frac{V_s}{2}\right)^2 dt} \Rightarrow V_{o rms} = \frac{V_s}{2}$$

$\Rightarrow V_{o rms} = 170$ volt.

$$I_{o max} = \frac{V_s/2}{R}\left(\frac{1-e^{\frac{-T_{on}}{\tau}}}{1+e^{\frac{-T_{on}}{\tau}}}\right)$$

$\Rightarrow \boxed{I_{o max} = 12.95A \quad I_{o min} = -12.95A}$

$T_{on} = \frac{T}{2} = 10\,msec.$

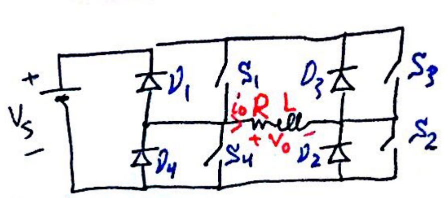
$\tau = L/R = 5\,msec.$

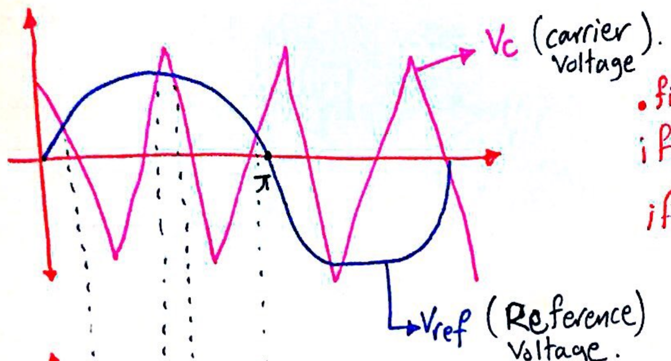
$I_{o max}$ & $I_{o min}$ same previous relation just replase V_s by $V_s/2$.

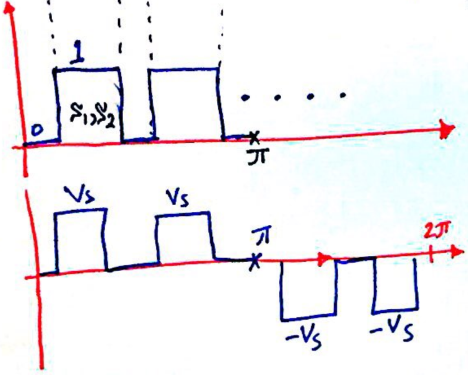
$$i(t) = \frac{V_s/2}{R} - \left(\frac{V_s/2}{R} - I_{min}\right)e^{-t/\tau}\ ;\ 0 < t \le 10\,msec.$$

*Pulse Width Modulation:

inverter (DC → AC) ⇒ sinusoidal.



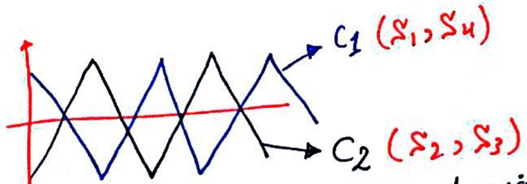




- first assumption:
  - if $V_{ref} > V_c \xRightarrow{\text{then}} 1$
  - if $V_{ref} < V_c \xRightarrow{\text{then}} 0$
- second assumption:
  - $S_1, S_2 \xRightarrow{ON} (0 \to \pi)$
  - $S_3, S_4 \xRightarrow{ON} (\pi \to 2\pi)$

⇒ This is Called: "Bipolar Inverter".

* Multi-level Inverter: 2-carriers ⇒

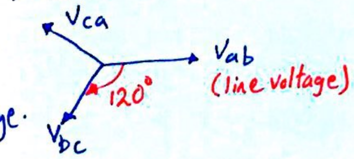


* shift of the harmonic ⟶ toward carrier frequancy.

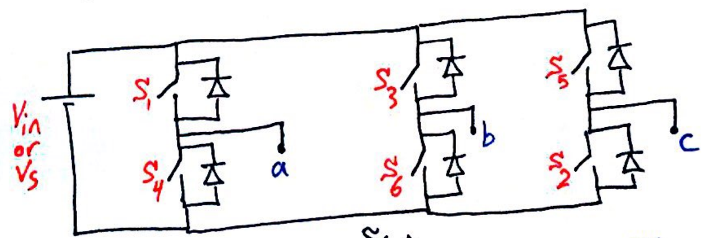
* 3-phase Inverter (DC/AC):

Y-connection.

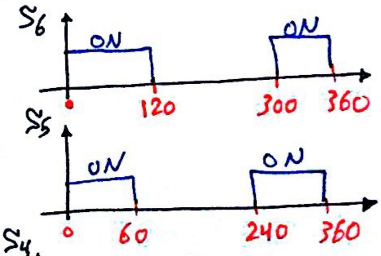
Need 3ϕ voltage.

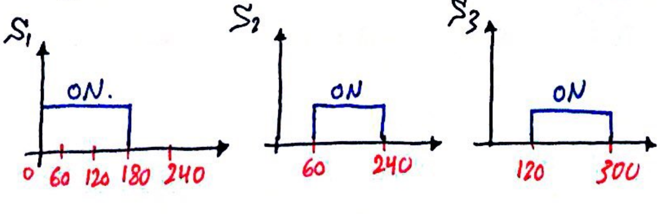


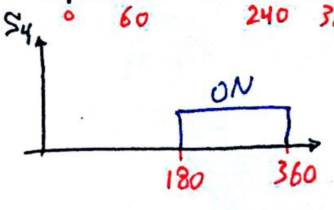
⇒ Configuration: will be with 3 Legs [6 switches].



* Control switches:
- each leg has one switch ON @ each time segment.
- Time Domain ⇒ 6 time segments (60°).
- Each switch will operate for 3 time segments (180°).
- Sequantially each switch will be delayed by (60°).





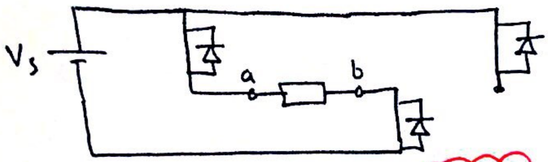


* for 3ϕ inverter: (6-cases).

① $0 \rightarrow 60°$ (S_1, S_5, S_6)
② $60° \rightarrow 120°$ (S_1, S_2, S_6)
③ $120° \rightarrow 180°$ (S_1, S_2, S_3)
④ $180° \rightarrow 240°$ (S_2, S_3, S_4)
⑤ $240° \rightarrow 300°$ (S_3, S_4, S_5)
⑥ $300° \rightarrow 360°$ (S_4, S_5, S_6)

Case ①: find V_{ab}, V_{bc}, V_{ca} (it will be like a load connected between the needed phases).

for V_{ab}:



$-V_s + V_{ab} = 0 \Rightarrow V_{ab} = V_s$

the same will be done for V_{bc} & V_{ca}:
you will find that: $V_{bc} = -V_s$, $V_{ca} = 0$

• Note: $V_{ab} + V_{bc} + V_{ca} = 0$ for all 6-cases.

** Do the same for the other 5-cases, the results will be as follows:

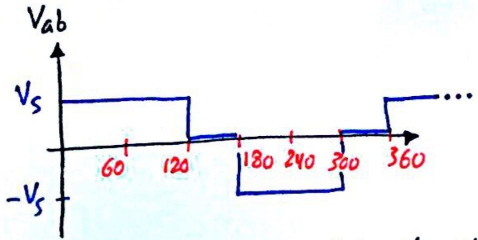
case ②: $V_{ab} = V_s$, $V_{bc} = 0$, $V_{ca} = -V_s$

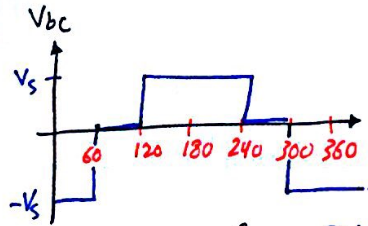
case ③: $V_{ab} = 0$, $V_{bc} = V_s$, $V_{ca} = -V_s$

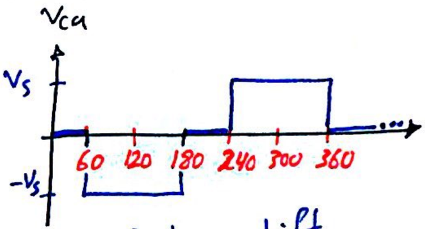
case ④: $V_{ab} = -V_s$, $V_{bc} = V_s$, $V_{ca} = 0$

case ⑤: $V_{ab} = -V_s$, $V_{bc} = 0$, $V_{ca} = V_s$

case ⑥: $V_{ab} = 0$, $V_{bc} = -V_s$, $V_{ca} = V_s$





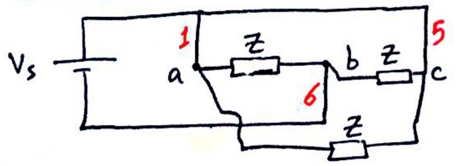


• Note: All of V_{ab}, V_{bc}, V_{ca} have the same figure BUT with $120°$ phase shift between each one of them with the other one.

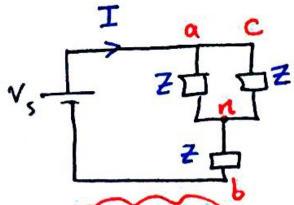
* Phase to Neutral Voltages:

case ①:

"Δ-connection"



we convert $\Longrightarrow$ to Y-connection



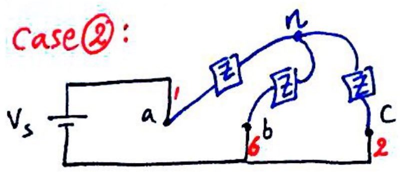
$I = \frac{V_s}{\frac{3}{2}Z}$

$V_{an} = V_{cn} = \frac{V_s}{\frac{3}{2}Z} \cdot \frac{Z}{2}$

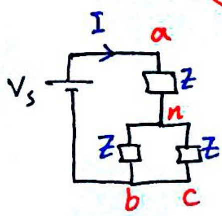
$\Rightarrow V_{an} = V_{cn} = \frac{V_s}{3}$

$V_{bn} = \frac{-2V_s}{3}$

case ②:



$\Longrightarrow$



$I = \frac{V_s}{\frac{3}{2}Z}$

$V_{an} = \frac{2}{3}V_s$

$V_{bn} = V_{cn} = \frac{-V_s}{3}$

** Do the same for the other 4-cases.

* Fourier transform for the Line & phase voltages:

V_{ab}: will have No DC value. (DC = 0), it will NOT have third harmonic (which is an advantage).
, it will have odd freq. 1, 5, ... → any factor of 3^n doesn't exist. (3, 9, 27, ...)

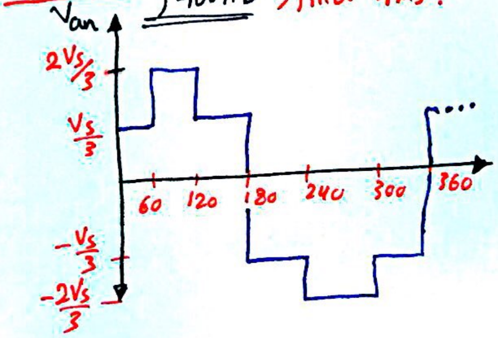
V_{an}: will have odd freq. 1, 3, 5, ...

The expressions are given as follows:

$$V_{an} = \frac{2V_s}{\pi}\left[\sin\omega t + \frac{1}{3}\sin 3(\omega t) + \frac{1}{5}\sin 5(\omega t) + \cdots\right]$$
$$V_{bn} = \frac{2V_s}{\pi}\left[\sin(\omega t - 120) + \frac{1}{3}\sin 3(\omega t - 120) + \cdots\right]$$
$$V_{cn} = \frac{2V_s}{\pi}\left[\sin(\omega t + 120) + \frac{1}{3}\sin 3(\omega t + 120) + \cdots\right]$$

$$V_{ab} = \sqrt{3}\,\frac{2V_s}{\pi}\left[\sin(\omega t + 30) - \frac{1}{5}\sin 5(\omega t + 30) + \cdots\right]$$
$$V_{bc} = \sqrt{3}\,\frac{2V_s}{\pi}\left[\sin(\omega t - 90) - \frac{1}{5}\sin 5(\omega t - 90) + \cdots\right]$$
$$V_{ca} = \sqrt{3}\,\frac{2V_s}{\pi}\left[\sin(\omega t + 150) - \frac{1}{5}\sin 5(\omega t + 150) + \cdots\right]$$

Example: f = 100 HZ, find V_{rms}?



$\Rightarrow T = 0.01\,\text{sec} \Rightarrow \frac{T}{6} = \frac{0.01}{6}$ sec.

$$V_{rms}^2 = \frac{2}{0.01}\left[\int_0^{\frac{0.01}{6}}\left(\frac{V_s}{3}\right)^2 dt + \int_{\frac{0.01}{6}}^{\frac{0.02}{6}}\left(\frac{2V_s}{3}\right)^2 dt + \int_{\frac{0.02}{6}}^{\frac{0.03}{6}}\left(\frac{V_s}{3}\right)^2 dt\right]$$

solving: $\boxed{V_{rms} = 0.47V_s}$

Example: 3Ø inverter is used to supply a 3Ø load using a DC voltage source if the desired freq at the load side 500 HZ.
Find the conduction period for each switch?

Solution:

$T = \frac{1}{500} = 2\,\text{msec}.$

$\Rightarrow$ conduction period $= 3 * \frac{T}{6}$ $\Rightarrow$ conduction $= 3 * \frac{2m}{6} = \underline{1\,\text{msec}}$.

* Voltage adjustment (Fixed Width Modulation [FWM]):

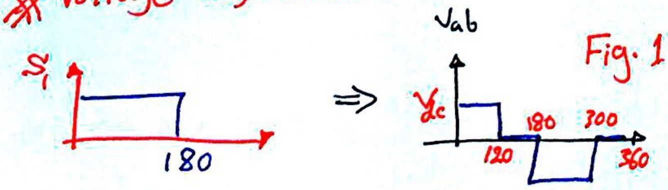
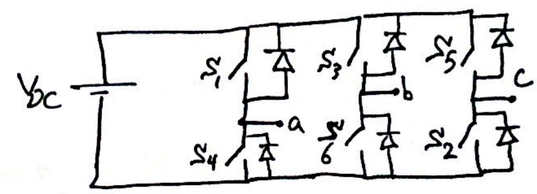


Fig. 1



FWM:

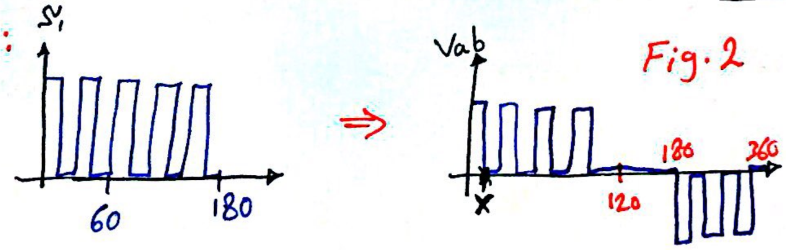


Fig. 2

• objective: Reduction for V_{ab}.

⟹ continue.

for Fig 1: $V_{ab_{rms}}\Big|_{\substack{\text{without}\\ FWM}} = \sqrt{\frac{1}{T}\int_0^{2*T/6} V_{dc}^2 dt + \int_{T/2}^{T\frac{300}{360}} V_{dc}^2 dt} = \sqrt{\frac{2}{T}\int_0^{2T/6} V_{dc}^2 dt}$

$= V_{dc}\sqrt{\frac{2}{T}\left(\frac{2T}{6}\right)}$ • without FWM: $V_{ab_{rms}} = \sqrt{\frac{2}{3}}V_{dc}$ ... ①

for Fig 2: $d \triangleq$ Duty Cycle. $= \frac{\sum \text{on time}}{\frac{120}{360}T}$ → conduction period.

$X = \frac{dT}{3N}$

$V_{ab_{rms}} = \sqrt{\frac{2*N}{T}\int_0^X V_{dc}^2 dt}$; where $N \equiv$ # of conduction time.

$d = \frac{N}{\text{conduction period}}$

$= \sqrt{\frac{2 \cdot N}{T}\left(\frac{dT}{3N}\right)} * V_{dc} = V_{dc}\sqrt{\frac{2}{3}d}$

$\Rightarrow d = \frac{N}{\frac{2T}{6}}$

• with FWM:

$V_{ab_{rms}} = \sqrt{\frac{3}{2}}V_{dc}\sqrt{d} = V_{ab_{\substack{rms\\ \text{without}\\ FWM}}} * \sqrt{d}$ ... ②

$d = \frac{X N}{2T/6}$

Divide ② by ① gives:

* Voltage reduction $\equiv \sqrt{d}$ *

*Summarize:

∴ V_{ab}

- without FWM: $V_{ab} = \sqrt{\frac{2}{3}}V_{dc}$
- with FWM: $V_{ab} = \sqrt{\frac{2}{3}}V_{dc}\sqrt{d}$

Example: FWM 3Ø inverter with duty ratio equals 25% is used to reduce the voltage if $V_s = V_{dc} = 150$ volt. Calculate RMS with & without FWM?

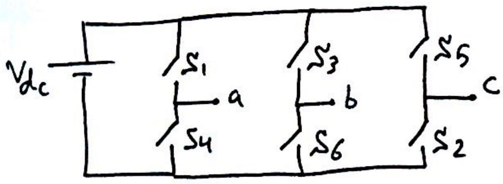
Solution: $V_{ab}\big|_{rms}$ (without) $= \sqrt{\frac{2}{3}}V_{dc} = \boxed{122.48 \text{ volt}}$

$V_{ab}\big|_{rms}$ (with) $= \sqrt{\frac{2}{3}}V_{dc}\sqrt{d} = \boxed{61.24 \text{ volt}}$

* Sequance Adjustment Using 3Ø inverter acb:

A swap for S_1 with S_5.
& S_2 with S_4

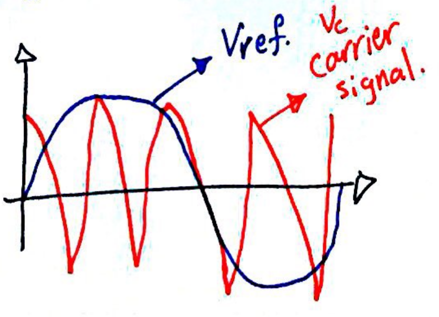
Do the same thing to find the voltages.



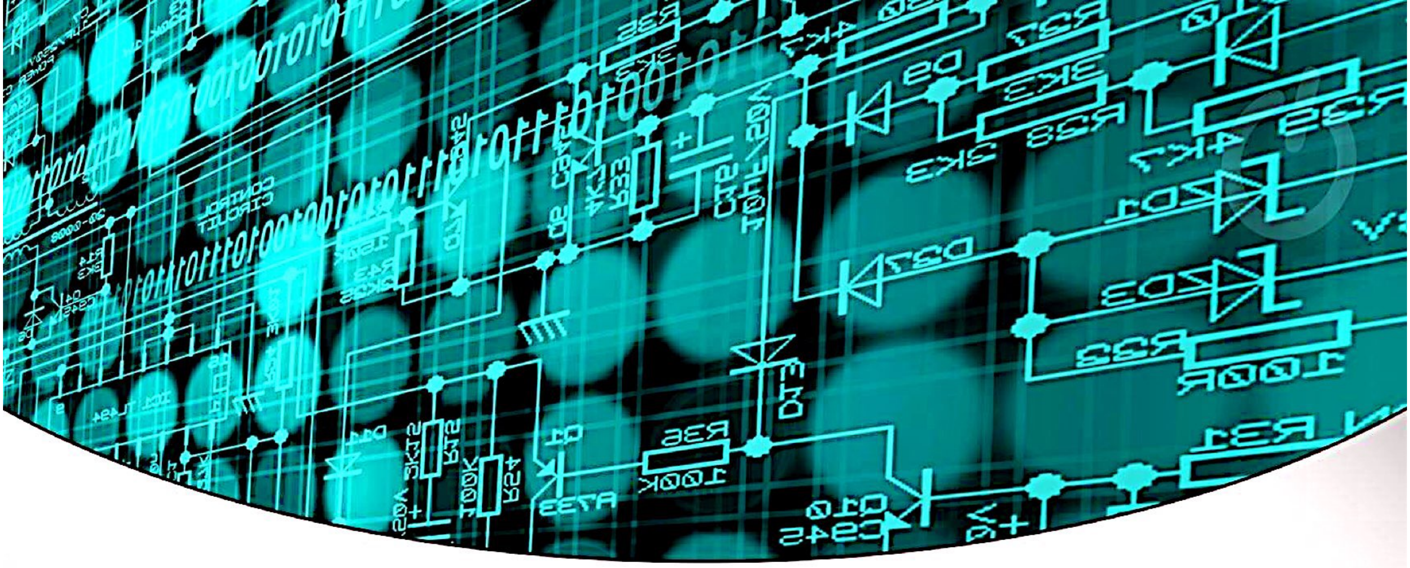
* Sinusoidal Pulse Width Modulation [SPWM]: * 3Ø *

• To control f, Mag., phase ⇒ To reduce the harmonics.

for Leg (1):



if Vref > Vc ⇒ ON ⇒ upper switch.
if Vref < Vc ⇒ ON ⇒ lower switch.



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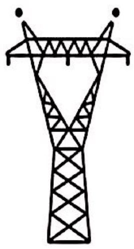
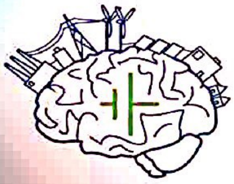
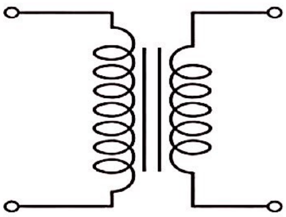


Dr.Sereen Althaher

By: Mhmd Abuhashya





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* Rectifier *

- Why we use it? To control the voltage.
- Application: Induction Motor (No load).
- from PIZ model: if we want to decrease V $\Rightarrow$ decrease the current.

$V\downarrow = \underbrace{Z}_{\text{constant.}} I\downarrow \Rightarrow P = IV$ (also decreased).

- H.W $\equiv$ Half-Wave.
- F.W $\equiv$ Full-Wave.

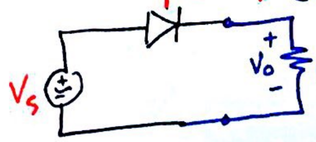
* We will study the following subjects:

$\Rightarrow$ Resistive 1ϕ-H.W.
$\Rightarrow$ Inductive 1ϕ F.W.
$\Rightarrow$ Capacitive. 3ϕ.

* Single-phase Half-Wave:

$\rightarrow$ Control cct SCR "silicon Controlled Rectifier".

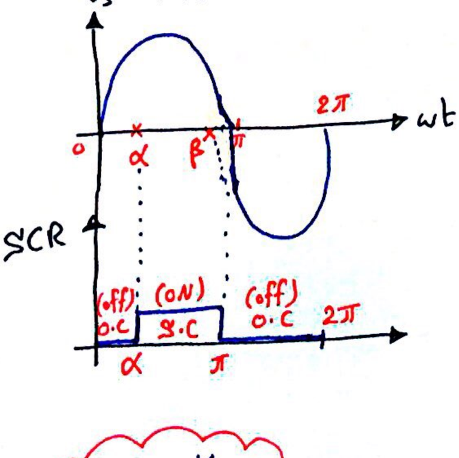
we will deal with the diode in ideal case. $(P=0)$



* Triggring Signal:

$\alpha \equiv$ triggring point.

$V_s = V_{max} \sin \omega t$



case (I): $\beta < \pi \Rightarrow V_{o_{avg}} = \frac{1}{T}\int_{\alpha}^{\beta} V_{max} \sin \omega t \, d\omega t$.

case (II): $\beta > \pi \Rightarrow V_{o_{avg}} = \frac{1}{T}\int_{\alpha}^{\pi} V_{max} \sin \omega t \, d\omega t$

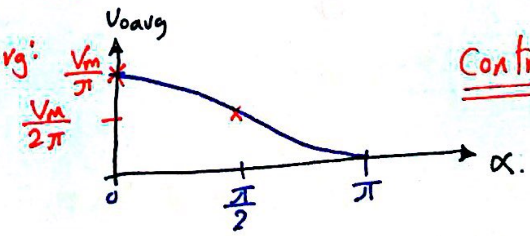
$\hookrightarrow$ we will take case (II):

$$V_{o_{avg}} = \frac{V_{max}}{2\pi}\left[\cos\alpha - \cos\pi\right]$$

$$\Rightarrow \boxed{V_{o_{avg}} = \frac{V_m}{2\pi}\left[\cos\alpha + 1\right]} *$$

$$* \boxed{I_{o_{avg}} = \frac{V_{o_{avg}}}{R}} *$$

. Draw $V_{o_{avg}}$:



Control α.

Example: 1ϕ - H.W rectifier is used to reduce the avg. voltage across a Resistive Load $R = 0.2 V_{avg}^2 + 5$ if $V_s = 110$ rms @ $\alpha = 90°$.

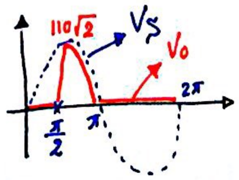
Find: a) $I_{o,avg}$? b) Plot the voltage across the SCR?

Solution:

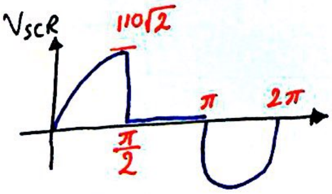
ⓐ $I_{o,avg} = \frac{V_{o,avg}}{R} \Rightarrow V_{o,avg} = \frac{V_m}{2\pi}[1+\cos\alpha] = \frac{110\sqrt{2}}{2\pi} = \underline{24.76}$ volt.

$\Rightarrow R = 0.2 V_{o\,avg}^2 + 5 \Rightarrow \underline{R = 127.6\,\Omega}$ $\boxed{I_{o,avg} = 0.194}$ A.

ⓑ



$V_{SCR} = V_s - V_o \Rightarrow$



*** Output voltage (RMS):**

$V_{o\,rms} = \sqrt{\frac{1}{T}\int_0^T (V_m \sin\omega t)^2 d\omega t} = \sqrt{\frac{V_m^2}{2\pi}\int_\alpha^\pi \sin^2\omega t\, dt}$ → $\sin^2\omega t = \frac{1}{2}[1-\cos 2\omega t]$

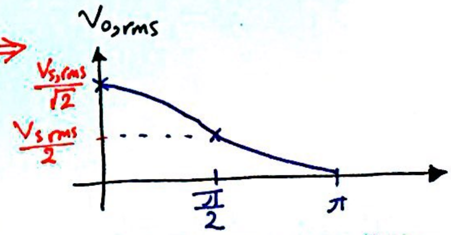
$= \frac{V_m}{\sqrt{2}\cdot\sqrt{2}}\sqrt{1-\frac{\alpha}{\pi}+\frac{\sin 2\alpha}{2\pi}} \Rightarrow V_{o\,rms} = \frac{V_{rms,source}}{\sqrt{2}}\sqrt{1-\frac{\alpha}{\pi}+\frac{\sin 2\alpha}{2\pi}}$

@ $\alpha = 0$: $V_{o\,rms} = \frac{V_{s,rms}}{\sqrt{2}}$

@ $\alpha = \frac{\pi}{2}$: $V_{o\,rms} = \frac{V_{s,rms}}{2}$

@ $\alpha = \pi$: $V_{o\,rms} = 0$

$\Rightarrow$



Example: AC source of 110 V_{rms} is connected to a resistive element 2 Ω through a H.W, 1ϕ rectifier. for $\alpha = \frac{\pi}{4}$ & $\alpha = \frac{\pi}{2}$ find the following:

a) $V_{o,rms}$. b) $I_{o\,rms}$. c) $V_{o\,avg}$ drop across SCR.

Solution:

ⓐ $\boxed{V_{o\,rms} = 74.13}$ volt. @ $\alpha = \frac{\pi}{4}$

$\boxed{V_{o\,rms} = 55}$ volt. @ $\alpha = \pi/2$.

ⓑ $\boxed{I_{o\,rms} = 37.07}$ A. @ $\alpha = \frac{\pi}{4}$

$\boxed{I_{o\,rms} = 27.5}$ A. @ $\alpha = \frac{\pi}{2}$.

ⓒ $V_{o\,SCR} = V_{s,avg} - V_{o,avg}$

$V_{o\,SCR} = 0 - V_{o,avg}$

$\Rightarrow \boxed{V_{o\,SCR} = -42.27}$ volt. @ $\alpha = \frac{\pi}{4}$

$\Rightarrow \boxed{V_{o\,SCR} = -24.76}$ volt. @ $\alpha = \frac{\pi}{2}$

Example: Design Rectifier to have a given V_s & a desired $V_{o\,avg}$?

Design $\Rightarrow$ find α by iterations.

$V_{o\,avg} = \frac{V_m}{2\pi}[1+\cos\alpha]$ → to find $V_{o\,avg}$ ⇒ find α from:

$V_{o\,rms} = \frac{V_{s,rms}}{\sqrt{2}}\sqrt{1-\frac{\alpha}{\pi}+\frac{\sin 2\alpha}{2\pi}}$

α by iterations

Take $V_{s,rms} = 110$ volt, $V_{o\,rms} = 80$ volt., Take $\alpha = 0, \frac{\pi}{2}, \pi$.

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* Average Power:

$$P_{avg} = V_{o,rms} I_{o_{rms}} = \frac{V_{o_{rms}}^2}{R} = I_{o_{rms}}^2 R$$

$$\Rightarrow \boxed{P_{avg} = \frac{V_m^2}{8\pi R}\left[2(\pi-\alpha)+\sin 2\alpha\right]}$$

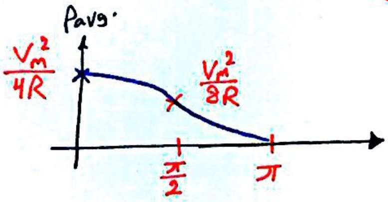
in rad.

→ Maximum Power @ $\alpha = 0$:

$$\boxed{P_{avg_{max}} = \frac{V_m^2}{4R}}$$

@ $\alpha = \frac{\pi}{2}$: $P_{avg} = \frac{V_m^2}{8R}$

@ $\alpha = \pi$: P_{avg} = zero.



$\alpha \equiv$ firing angle

*Power Factor:

$$PF = \frac{|P|}{|S|} \Rightarrow |S| = V_{rms} I_{rms} \Rightarrow \boxed{P = V_{rms_1} I_{rms_1} \cos\phi_1}$$

fundemental — Displacement Factor.

this for i_s ⇒ Not sinusoidal.
- → harmonics.
- → Fundemental.

***Fourier Transform for $i_s(t)$:**

→ $i(t) = C\sin(\omega t + \phi)$

$$\boxed{I_{1_{rms}} = C_1 = \sqrt{a_1^2 + b_1^2}}$$

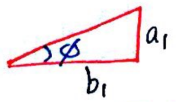
$$\boxed{a_1 = \frac{I_{s,max}}{2\pi}\left[\cos(2\alpha) - 1\right]}$$

→ $\boxed{I_{s_{1\,max}} = \frac{V_{s,max}}{R}}$

$$\boxed{b_1 = \frac{I_m}{4\pi}\left[\sin 2\alpha + 2(\pi-\alpha)\right]}$$

$i_s(t)$ has the same figure of $v_s(t)$ divided By R.

$$\boxed{\phi_1 = \tan^{-1}\left[\frac{a_1}{b_1}\right]}, \quad \boxed{\cos\phi_1 = \frac{b_1}{C_1}}$$



very Low.

$$\Rightarrow \boxed{pf = \frac{I_{1,rms}}{I_{rms}} \cos\phi}$$

$I_{rms} = \frac{V_{rms}}{R}$

Example: 1ϕ H.W.R is connected to 10Ω Resistor. $V_s = 110$ rms & $\alpha = 60°$.

Find: P_{load} & Pf ?

Solution: $P_{load} = \frac{V_{max}^2}{8\pi R}\left[2\pi - 2\alpha + \sin 2\alpha\right] \Rightarrow P_{Load} = 486.7W$.

$$PF \Rightarrow I_{o,rms} = \frac{V_{rms}}{R} = \frac{V_{s,rms}}{\sqrt{2}R}\sqrt{1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi}} \Rightarrow PF = \frac{I_{1rms}}{I_{o,rms}}\cos\phi$$

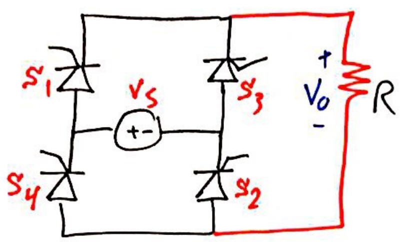
$I_{1,rms}$ → $\boxed{a_1 = -3.71,\ b_1 = 6.24}$

$$\Rightarrow \boxed{PF = 0.447} \text{ Lagging.}$$

$$\phi = \tan^{-1}\left(\frac{a_1}{b_1}\right)$$

※ Single-Phase-Full-Wave-Rectifier: "Resistive Load"

• Configuration: $V_s = V_m \sin\omega t$

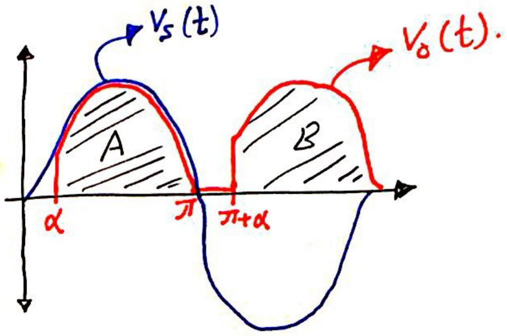


• Case ($V_s > 0$): S_1, S_2 (ON). from $\alpha \rightarrow \pi$.

By KVL: $-V_s + V_o = 0 \Rightarrow \boxed{V_o = V_s}$

• Case ($V_s < 0$): S_3, S_4 (ON).

By KVL: $+V_o + V_s = 0 \Rightarrow \boxed{V_o = -V_s}$



$$\boxed{\begin{aligned} V_{avg,FW} &= 2V_{avg,HW} \\ P_{FW} &= 2P_{H.W} \\ V_{rms,F.W} &= \sqrt{2}\, V_{rms,H.W} \end{aligned}}$$

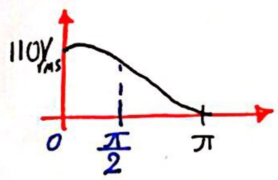
$$a_{1,F.W} = 2a_{1,H.W}$$
$$b_{1,F.W} = 2b_{1,H.W}$$
$$C_{1,F.W} = 2C_{1,H.W}$$

PF is the same: $PF_{H.W} = PF_{F.W.}$

A & B are symmetrical.

Example: F.W.R $R = 5\,\Omega$, $V_s = 110 V_{rms}$, $V_{o,rms} = 55\, V_{rms}$ Find α?

Solution: $V_{o,rms} = V_{s,rms}\sqrt{1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi}} \Rightarrow 55 = 110\sqrt{1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi}}$



• By Try & Error:

Try $\alpha = 0$

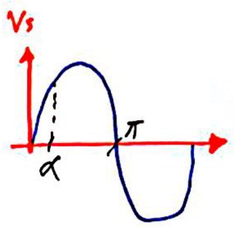
Try $\alpha = \pi/2$

Try $\alpha = \pi$

$\boxed{\alpha = 112°}$

※ 1-Ø H.W.R: "Inductive Load"

$\tau = \frac{L}{R}$



Assume Purely inductive Load: ($R=0$).

$V_L(t) = V_{max,o}\sin(\omega t) = V_{max}\cos(\omega t - \frac{\pi}{2})$

$I_L(t) = I_{max}\cos(\omega t - \frac{\pi}{2} - \frac{\pi}{2})$

$\Rightarrow I_L(t) = I_{max}\cos(\omega t - \pi)$

$= -I_{max}\cos(\omega t)$.

$P_L(t) = V(t)I(t)$

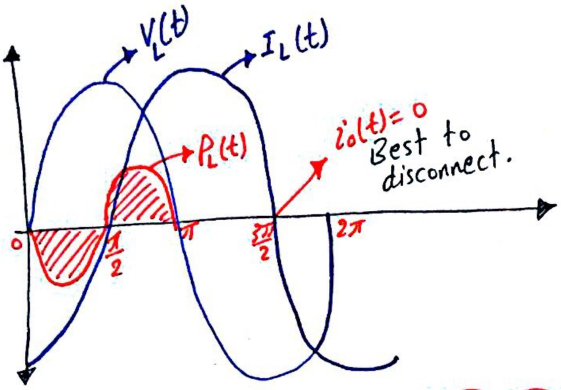
$= -V_{max}I_{max}\sin\omega t\cos\omega t$

$= -\frac{V_{max}I_{max}}{2}\sin(2\omega t)$

$V_L(t) = V_{max}\cos(\omega t - \frac{\pi}{2})$

$I_L(t) = -I_{max}\cos(\omega t)$

$P_L(t) = -\frac{V_{max}I_{max}}{2}\sin(2\omega t)$

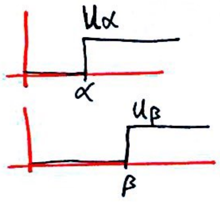


$(0 \to \pi)$

$\boxed{P_{avg} = Zero}$

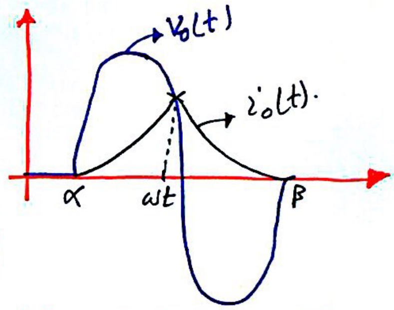
$V_o(t) = V_s \quad (\alpha \to \beta) \Rightarrow V_o = V_s [U_\alpha - U_\beta]$

$i(t) = \dfrac{V_o(t)}{Z}$



$\xRightarrow[i(t)]{laplace}$ $i(t) = \dfrac{V_{max}}{|Z|}\left[(U_\alpha - U_\beta)\sin(\omega t - \phi) + U_\alpha \sin(\phi - \alpha) e^{-\frac{(\omega t - \alpha)}{\omega \tau}}\right]$

$$\boxed{\begin{aligned} \tau &= L/R \\ Z &= \sqrt{R^2 + (\omega L)^2} \\ \phi &= \tan^{-1}\left(\frac{\omega L}{R}\right) \end{aligned}}$$



$V_{avg} = \dfrac{1}{2\pi}\displaystyle\int_\alpha^\beta V_{s,max} \sin\omega t \, d\omega t = \dfrac{V_{smax}}{2\pi}[\cos\alpha - \cos\beta]$

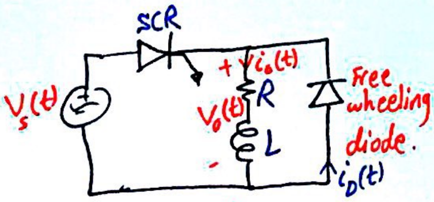
$\Rightarrow I_{o\,avg} = I_{o\,avg,L} + I_{o,avg,R} = Zero + \dfrac{V_{o,avg}}{R} \Rightarrow I_{o\,avg} = \dfrac{V_{smax}}{2\pi R}[\cos\alpha - \cos\beta]$

$V_{oavg} = V_{oavg\,L} + V_{oavg\,R} = Zero + I_{o\,avg} R \Rightarrow \boxed{V_{o\,avg} = I_{o\,avg} R}$

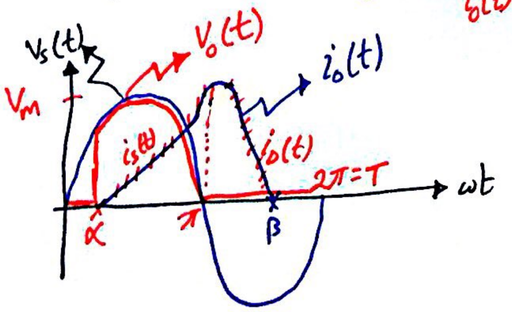
$\Rightarrow$ Find $V_{o_{rms}}$?!

* 1ϕ- H.W.R with inductive load + Free wheeling diode:

we want to find $\langle V_o \rangle$, $V_o(t)$ & $i_o(t)$!?



$$i_o(t) = \begin{cases} \bullet\ \alpha \to \pi: \\ \quad i_s(t) = \dfrac{V_m}{|Z|}\left[\sin(\omega t - \phi) + \sin(\phi - \alpha)\, e^{-\frac{(\omega t - \alpha)}{\omega\tau}}\right] \quad \cdots (1) \\ \bullet\ \pi \to \beta: \\ \quad i_D(t) = i_s(\pi)\, e^{-\frac{(\omega t - \pi)}{\omega\tau}}\, u(\omega\tau - \pi) \quad \cdots (2) \end{cases}$$



$V_{o\,avg} = \dfrac{1}{2\pi}\displaystyle\int_\alpha^\pi V_m \sin(\omega t)\, d\omega t = \dfrac{V_m}{2\pi}[\cos\alpha + 1]$

$i_{o\,avg} = i_{R,avg} = \dfrac{V_{o,avg}}{R}$

Example: 1Ø-H.W.R connected to inductive Load $R=10\Omega$, $L=20mH$.
$V_{s\,RMS}=110\,volt$ connected with F.W diode $\alpha=60°$, $f=60Hz$.

Find: ① Conduction period $\gamma=\beta-\alpha$? ② i_{max} ? ③ i_{avg} ?

Solution:

① Here we make assumptions: < 10% $\Rightarrow$ $i_d(\beta)=0.05\,i_s(\pi)$

so $0.05\,i(\pi)=i(\pi)\,e^{\frac{-(\beta-\pi)}{\omega\tau}}$, $\tau=2msec.$, $\omega=2\pi*60\ rad/s$ $\Rightarrow$ $Ln(0.05)=\frac{-(\beta-\pi)}{\omega\tau}$

solving: $\beta=309°$

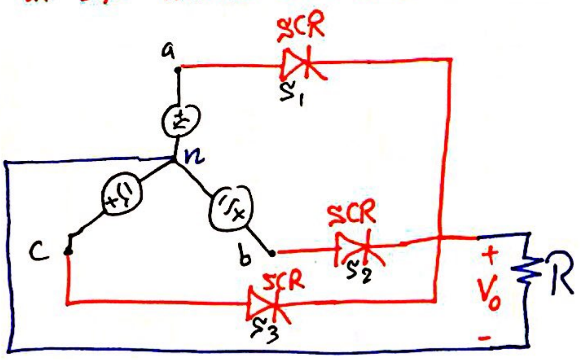
$\therefore\ \gamma=\beta-\alpha=309°-60°\Rightarrow \gamma=249°$

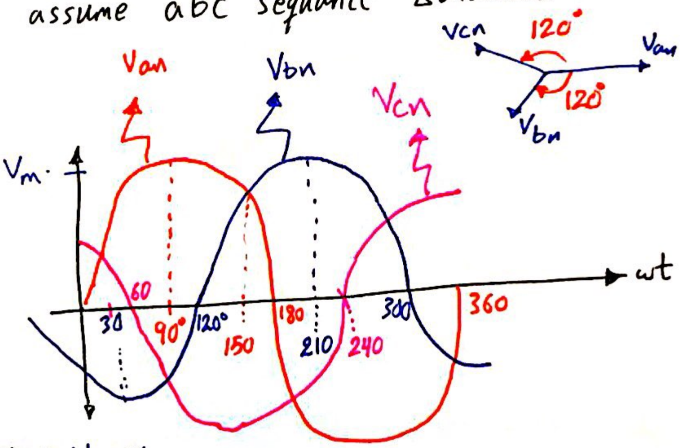
② The max value will occur @ $\omega t=\pi$: use equation ① in the previous page to find $i_{max}=i_s(\pi)$. $\Rightarrow$ $I_{max}=7.478\,A$.

③ $i_{o\,avg}=\frac{V_{avg}}{R}=\frac{\sqrt{2}\,V_{rms}}{2\pi R}[\cos\alpha+1]\Rightarrow I_{o\,avg}=3.714\,A$

※ 3Ø H.W.R with Resistive Load:

assume abc sequance "Balanced"

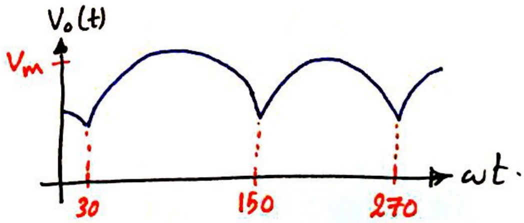




$V_{an}=V_m\sin\omega t$
$V_{bn}=V_m\sin(\omega t-120)$
$V_{cn}=V_m\sin(\omega t+120)$

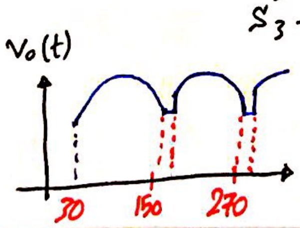
S_1 ON $\Rightarrow V_{an}>V_{bn},V_{cn}$
S_2 ON $\Rightarrow V_{bn}>V_{an},V_{cn}$
S_3 ON $\Rightarrow V_{cn}>V_{an},V_{bn}$

- $0\to30°\Rightarrow S_3$ ON. $V_o=V_{cn}$
- $30\to150°\Rightarrow S_1$ ON. $V_o=V_{an}$
- $150\to270°\Rightarrow S_2$ ON $V_o=V_{bn}$



$\Rightarrow$ All previous analysis for 3Ø done using $\alpha=0$.

if $\alpha=30°$ $\Rightarrow$ $S_1\to\alpha$.
$S_2\to\alpha+120$
$S_3\to\alpha+240$



* V_o avg: $V_{o\,avg\,(3Ø)}=3\,V_{o\,avg}(1Ø)$
* P_{avg}: $P_{avg\,3Ø}=3\,P_{avg\,1Ø}$
* I_o avg: $I_{o\,avg}=\frac{V_{o\,avg}}{R}$

$V_{o\,avg} = \frac{3V_m}{2\pi}\left[\cos\alpha - \cos\beta\right]$, $P_{avg} = \frac{3V_m^2}{8\pi R}\left[2(\beta-\alpha) + \sin 2\alpha - \sin 2\beta\right]$

Example: 3ϕ – H.W.R $V_{ab} = 208\,V_{rms}$, $R = 10\Omega$, $\alpha = 80°, 30°$ find P_{avg} ?

Solution: $V_m = \frac{208}{\sqrt{3}} * \sqrt{2}$ volt.

for $\alpha = 80° \Rightarrow \beta = \alpha + 120 = 200° \Rightarrow 180°$

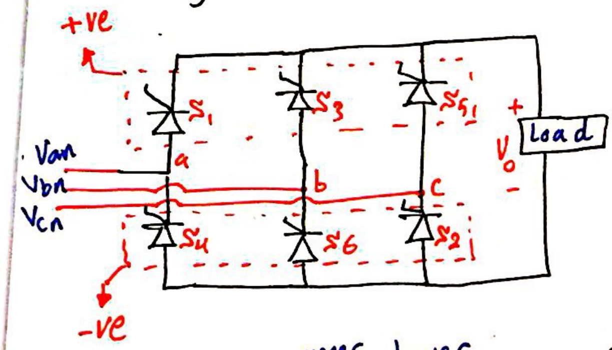
for $\alpha = 30° \Rightarrow \beta = 150°$

$P_{\alpha=80°,\,3\phi} = 1.32\,KW$, $P_{\alpha=30,\,3\phi} = 2.042\,KW$

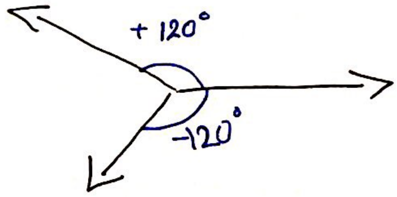
• **Note:** MAX P_{avg} @ $\alpha = 0$.

※ 3ϕ F.W.R :

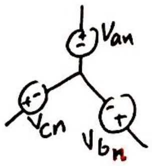
• Configuration:

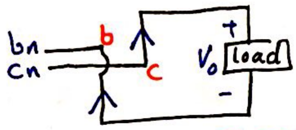


⇒ Y-connection (Balanced)
abc

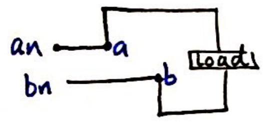


* $0 \rightarrow 30°$: upper $\underline{C}$ S_5 ; Lower $\underline{B}$ S_6





By KVL: $+V_o + V_{bn} - V_{cn} = 0 \Rightarrow V_o = V_{cn} - V_{bn} \Rightarrow V_o = V_{CB}$



* $30° \rightarrow 60°$: upper $\underline{A}$ S_1 ; Lower $\underline{B}$ S_6 $\Rightarrow V_o = V_{AB}$

* $60° \rightarrow 90°$: upper $\underline{A}$ S_1 ; Lower $\underline{B}$ S_6 $\Rightarrow V_o = V_{AB}$

* $90° \rightarrow 120°$: upper $\underline{A}$ S_1 ; Lower $\underline{C}$ S_2 $\Rightarrow V_o = V_{AC}$

* $120° \rightarrow 150°$: upper $\underline{A}$ S_1 ; Lower $\underline{C}$ S_2 $\Rightarrow V_o = V_{AC}$

* $150° \rightarrow 180°$: upper $\underline{B}$ S_3 ; Lower $\underline{C}$ S_2 $\Rightarrow V_o = V_{BC}$

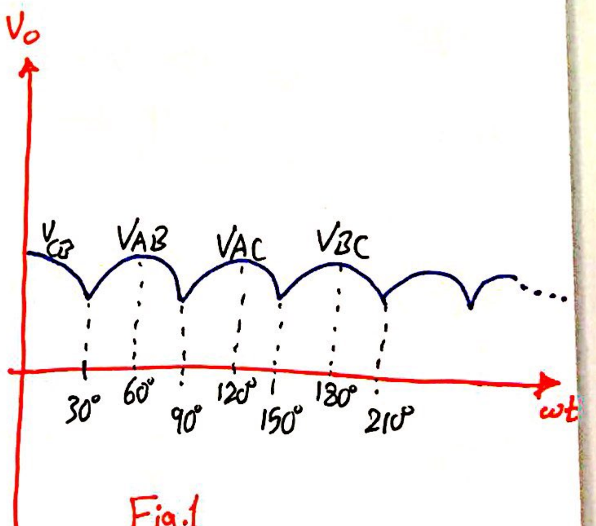


Fig.1

• Average for previous Fig.1: $V_{avg} = \frac{6}{2\pi}\int_{30}^{90} V_{AB}\, d\omega t$

$V_{an} = |V_{an}| \sin\omega t$

$V_{ab} = \sqrt{3}\,|V_{an}| \angle +30°$

$= \sqrt{3}\,|V_{an}| \sin(\omega t + 30°)$

$\Rightarrow V_{avg} = \frac{3}{\pi}\int_{30}^{90} \sqrt{3}\,|V_{an}| \sin(\omega t+30)$

$= \frac{3\sqrt{3}\,|V_{an}|}{\pi}\left[\cos 60 - \cos 120\right]$

$\Rightarrow \boxed{V_{avg} = \frac{3\sqrt{3}\,|V_{an}|}{\pi}}$ → peak value.

• for the current:

$\boxed{I_{avg} = \frac{V_{avg}}{R}}$

0 → 30 S_5, S_6
30 → 60 S_1, S_6
60 → 90 S_1, S_6
90 → 120 S_1 S_2
120 → 150 S_1 S_2
150 → 210 S_3 S_2

$\xrightarrow{\alpha = 30°}$ Conduction Period $S_1 \Rightarrow 120°$

S_5: 30°
S_1 60 → 180°
S_6 90 → 210

$S_1, S_6 \Rightarrow V_{AB}\,(60 \to 90)$

the same figure would be observed just shifted by 30°.

For V_{avg}: $V_{avg} = \frac{6}{2\pi}\int_{30+\alpha}^{90+\alpha} |V_{an}|\sqrt{3}\sin(\omega t+30)\, d\omega t = \frac{3|V_{an}|\sqrt{3}}{\pi}\left[\cos(60+\alpha) - \cos(120+\alpha)\right]$

$= \frac{3\sqrt{3}\,|V_{an}|}{\pi}\left[\cos 60\cos\alpha - \cancel{\sin 60 \sin\alpha} - \cos 120\cos\alpha + \cancel{\sin 120 \sin\alpha}\right]$

$\Rightarrow \boxed{V_{avg} = \frac{3\sqrt{3}\,|V_{an}|}{\pi}\cos\alpha}$ for $\alpha = 0 \to 60$

in case $\alpha = 60°$ $\Rightarrow$ $S_1, S_6 \Rightarrow V_{AB}\,(90 \to 120)$
$V_{AB}\,(120 \to 150)$

$S_5, S_6 \Rightarrow V_{CB}\,(60 \to 90)$

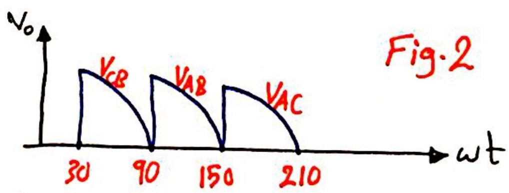


Fig.2

In case $\alpha = 90°$ $\Rightarrow$ $S_1, S_6 \Rightarrow V_{AB}\,(120 \to 150)$
$V_{AB}\,(150 \to 180)$

$S_1, S_2 \Rightarrow V_{AC}\,(180 \to 210)$
$V_{AC}\,(210 \to 240)$

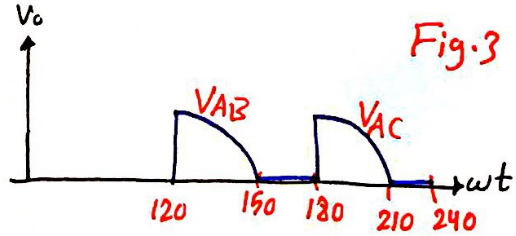


Fig.3

• For Fig.3: $V_{avg} = \frac{6}{2\pi}\int_{30+\alpha}^{150} |V_{an}|\sqrt{3}\sin(\omega t+30)\, d\omega t$

$= \frac{3\sqrt{3}\,|V_{an}|}{\pi}\left[\cos(60+\alpha) - \cos(180)\right]$ $\Rightarrow$ $\boxed{V_{avg} = \frac{3\sqrt{3}\,|V_{an}|}{\pi}\left[\cos(60+\alpha)+1\right]}$ for $\alpha = 90 \to 120$

for $\alpha = 60 \to 90$: $V_{avg} = \frac{6}{2\pi}\int_{30+\alpha}^{60+\alpha} |V_{an}|\sqrt{3}\sin(\omega t + 30^\circ)\,d\omega t \Rightarrow V_{avg} = \frac{3\sqrt{3}|V_{an}|}{\pi}\cos\alpha$

* The Average Voltage given by:

$$V_{avg} = \begin{cases} \frac{3\sqrt{3}|V_{an}|}{\pi}\cos\alpha & , 0 \leq \alpha < 90 \\ \frac{3\sqrt{3}|V_{an}|}{\pi}[\cos(60+\alpha)+1] & , 90 \leq \alpha < 120 \\ \text{Zero} & , \alpha \geq 120 \end{cases}$$

Example: 3ϕ F.W.R, $V_{ab_{rms}} = 208$ volt.

Find: i) Max. V_{avg}? ii) α @ which $V_{avg} = V_{phase_{peak}}$? iii) V_o, $\alpha = 30^\circ$?

Solution: (i) Max @ $\alpha = 0 \Rightarrow V_{avg} = \frac{3\sqrt{3}|V_{an}|}{\pi}\cos 0$, $V_{an} = \frac{\sqrt{2}(208)}{\sqrt{3}}$

$\Rightarrow V_{avg_{max}} = 280.9$ volt.

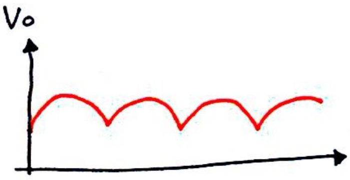
assume $0 \leq \alpha < 90$:

(ii) $V_{avg} = \frac{3\sqrt{3}V_{an}}{\pi}\cos\alpha \Rightarrow V_{an} = V_{avg}$

$\Rightarrow \cos\alpha = \frac{\pi}{3\sqrt{3}} \Rightarrow \alpha = 52.8$.

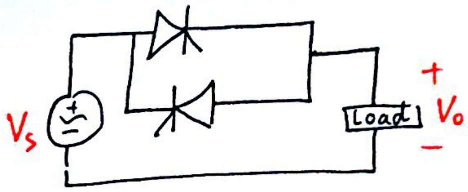
if α wasn't $0 \leq \alpha < 90$ Try other intervals.

(iii)



same Fig.1 shifted By 30°.

* AC/AC Converter:



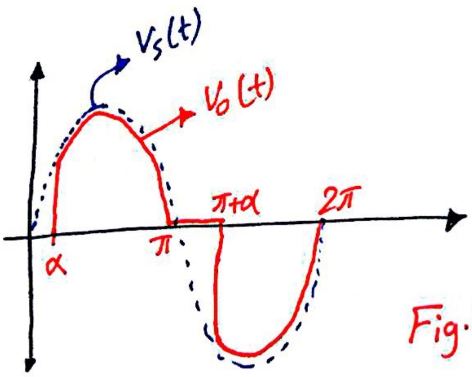


Fig.4.

* for Fig.4:

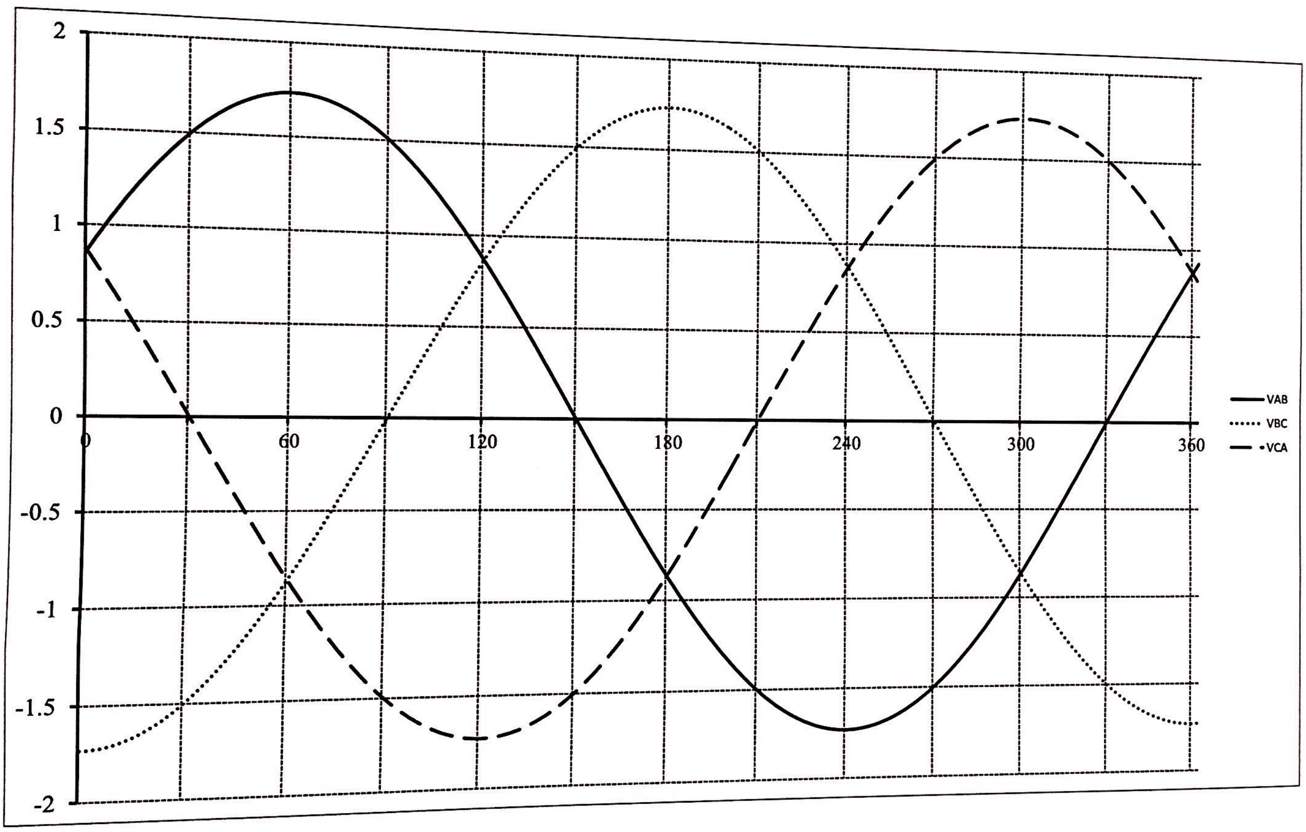
• Avg: $\boxed{V_{o_{avg}} = \text{Zero}}$

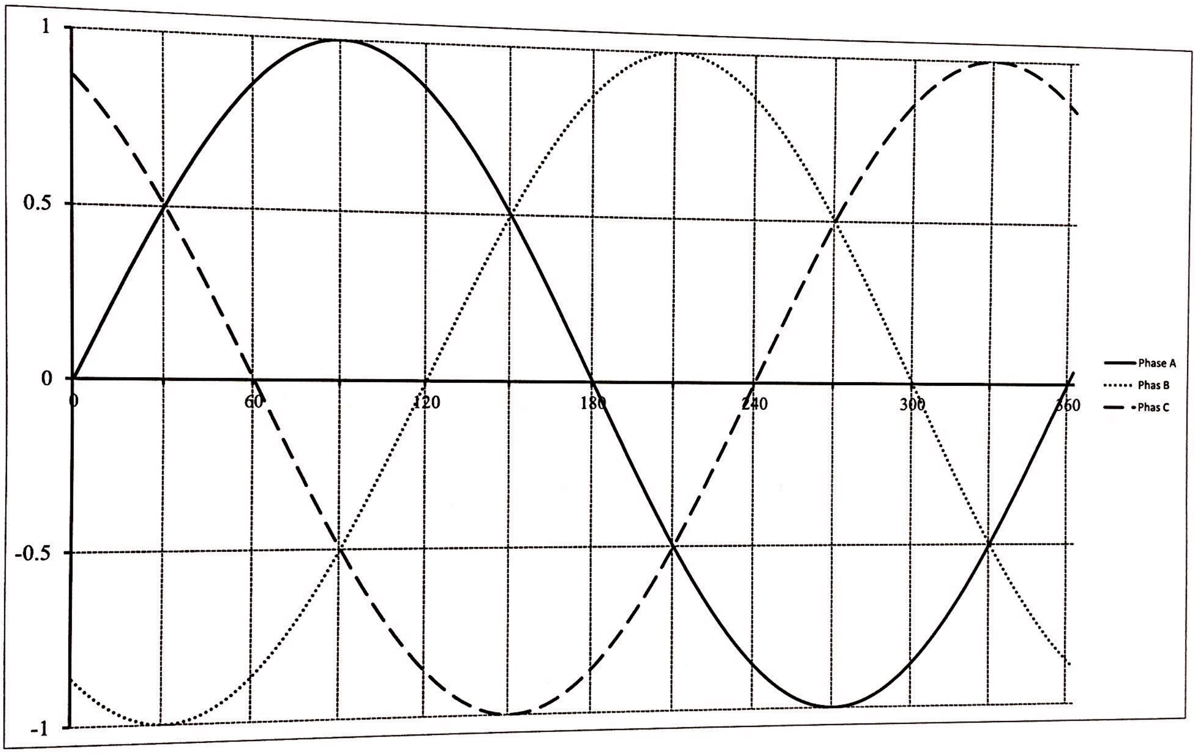
• RMS: $V_{o_{rms}} = \sqrt{\frac{2}{2\pi}\int_{\alpha}^{\pi}(V_m \sin\omega t)^2 d\omega t}$

solving: $V_{o_{rms}} = \frac{V_m}{\sqrt{2\pi}}\sqrt{(\pi-\alpha) + \frac{\sin 2\alpha}{2}}$

* * * * * * * * *
* End of Material.
* Best of Luck * * *

0




1
0.5
0
-0.5
-1
0
60
120
180
240
300
360
Phase A
Phas B
Phas C