Power System Analysis (I)

Dr. Daifallah Dalabeih

dalabeih@ju.edu.jo

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Notebook.

By. Mohammad Abu Hashya.

* * *

Power System Analysis (1)

Summary for Laws.

* **Bus Admittance Matrix:** element $y_{ii} = \sum$ of admittance Connected directly to i^{th} busbar.
  element $y_{ij} = -1 *$ Admittance between ith & jth busbars.

* **Bus Impedance Matrix:** $\boxed{[Z] = [Y]^{-1}}$

* **Reflection** → To (a) side ($* a^2$). for impedances.
  → To (1) side ($/ a^2$).

- $\boxed{a_{eff} = \text{Line Voltage Ratio.}}$
- $\boxed{X_{new} = X_{old} * \left(\frac{S_{new}}{S_{old}}\right) * \left(\frac{V_{old}}{V_{new}}\right)^2}$

- **Fault Current:** $\boxed{I_f = |E| \frac{1}{X} + |E| \left(\frac{1}{X'} - \frac{1}{X}\right) e^{-t/\tau'} + |E| \left(\frac{1}{X''} - \frac{1}{X'}\right) e^{-t/\tau''}}$

  where: $|E|$ RMS & phase.

  $\boxed{\frac{I_a}{\sqrt{2}} = \frac{|E|}{X}}$, $\boxed{\frac{I_b}{\sqrt{2}} = \frac{|E|}{X'}}$, $\boxed{\frac{I_c}{\sqrt{2}} = \frac{|E|}{X''}}$

* for Δ-Y connection ⇒ phase shift → +ve seq.: HV leads by 30°
  → -ve seq.: LV Leads by 30°

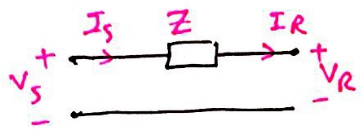
* **Transmission Line:** $\boxed{R_{dc} = \frac{\rho L}{A}}$, R_{ac} found from Tables.

- for Equilateral Lines: $\boxed{L = 2 * 10^{-7} \ln\left(\frac{D}{D_s}\right)}$ H/m , $\boxed{C = \frac{2\pi\epsilon}{\ln\left(\frac{D}{r}\right)}}$ F/m

- **Voltage Regulation:** $\boxed{VR\% = \frac{|V_{R,NL}| - |V_{R,FL}|}{|V_{R,FL}|} * 100\%}$

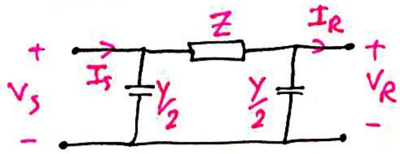
  where $|V_{R,NL}| = \frac{|V_S|}{|A|}$

- **Short Line:**



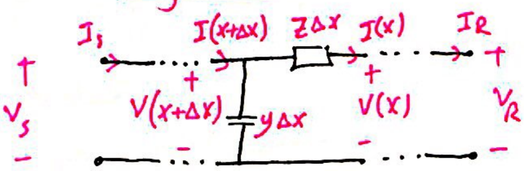
$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

- **Medium Line:**



$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} 1+\frac{ZY}{2} & Z \\ Y+\frac{ZY^2}{4} & 1+\frac{ZY}{2} \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

- **Long Line:**



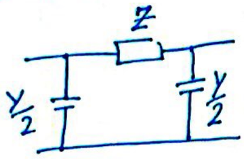
$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} \cosh\gamma\ell & Z_c\sinh\gamma\ell \\ \frac{1}{Z_c}\sinh\gamma\ell & \cosh\gamma\ell \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

$\boxed{\gamma = \sqrt{zy}}$, $\boxed{Z_c = \sqrt{\frac{z}{y}}}$, $\boxed{\gamma = \alpha + j\beta}$

* **Surge Impedance:** $R=0$, $Z_c = \sqrt{L/C}$

* **Surge Impedance Loading:** $\boxed{SIL = P = \frac{V^2}{\sqrt{L/C}}}$, $R = \sqrt{L/C}$ connected at the load.

* **π-cct for Long T.L:**



$Z = Z_c \sinh\gamma\ell$

$$\boxed{\frac{Y}{2} = \frac{\cosh\gamma\ell - 1}{Z} = \frac{1}{Z_c}\tanh\left(\frac{\gamma\ell}{2}\right)}$$

* **Power flow in T.L:**

$$S = \frac{|V_R||V_S|}{|B|}\angle\beta-\delta - \frac{|A||V_R|^2}{|B|}\angle\beta-\alpha$$

$$P = \frac{|V_R||V_S|}{|B|}\cos(\beta-\delta) - \frac{|A||V_R|^2}{|B|}\cos(\beta-\alpha)$$

$$Q = \frac{|V_R||V_S|}{|B|}\sin(\beta-\delta) - \frac{|A||V_R|^2}{|B|}\sin(\beta-\alpha)$$

$$P_{max} = \frac{|V_R||V_S|}{|B|} - \frac{|A||V_R|^2}{|B|}\cos(\beta-\alpha) \rightarrow @\ \delta=\beta \ \& \text{ Leading Pf.}$$

- series compensation factor $\equiv \frac{X_c}{X_L}$
- Shunt Compensation factor $\equiv \frac{B_L}{B_C}$

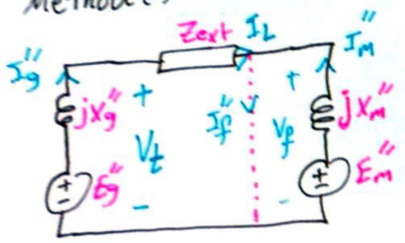
*** parameters of Compensation:**

- Series: $\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} 1 & X_c \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$
- Parallel: $\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{X_L} & 1 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$

* end of first material *

*** Balanced Fault:**

Method (1):



for E_g'' & E_m'' found from: (Before fault)

$$E_g'' = V_t + I_L jX_g''$$
$$E_m'' = V_t - I_L (Z_{ext} + jX_m'')$$

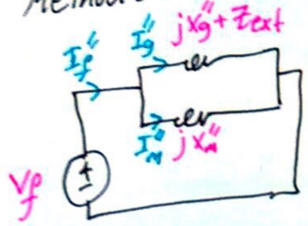
for V_f: $V_f = V_t - I_L Z_{ext}$

for I_g'' & I_m'':

$$\boxed{\begin{aligned} I_g'' &= I_L + \frac{V_f}{Z_{ext} + jX_g''} = \frac{E_g''}{Z_{ext} + jX_g''} \\ I_m'' &= -I_L + \frac{V_f}{jX_m''} = \frac{E_m''}{jX_m''} \end{aligned}}$$

$$I_f'' = I_g'' + I_m''$$

Method (2):



$$\boxed{\begin{aligned} V_{th} &= V_f \\ Z_{th} &= (jX_m'') // (jX_g'' + Z_{ext}) \\ I_f'' &= V_{th}/Z_{th} \end{aligned}}$$

*** Fault Calculations using Z_{bus} Method:**

- for 3-ph fault @ bus K:

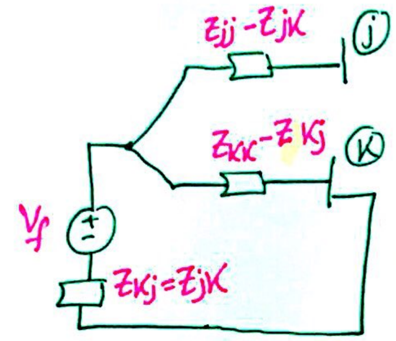
$$\boxed{I_f'' = \frac{V_f}{Z_{KK}}}$$

where: $\begin{bmatrix} V_1' \\ V_2' \\ V_3' \\ V_4' \end{bmatrix} = \begin{bmatrix} V_f \\ V_f \\ V_f \\ V_f \end{bmatrix}$

- New Voltages are: (assume fault @ bus 2) $\begin{bmatrix} V_1' + \Delta V_1 \\ 0 \\ V_3' + \Delta V_3 \\ V_4' + \Delta V_4 \end{bmatrix}$

and: $\begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \Delta V_3 \\ \Delta V_4 \end{bmatrix} = \begin{bmatrix} -I_f'' Z_{12} \\ -V_f \\ -I_f'' Z_{32} \\ -I_f'' Z_{42} \end{bmatrix} = \begin{bmatrix} -V_f Z_{12}/Z_{22} \\ -V_f \\ -V_f Z_{32}/Z_{22} \\ -V_f Z_{42}/Z_{22} \end{bmatrix}$

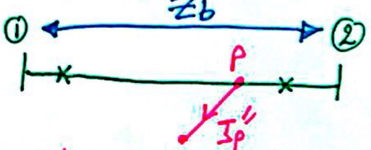
* Fault Calculations using Z_{bus} Equ. cct:



- Before fault: $V_f = V_K = V_j$
- After fault: $\boxed{I_f'' = V_f / Z_{KK} \\ V_j = V_f - I_f'' Z_{Kj}}$
- To find any current flowing in any Branch:

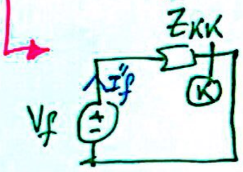
$$I_{ij} = \frac{V_i - V_j}{Z_b}$$ → impedance of that Branch.

* Fault 3-ph Balanced on T.L:



- Use Z_{th} Concept:
  - → Between bus & Ref: $\boxed{Z_{th} = Z_{KK}}$
  - → Between 2 busses: $\boxed{Z_{th} = Z_{jj} + Z_{ii} - 2Z_{ji}}$

equ. ↳

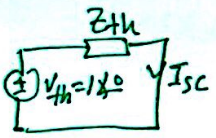


$$\boxed{I_f'' = \frac{V_f}{Z_{KK}}}$$ →

$$\boxed{Z_{th,1,2} = Z_{11} + Z_{22} - 2 Z_{12} \\ \Rightarrow Z_{KK} = Z_{11} + Z_b - \frac{(Z_{11} - Z_{21})^2}{Z_{th_{12}} - Z_b}}$$

* C.B Selection:

$$\boxed{SCMVA = \sqrt{3} * |\text{Nominal Voltage, KV}| * |I_{sc}| * 10^{-3}}, \quad SCMVA_{(PU)} = I_{sc(PU)}$$



$$Z_{th} = \frac{1}{I_{sc(PU)}} = \frac{1}{SCMVA_{(PU)}}$$

- Total instantaneous current $= I_f'' *$ factor
- interrupting KVA $= \sqrt{3} *$ KV of the bus connected to C.B * Current that C.B capable to interrupt.
- voltage Range factor $(K) = \dfrac{\text{Max. voltage}}{\text{Min voltage}}$.
- Rated S/C current * Operating Voltage ≡ Constant.

* Case when the givens are subtransient & Need transient:

Use E/X Method: Keep X'' the same for Gen & Trans. & use X' for the Motor as: $X_m' = 1.5 X_m''$

⇒ This Method Must satisfy 2-conditions:

1) voltage of the system must be within C.B operating Range.
2) Calculated Current $(I_f'') \leq 0.8 *$ Rated S/C current of C.B.

* Unbalanced Faults:

$$\boxed{\begin{aligned} a &= 1\angle 120^\circ \\ a^2 &= 1\angle 240^\circ \\ a^3 &= 1\angle 360^\circ \end{aligned}}$$

$$A_{matrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \qquad A^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}$$

* Symm. Components:

$$[V_{abc}] = [A][V_{012}]$$
$$[V_{012}] = [A]^{-1}[V_{abc}]$$

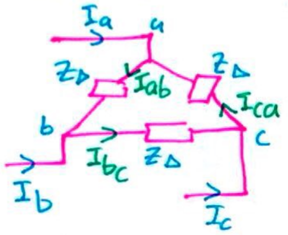
for line & phase voltages.

$$[I_{abc}] = [A][I_{012}]$$
$$[I_{012}] = [A]^{-1}[I_{abc}]$$

for line & phase currents.

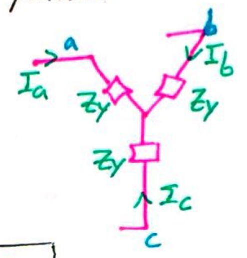
* Symm. Components of Δ & Y loads:

Δ-load:



$$I_a^{(1)} = \sqrt{3}\angle -30^\circ\, I_{ab}^{(1)}$$
$$I_a^{(2)} = \sqrt{3}\angle +30^\circ\, I_{ab}^{(2)}$$
$$I_a^{(0)} = 0$$

Y-load:



$$V_{ab}^{(0)} = 0$$
$$V_{ab}^{(1)} = \sqrt{3}\angle 30^\circ\, V_{an}^{(1)}$$
$$V_{ab}^{(2)} = \sqrt{3}\angle -30^\circ\, V_{an}^{(2)}$$

$$\boxed{Z_\Delta = 3Z_y}$$

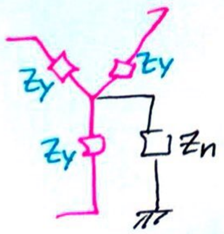
* Power:

$$S_{3\phi} = V_{an}I_{an}^* + V_{bn}I_{bn}^* + V_{cn}I_{cn}^* = 3V^{(0)}I^{(0)*} + 3V^{(1)}I^{(1)*} + 3V^{(2)}I^{(2)*} \longrightarrow (VA).$$

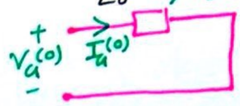
$$S_{(PU)} = \frac{S_{3\phi}}{S_b = 3S_{1\phi}} = \frac{V^{(0)}I^{(0)*}}{S_{1\phi}} + \frac{V^{(1)}I^{(1)*}}{S_{1\phi}} + \frac{V^{(2)}I^{(2)*}}{S_{1\phi}} \longrightarrow (PU).$$

* Seq. Impedance: (Earthing only Effect zero Seq.)

Y-load:

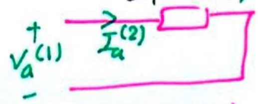


$Z_0 = Z_y + 3Z_n$



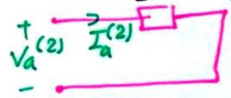
zero-seq-cct

$Z_1 = Z_y$



+ve-seq-cct

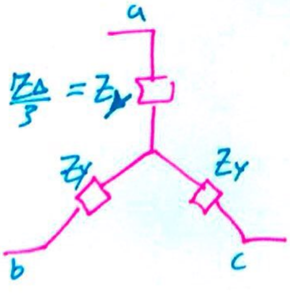
$Z_2 = Z_y$



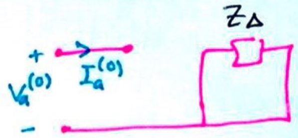
−ve-seq-cct.

for isolated n: $Z_n = \infty$ & solidly grounded: $Z_n = 0$

Δ-load: (converted to Y)



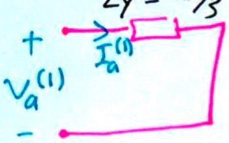
zero-seq-cct



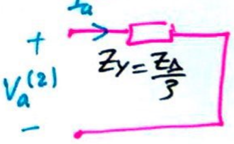
$V_a^{(0)} = 0$

$I_a^{(0)} = 0$

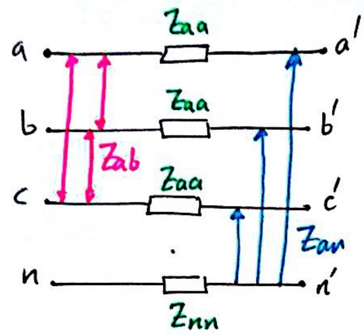
+ve-seq.-cct



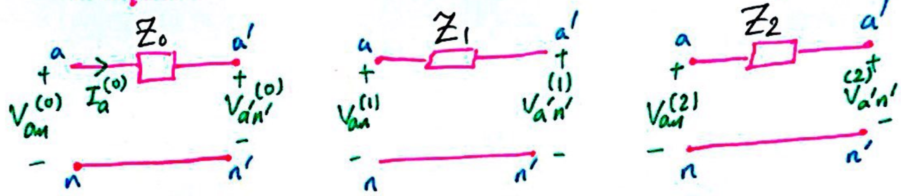
−ve-seq-cct



*Seq. ccts of T.L:



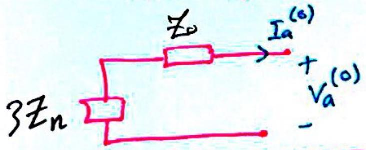
0-seq-cct   +ve-seq-cct   -ve-seq-cct



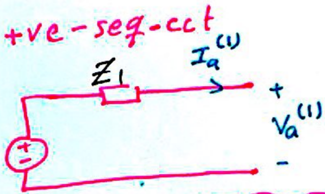
- $Z_0 = Z_s + 2Z_m$
- $Z_1 = Z_2 = Z_s - Z_m$
- $$\boxed{\begin{aligned} Z_s &= Z_{aa} - 2Z_{an} + Z_{nn} \\ Z_m &= Z_{ab} - 2Z_{an} + Z_{nn} \end{aligned}}$$

*Seq. ccts of Gen:

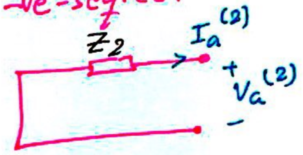
zero-seq.-cct



+ve-seq-cct



-ve-seq-cct

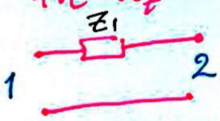


$Z_0 = R + j\omega(L_s - 2M_s)$

$Z_1 = Z_2 = R + j\omega(L_s + M_s)$

*Seq. ccts of Trans.:

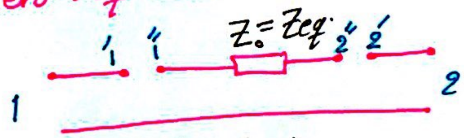
+ve-seq-cct



-ve-seq-cct

$Z_1 = Z_2 = Z_{eq}$.

zero-seq cct:

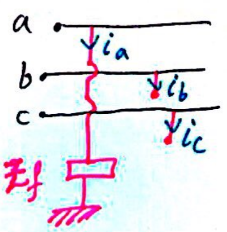


Between $(1' \& 1'')$, $(2' \& 2'')$:

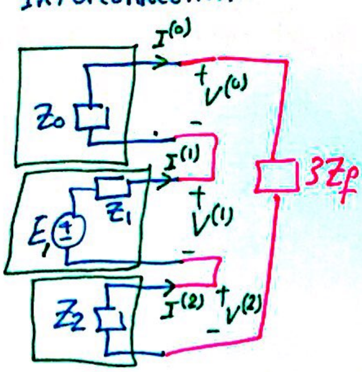
Y-type → $3Z_n$

Δ-type → S/C on $1''$ or/and $2''$.

*Line-Ground Fault:



Interconnection:

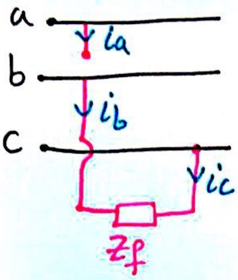


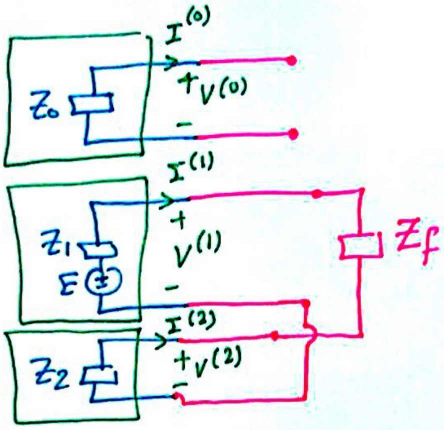
$$\boxed{I^{(0)} = I^{(1)} = I^{(2)} = \frac{E}{Z_0 + Z_1 + Z_2 + 3Z_f}}$$

$$\boxed{\begin{aligned} V^{(0)} &= -I^{(0)} Z_0 \\ V^{(1)} &= E_1 - I^{(1)} Z_1 \\ V^{(2)} &= -I^{(2)} Z_2 \end{aligned}}$$

$I_f = 3I^{(0)}$

* Line-Line Fault:



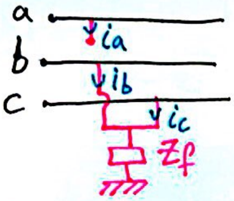


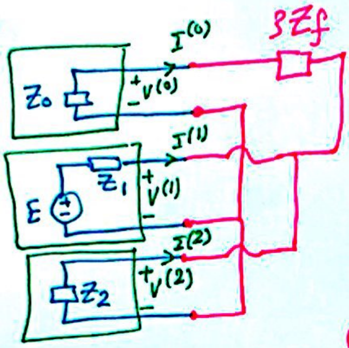
$$\boxed{I^{(0)} = V^{(0)} = 0}$$

$$\boxed{I^{(1)} = -I^{(2)} = \frac{E}{Z_1 + Z_2 + Z_f}}$$

$$\boxed{\begin{aligned} V^{(1)} &= E - I^{(1)} Z_1 \\ V^{(2)} &= -I^{(2)} Z_2 \end{aligned}}$$

* Line-Line-Ground Fault:





$$\boxed{\begin{aligned} I^{(1)} &= \frac{E}{Z_1 + [(Z_0 + 3Z_f) // Z_2]} \\ I^{(0)} &= -I^{(1)} \frac{Z_2}{Z_2 + Z_0 + 3Z_f} \\ I^{(2)} &= -(I^{(0)} + I^{(1)}) \end{aligned}}$$

$$V^{(0)} = -I^{(0)} Z_0, \quad V^{(1)} = V^{(2)} = E - I^{(1)} Z_1$$

* Finding Z_{bus} in the Unbalanced Fault:

- Find Seq. Network 0, +ve, -ve then find $Y_{bus}^{(0)}, Y_{bus}^{(1)}, Y_{bus}^{(2)}$ then find $Z_{bus}^{(0)}, Z_{bus}^{(1)}, Z_{bus}^{(2)}$ from the relation: $Z_{bus} = [Y_{bus}]^{-1}$
- Now you can find: $I^{(0)} = I^{(1)} = I^{(2)} = \dfrac{1\angle 0}{Z_0 + Z_1 + Z_2}$ "in case L-G fault with $Z_f = 0$"

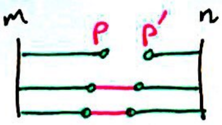
if the fault @ bus K:

$Z_1 = Z_2 = [Z_{KK}$ of $Z_{bus}^{(1)}]$

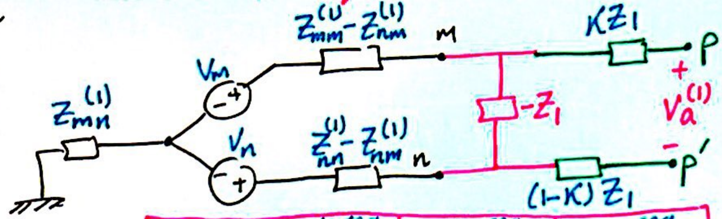
$Z_0 = [Z_{KK}$ of $Z_{bus}^{(0)}]$

* end of second Material *

* O/C Fault:



$\Rightarrow$ +ve seq. equ Thev. cct:

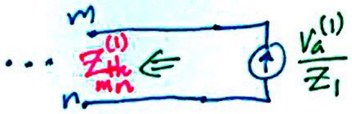


* -ve & zero seq the same BUT with sources S/C.

current injections:

	+ve seq.	-ve-seq	0-seq.
m	$V_a^{(1)}/Z_1$	$V_a^{(2)}/Z_2$	$V_a^{(0)}/Z_0$
n	$-V_a^{(1)}/Z_1$	$-V_a^{(2)}/Z_2$	$-V_a^{(0)}/Z_0$

After simplify:



$$Z_{pp'}^{(0)} = \frac{-Z_0^2}{(Z_{th_{mn}}^{(0)} - Z_0)}$$

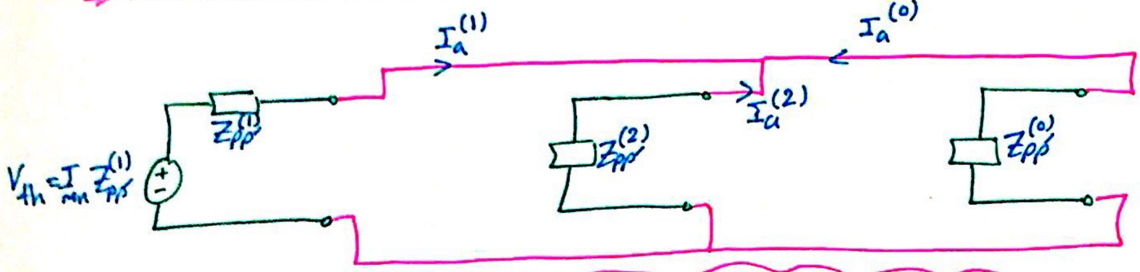
$$Z_{pp'}^{(1)} = \frac{-Z_1^2}{(Z_{th_{mn}}^{(1)} - Z_1)}$$

$$Z_{pp'}^{(2)} = \frac{-Z_2^2}{(Z_{th_{mn}}^{(2)} - Z_2)}$$

$$I_{mn} = \frac{V_m - V_n}{Z_1}$$

$$V_{th} = (V_m - V_n) \frac{Z_{pp'}^{(1)}}{Z_1} = I_{mn} Z_{pp'}^{(1)}$$

⇒ Interconnection between Networks :



$$I_a^{(1)} = \frac{I_{mn} Z_{pp'}^{(1)}}{Z_{pp'}^{(1)} + \left[Z_{pp'}^{(2)} // Z_{pp'}^{(0)}\right]} = \frac{I_{mn} Z_{pp'}^{(1)}\left[Z_{pp'}^{(0)} + Z_{pp'}^{(2)}\right]}{Z_{pp'}^{(1)} Z_{pp'}^{(0)} + Z_{pp'}^{(1)} Z_{pp'}^{(2)} + Z_{pp'}^{(0)} Z_{pp'}^{(2)}}$$

$$V_a^{(0)} = V_a^{(1)} = V_a^{(2)} = I_a^{(1)} \frac{Z_{pp'}^{(0)} Z_{pp'}^{(2)}}{Z_{pp'}^{(0)} + Z_{pp'}^{(2)}} \Longrightarrow V_a^{(0)} = V_a^{(1)} = V_a^{(2)} = \frac{I_{mn}}{\frac{1}{Z_{pp'}^{(0)}} + \frac{1}{Z_{pp'}^{(1)}} + \frac{1}{Z_{pp'}^{(2)}}}$$

⇒ change in voltages : @ i^{th} bus

$$\boxed{\begin{aligned} \Delta V_i^{(0)} &= V_a^{(0)} \left(Z_{im}^{(0)} - Z_{in}^{(0)}\right)/Z_0 \\ \Delta V_i^{(1)} = \Delta V_i^{(2)} &= V_a^{(1)} \left(Z_{im}^{(1)} - Z_{in}^{(1)}\right)/Z_1 \end{aligned}} \Longrightarrow \Delta V_i = \Delta V_i^{(0)} + \Delta V_i^{(1)} + \Delta V_i^{(2)}$$

* Power Flow :

⇒ Power flow equations:

$$S_i = \sum_{j=1}^{N} |V_i||V_j||Y_{ji}| \cancel{\angle \delta_i} - \delta_j - \theta_{ij}$$

$$P_i = \sum_{j=1}^{N} |V_i||V_j||Y_{ji}| \cos(\delta_i - \delta_j - \theta_{ij})$$

$$Q_i = \sum_{j=1}^{N} |V_i||V_j||Y_{ij}| \sin(\delta_i - \delta_j - \theta_{ij})$$

- if V_i for $(i = 1, N)$ are Known Then $P_{i,cal}$, $Q_{i,cal}$ obtained.
- Scheduled values:

$$\boxed{\begin{aligned} P_{i,sch} &= P_{Gi} - P_{Di} \\ Q_{i,sch} &= Q_{Gi} - Q_{Di} \end{aligned}}$$

- Mismatch values:

$$\boxed{\begin{aligned} \Delta P_i &= P_{isch} - P_{ical} \\ \Delta Q_i &= Q_{isch} - Q_{ical} \end{aligned}}$$

- Classifications:

[1] PQ Busbar: $P_{Gi} = Q_{Gi} = 0$
P_{Di} found using Load of Cast.
Q_{Di} found using a certain pf.
⇒ $\boxed{\begin{aligned} P_{isch} &= -P_{Di} \\ Q_{isch} &= -Q_{Di} \end{aligned}}$
P_{isch} & Q_{isch} are specified, $|V_j|$ & δ_j unknowns.

[2] PV Busbar: P_{Gi} & $|V_j|$ are specified, δ_j unknown.

[3] Slack Busbar: $\boxed{V_1 = 1\angle 0}$ ⇒ Reference Busbar is generator Bus.

- Losses: $\sum P_i = P_{Loss} = \sum P_{Gi} - \sum P_{Di}$, $\sum Q_i = Q_{loss} = \sum Q_{Gi} - \sum Q_{Di}$

• **Summary:**

Bus Type	# of Buses	specified quantities	# of available Equations	# of unknowns state variables.
slack $i=1$	1	$\lvert V_1\rvert, \delta_1$	0	0
Voltage controlled OR PV Bus. ($i=2,\dots,N_g+1$)	N_g	$P_i, \lvert V_i\rvert$	N_g	N_g
PQ Bus ($i=N_g+2,\dots,N$)	$N-N_g-1$	P_i, Q_i	$2(N-N_g-1)$	$2(N-N_g-1)$
Summation	N	$2N$	$2N-N_g-2$	$2N-N_g-2$

※ **Gauss Seidal Method:**

$$V_i^{(K)} = \frac{1}{Y_{ii}}\left[\frac{S_i^*}{V_i^{(K-1)*}} - \sum_{j=1}^{i-1} Y_{ji} V_j^{(K)} - \sum_{j=i+1}^{N} Y_{ji} V_j^{(K-1)}\right] \quad (1)$$

* steps:

(i) Specify a voltage for the slack bus ($V_1 = 1\angle 0$).

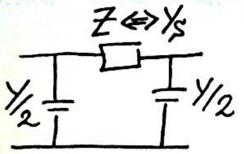
(ii) Assume initial value for other buses ($V_j^{(0)} = 1\angle 0$ for $j=2,\dots,N$).

(iii) Sub. V_1 & V_j into (1) to find $V_j^{(1)}$ for $j=2,\dots,N$.

(iv) find all voltages, then 1st iteration will be completed.

(v) Check that $\left|V_j^{(K)} - V_j^{(K-1)}\right| < \epsilon$ → if YES solution obtained. → if NO Go to 2nd iteration.

* For the Line DATA:



• if Z given, Then one can find Y_s: $Y_s = \frac{1}{Z}$

• from the given Total MVAR: $Y_s = \frac{MVAR}{3V_\phi^2} \Rightarrow \boxed{Y_{pu} = \frac{Y_s}{Y_b}}$ → $Y_b = \frac{1}{Z_b}$

⇒ Then find $\frac{Y}{2}$.

* from the Line DATA ⇒ Obtain $[Y_{bus}]$ matrix.

* Q of the load: $Q = P\tan[\cos^{-1} pf]$

⇒ Procedure:

1) start "assume the slack bus".

2) for PQ-Buses "find $V = |V|\angle\delta$"

⇒ use acceleration factor (α):

$1 < \alpha < 2$ ⇒ Typically: $\boxed{\alpha = 1.6}$

Use: $V_{j,acc}^{(K)} = V_j^{(K-1)} + \alpha\left(V_j^{(K)} - V_j^{(K-1)}\right)$

3) for PV Bus: "find @ first Q"

check: $Q_{min} \le Q \le Q_{max}$

→ if Yes: find $|V|\angle\delta$ using accelerated values.

→ if NO: set Q @ violated limit (e.g. $Q = Q_{max}$)

⇒ convert PV $\xrightarrow{to}$ PQ

Then calculate $|V|\angle\delta$.

⇒ where Q found from: $$Q_i^{(K)} = -Im\left[V_i^{(K-1)*}\left(\sum_{j=1}^{i-1} Y_{ij} V_j^{(K)} + \sum_{j=i}^{N} Y_{ij} V_j^{(K-1)}\right)\right]$$

* For the voltage obtained for PV Bus, one can find the corrected value of $V_j^{(1)}$:

$$V_{j,corr.}^{(1)} = \frac{V_j^{(1)}}{|V_j^{(1)}|} * |V_j^{(0)}|$$

※ Newton - Raphson - Method:

$$\boxed{\begin{aligned} g_1(x_1, x_2, u) &= h_1(x_1, x_2, u) - b_1 \\ g_2(x_1, x_2, u) &= h_2(x_1, x_2, u) - b_2 \end{aligned}}$$

⇒ Represent power flow equations.

$g_1, g_2 \equiv$ Mismatch Values.
$h_1, h_2 \equiv$ Calculated values.
$b_1, b_2 \equiv$ Specified Values.

These Power flow equ. found By:

$$g_1(x_1, x_2, u) = P_{i,cal} - P_{i,sch}.$$
$$g_2(x_1, x_2, u) = Q_{i,cal} - Q_{i,sch}.$$

⇒ $$P_{i,sch} = P_{G_i} - P_{D_i}$$
$$Q_{i,sch} = Q_{G_i} - Q_{D_i}$$

⇒ $$\boxed{\begin{aligned} P_{i,cal} &= \sum_{j=1}^{N} |V_i||V_j||Y_{ij}| \cos(\delta_i - \delta_j - \theta_{ij}) \\ Q_{i,cal} &= \sum_{j=1}^{N} |V_i||V_j||Y_{ij}| \sin(\delta_j - \delta_i - \theta_{ij}) \end{aligned}}$$

* Steps:

1) Find the correction values By: $$\begin{bmatrix} \Delta x_1^{(0)} \\ \Delta x_2^{(0)} \end{bmatrix} = \left[J^{(0)}\right]^{-1} \begin{bmatrix} \Delta g_1^{(0)} \\ \Delta g_2^{(0)} \end{bmatrix}$$

where: $$J^{(0)} = \begin{bmatrix} \frac{dg_1}{dx_1} & \cdots & \frac{dg_1}{dx_N} \\ \vdots & & \vdots \\ \frac{dg_N}{dx_1} & \cdots & \frac{dg_N}{dx_N} \end{bmatrix}$$ & $$\begin{bmatrix} \Delta g_1 \\ \Delta g_2 \end{bmatrix} = \begin{bmatrix} b_1 - h_1^{(0)}(x_1, x_2, u) \\ b_2 - h_2^{(0)}(x_1, x_2, u) \end{bmatrix}$$

2) Find the Correct answer By: $$x_1^{(1)} = x_1^{(0)} + \Delta x_1^{(0)}$$
$$x_2^{(1)} = x_2^{(0)} + \Delta x_2^{(0)}$$

3) CHECK: $\{x_1^{(1)} \,\&\, x_2^{(1)}\} < \epsilon$

→ if YES solution is Obtained.

→ if NO Re-iterate using $\begin{bmatrix} \Delta x_1^{(1)} \\ \Delta x_2^{(1)} \end{bmatrix} = \left[J^{(1)}\right]^{-1} \begin{bmatrix} \Delta g_1^{(1)} \\ \Delta g_2^{(1)} \end{bmatrix}$

⇒ Procedure:

1) start.

2) Assume initial solution for the unknowns $|V_i|$ & δ_i.

3) find $P_{i,cal}$ for $i = 2, \ldots, N$
$Q_{i,cal}$ for $i = 2, \ldots, N - N_g$

4) Evalnate Mismatches
$\Delta P_i = P_{i,sch} - P_{i,cal}$
$\Delta Q_i = Q_{i,sch} - Q_{i,cal}$.

5) Determine Elements of the Jacobian Matrix.

6) Evaluate for corrections ΔV_i, $\Delta \delta_i$

7) CHECK $|\Delta V_i| \,\&\, |\Delta \delta_i| < \epsilon$

If NOT: Go Next iteration
$V_i^{(K+1)} = V_i^{(K)} + \Delta V_i^{(K)}$
$\delta_i^{(K+1)} = \delta_i^{(K)} + \Delta \delta_i^{(K)}$

* if we have 4-buses ; bus#1 is slack & others PQ buses:

$$\begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \cdots & \frac{\partial P_2}{\partial |V_2|}|V_2| & \cdots & \frac{\partial P_2}{\partial |V_4|}|V_4| \\ \vdots & & & & \vdots \\ \frac{\partial P_4}{\partial \delta_2} & \cdots & \frac{\partial P_4}{\partial |V_3|}|V_2| & \cdots & \frac{\partial P_4}{\partial |V_4|}|V_4| \\ \vdots & & & & \vdots \\ \frac{\partial Q_4}{\partial \delta_4} & \cdots & \frac{\partial Q_4}{\partial |V_3|}|V_2| & \cdots & \frac{\partial Q_4}{\partial |V_4|}|V_4| \end{bmatrix} \begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta\delta_4 \\ \Delta|V_2|/|V_2| \\ \Delta|V_3|/|V_3| \\ \Delta|V_4|/|V_4| \end{bmatrix} = \begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta P_4 \\ \Delta Q_2 \\ \Delta Q_3 \\ \Delta Q_4 \end{bmatrix}$$

- **Note**: if bus#4 is PV bus ($\Delta V=0$), Q can't be calculated
  $\Rightarrow$ you will remove one row & one column.

- Order of J matrix $= 2N - N_g - 2$

* Corrections found from: $\begin{bmatrix} \Delta\delta_2 \\ \vdots \\ \frac{\Delta|V_4|}{|V_4|} \end{bmatrix} = [J]^{-1} \begin{bmatrix} \Delta P_2 \\ \vdots \\ \Delta Q_4 \end{bmatrix}$

* Elements of the Jacobian found as follows:

J_{11}: "Non Diagonal" $\frac{\partial P_i}{\partial \delta_j} = -|V_i||V_j||Y_{ij}|\sin(\theta_{ij}+\delta_j-\delta_i)$

"Diagonal" $\frac{\partial P_i}{\partial \delta_i} = \sum_{\substack{n=1 \\ n\neq i}}^{N} |V_i||V_n||Y_{in}|\sin(\theta_{in}+\delta_n-\delta_i)$

J_{21}: "Non Diagonal" $\frac{\partial P_i}{\partial \delta_j} = -|V_i||V_j||Y_{ij}|\cos(\theta_{ij}+\delta_j-\delta_j)$

"Diagonal" $\frac{\partial P_i}{\partial \delta_i} = \sum_{\substack{n=1 \\ n\neq i}}^{N} |V_i||V_n||Y_{in}|\cos(\theta_{in}+\delta_n-\delta_i)$

J_{12}: "Non Diagonal" $|V_j|\frac{\partial P_i}{\partial |V_j|} = |V_j||V_i||Y_{ij}|\cos(\theta_{ij}+\delta_j-\delta_i)$

"Diagonal" $|V_i|\frac{\partial P_i}{\partial |V_i|} = P_i + |V_i|^2 G_{ii}$

J_{22}: "Non Diagonal" $|V_j|\frac{\partial Q_i}{\partial |V_j|} = -|V_j||V_i||Y_{ij}|\sin(\theta_{ij}+\delta_j-\delta_i)$

"Diagonal" $|V_i|\frac{\partial Q_i}{\partial |V_i|} = Q_i - |V_i|^2 B_{ii}$

$\Rightarrow Y_{ii} = G_{ii} + jB_{ii}$

* Having Calculated the voltages, then one can:

① Find S_G for the slack Bus:

$$\boxed{S_G = S_{i\,(cal)} + S_D}$$

⇒ S_i found from $S_i = \sum_{j=1}^{N} |V_i||V_j||Y_{ij}| \angle \delta_i - \delta_j - \theta_{ij}$

⇒ S_D found from $S_D = P_D + j\,Q_D$

P_D: given.

$Q_D = P_D \tan[\cos^{-1} pf]$

② find Q_G for PV bus:

$$\boxed{Q_G = Q_D + Q_i}$$

⇒ $Q_D = P_D \tan[\cos^{-1} pf]$

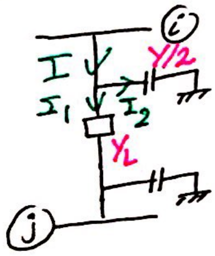
⇒ $Q_i = \sum_{j=1}^{N} |V_i||V_j||Y_{ij}| \sin(\delta_i - \delta_j - \theta_{ij})$

③ find Line flow:

To find S at a certain bus: (Bus i) $\boxed{S_i = V_i I^*}$

where $I = I_1 + I_2$

$= (V_i - V_j) Y_L + V_i \frac{Y}{2}$



* Losses $= \underline{\underline{S_i - S_j}}$

※ Violated Condition:

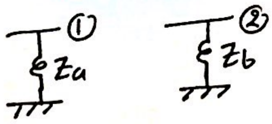
CHECK: $\boxed{1}$ specify Tolerance (a%) ⇒ $1-a\% \leq V_{accepted} \leq 1+a\%$

$\boxed{2}$ Limits: $P_{min} \leq P_G \leq P_{max}$ & $Q_{min} \leq Q_G \leq Q_{max}$

$\boxed{3}$ Capacity of lines.

※ Direct Determination of Zbus Matrix: "Building Algorithm".

1) start with Bus #1

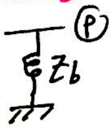


find $\underline{\underline{[Z_a]}}_{1\times1}$.

2) Go to Bus #2

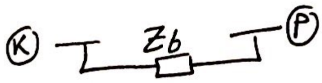
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \underbrace{\begin{bmatrix} Z_a & 0 \\ 0 & Z_b \end{bmatrix}}_{Z_{org}} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

3) Add Z_b to a new Bus ⓟ:



$$\begin{bmatrix} V_1 \\ \vdots \\ V_K \\ V_P \end{bmatrix} = \left[\begin{array}{c|c} Z_{org} & 0 \\ \hline 0 & Z_b \end{array}\right] \begin{bmatrix} I_1 \\ \vdots \\ I_K \\ I_P \end{bmatrix}$$

4) Add Z_b from a new bus (P) to an existing Bus (K):



Matrix:

$$\left[\begin{array}{c|c} Z_{org.} & \begin{matrix} Z_{1K} \\ Z_{2K} \\ \vdots \\ Z_{KK} \end{matrix} \\ \hline Z_{K1} \cdots Z_{KK} & Z_{KK}+Z_b \end{array}\right]$$

elements in the original matrix.

5) Add Z_b from existing Bus (K) to a Ref. Node:

- you will creat new row & column as previous step.
- eliminate (N+1) row & (N+1) column Then find:

$$\boxed{Z_{hi}(new) = Z_{hi}(old) - \frac{Z_{h(N+1)} * Z_{(N+1)i}}{Z_{KK}+Z_b}}$$

6) Add Z_b between two existing Buses (J) & (K):

Matrix:

$$\left[\begin{array}{c|c} Z_{org.} & \begin{matrix} col.j \\ - \\ col.K \end{matrix} \\ \hline row\,j - row\,K & Z_{bb} \end{array}\right]$$

$$\boxed{Z_{bb} = Z_{jj} + Z_{KK} - 2Z_{jK} + Z_b}$$

Then find:

$$\boxed{Z_{hi}(new) = Z_{hi}(old) - \frac{Z_{h(N+1)} * Z_{(N+1)i}}{Z_{bb}}}$$

* * *

End of Material.

* * *

Best of Luck.

* * * * * *