Power1

Fall017

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Power System Analysis (I)

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Fall 2017-2018

Notebook.

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* * *

※ Power System Components:

→ It consists of 3-subsystems:

Generation → Transmission → Distribution

* Generation :

Electrical energy is generated at power stations or plants :

Conventional power plants:

a) Thermal : Example: steam turbine, Gas turbine, Combined cycle.

b) Hydro.

c) Renewable: Example: solar, wind, Tidal.

(~) ⇒ This symbole for generator.

- Bonus Question:

What are the types of power stations in Jordan?

1) Fossil fuel power stations
   - → Natural gas-fired P.S.
     - → AL Quatrana P.S.
     - → Amman East P.S.
     - → Aqaba Thermal P.S.
   - → oil-shale-fired P.S
     - → Attarat P.S.
   - → oil-fired P.S → Aqaba Thermal P.S.

* Some of these P.S are still proposed.

2) Renewable Energy Power stations
   - → Hydrolic P.S.
   - → Wind Farms.

* Usually at conventional P.S voltage are generated in the range of (11-25) KV.

⇒ To transmit electrical energy this voltage is step-up by means of "3-ph transformers".

* In Jordan there are 2-levels of Transmission: (132 and 400) KV.

⇒ This voltage is then step-down at transmission substation to various levels.

Example: in Jordan, such voltages are (33 and 11) KV.

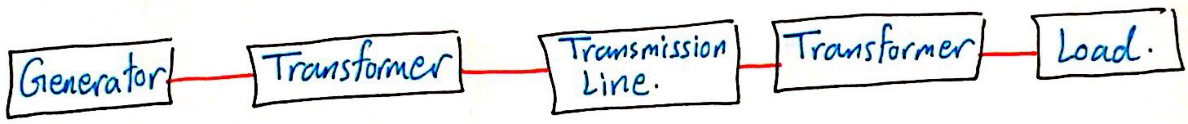
* After that, this voltage is steped-down for various consumers. e.g. (6.6KV, 3KV and 380V).

Q. How many Distribution levels in Jordan? [search in google].

Power System Representation:

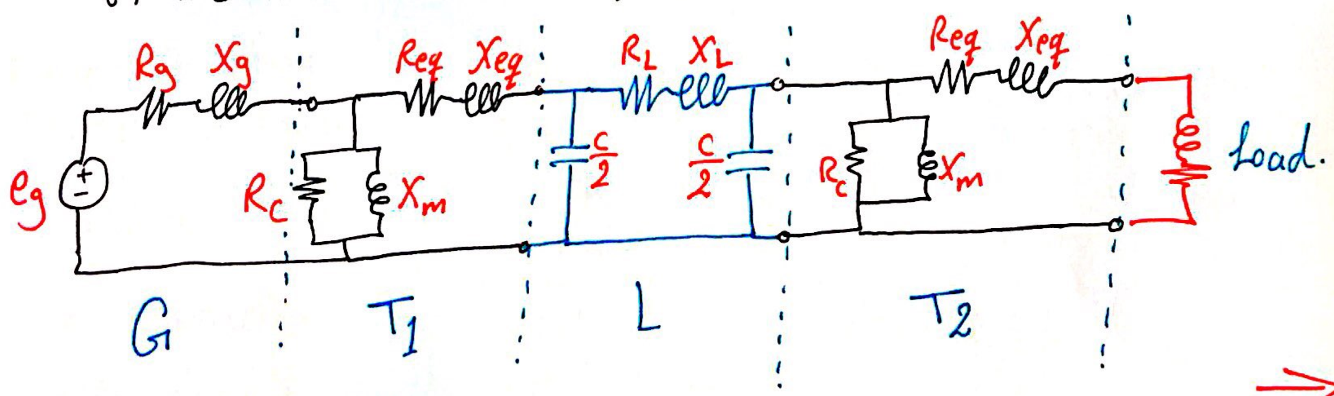
→ There are two approaches: 1) Graphical. 2) Mathematical.

* Graphical: This can be introduced by the following block system:



⇒ Since the system is balanced then one may use per-phase cct.

→ Here one make connections between the equivalent ccts of the various elements as follows:

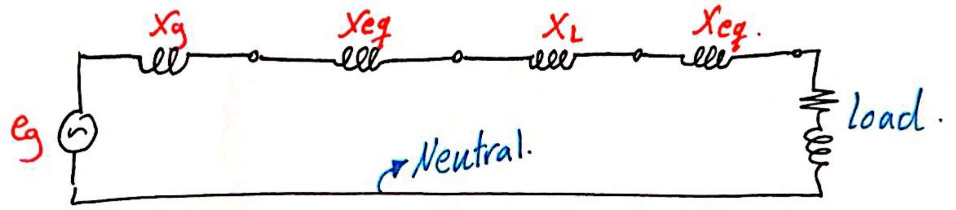


⇒

This is "the exact equivalent per-phase cct."

⇒ • For more simplification one may neglect parallel Branches:

• Also one may neglect resistance of variance elements:



⇒ This Diagram is Called: "Reactance Diagram"

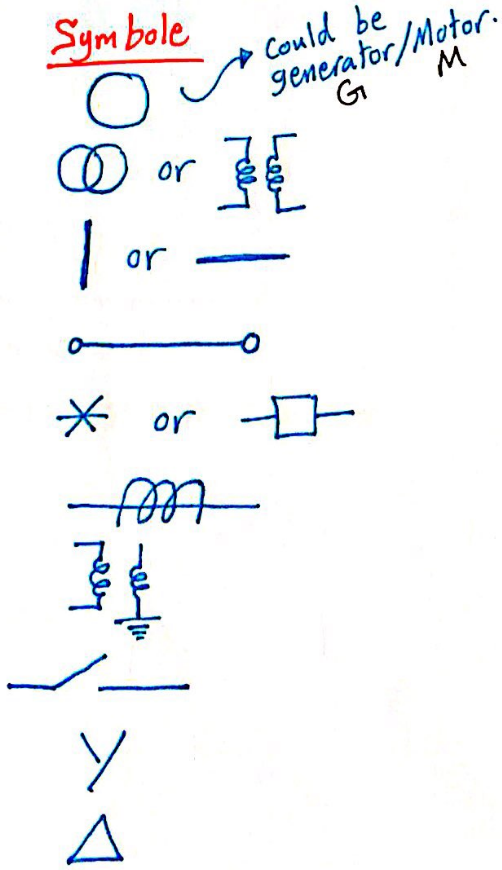
• Which can be used in the analysis of: a given Power system.

* Single/one Line Diagram:

→ since the Neutral is common to all elements, and if one is interested only in the connection between the various elements, Then each element or component is represented by a standard Symbole.

Component — **Symbole**

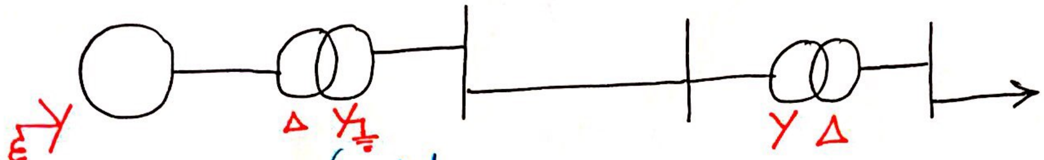
Rotating machinary.
2-winding Transformer.
Busbar (i.e node)
Transmission Line
Circuit Breaker.
Current Transformer
Potential Transformer
Isolator
Wye Connection.
Delta Connection.



* The amount of Components to be displayed depends on the Required application.

• For e.g: in Load flow analysis PT, CT and C.B are not shown.

* To Illustrate Consider the previous system:



Y-connected. Earthing Reactance.

solid Earthing.

This Called: "Single Line Diagram".

* See the given paper (Directed Power Flow Diagram)

↳ This SLD shows the power flow for a given load condition.

* See the given paper (Power Supply System & Key Diagram for 11/0.4KV substation.)

↳ Why we used the Capacitor Bank?

To Improve the pf. (pf Correction).

- C.B: It can make & Break.
- Isolator: It can't Break But It can make.

* Mathematical: ⇒ using Node Equations:

→ This is Illustrated as follows:

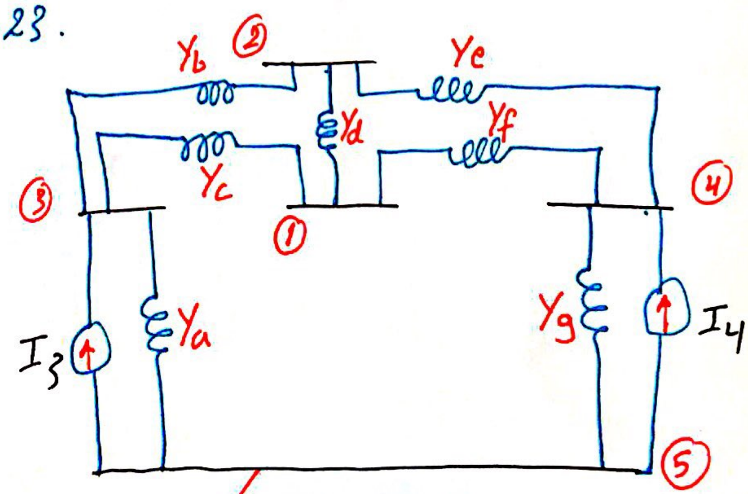
$Y_a, Y_b, \dots$ = Admittance of the line.

① → ⑤ Busbars.

Example in the Book Fig 1.23.

* Equate the currents leaving a busbar through admittances

To currents entering busbar through sources.



↳ Reference Busbar.

→ Continue ...

$(V_1 - V_3) Y_c + (V_1 - V_4) Y_f + (V_1 - V_2) Y_d = 0$

$\Rightarrow \quad V_1 (Y_c + Y_f + Y_d) - V_2 Y_d - V_3 Y_c - V_4 Y_f = 0 \quad \cdots ①$

$(V_2 - V_1) Y_d + (V_2 - V_3) Y_b + (V_2 - V_4) Y_e = 0$

$\Rightarrow \quad V_2 (Y_d + Y_b + Y_e) - V_1 Y_d - V_3 Y_b - V_4 Y_e = 0 \quad \cdots ②$

$(V_3 - V_1) Y_c + (V_3 - V_2) Y_b + (V_3 Y_a) = I_3$

$\Rightarrow \quad -V_1 Y_c - Y_b V_2 + (Y_c + Y_b + Y_a) V_3 = I_3 \quad \cdots ③$

$(V_4 - V_1) Y_f + (V_4 - V_2) Y_e + V_4 Y_g = I_4$

$\Rightarrow \quad -Y_f V_1 - Y_e V_2 + (Y_f + Y_e + Y_g) V_4 = I_4 \quad \cdots ④$

- Now Re-write the Equations in a Matrix Form.

$$\begin{bmatrix} Y_c+Y_f+Y_d & -Y_d & -Y_c & -Y_f \\ -Y_d & Y_d+Y_b+Y_e & -Y_b & -Y_e \\ -Y_c & -Y_b & Y_c+Y_b+Y_a & 0 \\ -Y_f & -Y_e & 0 & Y_f+Y_e+Y_g \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ I_3 \\ I_4 \end{bmatrix} \Rightarrow [Y][V] = [I]$$

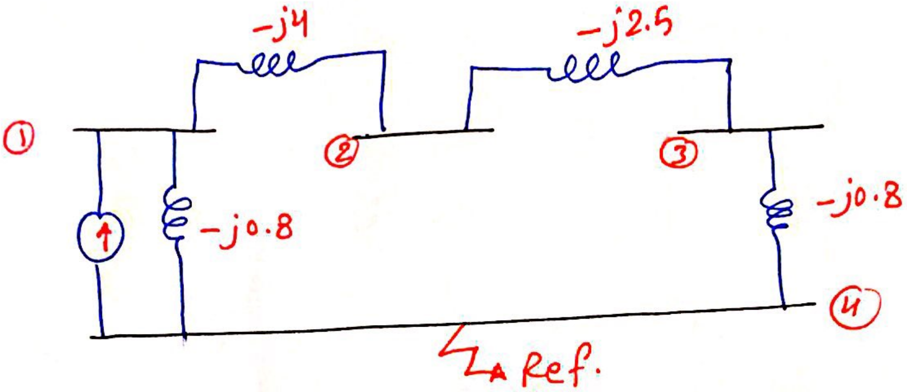
$\longrightarrow$ Symmetrical Matrix.

$[Y] \Rightarrow$ Bus Admittance Matrix.

* You Can Note the following:

element $y_{ii} = \sum$ of admittance Connected directly to i^{th} busbar.

element $y_{ij} = -1 *$ Admittance between the i^{th} & j^{th} busbars.



example: For the given system, evaluate [Y] bus matrix where the given data represent PU admittance ?

Solution:

$$Y = \begin{bmatrix} -j4.8 & j4 & 0 \\ j4 & -j6.5 & j2.5 \\ 0 & j2.5 & -j3.3 \end{bmatrix}$$

** Comment:

Bus Impedance Matrix $[Z]_{bus} \triangleq [Y]_{bus}^{-1}$

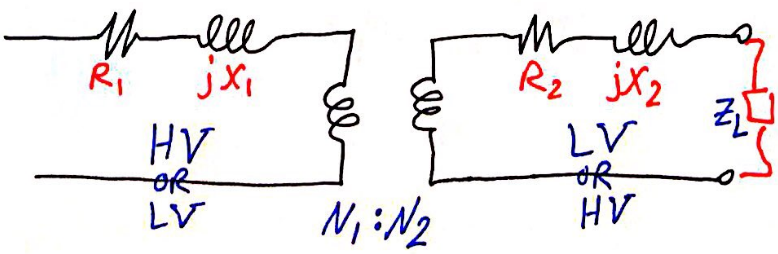
** Comment: 1) [Y] is used in the load flow analysis.
2) [Z] is used in the fault analysis.

※ Revision of equivalent ccts of Transformers & 3-ph synchronous generator:

- Transformer:

⇒ In 1-ph or 3-ph Transformers, Per-phase equ. cct. is used as follows:

(Neglect the parallel branches).



$N_1:N_2 \equiv$ Turns Ratio.

$1 : \frac{N_2}{N_1} = a_1$

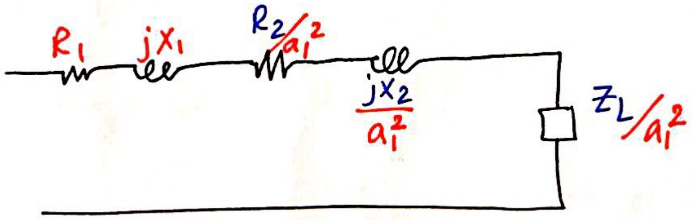
or

$\frac{N_1}{N_2} = a_2 : 1$ ⟹

⇒ By using the concept of Reflection:

- To Reflect to (a) side multiply impedance by a^2.
- To Reflect to (1) side divide impedance by a^2.

e.g.



$$R_{eq_1} = R_1 + R_2/a_1^2 \quad , \quad X_{eq_1} = X_1 + X_2/a_1^2$$

In using Ω units for resistance & Reactance
Then $R_{eq_1} \neq R_{eq_2}$ & $X_{eq_1} \neq X_{eq_2}$

However, By using the PU concept.
Then R_{eq} & X_{eq} are the same irrespective of the side being reflected to.

** Review the advantages of PU system.

1) Simplifies the process of solving system with many transformers.
2) The parameters of generators & transformers have the same range of values.
3) It simplifies the solution of 3-ph system, where $\sqrt{3}$ does not appear.

* For 3-ph Transformers, turns Ratio:

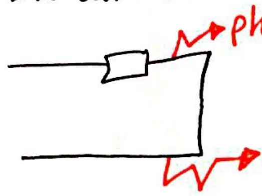
for Y-Y ⇒ $N_1 : N_2$
for Δ-Y ⇒ $N_1/\sqrt{3} : N_2$
for Δ-Δ ⇒ $N_1 : N_2$
for Y-Δ ⇒ $N_1 : N_2/\sqrt{3}$

Effective Turns Ratio $\equiv a_{eff}$.

⇒

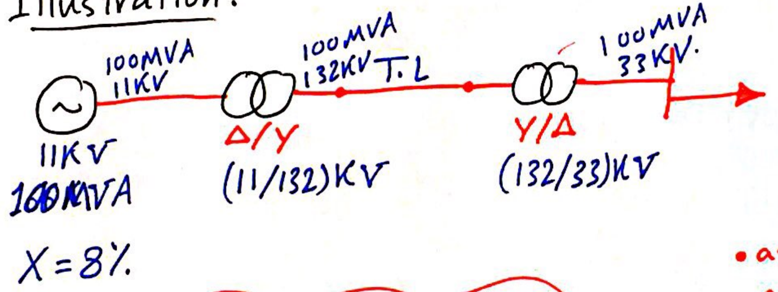
* It can be shown that: a_{eff} = Line Voltage Ratio
⇒ Irrespective of the type of connection.

• **Note:** In our solution procedure, we use per-phase cct concept.



• **Note:** For a given power system, which contain many transformers
Then:
1) This system has one base value for Apparent Power.
2) However, each section has a base value for the voltage, this base value changes according to Transformer Ratio.

Illustration:



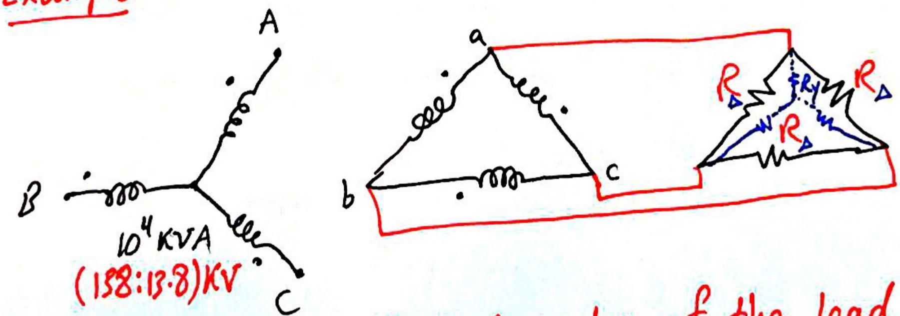
let $S_b = 100MVA$.
$V_b = 11KV$.

$$\boxed{X_{new} = X_{old} * \left(\frac{S_{new}}{S_{old}}\right) * \left(\frac{V_{old}}{V_{new}}\right)^2}$$

• assume we take a base value $V_b = 22KV$ the base value in the next transformer found as follows:

$$\frac{22K}{V_b} = \frac{11K}{132K}$$

Example:



8000KW Load.

For the given system, find the value of the load given between phase and Neutral on the HV side?

→

Solution:

since $P \triangleq \frac{V^2}{R} \Rightarrow R_\Delta = \frac{V^2}{P} = \frac{(13.8*10^3)^2}{\frac{8000*10^3}{3}} \Rightarrow \boxed{R_\Delta = 71.45}\,\Omega$

$R_Y = \frac{R_\Delta}{3} \Rightarrow \boxed{R_Y = 23.805}\,\Omega$

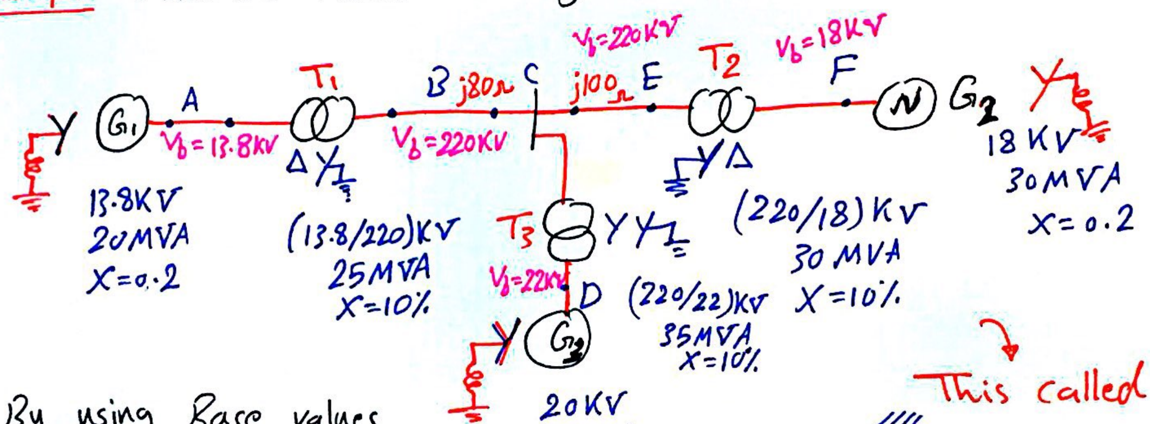
$R_{Y_{HV}} = a_{eff}^2\ R_Y = \left(\frac{138}{13.8}\right)^2 * 23.805 = \boxed{2380.5}\,\Omega$

$\Rightarrow Z_b = \frac{V_b^2}{S_b} = \frac{(138*10^3/\sqrt{3})^2}{10^4*10^3/3} = \boxed{1904.4}\,\Omega$

$Z_b = \frac{V_{b_{LL}}^2}{S_{b_{total}}}$

so $R_{Y_{HV}}(PU) = \frac{2380.5}{1904.4} = \boxed{1.25}$ PU.

Example: Find the Reactance diagram in PU for the following system:



By using Base values of 50MVA, 13.8KV.

This called: "+ve Sequance cct."

Note: the specified PU Reactance of each component is Based on the Ratings of that component.

⇒ These considered as old values.

50MVA, 13.8KV ⇒ New values.

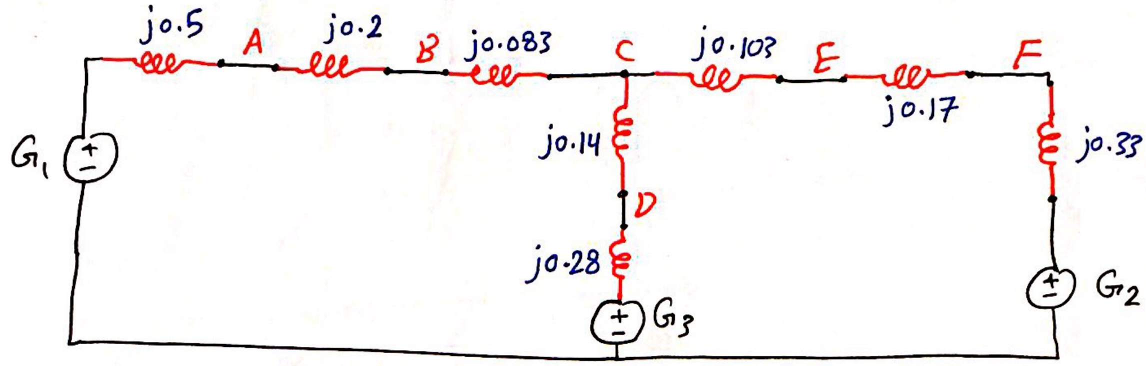
Solution: $X_{new} = X_{old}\left(\frac{S_{new}}{S_{old}}\right)\left(\frac{V_{old}}{V_{new}}\right)^2$

- for G_1: $G_1 = 0.2\left(\frac{13.8}{13.8}\right)^2\left(\frac{50}{20}\right) = \boxed{0.5}$
- for T_1: $T_1 = 0.1\left(\frac{13.8}{13.8}\right)^2\left(\frac{50}{25}\right) = \boxed{0.2}$

• Line BC : $Z_b = \frac{V_b^2}{S_b} = \frac{(220)^2}{50} = \boxed{968}\ \Omega$

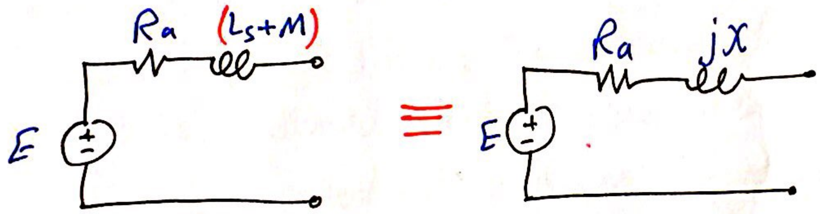
$\therefore Z_{line\,(PU)} = \frac{j80}{968} = \boxed{j0.083}$ PU.

• G_3 : $X_{new} = 0.2 \left(\frac{20}{22}\right)^2 \left(\frac{50}{30}\right) = \boxed{0.28}$



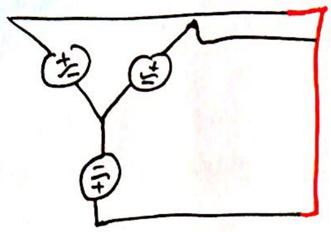
* Equivalent cct. of 3-ph synch. Gen. :

• Armature on stator & field on Rotor.



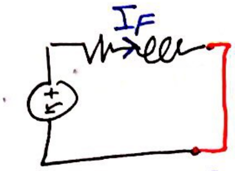
* Equivalent cct. of 3-ph Gen. under S/C conditions:

• objective : Find the Equivalent cct. under such condition.



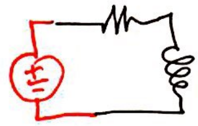
⇒ Under this condition, a very high current is going to flow, This current generates an opposing flux to the main field flux (i.e Armature Reaction) Causing the generated voltage To Decrease and Consequantly the fault current will decrease.

Scanned by CamScanner

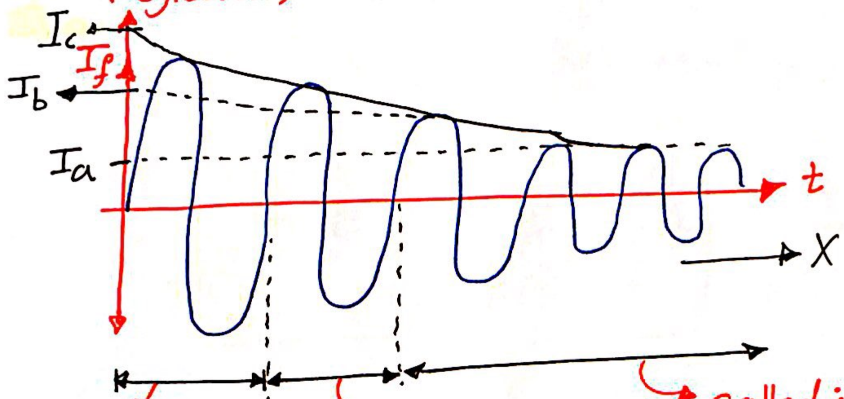


⇒ This looks like the case of RL cct where an AC source is applied.

$I_F \equiv$ Fault Current in the Armature Winding.



- If the DC component in the response is neglected, Then I_F will look like the following:



"figure (1)"

→ X increases.

$I_a \equiv$ peak current @ steady state.

called: "Subtransient." called: "Transient." called: "Steady State".

* Since it is assumed that No load emf of the generator doesn't change, Then To account for current decrease, it is assumed that the gen. Reactance changes. The so called "Subtransient Reactance X''", "Transient Reactance X'", and "Steady state Reactance X".

- I_f can be expressed as:

$$I_f = |E|\frac{1}{X} + |E|\left(\frac{1}{X'} - \frac{1}{X}\right)e^{-t/\tau'} + |E|\left(\frac{1}{X''} - \frac{1}{X'}\right)e^{-t/\tau''}$$

$|E| \equiv$ RMS value of the No-load phase voltage.

$\tau', \tau'' \equiv$ Time Constants of Transient and Subtransient Conditions.

* If figure(1) is given, Then it Can be used to find experimentally X, X', X'' as follows:

⟹

$$\frac{I_a}{\sqrt{2}} = \frac{|E|}{X} \quad \cdots (1)$$

$$\frac{I_b}{\sqrt{2}} = \frac{|E|}{X'} \quad \cdots (2)$$

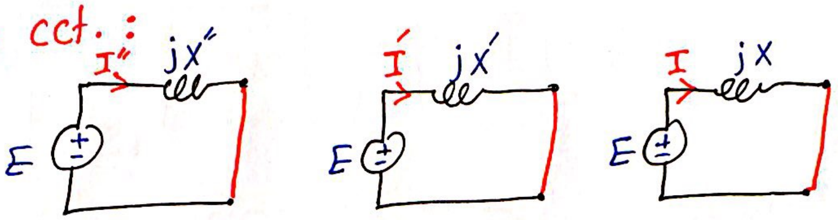
$$\frac{I_c}{\sqrt{2}} = \frac{|E|}{X''} \quad \cdots (3)$$

Comment:

1] If the graph of the fault current is given, Then (1), (2), & (3) can be used to evaluate X, X', X''.

2] If X, X', X'' are Known, Then the equivalent cct. can be found and Consequantly Fault current I_f'', I_f', I_f Can be Evaluated..

• **Equ. cct.:**



* Subtransient used in circuit breaker.
* Transient used in stability.
* Steady State used in steady state applications.

Example: A 300MVA, 13.8KV, 3-ph Y-connected 60 HZ generator. is adjusted to give rated voltage at O/C. a 3-ph fault is applied to its O/C terminals, the obtained fault current was as follows:

$$i = 10^4 \left[1 + e^{-t/\tau_1} + 6 e^{-t/\tau_2} \right] \text{, where } \tau_1 = 200\,mS,\ \tau_2 = 15\,mS$$

Evaluate: X, X', X'' in Ω & in PU.

Solution: $\tau_1 = 200\,mS$ (Transient) & $\tau_2 = 15\,mS$ (Subtransient)

By Comparing it with the given general expresion and Knowing that $|E| = \frac{13.8}{\sqrt{3}}$ KV $\Longrightarrow$

$$\boxed{\begin{aligned} X &= 0.797\,\Omega \\ X' &= 0.398\,\Omega \\ X'' &= 0.0996\,\Omega \end{aligned}}$$

$$Z_b = \frac{V_{LL}^2}{S_{3\phi}} = \frac{(13.8K)^2}{300M} = 0.635\,\Omega$$

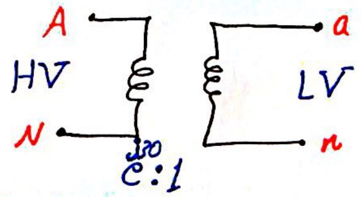
Per unit values:

$$\boxed{\begin{aligned} X &= 1.26 \text{ PU.} \\ X' &= 0.627 \text{ PU.} \\ X'' &= 0.157 \text{ PU} \end{aligned}}$$

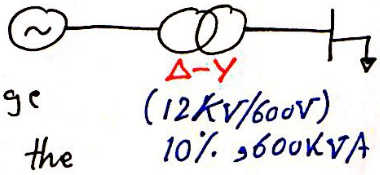
• **Note:** In the equ. cct. of 3-ph Transformer
in the case of Y-Y or Δ-Δ There is · No-phase shift.
in the case of Δ-Y or Y-Δ There is a 30°-phase shift.

⇒ In the Convention, in the +ve phase sequance the HV leads LV by 30°, in the -ve phase sequance the HV lags LV by 30°.

• +ve phase sequance, in PU :



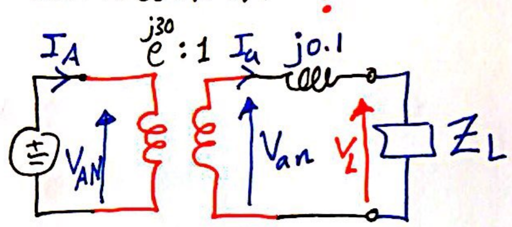
* Illustration:



Assume +ve phase sequance.

If the Line voltage & Line current at the gen. terminals are 11.9KV & 20A respectively, & the pf seen by the gen. is 0.8 lagging. Find the Line Current & voltage at LV side By using PU concept & using Base values of 12KV, 600KVA. ?

Solution:



$I_b = \frac{600*10^3}{\sqrt{3}*12*10^3} \Rightarrow \boxed{I_b = 28.87}$ A.

$|I_A|_{(PU)} = \frac{20}{28.87} \Rightarrow \boxed{|I_A| = 0.69}$ PU

$|V_{AN}| = \frac{11.9/\sqrt{3}}{12/\sqrt{3}} \Rightarrow \boxed{|V_{AN}| = 0.99}$ PU

$\Rightarrow I_A = 0.69\angle{-\cos^{-1}0.8} \Rightarrow I_A = 0.69\angle{-36.87^\circ}$ PU.

$\boxed{V_{AN} = 0.99\angle{0^\circ}}$ PU.

Now the values seen from LV : $V_{an} = 0.99\angle{-30^\circ}$ & $I_a = 0.69\angle{-66.87^\circ}$

$V_L = V_{an} - j0.1 * I_a \Rightarrow V_L = 0.95\angle{-33.4^\circ}$

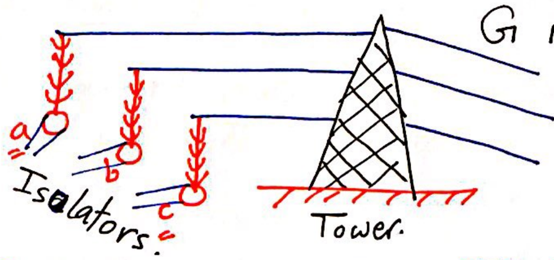
$Z_L = V_L / I_a \Rightarrow Z_L = 1.38\angle{33.5^\circ}$

* * *

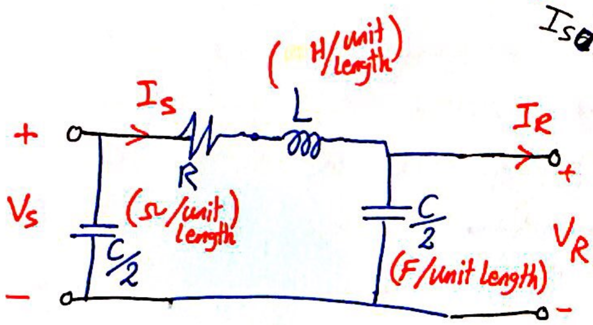
* Transmission Line:

- Objective: To find voltage, Current and Power Relationships.
- Procedure: Find the equivalent cct. of T.L.
- Parameters of Transmission Line:
  1) Resistance. 2) Inductance. 3) Capacitance. 4) Conductance.

$V_s \equiv$ Sending End Voltage.
$V_R \equiv$ Receiving End Voltage.



G is neglected.



* Since the line is made from a Conducting material (e.g AAC, ACSR) Then it has a Resistance, R.

Type classifications.
AAC: All Aluminum Conductor.

$$R_{dc} = \frac{\rho \ell}{A}$$

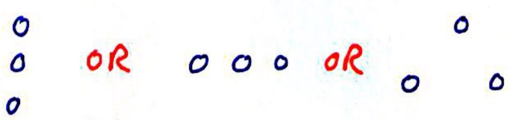
However, one use R_{ac}. ($R_{ac} > R_{dc}$), which can be found from standard Tables.

* Since the line carry ac current → generate ϕ → ϕ will induce a voltage in the Conductor. This is represented by L.

$C \equiv$ Used To Represent The Capacitance Effect Between phase & ground or Neutral.

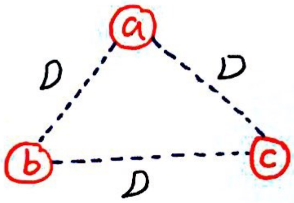
* Conductance, which represent the Leakage current along the line insulators of the line due to Pollution, it is very small and variable.

* The inductance & Capacitance of lines depend on the configuration of the lines.



For e.g: for Equilateral Lines L & C can be found as follows:

$$\boxed{L = 2 * 10^{-7} \ln\left(\frac{D}{D_s}\right)} \text{ H/m.}$$



$D_s \equiv$ Called Geometrical Radius (GMD) and can be found from standard tables.

$D \equiv$ Distance Between Lines & Conductors.

$$\boxed{C_n = \frac{2\pi\epsilon}{\ln\left(\frac{D}{r}\right)}} \text{ F/m}$$

$C_n \equiv$ Capacitance Between Phase & Neutral.
$r \equiv$ Radius of the Conductor.

Example: A 18Km, 60HZ, single-cct. 3-ph line composed of portridge conductors equally spaced with 1.6m. between centers. Assume wire tempreture of 50°C. Find the series impedance of the line? (i.e R jX)

Solution: from the given standard tables @ 50°C $R_{ac} = 0.3792\ \Omega/mi$

for this line: $R = 0.3792 * \frac{18}{1.609} \Rightarrow \boxed{R = 4.24\,\Omega}$.

$L = 2*10^{-7} \ln\left(\frac{D}{D_s}\right)$; $D = 1.6$ m, from standard tables $D_s = 0.0217$ ft.

$\Rightarrow L = 2*10^{-7} \ln\left(\frac{1.6}{0.0217*0.3048}\right) \Rightarrow \boxed{L = 10.98*10^{-7}}$ H/m.

Note:
1mile = 1.609 Km.
1 ft = 30.48 cm = 0.3048 m

$\therefore$ For this T.Line: $L = 10.98*10^{-7} * 18000$
$\Rightarrow \boxed{L = 0.0197}$ H.

$\therefore x = \omega L = 2\pi * 60 * 0.0197 = 7.43\,\Omega$

$\therefore Z = R + jX$

$\Rightarrow \boxed{Z = 4.24 + j7.43\ \Omega}$ #

※ Analysis:

- Lines Can be Classified into:

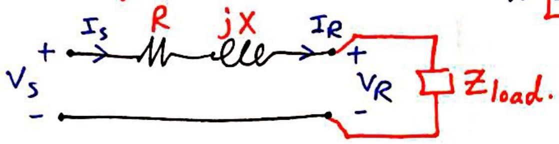
1) Short Line up to 80Km.
2) Medium Line $(80 < \ell < 240)$ Km.
3) Long Line $(\ell > 240)$ Km.

Note: There is 2 classifications for T.L : Types & Lengths.

• Short Line:

Hence the capacitance effect is neglected.

∴ Equivalent cct:



$$\therefore \boxed{I_s = I_R}\ ,\ \boxed{V_s = I_s(R+jX)+V_R}$$

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{array}{cc} A & B \\ \left[\begin{matrix} 1 & Z \\ 0 & 1 \end{matrix}\right] & \\ C & D \end{array} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

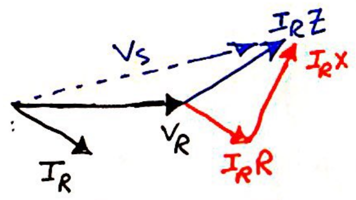
Called: ABCD parameters of short T. Line.

*Voltage Regulation: $VR\% = \dfrac{|V_{R,NL}| - |V_{R,FL}|}{|V_{R,FL}|} * 100\%$

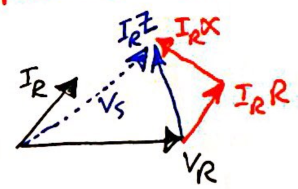
⇒ The Line will be analyzed under Inductive, Capacitive and Resistive Loading conditions. By using the concept of "phasor diagram".

- Here it will be assumed that $|V_R|$ & $|I_R|$ fixed and the pf is varied.

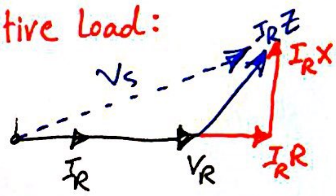
*Inductive load:



*Capacitive load:



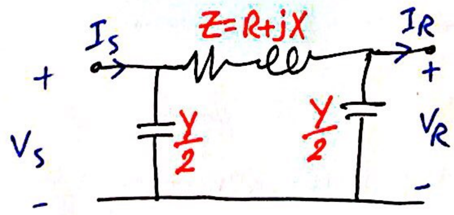
*Resistive Load:



$$V_{s\,(ind)} > V_{s\,(Res)} > V_{s\,(cap)}$$

*Ferranti-Effect: It is the increase in voltage occuring at the receiving end of a long T.L above the voltage at the sending end and it is more pronounced as how much the longer the line & the higher the voltage applied, also more pronounced in underground cables.

• Medium Line:



$Z \equiv$ Total series impedance of the line.

$Y \equiv$ Total shunt admittance of the line $= j\omega C$.

KVL:
$$V_s = Z\left(I_R + V_R\frac{Y}{2}\right) + V_R$$

$$\Rightarrow \boxed{V_s = V_R\left(1+Z\frac{Y}{2}\right) + Z I_R} \quad (1)$$

KCL: $$\boxed{I_s = V_s\frac{Y}{2} + \left(I_R + V_R\frac{Y}{2}\right)} \quad (2)$$

⇒

$\Rightarrow$ Substitute ① in ②:

$$I_S = \frac{Y}{2}\left[V_R\left(1+Z\frac{Y}{2}\right)+ZI_R\right]+I_R+V_R\frac{Y}{2}$$

$$= V_R\left[\frac{Y}{2}\left(1+Z\frac{Y}{2}\right)+\frac{Y}{2}\right]+I_R\left(1+\frac{ZY}{2}\right) = \boxed{V_R\left(Y+Z\frac{Y^2}{4}\right)+I_R\left(1+\frac{ZY}{2}\right)} \cdots ③$$

$\Rightarrow$ Re-write ① & ③ in Matrix Form:

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} \underbrace{1+\frac{ZY}{2}}_{A} & \underbrace{Z}_{B} \\ \underbrace{Y+\frac{ZY^2}{4}}_{C} & \underbrace{1+\frac{ZY}{2}}_{D} \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

As required $D=A$. #.

$\therefore V_S = AV_R + BI_R \cdots ④$

$I_S = CV_R + DI_R \cdots ⑤$

$$VR\% = \frac{|V_{R,NL}| - |V_{R,FL}|}{|V_{R,FL}|} * 100\%$$

$\Rightarrow$ $V_{R,NL}$ means $I_R = 0$

$\therefore ④ \Rightarrow V_{R,NL} = \frac{V_S}{A}$

so, we could write:

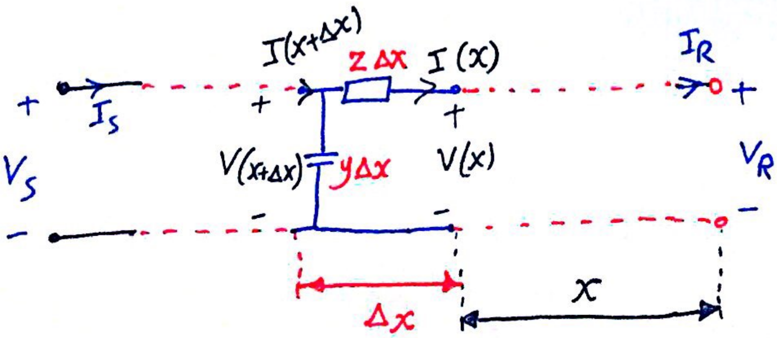
$$VR\% = \frac{|V_S/A| - |V_{R,FL}|}{|V_{R,FL}|} * 100\%$$

- **Long Transmission Line:**

In this case the Line is represented by:
a distributed Parameters as follows:

$z \equiv$ Series Impedance per unit length (Ω/mile)
$y \equiv$ Shunt Admittance per unit length (S/mile)

*Objective: To derive the equations. for $V(x)$ & $I(x)$.
* Procedure: Consider a small segment of the Line of the length Δx where x is measured from Receiving End.



By KVL: $V(x+\Delta x) = I(x)\,z\,\Delta x + V(x)$

$$\therefore \lim_{\Delta x \to 0} zI(x) = \frac{V(x+\Delta x)-V(x)}{\Delta x} \Rightarrow \boxed{\frac{dV}{dx} = zI(x)} \cdots \boxed{1} \Rightarrow$$

By KCL: $I(x+\Delta x) = V(x+\Delta x)\, y\Delta x + I(x)$

$$\therefore \lim_{\Delta x \to 0} \frac{I(x+\Delta x) - I(x)}{\Delta x} = y\, V(x+\Delta x) \Rightarrow \boxed{\frac{dI}{dx} = y\,V(x)} \quad \cdots \boxed{2}$$

* Diff. ② & ① with respect to x:

$$\frac{d^2V}{dx^2} = z\frac{dI}{dx} = zy\,V(x) \qquad \therefore \boxed{\frac{d^2V}{dx^2} = zyV(x)} \quad \cdots \boxed{3}$$

$$\frac{d^2I}{dx^2} = y\frac{dV}{dx} = yz\,I(x) \qquad \boxed{\frac{d^2I}{dx^2} = zy\,I(x)} \quad \cdots \boxed{4}$$

* To simplify let $\gamma^2 = zy$:

$\therefore \boxed{\gamma = \sqrt{zy}}$, and it is called $\equiv$ Propagation Constant.

$\boxed{Z_c = \sqrt{\frac{z}{y}}} \equiv$ Characteristic Impedance.

$$\Rightarrow \boxed{\frac{d^2V}{dx^2} = \gamma^2 V} \cdots \boxed{5}, \quad \boxed{\frac{d^2I}{dx^2} = \gamma^2 I} \cdots \boxed{6}$$

* Solve ⑤ & ⑥ To find $V(x)$ & $I(x)$:

$$\therefore \boxed{\begin{aligned} V(x) &= A_1 e^{\gamma x} + A_2 e^{-\gamma x} \quad \cdots \boxed{7} \\ I(x) &= B_1 e^{\gamma x} + B_2 e^{-\gamma x} \quad \cdots \boxed{8} \end{aligned}}$$

where A_1, A_2, B_1 & B_2 Constants to be found from initial conditions.

* Finding A_1 & A_2: $\boxed{V(0) = V_R = A_1 + A_2} \cdots \boxed{9}$

$$\frac{dV}{dx} = A_1\gamma e^{\gamma x} - A_2\gamma e^{-\gamma x} \Rightarrow z\,I(x) = A_1\gamma e^{\gamma x} - A_2\gamma e^{-\gamma x}$$

$$\boxed{z\,I(0) = A_1\gamma - A_2\gamma = z\,I_R} \cdots \boxed{10}$$

* Solve ⑨ & ⑩ To find A_1 & A_2:

$$\gamma * ⑨ + ⑩: \quad \gamma V_R + zI_R = 2A_1\gamma \Rightarrow \therefore A_1 = \frac{1}{2}\left(\frac{\gamma V_R + zI_R}{\gamma}\right) = \frac{1}{2}\left(V_R + \frac{z}{\gamma} I_R\right)$$

$$\Rightarrow A_1 = \frac{1}{2}\left(V_R + \sqrt{\frac{z}{y}}\, I_R\right) \Rightarrow \boxed{A_1 = \frac{1}{2}(V_R + Z_c I_R)} \cdots \boxed{11}$$

* from ⑨: $A_2 = V_R - A_1 \Rightarrow A_2 = V_R - \frac{1}{2}(V_R + Z_c I_R) \Rightarrow \boxed{A_2 = \frac{1}{2}(V_R - Z_c I_R)} \cdots \boxed{12}$

* Substitute ⑪ & ⑫ into ⑦:

$$V(x) = \frac{1}{2}(V_R + Z_c I_R) e^{\gamma x} + \frac{1}{2}(V_R - Z_c I_R) e^{-\gamma x}$$

$$= V_R\left(\frac{e^{\gamma x} + e^{-\gamma x}}{2}\right) + I_R Z_c\left(\frac{e^{\gamma x} - e^{-\gamma x}}{2}\right) \Rightarrow \boxed{V(x) = V_R \cosh\gamma x + Z_c I_R \sinh\gamma x.} \cdots \boxed{13}$$

$\therefore$ By Similar Procedures one can find B_1 & B_2, By substitution it can be found:

$$\boxed{I(x) = \frac{V_R}{Z_c}\sinh\gamma x + I_R\cosh\gamma x} \cdots \boxed{14}$$

$\therefore$ @ $x = \ell$ (i.e length of the Line)

$$V(\ell) = V_s = V_R \underbrace{\cosh\gamma\ell}_{A} + \underbrace{Z_c}_{B} I_R \sinh\gamma\ell$$

$$I(\ell) = I_s = \frac{V_R}{Z_c} \underbrace{\sinh\gamma\ell}_{C} + I_R \underbrace{\cosh\gamma\ell}_{D}$$

$\therefore$ Writing V_s & I_s in Matrix Form:

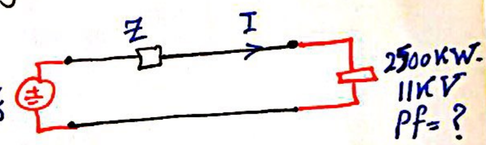
$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

where: $\boxed{\begin{aligned} A &= D = \cosh\gamma\ell \\ B &= Z_c \sinh\gamma\ell \\ C &= \frac{1}{Z_c}\sinh\gamma\ell \end{aligned}}$

Example: For the previous example of the 18Km Line. If this line delivers 2500 KW of 11KV to a balance Load. Find V_s & voltage Regulation when the pf is 1) 0.8 lagging. 2) unity. 3) 0.9 Leading.

Solution: $V_s = V_R + I Z \Rightarrow V_R = \frac{11}{\sqrt{3}} * 10^3 \angle 0^\circ$ volt.

$Z = 4.24 + j\,7.45 = 8.57 \angle 60.35^\circ\ \Omega$



1) 0.8 Lag.: $I = \frac{2500 * 10^3}{\sqrt{3} * 11 * 10^3 * 0.8} \angle -\cos^{-1}0.8 \Rightarrow \boxed{I = 164.02 \angle -36.87^\circ}$ A.

By substitution: $\boxed{V_s = 7660.66 \angle 4.19^\circ}$ volt. VR% = 20.6%

2) unity : $I = \frac{2500 * 10^3}{\sqrt{3} * 11 * 10^3 * 1} \angle 0^\circ \Rightarrow \boxed{I = 131.2 \angle 0}$ A $\therefore \boxed{V_s = 6975.9 \angle 8.05^\circ}$ volt.

VR% = 9.84%

3) 0.9 Lead: $I = \frac{2500 * 10^3}{\sqrt{3} * 11 * 10^3 * 0.9} \angle \cos^{-1}0.9 \Rightarrow \boxed{I = 145.8 \angle 25.8^\circ}$ A $\therefore \boxed{V_s = 6553.6 \angle 10.97^\circ}$ volt.

VR% = 3.19%

* Voltage Regulation found by: $VR\% = \frac{|V_s| - |V_R|}{|V_R|} * 100\%$

Example: A 60HZ, 3-ph Transmission 175 miles Long. It has a total series impedance of $Z = 35 + j140\ \Omega$, and a total shunt admittance of $Y = j930 * 10^{-6}$ S The line delivers 40MW at 220KV and 0.9 pf lagging.

Find: a) V_s, I_s and the pf @ the sending end.

b) VR% & η of the Line.

Solution:

a) $V_s = \underbrace{\cosh\gamma\ell}_{A}\, V_R + \underbrace{Z_c \sinh\gamma\ell}_{B}\, I_R$

$I_s = \underbrace{\frac{1}{Z_c}\sinh\gamma\ell}_{C}\, V_R + \underbrace{\cosh\gamma\ell}_{D}\, I_R$

$\Rightarrow Z_c = \sqrt{\frac{Z}{y}} = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{35 + j140}{j930 * 10^{-6}}}$ $Z_c = 393.9 \angle -7^\circ\ \Omega$

$\gamma\ell = \sqrt{zy}\,\ell = \sqrt{(35 + j140)(j930 * 10^{-6})}$

$\Rightarrow \gamma\ell = 0.3663 \angle 83^\circ$

So $e^{\gamma\ell} = e^{0.3663\angle 83^\circ} = e^{0.0446} \cdot e^{j0.3636}$ → always in rad.

$\Rightarrow$

$\Rightarrow \underline{e^{\gamma l} = 1.0456 \angle 20^\circ}$, $e^{-\gamma l} = \frac{1}{e^{\gamma l}} = \underline{0.9563 \angle -20^\circ}$

$A = D = \cosh \gamma l = \frac{1}{2}[e^{\gamma l} + e^{-\gamma l}] \Rightarrow \boxed{A = D = 0.9407 \angle 1^\circ}$

$B = Z_c \sinh \gamma l \Rightarrow \boxed{B = 135.9 \angle 76^\circ}$

$C = \frac{1}{Z_c} \sinh \gamma l = \frac{1}{2Z_c}[e^{\gamma l} - e^{-\gamma l}] \Rightarrow \boxed{C = 875 * 10^{-6} \angle 90^\circ}$

$\boxed{V_R = \frac{220}{\sqrt{3}} \angle 0 \ KV}$ $\Rightarrow I_R = \frac{40 * 10^6}{\sqrt{3} * 220 * 10^3 * 0.9} \angle -\cos^{-1} 0.9 \Rightarrow \boxed{I_R = 116.6 \angle -25.8^\circ}$ A.

By substitution it can be found: $\boxed{V_s = 130.4 \angle 6.3^\circ \ KV}$ $\boxed{I_s = 119 \angle 33^\circ}$

pf @ sending end $= \cos(6.3 - 33) = \boxed{0.8934}$ Leading.

b) $VR\% = \frac{|V_s/A| - |V_R|}{|V_R|} * 100\% = 9.15\%$, $\eta = \frac{P_{out}}{P_{in}} = \frac{40 * 10^6}{\sqrt{3} V_s I_s pf} \Rightarrow \eta = 96.5\%$

Note: See the report about the Losses in NEPCO.

* Comments:

- It was found that: $V_s(x) = \underbrace{\frac{V_R + I_R Z_c}{2} e^{\gamma x}}_{\text{Incident Component}} + \underbrace{\frac{V_R - I_R Z_c}{2} e^{-\gamma x}}_{\text{Reflected Component}}$

$I_s(x) = \frac{V_R/Z_c + I_R}{2} e^{\gamma x} - \frac{V_R/Z_c - I_R}{2} e^{-\gamma x}$

let $\gamma = \alpha + j\beta$

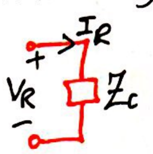
where: $\alpha \equiv$ Attenuation Constant.
$\beta \equiv$ Phase shift.

so $e^{\gamma x} = e^{\alpha x} \cdot e^{j\beta x}$
$e^{-\gamma x} = e^{-\alpha x} \cdot e^{-j\beta x}$

$\therefore V_s$ & I_s consists of Incident & Reflected components.

- If the Line is Terminated by its Characteristic Impedance (i.e Z_c)

$\therefore V_R = I_R Z_c$



$\therefore$ In this case, There is NO Reflected Component in V_s & I_s.

$\Rightarrow$ In this Case, The line is Called "Flat or ∞-line".

※ Surge Impedance:

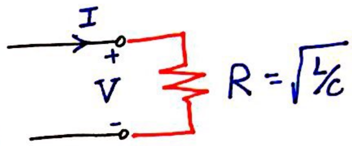
↳ it is the characteristic impedance of a Lossless Line $^{(R=0)}$ since $Z = R + jX$
$\Rightarrow Z = jX$

$Z_c = \sqrt{Z/y} = \sqrt{\frac{j\omega L}{j\omega C}} \Rightarrow Z_c = \sqrt{L/C}$

※ Surge Impedance Loading (SIL) of a Transmission Line:

SIL it is the power supplied by the line to a pure resistive load whose magnitude is equal to surge impedance.

$$|I| = \frac{|V|}{R} \Rightarrow \boxed{|I| = \frac{|V_p|}{\sqrt{L/C}}}$$ → phase voltage.



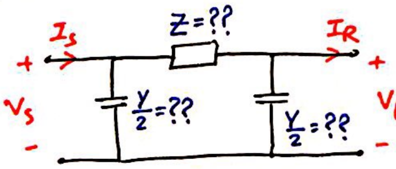
$$P = \sqrt{3}\, V I * 1 = \sqrt{3}\, V \frac{V}{\sqrt{3}} \frac{1}{\sqrt{L/C}} \Rightarrow \boxed{P = \frac{V^2}{\sqrt{L/C}}}$$ → Line voltage.

Line voltage

$|V_L| = \sqrt{3}\,|V_p|$

※ Equivalent Circuit of a long Transmission Line:

- objective: To represent by the π-equivalent.
- procedure: To find the equivalents of the series & shunt components. (i.e Z' & $\frac{Y'}{2}$).



* For this π-cct it was found that:

$$V_s = \left(1 + \frac{ZY}{2}\right) V_R + Z I_R \quad \cdots (1)$$

* For long T.L it was found that: $V_s = \cosh\gamma l \; V_R + Z_c \sinh\gamma l \; I_R \quad \cdots (2)$

$\therefore$ By equating (1) & (2): $\Rightarrow \boxed{Z = Z_c \sinh\gamma l} \quad \cdots (3)$

$$\Rightarrow 1 + Z\frac{Y}{2} = \cosh\gamma l \Rightarrow \therefore Z\frac{Y}{2} = \cosh\gamma l - 1 \Rightarrow \boxed{\frac{Y}{2} = \frac{\cosh\gamma l - 1}{Z}} \quad \cdots (4)$$

Substitute (3) into (4):

$$\frac{Y}{2} = \frac{\cosh\gamma l - 1}{Z_c \sinh\gamma l} = \frac{\frac{e^{\gamma l} + e^{-\gamma l}}{2} - 1}{\frac{e^{\gamma l} - e^{-\gamma l}}{2}} \cdot \frac{1}{Z_c} = \frac{1}{Z_c} \frac{\left(e^{\frac{\gamma l}{2}} - e^{\frac{-\gamma l}{2}}\right)^2}{\left(e^{\gamma l/2} + e^{\frac{-\gamma l}{2}}\right)\left(e^{\frac{\gamma l}{2}} - e^{\frac{-\gamma l}{2}}\right)}$$

$$\Rightarrow \frac{Y}{2} = \frac{1}{Z_c} \frac{e^{\frac{\gamma l}{2}} - e^{\frac{-\gamma l}{2}}}{e^{\frac{\gamma l}{2}} + e^{\frac{-\gamma l}{2}}} \Rightarrow \boxed{\frac{Y}{2} = \frac{1}{Z_c} \tanh\left(\frac{\gamma l}{2}\right)} \quad \cdots (5)$$

$\therefore$ (3) & (5) Can be used to find Equivalent π-cct.

Example: Find π-Equ. cct of the Long T.L in the previous example?

Solution:

By substitution it can be found that:

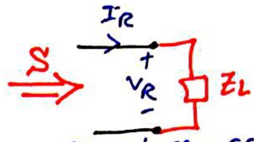
$$\boxed{\begin{aligned} Z' &= 135.9 \angle 76^\circ \\ \frac{Y'}{2} &= 4.64 * 10^{-4} \angle 90.9^\circ \end{aligned}}$$

※ Power Flow in a T. Line:

- Although one Can Calculate power flow at any point along the line by Knowing voltage, current & pf. However the objective: is to evaluate such power by using the $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ constants.

• procedure: it was found that:

$V_s = AV_R + BI_R \quad \therefore \quad I_R = \frac{V_s - AV_R}{B}$



per-phase-cct.

$S = V_R I_R^*$, Let: $V_s = |V_s| \angle \delta$, $A = |A| \angle \alpha$

$V_R = |V_R| \angle 0°$, $B = |B| \angle \beta$

$$\therefore I_R = \frac{|V_s| \angle \delta - |A||V_R| \angle \alpha}{|B| \angle \beta} = \frac{|V_s|}{|B|} \angle \delta - \beta - \frac{|A||V_R|}{|B|} \angle \alpha - \beta$$

$$S = V_R I_R^* = \frac{|V_R||V_s|}{|B|} \angle \beta - \delta - \frac{|A||V_R|^2}{|B|} \angle \beta - \alpha \quad \cdots (1)$$

since $S = P + jQ \cdots (2)$ * Applying (2) to (1):

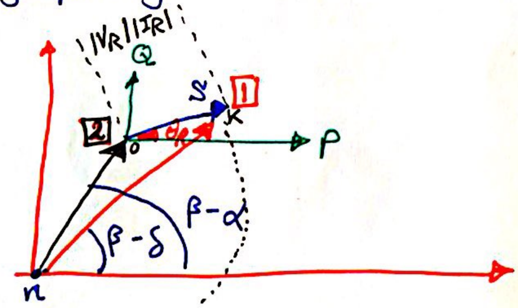
$$\Rightarrow P = \frac{|V_R||V_s|}{|B|} \cos(\beta - \delta) - \frac{|A||V_R|^2}{|B|} \cos(\beta - \alpha) \quad \cdots (3)$$

$$Q = \frac{|V_R||V_s|}{|B|} \sin(\beta - \delta) - \frac{|A||V_R|^2}{|B|} \sin(\beta - \alpha) \quad \cdots (4)$$

Hence by using (3) & (4) one may calculate P & Q transmitted by a T. Line.

equation (1) can be used To express S graphically.

$$S = \underbrace{\frac{|V_R||V_s|}{|B|} \angle \beta - \delta}_{\boxed{1}} - \underbrace{\frac{|A||V_R|^2}{|B|} \angle \beta - \alpha}_{\boxed{2}}$$



shift the orgin from point n to point o.

let the load changes by keeping $|V_s|$ & $|V_R|$ constants.

As the load changes by changing θ_R, then the line (nK) is going to move along a circle. Consequantly, P & Q are going to change.

P_{max} will occure when $\beta - \delta = 0 \quad \therefore \beta = \delta$

$$P_{max} = \frac{|V_R||V_s|}{|B|} - \frac{|A||V_R|^2}{|B|} \cos(\beta - \alpha)$$

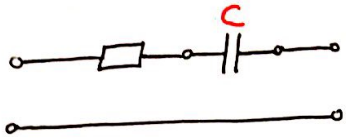
max @ Leading pf.

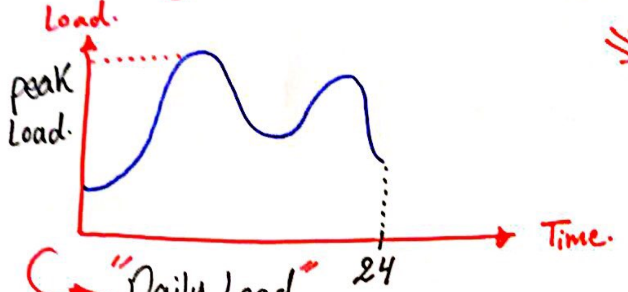
* Reactive Compensation:

At heavy Load, there will be Large voltage drop in the impeadance of the Line and Consequantly High value for VR. Since the major component of the impedance is inductive Reactance (X_L). Then this can be reduced by inserting Capacitance (X_C) in series with the line This is called "Series Compensation".

Ratio of $\frac{X_C}{X_L}$ is called "Series Compensation Factor".

* Under No-load or lightly Load Line, the effective Load will be Capacitive.
Causing V_R to increase to a high value.





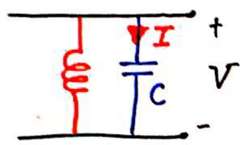
"Daily Load" Curve.

* To solve this problem, then an Inductor is placed in Parallel between phase & Neutral. This is called "Shunt Reactive Compensation".

Hence, shunt compensation factor $\triangleq \dfrac{B_L}{B_C}$

where: $B_L = \dfrac{1}{\omega L}$ & $B_C = \omega C$

* To reduce charging current:



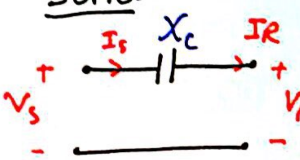
$I_B = j\omega C V$ Before

$I_A = V\left(j\omega C - \dfrac{j}{\omega L}\right) = V j\omega C\left(1 - \dfrac{1}{\omega C}\cdot\dfrac{1}{\omega L}\right)$; $\frac{1}{B_C}$, B_L

$I_A = Vj\omega C\left(1 - \dfrac{B_L}{B_C}\right)$ After.

* Parameters of Compensation:

• Series:

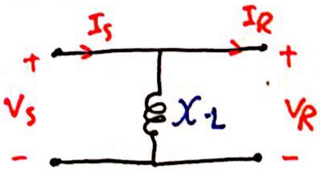


$V_s = V_R + I_R X_c$
$I_s = I_R$
$X_c = \dfrac{-j}{\omega C}$

$\therefore \begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} 1 & X_c \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$

$\therefore A = D = 1,\ B = X_c,\ C = 0$

• Parallel:



$X_L = j\omega L$
$V_s = V_R$
$I_s = I_R + \dfrac{V_R}{X_L}$

$\therefore \begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{X_L} & 1 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$

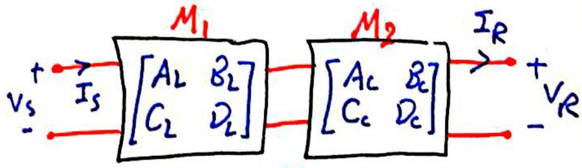
$\therefore A = D = 1,\ B = 0,\ C = \dfrac{1}{X_L}$

* Application:

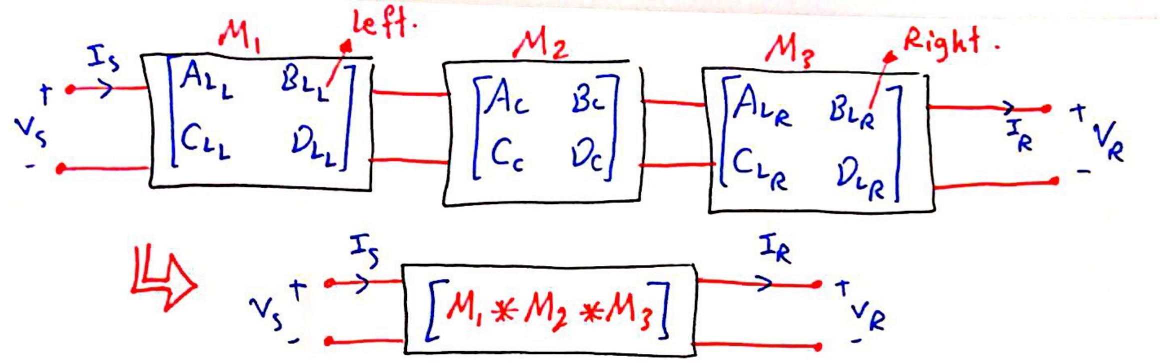
Compensation Could be:

1) At one of the terminals of T.L.

2) At a given Location in the Line.



⇓



Example: A 3-ph T.L is 300 mile long and serves (supply) a Load of 400MVA with 0.8 pf lagging at 345KV, if the line parameters as follows:

$A = D = 0.818 \angle 1.3^\circ$, $B = 172.2 \angle 84.2^\circ$, $C = 0.001933 \angle 90.4^\circ$

1) Determine V_s & I_s ?

$V_s = A V_R + B I_R \Rightarrow V_R = \frac{345}{\sqrt{3}} \angle 0$ KV.

$I_s = C V_R + D I_R \qquad I_R = \frac{400 * 10^6}{\sqrt{3} * 345 * 10^3} \angle -\cos^{-1} 0.8$

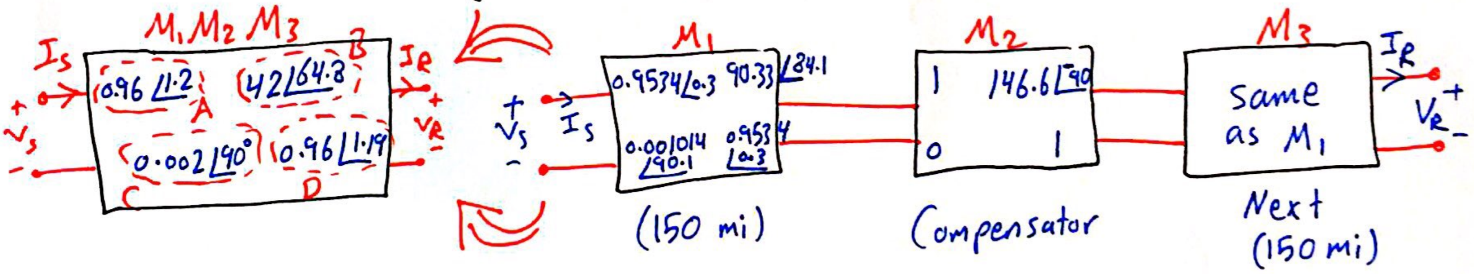
∴ By substitution: $I_s = 447.7 \angle 8.5^\circ$ A

$\boxed{V_s = 256.8 \angle 20.1^\circ}$ Kvolt.

$VR = \frac{\left|\frac{V_s}{A}\right| - |V_R|}{|V_R|} * 100\% = 57.6\%$

2) If a series Capacitor bank have a reactance (146.6) Ω is installed at the mid-point of the line & the $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ constant for each 150 mile are $A = D = 0.9534 \angle 0.3^\circ$, $B = 90.33 \angle 84.1^\circ$, $C = 0.001014 \angle 90.1^\circ$

for the capacitor $\begin{bmatrix} 1 & 146.6 \angle -90^\circ \\ 0 & 1 \end{bmatrix}$, Find [equ.] ?



$$\therefore V_s = A V_R + B I_R = (0.96 \angle 1.2 * 199.2 * 10^3 \angle 0) + (42 \angle 64.8 * 669.4 \angle -36.87)$$

$$\Rightarrow V_s = 216.7 \angle 4.5^\circ \text{ Kvolt.}$$

$$\therefore VR\% = \frac{\frac{|V_s|}{|A|} - |V_R|}{|V_R|} * 100\% \Rightarrow VR\% = 13.3\%$$

*Comment:

Parameter	Before Comp.	After Comp.
A = D	$0.818 \angle 1.3$	$0.96 \angle 1.2^\circ$
B	$172.2 \angle 84.2$	$42 \angle 64.8$
C	$0.001933 \angle 90.4$	$0.002 \angle 90.1$

⟹

Power1

Fall017

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⇛ It can be observed that the change in A, C, D is minimal.
, However there is a major change in B (i.e B↓).

* Since as see in the expression of power flow in the line $\propto \frac{1}{B}$

∴ Power capability of the line increases.

end of first material

※ Fault Analysis:

what!!

- Definition (Fault): It is an abnormal condition which may occur on the power system such as O/C & S/C.
  - In this course one is intersted in S/C.

why!!

- Objective: Results of fault Analysis can be used in the setting of protection system components: PT + CT, Relays & Circuit Breaker.

How!!

S/C Faults Can be Classified into:

→ Balanced or Symmetrical (3-ph fault).
use per phase Analysis.

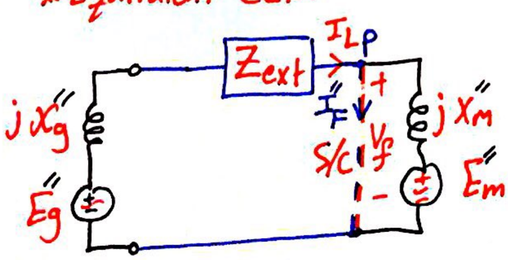
→ Unbalanced or Unsymmetrical could be: ⇒ Symmetrical Components concept is used.

1) Single Line to ground Fault.
2) Line to Line Fault.
3) Line to Line to ground Fault.

※ Balanced Fault:

To start consider a small system:

* Equivalent cct.:



Problem: S/C occur at the terminal of the fault.
- Objective: To find I''_F.

→ a Balanced 3-ph fault occur @ P.

$V_f \equiv$ Pre-Fault Voltage at point P.

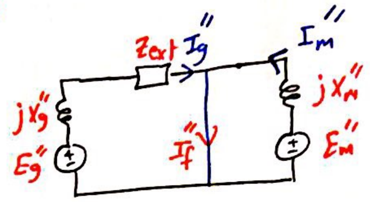
$I_L \equiv$ Load Current.

Before the fault: $E_g'' = I_L(jX_g'' + Z_{ext}) + V_F \quad \cdots (1)$

$E_m'' = V_F - I_L\, jX_m'' \quad \cdots (2)$

After the fault: $E_g'' = I_g''(jX_g'' + Z_{ext}) \quad \cdots (3)$

$E_m'' = I_m''\, jX_m'' \quad \cdots (4)$



where $I_g'', I_m'' \equiv$ The Contribution of the gen. & motor to fault current.

* Substitute (1) into (3) & (2) into (4):

$I_L(jX_g'' + Z_{ext}) + V_F = I_g''(jX_g'' + Z_{ext}) \quad \therefore \; I_g'' = I_L + \dfrac{V_F}{jX_g'' + Z_{ext}} \quad \cdots (5)$

$V_F - I_L\, jX_m'' = I_m''\, jX_m'' \quad \therefore \; I_m'' = \dfrac{V_f}{jX_m''} - I_L \quad \cdots (6)$

* Now for I_f'': $I_f'' = I_m'' + I_g'' \quad \cdots (7)$

* Substitute (5) & (6) into (7): $I_f'' = \dfrac{V_f}{jX_g'' + Z_{ext}} + \dfrac{V_f}{jX_m''} \quad \cdots (8)$

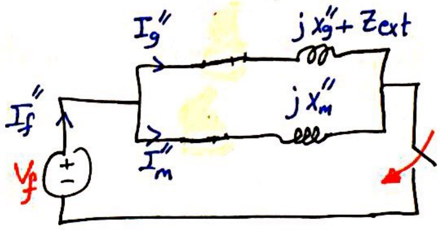
Hence, from (8) one can calculate the fault current I_f''.

* By observing (8), one can represent by a Thevenin Equivalent as follows:

(8) could be written as:

$$I_f'' = V_f \, \frac{jX_m'' + jX_g'' + Z_{ext}}{(jX_m'')(jX_g'' + Z_{ext})}$$

$$\Rightarrow \boxed{I_f'' = \frac{V_f}{jX_m'' \,//\, (jX_g'' + Z_{ext})}}$$



switch is closed at the fault.

$V_{th} = V_f$

$Z_{th} = (jX_m'') \,//\, (jX_g'' + Z_{ext}) \quad \cdots (9)$

$I_f'' = \dfrac{V_{th}}{Z_{th}}$

- Then one can use (8) or (9) To evaluate I_f''.

Example: A gen. is connected through a transformer to a synch. motor, for the same base values. the PU reactances are as follows: $X_g'' = j0.15$, $X_m'' = j0.35$, $Z_{ext} = j0.1$

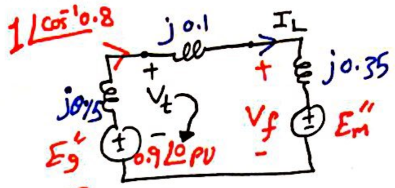
A 3-ph fault occur at the terminals of the motor when the terminal voltage of the generator is 0.9 PU & the output current is 1 PU @ 0.8 pf leading

Find I_f'', I_g'' & I_m'' ?

⇒ continue.

Solution:

This can be calculated by using E_g'' & E_m'' / thevenin.



- **method (1):**

$$E_g'' = I_L^* jX_g'' + V_t$$

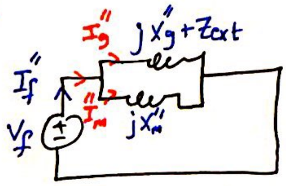
$$= (1\angle 36.87^\circ)(0.15\angle 90) + (0.9\angle 0) \Rightarrow \boxed{E_g'' = 0.82\angle 8.42}$$

$$E_m'' = V_t - I_L(Z_{ext} + jX_m'') = (0.9\angle 0) - (1\angle 36.87^\circ)(0.45\angle 90) \Rightarrow \boxed{E_m'' = 1.22\angle -17.1^\circ}$$

$$\therefore I_g'' = \frac{E_g''}{jX_g'' + Z_{ext}} \Rightarrow \boxed{I_g'' = 3.28\angle -81.58^\circ} \qquad I_m'' = \frac{E_m''}{jX_m''} \Rightarrow \boxed{I_m'' = 3.49\angle -107.1^\circ}$$

$$\therefore I_f'' = I_g'' + I_m'' \Rightarrow I_f'' = 6.6\angle -94.78^\circ \ \#$$

- **method (2):**



using Thevenin Equivalent cct. $I_f'' = \frac{V_f}{Z_{th}}$

$$Z_{th} = (jX_g'' + Z_{ext}) \,//\, (jX_m'')$$

$$V_f = V_t - I_L Z_{ext} = (0.9\angle 0) - (1\angle 36.87)(j0.1) \Rightarrow \boxed{V_f = 0.963\angle -4.76}$$

$$\therefore I_f'' = \frac{V_f}{Z_{th}} \Rightarrow \boxed{I_f'' = 6.6\angle -94.76}$$

* By current Division, it can be found: $\boxed{\begin{aligned} I_g'' &= 3.85\angle -94.76^\circ \\ I_m'' &= 2.75\angle -94.76^\circ \end{aligned}}$

Note in method (2) I_g'' & I_m'' are different in values with method (1) due to the load current. $(I_g'' + I_L)$, $(I_m'' - I_L)$

※ Fault Calculation Using Z_{bus} Method:

This is used to calculate fault currents & voltages due to a balanced 3-ph fault.

- procedure: Consider the following system:

The shown voltage on the elements give subtransient reactance.

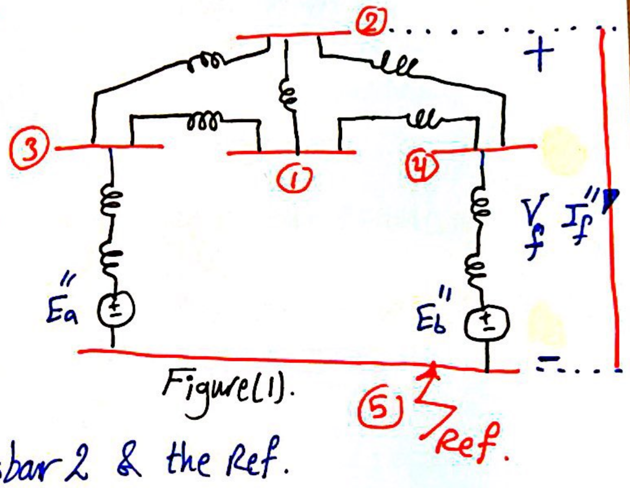


Figure (1).

- Objective:

if a s/c occure between a busbar & the ref evaluate I_f'' & the voltage at the busbars.

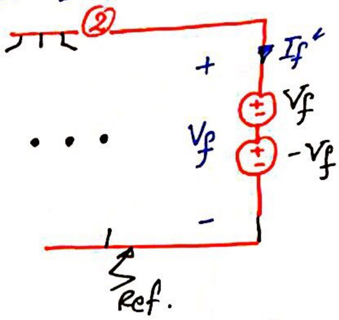
* for e.g let a fault occur between busbar 2 & the Ref.

$V_f \equiv$ Pre-Fault Voltage.

⟹

* I_f'' is going to flow from busbar 2 to the Ref. Then go inside the system Causing voltage changes @ each busbar.

* S/C between ② & the Ref Can be Simulated or represented by 2 voltage sources V_f & $-V_f$ connected in series.



* By Super Position:

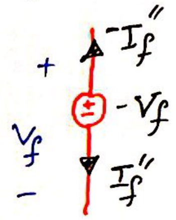
Consider:

1) $-V_f$ on its own. 2) V_f, E_a'', E_b'' together.

if V_f, E_a'', E_b'' are considered, these represent pre-fault condition, Then They DONOT Contribute to I_f''.

Hence the only source which contribute to I_f'' is $-V_f$.

(a S/C will be for E_a'', E_b'' & V_f)



* It was shown before: $[I] = [Y][V]$ ... ①
↳ Y_{bus}-matrix.

$\Rightarrow [V] = [Z][I]$ ... ②
↳ Z_{bus}-matrix $= [Y_{bus}]^{-1}$

* I_f'' is going to cause voltage changes at the busbars:

Apply ② to:
$$\begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \Delta V_3 \\ \Delta V_4 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{14} \\ Z_{21} & \cdots & \cdots & Z_{24} \\ Z_{31} & \cdots & \cdots & Z_{34} \\ Z_{41} & \cdots & \cdots & Z_{44} \end{bmatrix} \begin{bmatrix} 0 \\ -I_f'' \\ 0 \\ 0 \end{bmatrix}$$

Note that $\Delta V_2 = -V_f$.

$\underset{4\times4}{\text{matrix}} \cdot \underset{4\times1}{\text{matrix}} = \underset{4\times1}{\text{matrix}}$

$$\Rightarrow \begin{bmatrix} \Delta V_1 \\ -V_f \\ \Delta V_3 \\ \Delta V_4 \end{bmatrix} = \begin{bmatrix} Z_{11} & \cdots & Z_{14} \\ \vdots & \cdots & \vdots \\ Z_{41} & \cdots & Z_{44} \end{bmatrix} \begin{bmatrix} 0 \\ -I_f'' \\ 0 \\ 0 \end{bmatrix} = -I_f'' \begin{bmatrix} Z_{12} \\ Z_{22} \\ Z_{32} \\ Z_{42} \end{bmatrix}$$

$\therefore -V_f = -I_f'' Z_{22}$

$$\Rightarrow I_f'' = \frac{V_f}{Z_{22}}$$

- V_f is Known ≡ pre-fault voltage.
- Z_{22} is an element in the Z_{bus} matrix.

∴ In general, for a Balanced 3-ph fault at the K^{th} bus: $I_f'' = \dfrac{V_f}{Z_{KK}}$

$$\therefore \boxed{\begin{aligned} \Delta V_1 &= -I_f'' Z_{12} = -V_f \frac{Z_{12}}{Z_{22}} \\ \Delta V_2 &= -V_f \\ \Delta V_3 &= -I_f'' Z_{32} = -V_f \frac{Z_{32}}{Z_{22}} \\ \Delta V_4 &= -I_f'' Z_{42} = -V_f \frac{Z_{42}}{Z_{22}} \end{aligned}} \quad \cdots ③$$

↳ These are the voltages due to source $-V_f$.

** Reintroduce the 3-sources E_a'', E_b'' & V_f. Since they present pre-fault.

$$\begin{bmatrix} V_1' \\ V_2' \\ V_3' \\ V_4' \end{bmatrix} = \begin{bmatrix} V_f \\ V_f \\ V_f \\ V_f \end{bmatrix} \quad \cdots ④$$

⟹

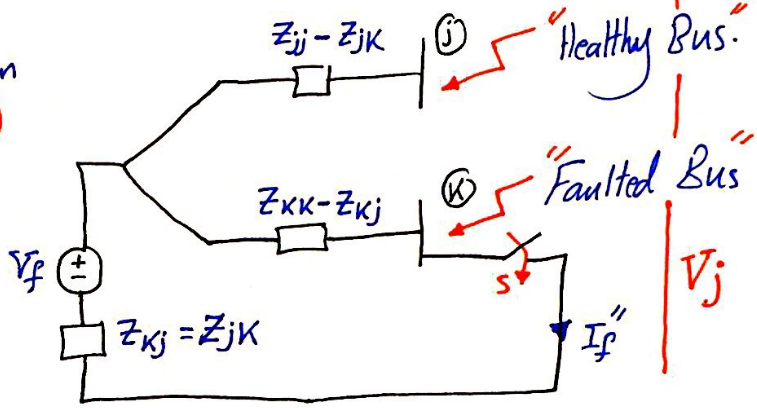
⇒ Voltages at the busbars ≡ ③ + ④

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} V_1' \\ V_2' \\ V_3' \\ V_4' \end{bmatrix} + \begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \Delta V_3 \\ \Delta V_4 \end{bmatrix} = \begin{bmatrix} V_f \\ V_f \\ V_f \\ V_f \end{bmatrix} + \begin{bmatrix} \Delta V_1 \\ -V_f \\ \Delta V_3 \\ \Delta V_4 \end{bmatrix} = \begin{bmatrix} V_1' + \Delta V_1 \\ 0 \\ V_3' + \Delta V_3 \\ V_4' + \Delta V_4 \end{bmatrix}$$

$\Delta V \equiv$ Voltage at the busbar due to I_f''.

※ Fault Calculations Using Z_{bus} Equ. cct.:

The previous results can be used to deduce the equ. cct. between any **2** buses of interest by using the elements of the Z-bus Matrix as follows:



• Before the fault: switch S is open
$V_K = V_j = V_f$ (i.e pre-fault voltage)
At the fault switch S is closed
Then I_f'' is going to flow.

∗ Apply KVL to Lower Loop:

$$I_f'' Z_{Kj} - V_f + I_f''(Z_{KK} - Z_{Kj}) = 0$$

$$\therefore\ V_f = I_f'' Z_{KK} \Rightarrow \therefore \boxed{I_f'' = \frac{V_f}{Z_{KK}}}$$

∗ Apply KVL to upper Loop: $-V_j + V_f - I_f'' Z_{Kj} = 0 \Rightarrow \therefore \boxed{V_j = V_f - I_f'' Z_{Kj}}$

Example: The Z_{bus} Matrix of a 4-bus system is as follows: (in PU)

$$Z_{bus} = j\begin{bmatrix} 0.15 & 0.08 & 0.04 & 0.07 \\ 0.08 & 0.15 & 0.06 & 0.09 \\ 0.04 & 0.06 & 0.13 & 0.05 \\ 0.07 & 0.09 & 0.05 & 0.12 \end{bmatrix}$$

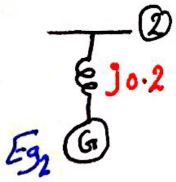
having generators @ buses ① & ② and their subtransient reactance are included in the Z_{bus}.

Find i) I_f'' for a 3-ph fault on bus ④ ?
ii) the current from gen.2 whose $X'' = j0.2$?

Solution: Solve the problem by using 1) The equations 2) Equ. cct.

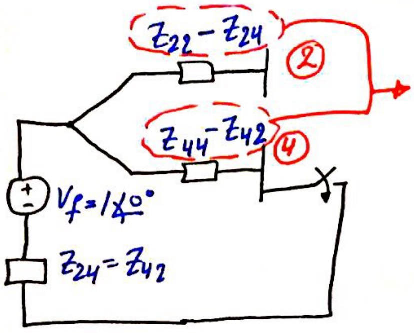
$$I_f'' = \frac{V_f}{Z_{44}} = \frac{1\angle 0}{j0.12} = \boxed{8.33\angle -90°}\ PU, \quad V_2 = V_f - I_f'' Z_{24} = (1\angle 0) - (8.33\angle -90°)(j0.09)$$

$$\boxed{V_2 = 0.25\angle 0}\ PU$$



$$I_{g_2} = \frac{E_{g_2} - V_2}{j0.2} = \boxed{3.745\angle -90°}\ PV$$

⟹ Continue



elements obtained from Z_{bus} Matrix.

*Comment: Calculating fault current within the system.
Having calculated the voltages at the busbars, then the current flow in any Branch, say between buses i & j can be found by:

$$I_{ij} = \frac{V_i - V_j}{b}$$

b → Impedance of the Branch between i & j

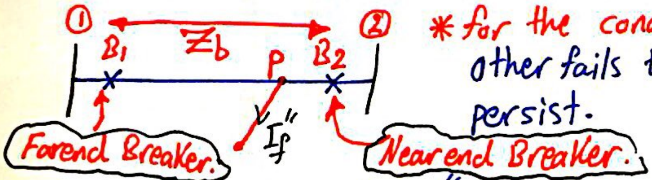
Becareful: it doesn't be found from Z_{bus} matrix. (found from Y_{bus} matrix).

Evaluate I_{34}:

$$I_{34} = \frac{V_3 - V_4}{\text{impedance between 3\&4 branch.}}$$

Balanced 3-ph Fault on T.L:

Transmission Lines are exposed to 3-ph faults more than that on Busbars at substations.



*for the condition where one breaker open & the other fails to open; under this condition, fault current persist.

- Objective: To Evaluate I_f'' Under this Condition.
- Procedure: Introduce a busbar at point P, say K^{th} bus, and Re-calculate Z_{bus} matrix.

*Here the concept of Z_{th} can be used.

i) Z_{th} between a bus & the Ref. OR ii) Z_{th} between any 2 buses.

$$Z_{th} = Z_{KK}$$

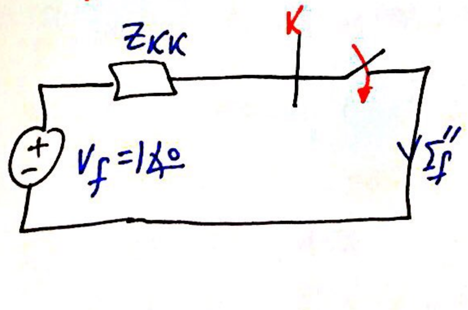
→ Z_{th} between K^{th} bus & the Ref.

$$Z_{th,ij} = Z_{jj} + Z_{ii} - 2Z_{ij}$$

All found from Z_{bus} Matrix.

$Z_b \equiv$ Impedance of the line.

*Equ. cct will be as follows:



$$I_f'' = \frac{V_f}{Z_{KK}}, \text{ where: } Z_{KK} = Z_{11} + Z_b - \frac{(Z_{11}-Z_{21})^2}{Z_{th,12} - Z_b} \quad \cdots ①$$

$$\Rightarrow Z_{th,12} = Z_{11} + Z_{22} - 2Z_{12} \quad \cdots ②$$

where the elements on the RHS (right hand side) of ① & ② can be found from $[Z_{bus}]$ matrix.

*Illustration: By using Z_{bus} of the previous example, it can be found:

Z_{th} for Bus 3 $= Z_{33} = \underline{j0.13}$

Z_{th} between BUSES 2 & 4 $= Z_{th,24} = Z_{22} + Z_{44} - 2Z_{24} = j(0.15 + 0.12 - 2*0.09)$
$= \underline{j0.09}$

* Introduction to C.B Selection:

If an industrial consumer or a consumer to be connected to a Network, Then the power distribution Company supply the consumer with SC MVA @ the terminal of connection.

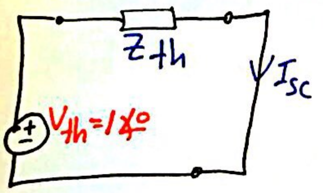
$SC\ MVA \triangleq \sqrt{3} * |\text{Nominal voltage, KV}| * |I_{sc}| * 10^{-3}$ ... (1)

$I_{sc} \equiv$ It is the RMS current in Ampers due to a Balanced 3-ph fault @ the point of the connection.

* If the Base MVA, $MVA_B = \sqrt{3} * |\text{Base voltage, KV}| * |I_b| * 10^{-3}$ ... (2)
* If the Base voltage = Nominal Voltage.

$\Rightarrow$ Divide (1) by (2): $SCMVA(PU) = I_{sc}(PU)$ ... (3)

* The Equ. cct @ s/c:



$\therefore\ I_{sc} = \frac{1}{Z_{th}}$, $SCMVA_{(PU)} = \frac{1}{Z_{th}}$

$\therefore\ Z_{th} = \frac{1}{SC\ MVA_{(PU)}}$

* Among the other Rating of C.B to be selected:

1] The Maximum Instantaneous Current which the C.B must with stand.

* In our Calculations, we have neglected DC component of I_f''. Then:

Total Instantaneous Current $= I_f'' *$ factor

* This Factor > 1 & depend on the Nominal voltage & Type of C.B.

* For Example: for 5K oil C.B $\Rightarrow$ $\boxed{\text{factor} = 1.6}$

2] Current to be interrupted by C.B

3] The Maximum Continuous Current.

$\hookrightarrow$ This is Related to the Load.

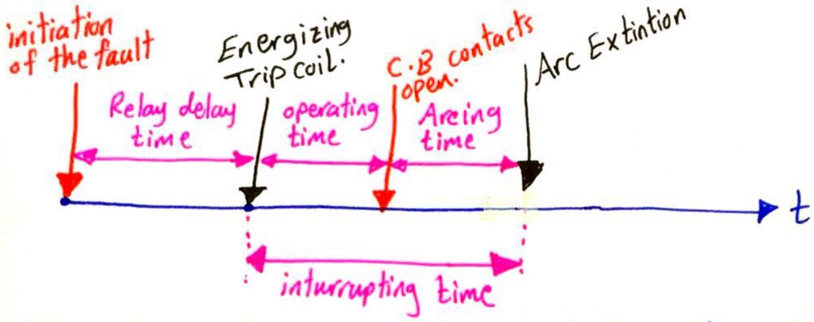
$\Rightarrow$

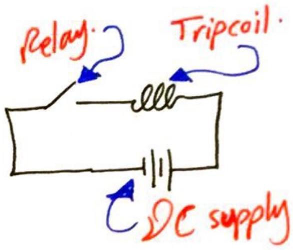
4] Interrupting, MVA or KVA & this is Defined as:

Interrupting KVA $= \sqrt{3} *$ (KV of the bus to which the Breaker is connected) * (The current which Breaker is Capable to interrupt).

⇒ This interrupting KVA depends on the speed of the Breaker.
(i.e interrupting time), usually measure in cycles (e.g 2, 4, 6, ... cycles).

* for 50HZ system ⇒ $T = \frac{1}{50} = 20\,msec$.





• Note: inturrupting time = operating time + Arcing time.

• Note: Arc is interrupted by: 1) oil 2) air 3) SF6 gas 4) Vaccum

5] Voltage Range Factor (K). ⇒ $K = \dfrac{\text{Max. operating voltage.}}{\text{Lower limit of operating voltage.}}$

Example: A C.B having: Nominal Rating of 34.5KV, Continuous Current of 1500A. , K = 1.65. Rated Max. voltage 38KV, Rated s/c current 22KA @ Rated Max voltage.

① Find the Lower Limit for operating voltage? $V_{min} = 38/1.65 = \boxed{23.03\,KV}$.

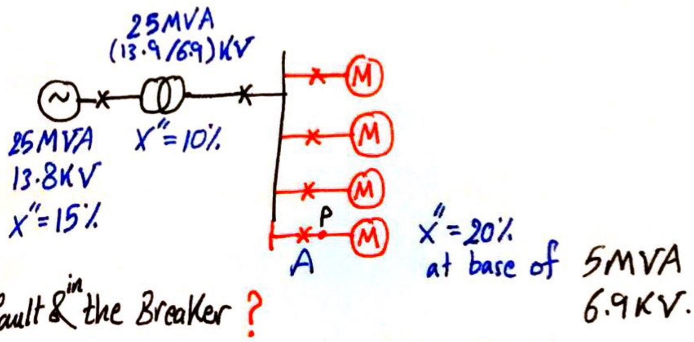
$K = \dfrac{\text{Max. Voltage}}{\text{Min Voltage}}$ → this is the voltage below which the breaker Tripcoil doesn't work.

K determining the Range over which the product (Rated s/c * operating voltage) = Constant.

② Find Rated s/c current @ 23.03 KV? $I_{sc_{23.03}} * 23.03 = 38 * 22 \Rightarrow \boxed{I_{sc_{23.03}} = 36.3\,KA}$

Example 10.7 page 406:

If a fault occure @ point P.



Find:

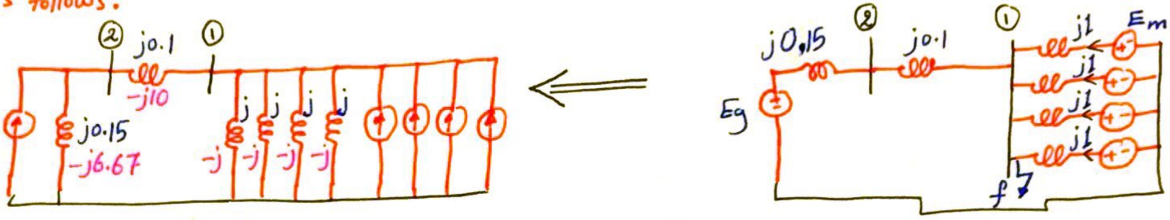
① The subtransient fault current, I_f''?

② Subtransient Current in Breaker A?

③ The symmetrical (S.S) s/c interrupting current in the fault & in the Breaker?

* Subtransient ≡ immediatly after the fault.

Solution: By using Base value of 25000 KVA & 13.8 KV, Then Reactance Diagram will be as follows:



Blue is a Reactance.
Pink is an Admittance.

$$[Y] = \begin{bmatrix} -j14 & j10 \\ j10 & -j16.67 \end{bmatrix} \Rightarrow [Z] = [Y]^{-1} = \begin{bmatrix} j0.125 & j0.075 \\ j0.075 & j0.105 \end{bmatrix}$$

① $I_f'' = \frac{V_f}{Z_{11}} = \frac{1\angle 0}{j0.125} = \boxed{-j8}$.

② To find Contribution of Gen. & Motor: $V_2 = V_f - \frac{Z_{21}}{Z_{11}} V_f$

$\Rightarrow V_2 = 1\angle 0 - \frac{j0.075}{j0.125} 1\angle 0 \Rightarrow V_2 = 0.4$

* all in PU *

* Contribution of gen: $= \frac{V_2 - V_1}{j0.1}$ → $V_1 = 0$ since at V_1 the fault occured. $= \frac{0.4 - 0}{j0.1} = \boxed{-j4}$

* Remeaning is supplied by the 4 Motors: $= -j8 - (-j4) = \boxed{-j4}$

⇒ so Each Motor Contributes by: $\frac{-j4}{4} = -j$

∴ The Current Passing through the Breaker $= -j4 + (3 * -j) = -j7$

③ Objective: is to select a C.B specification for a Breaker A.

- Among the Methods to do this is the so called E/X method.
  - ⇒ Here Keep the same X'' for the Gen., However for the Motor use X' which is equal to $1.5 X_m''$.

∴ In this Case in the Reactance Diagram $X_m' = 1.5 * j1 = j1.5$

⇒ Re-Calculate [Y] → then [Z].

- It can be found: $Z_{11} = j0.15$

$I_f'' = -j6.67$ PU, $I_b = \frac{25000}{\sqrt{3} * 6.9} = 2090A$ ∴ $\boxed{I_f'' = 13940\ A}$.

According to the E/X Method, The specification Can be specified as follows:

1] Select a C.B in such away that the voltage of the system falls within its operating range.

2] The Calculated Current ≤ 0.8 of the rated s/c current of the Breaker being selected.

* In this e.g: A 14.4KV C.B rated @ max Voltage of 15.5KV & K = 2.67.
Also @ 15.5KV rated s/c interrupting current = 8900A [from DATA sheet]

continue ⇒

⇒ * Lower voltage @ this selected C.B $= \frac{15.5}{2.67} = \underline{\underline{5.8}}$ KV.

∴ operating Range of the selected C.B (5.8 ⟶ 15.5) KV

Since the voltage of the system is 6.9 KV ∴ the first Condition Satisfied.

∴ The S/C interrupting current of 6.9 KV is : $I_{SC} * 6.9 = 15.5 * 8900$

$\Rightarrow I_{SC} = 19992.7 \Rightarrow I_{SC} \simeq 20000\,A$ & $I_{calculated} = 13940\,A$.

Since $13940 < 0.8 * 20000$ ∴ Selected C.B satisfy the second condition.

* Unbalanced (Unsymmetrical) Faults:

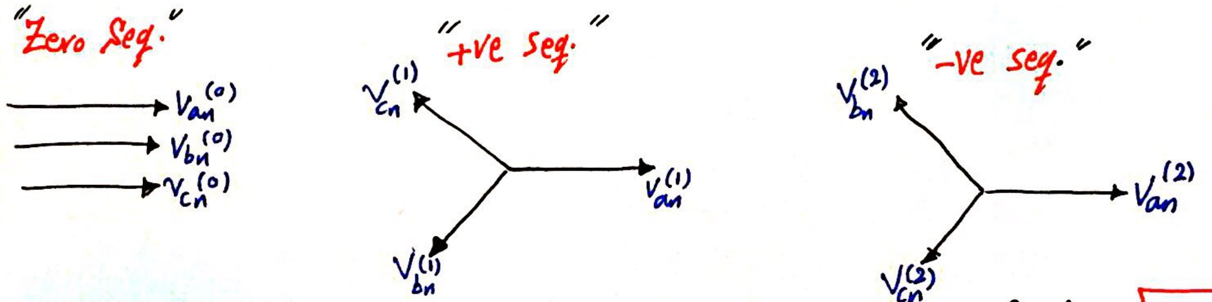
To analyze such faults, The Mathematical Concept of symmetrical Components used.

- Symmetrical Components: Here any Unbalanced voltage or current Can be expressed as the sum of 3 components, the so called Zero seq., +ve seq. & −ve seq.

⇒ where these sequances are represented by superscript of (0), (1), (2) respectively.

* Consider Unbalanced 3-ph voltages, Then: $V_{an} = V_{an}^{(0)} + V_{an}^{(1)} + V_{an}^{(2)}$

, $V_{bn} = V_{bn}^{(0)} + V_{bn}^{(1)} + V_{bn}^{(2)}$, $V_{cn} = V_{cn}^{(0)} + V_{cn}^{(1)} + V_{cn}^{(2)}$



* In this Concept, The Complex Operator (a) is introduced & defined as: $\boxed{a = 1\angle 120^\circ}$

⇒ This is introduced to simplify the process of Calculations as follows:

- V_{bn} & V_{cn} will be expressed in terms of the symmetrical components of phase a.

$$V_{an} = V_{an}^{(0)} + V_{an}^{(1)} + V_{an}^{(2)}$$
$$V_{bn} = V_{an}^{(0)} + a^2 V_{an}^{(1)} + a\,V_{an}^{(2)}$$
$$V_{cn} = V_{an}^{(0)} + a\,V_{an}^{(1)} + a^2 V_{an}^{(2)}$$

$$\Rightarrow \begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{an}^{(0)} \\ V_{an}^{(1)} \\ V_{an}^{(2)} \end{bmatrix}$$

$$\left\{ \begin{array}{l} a = 1\angle 120^\circ \\ a^2 = 1\angle 240^\circ \\ a^3 = 1\angle 360^\circ \end{array} \right\}$$

$$[V_{abc}] = [A]\,[V_{012}]$$

$$\therefore [V_{012}] = [A]^{-1}\,[V_{abc}]$$

* It can be shown that: $[A^{-1}] = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} * \frac{1}{3}$

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*Note: The sum definition can be applied to line voltages as follows:

$$\begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{ab}^{(0)} \\ V_{ab}^{(1)} \\ V_{ab}^{(2)} \end{bmatrix} \Rightarrow \therefore \begin{bmatrix} V_{ab}^{(0)} \\ V_{ab}^{(1)} \\ V_{ab}^{(2)} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix}$$

$\therefore V_{ab}^{(0)} = (V_{ab} + V_{bc} + V_{ca}) * \frac{1}{3}$ = Zero by KVL applied to double subscript notation.

$\therefore$ in line voltages there is NO zero-sequance component.

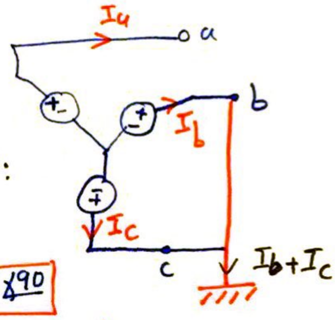
** Same Definitions apply to currents: $\begin{bmatrix} I_{an} \\ I_{bn} \\ I_{cn} \end{bmatrix} = [A] \begin{bmatrix} I_{an}^{(0)} \\ I_{an}^{(1)} \\ I_{an}^{(2)} \end{bmatrix}$

- process:

1) for each element find $Z^{(0)}, Z^{(1)}, Z^{(2)}$.
2) Find the 3-seq. Networks.
3) The connection between Networks depend on the type of fault.

Example: When a gen. has terminal a o/c & other 2 terminals suffered s/c to ground the symm. component of phase a, where $I_a^{(1)} = 600 \angle -90°$, $I_a^{(2)} = 250 \angle 90°$ and $I_a^{(0)} = 350 \angle 90°$ A. Find the current between each phase & current to ground?

Solution: $\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_a^{(0)} = 350 \angle 90° \\ I_a^{(1)} = 600 \angle -90° \\ I_a^{(2)} = 250 \angle 90° \end{bmatrix}$



Knowing $a = 1\angle 120°$, $a^2 = 1\angle 240°$ By multiplication it can be found that:

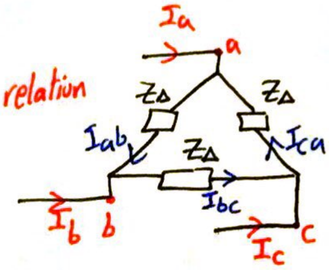
$\boxed{I_a = 0}$, $\boxed{I_b = 904.16 \angle 144.5°}$, $\boxed{I_c = 904.16 \angle 35.5°}$ $\therefore I_f = I_b + I_c$ $\boxed{I_f = 1050.1 \angle 90}$

* Symmetrical Components of Δ & Y loads:

- Objective: for a symm. load, and by using the concept of symm. components, find the relationships between: ① Line current & phase current for Δ-load. ② Line voltage & phase voltage for Y-load.

- Δ-load:

- objective: find relation between:
$I_a^{(1)}$ & $I_{ab}^{(1)}$
$I_a^{(2)}$ & $I_{ab}^{(2)}$



- Procedure: By KCL:

$I_a = I_{ab} - I_{ca}$ ... ①
$I_b = I_{bc} - I_{ab}$ ... ②
$I_c = I_{ca} - I_{bc}$ ... ③

⇒ Express ① by using symm. components:

$(I_a^{(0)} + I_a^{(1)} + I_a^{(2)}) = (I_{ab}^{(0)} + I_{ab}^{(1)} + I_{ab}^{(2)}) - (I_{ca}^{(0)} + I_{ca}^{(1)} + I_{ca}^{(2)})$ ... ④

since $[I_{012}] = [A]^{-1}[I_{abc}]$ ... ⑤

Then using ⑤ it can be found: $I_a^{(0)} = \frac{1}{3}(I_a + I_b + I_c)$ ... ⑥

By substituting ①, ② & ③ into ⑥: $\boxed{I_a^{(0)} = 0}$.

$\therefore$ ④: $I_a^{(1)} + I_a^{(2)} = \underbrace{(I_{ab}^{(0)} - I_{ca}^{(0)})}_{= \text{Zero}} + (I_{ab}^{(1)} - I_{ca}^{(1)}) + (I_{ab}^{(2)} - I_{ca}^{(2)})$ → since $I_{ab}^{(0)} = I_{bc}^{(0)} = I_{ca}^{(0)}$

$\therefore I_a^{(1)} + I_a^{(2)} = (I_{ab}^{(1)} - I_{ca}^{(1)}) + (I_{ab}^{(2)} - I_{ca}^{(2)})$ ... ⑦

⟹

7 : $\Rightarrow I_a^{(1)} + I_a^{(2)} = (I_{ab}^{(1)} - a I_{ab}^{(1)}) + (I_{ab}^{(2)} - a^2 I_{ab}^{(2)}) = I_{ab}^{(1)}(1-a) + I_{ab}^{(2)}(1-a^2) \quad \ldots 8$

* By similar approach, it can be found: $I_b^{(1)} + I_b^{(2)} = I_{bc}^{(1)}(1-a) + I_{bc}^{(2)}(1-a^2) \quad \ldots 9$

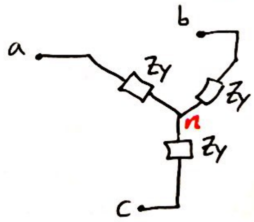
$\Rightarrow I_b^{(1)} + I_b^{(2)} = a^2 I_a^{(1)} + a I_a^{(2)} = I_{bc}^{(1)}(1-a) + I_{bc}^{(2)}(1-a^2) = a^2 I_{ab}^{(1)}(1-a) + a I_{ab}^{(2)}(1-a^2)$

$\Rightarrow a^2 I_a^{(1)} + a I_a^{(2)} = a^2 I_{ab}^{(1)}(1-a) + a I_{ab}^{(2)}(1-a^2) \quad \ldots 10$

$\Rightarrow$ Solving 8 & 10 To find $I_a^{(1)}$ & $I_a^{(2)}$: $\boxed{\begin{aligned} I_a^{(1)} &= \sqrt{3}\angle{-30^\circ}\, I_{ab}^{(1)} \\ I_a^{(2)} &= \sqrt{3}\angle{+30^\circ}\, I_{ab}^{(2)} \end{aligned}}$ ***

• Y-load :

• Objective: find relation between: $V_{ab}^{(1)}$ & $V_{an}^{(1)}$, $V_{ab}^{(2)}$ & $V_{an}^{(2)}$



• Procedure: by KVL or Double subscription Notation:

$V_{ab} = V_{an} + V_{nb} = V_{an} - V_{bn} \quad \ldots 1$

$V_{bc} = V_{bn} + V_{nc} = V_{bn} - V_{cn} \quad \ldots 2$

$V_{ca} = V_{cn} + V_{na} = V_{cn} - V_{an} \quad \ldots 3$

$\Rightarrow$ Express 1 interms of symm. components:

$(V_{ab}^{(0)} + V_{ab}^{(1)} + V_{ab}^{(2)}) = (V_{an}^{(0)} + V_{an}^{(1)} + V_{an}^{(2)}) - (V_{bn}^{(0)} + V_{bn}^{(1)} + V_{bn}^{(2)}) \quad \ldots 4$

* By using $[V_{012}] = [A]^{-1}[V_{abc}] \quad \ldots 5$

$V_{ab}^{(0)} = \frac{1}{3}(V_{ab} + V_{bc} + V_{ca}) \quad \ldots 6$

* Sub. 1, 2 & 3 into 6 $\Rightarrow \boxed{V_{ab}^{(0)} = 0}$

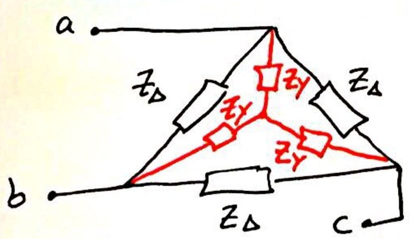
$\therefore$ 4 : $V_{ab}^{(1)} + V_{ab}^{(2)} = \underbrace{(V_{an}^{(0)} - V_{bn}^{(0)})}_{=0 \text{ since } V_{an}^{(0)} = V_{bn}^{(0)} = V_{cn}^{(0)}} + (V_{an}^{(1)} - V_{bn}^{(1)}) + (V_{an}^{(2)} - V_{bn}^{(2)})$

$\Rightarrow V_{ab}^{(1)} + V_{ab}^{(2)} = (V_{an}^{(1)} - a^2 V_{an}^{(1)}) + (V_{an}^{(2)} - a V_{an}^{(2)}) = V_{an}^{(1)}(1-a^2) + V_{an}^{(2)}(1-a)$

* Similarly : $V_{bc}^{(1)} + V_{bc}^{(2)} = V_{bn}^{(1)}(1-a^2) + V_{bn}^{(2)}(1-a)$

It can be shown: $\boxed{\begin{aligned} V_{ab}^{(1)} &= \sqrt{3}\angle{30^\circ}\, V_{an}^{(1)} \\ V_{ab}^{(2)} &= \sqrt{3}\angle{-30^\circ}\, V_{an}^{(2)} \end{aligned}}$ ***

*Relationship between Z_Δ & Z_Y :



$\Rightarrow Z_\Delta = \frac{V_{AB}^{(1)}}{I_{AB}^{(1)}} = \frac{\sqrt{3}\angle{30^\circ}\, V_{AN}^{(1)}}{\frac{I_{AN}^{(1)}}{\sqrt{3}}\angle{30^\circ}} = \frac{3\, V_{AN}^{(1)}}{I_{AN}^{(1)}} = 3 Z_Y$

$\boxed{Z_\Delta = 3 Z_Y}$

OR you can use $Z_\Delta = \frac{V_{AB}^{(2)}}{I_{AB}^{(2)}}$ To do the same proof.

*Power in terms of symmetrical components:

• objective: if symmetrical components are Known then one can calculate power as follows:

$$S = V_{an} I_{an}^* + V_{bn} I_{bn}^* + V_{cn} I_{cn}^* = [V_{an} \; V_{bn} \; V_{cn}] \begin{bmatrix} I_{an} \\ I_{bn} \\ I_{cn} \end{bmatrix}^* = \begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix}^T \begin{bmatrix} I_{an} \\ I_{bn} \\ I_{cn} \end{bmatrix}^*$$

$$= \Big[[A][V_{012}]\Big]^T \cdot \Big[[A][I_{012}]\Big]^* \Rightarrow S = [V_{012}]^T [A]^T [A]^* [I_{012}]^* \quad \ldots 1$$

where : $[V_{012}] = \begin{bmatrix} V^{(0)} \\ V^{(1)} \\ V^{(2)} \end{bmatrix}$, $[I_{012}] = \begin{bmatrix} I^{(0)} \\ I^{(1)} \\ I^{(2)} \end{bmatrix}$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \Longrightarrow [A]^T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} , [A]^* = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}$$

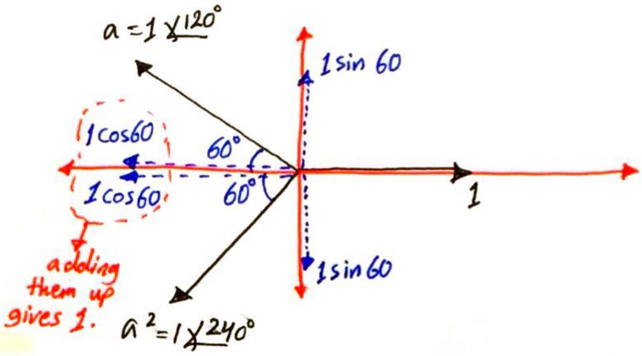
remember :

$a^2 = 1 \angle 240$

$(a^2)^* = 1\angle -240 = 1\angle 120^\circ = a$

Knowing : $1 + a + a^2 = 0$

(you can see that in) phasor diagram.



$$\therefore [A]^T[A]^* = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} = 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdots (2)$$

Substitute (2) in (1) :

$$[V^{(0)}\ V^{(1)}\ V^{(2)}] \cdot 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} I^{(0)*} \\ I^{(1)*} \\ I^{(2)*} \end{bmatrix} = 3\,V^{(0)}I^{(0)*} + 3\,V^{(1)}I^{(1)*} + 3\,V^{(2)}I^{(2)*} \cdots (3)$$

(in VA)

* if $S_b = 3\,S_{1\phi} \cdots (4)$, Then divide (3) by (4) : $\frac{S}{3S_{1\phi}} = \frac{V^{(0)}I^{(0)*}}{S_{1\phi}} + \frac{V^{(1)}I^{(1)*}}{S_{1\phi}} + \frac{V^{(2)}I^{(2)*}}{S_{1\phi}}$ (PU)

⇒ Hence, in PU factor (3) in equation (3) dissappears.

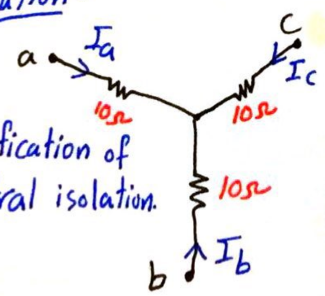
Example : A Balance Y-resistive Load of 10Ω have the following terminal voltages: $V_{ab} = 100\angle 0^\circ$ V, $V_{bc} = 80.8\angle -121.44^\circ$, and $V_{ca} = 90\angle 130^\circ$ V .

Assuming there is <u>NO Connection to Neutral</u> Find :

1) The Line Current from the sym. comp. of line voltages ?

2) The Complex Power supplied by using sym. comp. of voltages & currents ?

<u>Solution :</u>



signification of Neutral isolation.

1) By KVL : $I_a + I_b + I_c = 0$, $I^{(0)} \triangleq \frac{1}{3}[I_a + I_b + I_c] = 0$

$\therefore V_{ab}^{(1)} = \sqrt{3}\angle 30^\circ\, V_{an}^{(1)}$ $\quad \therefore V_{an}^{(1)} = \frac{V_{ab}^{(1)}}{\sqrt{3}\angle 30^\circ}$

$$\begin{bmatrix} V_{ab}^{(0)} \\ V_{ab}^{(1)} \\ V_{ab}^{(2)} \end{bmatrix} = [A^{-1}][V_{abc}] = \begin{bmatrix} 0 \\ 89.9\angle 2.9^\circ \\ 11.16\angle -23.96^\circ \end{bmatrix}$$

$\Rightarrow V_{an}^{(1)} = \frac{89.9\angle 2.9}{\sqrt{3}\angle 30^\circ}$

$\Rightarrow \boxed{V_{an}^{(1)} = 51.9\angle -27.1^\circ}$

⟹

$\Rightarrow V_{an}^{(2)} = \frac{V_{ab}^{(2)}}{\sqrt{3}\angle -30} = \frac{11.16\angle -23.96}{\sqrt{3}\angle -30^\circ} \Rightarrow \boxed{V_{an}^{(2)} = 6.4\angle 6.04^\circ}$

$\therefore I^{(1)} = \frac{V_{an}^{(1)}}{10} \Rightarrow \boxed{I^{(1)} = 5.19\angle -27.1^\circ}$ and $I^{(2)} = \frac{V_{an}^{(2)}}{10} \Rightarrow \boxed{I^{(2)} = 0.64\angle 6.04^\circ}$

$\therefore$ The Line Currents are: $\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = [A]\begin{bmatrix} I^{(0)} \\ I^{(1)} \\ I^{(2)} \end{bmatrix}$ By substitution:

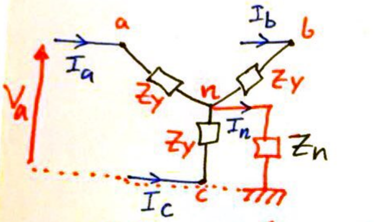
$$\boxed{\begin{aligned} I_a &= 5.73\angle -23.57^\circ \\ I_b &= 5.25\angle -14.8 \\ I_c &= 4.75\angle 87.5 \end{aligned}}$$

2) $S = 3\left[V^{(0)}I^{(0)*} + V^{(1)}I^{(1)*} + V^{(2)}I^{(2)*}\right] \Rightarrow \boxed{S = 820.371}$ VA $\angle 0^\circ$ since it is a resistive Load.

**** Sequance Impedances:**

- Objective: To find Zero, +ve & -ve seq. impedance of the various component in powes system.

- **3-ph Load:** Consider Y-connected load.



$I_n = I_a + I_b + I_c = \left(I_a^{(0)} + I_a^{(1)} + I_a^{(2)}\right) + \left(I_b^{(0)} + I_b^{(1)} + I_b^{(2)}\right) + \left(I_c^{(0)} + I_c^{(1)} + I_c^{(2)}\right)$

$= \left[I_a^{(0)} + I_b^{(0)} + I_c^{(0)}\right] + \underbrace{\left[I_a^{(1)} + I_b^{(1)} + I_c^{(1)}\right] + \left[I_a^{(2)} + I_b^{(2)} + I_c^{(2)}\right]}_{= \text{Zero (Because They are) Balanced Phasor.}}$

$\Rightarrow I_n = 3I_a^{(0)}$ since $I_a^{(0)} = I_b^{(0)} = I_c^{(0)}$

* Let our reference is the ground.

* Let V_a, V_b and $V_c \equiv$ The terminal voltages w.r.t Ref. $\therefore$ By KVL: $V_a = V_{an} + I_n Z_n$, $V_b = V_{bn} + I_n Z_n$, $V_c = V_{cn} + I_n Z_n$

$$\Rightarrow \begin{aligned} V_a &= I_a Z_y + 3I_a^{(0)} Z_n \\ V_b &= I_b Z_y + 3I_a^{(0)} Z_n \\ V_c &= I_c Z_y + 3I_a^{(0)} Z_n \end{aligned} \qquad \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix} + 3I_a^{(0)} Z_n \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

* By Using Symm. Components: $[A]\begin{bmatrix} V_a^{(0)} \\ V_a^{(1)} \\ V_a^{(2)} \end{bmatrix} = [A]\begin{bmatrix} V_{an}^{(0)} \\ V_{an}^{(1)} \\ V_{an}^{(2)} \end{bmatrix} + 3I_a^{(0)} Z_n \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \Rightarrow$ multiply By $[A]^{-1}$

$$\Rightarrow \begin{bmatrix} V_a^{(0)} \\ V_a^{(1)} \\ V_a^{(2)} \end{bmatrix} = \begin{bmatrix} V_{an}^{(0)} \\ V_{an}^{(1)} \\ V_{an}^{(2)} \end{bmatrix} + 3I_a^{(0)} Z_n * \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} V_{an}^{(0)} \\ V_{an}^{(1)} \\ V_{an}^{(2)} \end{bmatrix} + I_a^{(0)} Z_n \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix} \quad \ldots (1)$$

$\Rightarrow V_a^{(0)} = V_{an}^{(0)} + 3Z_n I_a^{(0)} = I_a^{(0)} Z_y + 3Z_n I_a^{(0)} \quad \ldots (2)$

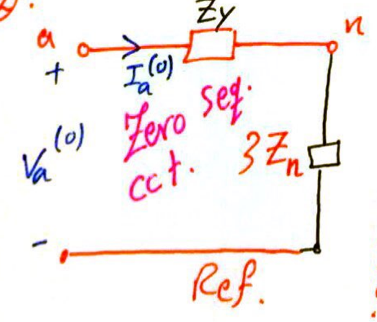
$V_a^{(1)} = V_{an}^{(1)} + 0 = I_a^{(1)} Z_y \quad \ldots (3)$

$V_a^{(2)} = V_{an}^{(2)} + 0 = I_a^{(2)} Z_y \quad \ldots (4)$

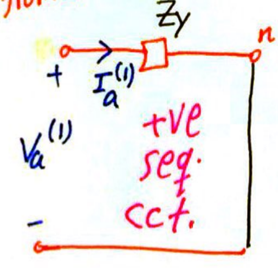
Each Equation in (2), (3) & (4) is an independent equation & Related only to one sequance (i.e 0, +ve, or -ve)

*** Use These Equations To deduce the Equ. Cct.

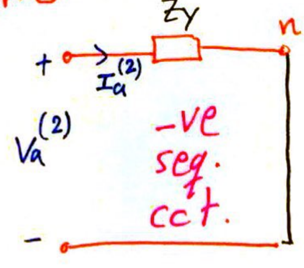
from (2):



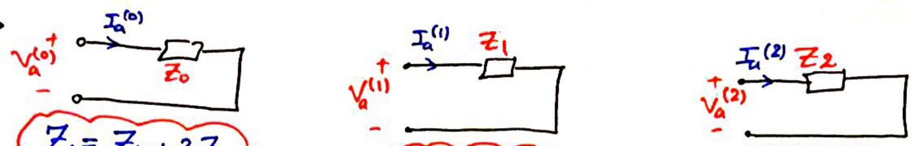
from (3):



from (4):



$\Rightarrow$



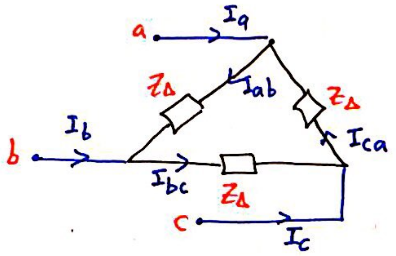
$Z_0 = Z_Y + 3Z_n$ $\quad$ $Z_1 = Z_Y$ $\quad$ $Z_2 = Z_Y$ $\quad$ $\boxed{Z_1 = Z_2}$

$Z_0, Z_1, Z_2 \Rightarrow$ Called Zero, +ve & −ve seq. impedance of the Y-connected Load.

*** Earthing Impedance Only Effect Zero-Seq-cct.

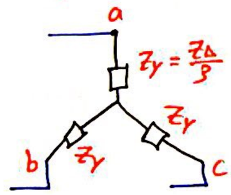
for e.g: if n is isolated $\therefore$ $Z_n = \infty$, if n is solidly grounded $\therefore$ $Z_n = 0$

• 3-ph load: Consider Δ-connected Load.

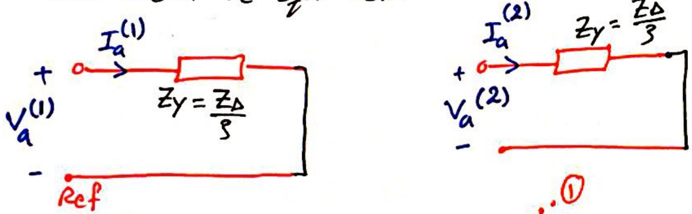


* Line currents CANNOT provide a path to Neutral

$\therefore$ The Equ. Y-load will be:



* The Equ. Y-connection Can be used to find the +ve & −ve equ. cct:



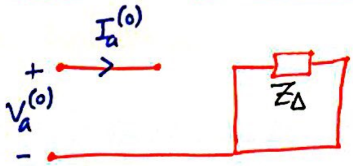
* For the Zero seq.: $V_{ab} = I_{ab} Z_\Delta$ ...①, $V_{bc} = I_{bc} Z_\Delta$ ...②, $V_{ca} = I_{ca} Z_\Delta$ ...③

① + ② + ③ : $V_{ab} + V_{bc} + V_{ca} = Z_\Delta \left(I_{ab} + I_{bc} + I_{ca}\right) = 0$

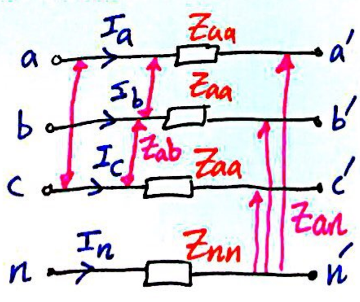
$\Rightarrow 0 = Z_\Delta \left(3 I_{ab}^{(0)}\right) = 3V_{ab}^{(0)}$ $\quad$ as we did in the previous page for $I_a + I_b + I_c$

$\therefore V_{ab}^{(0)} = 0$ $\quad$ $\therefore I_{ab}^{(0)} = 0$ $\quad$ $\therefore \boxed{I_{bc}^{(0)} = I_{ca}^{(0)} = 0}$ $\quad$ $\Rightarrow I_a^{(0)} = I_{ab}^{(0)} - I_{ca}^{(0)} \Rightarrow I_a^{(0)} = 0$

$\Rightarrow$ The Equ. Cct:



* Sequance ccts of T.L:



$\Rightarrow$ ALL phase conductors has the same impedance $= Z_{aa}$, Neutral has impedance $= Z_{nn}$.

* Mutual inductance between phase conductors is represented by an impedance $= Z_{ab}$

* Between Neutral & Conductors is represented by $= Z_{an}$

$\Rightarrow$

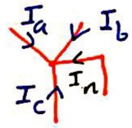
$Z_{aa} = R + j\omega L \quad , \quad Z_{ab} = j\omega M$

* Apply KVL: $V_{an} = I_a Z_{aa} + I_b Z_{ab} + I_c Z_{ab} + I_n Z_{an} + V_{a'n'} - (I_n Z_{nn} + I_a Z_{an} + I_b Z_{an} + I_c Z_{an})$

$\Rightarrow V_{an} - V_{a'n'} = I_n (Z_{aa} - Z_{an}) + I_b (Z_{ab} - Z_{an}) + I_c (Z_{ab} - Z_{an}) + I_n (Z_{an} - Z_{nn})$

$\therefore V_{an} - V_{a'n'} = I_a (Z_{aa} - Z_{an}) + (Z_{ab} - Z_{an})(I_b + I_c) + I_n (Z_{an} - Z_{nn}) \cdots (1)$

$\therefore$ By KCL: $I_n = -(I_a + I_b + I_c) \cdots (2)$



Sub. (2) into (1):

$$V_{an} - V_{a'n'} = I_a (\underbrace{Z_{aa} - 2Z_{an} + Z_{nn}}_{Z_s}) + (\underbrace{Z_{ab} - Z_{an} - Z_{an} + Z_{nn}}_{Z_m})(I_b + I_c)$$

$$\Rightarrow V_{an} - V_{a'n'} = I_a (Z_{aa} - 2Z_{an} + Z_{nn}) + I_b (Z_{ab} - 2Z_{an} + Z_{nn}) + I_c (Z_{ab} - 2Z_{an} + Z_{nn})$$

- The same procedure can be repeated for V_{bn} & V_{cn} To find $(V_{bn} - V_{b'n'})$ & $(V_{cn} - V_{c'n'})$:

It can be found that: $V_{an} - V_{a'n'} = I_a Z_s + I_b Z_m + I_c Z_m \cdots (1)$

$V_{bn} - V_{b'n'} = I_a Z_m + I_b Z_s + I_c Z_m \cdots (2)$

$V_{cn} - V_{c'n'} = I_a Z_m + I_b Z_m + I_c Z_s \cdots (3)$

$Z_s = Z_{aa} - 2Z_{an} + Z_{nn}$
$Z_m = Z_{ab} - 2Z_{an} + Z_{nn}$

- Rewrite (1), (2) & (3) in a matrix form:

$$\begin{bmatrix} V_{aa'} \\ V_{bb'} \\ V_{cc'} \end{bmatrix} = \begin{bmatrix} V_{an} - V_{a'n'} \\ V_{bn} - V_{b'n'} \\ V_{cn} - V_{c'n'} \end{bmatrix} = \begin{bmatrix} Z_s & Z_m & Z_m \\ Z_m & Z_s & Z_m \\ Z_m & Z_m & Z_s \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

* Introduce Symmetrical Components:

$$[A] \begin{bmatrix} V_{aa'}^{(0)} \\ V_{aa'}^{(1)} \\ V_{aa'}^{(2)} \end{bmatrix} = \begin{bmatrix} Z_s & Z_m & Z_m \\ Z_m & Z_s & Z_m \\ Z_m & Z_m & Z_s \end{bmatrix} [A] \begin{bmatrix} I_a^{(0)} \\ I_a^{(1)} \\ I_a^{(2)} \end{bmatrix} \xrightarrow{\text{multiply By } A^{-1}} \begin{bmatrix} V_{aa'}^{(0)} \\ V_{aa'}^{(1)} \\ V_{aa'}^{(2)} \end{bmatrix} = [A]^{-1} \begin{bmatrix} Z_s & Z_m & Z_m \\ Z_m & Z_s & Z_m \\ Z_m & Z_m & Z_s \end{bmatrix} [A] \begin{bmatrix} I_a^{(0)} \\ I_a^{(1)} \\ I_a^{(2)} \end{bmatrix}$$

By Multiplication, it can be found:

$$\begin{bmatrix} V_{aa'}^{(0)} \\ V_{aa'}^{(1)} \\ V_{aa'}^{(2)} \end{bmatrix} = \begin{bmatrix} Z_s + 2Z_m & 0 & 0 \\ 0 & Z_s - Z_m & 0 \\ 0 & 0 & Z_s - Z_m \end{bmatrix} \begin{bmatrix} I_a^{(0)} \\ I_a^{(1)} \\ I_a^{(2)} \end{bmatrix}$$

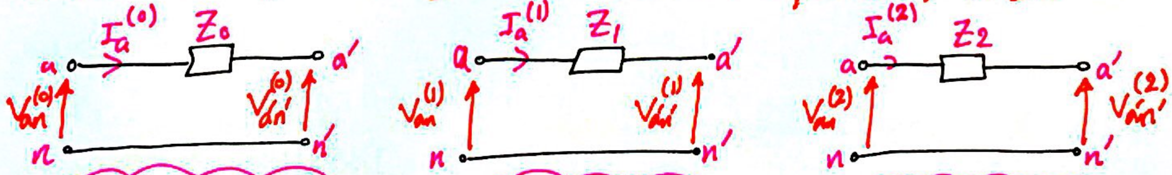
$\therefore$ The result is 3-independent Equations:

$V_{aa'}^{(0)} = \underbrace{(Z_s + 2Z_m)}_{Z_0} I_a^{(0)}$

$V_{aa'}^{(1)} = \underbrace{(Z_s - Z_m)}_{Z_1} I_a^{(1)}$

$V_{aa'}^{(2)} = \underbrace{(Z_s - Z_m)}_{Z_2} I_a^{(2)}$

* from these Equations, They can be used To find seq. ccts of T.L:



$Z_0 = Z_s + 2Z_m$ $\quad$ $Z_1 = Z_s - Z_m$ $\quad$ $Z_2 = Z_s - Z_m$

$\therefore Z_1 = Z_2$

where:

$Z_s = Z_{aa} - 2Z_{an} + Z_{nn}$

$Z_m = Z_{ab} - 2Z_{an} + Z_{nn}$

$\Rightarrow$ * $Z_1 = Z_2 = Z_{aa} - Z_{ab}$ *

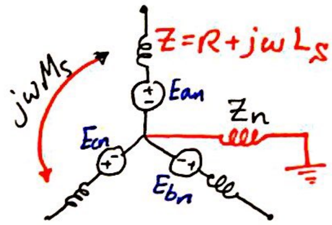
$\therefore$ Neutral only effect Z_0.
since it depend on Z_{nn}.

* Same General Procedure Can be applied to find seq. ccts for 3-ph Gen. + Transformer.

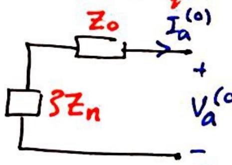
* Seq. cct. of 3-ph Generator:

By following the same previous procedure it can be found that:

$Z \equiv$ Self impedance of the gen.
$M_s \equiv$ Mutual Inductance between any two phase.
$L_s \equiv$ Self inductance.
$Z_n \equiv$ Earthing Impedance.

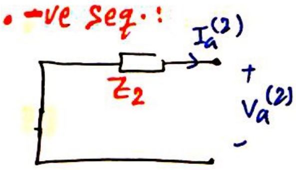


• Zero Seq.:

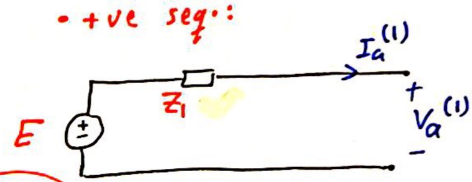


$$\boxed{Z_0 = R + j\omega(L_s - 2M_s)}$$

• −ve seq.:



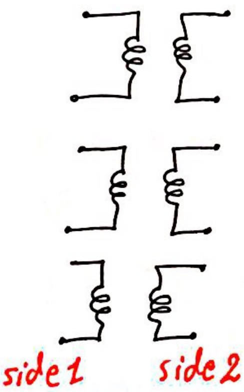
• +ve seq.:



$$\boxed{Z_1 = Z_2 = R + j\omega(L_s + M_s)}$$

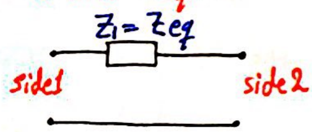
* The only active Network is the +ve seq of the Gen.

※ Seq. cct. of 3-ph Transformer:

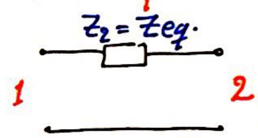


Each side Can be connected as Δ or Y.

• +ve seq.:

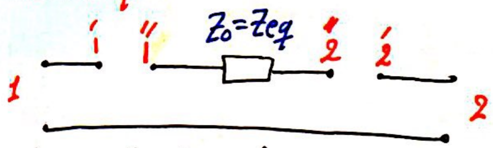


• −ve seq.:



$Z_{eq} \equiv$ Equivalent Series Impedance of the Transformer.

• Zero Seq.:



* Connection Between (1'-1'') & (2'-2'') depends on the type of connection (i.e Δ or Y).

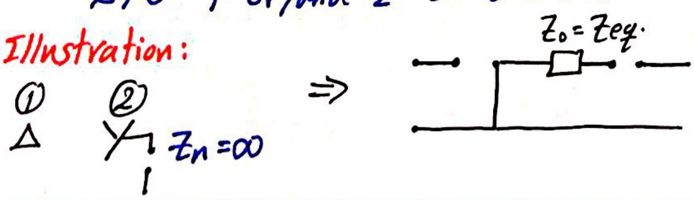
• For Y-type:

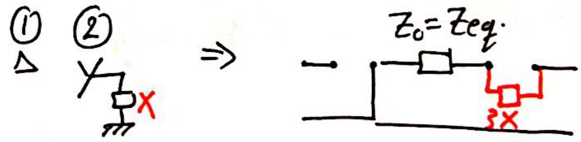
the connection is made by an impedance = $\boxed{3Z_n}$, where Z_n is the earthing impedance of Neutral.

• For Δ-type:

S/C 1'' or/and 2'' to the Ref..

* Illustration:



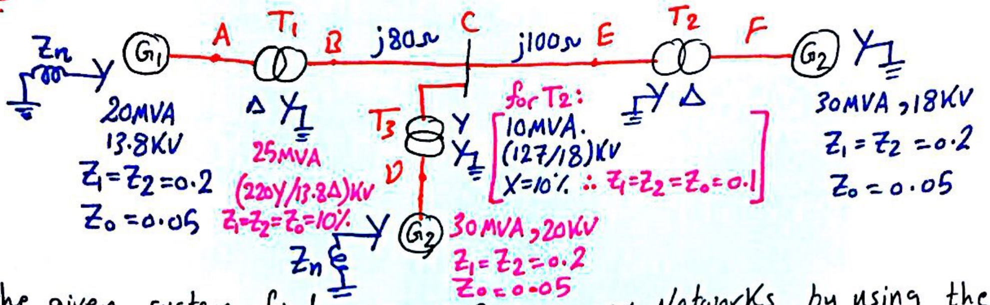


※ Seq. Networks:

Having produced seq. ccts of the individual components (i.e gen., Trans., T.L & Load) these ccts can be interconnected to produce:

+ve seq. Network, -ve seq. Network, Zero seq. Network.

Example:



For the given system, find +ve, -ve & zero seq Networks by using the given data & using Base value of 50MVA & 13.8KV @ the gen(1) ?

Given Information:

* The Reactance (Earthing Reactance) of G_1 & G_3 = 5%
* The zero-seq. Reactance of T.L is 210Ω for B-C & 250Ω for C-E.

Note: All the passive elements are Pure Reactance.

for T_3: 35MVA, (220Y/22Y)KV, X=10% ∴ $Z_1=Z_2=Z_0=0.1$

Note: 1) The given PU values of each component are based on the Ratings of the component.

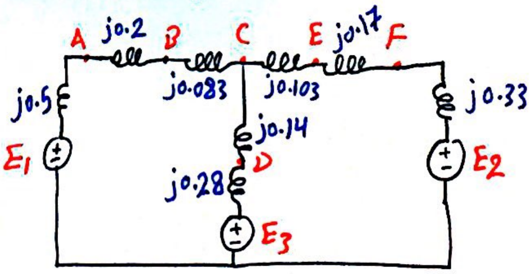
2) In this e.g Base voltage doesn't change only S_B change ∴ $Z_{new} = Z_{old} * \frac{S_{new}}{S_{old}}$

3) +ve & -ve seq. Networks are identical Exept that +ve seq. has source.

Solution:

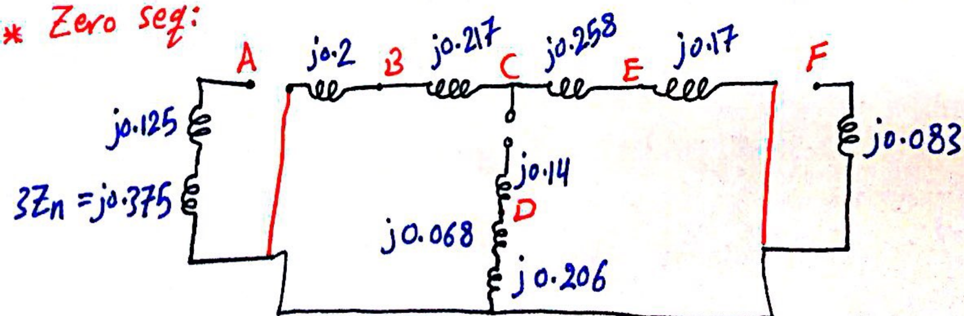
Component	$Z_1=Z_2$	Z_0	Z_n
G_1	0.5	0.125	0.125
T_1	0.2	0.2	
L_{BC}	0.083	0.217	
L_{CE}	0.103	0.258	
T_2	0.17	0.17	
G_2	0.33	0.083	
G_3	0.28	0.0689	0.0689
T_3	0.14	0.14	

* +ve seq:



* -ve seq: Same as +ve seq with E_1, E_2, E_3 replaced by S/C.

* Zero seq:



* Comments:

1) The interconnection between seq. Networks will depend on the type of the fault as will be shown later.

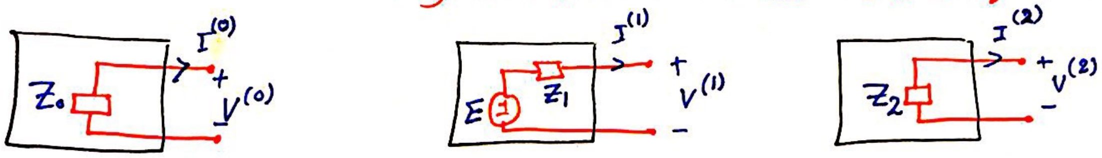
2) In the analysis, Each Network will be represented by its Thevenin equivalent according to the location of the fault.

For example: in the given example Assume the fault is on Bus C, Then one has to find Z_{eq} seen between Bus C & Bus Ref.

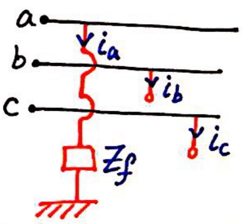
$\therefore$ from the previous Zero-seq cct: $Z_0 = (j0.217 + j0.2) // (j0.258 + j0.17)$

$\therefore$ The equivalent cct:

* By similar procedure one find +ve & -ve equivalents:



* Line-ground-fault:



$Z_f \equiv$ fault impedance.

- General Procedure:
  1) from schematic Diagram, find relationship for phase currents/voltages.
  2) Convert to symmetrical components.
  3) Deduce the interconnection.
  4) Evaluate symm. components.
  5) Evaluate corresponding phase voltages and currents under fault condition.

$\hookrightarrow$ for this cct: 1) $i_b = i_c = 0$ ... ① $\quad V_a = i_a Z_f$ ... ②

2) from equation ①: $\underbrace{I^{(0)} + a^2 I^{(1)} + a I^{(2)}}_{I_b} = \underbrace{I^{(0)} + a I^{(1)} + a^2 I^{(2)}}_{I_c} \Rightarrow I^{(1)}(a^2 - a) = I^{(2)}(a^2 - a)$

$\therefore I^{(1)} = I^{(2)}$

also from ①: $I_b = 0 = I^{(0)} + a^2 I^{(1)} + a I^{(2)} = I^{(0)} + I^{(1)}(a^2 + a)$ $\quad$ • remember: $a + a^2 + 1 = 0 \Rightarrow a^2 + a = -1$

$\Rightarrow \therefore 0 = I^{(0)} - I^{(1)} \quad \therefore I^{(0)} = I^{(1)} = I^{(2)}$ ... ③

from equation ②: $V_a^{(0)} + V_a^{(1)} + V_a^{(2)} = Z_f (I_a^{(0)} + I_a^{(1)} + I_a^{(2)})$ ... ④

substitute ③ into ④: $V_a^{(0)} + V_a^{(1)} + V_a^{(2)} = Z_f * 3 I_a^{(0)} \Rightarrow V_a^{(0)} + V_a^{(1)} + V_a^{(2)} = I_a^{(0)} 3 Z_f$ ... ⑤

3) $\therefore$ Seq. Networks should be connected in such away to satisfy ③ & ⑤ as follows:

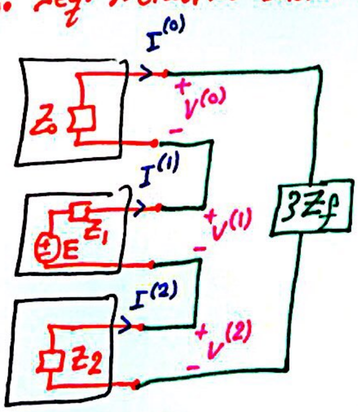


figure (1).

4) Symm. Components:

$$\therefore I^{(0)} = I^{(1)} = I^{(2)} = \frac{E}{Z_0 + Z_1 + Z_2 + 3Z_f} \quad \text{... ⑥}$$

$$V^{(0)} = -I^{(0)} Z_0 \quad \text{... ⑦}$$

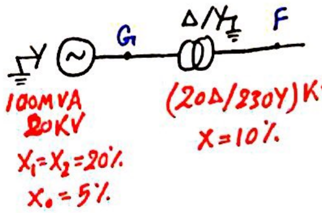
$$V^{(1)} = E_1 - I^{(1)} Z_1 \quad \text{... ⑧}$$

$$V^{(2)} = -I^{(2)} Z_2 \quad \text{... ⑨}$$

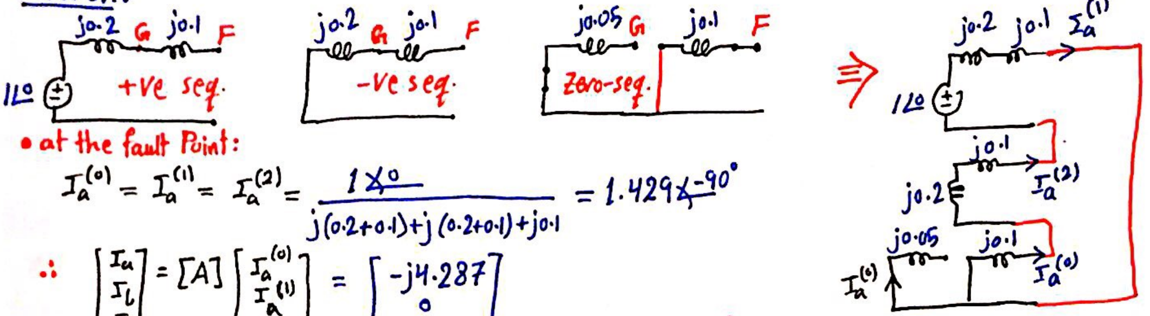
5) $[V_{abc}] = [A][V_{012}]$

$[I_{abc}] = [A][I_{012}]$

Example: For the given system (i.e gen. connected to transformer at No-load) the terminal voltage of the gen. is 20KV when a single-line-to-ground fault occurs on the o/c HV side of the transformer (i.e point F), Given the Base values at the generator 20KV & 100MVA, Find the phase currents @ points G&F ?



Solution:



• at the fault Point:

$$I_a^{(0)} = I_a^{(1)} = I_a^{(2)} = \frac{1\angle 0}{j(0.2+0.1)+j(0.2+0.1)+j0.1} = 1.429\angle -90^\circ$$

$$\therefore \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = [A] \begin{bmatrix} I_a^{(0)} \\ I_a^{(1)} \\ I_a^{(2)} \end{bmatrix} = \begin{bmatrix} -j4.287 \\ 0 \\ 0 \end{bmatrix}$$

• For the Gen.:

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = [A] \begin{bmatrix} 0 \\ 1.429 \angle -120 \\ 1.429 \angle -60 \end{bmatrix} = \begin{bmatrix} 2.475\angle -90^\circ \\ 2.475 \angle 90^\circ \\ 0 \end{bmatrix}$$

$I_a^{(1)} = 1.429 \angle -90-30 = 1.429 \angle -120^\circ$

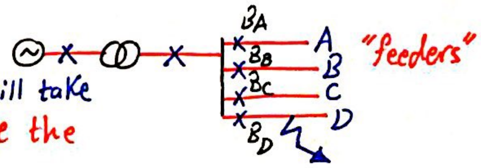
$I_a^{(2)} = 1.429 \angle -90+30 = 1.429 \angle -60^\circ$

$I_a^{(0)} = 0$ (from the cct. diagram).

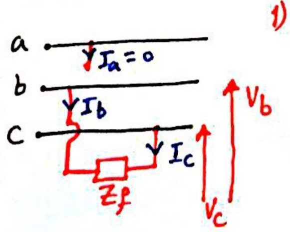
***Comment:** Base Current $I_B = \frac{100M}{\sqrt{3} * 20K} = 2886.8A$. $\therefore |I_a| = |I_b| = 2.475 * 2886.8 = \underline{\underline{7144.7\,A}}$.

***Comment:** Voltage Sag (i.e voltage drop)

for e.g: let a fault occur on feeder D, Then during the period taken for B_D to open, Then a voltage drop (i.e sag) will take a place on the healthy feeders A, B & C. which may cause the interruption of sensitive equipment.



※ Line-to-Line Fault:



1) $\boxed{I_a = 0}$ ... ① $\boxed{I_b = -I_c}$ ... ② $\boxed{V_b = V_c + I_b Z_f}$ ... ③

2) Convert to sym. comp.:

① ⇒ $I_a^{(0)} = \frac{1}{3}[I_a + I_b + I_c] = \frac{1}{3}[0 + I_b - I_b] = 0 \quad \therefore I_a^{(0)} = 0$ ... (3.1)

③ ⇒ $V_b - V_c = I_b Z_f \Rightarrow (V^{(0)} + a^2 V^{(1)} + aV^{(2)}) - (V^{(0)} + aV^{(1)} + a^2 V^{(2)}) = Z_f (I^{(0)} + a^2 I^{(1)} + a I^{(2)})$

$\Rightarrow V^{(1)}(a^2 - a) - V^{(2)}(a^2 - a) = Z_f (a^2 I^{(1)} + a I^{(2)})$ ... ④

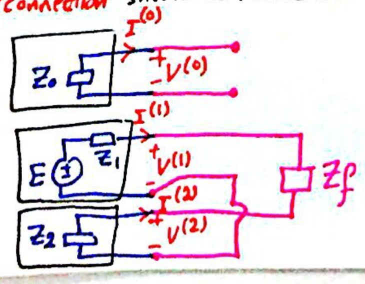
② ⇒ $I_b = -I_c \Rightarrow I^{(0)} + a^2 I^{(1)} + a I^{(2)} = -(I^{(0)} + a I^{(1)} + a^2 I^{(2)}) \Rightarrow I^{(1)}(a^2 + a) = I^{(2)}(-a^2 - a) \Rightarrow \therefore I^{(1)} = -I^{(2)}$ ... ⑤

Substitute ⑤ into ④:

$V^{(1)}(a^2 - a) - V^{(2)}(a^2 - a) = Z_f (a^2 I^{(1)} - a I^{(1)}) = Z_f I^{(1)}(a^2 - a) \quad \therefore V^{(1)} - V^{(2)} = Z_f I^{(1)}$ ... ⑥

3) interconnection should be made in such away to satisfy (3.1), ⑤ & ⑥:

Fig.(1)



4) from Fig.(1): $I^{(0)} = V^{(0)} = 0$, $I^{(1)} = -I^{(2)} = \frac{E}{Z_1 + Z_2 + Z_f}$

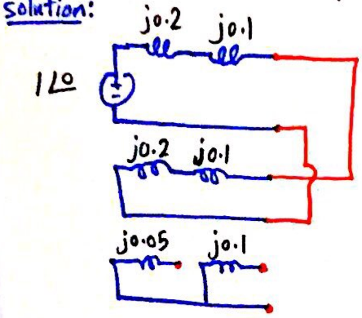
$V^{(1)} = E - I^{(1)} Z_1$

$V^{(2)} = -I^{(2)} Z_2$

5) $\therefore [V_{abc}] = [A][V_{012}]$, $[I_{abc}] = [A][I_{012}]$

Example: Solve the previous example for Line-to-line fault.

solution:



@ the fault: $I_a^{(0)}=0$, $I_a^{(1)}=-I_a^{(2)}=\dfrac{1\angle 0}{j0.3+j0.3}=1.67\angle -90^\circ$

$$\Rightarrow \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}=[A]\begin{bmatrix} 0 \\ 1.67\angle -90^\circ \\ 1.67\angle 90^\circ \end{bmatrix}=\begin{bmatrix} 0 \\ 2.893\angle 180^\circ \\ 2.893\angle 0 \end{bmatrix}$$

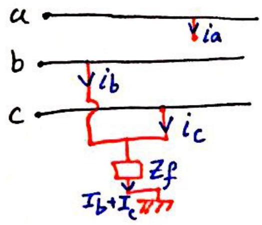
@ the Gen.: $I_a^{(0)}=0$, $I_a^{(1)}=1.67\angle -120^\circ$, $I_a^{(2)}=1.67\angle 120^\circ$

$$\Rightarrow \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}=[A]\begin{bmatrix} 0 \\ 1.67\angle -120^\circ \\ 1.67\angle 120^\circ \end{bmatrix}=\begin{bmatrix} 1.67\angle 180^\circ \\ 1.67\angle 180^\circ \\ 3.34\angle 0^\circ \end{bmatrix}$$

$|I_a|=|I_b|=1.67 * 2886.8 \simeq \underline{4821}$ A.

$|I_c|=3.34 * 2886.8 \simeq \underline{\underline{9642}}$ A

※ Line-to-Line-to-ground Fault:



1) $\boxed{i_a=0}$ ---① $\boxed{V_b=V_c}$ ···② $\boxed{V_b=Z_f(I_b+I_c)}$ ···③

2) ② ⇒ $V^{(0)}+a^2V^{(1)}+aV^{(2)}=V^{(0)}+aV^{(1)}+a^2V^{(2)} \Rightarrow V^{(1)}(a^2-a)=V^{(2)}(a^2-a)$

$\Rightarrow \boxed{V^{(1)}=V^{(2)}}$ ···④

③ ⇒ $V^{(0)}+a^2V^{(1)}+aV^{(2)}=Z_f(I^{(0)}+a^2I^{(1)}+I^{(2)}+I^{(0)}+aI^{(1)}+a^2I^{(2)})$

$\Rightarrow V^{(0)}+V^{(1)}(a^2+a)=Z_f[2I^{(0)}+I^{(1)}(a^2+a)+I^{(2)}(a^2+a)]$

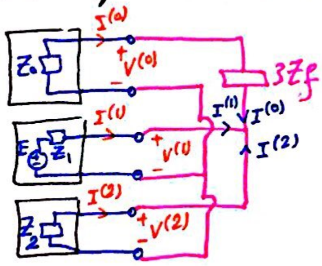
$\therefore \boxed{V^{(0)}-V^{(1)}=Z_f[2I^{(0)}-I^{(1)}-I^{(2)}]}$ ···⑤

① ⇒ $I^{(0)}+I^{(1)}+I^{(2)}=0 \Rightarrow \boxed{I^{(0)}=-(I^{(1)}+I^{(2)})}$ ···⑥

sub ⑥ into ⑤: $V^{(0)}-V^{(1)}=Z_f[2I^{(0)}+I^{(0)}]$

$\therefore \boxed{V^{(0)}-V^{(1)}=I^{(0)}3Z_f}$ ···⑦

3) Connect the seq. Networks in such away that ④, ⑥ & ⑦ are satisfied:



4) $\therefore I^{(1)}=\dfrac{E}{Z_1+(Z_0+3Z_f)//Z_2}$ $V^{(1)}=V^{(2)}=E-I^{(1)}Z_1$

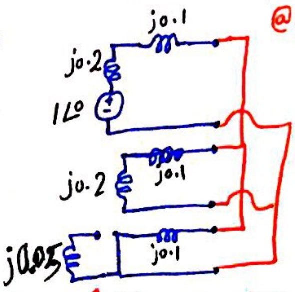
By current division: $-I^{(0)}=I^{(1)}\dfrac{Z_2}{Z_0+Z_2+3Z_f}$

$V^{(0)}=-I^{(0)}Z_0$, $I^{(0)}+I^{(1)}+I^{(2)}=0 \therefore \underline{\underline{I^{(0)}=-(I^{(1)}+I^{(2)})}}$

5) $[V_{abc}]=[A][V_{012}]$, $[I_{abc}]=[A][I_{012}]$

Example: Solve the Previous example for L-L-G fault.

solution:



@F: $I^{(1)}=\dfrac{1\angle 0}{j0.3+(j0.3//j0.1)}=\underline{\underline{2.67\angle -90^\circ}}$

$V^{(0)}=V^{(1)}=V^{(2)}=1\angle 0-I^{(1)}(j0.3)=\underline{\underline{0.2}}$

$I^{(2)}=\dfrac{-V^{(2)}}{j0.3}=\underline{\underline{0.67\angle 90^\circ}}$

$I^{(0)}=\dfrac{-V^{(0)}}{j0.1}=\underline{\underline{2\angle 90^\circ}}$

@G: $I^{(0)}=0$, $I^{(1)}=2.67\angle -120^\circ$, $I^{(2)}=0.67\angle 120^\circ$

@F: $\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}=\begin{bmatrix} 0 \\ 4.17\angle 134 \\ 4.17\angle 46 \end{bmatrix}$ & @G: $\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}=\begin{bmatrix} 2.41\angle -134 \\ 2.41\angle 134 \\ 3.34\angle 0 \end{bmatrix}$

※ Application of Z_{bus} matrix in Unbalanced Fault:

- objective: is to use this matrix for finding Z_0, Z_1, Z_2 of a given system.
- It was found that: $Z_{bus}=[Y_{bus}]^{-1}$, hence one has to find $[Y_{bus}]$ matrix for each seq. Network Then find Z_{bus} for each seq Network.

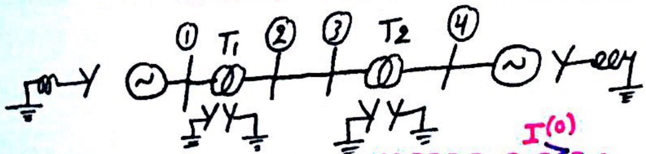
Power1

Fall017

Dr.Deefallah Dalabeeh

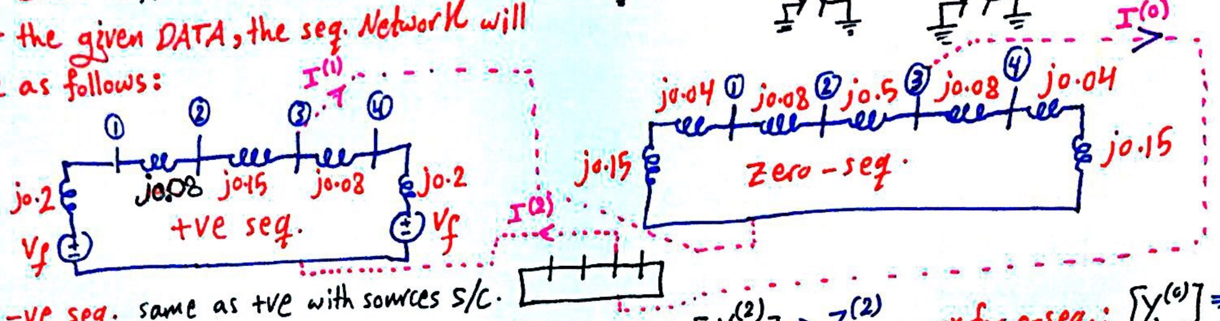
By: Mhmd Abuhashya

• Illustration: Consider the following system:



in Book: Ex 12.1/479 + 12.2/485

for the given DATA, the seq. Network will be as follows:



-ve seq. same as +ve with sources s/c.

* for +ve seq: $[Y^{(1)}_{bus}] \Rightarrow Z^{(1)}_{bus}$

* for -ve seq.: $[Y^{(2)}_{bus}] \Rightarrow Z^{(2)}_{bus}$

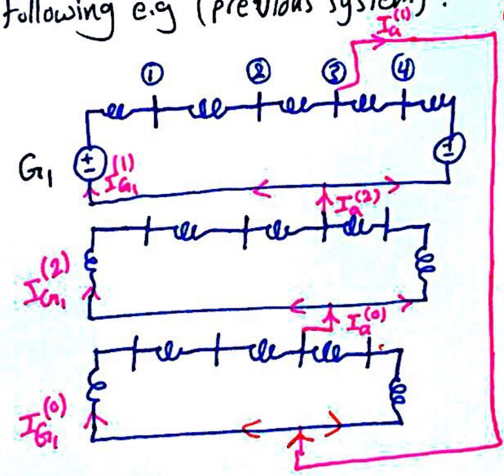
* for 0-seq.: $[Y^{(0)}_{bus}] \Rightarrow Z^{(0)}_{bus}$.

* It can be found: $Z^{(1)}_{bus} = Z^{(2)}_{bus} = \begin{bmatrix} 0.1437 & 0.1211 & 0.0789 & 0.0563 \\ 0.1211 & 0.1696 & 0.1104 & 0.0789 \\ 0.0789 & 0.1104 & 0.1696 & 0.1211 \\ 0.0563 & 0.0789 & 0.1211 & 0.1437 \end{bmatrix}$, $Z^{(0)}_{bus} = \begin{bmatrix} 0.1553 & 0.1407 & 0.0493 & 0.0347 \\ 0.1407 & 0.1999 & 0.0701 & 0.0943 \\ 0.0493 & 0.0701 & 0.1999 & 0.1407 \\ 0.0347 & 0.0493 & 0.1407 & 0.1553 \end{bmatrix}$

* for e.g Consider a L-G fault @ bus ③: (the connection will be as shown above).

$$I^{(0)} = I^{(1)} = I^{(2)} = \frac{1\angle 0}{Z_0 + Z_1 + Z_2} \; ; \; Z_0 = [Z_{33} \text{ of } Z^{(0)}_{bus}] \; ; \; Z_1 = [Z_{33} \text{ of } Z^{(1)}_{bus}] = Z_2$$

* Having Calculated the symm. components of a given fault current, then cct techniques Can be used to find the fault current through any element as illustrated by the following e.g (previous system): [for L-G fault @ bus ③]



⇒ find current supplied by G_1?

• find $I_{G_1}^{(0)}$, $I_{G_1}^{(1)}$, $I_{G_1}^{(2)}$ By using Current Division.

$I_{G_1}^{(1)} = 0.73\angle{-90^\circ}$ $\quad I_{G_1}^{(2)} = I_{G_1}^{(1)} = 0.73\angle{-90^\circ}$

$I_{G_1}^{(0)} = 0.519\angle{-90^\circ}$

∴ for G_1:

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = [A] \begin{bmatrix} I_{G_1}^{(0)} \\ I_{G_1}^{(1)} \\ I_{G_1}^{(2)} \end{bmatrix} = \begin{bmatrix} 1.99\angle{-90^\circ} \\ 0.21\angle{+90^\circ} \\ 0.21\angle{90^\circ} \end{bmatrix}$$

* (end of second material) *

* Open-Circuit Fault:

if an o/c occur on one phase or 2-phases (for e.g due to an accident), then the system becomes Unsymmetrical OR Unbalanced.

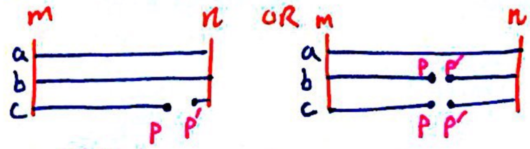


Fig.1

Then the objective is to evaluate the voltage across the points p, p'.

• Procedure: The +ve, -ve & Zero seq networks concept can be used.

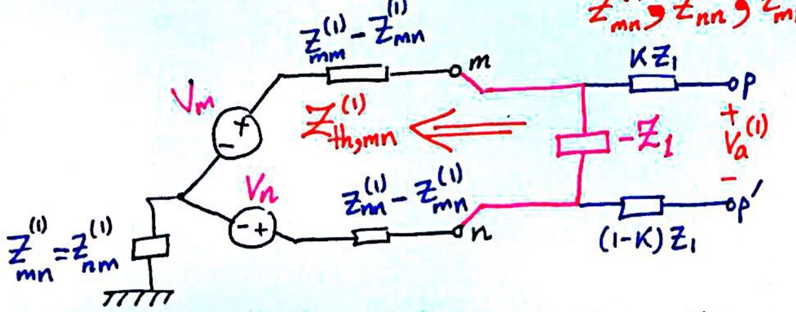
* This problem can be simulated by: ① Removing the Line between m & n.

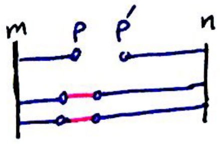
② Introduce s/c for other phases (e.g s/c is introduced for phases a & b).

* if Z_1, Z_2 & Z_0 are the seq. impedances of the line between m & n.
Hence, the removal of the line is simulated by adding: $-Z_1, -Z_2, -Z_0$
to the Thevenin equivalent between buses m & n, for the +ve, -ve & zero seq.
And Adding the fraction of the lines as follows:

Consider the +ve Seq. Thev. Equ.:

$Z^{(1)}_{mn}, Z^{(1)}_{nn}, Z^{(1)}_{mm} \equiv$ The elements in Z_{bus} of the +ve seq. Network of the original system.



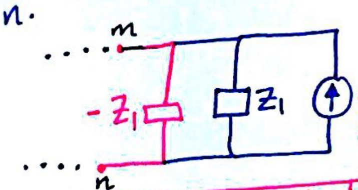


$V_a^{(1)} \equiv$ +ve symm. component of phase a for $V_{pp'}$.

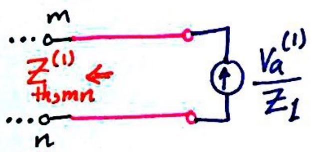
* we would like to evaluate: $V_{pp',a}$, $V_{pp',b}$, $V_{pp',c}$.

• Note: the -ve seq. Thev. Equ. is the same, BUT replacing the sources by S/C and superscript (1) by (2).
Zero seq. Thev. Equ. is the same, BUT replacing the sources by S/C & superscript (1) by (0).

* Use source transformation (for the above cct) To find current injections at the busbars m & n.



$$\frac{V_a^{(1)}}{KZ_1+(1-K)Z_1} = \frac{V_a^{(1)}}{Z_1} \Rightarrow$$



• current injections:

	+ve seq.	-ve seq.	Zero seq.
m	$V_a^{(1)}/Z_1$	$V_a^{(2)}/Z_2$	$V_a^{(0)}/Z_0$
n	$-V_a^{(1)}/Z_1$	$-V_a^{(2)}/Z_2$	$-V_a^{(0)}/Z_0$

∴ By Using Current injections one can find change of voltages at busbar m & n as explained before.

* To find the method of connection between seq. Networks one has to find Thevenin Equivalent for each seq.

* o/c fault was simulated by Current injections at busbars m & n.

* objective: To evaluate voltage changes at busbars due to this fault.

For e.g at the i^{th} bus, the changes will be as follows:

$$\Delta V_i^{(0)} = (Z_{im}^{(0)} - Z_{in}^{(0)})\, V_a^{(0)}/Z_0 \quad \cdots ①$$
$$\Delta V_i^{(1)} = (Z_{im}^{(1)} - Z_{in}^{(1)})\, V_a^{(1)}/Z_1 \quad \cdots ②$$
$$\Delta V_i^{(2)} = \underbrace{(Z_{im}^{(2)} - Z_{in}^{(2)})}_{\text{element of } Z_{bus}\ 0,1,2.}\, V_a^{(2)}/Z_2 \quad \cdots ③$$

$Z_1, Z_2, Z_0 \equiv$ +ve, -ve, zero impedances of T. Line.

* ①, ② & ③ can be solved by finding $V_a^{(0)}, V_a^{(1)}$ & $V_a^{(2)}$ as follows:

Finding Thev. equ. for the +ve, -ve, zero equivalent networks between buses m & n.

$$\text{+ve seq. } Z_{pp'}^{(1)} = KZ_1 + \left[(-Z_1) // Z_{th,mn}^{(1)}\right] + (Z_1 - KZ_1) \Rightarrow Z_{pp'}^{(1)} = -Z_1^2/(Z_{th,mn}^{(1)} - Z_1) \quad \cdots ④$$

$$\Rightarrow V_{th} = (V_m - V_n)\,\frac{(-Z_1)}{Z_{th,mn}^{(1)} - Z_1} \Rightarrow V_{th} = (V_m - V_n)\,\frac{Z_{pp'}^{(1)}}{Z_1} \quad \cdots ⑤$$

* Before the o/c of the line, the current flowing from m to n is I_{mn}:

$$I_{mn} = \frac{V_m - V_n}{Z_1} \quad \cdots ⑥$$

∴ substituting ⑥ into ⑤: $V_{th} = I_{mn} Z_{pp'}^{(1)} \quad \cdots ⑦$

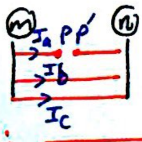
* By Similar Approach it can be found that:

$$Z_{pp'}^{(2)} = -Z_2^2/(Z_{th,mn}^{(2)} - Z_2) \quad \cdots ⑧$$
$$Z_{pp'}^{(0)} = -Z_0^2/(Z_{th,mn}^{(0)} - Z_0) \quad \cdots ⑨$$

④ + ⑦ ⟹ +ve seq. Network.
⑧ ⟹ -ve seq. Network.
⑨ ⟹ Zero seq. Network.

* The interconnection between these Networks, depends on the Type of Fault.

⟹ Consider the Case of One Open Conductor.



$$\boxed{I_a = 0} \quad \cdots ⑩$$

$$\therefore I_a^{(0)} + I_a^{(1)} + I_a^{(2)} = 0 \quad \cdots ⑪$$

• for phase b & c: $\boxed{V_{pp',b} = V_{pp',c} = 0} \quad \cdots ⑫$

* Transpose ⑫ into symm. components:

$$\begin{bmatrix} V_a^{(0)} \\ V_a^{(1)} \\ V_a^{(2)} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_{pp',a} \\ V_{pp',b} = 0 \\ V_{pp',c} = 0 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} V_{pp',a} \\ V_{pp',a} \\ V_{pp',a} \end{bmatrix} \quad \cdots ⑬$$

$$\therefore V_a^{(0)} = V_a^{(1)} = V_a^{(2)}$$

∴ Connection between Seq. Networks should Satisfy ⑪ & ⑬:

Fig.(1)

$Z_{pp'}^{(1)}, Z_{pp'}^{(2)}, Z_{pp'}^{(0)} \equiv$ were evaluated from thev. eq. between buses m & n.

$\therefore$ From Fig. (1):

$$I_a^{(1)} = \frac{V_{th} = I_{mn} Z_{pp'}^{(1)}}{Z_{pp'}^{(1)} + (Z_{pp'}^{(2)} // Z_{pp'}^{(0)})} \Rightarrow I_a^{(1)} = \frac{I_{mn} Z_{pp'}^{(1)} (Z_{pp'}^{(2)} + Z_{pp'}^{(0)})}{Z_{pp'}^{(1)} Z_{pp'}^{(2)} + Z_{pp'}^{(1)} Z_{pp'}^{(0)} + Z_{pp'}^{(2)} Z_{pp'}^{(0)}} \quad \ldots (14)$$

Now the symm. Components $V_a^{(0)} = V_a^{(1)} = V_a^{(2)} = I_a^{(1)} \dfrac{Z_{pp'}^{(2)} Z_{pp'}^{(0)}}{Z_{pp'}^{(2)} + Z_{pp'}^{(0)}}$ ... (15)

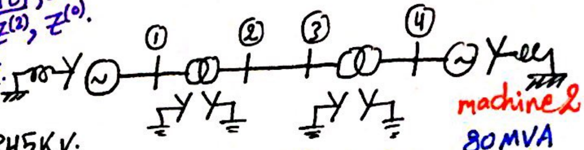
* Substitute (14) into (15) & Re-arrange, it can be found: $V_a^{(0)} = V_a^{(1)} = V_a^{(2)} = \dfrac{I_{mn}}{\frac{1}{Z_{pp'}^{(0)}} + \frac{1}{Z_{pp'}^{(1)}} + \frac{1}{Z_{pp'}^{(2)}}}$ ... (16)

* Finally, Substitute (16) into (1), (2) & (3) To find $\Delta V_i^{(0)}, \Delta V_i^{(1)}, \Delta V_i^{(2)}$.

Example: for the shown system:

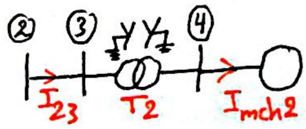
$Z_{bus}^{(1)}, Z_{bus}^{(2)}, Z_{bus}^{(0)}$ are given page 482 & 486 in Book. see page [46] for $Z^{(1)}, Z^{(2)}, Z^{(0)}$.

PU values are according to base values 100MVA, 345KV.



Determine the change in voltages at bus (3), when T.L between Buses (2) & (3) undergoes one-open-conductor fault ?

- choose a base value of 100MVA, 345K volt.
- Given for the line $X_1 = X_2 = 15\%$ & $X_0 = 50\%$.



Solution:

objective is To find ΔV_3. $\Rightarrow$ load in our case is the motor.

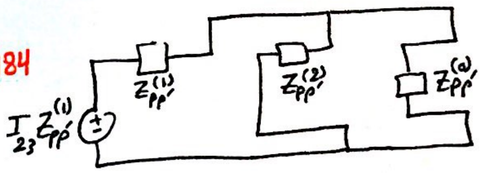
$$I_{mch2} = \frac{80000 * 10^3}{\sqrt{3} * 20 * 10^3} \angle -\cos^{-1}(0.85) \Rightarrow \boxed{I_{mch2} = 2309 \angle -31.8^\circ} A.$$

$$I_{23} = 2309 \angle -31.8^\circ * \frac{20}{345} \Rightarrow \boxed{I_{23} = 133.9 \angle -31.8^\circ} A \qquad I_b = \frac{100 * 10^6}{\sqrt{3} * 345 * 10^3} \Rightarrow \boxed{I_b = 167.3} A.$$

$$Z_{pp'}^{(1)} = \frac{-Z_1^2}{Z_{th,23}^{(1)} - Z_1} \; ; \; Z_1 = j0.15$$

$$Z_{th,23}^{(1)} = Z_{22}^{(1)} + Z_{33}^{(1)} - 2Z_{23}^{(1)} = j0.1184$$

$Z_{pp'}^{(1)} = j0.712$



By similar approach it can be found:

$Z_{pp'}^{(1)} = Z_{pp'}^{(2)} = j0.712$, $Z_{pp'}^{(0)} = j\,1.039$

$$V_a^{(1)} = V_a^{(0)} = V_a^{(2)} = \frac{I_{23}}{\frac{1}{Z_{pp'}^{(0)}} + \frac{1}{Z_{pp'}^{(1)}} + \frac{1}{Z_{pp'}^{(2)}}} \Rightarrow \boxed{V_a^{(0)} = V_a^{(1)} = V_a^{(2)} = 0.2122 \angle 58.2^\circ}$$

$$\Delta V_3^{(1)} = \Delta V_3^{(2)} = \frac{Z_{32}^{(1)} - Z_{33}^{(1)}}{Z_1} V_a^{(1)} = \boxed{0.0837 \angle -121.8^\circ}, \quad \Delta V_3^{(0)} = \frac{Z_{32}^{(0)} - Z_{33}^{(0)}}{Z_0} V_a^{(0)} = \boxed{0.0551 \angle -121.8^\circ}$$

$$\therefore \Delta V_3 = \Delta V_3^{(0)} + \Delta V_3^{(1)} + \Delta V_3^{(2)} \Rightarrow \Delta V_3 = 0.2225 \angle -121.8^\circ \; \#$$

* in case of o/c fault:

$$\begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \Delta V_3 \\ \vdots \\ \Delta V_n \end{bmatrix} = \begin{bmatrix} Z_{11} & \cdots & Z_{1n} \\ \vdots & & \vdots \\ Z_{n1} & \cdots & Z_{nn} \end{bmatrix} \begin{bmatrix} 0 \\ V_a/Z_1 \\ -V_a/Z_1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$\Rightarrow$ in case $m = 2$ & $n = 3$

i.e: o/c fault between (2) & (3).

* Power flow (Load flow) *

What!!

- **Power Flow**: It is the calculation of voltages at busbars for a given load condition.

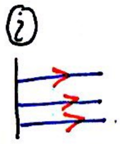
* Consequantly, the following may calculated:
  - i) flow of current in Lines.
  - ii) flow of Power in lines.
  - iii) Losses in Lines.

Why!!

* Power flow analysis is used:
  - (i) In Planning or in expanssion.
  - (ii) In operation; to find the optimal solution.

How!!

* Load Flow Formulation Problem:
  - Consider the i^{th} bus in agiven system.



Current injected into the Network: $I_i = Y_{i1}V_1 + Y_{i2}V_2 + \dots + Y_{iN}V_N$

where $N \equiv$ Number of busbars.

$$\Rightarrow I_i = \sum_{j=1}^{N} Y_{ij} V_j$$

$$[I] = [Y][V]$$

$$\begin{bmatrix} I_1 \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} Y_{11} & \dots \\ \vdots & Y_{NN} \end{bmatrix} \begin{bmatrix} V_1 \\ \vdots \\ V_N \end{bmatrix}$$

- Complex Power injected into Network from bus i:

$$S = P + jQ = V_i I_i^* \quad \therefore \quad S = |V_i| \angle \delta_i \left(\sum_{j=1}^{N} Y_{ij} V_j \right)^*$$

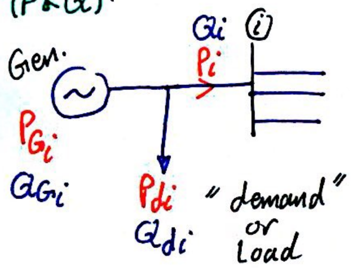
$\Rightarrow$ Let $V_i = |V_i| \angle \delta_i$, $Y_{ij} = |Y_{ij}| \angle \theta_{ij}$, $V_j = |V_j| \angle \delta_j$

$$\Rightarrow S_i = \sum_{j=1}^{N} |V_i||V_j||Y_{ij}| \angle \delta_i - \delta_j - \theta_{ij}$$

$\Rightarrow$ Power Flow Equation (P & Q).

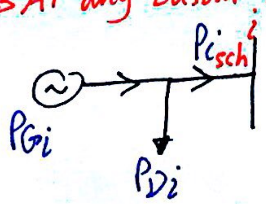
$$P_i = \sum_{j=1}^{N} |V_i||V_j||Y_{ij}| \cos(\delta_i - \delta_j - \theta_{ij}) \quad \dots (1)$$

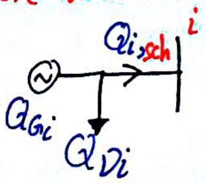
$$Q_i = \sum_{j=1}^{N} |V_i||V_j||Y_{ij}| \sin(\delta_i - \delta_j - \theta_{ij}) \quad \dots (2)$$



* Equations (1) & (2) gives the Active & Reactive power injected to the Network from bus i.
* Equations (1) & (2) Can be used to calculate P_i & Q_i. if $V_i \Rightarrow i = 1, N$ are Known, then they are called $P_{i,cal}$ & $Q_{i,cal}$.

$\Rightarrow$ At any busbar there are 3-parameters:





$P_{Gi}, P_{Di} \equiv$ are called scheduled generation and load at bus i.

$P_{Di} \equiv$ Demand OR Load at bus i.

$$\therefore P_{i sch} = P_{Gi} - P_{Di} \quad \dots (3)$$

$$\therefore Q_{i sch} = Q_{Gi} - Q_{Di} \quad \dots (4)$$

∴ In the process of calculations, there will be mismatch between $P_{i,cal}$ & $P_{i,sch}$.

∴ $\Delta P_i = P_{isch} - P_{ical}$ $\Delta Q_i = Q_{isch} - Q_{ical}$

* Solution will be obtained when Power mismatch = 0 (i.e $\Delta P_i = \Delta Q_i = 0$).
* By observing ① & ② it can be found that there are 4 unknowns $P_i, Q_i, |V_j|$ & δ_j

Hence one has to specify 2 parameters and calculates for the other 2 variables.

⇒ This selection depends on the type of busbar, and classify as follows:

① P-Q or Load Bus: • This is a Non-generator bus. ⇒ ∴ $P_{G_i} = Q_{G_i} = 0$.

- P_{D_i} can be estimated by using (for e.g) "Load for Cast".
- Q_{D_i} can be calculated by assuming a certain Pf.

∴ $P_{isch} = P_{G_i} - P_{D_i} \Rightarrow P_{isch} = -P_{D_i}$ ∴ $Q_{isch} = Q_{G_i} - Q_{D_i} \Rightarrow Q_{i_{sch}} = -Q_{D_i}$

here P_{isch} & Q_{isch} are the specified values for P_i & Q_i.

∴ ① & ② Can be used To calculate $|V_j|$ & δ_j.

② Voltage-Controlled or PV bus:

- This is a generator bus. ⇒ here one can specify P_{G_i} & $|V_i|$. $P_{isch} = P_{G_i} - P_{D_i}$
- Q_i Can be calculated when ALL the voltages of the busbars are found.

Hence, The only unknown is δ_i.

③ SLacK or Reference Busbars:

- Since one calculated phasor voltages then one busbar should be selected as a Reference. Usually, This busbar given Number 1 & specified as follows: $V_1 = 1\angle 0°$ PU. Usually, This busbar it is a generator bus.

Why (generator bus) ?!

$\sum P_i = \sum P_{G_i} - \sum P_{D_i} = P_{Loss}$ "Active Losses in Resistive Component".

$\sum Q_i = \sum Q_{G_i} - \sum Q_{D_i} = Q_{Loss}$ "Reactive Losses in Reactive Component".

• Summary:

Bus Type	# of Buses	specified quantities	# of available equations	# of Unknowns state variables
Slack, $i=1$	1	$\lvert V_1\rvert, \delta_1$	0	0
Voltage Controlled or PV bus. ($i=2,\dots,N_g+1$)	N_g	$P_i, \lvert V_i\rvert$	N_g	N_g
Load or PQ bus ($i=N_g+2,\dots,N$)	$N-N_g-1$	P_i, Q_i	$2(N-N_g-1)$	$2(N-N_g-1)$
SUM ⇒	N	$2N$	$2N-N_g-2$	$2N-N_g-2$

* Unknowns: are called dependent or state variables.

• Illustration: (Example 9.1) If a power system has 9-buses, at each bus there is a load. Generators are connected to buses 1, 2, 5, 7.

* Identify bus types & unknowns to be evaluated?

Solution: Types:
• Slack bus ⇒ 1
• Voltage Controlled or PV bus ⇒ 2, 5, 7
• Load or PQ bus ⇒ 3, 4, 6, 8, 9

Unknowns: voltage controlled ⇒ $\delta_2, \delta_5, \delta_7$ Load ⇒ V_3, V_4, V_6, V_8, V_9.
where $V_x = \lvert V_x\rvert \angle \delta_x$

$\boxed{N=9}$, $\boxed{N_g=3}$ ⇒ ∴ # of equations $= 2*9-3-2 = 13$ equations.

*** Comment: Since Power flow equations are Non-linear functions of the state variables, Then Iterative Methods are used to solve them

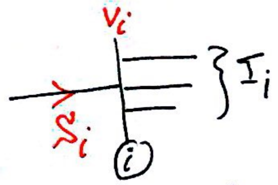
⇒ Here 2-methods will be considered:

1] Gauss-Seidal Method: This uses Power flow equations to Calculate Voltages.

2] Newton-Raphson Method: This uses Power mismatch ($\Delta P, \Delta Q$) to Calculate Voltages.

※ Gauss-Seidal Method:

This is introduced as follows:



$$S_i = V_i I_i^* \Rightarrow S_i^* = V_i^* I_i = V_i^* \sum_{n=1}^{N} Y_{in} V_n$$

$Y_{in} \equiv$ element in the $[Y]_{bus}$ matrix..

$$\therefore \frac{S_i^*}{V_i^*} = \sum_{n=1}^{N} Y_{in} V_n = Y_{ii} V_i + \sum_{\substack{n=1 \\ n\neq i}}^{N} Y_{in} V_n$$

$$\therefore\ V_i^{(K)} = \frac{1}{Y_{ii}}\left[\frac{S_i^*}{V_i^*} - \sum_{\substack{n=1\\n\neq i}}^{N} Y_{in}V_n\right] = \frac{1}{Y_{ii}}\left[\frac{S_i^*}{V_i^*} - \sum_{j=1}^{i-1} Y_{ij}V_j - \sum_{j=i+1}^{N} Y_{ij}V_j\right]$$

$K \equiv$ Number of Iterations.

$\boxed{S_i = P_i + jQ_i}$; The complex transmitted power.

$V^{(0)} \equiv$ initial. $V_1^{(0)} = 1\angle 0$ Let: $V_2^{(0)} = V_3^{(0)} = \cdots = V_N^{(0)} = 1\angle 0$

$$\Rightarrow \boxed{V_i^{(K)} = \frac{1}{Y_{ii}}\left[\frac{S_i^*}{V_i^{(K-1)*}} - \sum_{j=1}^{i-1} Y_{ij}V_j^{(K)} - \sum_{j=i+1}^{N} Y_{ij}V_j^{(K-1)}\right]} \quad \cdots (1)$$

Equation (1) is used to Calculate New values for voltages as follows:

• Procedure:

(i) Specify a voltage for the slack bus, $V_1 = 1\angle 0$. This value is Kept constant during calculations.

(ii) Assume initial value for voltage for all other buses. let $V_j^{(0)} = 1\angle 0$ for $j = 2, \dots, N$.

(iii) Substitute V_1 & V_j in (1) to calculate New values for voltages $V_j^{(1)}$, $j = 2, \dots, N$.

• Note: on the RHS of (1), one always substitute the most recent values.

(iv) When all voltages are calculated, then one iteration is completed.

(v) At the end of iteration, Check that: $|V_i^{K} - V_i^{K-1}| < \epsilon$ for all buses.

$\epsilon \equiv$ Specified Tolerance e.g: $\epsilon = 10^{-5}$.

After check if → Yes, Solution is Obtained.
→ Not, go to next iteration.

• To illustrate the application of equation (1), Consider 4-bus system, with bus (1) as a slack. (2), (3) & (4) as PQ-buses. with initial values given as $V_2^{(0)}, V_3^{(0)}, \& V_4^{(0)}$.

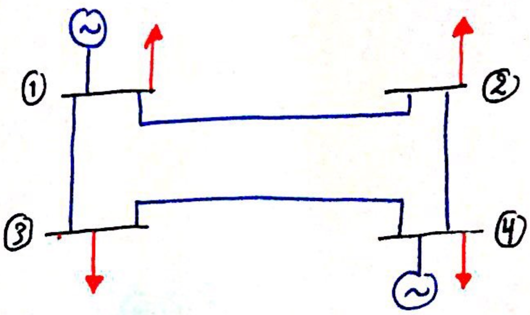
• Note: V_1 is fixed $V_1 = 1\angle 0$.

$$\left.\begin{aligned}
V_2^{(1)} &= \frac{1}{Y_{22}}\left[\frac{P_{2,sch} - jQ_{2,sch}}{V_2^{(0)*}} - \left(Y_{21}V_1 + Y_{23}V_3^{(0)} + Y_{24}V_4^{(0)}\right)\right] \\
V_3^{(1)} &= \frac{1}{Y_{33}}\left[\frac{P_{3,sch} - jQ_{3,sch}}{V_3^{(0)*}} - \left(Y_{31}V_1 + Y_{32}V_2^{(1)} + Y_{34}V_4^{(0)}\right)\right] \\
V_4^{(1)} &= \frac{1}{Y_{44}}\left[\frac{P_{4,sch} - jQ_{4,sch}}{V_4^{(0)*}} - \left(Y_{41}V_1 + Y_{42}V_2^{(1)} + Y_{43}V_3^{(1)}\right)\right]
\end{aligned}\right\} \text{1st iteration is Completed.}$$

⇒ Check: $|V_j^{(1)} - V_j^{(0)}| < \epsilon$ for all $j = 2, 3, 4$ → if yes solution is obtained.
→ if NOT go to 2nd iteration.

Example 9.2 page 337 :

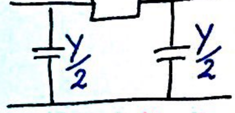
system has : 4-buses, 2 gen., 4 Lines.
with the following PU data for the line and buses by using Base value of 100 MVA, 230 KV.



• Objective : Use G.S method to evaluate voltages @ Busbars.

Line :

$Z = R + jX \Rightarrow Y_s$.



Line DATA:

Bus-Bus	serie Z R_{pu}	X_{pu}	$Y_s = \frac{1}{Z}$ G_{pu}	B_{pu}	Total charging MVAR	$Y/2$
1 - 2	0.01008	0.05040	3.815629	-19.078144	10.25	0.05125
1 - 3	0.00744	0.03720	5.169561	-25.847809	7.75	0.03875
2 - 4	0.00744	0.03720	5.169561	-25.847809	7.75	0.03875
3 - 4	0.01272	0.06360	3.023705	-15.128528	12.75	0.06375

Note : G, B, $Y/2$ found from the Calculations.

• Calculations :

$$Y_{12} = \frac{1}{Z_{12}} = \frac{1}{0.01008 + j0.0504}$$

$$\Rightarrow Y_{12} = 3.815629 - j19.078144$$

Total VAR $= 3 V_\phi I_\phi$

$= 3 V_\phi (V_\phi \omega C)$

$= 3 V_\phi^2 Y$

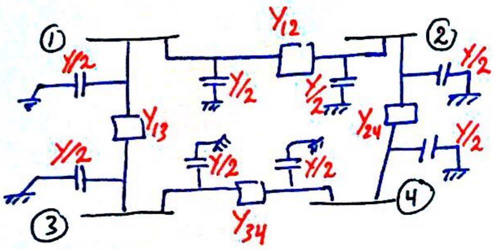
$$Y = \frac{VAR}{3 V_\phi^2} = \frac{10.25 * 10^6}{3\left(\frac{230}{\sqrt{3}}\right)^2 * 10^6}$$

$Y_{pu} = \frac{Y}{Y_b} \longrightarrow Y_b = \frac{1}{Z_b}$

$\Rightarrow Z_b = \frac{(230)^2}{100}$

$\Rightarrow Y_{pu} = 0.1025$

so $\frac{Y}{2} = 0.05125$



• from the given & calculated values for the lines one can evaluate $[Y]_{Bus}$.

Bus DATA: Bus 1 is slack. ; P_G & Q_G will be calculated at the end.

	Bus	Generation P_G	Q_G	MV Load P	Q	V
slack.	1	—	—	50	30.99	$1\angle 0°$
PQ or load	2	0	0	170	105.35	$1\angle 0°$
PQ or load	3	0	0	200	123.94	$1\angle 0°$
PV.	4	318	—	80	49.58	$1.02\angle 0°$

Q → Calculated from : $Q = P \tan[\cos^{-1} 0.85]$

Bus 1 V → specified value.

Bus 2, 3 V → Initial assumed values for the unknowns.

Bus 4 V → voltage "given" Control.

• Procedure as follows :

start → Input DATA → start with PQ-buses To calculate $V = |V| \angle \delta$ use acceleration factor → start with PV buses for each bus: first Calculate Q → check that: $Q_{min} \leq Q \leq Q_{max}$.

If Yes → Calculate V

If No → set Q at the violated Limit: e.g. if $Q = Q_{max}$ ∴ The busbar is converted (PV) ⟶ (PQ).

⇒ for the Example:

- start with Bus 2:

$$V_2^{(1)} = \frac{1}{Y_{22}}\left[\frac{P_{2,sch} - jQ_{2,sch}}{V_2^{(0)*}} - \left(Y_{21}V_1 + Y_{23}V_3^{(0)} + Y_{24}V_4^{(0)}\right)\right]$$

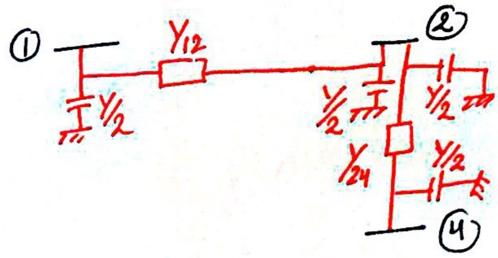
- from the Line DATA: (There are Two Lines) for Y_{22}.

Line 1-2: $Y_{12} = 3.815629 - j19.078144$ ... ①

$\frac{Y}{2} = j0.05125$ ... ②

Line 2-4: $Y_{24} = 5.169561 - j25.847809$ ... ③

$\frac{Y}{2} = j0.03875$ ... ④



Y_{22} = ① + ② + ③ + ④ ⇒ $\boxed{Y_{22} = 8.98519 - j44.835953}$ → See the Table in the Book.

$$P_2 - jQ_2 = (P_{g_2} - P_{L_2}) - j(Q_{g_2} - Q_{L_2}) = \frac{(0-170)}{100} - j\frac{(0-105.35)}{100}$$

⇒ $\boxed{P_2 - jQ_2 = -1.7 + j1.0535}$

∴ By Substitution: $V_2^{(1)} = 0.983 - j0.032$

* Acceleration Factor (α): $1 < \alpha < 2$, used in order to reduce the number of iterations, usually they use $\alpha = 1.6$.

$$V_{2,acc.}^{(1)} = V_2^{(0)} + \alpha\left(V_2^{(1)} - V_2^{(0)}\right)$$

∴ By Substitution: $V_{2,acc}^{(1)} = 0.973 - j0.0517$.

*** Repeat the same procedure for Bus 3.
it Can be found that: $V_{3,acc}^{(1)} = 0.953 - j0.066$

- Note: In iteration use V_{acc}.

- Bus 4: (PV bus).

⇒ first Calculate Q_4.

⇒ In general: $S_i = V_i I_i^* \Rightarrow S_i^* = V_i^* I_i = P_{G_i} - jQ_{G_i} = \left[V_i^* \sum_{j=1}^{n} V_j Y_{ij}\right]$

$$Q_i^{(K)} = -Im\left[V_i^{(K-1)*}\left(\sum_{j=1}^{i-1} Y_{ij}V_j^{(K)} + \sum_{j=i}^{N} Y_{ij}V_j^{(K-1)}\right)\right]$$

imaginary.

⇒ for Bus 4: $Q_4^{(1)} = -I_m\left[V_4^{(0)*}\left[Y_{42}V_{2,acc}^{(1)} + Y_{43}V_{3,ac}^{(1)} + Y_{44}V_4^{(0)}\right]\right]$

∴ By Substitution: $\boxed{Q_4^{(1)} = 1.654151}$

- in this example: it is assumed that the calculated Q_4 is within the limit
⇒ so Calculate $V_4^{(1)}$.

$$V_4^{(1)} = \frac{1}{Y_{44}}\left[\frac{P_{4,sch} - jQ_4^{(1)}}{V_4^{(0)*}} - (Y_{42}V_{2,ac}^{(1)} + Y_{43}V_{3,acc}^{(1)})\right] = \underline{1.017874 - j0.010604}$$

$|V_4^{(1)}| = 1.017929$ $\quad \therefore V_{4,corr.}^{(1)} = \frac{1.02}{1.017929}\underbrace{(1.017874 - j0.010604)}$ → from this δ_4 Can be Calculated.

Corrected Value.

$$\therefore \underline{V_4^{(1)} = 1.02\,\angle\delta_4^{(1)}}$$

@ the end of 1st iteration, check for each busbar 2,3,4:

$|V_i^{(1)} - V_i^{(0)}| \leqslant \epsilon$ → specified Tolerance is satisfied → if NOT Go to Next iteration.

- Among the input data: ϵ and max. No. of iterations.

※ Newton-Raphson Method:

This is based on the Taylor's Expansion for 2 functions with 2 or more variables. It is illustrated as follows:

$h_1(x_1, x_2, u) = b_1 \quad \cdots ①$ $\qquad h_1, h_2 \equiv$ 2 functions. $\quad x_1, x_2 \equiv$ UnKnown variables.

$h_2(x_1, x_2, u) = b_2 \quad \cdots ②$ $\qquad b_1, b_2 \equiv$ Constants represent specified values of h_1 & h_2.

$u \equiv$ Called Control variable, and to be taken as constant Later on.

*Hence, for initial assumed values for x_1 & x_2, h_1, h_2 represent Calculated values.

∴ Difference between Calculated & specified gives mismatch. Consequantly the following 2 functions are interduced:

$g_1(x_1, x_2, u) = h_1(x_1, x_2, u) - b_1 \quad \cdots ③$

$g_2(x_1, x_2, u) = h_2(x_1, x_2, u) - b_2 \quad \cdots ④$

∴ one assumes initial solutions: $x_1^{(0)}$ & $x_2^{(0)}$.

- Objective: is to find the amount of corrections $\Delta x_1^{(0)}$ & $\Delta x_2^{(0)}$ to the assumed solution, in order to obtain the actual or required answers x_1^* & x_2^*. → stand for the correct answer.

$$\therefore g_1(x_1^*, x_2^*, u) = g_1(x_1^{(0)} + \Delta x_1^{(0)}, x_2^{(0)} + \Delta x_2^{(0)}, u) = 0 \quad \cdots ⑤$$

$$g_2(x_1^*, x_2^*, u) = g_2(x_1^{(0)} + \Delta x_1^{(0)}, x_2^{(0)} + \Delta x_2^{(0)}, u) = 0 \quad \cdots ⑥$$

⇒ Now Taylor's Expansion is applied to ⑤ & ⑥ as follows:

$$g_1(x_1^*, x_2^*, u) = g_1(x_1^{(0)}, x_2^{(0)}, u) + \Delta x_1^{(0)} \left.\frac{dg_1}{dx_1}\right|^{(0)} + \Delta x_2^{(0)} \left.\frac{dg_1}{dx_2}\right|^{(0)} + \cdots = 0 \quad \cdots ⑦$$

$$g_2(x_1^*, x_2^*, u) = g_2(x_1^{(0)}, x_2^{(0)}, u) + \Delta x_1^{(0)} \left.\frac{dg_2}{dx_1}\right|^{(0)} + \Delta x_2^{(0)} \left.\frac{dg_2}{dx_2}\right|^{(0)} + \cdots = 0 \quad \cdots ⑧$$

- Partial Derivatives are evaluated at initial values.
- Higher Derivatives are neglected.

$g_1(x_1, x_2, u) = h_1(x_1, x_2, u) - b_1) \quad \cdots$ ⓐ

$\Rightarrow \left.\frac{dg_1}{dx_1}\right|^{(0)}$ 1) Diff. ⓐ w.r.t x_1. 2) Substitute initial value of x_1 & x_2 to get $\left.\frac{dg_1}{dx_1}\right|^{(0)}$.

• Re-write ⑦ & ⑧ in a matrix form:

$$\begin{bmatrix} \frac{dg_1}{dx_1} & \frac{dg_1}{dx_2} \\ \frac{dg_2}{dx_1} & \frac{dg_2}{dx_2} \end{bmatrix}^{(0)} \begin{bmatrix} \Delta x_1^{(0)} \\ \Delta x_2^{(0)} \end{bmatrix} = \begin{bmatrix} -g_1(x_1^{(0)}, x_2^{(0)}, u) = b_1 - h_1(x_1^{(0)}, x_2^{(0)}, u) = \Delta g_1 \\ -g_2(x_1^{(0)}, x_2^{(0)}, u) = b_2 - h_2(x_1^{(0)}, x_2^{(0)}, u) = \Delta g_2 \end{bmatrix} \quad \cdots ⑨$$

This matrix Called "Jacobian Matrix, (J)"

from ⑨: $J^{(0)} \begin{bmatrix} \Delta x_1^{(0)} \\ \Delta x_2^{(0)} \end{bmatrix} = \begin{bmatrix} \Delta g_1 \\ \Delta g_2 \end{bmatrix}$ $\therefore$ $\begin{bmatrix} \Delta x_1^{(0)} \\ \Delta x_2^{(0)} \end{bmatrix} = \left[J^{(0)}\right]^{-1} \begin{bmatrix} \Delta g_1 \\ \Delta g_2 \end{bmatrix} \quad \cdots ⑩$

• Since the RHS of ⑩ is Calculated, then the corrections $\begin{bmatrix} \Delta x_1^{(0)} \\ \Delta x_2^{(0)} \end{bmatrix}$ can be evaluated.

$\Rightarrow$ CHECK: if Corrections $< \epsilon$ (certain specified tolerance, e.g 10^{-4}).

- if $\boxed{\text{Yes}}$, solution is obtained. $\boxed{x_1^{(1)} = x_1^{(0)} + \Delta x_1^{(0)} \quad x_2^{(1)} = x_2^{(0)} + \Delta x_2^{(0)}}$
- if $\boxed{\text{NO}}$, Calculate New Corrections $\begin{bmatrix} \Delta x_1^{(1)} \\ \Delta x_2^{(1)} \end{bmatrix} = \left[J^{(1)}\right]^{-1} \begin{bmatrix} \Delta g_1 \\ \Delta g_2 \end{bmatrix} \quad \cdots ⑪$

* To Illustrate the application of this procedure:

see example 9.4 (page 344).

- Let $g_1(x_1, x_2, u) = 4u x_2 \sin x_1 + 0.6 = 0$ $\overset{\text{so}}{\Rightarrow}$ $h_1(x_1, x_2, u) = 4u x_2 \sin x_1$ & $b_1 = 0.6$

  $g_2(x_1, x_2, u) = 4x_2^2 - 4u x_2 \cos x_1 + 0.3 = 0$ $\Rightarrow h_2(x_1, x_2, u) = 4x_2^2 - 4u x_2 \cos x_1$ & $b_2 = -0.3$
- let in this e.g: $u = 1$, $x_1^{(0)} = 0$, $x_2^{(0)} = 1$, $\epsilon = 10^{-5}$

$$J^{(0)} = \begin{bmatrix} \frac{dg_1}{dx_1} & \frac{dg_1}{dx_2} \\ \frac{dg_2}{dx_1} & \frac{dg_2}{dx_2} \end{bmatrix} = \begin{bmatrix} 4x_2 \cos x_1 & 4 \sin x_1 \\ 4x_2 \sin x_1 & 8x_2 - 4\cos x_1 \end{bmatrix}$$

substitute $x_1^{(0)} = 0$, $x_2^{(0)} = 1$

$\Rightarrow J^{(0)} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$

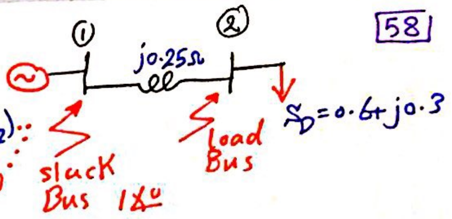
$\therefore \begin{bmatrix} \Delta x_1^{(0)} \\ \Delta x_2^{(0)} \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}^{-1} \begin{bmatrix} -0.6 \\ -0.3 \end{bmatrix} = \begin{bmatrix} -0.15 \text{ rad} \\ -0.075 \end{bmatrix}$ $\Rightarrow$ check: Correction $\overset{?}{<} \epsilon$

[NO] $\Rightarrow$ so Re-iterate.

• Note: The given equations represent:
The Power flow Equations of the system shown in Fig 9.3. (see the Book).

* solved e.g. represent the load-flow problem of a 2-bus system as follows:
(Fig. 9.3) $\Longrightarrow$

$$P_{2,cal} = \sum_{j=1}^{2} |V_2||V_j||Y_{2j}| \cos(\delta_2 - \delta_j - \theta_{2j}) \cdots ⓪$$



$$= |V_2||V_1||Y_{21}|\cos(\delta_2 - \delta_1 - \theta_{21}) + |V_2||V_2||Y_{22}|\cos(\delta_2 - \delta_2 - \theta_{22})$$

$$Y_{21} = \frac{-1}{j0.25} = 4\angle 90° \cdots ② \qquad Y_{22} = \frac{1}{j0.25} = 4\angle -90° \cdots ③$$

• By sub ② & ③ into ①: $P_{2,cal} = 4|V_1||V_2|\sin\delta_2 \cdots ④$

$$\Rightarrow g_1 = P_{2,cal} - P_{2,sch} = P_{2,cal} - (P_g - P_L) = P_{2,cal} - (0 - 0.6) = 4|V_1||V_2|\sin\delta_2 + 0.6$$

$$g_1(x_1, x_2, u) = 4|V_1||V_2|\sin\delta_2 + 0.6$$

Do the same thing By replacing $P_{2,cal}$ By $Q_{2,cal}$ & cos By sin in equ. ⓪.

∴ By Substituting: $Q_{2,cal} = 4|V_2|^2 - 4|V_2||V_1|\cos\delta_2$

$$\therefore g_2 = Q_{2,cal} - Q_{2,sch} = Q_{2,cal} - (0 - 0.3)$$

$$g_2(x_1, x_2, u) = 4|V_2|^2 - 4|V_1||V_2|\cos\delta_2 + 0.3$$

⇒ The Equations of g_1 & g_2 represent the power-flow equations with the unknowns: $x_2 = |V_2|$ & $x_1 = \delta_2$

* N.R Power Flow Solution:

Start

Assume initial solutions for the unknowns $|V_i|$ & δ_i

Evaluate $P_{i,cal}$ for $i = 2, \dots, N$
$Q_{i,cal}$ for $i = 2, \dots, (N - N_g)$

Evaluate Power Mismatch
$\Delta P_i = P_{i,sch} - P_{i,cal}$
$\Delta Q_i = Q_{i,sch} - Q_{i,cal}$

Evaluate the elements of Jacobian Matrix.

Evaluate for Corrections ΔV_i & $\Delta \delta_i$

Check that: $|\Delta V_i|$ & $|\Delta \delta_i| < \epsilon$

If NO

Go to Next iteration.
$V_i^{(K+1)} = V_i^{(K)} + \Delta V_i^{(K)}$
$\delta_i^{(K+1)} = \delta_i^{(K)} + \Delta \delta_i^{(K)}$

• As found before:

$$P_{i,cal} = \sum_{j=1}^{N} |V_i||V_j||Y_{ij}|\cos(\delta_i - \delta_j - \theta_{ij}) \cdots ① \qquad \Delta P_i = P_{i,sch} - P_{i,cal} \cdots ③$$

$$Q_{i,cal} = \sum_{j=1}^{N} |V_i||V_j||Y_{ij}|\sin(\delta_i - \delta_j - \theta_{ij}) \cdots ② \qquad \Delta Q_i = Q_{i,sch} - Q_{i,cal} \cdots ④$$

• To Formulate Jacobian Matrix:

Assume that there are 4-Buses where #1 is the "slack" & the other 3 buses are "Load buses" or "PQ-buses."

$$\begin{array}{c} \begin{array}{cc} \quad J_{11} \quad & \quad J_{12} \end{array} \\ \left[\begin{array}{ccc|ccc} \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & \frac{\partial P_2}{\partial \delta_4} & \frac{\partial P_2}{\partial |V_2|} & \frac{\partial P_2}{\partial |V_3|} & \frac{\partial P_2}{\partial |V_4|} \\ \frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} & \frac{\partial P_3}{\partial \delta_4} & \frac{\partial P_3}{\partial |V_2|} & \frac{\partial P_3}{\partial |V_3|} & \frac{\partial P_3}{\partial |V_4|} \\ \frac{\partial P_4}{\partial \delta_2} & \frac{\partial P_4}{\partial \delta_3} & \frac{\partial P_4}{\partial \delta_4} & \frac{\partial P_4}{\partial |V_2|} & \frac{\partial P_4}{\partial |V_3|} & \frac{\partial P_4}{\partial |V_4|} \\ \hline \frac{\partial Q_2}{\partial \delta_2} & \frac{\partial Q_2}{\partial \delta_3} & \frac{\partial Q_2}{\partial \delta_4} & \frac{\partial Q_2}{\partial |V_2|} & \frac{\partial Q_2}{\partial |V_3|} & \frac{\partial Q_2}{\partial |V_4|} \\ \frac{\partial Q_3}{\partial \delta_2} & \frac{\partial Q_3}{\partial \delta_3} & \frac{\partial Q_3}{\partial \delta_4} & \frac{\partial Q_3}{\partial |V_2|} & \frac{\partial Q_3}{\partial |V_3|} & \frac{\partial Q_3}{\partial |V_4|} \\ \frac{\partial Q_4}{\partial \delta_2} & \frac{\partial Q_4}{\partial \delta_3} & \frac{\partial Q_4}{\partial \delta_4} & \frac{\partial Q_4}{\partial |V_2|} & \frac{\partial Q_4}{\partial |V_3|} & \frac{\partial Q_4}{\partial |V_4|} \end{array}\right] \\ \begin{array}{cc} \quad J_{21} \quad & \quad J_{22} \end{array} \end{array} \begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta\delta_4 \\ \Delta|V_2| \\ \Delta|V_3| \\ \Delta|V_4| \end{bmatrix} = \begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta P_4 \\ \Delta Q_2 \\ \Delta Q_3 \\ \Delta Q_4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \Delta\delta_2 \\ \vdots \\ \Delta|V_4| \end{bmatrix} = [J]^{-1} \begin{bmatrix} \Delta P_2 \\ \vdots \\ \Delta Q_4 \end{bmatrix}$$

• *Note*: Elements of the Jacobian can be found from 1 & 2 as will be shown later on.

• For simplification: each element say $\frac{\partial P_2}{\partial |V_2|} * \frac{|V_2|}{|V_2|}$, $\frac{\partial P_3}{\partial |V_3|} * \frac{|V_3|}{|V_3|}$, ... and so on.

$$V_i^{(K+1)} = V_i^{(K)} + \Delta V_i^{(K)} = V_i^{(K)}\left(1 + \frac{\Delta V_i^{(K)}}{V_i^{(K)}}\right)$$

↳ for J_{12} & J_{22}.

• The following can be found:

J_{11}: $$\frac{\partial P_i}{\partial \delta_j} = -|V_i||V_j||Y_{ij}|\sin(\theta_{ij}+\delta_j-\delta_i) \quad \text{"Non diagonal elements".} \quad \ldots (9.52)$$

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{\substack{n=1 \\ n\neq i}}^{N} |V_i||V_n||Y_{in}|\sin(\theta_{in}+\delta_n-\delta_i) \quad \text{"Diagonal elements".} \quad \ldots (9.53)$$

J_{21}: $$\frac{\partial Q_i}{\partial \delta_j} = -|V_i||V_j||Y_{ij}|\cos(\theta_{ij}+\delta_j-\delta_i) \quad \text{"Non diagonal elements".} \quad \ldots (9.55)$$

$$\frac{\partial Q_i}{\partial \delta_i} = \sum_{\substack{n=1 \\ n\neq i}}^{N} |V_i||V_n||Y_{in}|\cos(\theta_{in}+\delta_n-\delta_i) \quad \text{"Diagonal elements".} \quad \ldots (9.56)$$

J_{12}: $$|V_j|\frac{\partial P_i}{\partial |V_j|} = |V_j||V_i||Y_{ij}|\cos(\theta_{ij}+\delta_j-\delta_i) \quad \text{"Non diagonal elements".} \quad \ldots (9.58)$$

$$|V_i|\frac{\partial P_i}{\partial |V_i|} = P_i + |V_i|^2 G_{ii} \quad \text{"Diagonal elements".} \quad \ldots (9.61)$$

where $Y_{ii} = G_{ii} + jB_{ii}$

J_{22}: $$|V_j|\frac{\partial Q_i}{\partial |V_j|} = -|V_j||V_i||Y_{ij}|\sin(\theta_{ij}+\delta_j-\delta_i) \quad \text{"Non diagonal elements".} \quad \ldots (9.62)$$

$$|V_i|\frac{\partial Q_i}{\partial |V_i|} = Q_i - |V_i|^2 B_{ii} \quad \text{"Diagonal elements".} \quad \ldots (9.63)$$

• **Note**: in the case of PV-Bus, Then: there is NO correction for the voltage magnitude (i.e $\Delta V = 0$). also Q is NOT Known. Hence ΔQ can NOT be calculated.

⇒ for example: in the previous system if Bus #1 slack, Bus #2 & 3 Load buses, Bus #4 PV.

∴ In this case: $\Delta V_4 = 0$, and ΔQ_4 can not be calculated.
therefore, in this case Eliminate 6^{th} row & 6^{th} column.

∴ Matrix (6x6) becomes (5x5).

∴ The order of J matrix $2N - N_g - 2$ in this e.g: $2*4-1-2 = \underline{5}$

Example: Solve the previous system (e.g 9.2) By using N.R Method.

• Find order of J?
$2N - N_g - 2 = 8 - 1 - 2 = 5$ ∴ Jacobian ≡ 5×5.

• Express the Jacobian Matrix?

$$\begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \cdots & \frac{\partial P_2}{\partial \delta_4} & \cdots & \frac{\partial P_2}{\partial |V_3|}|V_3| \\ \vdots & & & & \\ \frac{\partial P_4}{\partial \delta_2} & \cdots & \cdots & \cdots & \frac{\partial P_4}{\partial |V_3|}|V_3| \\ \vdots & & & & \\ \frac{\partial Q_3}{\partial \delta_2} & \cdots & \cdots & \cdots & \frac{\partial Q_3}{\partial |V_3|}|V_3| \end{bmatrix} \begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_3 \\ \Delta \delta_4 \\ \frac{\Delta |V_2|}{|V_2|} \\ \frac{\Delta |V_3|}{|V_3|} \end{bmatrix} = \begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta P_4 \\ \Delta Q_2 \\ \Delta Q_3 \end{bmatrix}$$

• Initial voltages or state variables: specified: $V_1^{(0)} = 1\angle 0$, $V_4^{(0)} = 1.02\angle 0$ assumed: $V_2^{(0)} = 1\angle 0$, $V_3^{(0)} = 1\angle 0$

These called Flat start.

• Find initial mismatch at Bus 3?
⇒ $\Delta P_3^{(0)}$ & $\Delta Q_3^{(0)}$.

⇒ $\Delta P_3 = P_{3,sch} - P_{3,cal}$; $P_{3,sch} = P_{3,gen.} - P_{3,load} = 0 - 200 = -200\,MW = \underline{-2\,PU}$.

$P_{3,cal}^{(0)} = \sum_{j=1}^{N} |V_3||V_j||Y_{3j}|\cos(\delta_3 - \delta_j - \theta_{3j})$ By using the given $[Y_{bus}]$ matrix ⇒ find $Y_{31}, Y_{32}, Y_{33}, Y_{34}$

and substituting initial & specified voltages: $P_{3,cal}^{(0)} = \underline{-0.0604731\,PU}$.

∴ $\Delta P_3^{(0)} = -2 + 0.0604731$ ⇒ $\boxed{\Delta P_3^{(0)} = -1.94}$; initial mismatch.

⇒ $\Delta Q_3 = Q_{3,sch} - Q_{3,cal}$; $Q_{3,sch} = Q_{3,gen.} - Q_{3,load} = 0 - 123.94 = \underline{-1.2394\,PU}$.

$Q_{3,cal}^{(0)} = \sum_{j=1}^{N} |V_3||V_j||Y_{3j}|\sin(\delta_3 - \delta_j - \theta_{3j})$ ⇒ $\underline{Q_{3,cal}^{(0)} = -0.4048696\,PU}$.

∴ $\boxed{\Delta Q_3^{(0)} = -0.8345304 \simeq -0.835}$

⇒ By similar approach one can find $\Delta P_4, \Delta P_2$ & ΔQ_2.

Continue ......

• Find the element of the Jacobian ?

* for e.g.: Initial value of the element in 2^{nd} row & 3^{rd} column.

∴ The required element is: $\frac{\partial P_3}{\partial \delta_4}$ which is Non Diagonal element.

$$\Rightarrow \frac{\partial P_3}{\partial \delta_4} \triangleq -|V_3||V_4||Y_{34}|\sin(\theta_{34}+\delta_4-\delta_3) \xrightarrow{\text{By sub.}} \boxed{\frac{\partial P_3}{\partial \delta_4} = -15.42}$$

* Initial value of the element in 5^{th} row & 5^{th} column.

$$\therefore \text{element} = |V_3|\frac{\partial Q_3}{\partial |V_3|} \triangleq \underbrace{Q_3}_{\text{transmitted "calculated"}} - |V_3|^2 \underbrace{B_{33}}_{Y_{bus}\text{ matrix.}} \Longrightarrow \boxed{|V_3|\frac{\partial Q_3}{\partial |V_3|} = 40.459}$$

⇒ The same procedure can be repeated to find all elements of [J].

∴ Now one can proceed to calculate voltages:

⇒ Calculate corrections, repeat iterations

Until ΔP & $\Delta Q < \epsilon$.

OR $|V_i^K - V_i^{K-1}| < \epsilon$.

$$\begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta\delta_4 \\ \frac{\Delta V_2}{|V_2|} \\ \frac{\Delta V_3}{|V_3|} \end{bmatrix} = [J]^{-1} \begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta P_4 \\ \Delta Q_2 \\ \Delta Q_3 \end{bmatrix}$$

** Elements of Jacobian Matrix are shown @ the top of Page 355.

• The results of this analysis are shown in Figures 9.4 ⇒ Table.
9.5 ⇒ Results on single-Line diagram.

Fig. 9.4

Bus No.	|V|	$\delta^{(deg)}$	Gen P	Gen Q	Load P	Load Q
1	1	0	186.81	114.5	50	30.99
2	0.982	-0.976	0	0	170	105.35
3	0.969	-1.872	0	0	200	123.94
4	1.02	1.523	318	181.43	80	49.58

These calculated voltages can be used to find Any unknowns.

Load Q → found from $Q = P\tan[\cos^{-1} pf]$

also P_{G_1}, Q_{G_1}, Q_{G_4} found as shown in the coming analysis.

• Comment: Having Calculated the voltages at busbars, then one can:

1] Find $S_G = P_G + jQ_G$ @ the slack bus.

2] Find Q_G at PV bus.

3] Line flow.

4] Any Violated Variable.

1] * Slack Bus: $S_G = \underbrace{S_i}_{\text{power transmitted or calculated @ the Busbar.}} + \underbrace{S_D}_{\text{Demand or Load at the busbar.}}$

Continue...

$S_1 = P_{1,cal} + jQ_{1,cal}$.

By using $P_i = \sum_{j=1}^{N} |V_i||V_j||Y_{ij}| \cos(\delta_i - \delta_j - \theta_{ij})$, where $i = 1$ in case of slack Bus.

By using the results in Fig 9.4 $\Rightarrow V_1 = 1\angle 0$, $V_2 = 0.982\angle -0.976$, $V_3 = 0.969\angle -1.872$, $V_4 = 1.02\angle 1.523$ and in Table 9.2 of $[Y_{bus}]$ matrix.

P_D was given: $P_D = 50MW$.

$\Rightarrow P_1 = 1.36\ PU = \underline{136\,MW}$. $\Rightarrow P_G = 136 + 50 = \underline{186\,MW}$.

- Similarly one can calculate Q_1, it can be found: $\underline{Q_1 = 83.51\,MVAR}$.

$Q_D = 30.99\,MVAR$. $\therefore Q_G = Q_1 + Q_D = \underline{114.5\,MVAR}$

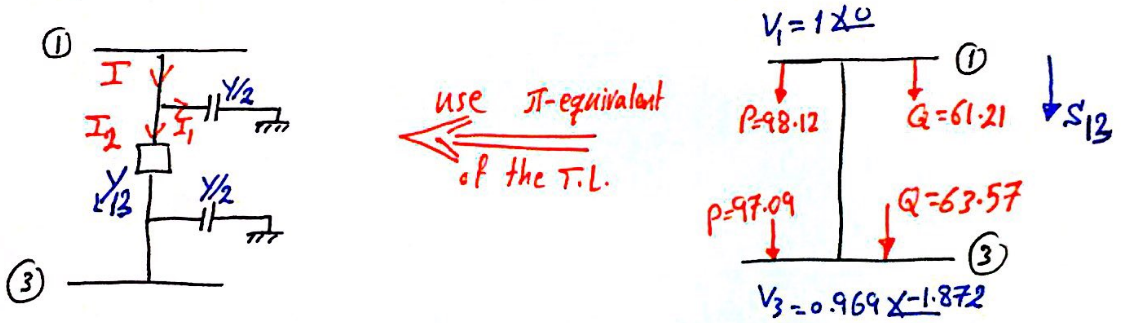
2) * PV Bus#4:

$Q_G = Q_4 + Q_D$ By Calculations: $Q_D = 49.58\,MVAR$, $Q_4 = 131.85\,MVAR$.

$\therefore Q_G = 181.43\ MVAR$.

3) * Line Flow:

Consider for e.g line between buses 1&3 as shown in Fig 9.5:



$\therefore I = I_1 + I_2 = V_1 \frac{Y}{2} + (V_1 - V_3) Y_{L13}$; $\frac{Y}{2}$ & Y_{L13} deduced from input DATA of the Line.

solving: $I = 1.165\angle -32.1$ PU. $Y_{L13} = \frac{1}{Z_{L13}}$

$\therefore S_{13} = V_1 I^* = 0.987 + j0.619$ PU $\Rightarrow S_{13} = 98.7 + j61.9$ MVA.

- By similar approach one can find S entering Bus#3.
- Having calculated power flow in the lines, one can calculate losses in the lines.

For e.g: for the Line (1-3) $S_{losses} = [98.12 + j61.21] - [97.09 + j63.57]$

$\Rightarrow S_{losses} = 1.03 - j2.36$ MVA.

- By summation over all Lines, one can find the Losses in the system.

Continue...

4] * Violated Conditions:

- The power flow study under the Normal or forcast Conditions is Called "Base Study".
- Check for any violations, here in the input DATA:
  - i) Specify Tolerence for voltage eg $\mp 5\%$ $\Rightarrow V_{accepted} = 1 \pm 0.05 = 0.95 \to 1.05$

    In our 4-bus system; There were No-Voltage Violations.
  - ii) $P_{min} \leq P_G \leq P_{max}$, In our example; Limits were NOT specified.

    $Q_{min} \leq Q_G \leq Q_{max}$
  - iii) Capacity of Lines, for e.g MVA.

*If there is any violation, perform "Sensitivity Analysis".
for example: 1) Adding Lines. 2) Modifying Loads.

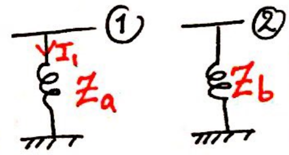
* * *

* Direct Determination of Z_{bus} Matrix: "see the Book Page 301/section 8.4."

1] Number the Busbars & Branches of the given Network.
- A Branch could be between a busbar & the Ref. or between 2 busbars.

2] Start by writing an equation for bus #1 which a branch Connected to Ref with impedance $\equiv Z_a$. $\therefore$ $[V_1] = [Z_a][I_1]$

This equation can be considered as 3 matrices, each one is of the order (1x1).



2.1 Now one may add a new bus connected to first bus or to the Ref.
for e.g: let the second bus connected to Ref. through impedance $\equiv Z_b$
Then one have the following matrix equations:

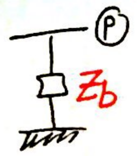
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{array}{c|cc} & ① & ② \\ \hline ① & Z_a & 0 \\ ② & 0 & Z_b \end{array} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

This matrix Called Original Matrix, To be modified By adding one Branch at a time.

3] One may proceed to modify the original matrix by adding other buses & branches following the procedures. [see 8.4/294] which may be:

3.1 Adding Z_b from a new bus (P) to the ref. Node as follows:

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_K \\ V_P \end{bmatrix} = \left[\begin{array}{c:c} Z_{orginal} & \text{zeros} \\ \hdashline \text{zeros} & Z_b \end{array}\right] \begin{bmatrix} I_1 \\ \vdots \\ I_K \\ I_P \end{bmatrix} \quad \ldots\ldots (3.1)$$



Continue...

3.2 Adding Z_b from a new bus (P) to an existing bus (K)

Then the matrix will be as follows:

$$\begin{bmatrix} V_1 \\ \vdots \\ V_K \\ \hdashline V_P \end{bmatrix} = \left[\begin{array}{c:c} Z_{original} & \begin{matrix} Z_{1K} \\ Z_{2K} \\ \vdots \\ Z_{KK} \end{matrix} \\ \hdashline Z_{K1} \cdots\cdots Z_{KK} & Z_{KK}+Z_b \end{array}\right] \begin{bmatrix} I_1 \\ \vdots \\ I_K \\ \hdashline I_P \end{bmatrix} \quad \cdots (3.2)$$

elements of the original matrix.

$Z_{KK} \equiv$ Element of the the original Matrix.

3.3 Adding Z_b from existing bus (K) to a Ref. Node:

a) Create a new row & a new column exactly as in 3.2.

b) Eliminate the $(N+1)$ row & the $(N+1)$ column, To find each element $Z_{hi}(new)$ in the new Matrix as follows:

row, column

$$Z_{hi}(new) = Z_{hi}(old) - \frac{Z_{h(N+1)} * Z_{(N+1)i}}{Z_{KK} + Z_b} \quad \cdots (3.3)$$

$(N+1)$ is the new row or column.

3.4 Adding Z_b between two existing buses (j) & (K):

• Formulate the following matrix:

$$\begin{bmatrix} V_1 \\ \vdots \\ V_P \end{bmatrix} = \left[\begin{array}{c:c} Z_{original} & col.j - col.K \\ \hdashline Row.j - Row.K & Z_{bb} \end{array}\right] \begin{bmatrix} I_1 \\ \vdots \\ I_P \end{bmatrix} \quad \cdots (3.4)$$

where: $Z_{bb} = Z_{jj} + Z_{KK} - 2 Z_{jK} + Z_b$

found from original matrix.

• Modify the produced matrix By using:

$$Z_{hi}(new) = Z_{hi}(old) - \frac{Z_{h(N+1)} * Z_{(N+1)i}}{Z_{bb}} \quad \cdots (3.5)$$

original.

Example: Find Z_{bus} for the given system shown in Fig.1 By using the Building Algorithm?

Note: Subscript is used for Z_{bus} at each step:

1) Starting with Bus 1 $\Rightarrow$ $Z_{bus1} = [j1.25]$.

Note: all elements have j so it will not be written in the following steps.

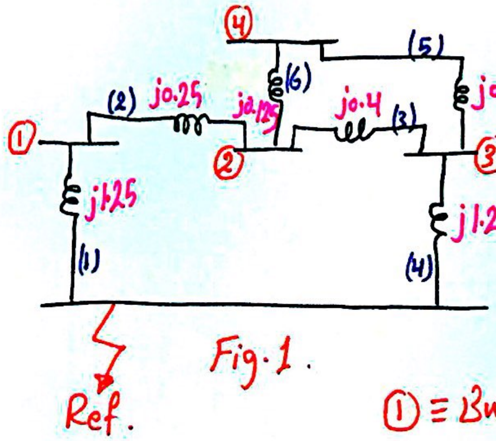


Fig. 1.

① ≡ Bus No.

(2) ≡ Branch No.

Continue ...

2) Go to bus ② with its impedance of j0.25.

$$\Rightarrow Z_{bus2} = \begin{bmatrix} 1.25 & 1.25 \\ 1.25 & 1.25+0.25=1.5 \end{bmatrix}$$

3) Bus③ with impedance j0.4 is connected to Bus②.

$$\Rightarrow Z_{bus3} = \begin{bmatrix} 1.25 & 1.25 & 1.25 \\ 1.25 & 1.5 & 1.5 \\ 1.25 & 1.5 & 1.5+0.4=1.9 \end{bmatrix} \rightarrow Z_{original\ 3\times3}.$$

4) If one decide to add impedance $Z_b = j1.25$ from Bus③ to Ref. to obtain the following matrix: Then one follow equation $\boxed{3.2}$ to connect a new bus Ⓟ

$$\begin{bmatrix} 1.25 & 1.25 & 1.25 & 1.25 \\ 1.25 & 1.5 & 1.5 & 1.5 \\ 1.25 & 1.5 & 1.9 & 1.9 \\ 1.25 & 1.5 & 1.9 & 3.15 \end{bmatrix} \rightarrow Z_{33}+Z_b = 1.9+1.25 = \underline{\underline{3.15}}$$

• The created new column & new row: By using equation $\boxed{3.3}$:

$$Z_{hi}(new) = Z_{hi}(old) - \frac{Z_{h(N+1)}\,Z_{(N+1)i}}{Z_{KK}+Z_b}$$

for e.g.: Z_{11} $h=1$, $i=1$ $\Rightarrow N=3, K=3$ $\therefore Z_{11}(new) = Z_{11}(old) - \dfrac{Z_{14}\,Z_{41}}{Z_{33}+Z_b}$

$$\Rightarrow Z_{11}(new) = 1.25 - \frac{(1.25)*(1.25)}{1.9+1.25} \Rightarrow \underline{Z_{11}(new) = 0.75397}$$

* The same procedure can be repeated for the elements $Z_{12}, Z_{13}, Z_{21}, Z_{22}, Z_{23}, Z_{31}, Z_{32}, Z_{33}$.

$\Rightarrow$ To Give:

$$\begin{bmatrix} 0.75397 & \cdots & 0.49603 \\ \vdots & & \vdots \\ 0.49603 & \cdots & 0.75397 \end{bmatrix}_{3\times3\ matrix.} \rightarrow Z_{original\ 3\times3}.$$

5) One may now add impedance $Z_b = j0.2$ from Bus③ to Bus④.

$$Z_{bus6} = \begin{bmatrix} \text{Original} & \begin{matrix} 0.49603 \\ 0.59524 \\ 0.75397 \end{matrix} \\ 0.49603 \ \ 0.59524 \ \ 0.75397 & 0.95397 \end{bmatrix} \rightarrow Z_{33}+Z_b = 0.75397 + 0.2 = \underline{\underline{0.95397}}.$$

Continue...

6). Finally one add impedance $Z_b = j0.125$ between ② & ④.

$$Z'_{bus,6} = \left[\begin{array}{c|c} Z_{bus,6} & \text{Col.2} - \text{Col 4 of } Z_{bus,6} \\ \hline \text{Row 2} - \text{Row 4 of } Z_{bus,6} & Z_{55} \end{array}\right]$$

; where: $Z_{55} = Z_{22} + Z_{44} - 2Z_{24} + Z_b$

$\Rightarrow$ $Z_{55} = 0.67421$

* Eliminate the 5th row & 5th column By using equation (3.5):

$$Z_{hi}(new) = Z_{hi}(old) - \frac{Z_{h(N+1)}\, Z_{(N+1)i}}{Z_{bb}} \quad ; \; Z_{bb} = Z_{55}.$$

for e.g: $Z_{11}(new) = 0.7166$

$\Rightarrow$ Finally the Z_{bus} will be as follows:

$$Z_{bus} = j\begin{bmatrix} 0.7166 & \cdots & 0.5805 \\ \vdots & & \vdots \\ 0.5805 & \cdots & 0.7631 \end{bmatrix}$$ 4x4 Matrix.

* * *

End of Material

* * *

Best * * *

* * of * *

* * * Luck.