13.5

Jordan University
Mathematics Department
Mathematics for Engineering (II) . Second Exam. 22/04/2013

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Lecture time: 9:30 – 11:00

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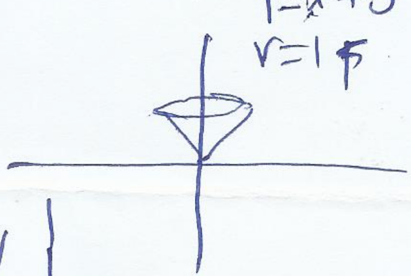
1. Use the Divergence Theorem to evaluate $\iint_S \vec{F}\cdot\vec{n}\,dA$, where
$\vec{F}(x,y,z) = (xz\sin(yz)+x^3)i + (\cos(yz))j + (3zy^2)k$ and
S is the surface of the cone $z=\sqrt{x^2+y^2}$, $0 \le z \le 1$. [8 points]

$$\iint_S \vec{F}\cdot n\,dA = \iiint_T \operatorname{Div}(F)\,dA$$

$$\operatorname{Div}(F) = \cancel{z\sin(yz)} + 3x^2 - \cancel{z\sin(yz)} + 3y^2$$

$1 = x^2+y^2$
$r = 1$

$$\iiint_T 3(x^2+y^2)\,dA$$



$$\int_0^{2\pi}\int_0\int_0^1 3\cdot r^3\,dr\,du\,du = \int_0^{2\pi}\int_0 \frac{3r^4}{4}\Big|_0 \,du\,du$$

$$= \int_0^{2\pi}\int_0 \frac{3}{4}\,d\theta = \int \frac{3}{4}\theta\Big|_0^{2\pi} = \cancel{\int 3\pi}$$

3.5

2. Use Stokes's theorem to evaluate $\iint_S (curl\vec{F})\cdot\vec{n}dA$, where S is the surface of the hemisphere $x^2+y^2+z^2=9$ with $z\geq 0$, oriented outward; $\vec{F}(x,y,z)=(2y\cos z)i+(e^x\sin z)j+(xe^y)k$ [7 points]

$\nabla \vec{F} = 0 + 0 + 0$

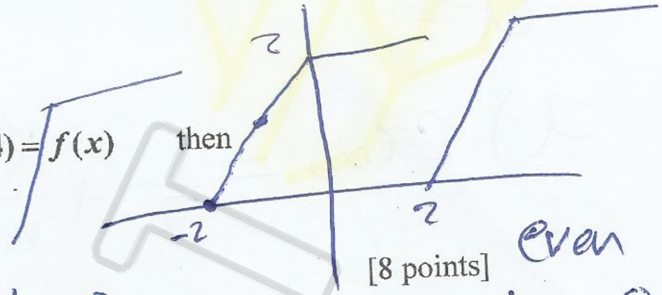
zero

3. If $f(x)=\begin{cases} x+2, & -2\le x<0, \\ 2, & 0\le x<2, \end{cases}$, $f(x+4)=f(x)$ then

a) Find the Fourier series for $f(x)$,

b) Find $\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}$.

[8 points]



even

$b_n=0$

$L=2$

$a_0=\frac{1}{2L}\int_{-L}^{L} f(x)\,dx$

$a_0=\frac{1}{4}\int_{-2}^{0} x+2\,dx+\frac{1}{4}\int_{0}^{2} 2dx$

$=\frac{1}{4}\left(\frac{x^2}{2}+2x\Big|_{-2}^{0}\right)+\frac{1}{4}\,2x\Big|_{0}^{2}=\frac{1}{4}(2-4)+\frac{1}{4}\times 4$

$=-\frac{1}{2}+1=\frac{1}{2}$

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$a_n=\frac{1}{2}\int_{-2}^{0}(x+2)\cos\frac{n\pi x}{2}\,dx+\int_{0}^{2}2\cos\frac{n\pi x}{2}\,dx$

$x+2 \Rightarrow \quad dx$

$\cos\frac{n\pi x}{2}\longrightarrow\frac{2}{n\pi}\sin\frac{n\pi x}{2}$

$=\frac{2(x+2)}{n\pi}\sin\frac{n\pi x}{2}-\int\frac{2}{n\pi}\sin\frac{n\pi x}{2}$

$+\frac{4}{n^2\pi^2}\cos\frac{n\pi x}{2}$

$a_n=\frac{1}{2}\left(\frac{2(x+2)}{n\pi}\sin\frac{n\pi x}{2}+\frac{4}{n^2\pi^2}\cos\frac{n\pi x}{2}\right)\Big|_{-2}^{0}+\frac{4}{n\pi}\sin\frac{n\pi x}{2}\Big|_{0}^{2}$

0 $\quad$ 0

$b_n \; 2$

$a_n=\frac{4}{n^2\pi^2}-\frac{4}{n^2\pi^2}\cos(n\pi)=\frac{4}{n^2\pi^2}\left(1-(-1)^n\right)\frac{1}{2}$ if n even $1-1=0$

$=\frac{8}{(2n-1)^2\pi^2}\;\frac{1}{2}$

$f(x)=\frac{1}{2}+\frac{8}{(2n-1)^2\pi^2}\frac{1}{2}\cos\frac{n\pi x}{2}$

$f(0)=\frac{1}{2}+\frac{8}{(2n-1)^2\pi^2}$

$f(0)=2$

$2=\frac{1}{2}+\frac{8}{(2n-1)^2\pi^2}$

$1.5=\frac{8}{(2n-1)^2\pi^2}\rightarrow\sum\frac{1}{(2n-1)^2}=1.\ldots$

4. Find the Fourier transform of $f(x)=e^{-c|x|}$, $-\infty<x<\infty,\ c>0$

[7 points]

$$= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\, e^{-jxw}\, dx \qquad f(x)=\begin{cases} e^{-cx} & x>0 \\ e^{cx} & x<0 \end{cases}$$

$$= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{0} e^{cx}\, e^{-ixw}\, dx = \frac{1}{\sqrt{2\pi}}\int e^{(cx+jxw)}\, dx$$

$$= \frac{1}{\sqrt{2\pi}} \frac{e^{(cx+jxw)}}{(c+jw)}\Bigg|_{-\infty}^{0} =$$

$$= \frac{1}{\sqrt{2\pi}\,(c+jw)}$$

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$$\frac{1}{\sqrt{2\pi}}\int_0^{\infty} e^{-cx}\, e^{jxw}\, dx = \frac{1}{\sqrt{2\pi}}\int_0^{\infty} e^{(jw-c)x}\, dx$$

$$= \frac{1}{\sqrt{2\pi}\,(jw-c)}\, e^{(jw-c)x}\Bigg|_0^{\infty} = \frac{-1}{\sqrt{2\pi}\,(jw-c)}$$

$$= \frac{1}{\sqrt{2\pi}\,(c+jw)} + \frac{1}{\sqrt{2\pi}\,(jw-c)}$$