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The University of Jordan
Mathematics Department

Eng. Mathematics (II)

First exam
17/03/2013

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Lecture time: 9:30 – 11:00

Q1) Let $f(x,y,z)=e^{yx}z$, $\vec{F}(x,y,z)=xyi+yzj+zxk$ and $\vec{r}=xi+yj+zk$ .State whether each expression is meaningful, then find:

a) $\nabla\cdot(f\nabla f)$;

b) $(\vec{F}\cdot\nabla)f$;

c) $\nabla f\times(\nabla\cdot f)$;

d) $div(\nabla\times\nabla f)$;

e) $\nabla(\vec{r}\cdot\vec{F})$.

(5 points)

$\nabla = \frac{\partial}{\partial x}i + \frac{\partial}{\partial y}j + \frac{\partial}{\partial z}k$

$\nabla f = ye^{yx}zi + xe^{yx}zj + e^{yx}$

a) $f\nabla f = z^2ye^{2yx}i + xz^2e^{2yx}j + ze^{2yx}$

$\nabla\cdot(f\nabla f) = 2z^2y^2e^{2yx} + 2x^2z^2e^{2yx} + e^{2yx}$

$1+0+1+1+0.5 = 3.5$

b) ~~$(\vec{F}\cdot\nabla)f = xy$~~

$(\vec{F}\cdot\nabla) = xy\frac{\partial f}{\partial x} + xz\frac{\partial f}{\partial y} + zx\frac{\partial f}{\partial z}$

$(\vec{F}\cdot\nabla)f = xy^2e^{yx} + xz^2e^{yx} + zxe^{yx}$

c) $(\nabla\cdot f) \rightarrow$ vector $\cdot$ scalar X

d) $\nabla\times\nabla f = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ ye^{yx}z & xze^{yx} & e^{yx} \end{vmatrix} = \left(\frac{\partial}{\partial y}e^{yx} - \frac{\partial}{\partial z}xze^{yx}\right)i - \left(\frac{\partial}{\partial x}e^{yx} - \frac{\partial}{\partial z}yze^{yx}\right)j + \left(\frac{\partial}{\partial x}xze^{yx} - \frac{\partial}{\partial y}yze^{yx}\right)k$

$=(xe^{yx} - xe^{yx})i - (ye^{yx} - ye^{yx})j + (ze^{yx} + xyze^{yx} - ze^{yx} - yzxe^{yx})k$

$div(\nabla\times\nabla f) = 0$

e) $\vec{r}\cdot\vec{F} = x^2y + y^2z + z^2x$

$\nabla(\vec{r}\cdot\vec{F}) = (2xy + 2zx)i + (x^2 + 2yz)j + (y^2 + 2xz)k$

Q2) Evaluate $\oint_C (y\sin xy - 3y)dx + (2x + x\sin xy)dy$, where C is the square with vertices (1,0), (0,1), (-1,0), (0,-1). (4 points)

$\frac{\partial f_2}{\partial x} = 2 + \sin xy + xy\cos xy$

$\frac{\partial f_1}{\partial y} = yx\cos xy + \sin xy - 3$

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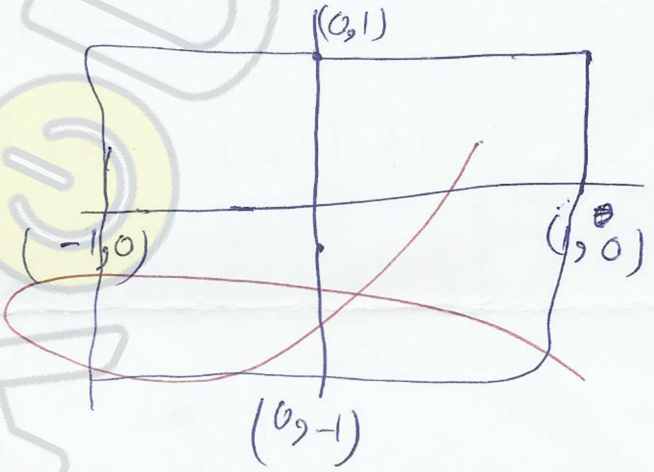
$\iint_R 2 + \sin xy + xy\cos xy - (yx\cos yx - \sin yx + 3)$

$= \int_{-1}^{1}\int_{-1}^{1} 5\, dx\, dy$

$= \int_{-1}^{1} 5x \Big|_{-1}^{1} dy$

$= \int_{-1}^{1} 10\, dy = 10y \Big|_{-1}^{1}$

$= 20$



Q3) Find the value of $\int_C \vec{F}\cdot dr$ from the point (0, 0, 0) to the point (1, 1, 1),

where C: $\vec{r}(t) = ti + t^2 j + t^3 k$

$$\vec{F}(x,y,z) = (2-e^z)i + (2y-1)j + (2-xe^z)k$$

(5 points)

$r'(t) = 1i + 2tj + 3t^2 k$

$$= \int_0^1 \vec{F}(r(t)) \cdot r'(t)\,dt \qquad F(r(t)) = (2-e^{t^3})i + (2t^2-1)j + (2-te^{t^3})k$$

$$= \int_0^1 (2-e^{t^3}) + 4t^3 - 2t + 6t^2 - 3t^3 e^{t^3}\,dt$$

$$= \int_0^1 -3t^3 e^{t^3} + \cancel{4} - e^{t^3} + 4t^3 + 6t^2 - 2t + 2)\,dt$$

$$= -\int_0^1 e^{t^3}(-3t^3+1)\,dt + (t^4 + 2t^3 - t^2 + 2t)\Big|_0^1$$

$= 1 + 2 - 1 + 2$

$= 4$

$u = 3t^3 + 1$

$dv\, e^{t^3} \rightarrow e^{-t^3}$

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Q4) Evaluate $\iint_S \vec{F}\cdot n\,dA$, where $\vec{F}(x,y,z) = y^2 i + x^2 j + 5z\,k$,

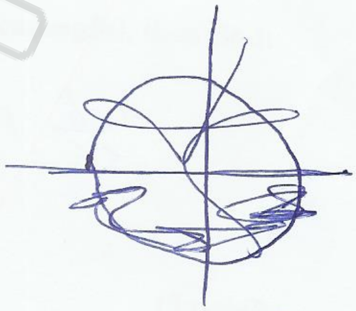
S is the boundary of the solid enclosed by the paraboloid $z = x^2 + y^2$ and the plane $z = 1$, oriented outward.

(6 points)

$\cancel{N = \nabla f} = x^2 + y^2 - z = 0$

$\nabla s = 2x\,i + 2y\,j - 1k = N$ ✓

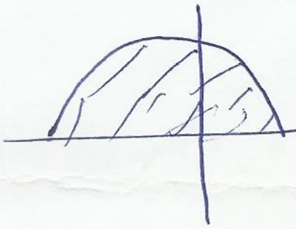
$\vec{F}(x,y,z) = y^2 i + x^2 j + (x^2+y^2)k$



$\cancel{x^2+y^2} = 1$

$= \iint \vec{F}\cdot n\,dA$

$= \iint_S 2xy^2 + 2yx^2 - (x^2+y^2)\,dA$ ✓



$x^2+y^2=1$

$\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}} 2xy^2 + 2yx^2 - (x^2+y^2)\,dy\,dx$

$\cancel{2\sin\cos + 2x\cos\ldots}$

$= \int_{-1}^{1} \frac{2xy^3}{3} + y^2x^2 - \frac{y^3}{3}\Big|_0^{\sqrt{1-x^2}} dx$

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$= \int_{-1}^{1} \frac{2}{3}x(\sqrt{1-x^2})^3 + (1-x^2)x^2 - \frac{(\sqrt{1-x^2})^3}{3}\,dx$

$1-x^2=u$

$-2x = \frac{du}{dx}$

$dx = \frac{du}{-2x}$

$+ x^2\sqrt{1-x^2}$

$= \frac{1}{3}\int -u^{\frac{3}{2}}du + \left(\frac{x^3}{3} - \frac{x^5}{5}\right)\Big|_{-1}^{1} + \int (1-u)(u^{\frac{1}{2}})\,du + \int_{-1}^{1}\frac{\sqrt{1-x^2}}{3}\,dx$

$= \left(\frac{1}{3} - u^{\frac{5}{2}}\times\frac{2}{5}\right) + \left(\frac{1}{3}-\frac{1}{5}\right) - \left(\frac{-1}{3}+\frac{1}{5}\right) + \frac{2u^{\frac{3}{2}}}{3} + u^{\frac{5}{2}} + \frac{2}{5}$

$u = 1-x^2$

$= \frac{1}{3}\left(-(1-x^2)^{\frac{5}{2}} \times \frac{2}{5} + \frac{2}{3} + \frac{2}{5} + \right.$