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Q1: (a) Find the basic Fourier series associated with $f(x) = x^2,\ x\epsilon[-1,1]$

(b) Show that $1+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+\cdots=\frac{\pi^2}{6}$

a) $f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi}{L}x + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi}{L}x$

$L=1$

$f(x)=x^2 \Rightarrow$ even

$a_0 = \frac{1}{1}\int_{-1}^{1} x^2 dx = \frac{1}{3}x^3\Big|_{-1}^{1} = \frac{1}{3}(1+1)$

$\boxed{a_0 = \frac{2}{3}}$

$b_n =$ zero $\Rightarrow \int_{-L}^{L}$ odd $=$ zero

$a_n = \frac{2}{1}\int_0^1 \underbrace{x^2}_{u}\underbrace{\cos n\pi x\, dx}_{dv}$

$u.v-\int v\, du$

$= 2\left[\frac{1}{\pi n}x^2\sin n\pi x\Big|_0^1 - \frac{1}{\pi n}\int_0^1 2x\sin n\pi x\, dx\right]$

$= 2\left[\frac{1}{\pi n}x^2\sin n\pi x\Big|_0^1 - \frac{2}{\pi n}\left[\frac{\cos n\pi x}{n\pi}\cdot x\Big|_0^1 + \frac{1}{\pi n}\int_0^1 \cos n\pi x\, dx\right]\right]$

$\cancel{= \frac{2}{n}\left[\sin n\pi - \frac{2}{n}\left[-\frac{\cos n\pi}{n} + \frac{1}{n^2}\int_0^1 \cos n\pi x\right]\right]}$ zero

$\cancel{= -\frac{4}{n^2}\left[\frac{(-1)^{n+1}}{n} + \frac{1}{n^3}\sin n\pi x\Big|_0^1\right]}$

$a_n = 2\left[\frac{1}{\pi n}\underbrace{\sin n\pi}_{zero} - \frac{2}{\pi n}\left[-\frac{\cos n\pi}{n\pi} + \frac{1}{(\pi n)^2}\underbrace{\sin n\pi x\Big|_0^1}_{zero}\right]\right]$

$a_n = -\frac{4}{\pi n}\left(\frac{(-1)^{n+1}}{n\pi}\right)$

$a_n = (-1)^n * \frac{4}{(n\pi)^2}$

$\cos n\pi = \begin{cases} 1 & , n=2m \\ -1 & , n=2m+1 \end{cases}$

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$$\boxed{f(x) \approx \frac{1}{3} + \sum_{n=1}^{\infty}(-1)^n * \frac{4}{(n\pi)^2}\cos n\pi x}$$

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Q2:Verify Stoke's theorem.

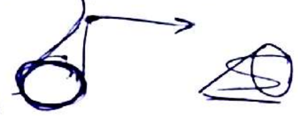
Where S is the surface of the cone $\sqrt{z^2+y^2}=x$, bounded by the two planes $x=0$, and $x=h$.

$\vec{F} = x\hat{i} - z\hat{j} + y\hat{k}$

$$\iint_S \nabla\times\vec{F}\cdot\hat{n}\,ds = \oint_C \vec{F}\cdot d\vec{r}$$

$S: \sqrt{z^2+y^2} = x$

$x \in [0, h]$



L.H.S

$$\nabla\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & -z & y \end{vmatrix} = \hat{i}(1+1) - \hat{j}(0-0) + \hat{k}(0-0)$$

$$\nabla\times\vec{F} = 2\hat{i}$$

$$\hat{n} = \frac{\langle 1, 2y, 2z\rangle}{|\sqrt{1+4y^2+4z^2}|}$$

h

$y^2+z^2=h$

$y = r\cos\theta$

$z = r\sin\theta$

$dz\,dy = r\,dr\,d\theta$

$r \in [0, \sqrt{h}]$

$\theta \in [0, 2\pi]$

$$= \iint_S 2\,dy\,dz$$

$$= \int_0^{2\pi}\int_0^{\sqrt{h}} 2r\,dr\,d\theta$$

$$= 2\pi\, r^2 \Big|_0^{\sqrt{h}}$$

$$\boxed{\iint_S \nabla\times\vec{F}\cdot\hat{n}\,ds = 2\pi h}$$

R.H.S

$$\oint_C F\cdot dr = \int_C x\,dx - z\,dy + y\,dz$$

$x = h$

$dx = 0$

$$= \int_C -z\,dy + y\,dz$$

$z = \sqrt{h}\sin\theta \qquad dz = \sqrt{h}\cos\theta\,d\theta$

$y = \sqrt{h}\cos\theta \qquad dy = -\sqrt{h}\sin\theta\,d\theta$

$$= \int_0^{2\pi} h\sin^2\theta\,d\theta \qquad \theta \in [0, 2\pi]$$

$$+ \int_0^{2\pi} h\cos^2\theta\,d\theta$$

⇒ cont.

$$= h\int_0^{2\pi} \left(\overbrace{\cos^2\theta + \sin^2\theta}^{1}\right) d\theta = 2\pi h$$

$$\cancel{= h\int_0^{2\pi} (2\cos^2\theta - 1)\, d\theta}$$

$$\boxed{\oint_C \vec{F} \cdot d\vec{r} = 2\pi h}$$

$$\cancel{= h\int_0^{2\pi} (1+\cos 2\theta - 1)\, d\theta}$$

$$\cancel{= h * \left[\frac{\sin 2\theta}{2}\right]_0^{2\pi}}$$

$$\cancel{= \text{zero}}$$

Q3: (a)Find the Fourier integral representation for

$$f(x) = \frac{\pi}{2}\begin{cases} x, & -1 < x < 1 \\ 0, & otherwise \end{cases}$$

(b) Evaluate $\int_0^\infty \frac{\sin\alpha - \alpha\cos\alpha}{\alpha^2}\sin\alpha\, d\alpha$.

a) $f(x) \sim \int_0^\infty (A_\alpha \cos\alpha x + B_\alpha \sin\alpha x)\, d\alpha$

$A_\alpha = \frac{1}{\pi}\int_{-\infty}^{\infty} f(x) \cos\alpha x\, dx = \frac{1}{\pi}\int_{-1}^{1} \frac{\pi}{2} x \cos\alpha x\, dx$

$= \frac{1}{2}\left[\frac{1}{\alpha} x \sin\alpha x \Big|_{-1}^{1} - \frac{1}{\alpha}\int_{-1}^{1} \sin\alpha x\, dx\right]$

$= \frac{1}{2}\left[\frac{1}{\alpha}(\cancel{\sin\alpha - \sin\alpha}) + \frac{1}{\alpha^2}\cos\alpha x \Big|_{-1}^{1}\right] = \text{zero}$

$\cos\alpha - \cos\alpha$

$A_\alpha = \text{zero}$

$B_\alpha = \frac{1}{\pi}\int_{-1}^{1} \frac{\pi}{2} x \sin\alpha x\, dx$

$= \frac{1}{2}\left[\frac{-1}{\alpha} x\cos\alpha x \Big|_{-1}^{1} + \frac{1}{\alpha}\int_{-1}^{1}\cos\alpha x\, dx\right] = \frac{1}{2}\left[\frac{-1}{\alpha}(\cos\alpha + \cos\alpha) + \frac{1}{\alpha^2}\sin\alpha x \Big|_{-1}^{1}\right]$

$\frac{1}{\alpha^2}(\sin\alpha -$

$B_\alpha = \frac{1}{2}\left(-\frac{2}{\alpha}\cos\alpha + \frac{2}{\alpha^2}\sin\alpha\right)$

$$\boxed{f(x) \sim \int_0^\infty \left(-\frac{\cos\alpha}{\alpha} + \frac{\sin\alpha}{\alpha^2}\right)\sin\alpha x\, d\alpha}$$

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Cont. ⇒