* دكتور الله أرجو مراجعة السؤال الأخير
أنا لم أخطأ لا في طريقة الحل و لا في الحسابات

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الجامعة الأردنية	الرياضيات الهندسية ٢: الامتحان الأول
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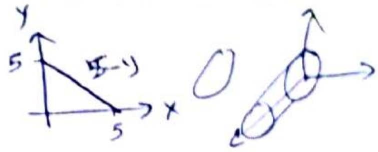
Q1: Evaluate $\iint_S xz\,dS$

Where S is the surface of the cylinder $z^2 + y^2 = 9$, bounded by the two planes $x = 0$, and $x = 5 - y$.

$z^2+y^2=9$ $\qquad 0 < x < 5-y$



$\iint G(r)\,ds = \iint G(\vec r(u,v)) \cdot |\vec r_u \times \vec r_v| \, du\,dv$

$\vec r(u,v) = \langle v, 3\sin u, 3\cos u \rangle \qquad u \in [0, 2\pi]$

$v \in [0, 5]$

$\vec r_u \times \vec r_v = \begin{vmatrix} \hat i & \hat j & \hat k \\ 0 & +3\cos u & -3\sin u \\ 1 & 0 & 0 \end{vmatrix}$ $= \cancel{\hat i(0-0) - \hat j(0+3\cos u) + \hat k(0-3\sin u)}$ 3

$= \hat i(0) - \hat j(0 + 3\sin u) + \hat k(0 - 3\cos u)$ $\qquad \cancel{= 3\cos u\,\hat j + 3\sin u\,\hat k}$

$|\vec r_u \times \vec r_v| = \sqrt{9\sin^2 u + 9\cos^2 u}$ $\qquad \cancel{= \sqrt{9} = 3}$

$\boxed{|\vec r_u \times \vec r_v| = \sqrt{9} = 3}$

$\cancel{G(r) = xz}$

$\cancel{G(\vec r(u,v)) = 3v\sin u}$

$\cancel{\int_0^2\int_0^{2\pi} 9v\sin u\,du\,dv = \int_0^2 -9v\cos u\Big|_0^{2\pi} dv}$

$\cancel{= \int_0^2 -9v(1-1)\,dv = zero}$

$G(r) = xz$

$G(\vec r(u,v)) = 3v\cos u$

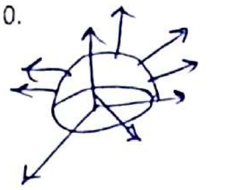
$\int_0^{5}\int_0^{2\pi} 9v\cos u\,du\,dv = \int_0^5 9v\sin u\Big|_0^{2\pi} dv$

$\boxed{\iint_S xz\,ds = \underline{zero}}$

Q2: Evaluate $\iint_S \vec{F}\cdot\hat{n}\,dS$

Where S is the surface of the hemisphere $z^2+y^2+x^2=25,\ y\ge 0$.

And $\vec{F}=x\hat{i}-z\hat{j}+y\hat{k}$.



$\vec{F}(x,-z,y)$ $\quad a=5$

$\vec{r}(u,v)=\langle 5\cos v\sin u,\ 5\sin v\sin u,\ 5\cos u\rangle$

~~$u\in[0,2\pi]$~~ ~~$v\in[0,\pi]$~~ $\quad u\in[0,\pi]$, $v\in[0,2\pi]$

$\iint F\cdot\hat{n}\,ds = \iint F(\vec{r}(u,v))\cdot\vec{N}\,du\,dv$

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$\vec{N}=\vec{r}_u\times\vec{r}_v=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 5\cos v\cos u & 5\sin v\cos u & -5\sin u\\ -5\sin v\sin u & 5\cos v\sin u & 0\end{vmatrix}=\hat{i}(25\cos v\sin^2 u)+\hat{j}(0+25\sin v\sin^2 u)+\hat{k}(25\cos u\sin u)$

$\vec{F}(\vec{r}(u,v))=\langle 5\cos v\sin u,\ -5\cos u,\ 5\sin v\sin u\rangle$

$\int_0^{2\pi}\int_0^{\pi}\left(125\cos^2 v\sin^3 u-\underbrace{125\sin v\cos u\sin^2 u}_{0}+\underbrace{125\sin v\cos u\sin^2 u}_{0}\right)du\,dv$

$=125\int_0^{2\pi}\frac{1}{2}(1+\cos 2v)\,dv\int_0^{\pi}\sin u\,(1-\cos^2 u)\,du$

$=125\left[\pi+\frac{\sin 2v}{2}\Big|_0^{2\pi}\right]*\left[-\cos u\Big|_0^{\pi}-\int_0^{\pi}\sin u\cos^2 u\,du\right]$

$-(-1-1)$

$\cos u=n \quad \sin u\,du=\frac{dn}{-\sin u}$

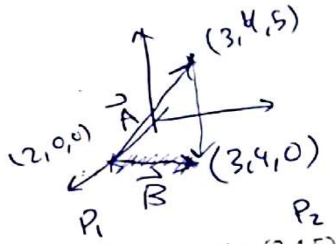
$125\pi*\left[2+\left(\int n^2\,dn\right.\right.$

$=125\pi*\left(2+\frac{\cos^3 u}{3}\Big|_0^{\pi}\right)$

$=125\pi\left(2+\frac{(-1-1)}{3}\right)$

$=125\pi\left(\frac{6}{3}-\frac{2}{3}\right)=\frac{4\times125}{3}\pi=\frac{500\pi}{3}$

$\boxed{\iint_S \vec{F}\cdot\hat{n}\,ds=\frac{500\pi}{3}}$



Q3: Evaluate $\int_C \vec{F}\cdot d\vec{r}$

Where C is the line segments from the point, (2,0,0) to the point (3,4,5) then to the point (3,4,0). And $\vec{F} = x\hat{\imath} - z\hat{\jmath} + y\hat{k}$.

P_3

$\overrightarrow{P_1P_2} = \cancel{3}\langle 1, 4, 5\rangle = \vec{A}$

$\overrightarrow{P_1P_3} = \langle 1, 4, 0\rangle = \vec{B}$

$\vec{F} = \langle x, -z, y\rangle$

$\vec{r}(t) = r_0 + \cancel{F}\vec{v}\,t$

$\int \vec{F}(\vec{r}(t)) \cdot \vec{r}'(t)\,dt$

on C_1 $\cancel{0}$

$\vec{r}_1(t) = (2,0,0) + [1,4,5]t$

$\vec{r}_1(t) = (2+t)\hat{\imath} + 4t\hat{\jmath} + 5t\hat{k}$

$\vec{F}(\vec{r}_1(t)) = \langle (2+t), -5t, 4t\rangle$

$\vec{r}_1'(t) = \langle 1, 4, 5\rangle$

$\int_0^1 2+t - \cancel{20t} + \cancel{20t}\,dt$

$= 2 + \frac{1}{2}$

$\int_{C_1} F.dr = \frac{5}{2}$

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on C_2

$\vec{r}_2(t) = (2,0,0) + [1,4,0]t$

$= (2+t)\hat{\imath} + 4t\hat{\jmath}$

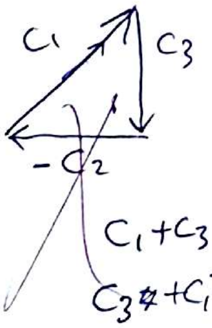
$\vec{F}(\vec{r}_2(t)) = \langle (2+t), 0, 4t\rangle$

$\vec{r}_2'(t) = \langle 1, 4, 0\rangle$

$\int_0^1 2+t\,dt$

$\int_{C_2} F dr = \frac{5}{2}$

$$\boxed{\int_C F.dr = \int_{C_1} F.dr + \int_{C_3} F.dr = \cancel{\tfrac{5}{2} + \tfrac{5}{2} = 2.5}\ \frac{5}{2}}$$



$C_1 + C_3 = C$

$C_3 \cancel{+} + C_1 \cancel{-} C_2 = 0$

$C_3 = -C_1 + C_2$

$\int_{C_3} F.dr = -\int_{C_1} F.dr + \int_{C_2} F.dr$

$-\frac{5}{2} + \frac{5}{2}$

$\int_{C_3} F.dr = \text{zero}$

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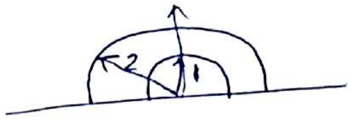
Q4: Evaluate $\oint_C y^2 dx + 3xy\, dy$.

Where C is the semi circles $y^2 + x^2 = 1$, $y^2 + x^2 = 4$, in upper half plane

$\vec{F} = \langle y^2, 3xy, 0 \rangle$



Polar (r, θ, z)

$x = r\cos\theta$ $\qquad dxdy = r\,dr\,d\theta$

$y = r\sin\theta$ $\qquad r \in [1,2]$

$z = 0$ $\qquad \theta\ [0, \pi]$

$$\oint_C y^2 dx + 3xy\,dy = \iint \nabla \times \vec{F}\ dxdy$$

$$\nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & 0 \\ y^2 & 3xy & 0 \end{vmatrix} = \hat{k}(3y - 2y)$$

$$= y\hat{k}$$

$$\int_0^\pi \int_1^2 r^2 \sin\theta\, dr\, d\theta = \int_0^\pi \frac{r^3}{3}\Big|_1^2 \sin\theta\, dr\, d\theta$$

$$= \frac{7}{3}(-\cos\theta)\Big|_0^\pi$$

$$\boxed{\oint_C y^2dx + 3ydy = \frac{14}{3}}$$