Partial

Dr:Nafeth Abujaradeh

By: Hamza Abedrabbo

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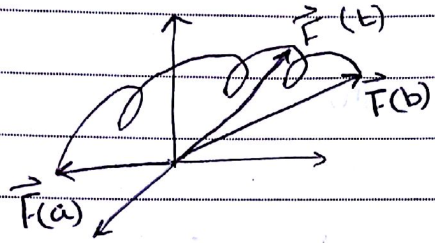
POWERUNIT

Partial   20/6/2016

No.

* Vector Valued functions:

$$\vec{F}(t) = f_1(t)\,\hat{i} + f_2(t)\,\hat{j} + f_3(t)\,\hat{k} \qquad t \in [a,b]$$



$$\vec{r}(t) = x(t)\,\hat{i} + y(t)\,\hat{j} + z(t)\,\hat{k}$$

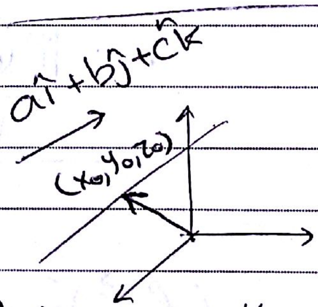
* Vectors:

$$\lim_{t\to t_0} \vec{r}(t) = \lim_{t\to t_0} x(t)\,\hat{i} + \lim_{t\to t_0} y(t)\,\hat{j} + \lim_{t\to t_0} z(t)\,\hat{k}$$

$$\vec{r}\,'(t) = \lim_{h\to 0} \frac{1}{h}\left(\vec{r}(t+h) + \vec{r}(t)\right)$$

$\vec{v}(t) \to$ Velocity

$$\vec{a}(t) = \vec{r}\,''(t) \to \text{accelaration}$$



$$\begin{cases} x = x_0 + at \\ y = y_0 + bt \\ z = z_0 + ct \end{cases}$$

(1) Lines in the space:

$$\vec{r}(t) = (x_0 + at)\,\hat{i} + (y_0 + bt)\,\hat{j} + (z_0 + ct)\,\hat{k}$$

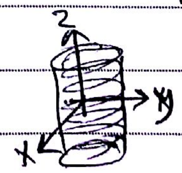
$$\hat{T}(t) = \frac{\vec{r}\,'(t)}{|\vec{r}\,'(t)|} \Rightarrow \text{unit tangent Vector}$$

$|\vec{v}(t)| \Rightarrow$ Speed

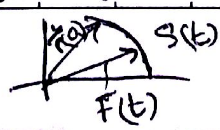
$$S(t) = \int_0^t |\vec{r}\,'(\tau)|\,d\tau \quad \text{(Arc length)}$$

(2) Helix

$$\vec{r}(t) = \cos(t)\,\hat{i} + \sin(t)\,\hat{j} + t\,\hat{k}$$



$x = \cos t$
$y = \sin t$ $\Big\rangle\ x^2+y^2$
$z = t$



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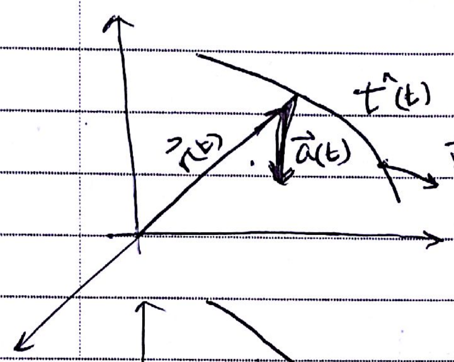
No.

$S = S(t)$

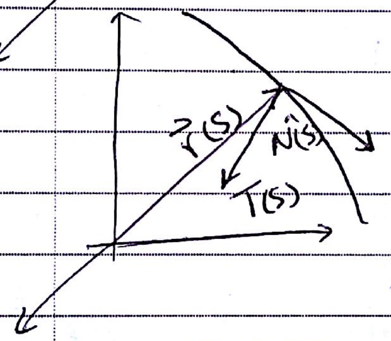
$\Downarrow$

$t = t(S)$

$\vec{r} = \vec{r}(s) \rightarrow \vec{r}' = \vec{r}'(s) = \hat{T}(s) \Rightarrow$ arc length parametrization



$\vec{r}'(t) = \vec{v}(t)$



$\boxed{\vec{r}'(s) = \hat{T}(s)}$, $\hat{T}$ unit vector

$$\boxed{\hat{N}(s) = \frac{\hat{T}'(s)}{|\hat{T}'(s)|}}$$

Normal

Tangent

$|\hat{T}'(s)| \rightarrow$ curvature

$$\boxed{K = \frac{|\vec{r}'(t) \times \vec{r}''(t)|}{|\vec{r}'(t)|^3}} \rightarrow \text{curvature}$$

---

① $(\ell \vec{F})'(t) = \ell \vec{F}'(t)$

② $(\vec{F} \cdot \vec{g})'(t) = \vec{F}(t) \cdot \vec{g}'(t) + \vec{F}'(t) \cdot \vec{g}(t)$

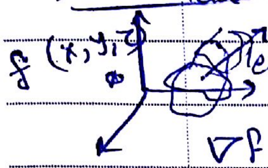
③ $(\vec{F} \times \vec{g})'(t) = \vec{F}(t) \times \vec{g}'(t) + \vec{F}'(t) \times \vec{g}(t)$

Vector & Scalar field

Scalar field

$f = f(x,y,z)$

$f(x,y,z)$



level surface $f(x,y,z) = C$

$\nabla f = f_x \hat{i} + f_y \hat{j} + f_z \hat{k}$

$f = f(x,y)$

∇f عمودي على

(x,y)

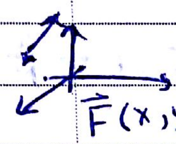
level curve

$\nabla f = f_x \hat{i} + f_y \hat{j}$

vector field

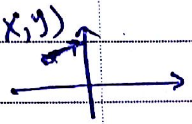
$$\boxed{D_{\hat{u}} f = \nabla f \cdot \vec{u}}$$

Directional derivative of f to $\hat{u}$



$$\vec{F}(x,y,z) = f_1(x,y,z)\hat{i} + f_2(x,y,z)\hat{j} + f_3(x,y,z)\hat{k}$$

(x,y)



$$\vec{F}(x,y) = f_1(x,y)\hat{i} + f_2(x,y)\hat{j}$$

~~Gradient of a vector~~

$$\boxed{\text{max rate of change} = |\nabla f|^2}$$

$$\boxed{\text{min rate of change} = -|\nabla f|^2}$$

$$\boxed{-|\nabla f|^2 \leq D_{\hat{u}} f \leq |\nabla f|^2}$$

$$\boxed{\hat{U} = \frac{\nabla f}{|\nabla f|}} \rightarrow \text{Em: chB}$$

No.

$\nabla = \frac{d}{dx}\hat{i} + \frac{d}{dy}\hat{j} + \frac{d}{dz}\hat{k}$ (Differential operator ∇)

$\vec{F} = F_1\hat{i} + F_2\hat{j} + F_3\hat{k}$

$$\boxed{\nabla\cdot\vec{F} = \frac{dF_1}{dx} + \frac{dF_2}{dy} + \frac{dF_3}{dz}}$$

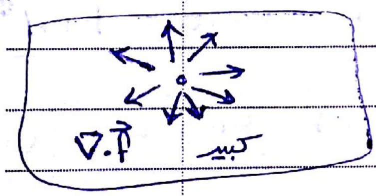
Ex 1 $\vec{F}(x,y,z) = xyz\hat{i} + xy\hat{j} + y^2z\hat{k}$ find $\nabla\cdot\vec{F}$

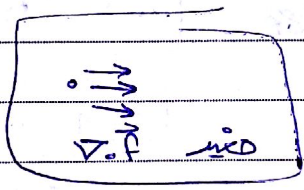
Sol

$$\boxed{\nabla\cdot\vec{F} = yz + x + y^2}$$

$\nabla\cdot\vec{F}$ (Divergance of a Vector Field)

مدى تشتت المتجه





$\vec{F}\cdot\vec{\nabla} = f_1\frac{d}{dx} + f_2\frac{d}{dy} + f_3\frac{d}{dz}$

Ex2 $\vec{F}(x,y,z) = xyz\hat{i} + xy\hat{j} + y^2z\hat{k}$

find $\vec{F}\cdot\vec{\nabla}$??

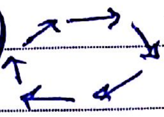
$\vec{F}\cdot\nabla = xyz\frac{d}{dx} + xy\frac{d}{dy} + y^2z\frac{d}{dz}$ = zero

Ex3 find $(\vec{F}\cdot\nabla)((\sin x)ye^z)$

$= xyz((\cos x)ye^z) + xy((\sin x)e^z) + y^2z((\sin x)ye^z)$

No.

$\nabla \times \vec{F}$ (curl of a vectorField)



مدى الالتفاف في المتجهة

$$\nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix}$$

$\frac{\partial F_3}{\partial y}$ $\hat{i}$

$$\nabla \times \vec{F} = \left(\frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z}\right)\hat{i} - \hat{j}\left(\frac{\partial F_3}{\partial x} - \frac{\partial F_1}{\partial z}\right) + \hat{k}\left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right)$$

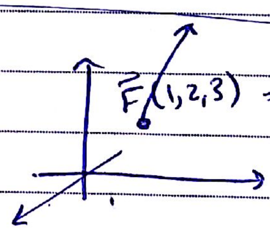
Ex4 $\vec{F}(x,y,z) = xyz\,\hat{i} + xy\,\hat{j} + y^2 z\,\hat{k}$

$\nabla \times \vec{F}$??

sol

$$\nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xyz & xy & y^2 z \end{vmatrix} = \hat{i}(2yz - 0) - \hat{j}(0 - xy) + \hat{k}(y - xz)$$

find $\vec{F}(1,2,3)$ for Ex 4



$\vec{F}(1,2,3) = 6\hat{i} + 2\hat{j} + 12\hat{k}$

$\vec{F}(1,2,3) = 6\hat{i} + 2\hat{j} + 12\hat{k}$

22/6/2016

3rd lecture

No.

* $f = f(x,y,z)$ f is a scalar field.

$$\nabla f = \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}$$

$$\nabla \cdot \nabla f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\boxed{\nabla^2 f = \nabla \cdot \nabla f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}}$$

~~laplace~~ (laplacian of f)

$\boxed{\Delta f = \nabla^2 f}$ (Divergance of a gradient) (Scalar field)

* $u = U(x,y,z)$

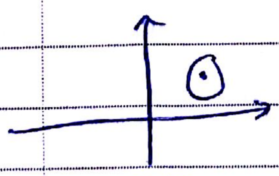
$\boxed{\Delta u = 0} \rightarrow$ (Steady state Heat equation)
(laplace equation)

مثال

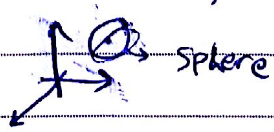
$\Delta u > 0$ * إذا درجات الحرارة في النقطة أكبر من درجات حرارة المحيط بها

$\Delta u = 0$ * // // // // // يساوي درجات حرارة المحيط بها

$\Delta u < 0$ * // // // // // أقل من درجات حرارة المحيط بها



2-D



3D

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No.

* Divergance of the curl $(\nabla \cdot \nabla \times \vec{F})$

$$\nabla \cdot (\nabla \times \vec{F}) = \nabla \cdot \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f_1 & f_2 & f_3 \end{vmatrix}$$

$$= \nabla \cdot \left[\left(\frac{\partial f_3}{\partial y} - \frac{\partial f_2}{\partial z} \right) \hat{i} - \hat{j} \left(\frac{\partial f_3}{\partial x} - \frac{\partial f_1}{\partial z} \right) + \hat{k} \left(\frac{\partial f_2}{\partial x} - \frac{\partial f_1}{\partial y} \right) \right]$$

$$= \frac{\partial^2 f_3}{\partial x \partial y} - \frac{\partial^2 f_2}{\partial x \partial z} - \frac{\partial^2 f_3}{\partial y \partial x} + \frac{\partial^2 f_1}{\partial y \partial z} + \frac{\partial^2 f_2}{\partial z \partial x} - \frac{\partial^2 f_1}{\partial z \partial y} = \text{zero (Always)}$$

* Some properties:

① $\nabla (fg) = f \nabla g + g \nabla f$

② $\nabla \left(\frac{1}{g} \right) = \frac{-\nabla g}{g^2}$

③ $\nabla \left(\frac{f}{g} \right) = \frac{g \nabla f - f \nabla g}{g^2}$

26/6/2016
4th lec

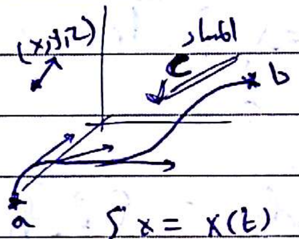
No.

Line Integral

$$\boxed{W = \int_C \vec{F} \cdot \vec{dr}}$$

Vector field

$$\vec{F}(x,y,z) = f_1(x,y,z)\,\hat{i} + f_2(x,y,z)\,\hat{j} + f_3(x,y,z)\,\hat{k}$$



$$\begin{cases} x = x(t) \\ y = y(t) \\ z = z(t) \end{cases} \quad t \in [a,b]$$

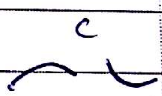
$$\vec{r}(t) = x(t)\,\hat{i} + y(t)\,\hat{j} + z(t)\,\hat{k}, \quad t \in [a,b]$$

$$\vec{dr} = (x'(t)\,\hat{i} + y'(t)\,\hat{j} + z'(t)\,\hat{k})\,dt$$

* الشروط

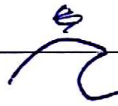
① C cont.

⇐ مسار ~~مت~~ متصل



C not cont.

② C must be ~~differantiable~~ differantiable (smooth)



⇒ C not differantiable

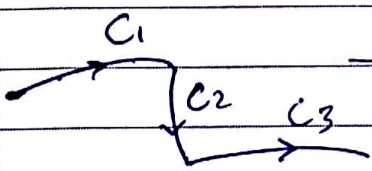
③ المسار C يجب أن لا يقطع نفسه.

اذا الشروط تحققت

* C is a Smooth Curve

④ must not → $\vec{r} = \vec{r}(t), \quad t \in [c,d]$

⑤

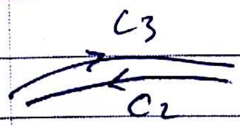


⟶ $\boxed{C = C_1 + C_2 + C_3}$ must be continuos.

* هاي مش زي اللي بالشرط رقم ②

⇐ (differantiable)

⑥



⟶ $\boxed{C_3 = -C_2}$

No.

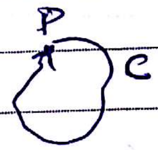
* $$\int_{C_1+C_2} \vec{F}\cdot\vec{dr} = \int_{C_1} \vec{F}\cdot\vec{dr} + \int_{C_2} \vec{F}\cdot\vec{dr}$$

* $$\int_{-C} \vec{F}\cdot\vec{dr} = -\int_{C} \vec{F}\cdot\vec{dr}$$

* $$\int_C (\vec{F}\mp\vec{G})\cdot\vec{dr} = \int_C \vec{F}\cdot\vec{dr} \mp \int_C \vec{G}\cdot\vec{dr}$$

* $$\int_C \ell\vec{F}\cdot\vec{dr} = \ell\int_C \vec{F}\cdot\vec{dr}$$

* for a closed curve $\int_C \vec{F}\cdot\vec{dr} = \text{zero}$



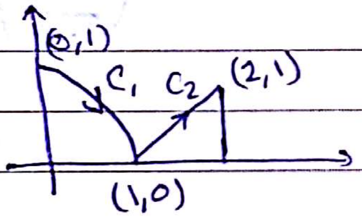
~~Line Integ~~

Line Integral (cont.):

$$W = \int_C \vec{F} \cdot \vec{dr}$$

$$= \int_a^b \big[f_1(x(t), y(t), z(t))\, x'(t) + f_2(x(t), y(t), z(t))\, y'(t) + f_3(x(t), y(t), z(t))\, z'(t) \big]\, dt$$

ex



find Line Integral ??

$$\vec{F} = xy\hat{i} + 2x^2y\hat{j}$$

sol: $C = C_1 + C_2$

$$\int_{C_1+C_2} \vec{F} \cdot \vec{dr} = \int_{C_1}$$

$\boxed{1}$ On C_1

$x^2 + y^2 = 1$

$\Updownarrow$

(1) $\begin{cases} x = t \\ y = \sqrt{1 - t^2} \end{cases}$

or

(2) $\begin{cases} x = \sin t \\ y = \cos t \end{cases}$ $t \in [0, 1]$

$t = 0$ $\rightarrow x = 0 \rightarrow (0,1)$, $y = 1$

$t = \pi/2$ $x = 1 \rightarrow (1,0)$, $y = 0$

$\vec{r} = \sin t\,\hat{i} + \cos t\,\hat{j}$

$\vec{dr} = (\cos t\,\hat{i} - \sin t\,\hat{j})\, dt$

$$\int_{C_1} \vec{F} \cdot \vec{dr} = \int_0^{\pi/2} \big[\sin t \cos^2 t \cos t + 2\sin^2 t \cos t (-\sin t)\big]\, dt$$

No.

طريقة ٢

on C_1

$x^2+y^2=1$

$$\begin{cases} x=t \\ y=\sqrt{1-t^2} \end{cases}$$

$$\vec{dr} = \left(\hat{i} - \frac{t}{\sqrt{1-t^2}}\hat{j}\right) dt$$

$$\int_{C_1} \vec{F}\cdot\vec{dr} = \int_0^1 \left[t(1-t^2) + 2t^2\sqrt{1-t^2}\left(\frac{-t}{\sqrt{1-t^2}}\right) \right] dt$$

$\boxed{2}$ On C_2

$y = x-1$ (from graph)

$$\begin{cases} x=t \\ y=t-1 \end{cases} \text{— Parametric eq.}$$

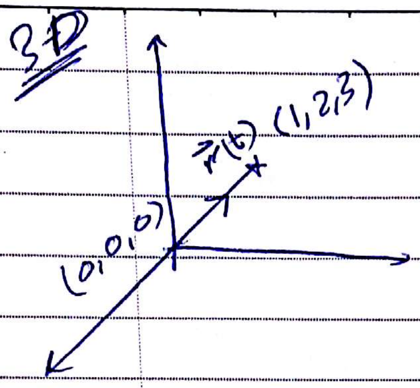
$\vec{dr} = (\hat{i}+\hat{j})\, dt$ ← اشتقاق الـ Parametric eq.

$$\vec{F}(x(t),y(t)) = t(t-1)^2\,\hat{i} + 2t^2(t-1)\,\hat{j}$$

$$\int_{C_2} \vec{F}\cdot dr = \int_1^2 \left[t(t-1)^2 + 2t^2(t-1) \right] dt$$

No.

3D



$$\vec{r}(t) = t\hat{i} + 2t\hat{j} + 3t\hat{k} \qquad t \in [0,1]$$

$$d\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k})\,dt$$

$$\vec{F} = y\hat{i} + z\hat{j} + x\hat{k}$$

$$\vec{F}(\vec{r}(t)) = 2t\hat{i} + 3t\hat{j} + t\hat{k}$$

$$\int_C \vec{F}\cdot d\vec{r} = \int_0^1 (2t + 6t + 3t)\,dt$$

27/6/2016

lec 5

No.

Line Integration:

$$\boxed{W = \int_c \vec{F} \cdot d\vec{r}}$$

$\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k}$

$\vec{F} = f_1(x,y,z)\hat{i} + f_2(x,y,z)\hat{j} + f_3(x,y,z)\hat{k}$ $\quad \vec{dr} = x'\hat{i} + y'\hat{j} + z'\hat{k}$

$\vec{F}(\vec{r}(t)) = f_1(x(t), y(t), z(t))\hat{i} + \dots$

$$W = \int_a^b \left[f_1(x(t), y(t), z(t))\, x' + f_2(x(t), y(t), z(t))\, y' + \dots \right] dt$$

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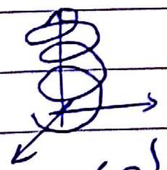
ex: $\vec{F} = xy\hat{i} + yz\hat{j} + xz\hat{k}$

C: $\vec{r}(t) = \cos t\,\hat{i} + \sin t\,\hat{j} + t\hat{k}$

$t \in [0, 2\pi]$



Find $\int_c \vec{F} \cdot \vec{dr}$??

sol:

$\vec{dr} = (-\sin t\,\hat{i} + \cos t\,\hat{j} + \hat{k})\, dt$

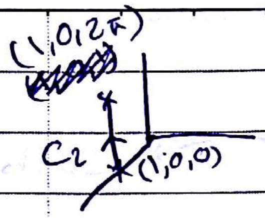
$x = \cos t \quad y = \sin t \quad z = t$

$\vec{F}(\vec{r}(t)) = \cos t \sin t\,\hat{i} + t \sin t\,\hat{j} + t\cos t\,\hat{k}$

$$\int_0^{2\pi} (-\sin^2 t \cos t + t \sin t \cos t + t\cos t)\, dt$$

No.

same ex different C_2



Sol:

$$\left.\begin{aligned} x &= 1 \\ y &= 0 \\ z &= t \quad t \in [0, 2\pi] \end{aligned}\right\}\ \text{parametric equations}$$

$$\vec{r}(t) = i + 0\hat{j} + t\hat{k}$$

$$\vec{dr} = \hat{k}\,dt$$

$$\vec{F} = xy\,\hat{i} + yz\,\hat{j} + xz\,\hat{k}$$

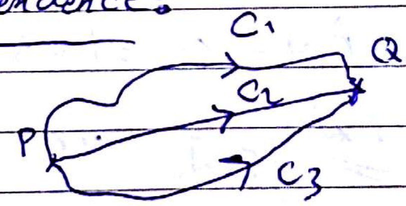
$$\vec{F}(\vec{r}(t)) = 0\hat{i} + 0\hat{j} + t\hat{k}$$

$$\int_C \vec{F}\cdot dr = \int_0^{2\pi} t\,dt = \frac{1}{2}t^2\Big|_0^{2\pi} = \frac{1}{2}(4\pi^2)$$

$$= 2\pi^2$$

---

path Independence:



(1) $\int_{C: P \to Q} \vec{F}\cdot\vec{dr}$ is indep. of the path

(2) $\int_{C_1} \vec{F}\cdot\vec{dr} = \int_{C_3} \vec{F}\cdot dr$ in path Indep.

$$\Rightarrow \int_{C_1} \vec{F}\cdot dr + \int_{-C_3} \vec{F}\cdot dr = 0 \quad \text{(closed path)}$$

$$\Rightarrow \oint \vec{F}\cdot\vec{dr} = \text{zero}$$

(closed path)

$\boxed{14}$

No.

→ (3) $\nabla \times \vec{F} = 0$ (irrotational)

→ (4) $\vec{F} = \nabla \phi$ ⇐ (vector field $\vec{F}$ is a gradient of a scalar field)

$\nabla \times \nabla \phi =$ zero [curl of a gradient is always zero]

ex $\vec{F} = xy\hat{i} + yz\hat{j} + xz\hat{k}$ find $\nabla \times \vec{F}$??

sol

$$\nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & yz & xz \end{vmatrix} = \hat{i}(0-y) - \hat{j}(z-0) + \hat{k}(0-x)$$

$\nabla \times \vec{F} \neq 0 \Rightarrow$ (rotational)

$\nabla \times \vec{F} = -y\hat{i} - z\hat{j} - x\hat{k}$

this means that it's path dependent

⇐ كل مسار اله قيمة مختلفة عن المسار الثاني لنفس النقطتين

ex $\phi(x,y,z) = xy^2z^3$ Find $\vec{F}, \nabla \times \vec{F}$??

sol $\vec{F} = \nabla \phi = y^2z^3\hat{i} + 2xyz^3\hat{j} + 3xy^2z^2\hat{k}$

$\nabla \times \vec{F} =$ zero (path independent)

* $\phi(x,y,z)$ is a potential function of $\vec{F}$.

(5) $\int_{C:P\to Q} \vec{F} \cdot d\vec{r} = \phi(Q) - \phi(P)$

No.

ex

$\int \vec{F} \cdot \vec{dr}$?? $\therefore$ $\phi(x,y,z) = xy^2z^3$ $\therefore$

$C: (1,0,0) \to (1,2,3)$
P Q

sol:

$$\int_{C:(1,0,0)\to(1,2,3)} \vec{F} \cdot dr = \phi(1,2,3) - \phi(1,0,0)$$

$$= 1*4*27 - 0$$

$$= 108$$

ex: $\int_c \vec{F} \cdot \vec{dr}$?? $\vec{F} = y^2z^3\hat{i} + 2xyz^3\hat{j} + 3xy^2z^2\hat{k}$

sol

if $\nabla \times \vec{F} = 0$

$\Rightarrow \vec{F} = \nabla\phi = \phi_x\hat{i} + \phi_y\hat{j} + \phi_z\hat{k} \Rightarrow \phi_x = y^2z^3$

$\phi_y = 2xyz^3$

$\phi = \int \phi_x\, dx = \int \phi_y\, dy = \int \phi_z\, dz$ $\quad \phi_z = 3xy^2z^2$

$\cancel{= xy^2z^3 = xy^2z^3 = xy^2z^3}$

$\phi = xy^2z^3 + f(y,z) = xy^2z^3 + g(x,z) = xy^2z^3 + h(x,y)$

f(z) g(z) zero

$f(y,z) = g(x,z) = h(x,y) = 0$

$$\boxed{\phi = xy^2z^3}$$

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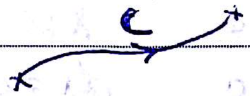
lec 6

No.

line Integrals

$$\int_C \vec{F}\cdot\vec{dr}$$



TFAE: (The following are equivalent)

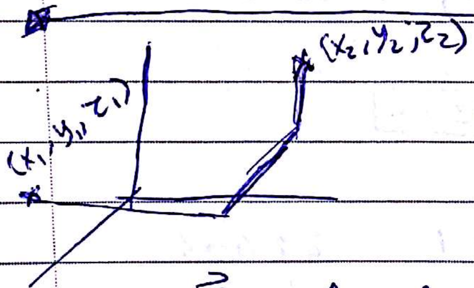
(1) the line integral is independent of the path

(2) the line integral around any closed curve is zero.

(3) $\nabla \times \vec{F} = 0$

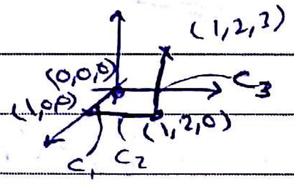
(4) $\vec{F} = \nabla \phi$ $\quad$ ϕ: Potential function

(5) $\int_C \vec{F}\cdot\vec{dr} = \phi(Q) - \phi(P)$



ex: $\vec{F} = xy\hat{i} + yz\hat{j} + xz\hat{k}$

find $W_{(0,0,0)\to(1,2,3)}$



$$\int_{C_1+C_2+C_3} \vec{F}\cdot\vec{dr} = \int_{C_1} \vec{F}\cdot\vec{dr} + \int_{C_2} \vec{F}\cdot\vec{dr} + \int_{C_3} \vec{F}\cdot\vec{dr}$$

$$\cancel{= \int_0^1 \vec{F}\cdot dx + \int_0^2 \vec{F}\cdot dy + \int_0^3 \vec{F}\cdot dz}$$

$$= \int_0^1 \quad \Rightarrow$$

No.

On C_1

$y = z = 0 \qquad x : 0 \to 1$

$dy = dz = 0 \qquad dx \neq 0$

$$\int_0^1 xy\,dx + \cancel{\int_0^0 yz\,dy} + \cancel{\int_0^0 xz\,dz}$$

$= \text{zero} \;,\; \boxed{y=0}$

On C_2 $\quad x = 1 \;,\; z = 0 \qquad y : 0 \to 2$

$dx = dz = 0 \qquad dy \neq 0$

$$\cancel{\int_1^1 xy\,dx} + \int_0^2 yz\,dy + \cancel{\int_0^0 xz\,dz}$$

$= \text{zero} \;,\; \boxed{z=0}$

On C_3 $\quad y = 2 \quad x = 1 \qquad z : 0 \to 3$

$dy = dx = 0 \qquad dz \neq 0$

$$\cancel{\int xy\,dx} + \cancel{yz\,dy} + \int_0^3 xz\,dz$$

$$= \frac{1}{2} z^2 \Big|_0^3 = \frac{9}{2}$$

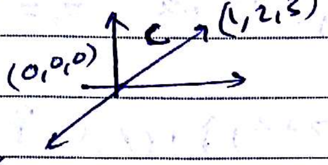
$$\int_C F \cdot dr = \frac{9}{2}$$

No.

ex: $\vec{F} = xy\,\hat{i} + yz\,\hat{j} + xz\,\hat{k}$

find $\int_{(0,0,0)}^{(1,2,3)} \vec{F}\cdot dr$??

Another way of solving



Sol

$$\int \vec{F}\cdot dr = \int xy\,dx + \int yz\,dy + \int xz\,dz$$

parametric equations:

$$\begin{cases} x = t\ , \quad t\in[0,1] \Rightarrow dx = dt \\ y = 2t \qquad\qquad\qquad dy = 2dt \\ z = 3t \qquad\qquad\qquad dz = 3dt \end{cases}$$

$$\int \vec{F}\cdot dr = \int_0^1 \left(2t^2 \cancel{dt} + 12t^2 \cancel{dt} + 9t^2\right) dt$$

$$= \int_0^1 23t^2\,dt$$

$$= \frac{23}{3}$$

By that $\nabla\times\vec{F} \neq 0$ (dependent of the path) for the previous 2 examples

* الجواب طلع مختلفة لنفس ال function باستخدام مسارات مختلفة.

No.

ex: $\phi = xyz + 2x + 3y + z$ $(0,0,0) \to (1,2,3)$

$\phi_x = yz + 2$, $\phi_y = xz + 3$, $\phi_z = xy + 1$

$\vec{F} = \nabla\phi$

$\vec{F} = (yz+2)\,\hat{i} + (xz+3)\,\hat{j} + (xy+1)\,\hat{k}$

Sol:

$\int (yz+2)\,dx + (xz+3)\,dy + (xy+1)\,dz$

On C_1 $dy = dz = 0$ $dx \neq 0$

$x: 0 \to 1$ $y = z = 0$

$\int_0^1 2\,dx = 2$

On C_2 $dx = dz = 0$ $dy \neq 0$

$y: 0 \to 2$ $x = 1$, $z = 0$

$\int_0^2 3\,dy = 6$

On C_3 $dy = dx = 0$ $dz \neq 0$

$z: 0 \to 3$ $x = 1$ $y = 2$

$\int_0^3 (2+1)\,dz = 9$

$\int \vec{F} \cdot \vec{dr} = 2 + 6 + 9 = 17$

Another way for solving previous ex

ex Sol:

$$\int \vec{F} \cdot dr = \int (yz+2)\,dx + (xz+3)\,dy + (xy+1)\,dz$$

Parametric equations

$(0,0,0) \to (1,2,3)$

$$\begin{cases} x = t \to & dx = dt \\ y = 2t & dy = 2dt \\ z = 3t & dz = 3dt \end{cases}$$

$t \in [0,1]$

$$= \int_0^1 \left[(6t^2+2) + 2(3t^2+3) + 3(2t^2+1) \right] dt$$

$$\cancel{= \int_0^1 (11t^2+6)\,dt} \quad \cancel{= \int_0^1 (18t^2+11)}$$

$$\cancel{= \frac{11}{3} + 6}$$

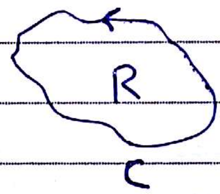
$$= \int_0^1 (18t^2+11)\,dt$$

$$= 6 + 11 = 17$$

---

* Green's theorem:

~~Special Case~~



$\vec{F} = f_1(x,y)\,\hat{i} + f_2(x,y)\,\hat{j}$

$C = \partial R$

$$\oint_C \vec{F} \cdot \vec{dr}$$

$$= \oint_C \left[f_1(x,y)\,dx + f_2(x,y)\,dy \right]$$

$$= \iint_R \left(\frac{\partial f_2}{\partial x} - \frac{\partial f_1}{\partial y} \right) dx\,dy$$

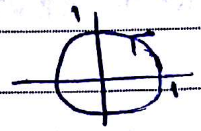
Green's theorem conditions:

(1) closed path (C)

(2) only two components (f (x,y))

No.

ex $\vec{F} = -y\hat{i} + x\hat{j}$

C  find $\oint F \cdot dr$??

Parametric eq. : ط١

Green's thm : ط٢

ط٢ : Green's thm

$$\oint_C F \cdot dr = \iint_R 2 \, dxdy$$

(1) $2 * \pi r^2 = 2\pi$

OR $\iint_R 2 \, dxdy =$ (2) $\int_0^{2\pi}\int_0^1 2r \, dr \, d\theta$

$$= \int_0^{2\pi} r^2 \Big|_0^1 d\theta$$

$$= 2\pi$$

OR ط١ : Parametric eq.

$x = \cos t$

$y = \sin t \quad t \in [0, 2\pi]$

$dx = -\sin t \, dt$

$dy = \cos t \, dt$

$$\oint F \cdot dr = \int -y\,dx + x\,dy$$

$$= \int_0^{2\pi} (\sin^2 t + \cos^2 t)\, dt$$

$$= \int_0^{2\pi} 1 \, dt = 2\pi$$

No.

$\oint_C \oint -y\,dx + x\,dy = 2 * \text{Area inclosed by } C$

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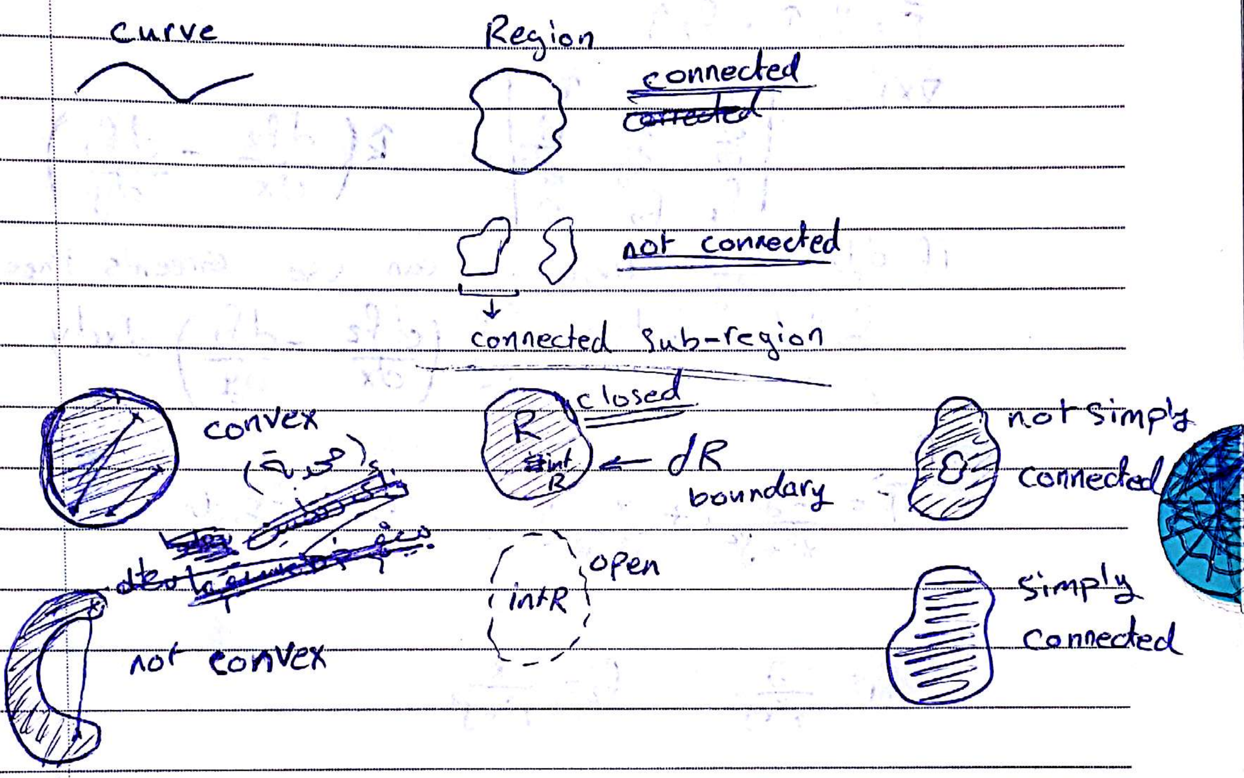
No.



convex ⇒ 2 pts that can be connected with a Straight line without getting out of the Region
(محدب)

$$\vec{F} = \frac{-y}{x^2+y^2}\,\hat{i} + \frac{x}{x^2+y^2}\,\hat{j}$$

مو معرف عليها
على (0,0)

⇒ not simply connected

⇒ Can't use Green's theorem.

$\vec{F} = f_1\hat{i} + f_2\hat{j}$

$\Rightarrow \nabla \times \vec{F} = 0$ to use Green's theorem

No.

$\vec{F} = f_1 \hat{i} + f_2 \hat{j}$

$$\nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f_1 & f_2 & 0 \end{vmatrix} = \hat{k}\left(\frac{\partial f_2}{\partial x} - \frac{\partial f_1}{\partial y}\right)$$

if $\frac{\partial f_2}{\partial x} = \frac{\partial f_1}{\partial y}$ then we can use Green's theorem

$$\oint_C f_1 dx + f_2 dy = \iint_R \left(\frac{\partial f_2}{\partial x} - \frac{\partial f_1}{\partial y}\right) dx\,dy$$

ex A

$$\vec{F} = \frac{-y}{x^2+y^2}\hat{i} + \frac{x}{x^2+y^2}\hat{j} \qquad \text{find } \oint F \cdot dr$$

$$f_1 = \frac{-y}{x^2+y^2} \qquad f_2 = \frac{x}{x^2+y^2}$$

$$\frac{\partial f_1}{\partial y} = -\frac{(x^2+y^2) + y(2y)}{(x^2+y^2)^2} = \frac{-x^2+y^2}{(x^2+y^2)^2}$$

$$\frac{\partial f_2}{\partial x} = \frac{(x^2+y^2) - x(2x)}{(x^2+y^2)^2} = \frac{y^2-x^2}{(x^2+y^2)^2}$$

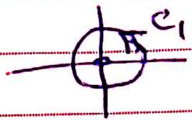
$\frac{\partial f_1}{\partial y} = \frac{\partial f_2}{\partial x}$ then we use Green's theorem

$$\iint_R \underbrace{\frac{y^2-x^2}{(x^2+y^2)^2} - \frac{y^2-x^2}{(x^2+y^2)^2}}_{\text{zero}} dx\,dy = \text{zero}$$

غلط لأن معرف
(0,0)

No.

ex $\oint_c \frac{-y}{x^2+y^2}\,dx + \frac{x}{x^2+y^2}\,dy = ??$



Parametric eq.

$x = \cos t$

$y = \sin t$

$t \in [0, 2\pi]$

$dx = -\sin t\,dt \qquad x^2+y^2 = 1$

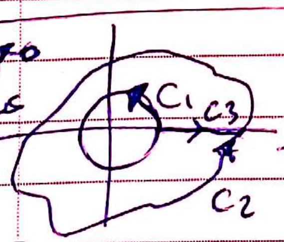
$dy = \cos t\,dt$

$$\oint = \int_0^{2\pi} +\sin^2 t\,dt + \int_0^{2\pi} \cos^2 t\,dt$$

$$= \int_0^{2\pi} (\sin^2 t + \cos^2 t)\,dt$$

$$= 2\pi$$

معرف ما عدا على $(0,0)$



$\oint_{c_1} = 2\pi$

then $\oint_{c_2} = 2\pi$

$\int_{C_1} = \int_{C_2}$

اثباتا

$\oint = \text{zero}$

$C_1 + C_3 - C_2 - C_3$

عشان نرجعنا لنفس النقطة ليكون closed

$\int_{c_1 - c_2} = 0$

Remember **ex A** in the Previous page

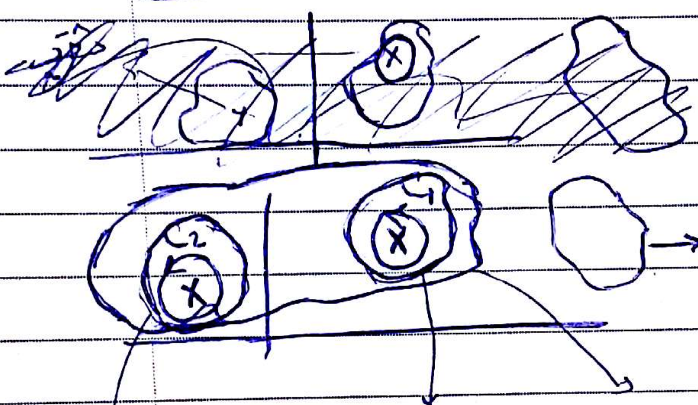
$\Rightarrow \int_{c_1} = \int_{c_2}$

No.

Conservative in 2-D

$\frac{df_2}{dx} - \frac{df_1}{dy} = \text{zero}$ from $\iint_R \left(\frac{df_2}{dx} - \frac{df_1}{dy}\right) dx\,dy$

in 3-D $\Rightarrow \nabla \times \vec{F} = \text{zero}$



$\oint = 0$

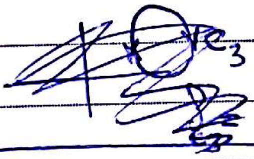
$\oint = \oint_{C_1} + \oint_{C_2}$

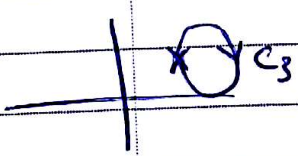
$\oint = \oint_{C_1}$

$\oint = \oint_{C_2}$

X: ثقب

C_2, C_1 : دائرة الوحدة





$\oint_{C_3}$ undefined

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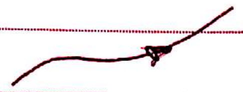
No.

Curve:



$\vec{r}(t) = x(t)\hat{\imath} + y(t)\hat{\jmath} + z(t)\hat{k}$

Parametric equations:
$$\begin{cases} x = x(t) \\ y = y(t) \\ z = z(t) \end{cases} \qquad t \in [a,b]$$

~~Surf~~

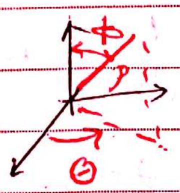
Surface:-

$\vec{r} = \vec{r}(u,v) = x(u,v)\hat{\imath} + y(u,v)\hat{\jmath} + z(u,v)\hat{k}$

$u \in [a,b]$
$v \in [c,d]$

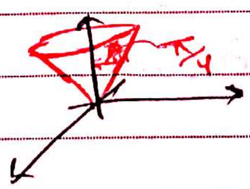
Parametric equations
$$\begin{cases} x = x(u,v) \\ y = y(u,v) \\ z = z(u,v) \end{cases}$$

Spherical coordinates:



$x = \rho \cos\theta \sin\phi$
$y = \rho \sin\theta \sin\phi \quad \Rightarrow \quad \theta \in [0, 2\pi]$
$z = \rho \cos\phi \qquad \phi \in [0, \pi]$

ex: $\phi = \pi/4$



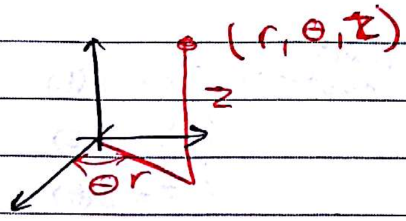
~~Ex: $\phi = \pi/4$~~

cylindrical coordinates

$x = r\cos\theta$

$y = r\sin\theta$

$z = z$



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ex: $x = u - v$, $u \in [0,1]$

$y = u + v$ $\quad v \in [2,5]$

$z = uv$

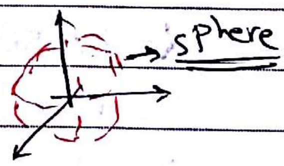
→ surface equation

⇒ $\frac{dr}{du}$: مماس ل u

$\frac{dr}{dv}$: مماس ل v

$\frac{dr}{du} \times \frac{dr}{dv}$: Normal of the surface.

Ex: $\theta = \pi/6$, $\phi = \pi/3$



$x = \cos\theta \sin\phi$

$y = \sin\theta \sin\phi$

$z = \cos\phi$

~~θ curve~~

No.

ex Cont.

θ curve

$x = \frac{\sqrt{3}}{2}\cos\theta$

$y = \frac{\sqrt{3}}{2}\sin\theta$

$z = \frac{1}{2}$

$\vec{r} = \left(\frac{\sqrt{3}}{2}\cos\theta\,\hat{i} + \frac{\sqrt{3}}{2}\sin\theta\,\hat{j} + \frac{1}{2}\hat{k}\right)$

ϕ curve

$x = \frac{\sqrt{3}}{2}\sin\phi$

$y = \frac{1}{2}\sin\phi$

$z = \cos\phi$

$\vec{r} = \left(\frac{\sqrt{3}}{2}\sin\phi\,\hat{i} + \frac{1}{2}\sin\phi\,\hat{j} + \cos\phi\,\hat{k}\right)$

$\cancel{\frac{d\vec{r}}{d\theta} = \langle -\frac{\sqrt{3}}{2}\sin\theta, \frac{\sqrt{3}}{2}\cos\theta}$

$\frac{d\vec{r}}{d\theta} = \left\langle -\frac{\sqrt{3}}{2}\sin\theta, \frac{\sqrt{3}}{2}\cos\theta, 0\right\rangle$

$\frac{d\vec{r}}{d\phi} = \left\langle \frac{\sqrt{3}}{2}\cos\phi, +\frac{1}{2}\cos\phi, -\sin\phi\right\rangle$

$\left.\frac{d\vec{r}}{d\theta}\right|_{\theta=\pi/6} = \left\langle -\frac{\sqrt{3}}{4}, \frac{3}{4}, 0\right\rangle$

$\left.\frac{d\vec{r}}{d\phi}\right|_{\phi=\pi/3} = \left\langle \frac{\sqrt{3}}{4}, \frac{1}{4}, -\frac{\sqrt{3}}{2}\right\rangle$

$\frac{d\vec{r}}{d\theta} \times \frac{d\vec{r}}{d\phi}$ = Normal to the Surface

$\Rightarrow$ Ex $(u, v) = (\frac{1}{2}, 3)$

$u \rightarrow v$ - curve

$v \rightarrow u$ - curve

محاضرة الاثنين ماقبل العيد

No.

حاليا مش موجودة

11/7

No.

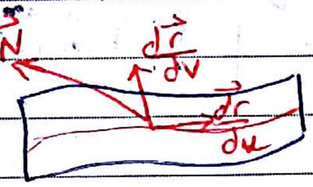
$x = x(u,v)$

$y = y(u,v) \qquad u \in [a,b]$

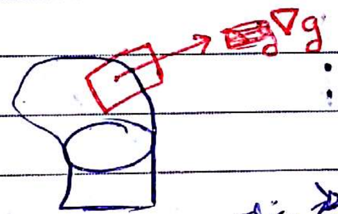
$z = z(u,v) \qquad v \in [c,d]$

$\vec{r}(u,v) = x(u,v)\hat{i} + y(u,v)\hat{j} + z(u,v)\hat{k}$

$\vec{N}(u,v) = \frac{d\vec{r}}{du} \times \frac{d\vec{r}}{dv}$

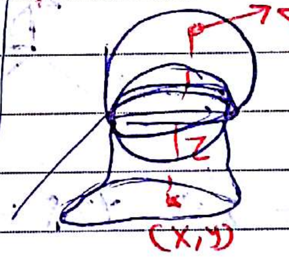


$g = g(x,y,z) = c$



سطح مغلق

$z = f(x,y) \rightarrow g = z - f = 0$

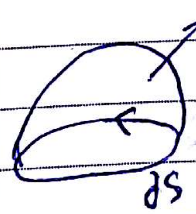


غير متعاكسات

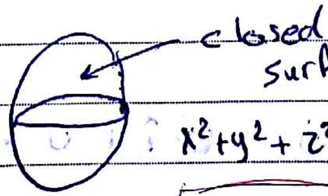
~~OR~~ orientable : له سطح داخلي وسطح خارجي ex: Sphere

اذا ما استطعت الانتقال من نقطة أعلى السطح إلى النقطة أسفل السطح

~~non orientable : ex: Half of a Sphere~~

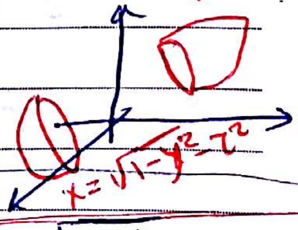


$z = \sqrt{1-x^2-y^2}$



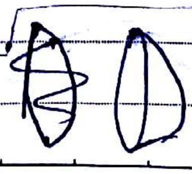
$x^2+y^2+z^2=1$

$x = -\sqrt{1-y^2-z^2}$



$x = \sqrt{1-y^2-z^2}$

$y = -\sqrt{1-x^2-z^2}$



$z = -\sqrt{1-x^2-y^2}$

$y = \sqrt{1-x^2-y^2}$

No.

Surface Integral

ex $\vec{F} = f_1(x,y,z)\hat{i} + f_2(x,y,z)\hat{j} + f_3(x,y,z)\hat{k}$

Defined on S

$$\boxed{\iint_S \vec{F}\cdot\vec{n}\, ds}$$ out flux

$x = x(u,v)$

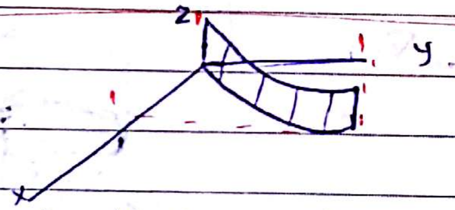
$y = y(u,v)$ $\quad u \in [a,b]$

$z = z(u,v)$ $\quad v \in [c,d]$

$\vec{N}(u,v) = \frac{d\vec{r}}{du} \times \frac{d\vec{r}}{dv}$

$\vec{r}(u,v) = x(u,v)\hat{i} + y(u,v)\hat{j} + z(u,v)\hat{k}$

ex



$y = x^2$

$z = z$

$\vec{F} = xy\hat{i} + yz\hat{j} + xy\hat{k}$ $\quad x,y \in [0,1]$

sol:

X و Z المتغيرات هم ال u و V

هلا مرحلة ال Parametric equations

$x = u$

$y = u^2$ $\quad u,v \in [0,1]$

$z = v$

$\vec{F}(u,v) = u^3\hat{i} + u^2 v\hat{j} + u^3\hat{k}$

$\cancel{\vec{N} = \frac{dz}{du} \times \frac{dr}{dv}}$

No.

$$\vec{N} = \frac{d\vec{r}}{du} \times \frac{d\vec{r}}{dv}$$

$\cancel{\frac{d\vec{r}}{du} =}$ $\quad \vec{r} = x(u,v)\,\hat{i} + y(u,v)\,\hat{j} + z(u,v)\,\hat{k}$

$$\vec{r} = u\,\hat{i} + u^2\,\hat{j} + v\,\hat{k}$$

$$\frac{d\vec{r}}{du} = \hat{i} + 2u\,\hat{j} \qquad \frac{d\vec{r}}{dv} = \hat{k}$$

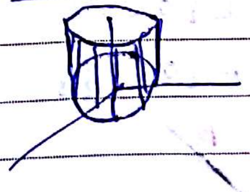
$$\vec{N} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2u & 0 \\ 0 & 0 & 1 \end{vmatrix} = -\hat{j} + 2u\,\hat{i}$$

$$\iint_S \vec{F}\cdot\vec{n}\,ds = \int_0^1\int_0^1 (2u^4 - u^2 v)\,du\,dv$$

$$= \int_0^1 \frac{2u^5}{5} - \frac{u^3}{3}v \Big|_0^1 dv$$

$$= \frac{2}{5} - \frac{1}{6}$$

ex: $\cancel{\vec{F} = f_1(x,y,z)\,\hat{i} + f_2(x,y,z)\,\hat{j} + f_3(x,y,z)\,\hat{k}}$



$$\vec{F} = xy\,\hat{i} + yz\,\hat{j} + xy\,\hat{k}$$

Sol:

$x = \cos\theta \qquad \theta \in [0, 2\pi]$

$y = \sin\theta \qquad v \in [0, 1]$

$z = v$

$$\vec{F}(\theta, v) = \sin\theta\cos\theta\,\hat{i} + v\sin\theta\,\hat{j} + v\cos\theta\,\hat{k}$$

$$\vec{r}(\theta, v) = \cos\theta\,\hat{i} + \sin\theta\,\hat{j} + v\,\hat{k}$$

No.

$$\vec{N} = \frac{dr}{d\theta} \times \frac{dr}{dv}$$

$$\frac{dr}{d\theta} = -\sin\theta\,\hat{i} + \cos\theta\,\hat{j} + 0\hat{k}$$

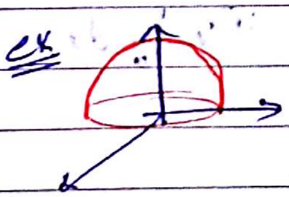
$$\frac{dr}{dv} = \hat{k}$$

$$\vec{N} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} = \sin\theta\,\hat{j} + \cos\theta\,\hat{i}$$

$$\iint_S \vec{F}\cdot\vec{n}\,ds = \int_0^{2\pi}\int_0^{1} (\sin\theta\cos^2\theta + v\sin^2\theta)\,dv\,d\theta$$

---

ex



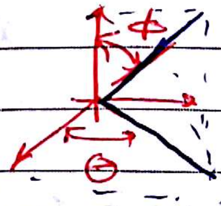
$$\vec{F} = xy\,\hat{i} + yz\,\hat{j} + xz\,\hat{k}$$

Sol

$x = \cos\theta\,\sin\phi$

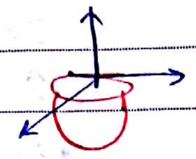
$y = \sin\theta\,\sin\phi$ $\quad \theta \in [0, 2\pi]$

$z = \cos\phi$ $\quad \phi \in [0, \frac{\pi}{2}]$



No.

$$\vec{F}(\theta,\phi) = \sin\theta\cos\theta\sin^2\phi\,\hat{i} + \sin\theta\sin\phi\cos\phi\,\hat{j} + \cos\theta\sin\phi\cos\phi\,\hat{k}$$



$\phi \in [-\frac{\pi}{2}, \pi]$

$$\vec{r}(\theta,\phi) = \cos\theta\sin\phi\,\hat{i} + \sin\theta\sin\phi\,\hat{j} + \cos\phi\,\hat{k}$$

$$\vec{N} = \frac{dr}{d\theta} \times \frac{dr}{d\phi}$$

$$\frac{dr}{d\theta} = -\sin\theta\sin\phi\,\hat{i} + \cos\theta\sin\phi\,\hat{j}$$

$$\frac{dr}{d\phi} = \cos\theta\cos\phi\,\hat{i} + \sin\theta\cos\phi\,\hat{j} + -\sin\phi\,\hat{k}$$

$$\vec{N} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -\sin\theta\sin\phi & \cos\theta\sin\phi & 0 \\ \cos\theta\cos\phi & \sin\theta\cos\phi & -\sin\phi \end{vmatrix}$$

$$\cancel{= 1 - \sin^2\theta \cdot \tfrac{1}{2}\sin 2\phi}$$

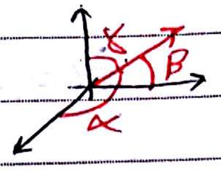
$$\iint_S \vec{F}\cdot\vec{N}\,ds = \int_0^{\pi/2}\int_0^{2\pi} \vec{F}\cdot\vec{N}\,d\theta\,d\phi$$

12/7

No.

$z = f(x,y)$

$\hat{n} = \cos\alpha\,\hat{i} + \cos\beta\,\hat{j} + \cos\gamma\,\hat{k}$

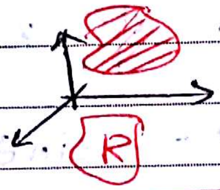


$\cancel{*\cos^2\alpha + \cos^2\beta}$

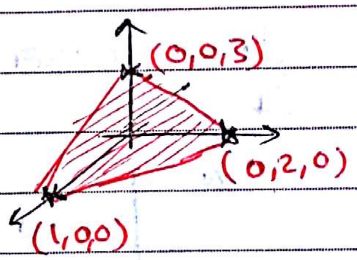
$* \cos^2\alpha + \cos^2\beta + \cos\gamma = 1$

$* \iint_S \vec{F}\cdot\hat{n}\,ds = \iint_S \vec{F}\cdot\vec{ds}, \quad \vec{ds} = \hat{n}\,ds$

$= \iint_R \vec{F}\cdot\hat{n}\,\frac{dx\,dy}{\cos\gamma}$



Ex: $\vec{F} = x^2\hat{i} + yx\hat{j} + xz\hat{k}$



Sol:

معادلة ال plane ← $\boxed{g = \frac{x}{1} + \frac{y}{2} + \frac{z}{3} = 1}$

$g = 6x + 3y + 2z = 6$

$\boxed{\hat{n} = \frac{\nabla g}{|\nabla g|}} = \frac{6\hat{i} + 3\hat{j} + 2\hat{k}}{\sqrt{36+9+4}} = \frac{6}{7}\hat{i} + \frac{3}{7}\hat{j} + \frac{2}{7}\hat{k}$

$\iint_S \vec{F}\cdot\hat{n}\,ds = \iint \vec{F}\cdot\vec{ds}$

$* \vec{F}\cdot\hat{n} = \underbrace{\frac{6}{7}x^2 + \frac{3}{7}xy + \frac{2}{7}xz}$ → بقسم على cosγ عشان أسقطها على ال xy-plane

$\frac{\vec{F}\cdot\hat{n}}{\cos\gamma} = \frac{7}{2}(\text{''''''''''})$

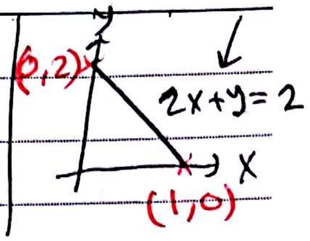
$z = \frac{1}{2}(6-6x-3y)$

لأنه اسقطها على $z=0$

$6x+3y=6$

No.

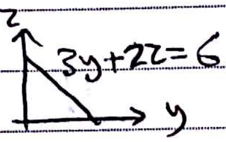
$$\iint \vec{F}\cdot\hat{n}\,ds = \int_0^2\int_0^{\frac{1}{2}(2-y)} \left(3x^2 + \frac{3}{2}xy + xz\right)dx\,dy$$



* اذا بدي اسقطها على YZ plane

$$\iint \vec{F}\cdot\hat{n}\,ds = \iint \vec{F}\cdot\hat{n}\,\frac{dy\,dz}{\cos\alpha}$$

$x = \frac{1}{6}(6-3y-2z)$

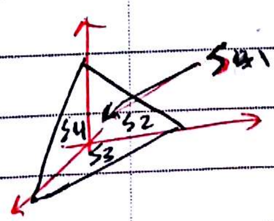


* $\dfrac{\vec{F}\cdot\hat{n}}{\cos\alpha} = \dfrac{7}{6}\left(\dfrac{6}{7}x^2 + \dfrac{3}{7}xy + \dfrac{2}{7}xz\right)$

$$\iint \vec{F}\cdot\hat{n}\,ds = \int_0^3\int_0^{\frac{1}{6}(6-2z)} \left(x^2 + \frac{3}{6}xy + \frac{1}{3}xz\right)dy\,dz$$

* اذا بدي اسقطها على xz-plane

$$\iint \vec{F}\cdot\hat{n}\,ds = \iint \vec{F}\cdot\hat{n}\,\frac{dx\,dz}{\cos\beta}$$



$$\iint_S \vec{F}\cdot\vec{n}\,ds = \iint_{S_1} \vec{F}\cdot\vec{ds} + \iint_{S_2} \vec{F}\cdot\vec{ds} + \iint_{S_3} \vec{F}\cdot\vec{ds} + \iint_{S_4} \vec{F}\cdot\vec{ds}$$

* S_1 حلها up

* S_2 :
1. $x=0$
2. $\hat{n} = -\hat{i}$
3. $\vec{F}\cdot\vec{n} = -x^2$

$-x^2 = 0$

* $\iint_{S_2} \vec{F}\cdot\hat{n}\,ds = \text{zero}$

* S_3
1. $z=0$
2. $\hat{n} = -\hat{k}$
3. $\vec{F}\cdot\hat{n} = -xz = 0$
4. $ds = dx\,dy$

* $\iint_{S_3} 0\,dx\,dy = \text{zero}$

No.

* S_4 ① $y=0$

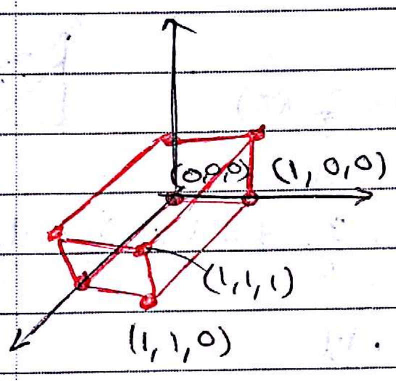
② $\hat{n} = -\hat{j}$

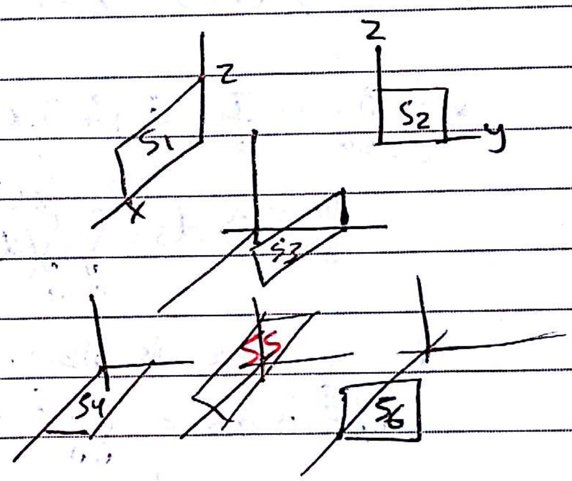
③ $\vec{F} \cdot \hat{n} = -yx = 0$

④ $ds = dxdy$

$\iint 0 \, dxdz = 0$

Ex: $\vec{F} = y\hat{i} + \hat{k}$ **H.W**

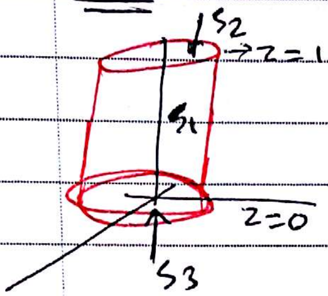




~~Ex: $\vec{F}$~~

No.

$\underline{ex}$: $\vec{F} = x\hat{i} + y\hat{j} + (z^2-1)\hat{k}$



$$\iint \vec{F}\cdot\hat{n}\,ds = \iint_{S_1} \vec{F}\cdot\hat{n}\,ds + \cancel{\iint_{S_2} \vec{F}\cdot\hat{n}\,ds}^{\,0}$$
$dxdy$

$$+ \cancel{\iint_{S_3} \vec{F}\cdot\hat{n}\,ds}$$
π^2 $dxdy$

مساحة الدائرة

$\hat{n} = \nabla g = 2x\hat{i} + 2y\hat{j}$

$\hat{n} = x\hat{i} + y\hat{j}$

$$\iint_{S_1} (x^2+y^2)\,\frac{dz\,dx}{\cos\beta}\ ,\ \cos\beta = y\ ,\ y = \sqrt{1-x^2}$$

$$= \iint \left(x^2 + (1-x^2)\right) \frac{dz\,dx}{\sqrt{1-x^2}}$$

$$= \int_{-1}^{1} \frac{dx}{\sqrt{1-x^2}}$$