أحمد غالب محمد ، الفقهاء 0110443

University of Jordan

ELECT. ENG DEPT.

Numerical Methods EE301 First Exam Sun. 27th Oct. 2013 Mark out of 20 11

Answer all questions in ink

2 bit → signf.
1 bit → exp.
1 bit → sign
1 bit → sign exp.

Q1 A decimal computer of **2** digits for the significand, **1** digit for the exponent ,**1** digit for the number sign, and **1** digit for the exponent sign . Furthermore, this computer uses **number normalization** and does number chopping. Use this computer to answer the following questions (4 marks).

0	0	9	9	9

What is the **positive floating point number** range given by this computer? $+1.0*10^{-9}$ to $+9.9*10^{+9}$

What result this computer gives for $(1/13)^2$ $+7.6\cancel{\ldots}*10^{-2}$

What result this computer gives for $(10^7 \times 10^8) \times 10^{-9}$ NAN overflow 3

10^{+5}

∞

Q2 A **hypothetical decimal** computer is allowed a **total of 5 decimal** digits for storage. (3marks).

0	0	9	9	9

Do the proper digit allocation to get **maximum natural** number, which is $+9.9*10^{+9}$

Do the proper digit allocation to get **maximum positive integer** number, which is $+9.999$ 0

Do the proper digit allocation to get **minimum positive floating point** number, which is $+9.999$

0 9 9 9 9

Q3 A convergent iterative process to calculate the cosine of an angle ends with following three last estimates for $\cos(\frac{\pi}{3})$, which are 0.44 , 0.46, and 0.48. (3 marks).

$= 0.5 - 0.48$

What is the **absolute minimum true** error ? in 0.48 ⟹ absolue true $= 0.02$

What is the **minimum relative true** error ? in 0.48 ⟹ relative true error $= \frac{0.5-0.48}{0.5} * 100\%$

What is the **percent minimum approximate relative** error ? $= 4\%$

$$= \frac{0.48 - 0.44}{0.48} * 100\% = 4.167\%$$

2.5

Q4 Given the expansion about the **origin** for the exponential function e^x (5 marks)

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!} + \dots$$

i) Use the first 4 terms to estimate $e^{0.2}$ what is the best possible **percent true relative error**?.

4 termes is the best possible

true value $= 1.221402758$

① $\boxed{e^{0.2} = 1}$ $|\epsilon_t| = 18.13\%$

② $\boxed{e^{0.2} = 1 + 0.2 = 1.2}$ $|\epsilon_t| = 1.75\%$

③ $e^{0.2} = 1 + 0.2 + \frac{(0.2)^2}{2!} = 1.22$

$|\epsilon_t| = 0.115\%$

④ $e^{0.2} = 1 + 0.2 + \frac{(0.2)^2}{2!} + \frac{(0.2)^3}{3!} = 1.2213333$

$\epsilon_t = 5.68 * 10^{-3}\%$

2

ii) If the first 4 terms were used to estimate $e^{0.2}$, use the error term R_n to determine the **maximum** error involved, and calculate the value of ζ which accounts for this error.

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{(\xi)^4}{4!}$$

$$e^{0.2} = 1 + 0.2 + \frac{(0.2)^2}{2!} + \frac{(0.2)^3}{3!} + (e^{0.2} - x)$$

$$x = 1.221333333$$

$$\frac{(\xi)^4}{24} = e^{0.2} - x \Rightarrow \xi = \sqrt[4]{(e^{0.2} - x) * 24}$$

$$\boxed{\xi = 0.2020\cancel{37}2794}$$

$R_n = 6.9425 * 10^{-5}$

1

Q5 Given $f(x) = e^{-x} - x$, and the intervals [-0.5 0.1] and [0 1] (5 marks)

i)Which interval encloses a zero of $f(x)$? Why?

$\cancel{[-0.5, 0.1]}$ because ~~f(0) = 0.80 f(1) = -0.63212~~

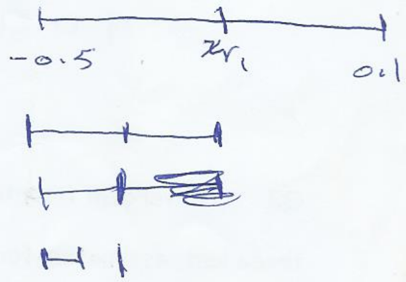
$f(0.1) = 0.80483$ | $f(-0.5) = 2.1487$

$f(0.1)\,f(-0.5) = -ve$ ✓

& $\not\propto f(0)\,f(0.1) = 0$

ii)Use the **false position** method in that interval to estimate the zero after performing **three** iterations. What is the **percent approximate relative** error after the third iteration.

$$x_r = x_u - \frac{f(x_u)\,(x_l - x_u)}{f(x_l) - f(x_u)}$$



$x_{r_1} = 0.459333472 1$

$f(x_{r_1}) = +0.172371082$

$x_{r_2} = -0.375634$

$x_{r_3} = 0.3425782238$

$f(x_{r_3}) = 0.367359 3594$

$x_{r_4} = 0.54112$

0

2

$e_a = ?$