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University of Jordan

Electrical Engineering Dept.

Engineering Numerical Methods (0903301)

Fall Term 2014 - 2015

First term Exam (25%)

Instructor: Dr. Andraws Swidan

Date: Monday 3/11/2014

Time allowed: One hour

Student Name: _____ Student Number: 0110 494

Select the correct answer for each of the MCQ and write the answer for short answer questions:Consider the function $f(x) = x^2$. The approximation of $f(0.9)$ using linear interpolation between the points $x = 0$ and $x = 2$ gives;

Secant
 $x_0 = 1$
 $x_1 = 0$

a) 1.2

b) 1.4

c) 1.6

d) 1.8

$$P_1(x) = f(x_1) + \frac{f(x_2) - f(x_1)}{(x_2 - x_1)}(x - x_1)$$

$$f(0.9) = f(0) + \frac{f(2) - f(0)}{(2 - 0)}(0.9 - 0)$$

(2) *

$$\frac{4}{2} \times (0.9 - 0)$$

Consider the following data:

X	2	2.2	2.4	2.6	2.8	3
Y	4	3	2	4	6	12

What is the highest degree of [Lagrange polynomial] that can be constructed?

A

5 B 6

$$L_i = \frac{x - x_2}{x_i - x_2} \quad L = L_1(x-x_1)$$

If cubic Lagrange interpolating polynomial is used, choose the appropriate points and estimate the value of y at $x = 2.9$ is;

A

0.25

$$P(x) = L_1 [] + L_2 [(x-x_1)] + L_3 [(x-x_1)(x-x_2)]$$

For the given data an approximation function of the form $f(x) = axe^{bx}$ is suggested.

Using least-square regression, find a and b.

$\rightarrow x$	1.0	2.0	3.0	4.0
$\rightarrow y$	1.5	3.5	6.9	11
$\rightarrow \ln y$	0.405	1.25	1.93	2.397

$$\frac{\beta}{\alpha} \propto e^{\beta x}$$

a =	b =
-----	-----

1

log

$$\begin{cases} a = a_0 \\ b = 10^{a_0} \end{cases}$$

$$a_0 = \bar{a}_0 Y + \bar{b}_0 X$$

(1)

Referring to the table below, the value of M corresponding to T=25 and W=0 is:

A

T=25

	W=-10	W=5
T=15	M=160	M=180
T=40	M=130	M=140

$$M = \boxed{167}$$

$$S(x) = a_0 + b_0 (x_{i+1})$$



Using Newton-Raphson method, the 1st iteration approximation of $\tan^{-1}(0.3)$ starting with $x_0 = 0.1000$ is:

$$\boxed{0.9676}$$

$$f(x) = -\frac{f(x)}{f'(x)}$$

$$\frac{f(x) - f(x_i)}{x - x_i} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$\frac{1}{1 + (0.1)^2} = \frac{1}{1}$$

Using Bisection method, if it is desired to reduce approximation error (E_a) below 10%, the maximum number of required iterations is: (assume a unit interval)

a) 5

b) 3

c) 10

d) 4

$$E_a = \frac{\text{new} - \text{old}}{\text{new}}$$

x; x_{i+1} Using secant method with $x_0=1$, the 1st iteration estimate of $x=e^{-x}$ is:

$$\boxed{a) -0.6458}$$

$$\boxed{b) -0.6321}$$

$$\boxed{c) 0.5373}$$

$$\boxed{d) 1.01}$$

$$x_{i+1} = e^{-x_i}$$

$$x_1 = e^{-x_0}$$

$$x_1 = e^{-1}$$

Solving $e^x - 2x = 1$, with $x_0=1$ by fixed point iteration, the 2nd iteration estimate of x is:

$$\boxed{-0.537349}$$

e^x

$$\begin{aligned} x_{i+1} &= e^{-x_i} \\ x_0 &= e^{-x_0} \end{aligned}$$

$$x_{i+1}$$

$$x = e^{-x/2}$$

$$\boxed{1.3611}$$

Solve the following equation using three fixed point iterations:

$$\boxed{e^x - 2x = 1}$$

$$\boxed{x_0 = 1}$$

$$x_{i+1}$$

$$x_0 = e^{-x_i}$$

$$x_1 = e^{-x_0}$$

$$\begin{aligned} x_{i+1}, x_0 &\\ 0 & 0.361 \\ 1 & 0.6 - 0.3 \\ 2 & x_{i+1} = e^{-x_i/2} \\ 3 & x_{i+1} = e^{-x_i/2} \\ 4 & x_{i+1} = e^{-x_i/2} \end{aligned}$$

$$\begin{aligned} x_{i+1} &= e^{-x_i/2} \\ x_0 &= 1 \\ x_1 &= e^{-1/2} \\ x_2 &= e^{-0.5/2} \\ x_3 &= e^{-0.25/2} \\ x_4 &= e^{-0.125/2} \\ x_5 &= e^{-0.0625/2} \\ x_6 &= e^{-0.03125/2} \\ x_7 &= e^{-0.015625/2} \\ x_8 &= e^{-0.0078125/2} \\ x_9 &= e^{-0.00390625/2} \\ x_{10} &= e^{-0.001953125/2} \\ x_{11} &= e^{-0.0009765625/2} \\ x_{12} &= e^{-0.00048828125/2} \\ x_{13} &= e^{-0.000244140625/2} \\ x_{14} &= e^{-0.0001220703125/2} \\ x_{15} &= e^{-0.00006103515625/2} \\ x_{16} &= e^{-0.000030517578125/2} \\ x_{17} &= e^{-0.0000152587890625/2} \\ x_{18} &= e^{-0.00000762939453125/2} \\ x_{19} &= e^{-0.000003814697265625/2} \\ x_{20} &= e^{-0.0000019073486328125/2} \\ x_{21} &= e^{-0.00000095367431640625/2} \\ x_{22} &= e^{-0.000000476837158203125/2} \\ x_{23} &= e^{-0.0000002384185791015625/2} \\ x_{24} &= e^{-0.00000012020928955078125/2} \\ x_{25} &= e^{-0.000000060104644775390625/2} \\ x_{26} &= e^{-0.0000000300523223876953125/2} \\ x_{27} &= e^{-0.00000001502616119384765625/2} \\ x_{28} &= e^{-0.000000007513080596923828125/2} \\ x_{29} &= e^{-0.0000000037565402984619140625/2} \\ x_{30} &= e^{-0.00000000187827014923095703125/2} \\ x_{31} &= e^{-0.000000000939135074615478515625/2} \\ x_{32} &= e^{-0.0000000004695675373077392578125/2} \\ x_{33} &= e^{-0.00000000023478376865386962890625/2} \\ x_{34} &= e^{-0.000000000117391884326934814453125/2} \\ x_{35} &= e^{-0.0000000000586959421634674072265625/2} \\ x_{36} &= e^{-0.00000000002934797108173370361328125/2} \\ x_{37} &= e^{-0.0000000000146739855408668518059375/2} \\ x_{38} &= e^{-0.0000000000073369927704334259029688/2} \\ x_{39} &= e^{-0.0000000000036684963852167129514843/2} \\ x_{40} &= e^{-0.0000000000018342481926083564757421/2} \\ x_{41} &= e^{-0.000000000000917124096304178237871/2} \\ x_{42} &= e^{-0.000000000000458562048152089118955/2} \\ x_{43} &= e^{-0.000000000000229281024076044559477/2} \\ x_{44} &= e^{-0.000000000000114640512038022279738/2} \\ x_{45} &= e^{-0.000000000000057320256019011139869/2} \\ x_{46} &= e^{-0.000000000000028660128009505569934/2} \\ x_{47} &= e^{-0.000000000000014330064004752784969/2} \\ x_{48} &= e^{-0.000000000000007165032002376392484/2} \\ x_{49} &= e^{-0.000000000000003582516001188196242/2} \\ x_{50} &= e^{-0.000000000000001791258000594098121/2} \\ x_{51} &= e^{-0.000000000000000895629000297004806/2} \\ x_{52} &= e^{-0.000000000000000447814500148502003/2} \\ x_{53} &= e^{-0.000000000000000223907250074251001/2} \\ x_{54} &= e^{-0.000000000000000111953625037125500/2} \\ x_{55} &= e^{-0.000000000000000055976812518562750/2} \\ x_{56} &= e^{-0.000000000000000027988406259281375/2} \\ x_{57} &= e^{-0.0000000000000000139942031254916875/2} \\ x_{58} &= e^{-0.00000000000000000699710156252484375/2} \\ x_{59} &= e^{-0.0000000000000000034985507812512421875/2} \\ x_{60} &= e^{-0.00000000000000000174927539687506209375/2} \\ x_{61} &= e^{-0.0000000000000000008746376984375290475/2} \\ x_{62} &= e^{-0.0000000000000000004373188492187512375/2} \\ x_{63} &= e^{-0.00000000000000000021865942460937568875/2} \\ x_{64} &= e^{-0.000000000000000000109329712304687534375/2} \\ x_{65} &= e^{-0.00000000000000000005466485615234375175/2} \\ x_{66} &= e^{-0.0000000000000000000273324280761718750875/2} \\ x_{67} &= e^{-0.00000000000000000001366621403808593754375/2} \\ x_{68} &= e^{-0.0000000000000000000068331070219042937521875/2} \\ x_{69} &= e^{-0.000000000000000000003416553510952146875109375/2} \\ x_{70} &= e^{-0.0000000000000000000017082767554760734375546875/2} \\ x_{71} &= e^{-0.00000000000000000000085413837773803671875234375/2} \\ x_{72} &= e^{-0.0000000000000000000004270691888690183593751171875/2} \\ x_{73} &= e^{-0.000000000000000000000213534594434509179687505859375/2} \\ x_{74} &= e^{-0.0000000000000000000001067672972172545898437529734375/2} \\ x_{75} &= e^{-0.000000000000000000000053383648608627394921875148734375/2} \\ x_{76} &= e^{-0.0000000000000000000000266918243043136974609375743671875/2} \\ x_{77} &= e^{-0.0000000000000000000000133459121521568487304687537334375/2} \\ x_{78} &= e^{-0.00000000000000000000000667295607607842436523437518671875/2} \\ x_{79} &= e^{-0.00000000000000000000000333647803803921218267187509334375/2} \\ x_{80} &= e^{-0.000000000000000000000001668239019019606091359375046671875/2} \\ x_{81} &= e^{-0.0000000000000000000000008341195095098030475796875023334375/2} \\ x_{82} &= e^{-0.00000000000000000000000041705975475490152378984375016671875/2} \\ x_{83} &= e^{-0.000000000000000000000000208529877377495769894921875008334375/2} \\ x_{84} &= e^{-0.00000000000000000000000010426493868874788494746093750041671875/2} \\ x_{85} &= e^{-0.000000000000000000000000052132469234373942473730468750020834375/2} \\ x_{86} &= e^{-0.000000000000000000000000026066234617186971236865234375001041875/2} \\ x_{87} &= e^{-0.000000000000000000000000013033117308593485618332656250005209375/2} \\ x_{88} &= e^{-0.00000000000000000000000000651655865429674280916312500260484375/2} \\ x_{89} &= e^{-0.000000000000000000000000003258279327148371404581562500130241875/2} \\ x_{90} &= e^{-0.0000000000000000000000000016291396635741857022907812500065121875/2} \\ x_{91} &= e^{-0.00000000000000000000000000081456983178709285114539843750032561875/2} \\ x_{92} &= e^{-0.00000000000000000000000000040728491589354642557274843750016281875/2} \\ x_{93} &= e^{-0.00000000000000000000000000020364245794677321278637437500081341875/2} \\ x_{94} &= e^{-0.0000000000000000000000000001018212289733866063931875000406709375/2} \\ x_{95} &= e^{-0.000000000000000000000000000050910614486693303196593750020345484375/2} \\ x_{96} &= e^{-0.000000000000000000000000000025455307243346651598296875001017274375/2} \\ x_{97} &= e^{-0.00000000000000000000000000001272765362167332579914843750005086375/2} \\ x_{98} &= e^{-0.00000000000000000000000000000636382681083666289957437500025431875/2} \\ x_{99} &= e^{-0.000000000000000000000000000003181913405418331449787187500127159375/2} \\ x_{100} &= e^{-0.000000000000000000000000000001590956702709165724893750000635796875/2} \\ x_{101} &= e^{-0.0000000000000000000000000000007954783513545828624468750000317896875/2} \\ x_{102} &= e^{-0.00000000000000000000000000000039773917567729143122343750001589484375/2} \\ x_{103} &= e^{-0.0000000000000000000000000000001988695878386457156117812500079474375/2} \\ x_{104} &= e^{-0.00000000000000000000000000000009943479391932285780589375000397371875/2} \\ x_{105} &= e^{-0.0000000000000000000000000000000497173969596614289529484375001986875/2} \\ x_{106} &= e^{-0.0000000000000000000000000000000248586984798307144764743750009934375/2} \\ x_{107} &= e^{-0.00000000000000000000000000000001242934923991535723823750000496721875/2} \\ x_{108} &= e^{-0.00000000000000000000000000000000621467461995767861911875000248364375/2} \\ x_{109} &= e^{-0.00000000000000000000000000000000310733730997883930955937500124181875/2} \\ x_{110} &= e^{-0.000000000000000000000000000000001553668654989419654779687500062090625/2} \\ x_{111} &= e^{-0.000000000000000000000000000000000776834327494709827389687500031049375/2} \\ x_{112} &= e^{-0.00000000000000000000000000000000038841716374735491369484375000155234375/2} \\ x_{113} &= e^{-0.000000000000000000000000000000000194208581873677456847437500007761875/2} \\ x_{114} &= e^{-0.00000000000000000000000000000000009710429093683872842375000038809375/2} \\ x_{115} &= e^{-0.0000000000000000000000000000000000485521454683193642187500001940484375/2} \\ x_{116} &= e^{-0.0000000000000000000000000000000000242760727341596821093750000097024375/2} \\ x_{117} &= e^{-0.00000000000000000000000000000000001213803636707984105468750000485121875/2} \\ x_{118} &= e^{-0.000000000000000000000000000000000006069018183539920527343750000242561875/2} \\ x_{119} &= e^{-0.000000000000000000000000000000000003034509091769960263687500001212809375/2} \\ x_{120} &= e^{-0.0000000000000000000000000000000000015172545458849801318437500000606404375/2} \\ x_{121} &= e^{-0.00000000000000000000000000000000000075862727294449006592187500003032021875/2} \\ x_{122} &= e^{-0.0000000000000000000000000000000000003793136364722450329609375000015160109375/2} \\ x_{123} &= e^{-0.00000000000000000000000000000000000018965681823612250148048437500007580054375/2} \\ x_{124} &= e^{-0.00000000000000000000000000000000000009482840911806125074024375000037900271875/2} \\ x_{125} &= e^{-0.0000000000000000000000000000000000000474142045590306253701218750000189501375/2} \\ x_{126} &= e^{-0.00000000000000000000000000000000000002370710227951531251853437500000947506875/2} \\ x_{127} &= e^{-0.000000000000000000000000000000000000011853551139757656259267187500004737534375/2} \\ x_{128} &= e^{-0.0000000000000000000000000000000000000059267755698788281254634375000023687571875/2} \\ x_{129} &= e^{-0.00000000000000000000000000000000000000296338778493941406252318750000118437571875/2} \\ x_{130} &= e^{-0.000000000000000000000000000000000000001481693892469707031251158750000592187571875/2} \\ x_{131} &= e^{-0.00000000000000000000000000000000000000074084694623495351562557937500002960937571875/2} \\ x_{132} &= e^{-0.000000000000000000000000000000000000000370423473117476753125289375000014804687571875/2} \\ x_{133} &= e^{-0.00000000000000000000000000000000000000018521173655873837562514453750000074023437571875/2} \\ x_{134} &= e^{-0.0092605868279369187531257223750000370117187571875/2} \\ x_{135} &= e^{-0.0046302934139684593751562536187500001850585937571875/2} \\ x_{136} &= e^{-0.000$$

The formula is

$$P(x) = 1 + (x) + \frac{x^2}{2!} + \frac{x^3}{3!}$$

1.5

Write the third degree Taylor polynomial for $f(x) = \cos x$ about $x_0=0$

$$\cos x = 1$$

$$\cos(x) = 1 + x$$

$$\cos(x) = 1 + \cancel{\frac{\pi^2}{2!}} + \cancel{\frac{(\pi^2)^2}{2!}} + \cancel{\frac{(\pi^2)^3}{3!}}$$

$$\cos(0) = 1$$

$$1 \rightarrow 1$$

$$\cos(x) = 1 \Rightarrow x = 2\pi$$

The number π has the form $\pi = 3.14159265\dots$. Write it using 5 significant digits using rounding and chopping

Rounded π	Chopped π
3.14159	3.14159

$$\cos(x) = \frac{2\pi}{2\pi} + 2\pi \Rightarrow x = \frac{\pi}{2}$$

$$\cos(x) = 1 + \frac{\pi}{2} + \frac{(\pi)^2}{2!}$$

The bisection algorithm was used to find a root of a function. The calculations revealed the following:

Estimate P3

P1	P2	P3
1.5	1.25	1.375

$$\frac{P_2 + P_1}{2} = \frac{1.5 + 1.25}{2} = 1.375$$

The equation $x^3 + 4x^2 - 10 = 0$ has a unique root in $[1,2]$. The fixed point form suggested is

$$G(x) = x - (x^3 + 4x^2 - 10)/(3x^2 + 8x)$$

If $P_0 = 1.5$, find P_1 with 3 decimal digits

1.375

Given the following table:

x	F(x)	Zero divided difference	First divided difference	Second divided difference
8.1	16.94			
8.3	17.56			
8.6	18.50			
8.7	18.282			

- A) Shadow the cells that will be filled
 B) What is the highest degree of divided difference possible?

4	X
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- C) Write the second degree interpolating polynomial using Newton's forward divided difference formula and use it to estimate $f(8.2)$.

$$F(x) = f(x_0) + \frac{f(x_1) - f(x_0)}{x_1 - x_0} x + \frac{f(x_2) - f(x_1)}{x_2 - x_1} x^2$$

Given a table of three points $(X_0, X_1, \text{ and } X_2)$ and the value of the function at these points $(f(X_0), f(X_1) \text{ and } f(X_2))$ and the value of $f'(X_0)$ and $f'(X_2)$

- a) How many quadratic splines can be constructed?

A

9	8	6
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x_0, x_1, x_2
 $f(x_0), f(x_1), f(x_2)$



- b) How many cubic splines can be constructed?

A

27	8
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- c) Write the equations governing the construction of quadratic splines.

$$S_i(x) = a_i + b_i(x_{i+1} - x_i) + c_i(x_{i+1} - x_i)^2$$

$$a_i = f_i$$

$$S_i(x_0) = 0$$

$$S_i(x_2) = S_3(x_2)$$

$$S_3(x_3) = S_0(x_3)$$

$$h_i = x_{i+1} - x_i$$

a_i
 b_i
 4

d) Write the equations governing the construction of clamped cubic splines.

$$S_3(x) = a_i + b_i(x - x_i) + c_i(x - x_i)^2 + d_i(x - x_i)^3$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ h_1 & 2(h_1+h_2) & h_2 & h_2 & h_2 \\ 0 & h_2 & 2(h_2+h_3) & h_3 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 3f(x_2) - 3f(x_1) \\ 3f(x_3) - 3f(x_2) \\ f(x_1) \end{bmatrix}$$

$$a_i = f_i$$

$$h_i = x_{i+1} - x_i$$

$$b_i + 2c_i h_i = b_{i+1}$$

$$f_i + b_i h_i + c_i h_i^2 = f_{i+1}$$

$$b_i = \frac{f_{i+1} - f_i}{h_i} - \frac{h_i}{3}(2c_i + c_{i+1})$$