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University of Jordan

Electrical Engineering Dept.

Engineering Numerical Methods (0903301)
First term Exam (25%)

Date: Monday 3/11/2014

Fall Term 2014 - 2015
Instructor: Dr. Andraws Swidan

Time allowed: One hour

Student Name: [illegible] Student Number: 0110494

Select the correct answer for each of the MCQ and write the answer for short answer questions:

← أحتاج إلى القوانين لحل الأسئلة

Consider the function $f(x) = x^2$. The approximation of $f(0.9)$ using linear interpolation between the points $x = 0$ and $x = 2$ gives;

Secant $X_0 = 1$ $X_1 = 0$

a) 1.2 b) 1.4 c) 1.6 d) 1.8

$P_1(x) = f(x_1) + \frac{f(x_2) - f(x_1)}{(x_2 - x_1)} * (x - x_1)$

$f(0.9) = \cancel{f(0)} + \frac{f(2) - f(0)}{(2)} *$

$\frac{4}{2} * (0.9 -)$

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Consider the following data ;

X	2	2.2	2.4	2.6	2.8	3
Y	4	3	2	4	6	12

What is the highest degree of Lagrange polynomial that can be constructed?

A $\boxed{5 \; \cancel{6}}$

$L_1 = \frac{X - X_2}{X_1 - X_2}$

If cubic Lagrange interpolating polynomial is used, choose the appropriate points and estimate the value of y at $x = 2.9$ is;

A $\boxed{0.25}$

$f(x) = L_1[\quad] + L_2[(x - x_1)] + L_3(x - x_1)(x - x_2)$

For the given data an approximation function of the form $f(x) = axe^{bx}$ is suggested. Using least-square regression, find a and b.

$\beta . x e^{\beta x}$

x	1.0	2.0	3.0	4.0
y	1.5	3.5	6.9	11
$Ln\, y$	0.405	1.25	1.93	2.397

a =	b =

log

$a = a_1$
$b = 10^{a_0}$

$a_0 = a_1\bar{Y} + B\bar{X}$

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Referring to the table below, the value of M corresponding to T=25 and W=0 is:

	W=-10	W=5
T=15	M=160	M=180
T=40	M=130	M=140

A

M = 167

Using Newton-Raphson method , the 1st iteration approximation of $tan^{-1}(0.3)$ starting with x_0 = 0.1000 is:

0.0678

Using Bisection method, if it is desired to reduce approximation error (E_a) below 10%, the maximum number of required iterations is: (assume a unit interval)

a) 5 b) 3 c) 10 d) 4

Using secant method with x_0=1, the 1st iteration estimate of $x=e^{-x}$ is:

a) -0.6458 b) -0.6321 c) 0.5373 d) 1.01

Solving e^x−2x=1, with x_0=1 by fixed point iteration, the 2nd iteration estimate of x is:

~~0.418749~~

1.3611

$x = \frac{e^x - 1}{2}$

Solve the following equation using three fixed point iterations:

$e^x - 2x = 1$ $x_0 = 1$

$x_{i+1} = \frac{e^{x_i} - 1}{2}$

0.559140

0.68056

1.5

The Formella:
$\Rightarrow f(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$

$\cos x = 1$
$x = 2\pi$

Write the third degree Taylor polynomial for f(x) = cos x about x_0=0

$\cos(x) = 1 + x$
$f(x) = 1 + x$

$\cos(x) = 1 + \frac{\pi}{2} + \frac{(\pi/2)^2}{2!} + \frac{(\pi/2)^3}{3!}$

$\cos(0) = 1$ $\quad 1 \to 1$ $\quad \cos(x) = 1 \Rightarrow x = 2\pi$

The number π has the form π = 3.14159265… . Write it using 5 significant digits using rounding and chopping

$\cos(x) = \frac{2\pi}{2\pi} + 2\pi \Rightarrow x = \frac{\pi}{2}$

$\frac{4\pi}{2}$ 2π

Rounded π	Chopped π
3.14159	3.14159

$\cos(x) = 1 + \frac{\pi}{2} + \frac{(\pi/2)^2}{2!}$

The bisection algorithm was used to find a root of a function. The calculations revealed the following:
Estimste P3

P1	P2	P3
1.5	1.25	1.375

$\frac{P_2+P_1}{2} = \frac{1.5+1.25}{2} = 1.375$

The equation x3 +4x2 -10 = 0 has a unique root in [1,2]. The fixed point form suggested is
G(x) = x – (x3 +4x2 -10)/(3x2+8x)
If P0=1.5 , find P1 with3 decimal digits

1.375

Given the following table:

x	F(x)	Zero divided difference	First divided difference	Second divided difference
8.1	16.94			
8.3	17.56			
8.6	18.50			
8.7	18.282			

A) Shadow the cells that will be filled

B) What is the highest degree of divided difference possible?

$\boxed{4}$

C) Write the second degree interpolating polynomial using Newton's forward divided difference formula and use it to estimate f(8.2).

$$F_1(x) = F(x_1) - \frac{f(x)}{f'(x)}$$

$$f'(x) = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Given a table of three points (X_0, X_1, and X_2) and the value of the function at these points ($f(X_0)$, $f(X_1)$ and $f(X_2)$) and the value of $f'(X_0)$ and $f'(X_2)$

a) How many quadratic splines can be constructed?

$x_0 \quad x_1 \quad x_2$

$f(x_0) \quad f(x_1) \quad f(x_2)$

A $\boxed{9 \; \cancel{5} \; \cancel{6}}$

b) How many cubic splines can be constructed?

A $\boxed{27 \; \cancel{8}}$

c) Write the equations governing the construction of quadratic splines.

$$S_2(x) = a_i + b_i (x_{i+1} - x_i) + c_i (x_{i+1} - x_i)^2$$

$$\boxed{a_i = f_i}$$

$S(x_0) = 0$

$S_2(x_2) = S_3(x_2)$

$S_3(x_3) = S_8(x_3)$

c_i

b_i

$$\boxed{h_i = x_{i+1} - x_i}$$

d) Write the equations governing the construction of clamped cubic splines.

$$S_3(x) = a_i + b_i (x_{i+1} - x_i) + c_i (x_{i+1} - x_i)^2 + b_i (x - x_i)^3$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ h_1 & 2(h_1+h_2) & h_2 & h_2 & h_2 \\ 0 & h_2 & 2(h_2+h_3) & h_3 & \\ 0 & 0 & 0 & 1 & \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 3f(x_3, x_2) - 3f(x_2, x_1) \\ 3f(x_4 x_3) - 3f(x_3 x_1) \\ \cancel{3f(x_4x_3)} x.1 \end{bmatrix}$$

$$a_0 = f_i$$

✓ $h_i = x_{i+1} - x_i$

✓ $b_i + 2c_i h_i = b_{i+1}$

✓ $f_i + b_0 h_i + c_0 h_i^2 = f_{i+1}$

$$b_i = \frac{f_{i+1} - f_i}{h_i} - \frac{h_i}{3}(2c_i + c_{i+1})$$