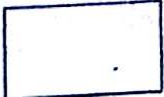


Numerical Methods 903301

Student Name:

Answers should be written in ink



Power unit-

Q1) From numerical methods point of view, in not more than one line, briefly answer

i) The advantage of **direct methods** for solving systems of equations over **iterative methods**.

.....

ii) The advantage of the **Newton method** over the **successive substitution method**

.....

iii) The advantage of the **secant method** over the **Newton Raphson method**.

.....

iv) A **difficulty** when calculating the **condition number**.

by a line

.....

v) A use of eigenvalues.....

vi) A use of matrix norms.....

vii) Write down the error formula involved with the **Newton-Raphson method** relating E_{i+1} to E_i .

Q2) a) Formulate the solution of $x^3 = \cos(x)$ as a zero finding problem, hence use the **fixed point method** to find the solution starting with $x^0=0.8$. Perform as many iteration needed to confirm convergence. What answer do you get?. Now, repeat the process once more starting with $x^0=0.8$ and write down the first 5 iterations to 10 significant digits.

b) To check the result obtained above, use the **Newton-Raphson method** starting with $x^0=0.8$. Write down the first 5 iterations to 10 significant digits .

Inspect the numbers obtained now to comment on the rate of convergence. What is it? Is it expected ?.

Q3) Consider the equations $Ax = b$; i.e.
$$\begin{bmatrix} 120 & 60 & 40 \\ 60 & 40 & 30 \\ 40 & 30 & 24 \end{bmatrix} x = \begin{bmatrix} 220 \\ 130 \\ 94 \end{bmatrix}$$

a) Use the naïve Gauss elimination method to determine the solution x .

b) Suppose now that **220** in the b matrix is changed to **200**, obtain the solution once more. Note that you can avoid repeating all the calculation done before. Be wise.

Is the new solution significantly different?. If so, why is that?. You may need the fact that the infinity norm of the inverse of A is **3.4**..

Q4) Given the system of equations $Ax = b$; i.e.
$$\begin{bmatrix} 0 & -1 & 4 \\ 8 & -2 & -1 \\ -1 & 5 & -2 \end{bmatrix} x = \begin{bmatrix} 10 \\ 1 \\ 3 \end{bmatrix}$$

for what reasons do we need to rearrange the equations when using iterative methods?

.....
.....

After rearrangement can you **confirm** convergence of your iterative solution?
List down linear algebra facts (not demonstrate by calculation).

Now **Calculate** by performing **two** iterations only using **Gauss-Seidel method** starting with $x^0 = [0.8 \ 1.8 \ 2.8]^T$.

Using vector norms, calculate the **relative approximate error** at the end of the **second** iteration.