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BY:

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* Revision :-

(ex) Calculate & sketch $i_c(t)$ when $v_c(t) = 5(1 - e^{-t/10^{-6}})$

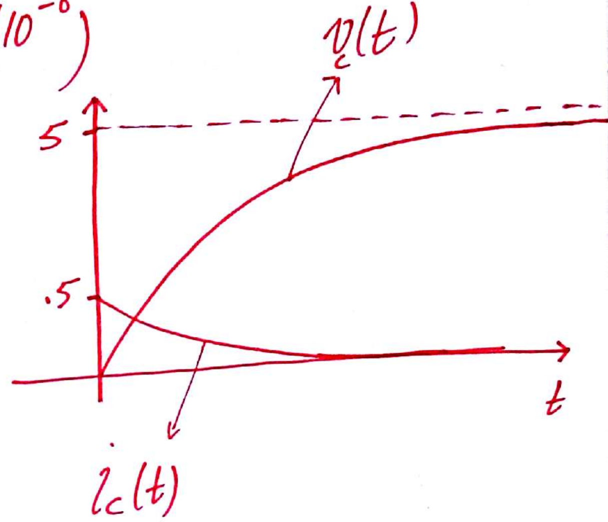
* $C = .1\,\mu F$

Sol:

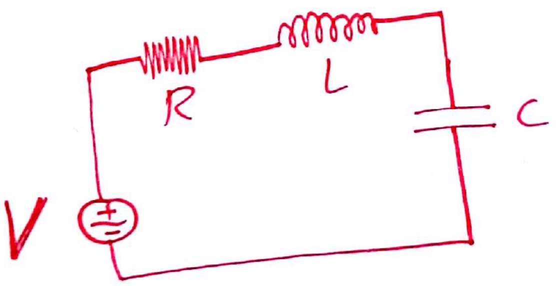
$$i_c(t) = C \cdot \frac{dv}{dt}$$

$$i_c(t) = (0.1\mu) * (5 \cdot \frac{1}{\mu} \cdot e^{-t/10^{-6}})$$

$$i_c(t) = .5\, e^{-t/10^{-6}}\ A$$

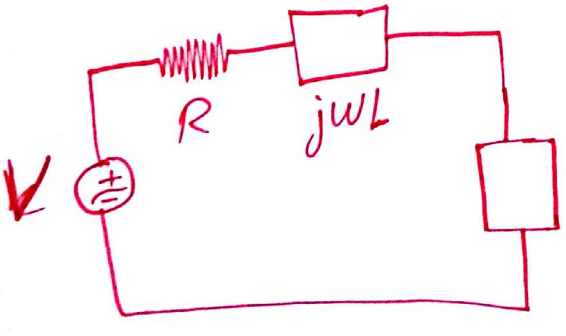


Impedance & Phasor Domain :-



$$* \; v(t) = i(t) * R + L\frac{di(t)}{dt} + \frac{1}{C}\int_{-\infty}^{t} i_c(t)\, dt$$

- changing to phasor Domain :-



$\frac{1}{jwC}$

$$* \; V = I\left(R + jwL + \frac{1}{jwC}\right)$$

to change From time Domain to phasor Domain:-

① Function in time domain must be periodic source.

② must be in the form of $\begin{cases} V_m \cos(\omega t + \theta_v) \\ I_m \cos(\omega t + \theta_i) \end{cases}$

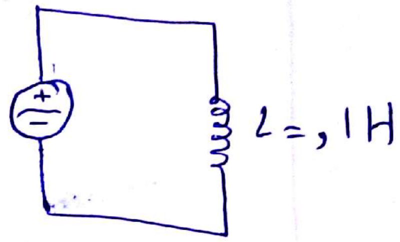
③ the phasor domain is in the form:-

- $V_m \angle \theta_v$

- $I_m \angle \theta_i$

ex calculate and sketch $v_L(t)$ when $i_L(t) = 20te^{-2t}$ A and $L = 0.1H$

$I_L = 20te^{-2t}$



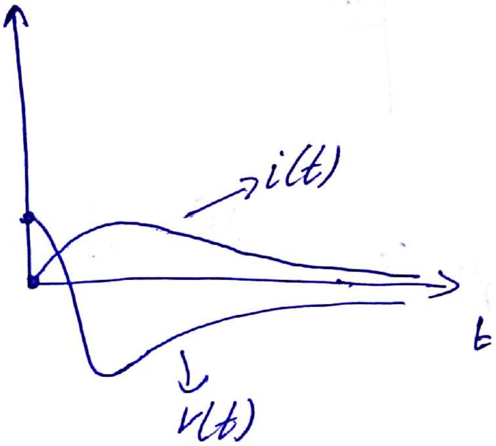
$V_L = j\omega L I_L$

$$v_L(t) = \frac{L\, di_L(t)}{dt}$$

⇒ [the inductor current can not be changed Instantaneously]

$= .1 * [-40te^{-2t} + 20e^{-2t}]$

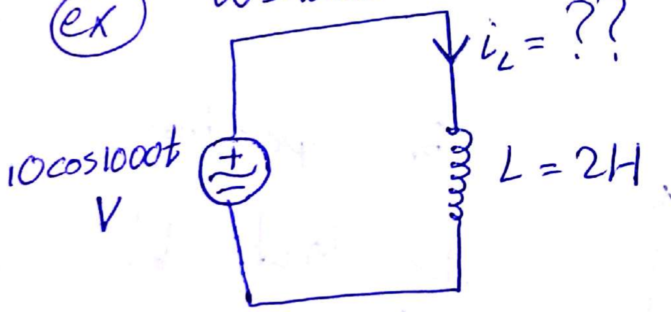
$= 2e^{-2t}[1-2t]$



$i(t) = C\frac{dv}{dt}$ ⇒ the capacitor ~~current~~ voltage can not changed instantaneously

---

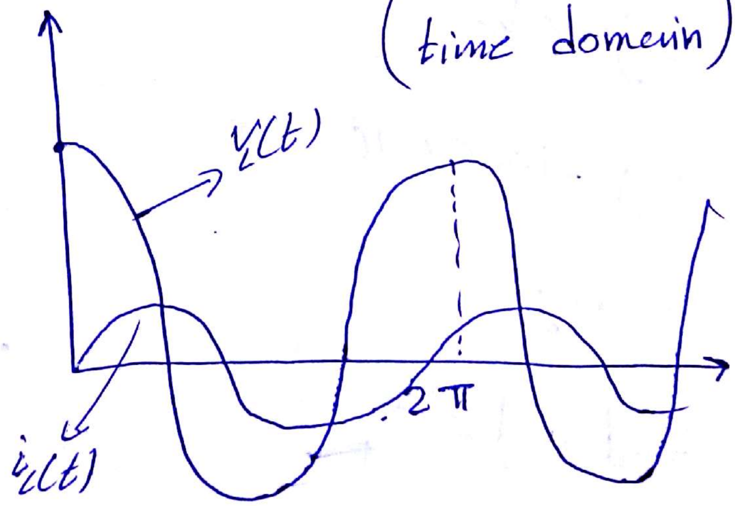
ex $\omega = 1000$



$i_L(0) = 0$ ⇒ For inductor V leads I by $\frac{\pi}{2}$ (time domain)

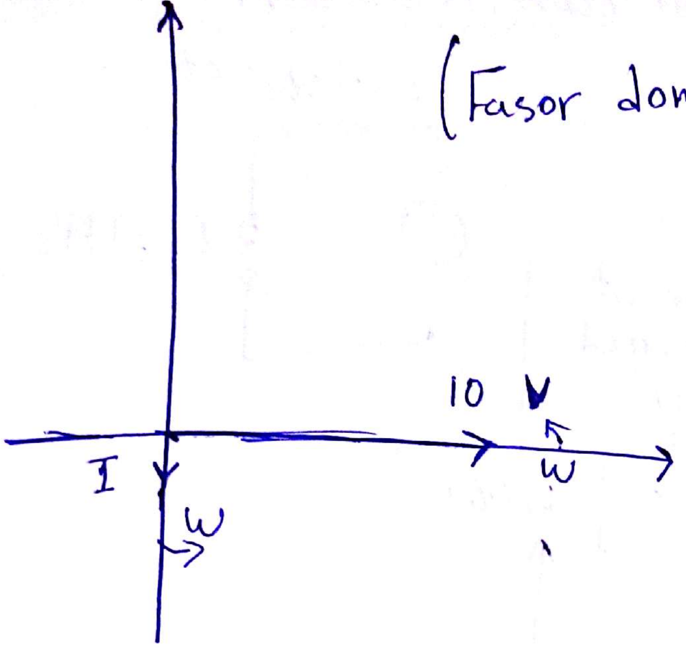
$$v_L(t) = \frac{L\, di(t)}{dt}$$

$$i_L(t) = \frac{1}{L}\int_0^t v_L(t)\, dt$$



$= .5 * \frac{10}{1000} \sin 1000t = .05 \sin 1000t$ A

(Fasor domain)



$V = 10\angle 0$

$I = .05\angle -\frac{\pi}{2}$

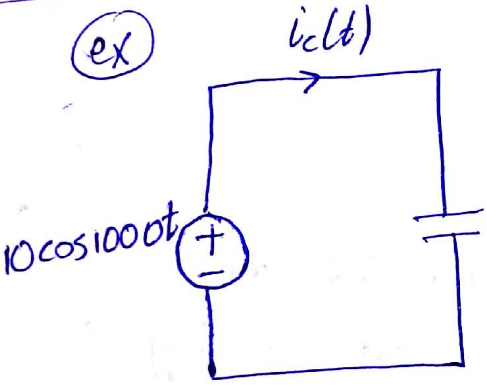
$A\angle\theta = Ae^{j\theta} = A\cos\theta + jA\sin\theta$

$j = 1\angle\frac{\pi}{2}$

$-j = 1\angle -\frac{\pi}{2}$

---

ex



$V_c(t) = 10\cos 1000t$

$V_c = 10\angle 0$

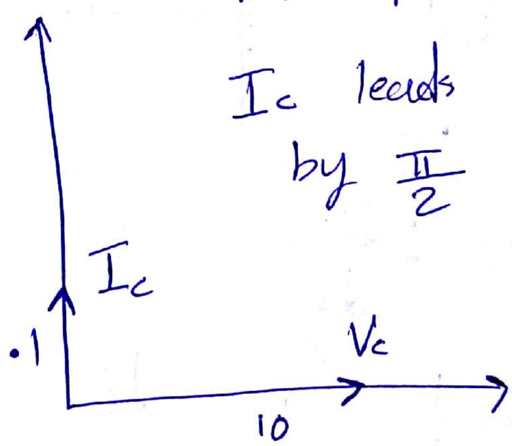
$= 10$

$I_c = j\omega c V_c$

$= j * 1000 * 10^{-5} * 10$

$= .1j$

$= .1 * 1\angle\frac{\pi}{2}$

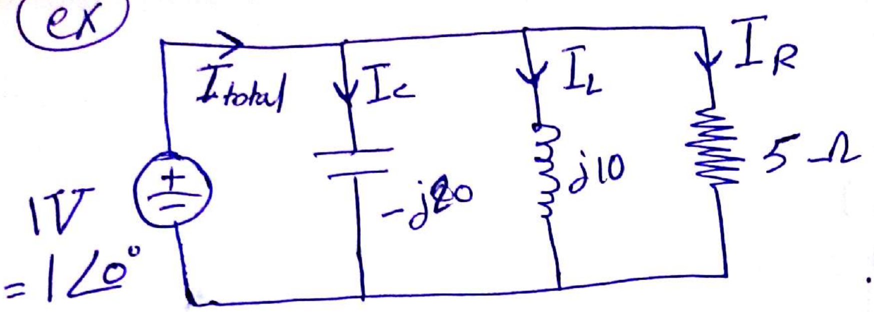


For capacitors I_c leads V_c by $\frac{\pi}{2}$

↳ time domain

$i_c(t) = .1\cos(1000t + \frac{\pi}{2}) = .1\sin(\omega t)$

(ex)



Find the current every where?

$$I_{total} = I_R + I_L + I_C.$$

$$I_R = \frac{1\angle 0°}{5} = .2\angle 0°$$

$$I_L \quad \frac{1\angle 0°}{10j} = -j.1 = .1\angle -\frac{\pi}{2}$$

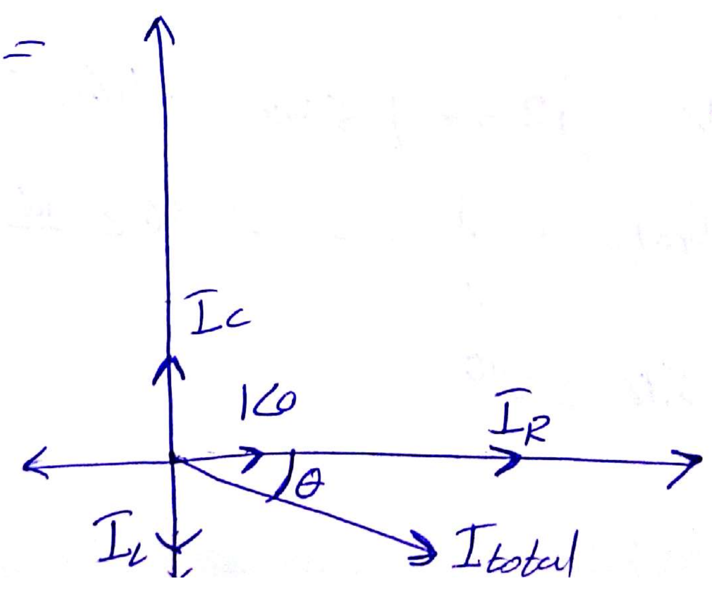
$$I_C = \frac{1\angle 0°}{-j20} = j.05 = 0.05\angle \frac{\pi}{2}$$

$$I_{total} = .2 + \cancel{.1\angle -\tfrac{\pi}{2}} + .1j + .05j = .2 - j0.05$$

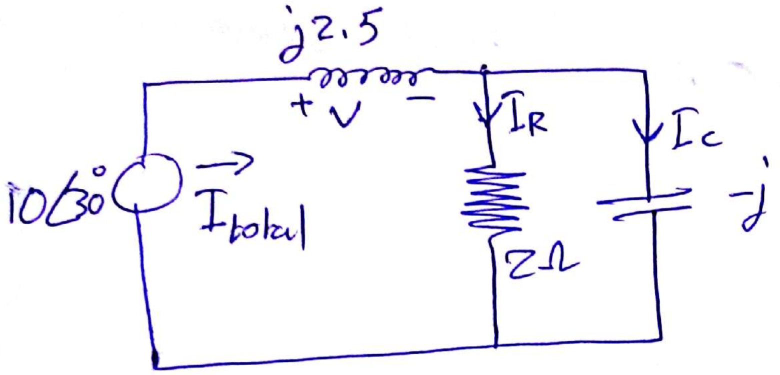
in polar form

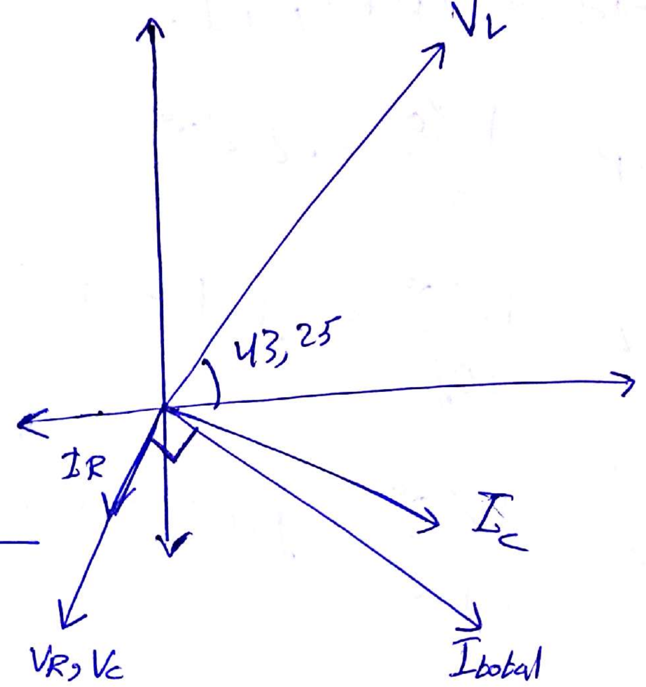
$$A = \sqrt{(Re)^2 + (Im)^2} = \sqrt{(.2)^2 + (.05)^2}$$

$$\theta = \tan^{-1}\left(\frac{-.05}{.2}\right) =$$



ex Draw all phasers:-





$$I_{total} = \frac{10\angle 30}{j2.5 + 2\Omega \,//\, -j}$$

$$2 // -j = \frac{-j2}{2-j}$$

$$I_{total} = \frac{10\angle 30^\circ}{j2.5 + \frac{-j2}{2-j}} \cdot \frac{(2-j)}{(2-j)}$$

$$= \frac{10\angle 30^\circ (2-j)}{j5 - 2.5 - j2} = \frac{10\angle 30^\circ (2-j)}{2.5 + j3}$$

$$\boxed{I_{total} = 3.92 - j4.14 = 5.72\angle -46.75}$$

$$\boxed{V_L = j2.5 * [5.72\angle -46.75] = 14.3\angle 43.25}$$

$$I_R = I_{total} * \frac{-J}{2-j} = 2.56\angle -110$$

$$V_R = 5.12\angle -110$$

$$V_c = V_R$$

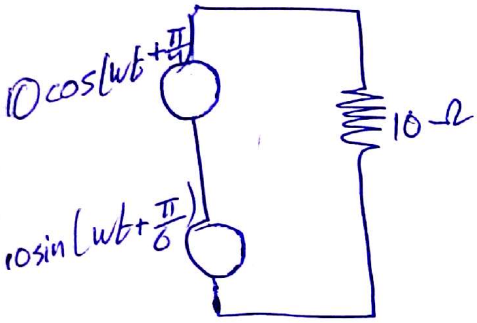
$$I_c = \frac{V_c}{Z_c} = \frac{5.12\angle -110}{-j1} = 5.12\angle -20$$

Find $V_1 + V_2$ in time domeain

$$V_1(t) = 15\cos\left(377t + \frac{\pi}{4}\right) = 15\angle\frac{\pi}{4}$$

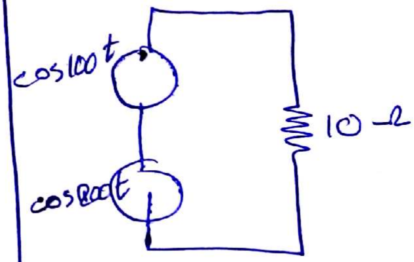
$$v_2(t) = 15\cos\left(377t + \frac{\pi}{6}\right) = 15\angle\frac{\pi}{6}$$

$$\underbrace{V_1 + V_2}_{\text{Rectangular}} = 15\cos\frac{\pi}{4} + j15\sin\frac{\pi}{4} + 15\cos\frac{\pi}{6} + j\,15\sin\frac{\pi}{6}$$



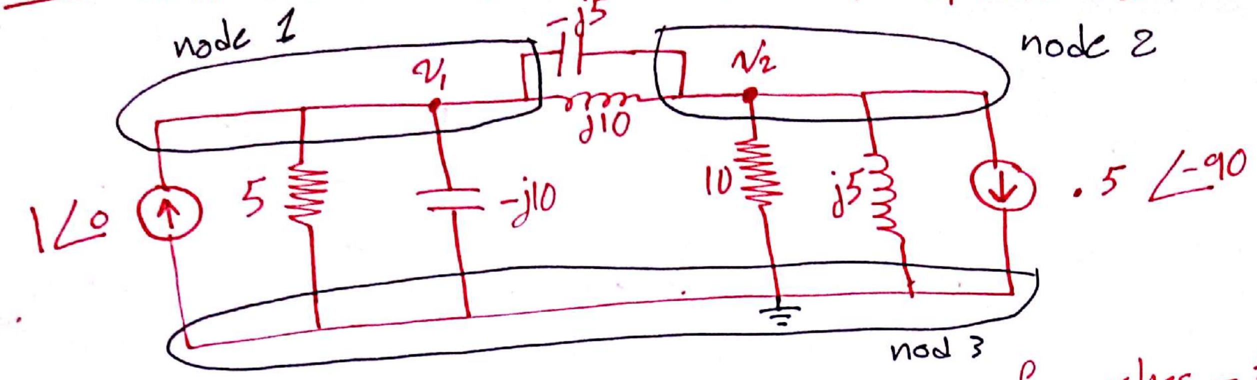
$$V_1 = 10\angle\frac{\pi}{4}$$

$$V_2 = 10\angle\frac{\pi}{6} - \frac{\pi}{2}$$



No phasor Domeain

ex Find the time domain value of v_1 and v_2 :-



# of equations for mesh analysis : # of meshes − # of current sourses

# of equations for nodal analysis : # of nodes − # of voltage sourses − 1

* independent sourses

Reference node

@ node 1 :-

$$1\angle 0 + \frac{0 - v_1}{5} + \frac{0 - v_1}{-j10} + \frac{v_2 - v_1}{j10} + \frac{v_2 - v_1}{-j5} = 0$$

at node 2 :-

$$-.5\angle -90 + \frac{0 - v_2}{j5} + \frac{-v_2}{10} + \frac{v_1 - v_2}{j10} + \frac{v_1 - v_2}{-j5} = 0$$

solve 2eq :-

$$v_1 = 1 - j2 = 2.24\angle -63.4 = 2.24\cos(\omega t - 63.4)\ V$$

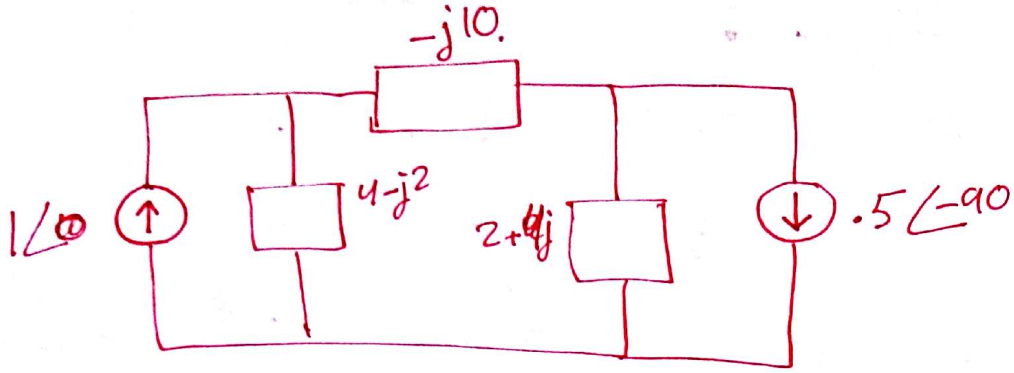
$$v_2 = -2 + j4 = 4.47\angle 116.6 = 4.47\cos(\omega t + 116.6)\ V$$

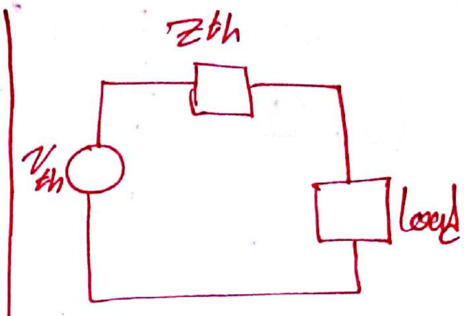
v_1 leads v_2 by $180°$

v_2 leads v_1 by $180°$

اذا كانت الزاوية $180°$ اي phasor يكون lead

(10) Find thevenin eq seen by "-j10"





$\longrightarrow$ voltage $\rightarrow$ short ~~open~~ circuit

* Z_{Th} : (1) Kill all sourses $\boxed{\text{and find Z equiv}}$

Z_{norton} $\longrightarrow$ current $\rightarrow$ ~~short~~ open circuit

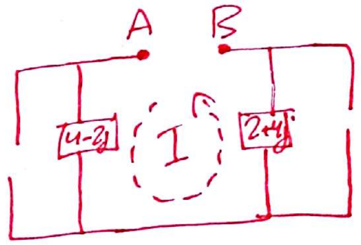
(2) $\dfrac{V_{o.c}}{I_{s.c}}$

* V_{Th} : $V_{o.c}$

* I_{norton}: $I_{s.c}$

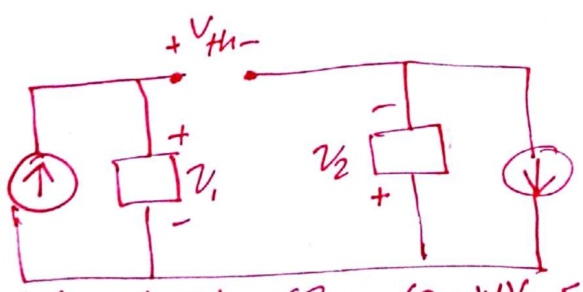
sol

Z_{Th}:



$Z_{Th} = 4-2j + 2+4j = (6+2j)\ \Omega$

V_{th}

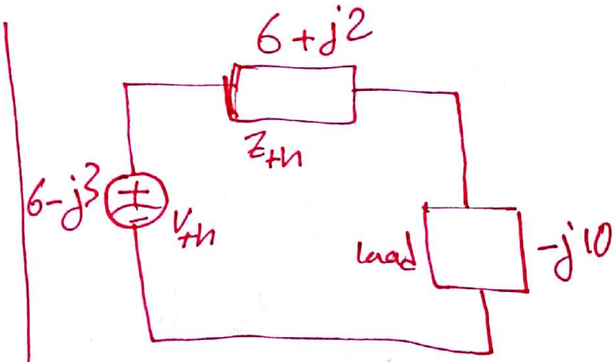


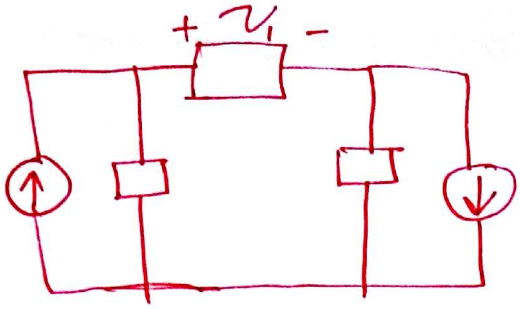
$(V_1 = 4-2j)$ $(V_2 = (2+j4)(.5\angle -90))$

KVL

$-V_1 + V_{th} - V_2 = 0$

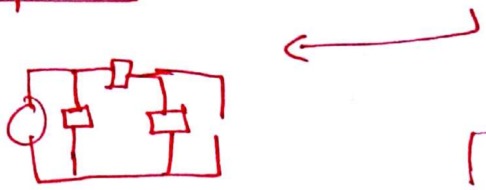
$V_{th} = V_1 + V_2 = 6 - j3$

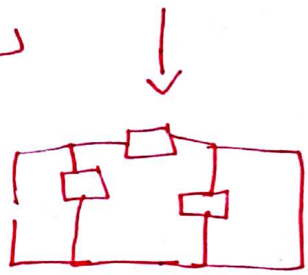




By soperposition

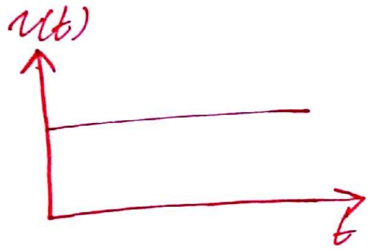
$$v_1 = v_1' + v_1''$$

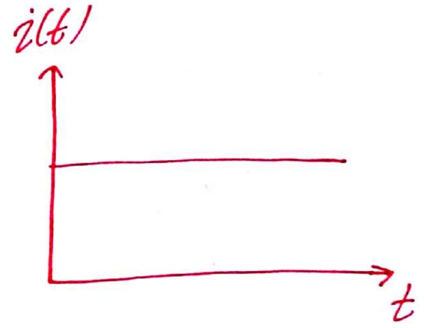




CHp -11- : power analysis :-

Review :-





$\boxed{DC}$

$$p(t) = v(t) * i(t)$$

$$P = vi = VI$$

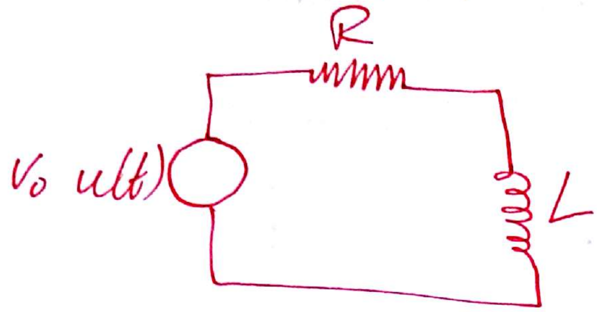
Instantaneus power :

$$R \rightarrow p(t_0) = v(t)\, i(t)$$

$$L \rightarrow p(t_0) = v(t)\, i(t) = \frac{L\, di(t_0)}{dt_0} * i(t_0)$$

$$c \rightarrow p(t_0) = v(t)\, i(t) = v(t) * \frac{c\, dv_c(t_0)}{d(t_0)}$$

ex) Find the inst power across the inductor



$$i(t) = \frac{V_0}{R}\left(1 - e^{\frac{-Rt}{L}}\right) u(t)$$

$$= \frac{V_0}{R} u(t) - \frac{V_0}{R} e^{\frac{-Rt}{L}} u(t)$$

source Ins. power :

$$p(t) = v(t)\, i(t) = \frac{V_0^2}{R}\left(1 - e^{\frac{-Rt}{L}}\right) u(t)$$

ins. power across R :-

$$P_R(t) = v(t)\, i(t) = R\, i^2(t) = \frac{v(t)^2}{R}$$

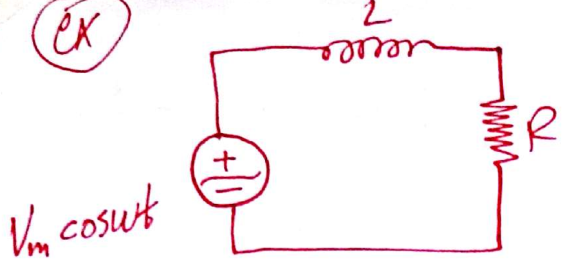
$$= R\left[\left(\frac{V_0}{R}\right)^2 \left(1 - e^{\frac{-Rt}{L}}\right)^2 u(t)\right] = \frac{V_0^2}{R}\left(1 - e^{\frac{-Rt}{L}}\right)^2 u(t)$$

Inst. power across L :-

$$p(t) = L\frac{di(t_0)}{d(t_0)} * i(t) = L * \frac{-R}{L} * e^{\frac{-Rt}{L}} * \frac{V_0}{R} * \frac{V_0}{R}\left(1 - e^{\frac{-Rt}{L}}\right) u(t)$$

$$p(t) = \frac{V_0^2}{R}\left(e^{\frac{-Rt}{L}}\right)\left(1 - e^{\frac{-Rt}{L}}\right) u(t)$$

ex



$*$ sourse inst. power

$p(t) = v(t)\, i(t)$

$p(t) = V_m \cos wt * I_m \cos(wt + \phi)$

$\# \cos x * \cos y = \dfrac{\cos(x+y) + \cos(x-y)}{2}$

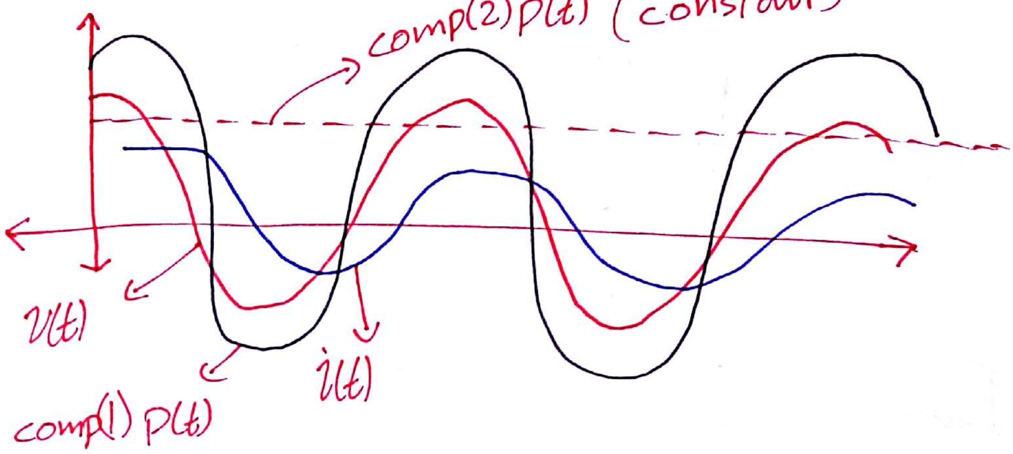
$p(t) = \dfrac{V_m I_m}{2}\left[\cos(2wt + \phi + \cos\phi\right]$

$v(t) = V_m \cos wt$

$i(t) = I_m \cos(wt + \phi)$

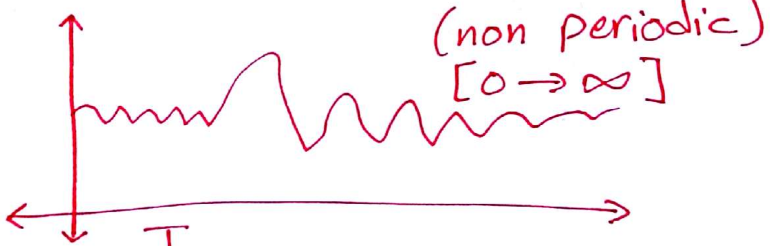
$I_m = \dfrac{V_m}{\sqrt{R^2 + (wL)^2}}$

$\phi = \tan^{-1}\left(\dfrac{wL}{R}\right)$



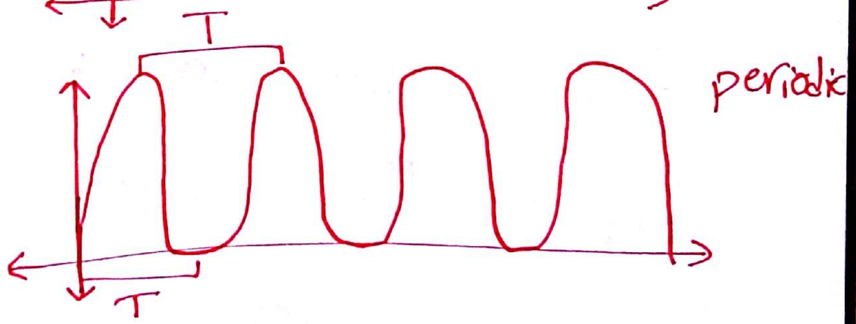
Average power : more accurate & efficient

$P_{avg} = \dfrac{1}{t_2 - t_1} \displaystyle\int_{t_1}^{t_2} p(t)\, dt$



- for a periodic signal

$P_{avg} = \dfrac{1}{T} \displaystyle\int_{0}^{T} p(t)\, dt$



$*$ lets ~~assume~~ assume :-

$v(t) = V_m \cos(wt + \theta)$
$i(t) = I_m \cos(wt + \phi)$
} for any element

- Find the average power

$$p(t) = v(t) * i(t) = V_m \cos(\omega t + \theta) * I_m \cos(\omega t + \phi)$$

$$p(t) = \frac{V_m I_m}{2}\left[\cos(2\omega t + \theta + \phi) + \cos(\theta - \phi)\right]$$

$$P_{avg} = \frac{1}{T}\int_0^T p(t)\,dt$$

$$P_{avg} = \frac{1}{T}\left[\overset{0}{\cancel{\int_0^T \frac{V_m I_m}{2}\cos(2\omega t + \theta + \phi)\,dt}} + \int_0^T \frac{V_m I_m}{2}\cos(\theta - \phi)\,dt\right]$$

* the integration for any sinusoidal function for any period equal zero

$$P_{avg} = \frac{1}{T}\int_0^T \frac{V_m I_m}{2}\cos(\theta - \phi)\,dt$$

$$P_{avg} = \boxed{\frac{V_m I_m}{2}\cos(\theta - \phi)}$$

For a resistive load :-

$$P_{avg} = \frac{V_m I_m}{2} \rightarrow (\theta - \phi) = 0^\circ = \frac{I_m^2 R}{2} = \frac{V_m^2}{2R}$$

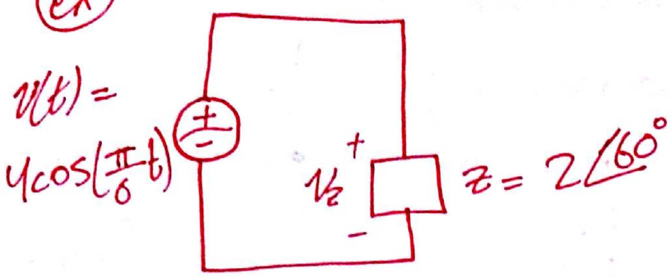
For an inductor :-

$$P_{avg} = 0 \rightarrow (\theta - \phi) = 90^\circ$$

For an capacitor :-

$$P_{avg} = 0 \rightarrow (\theta - \phi) = -90^\circ$$

ex



*find p(t), P_{avg} for the element

$$v(t) = 4\cos\left(\frac{\pi}{6}t\right) = 4\angle 0^\circ \quad , \quad \omega = \frac{\pi}{6}$$

sol

$$I = \frac{V}{Z} = \frac{4\angle 0^\circ}{2\angle 60^\circ} = 2\angle -60^\circ = 2\cos\left(\frac{\pi}{6}t - 60^\circ\right)$$

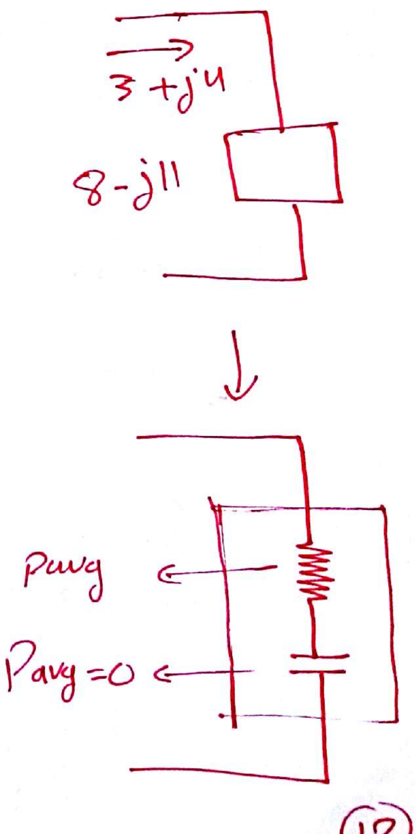
$$p(t) = v(t)\, i(t) = 4\cos\left(\frac{\pi}{6}t\right) * 2\cos\left(\frac{\pi}{6}t - 60^\circ\right)$$

$$= 4\left[\cos\left(\frac{\pi}{3}(t - 60) + \cos(60^\circ)\right]$$

$$= \left[2 + 4\cos\left(\frac{\pi}{3}t - 60^\circ\right)\right] W$$

$$P_{avg} = 2\ W$$

ex Find the average power delivered to the Impedance :-

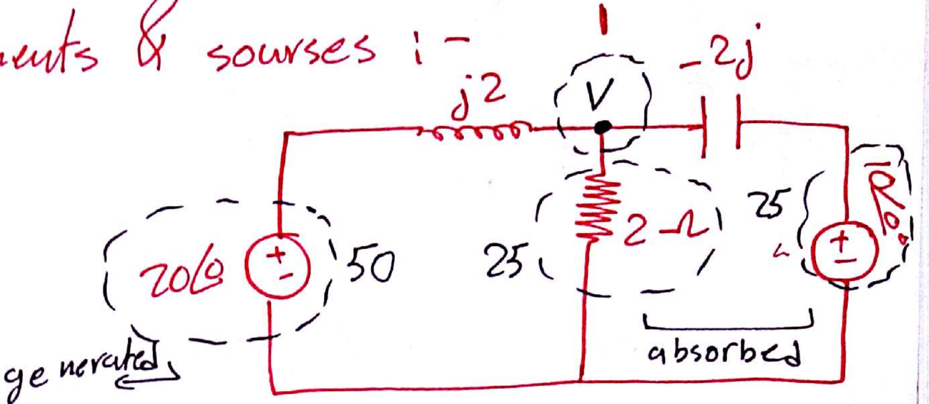


$$I = 3 + j4 = \sqrt{9 + 16}\ \angle \tan^{-1}\left(\frac{4}{3}\right)$$

$$P_{avg} = \frac{V_m I_m}{2} = \frac{I_m^2 R}{2}$$

$$= \frac{(5)^2 * 8}{2} = 100\ W$$

(ex) Find the average power absorbed or generated by elements & sourses :-



$$\boxed{P_{avg}(j2) = 0}$$

$$\boxed{P_{avg}(-j2) = 0}$$

at node -1-

$$\frac{20\angle 0 - V}{j2} + \frac{0 - V}{2} + \frac{10\angle 0 - V}{-j2} = 0$$

$$V = 10\angle -90 = -j10$$

$$P_{avg}(R) = \frac{V_m^2}{2R} = \boxed{25\ W}$$

$$I_1 = \frac{20\angle 0 - 10\angle -90}{j2} = 5 - j10 = 11.18\angle -63.43$$

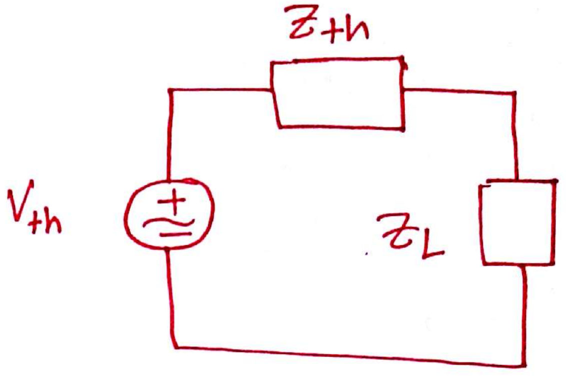
$$P_{avg}(\text{sourse}) = \frac{V_m I_m}{2}\cos(\theta - \phi)$$

$$= \frac{20 * 11.18}{2}\cos(0 + 63 + 43) = \boxed{50\ W}$$

$$I_2 = \frac{10\angle 0 - 10\angle -90}{-j2} = 5 - j5 = 7.07\angle -45$$

$$P_{avg}(10\angle 0) = \frac{10 * 7.07}{2}\cos(0 + 45) = \boxed{25\ W}$$

Maximum power transfer :-



$$I_L = \frac{V_{th}}{Z_{th} + Z_L}$$

$$I_L = \frac{V_{th}}{R_{th} + jX_{th} + R_L + jX_L}$$

$$V_L = V_{th} * \frac{Z_{\cancel{th}L}}{Z_{th} + Z_L} = \frac{R_L + jX_L}{R_{th} + jX_{th} + R_L + jX_L} * V_{th}$$

$$|V_L| = |V_{th}| * \frac{\sqrt{(R_L)^2 + (X_L)^2}}{\sqrt{(R_L + R_{th})^2 + (X_L + X_{th})^2}}$$

$$\angle V_L = \angle V_{th} + \tan^{-1}\left(\frac{X_L}{R_L}\right) - \tan^{-1}\left(\frac{X_L + X_{th}}{R_L + R_{th}}\right)$$

$$|I_L| = \frac{|V_{th}|}{\sqrt{(R_L + R_{th})^2 + (X_L + X_{th})^2}}$$

$$\angle I_L = \angle V_{th} - \tan^{-1}\left(\frac{X_L + X_{th}}{R_L + R_{th}}\right)$$

$$P_{avg}(Z_L) = \frac{V_m I_m}{2} \cos(\theta - \phi)$$

$$= \frac{|V_L||I_L|}{2} \cos(\angle V_L - \angle I_L)$$

$$\frac{dP_{avg}}{dR_L} = 0 \rightarrow R_L = R_{th}$$

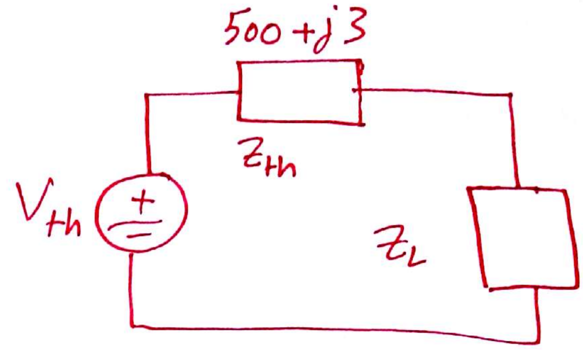
$$\frac{dP_{avg}}{dX_L} = 0 \rightarrow X_L = -X_{th}$$

For maximum power transfer

$$Z_L = Z_{th}^*$$

To maximite the power transfer :-

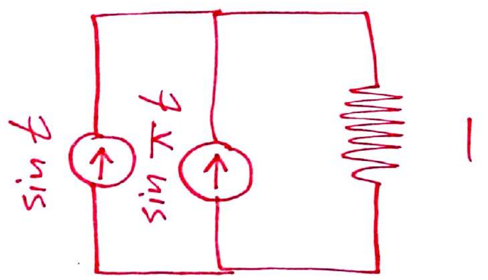
(ex) Find Z_L that maximite the power transfer ?



sol

$$Z_L = 500 - j3$$

(ex) if $i(t) = \sin t + \sin \pi t$, $R = 1$, find the average power absorbed by the load :-



$$p(t) = v(t) * i(t)$$
$$= i^2(t) * R$$
$$= (\sin t + \sin \pi t)^2 * R$$

$$P_{avg} = \frac{1}{T} \int_0^T p(t)\, dt$$
$$= \frac{1}{T} \int_0^T (\sin t + \sin \pi t)^2 R\, dt$$
$$= \frac{R}{T} \int_0^T (\sin^2 t + 2 \sin t \sin \pi t + \sin^2 \pi t)\, dt$$
$$\left(\frac{1}{2} - \cancel{\frac{1}{2}\cos 2t}\right) + \left(\cancel{\frac{1}{2}\cos(t+\pi t)} + \cancel{\frac{1}{2}\cos(t-\pi t)}\right) + \frac{1}{2} - \cancel{\frac{1}{2}\cos \pi t}.$$

$$= \frac{R}{T} * \left[\frac{1}{2}T + \frac{1}{2}T\right] = 1\,W$$

- For a multipile frequency source: the Avg power across a resister:

$$P_{avg} = \frac{1}{2}\left(I_{m_1}^2 R + I_{m_2}^2 R + I_{m_3}^2 R + \text{---}\right)$$

$$P_{avg} = P_{avg_1} + P_{avg_2} + P_{avg_3} + \text{-----}$$

$p(t) = P_1(t) + P_2(t) \Rightarrow \times$

$$\begin{array}{cc} P_1(t) & P_2(t) \\ \downarrow & \downarrow \\ i_1(t)^2 R & i_2(t)^2 R \end{array}$$

$$p(t) = i(t)^2 * R = (i_1(t) + i_2(t))^2 * R$$

$$= i_1(t)^2 * R + i_2(t)^2 * R + 2\, i_1(t)\, i_2(t) * R \neq P_1(t) + P_2(t)$$

ex $i(t) = 2\cos(10t) - 3\sin(20t)$, $R = 4\,\Omega$

Find the Avg power?

- Different frequency

$$P_{avg} = \frac{1}{2} I_{m_1}^2 * R + \frac{1}{2} I_{m_2}^2 * R$$

$$= \frac{1}{2}(2)^2 * 4 + \frac{1}{2}(-3)^2 * 4$$

$$= 8 + 18 = 26\ W$$

✗- طريقة ال phasor
او ال frequency
في Avg power

ex $i(t) = 2\cos(10t) - 3\cos(10t)$, $R = 4$

Find Avg power? # same frequency

sol

$i(t) = -\cos(10t)$

$$P_{avg} = \frac{I_m^2}{2} * R = \frac{(-1)^2}{2} * 4 = 2\,W$$

---

ex $i(t) = 2\cos(wt) + 3\cos(wt + \frac{\pi}{6})$, $R = 4$

sol Find Avg power?

$I_1 = 2\angle 0^\circ = 2$

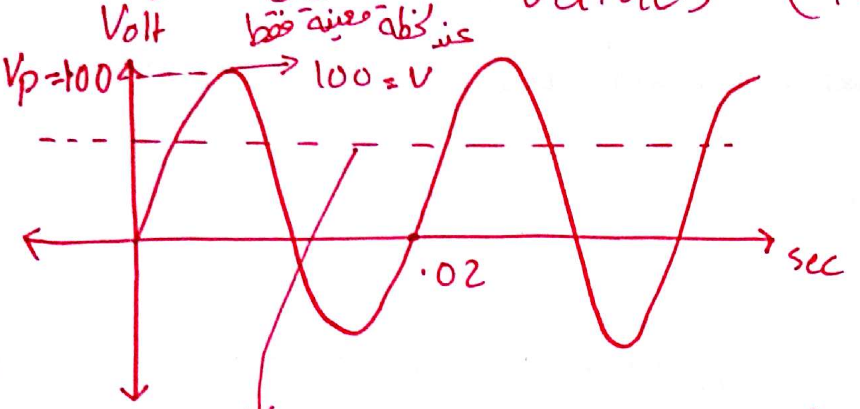
$I_2 = 3\angle \frac{-\pi}{6} = 3\cos(-\frac{\pi}{6}) + j\,3\sin(-\frac{\pi}{6})$

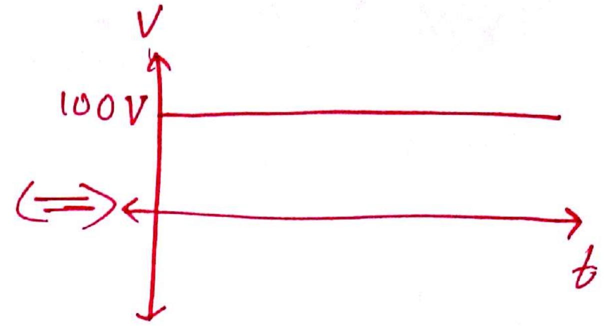
$I = 2 + \frac{3\sqrt{3}}{2} - j * 3 * \frac{1}{2}$

$$I_m = \sqrt{\left(2 + \frac{3\sqrt{3}}{2}\right)^2 + \left(\frac{3}{2}\right)^2} = \boxed{}$$

$$P_{avg} = \frac{1}{2} I_m * R = \frac{1}{2} * \downarrow * 4$$

Effective values (RMS) :-



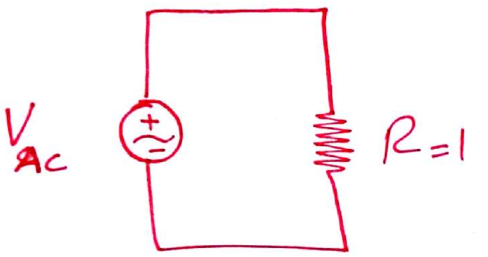


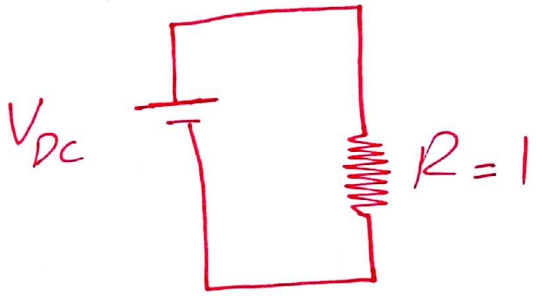
$$RMS = \frac{V_p}{\sqrt{2}}$$

* (DC) اكثر ضرراً من (AC)

(DC) ← اكثر ضرر

(AC) ← اقل ضرر





$$P_{avg} = \frac{1}{2} * \frac{V_{ac}^2}{R}$$

$$P_{avg} = \frac{V_{DC}^2}{R}$$

$$P_{avg} = P_{avg}$$

$$\frac{1}{2} \frac{V_{ac}^2}{\cancel{R}} = \frac{V_{DC}^2}{\cancel{R}}$$

$$V_{ac} = \sqrt{2} * V_{DC}$$

- اذا وجد مصدر للطاقة (DC) فإن المصدر المكافئ له بالطاقة

في (AC) = ($\sqrt{2} * V_{DC}$)

ex) if there is a (DC) source with 100 V, what is (RMS) of an (AC) source that will produce the same amount of power?

$$V_{ac} = \sqrt{2} * V_{DC} = \sqrt{2} * 100 = 141.2\ V$$

---

if there general (AC) source (not nesscarly sin cos)

$$P_{avg(AC)} = \frac{1}{T}\int_0^T \frac{V_{AC}^2}{R}\cdot dt \quad , \quad P_{avg(DC)} = \frac{V_{DC}^2}{R}$$

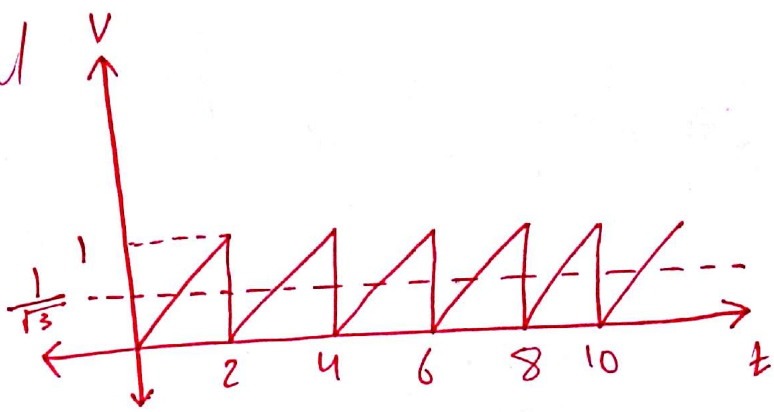
$$P_{avg(AC)} = P_{avg(DC)}$$

$$\frac{1}{T}\int_0^T \frac{V_{AC}^2}{\cancel{R}}\cdot dt = \frac{V_{DC}^2}{\cancel{R}}$$

$$V_{DC} = V_{eff} = \sqrt{\frac{1}{T}\int_0^T V_{AC}(t)^2 \cdot dt}$$

Root Mean Square Value RMS

ex Find the (RMS) value in the shown signal



$$V_{eff} = \sqrt{\frac{1}{T}\int_0^T v(t)^2 \, dt}$$

$$T = 2 \rightarrow \begin{pmatrix} v(t) = \frac{1}{2} t \\ 0 < t < 2 \end{pmatrix}$$

$$V_{eff} = \sqrt{\frac{1}{2}\int_0^2 \frac{1}{4} t^2 \, dt} = \sqrt{\frac{1}{2} * \frac{1}{12} t^3 \Big|_0^2} = \sqrt{\frac{1}{2} * \frac{8}{12}} = \frac{1}{\sqrt{3}} \text{ V}$$

---

ex Find (RMS) value at an (AC) signal :-

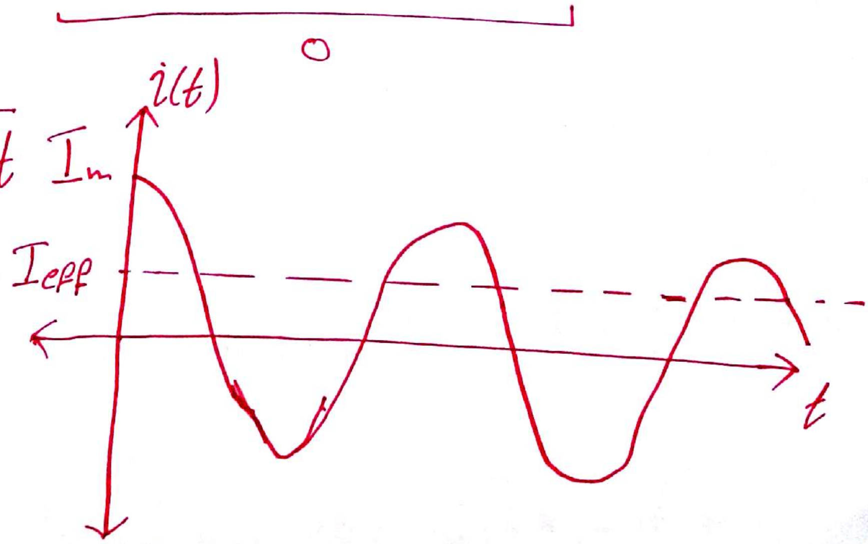
$i(t) = I_m \cos(\omega t + \phi)$?

$$I_{eff} = \sqrt{\frac{1}{T}\int_0^T i(t)^2 \, dt}$$

$$I_{eff} = \sqrt{\frac{1}{T}\int_0^T I_m^2 \cos^2(\omega t + \phi) \, dt} \qquad * \cos^2(\theta) = \frac{1}{2} + \frac{1}{2}\cos(2\theta)$$

$$I_{eff} = \sqrt{\frac{1}{T}\int_0^T \frac{1}{2} I_m^2 \, dt + \underbrace{\int_0^T \frac{1}{2} I_m^2 \cos(2\omega t + 2\phi) \, dt}_{0}}$$

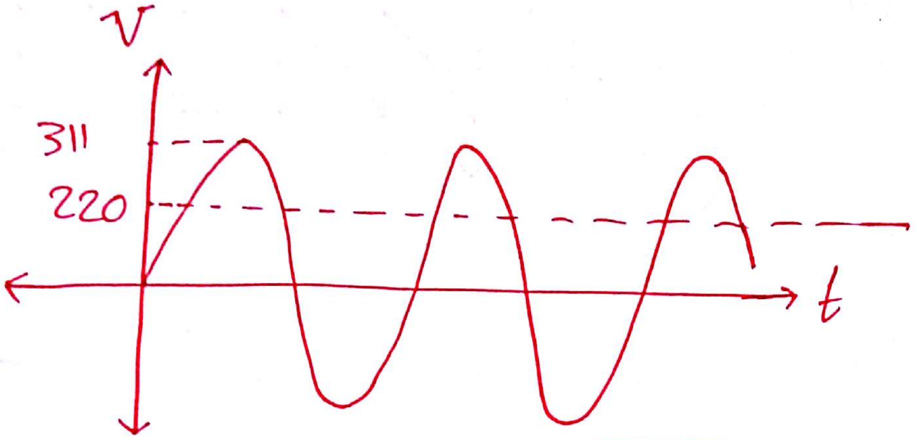
$$I_{eff} = \sqrt{\frac{1}{T}\int_0^T \frac{1}{2} I_m^2 \, dt}$$



$$I_{eff} = \frac{I_m}{\sqrt{2}}$$

- in jordan → 220 V (RMS) value

so $V_p = 220 * \sqrt{2} = 311\ V$



- For a resistive load with a sinusoidal source:-

$$P_{avg} = \frac{1}{2} V_m I_m = \frac{1}{2} I_m^2 R = \frac{V_m^2}{2R}$$

$$\left.\begin{aligned} - V_m &= \sqrt{2}\ V_{eff} = \sqrt{2}\ V_{RMS} \\ - I_m &= \sqrt{2}\ I_{eff} = \sqrt{2}\ I_{RMS} \end{aligned}\right\} \text{(RMS)}$$

(تكون ال I او V)

$$P_{avg} = V_{RMS} I_{RMS} = I_{RMS}^2 * R = \frac{V_{RMS}^2}{R}$$

* In general

voltage → current

$$P_{avg} = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i)$$

$$= V_{RMS} * I_{RMS} \cos(\theta_v - \theta_i)$$

ex - $i(t) = 5\cos(wt) \rightarrow I_{eff} = \frac{5}{\sqrt{2}}$

- $v(t) = 3\sin(wt) \rightarrow V_{eff} = \frac{3}{\sqrt{2}}$

any sinusoidal signal $X_{eff} = \frac{X_m}{\sqrt{2}}$

---

ex - $v(t) = 10\cos(wt - \frac{\pi}{3}) \rightarrow V_{eff} = \frac{10}{\sqrt{2}}$

- $i(t) = 4\sin(wt) + 3\cos(wt) \rightarrow I_{eff} = \frac{5}{\sqrt{2}}$

* $A\cos(x) + B\sin(x) = \sqrt{A^2+B^2}\cos(x - \tan^{-1}(\frac{B}{A}))$

or: $I = 4\angle -\frac{\pi}{2} + 3\angle 0° = 3 - j4$

- $i(t) = 5\cos(ut) + 3\sin(2ut)$

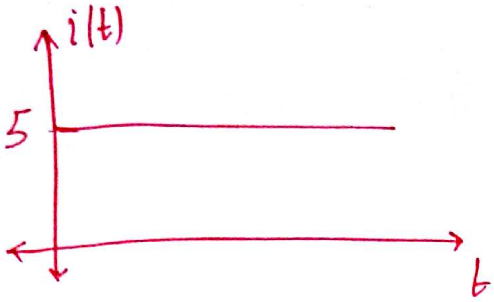
* for a signal with multiple frequency :-

$I_{eff} = \sqrt{I_{eff_1}^2 + I_{eff_2}^2 + I_{eff_3}^2 + \cdots}$

$I_{eff} = \sqrt{(\frac{5}{\sqrt{2}})^2 + (\frac{3}{\sqrt{2}})^2} = \sqrt{\frac{34}{2}}$

$P_{avg} = I_{eff_1}^2 * R + I_{eff2}^2 * R + I_{eff3}^2 * R + \cdots$

ex) $i(t) = 5 \longrightarrow I_{RMS} = ?$ $\quad \frac{5}{\sqrt{2}}$ X



$$\rightarrow I_{RMS} = \sqrt{\frac{1}{T}\int_0^T (5)^2 \, dt}$$

$$= \sqrt{\frac{1}{T}\, 25t \Big|_0^T} = 5$$

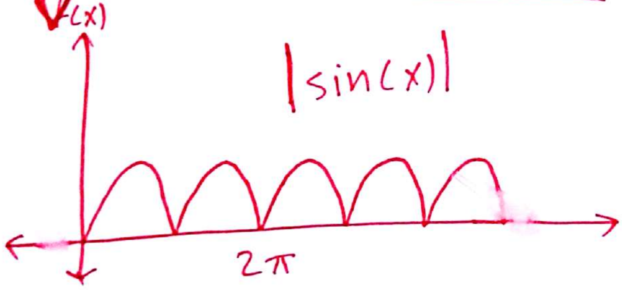
$$i(t) = 5\cos(wt) \longrightarrow I_{RMS} = \frac{5}{\sqrt{2}} \checkmark$$

---

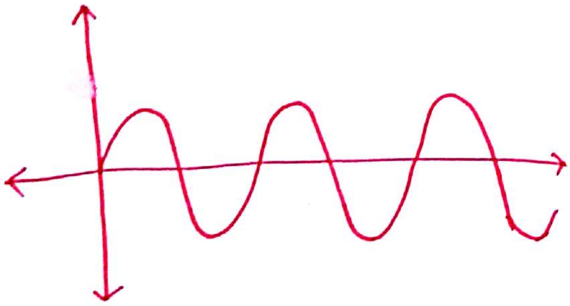
ex) $i(t) = 5 + 3\cos\left(wt - \frac{\pi}{2}\right)$

$$I_{RMS} = 5 + \frac{3}{\sqrt{2}} \quad \text{X}$$

$$I_{RMS} = \sqrt{(5)^2 + \left(\frac{3}{\sqrt{2}}\right)^2} = \sqrt{25 + \frac{9}{2}}$$

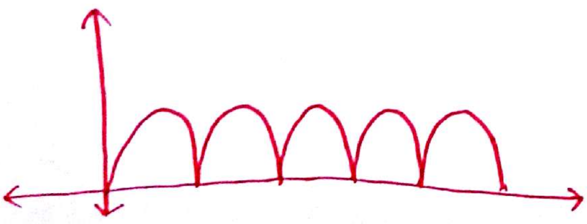


$$V_{RMS} = \sqrt{\frac{1}{T}\int_0^T v(t)^2 \, dt}$$

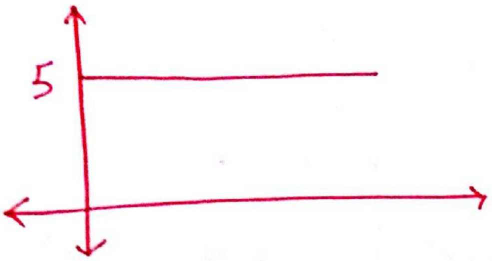


$\therefore V_{RMS} = \frac{5}{\sqrt{2}}$

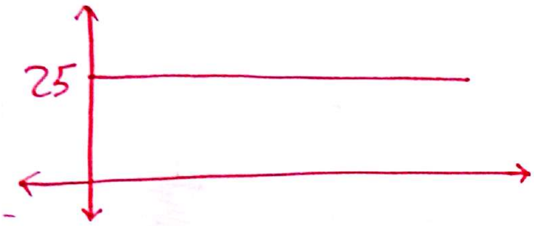
تربيعهم
متساوي

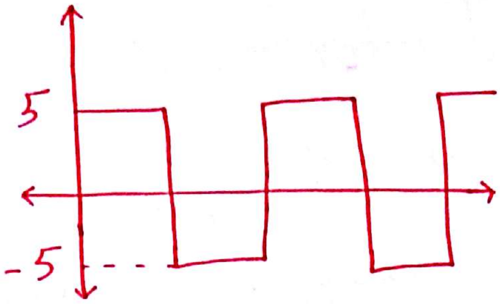


$\therefore V_{RMS} = \frac{5}{\sqrt{2}}$

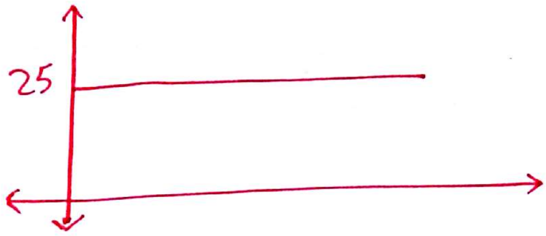


↓ تربيع



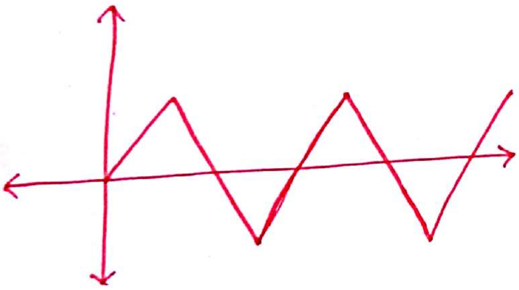


↓ تربيع

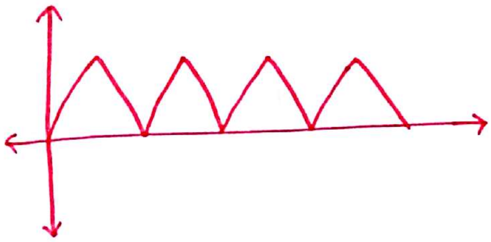


V_{RMS} for Both are 5

$$V_{RMS} = \sqrt{\frac{1}{T}\int_0^{\frac{1}{2}T} (5)^2\,dt + \int_{\frac{1}{2}T}^{T} (-5)^2\,dt} = 5$$



$\Longleftrightarrow$



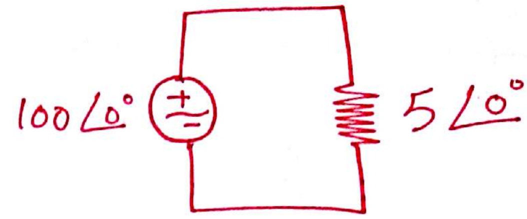
$$V_{avg} = \frac{1}{T}\int_0^T v(t)\,dt = 0$$

$$V_{RMS} = \sqrt{\frac{1}{T}\int_0^T v(t)^2\,dt} \neq 0$$

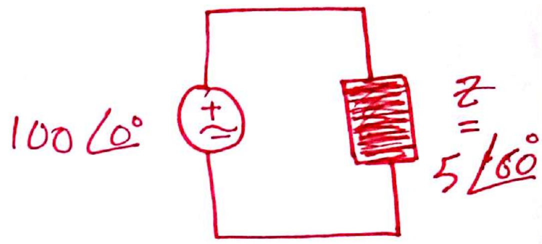
power factor

$I = 20\angle 0°$

$P_{avg} = 1000\ W$ (apparent) → resistor



$I = \dfrac{100\angle 0°}{5\angle 60°} = 20\angle -60°$

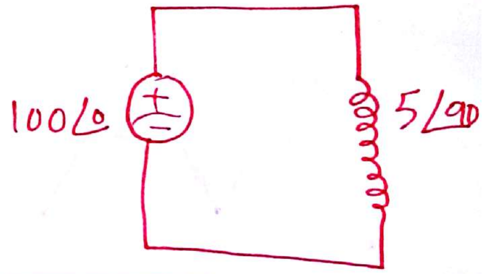


$P_{avg} = \dfrac{V_m I_m}{2} \cos(\theta_v - \theta_i)$

$= \dfrac{100 * 20}{2} * \cos(60°) = \underline{500\ W}$ (Real)

There is alost power

$P_{avg} = \dfrac{V_m\ I_m}{2} \cos(90) = 0$??



---

power factor (just for a sinusoidal system)

$= \dfrac{\text{Real power}}{\text{apparent power}} = \dfrac{\cancel{V_{RMS}}\ \cancel{I_{RMS}} \cos(\theta_v - \theta_i)}{\cancel{V_{RMS}}\ \cancel{I_{RMS}}}$

$P.F = \cos(\theta_v - \theta_i)$, $(\theta_v - \theta_i) \Rightarrow$ P.F angle

- for (R) load $\rightarrow \theta_v - \theta_i = 0 \rightarrow P.F = 1$
- for (L/C) load $\rightarrow \theta_v - \theta_i = \pm 90 \rightarrow P.F = 0$

- for RX load → $-\frac{\pi}{2} < \theta_v - \theta_i < \frac{\pi}{2}$ → P.F → $0 < P.F < 1$)

X: C L

ex) let $V = 5\angle 30$, $I = 3\angle -30$ } P.F = cos(30 − −30) = .5 (lagging RL)

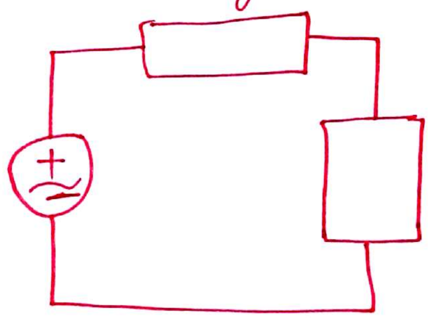
I lags V

ex) let $V = 5\angle -30$, $I = 3\angle 30$ } P.F = cos(−30 − 30) = .5 (leading RC)

I leads V

---

ex) Find the source power factor ?



اذا لم يذكر السؤال نوع ال source نعتبرها RMS

sol - find θ_i → find I

$$I_{RMS} = \frac{V_{RMS}}{Z_{eq}}$$

$$= \frac{60\angle 0}{(2-j)+(1+j5)} = 12\angle -53.13$$

$$P.F = \cos(\theta_v - \theta_i)$$

$$= \cos(0 - -53.13) = .6$$

(lagging)

- $v(t) = \underline{5}\cos(\omega t)$

peak value

→ follow

$- P_{avg}(2-j) = \frac{1}{2} I_m^2 R = I_{RMS}^2 R$

power of imaginary = 0

$= (12)^2 * 2 = 288$ W

$- P_{avg}(1+j5) = (12)^2 * 1 = 144$ W

- power (apparent) prudused By the source :-

$= V_{RMS} * I_{RMS} = 60 * 12 = 720$ $\boxed{}$ unit ??

$442 \neq 720$?

complex power (S)

↳ some power is lost

* to represent the combined power in R & X

R ⟶ absourbed real power

X → L, → C] ⟶ stored reactive power

طاقة ضائعة ← عمليات الشحن والتفريغ والمجالات المغناطيسية.

- complex power (S)

↳ P ⟶ Real power (R)

↳ Q ⟶ Reactive power (L/C)

↳ charging and discharging of (L) and (C)

(L) ↓ current

(C) ↓ voltage

take :-

$$V = |V_{eff}| \angle \theta_v \quad \& \quad I = |I_{eff}| \angle \theta_i$$

$$P = |V_{eff}|\,|I_{eff}| \cos(\theta_v - \theta_i)$$

$$= |V_{eff}|\,|I_{eff}| \, Re\left\{ e^{j(\theta_v - \theta_i)} \right\}$$

$$-e^{j(\theta_v - \theta_i)} = \cos(\theta_v - \theta_i) + j\sin(\theta_v - \theta_i)$$

$$Re\left\{ e^{j(\theta_v - \theta_i)} \right\} = \cos(\theta_v - \theta_i)$$

$$Im\left\{ e^{j(\theta_v - \theta_i)} \right\} = j\sin(\theta_v - \theta_i)$$

$$\Rightarrow P = Re\left\{ |V_{eff}|\, e^{j\theta_v} * |I_{eff}|\, e^{-j\theta_i} \right\}$$

(real power) $P = Re\{ V I^* \}$

$Q = Im\{ V I^* \}$ (reactive power)

$\vec{S} = V I^*$ (complex power)

$$\underset{\text{(complex power)}}{S} = VI^* = \underset{\text{(Real power)}}{P} + j\underset{\text{(Reactive power)}}{Q}$$

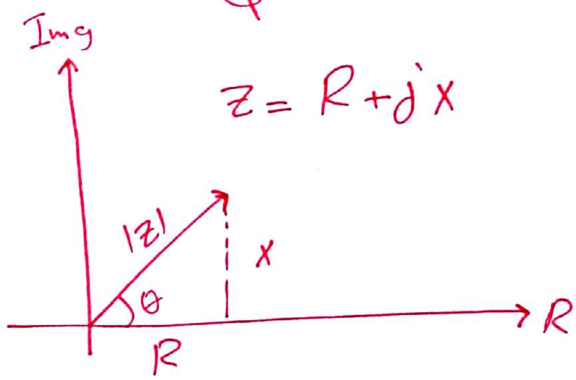
(unit V.A) (unit W) $|R|$ (unit VAR) (L/C) ↳ (volt Amper reactive)

$$- S = VI^* = |V_{eff}|\,|I_{eff}|\, e^{j(\theta_v - \theta_i)}$$

$$S = |V_{eff}|\,|I_{eff}| \left[\cos(\theta_v - \theta_i) + j\sin(\theta_v - \theta_i)\right]$$

$$S = \underbrace{|V_{eff}|\,|I_{eff}|\cos(\theta_v - \theta_i)}_{P} + j\underbrace{|V_{eff}|\,|I_{eff}|\sin(\theta_v - \theta_i)}_{Q}$$

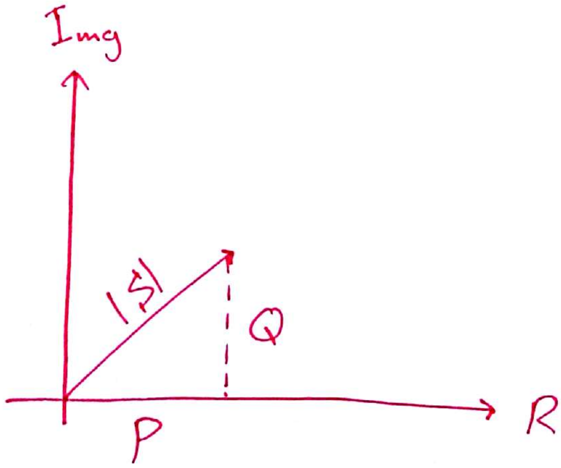
$$Z = R + jX$$



\# $P = |V_{eff}|\,|I_{eff}|\cos(\theta_v - \theta_i)$

$P = V_{RMS}\, I_{RMS} \cos(\theta_v - \theta_i)$

$P = \dfrac{V_m I_m}{2} \cos(\theta_v - \theta_i)$



\# $Q = |V_{eff}|\,|I_{eff}|\sin(\theta_v - \theta_i)$

$Q = V_{RMS}\, I_{RMS} \sin(\theta_v - \theta_i)$

$Q = \dfrac{V_m I_m}{2} \sin(\theta_v - \theta_i)$

$$S = P + jQ$$

$* \; S = P + jQ$

$S \downarrow$ complex, VA

$P \downarrow$ Real, W

$Q \downarrow$ Reactive, VAR

$$* \; P = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i)$$

$$= V_{RMS} I_{RMS} \cos(\theta_v - \theta_i)$$

$$* \; Q = \frac{V_m I_m}{2} \sin(\theta_v - \theta_i)$$

$$= V_{RMS} I_{RMS} \sin(\theta_v - \theta_i)$$

$$* \; |S| = \sqrt{P^2 + Q^2}$$

$\downarrow$

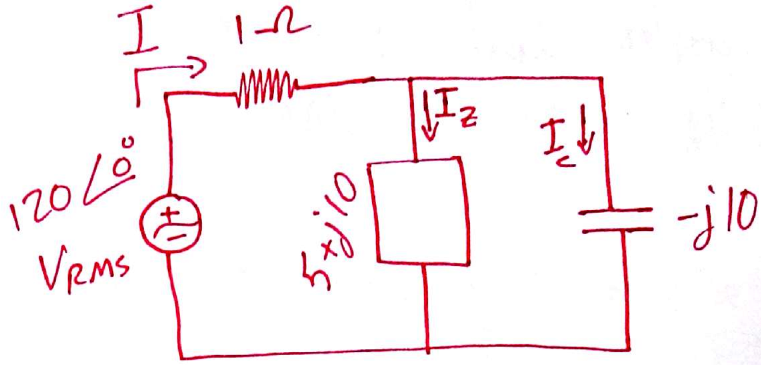
apparent

VA

$$* \; 0 \leq PF = \frac{P}{|S|} \leq 1$$

(ex) Find the complex power (absorbed/produced) by each element?

sol



$$I = \frac{120\angle 0^\circ}{1 + \frac{(5+j10)(-j10)}{5}}$$

$$I = 5.16\angle 25.46^\circ \text{ A}$$

$S_{source} = V I^* = (120\angle 0^\circ)(5.16\angle -25.46)$

$$= 558.98 - j\,266.14 \text{ VA}$$

* For sources: +ve sign means power generated, −ve sign means power absorbed.

* For elements: the opposite of sources

$S_{(1\Omega)} = V I^* = (I R) I^*$ * $P_{(1\Omega)} = 26.6$ W

$$= (5.16\angle -25.46^\circ) * (5.16\angle 25.46^\circ * 1) = 26.6 + j0 \text{ VA}$$

$S_{(-j10)} = V I_c^* = (I_c * (-j10)) * I_c^*$

$$I_c = I * \frac{(5+j10)}{5} = 11.53\angle 88.89 \text{ A}$$

* the capacitor works as a generator of reactive power

$$S_{(-j10)} = ((11.53\angle 88.89) * (-j10)) * (11.53\angle -88.89)$$

$$= -j\,1331 \text{ VA}, \quad Q = 1331 \text{ VAR}$$

$S_{(5+j10)} = V I_z^* = (I_z * (5+j10)) * I_z^*$

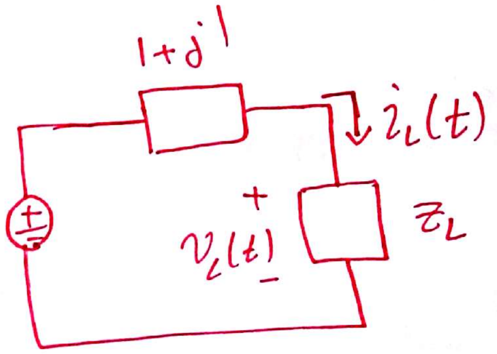
$I_z = I - I_c = 10.31\angle -64.53$

$S_{(5+j10)} = ((10.31\angle -64.53) * (5+j10)) * (10.31\angle 64.53)$

$= 531.48 + j1062.96$ VA

---

ex) Find (PF) and (complex power) For both source and load?



$v_L(t) = 60\cos(wt - 10)$ V

$i_L(t) = 1.5\cos(wt + 50)$ A

$PF_{(load)} = \cos(\theta_{v_L} - \theta_{i_L})$

$= \cos(-10 - 50) = .5$ leading

$V_{RMS_L} = \frac{60}{\sqrt{2}}\angle -10$ V , $I_{RMS_L} = \frac{1.5}{\sqrt{2}}\angle 50$ A

$S_{Load} = V_L I_L^* = \left(\frac{60}{\sqrt{2}}\angle -10\right) * \left(\frac{1.5}{\sqrt{2}}\angle -50\right) = 45\angle -60$ VA

$= \underbrace{22.5}_{P} - j\underbrace{38.97}_{Q}$

$|S| = \sqrt{(22.5)^2 + (38.97)^2} = 45$

$P = |S|\cos(\theta)$

PF angle $= \cancel{\theta}(\theta_v - \theta_i)$

$Q = |S|\sin(\theta)$

$$V_{RMS(source)} = V_{L(RMS)} + I_{L(RMS)} * (1+j1)$$

$$= \frac{60}{\sqrt{2}} \angle{-10} + \frac{1.5}{\sqrt{2}} \angle{50} (1+j1)$$

$$= 41.65 + j5 = 42 \angle{-8} \ V$$

$$PF_{(source)} = \cos(\theta_v - \theta_i)$$

$$= \cos(-8-50) = 0.53 \text{ leading}$$

$$S_{(source)} = V * I^* = (42 \angle{-8}) * \left(\frac{1.5}{\sqrt{2}} \angle{-50}\right)$$

$$= 44.54 \angle{-58} = 23.6 - j\,37.7 \ VA$$

- $P_{losse\ T.L} = 23.6 - 22.5 = 1.1 \ W$

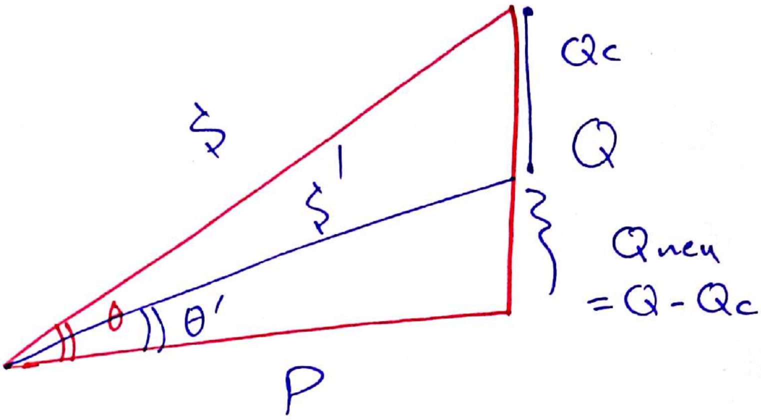
- $Q_{losse\ T.L} = 38.97 - 37.7 = 1.27 \ VAR$

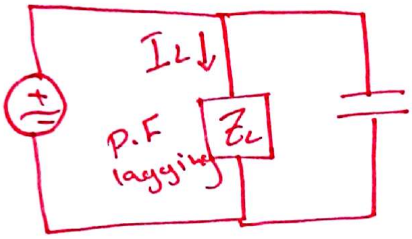
- $S_{T.L} = 1.1 + j\,1.27 \ VA$

* PF correction :-

- increasing the PF of the load without changing it's current or voltage

- it could be done by adding a capacitor in parallel with the load.





$I_L = \frac{V}{Z_L}$

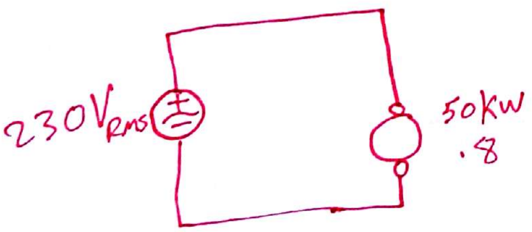
$I_L' = \frac{V}{Z_L}$

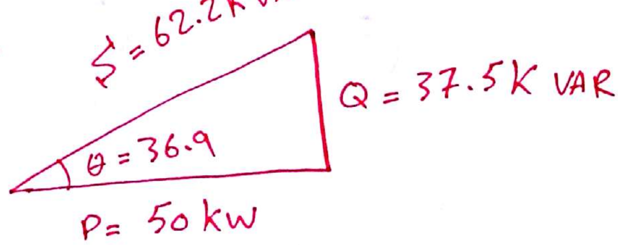
$\Big\}\ I_s' \neq I_s$

(ex) A 50 kw, 0.8 lagging PF motor is aporated a 230 V, it is required to increase the PF to 0.95 lagging.

sol: - draw the power tringle before adding the capacitor.

Before :





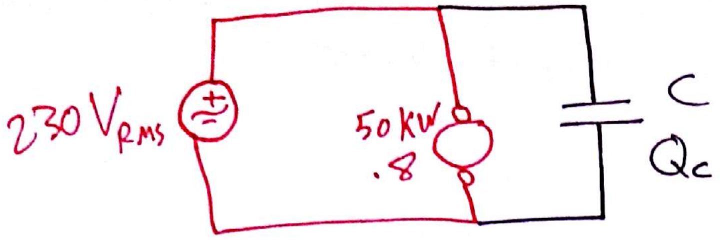
$PF = \cos\theta = .8$

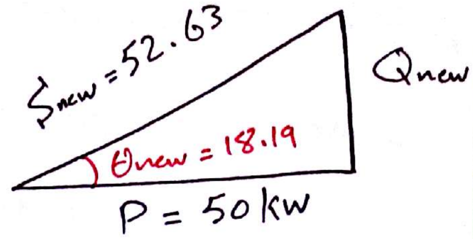
$S = \frac{P}{.8} = 62.5$

$\tan\theta = \frac{Q}{P}$

→ follow

After





$\theta_{new} = \cos^{-1}(0.95) = 18.19°$

$Q_c = Q_{new} - Q_{old}$

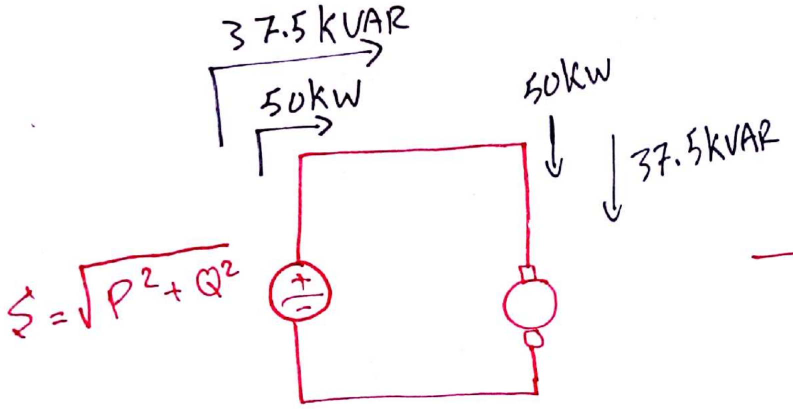
$|Q_c| = (16.43 - 37.5)\ k\,VAR$

$= -21.07\ k\,VAR$

$S_{new} = \dfrac{P}{0.95} = 52.63$

$\tan(\theta_{new}) = \dfrac{Q_{new}}{P}$

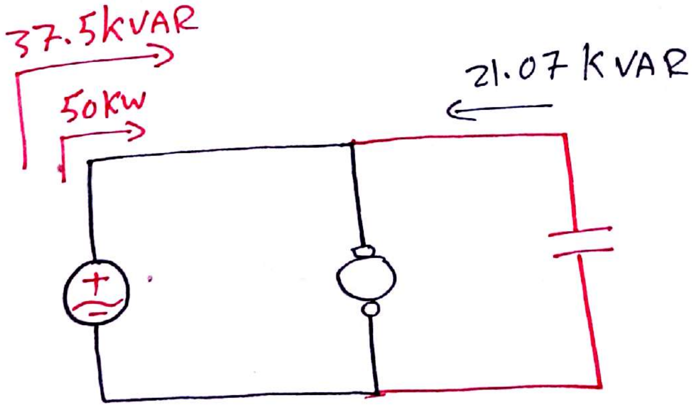
$Q_{new} = 16.43\ k\,VAR$



$\rightarrow$

$PF = \dfrac{P}{S}$

$= \dfrac{50}{62.5} = .8$



$PF = \dfrac{P}{S} = \dfrac{50}{\sqrt{(50)^2 + (16.43)^2}} = 0.95$

$$Q_c = V_{RMS} \, I_{RMS} \, \cancel{\sin(\theta_v - \theta_i)}^{\,1}$$

$$Q_c = V_{RMS} \, I_{RMS} = V_{RMS} * \frac{V_{RMS}}{X_c \rightarrow \frac{1}{wc}}$$

$$Q_c = (V_{RMS})^2 * wc$$

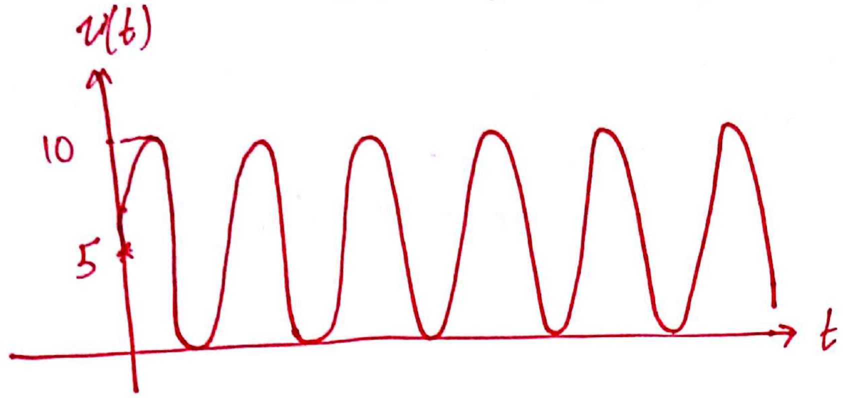
$$C = \frac{Q_c}{(V_{RMC})^2 * w}$$

In the previous ex :-

$$C = \frac{21070}{(230)^2 * 2\pi * f} \rightarrow$$

تكون معطاه بالسؤال

ex) Find the (RMS) value for the shown voltage waveform



$$v(t) = 5\sin(\omega t) + 5 \text{ V}$$

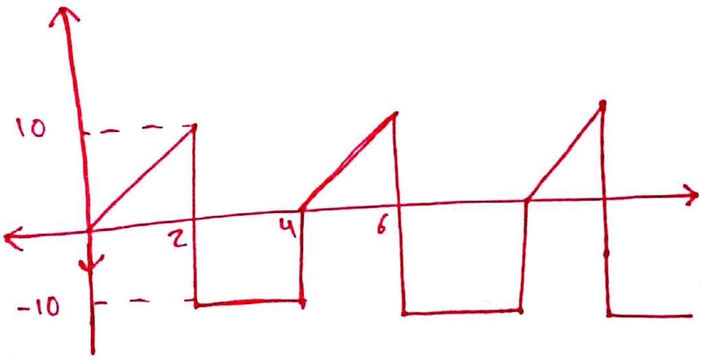
$$V_{eff} = \sqrt{\left(\frac{5}{\sqrt{2}}\right)^2 + (5)^2} = 6.123 \text{ V}$$

---

ex) a) Find the (RMS) value :-

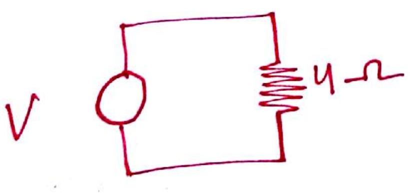
$T = 4$

$$v(t) = \begin{cases} 5t, & 0 \le t < 2 \\ -10, & 2 \le t < 4 \end{cases}$$



$$V_{RMS} = \sqrt{\frac{1}{4}\left[\int_0^2 25t^2\,dt + \int_2^4 (-10)^2\,dt\right]} = 8.165 \text{ V}$$

b) what is the absorbed power by a (4 Ω) resistor connected to this voltage source.

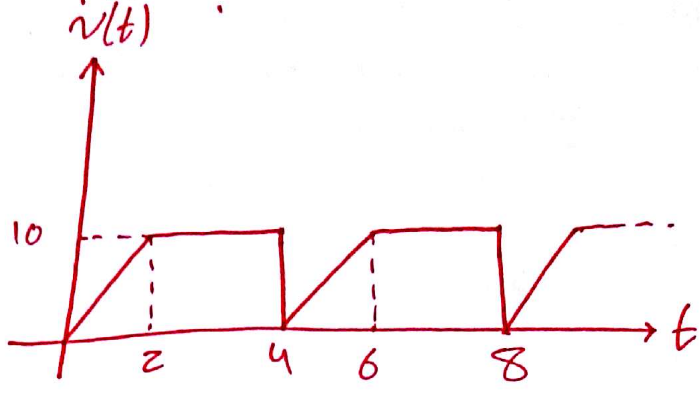


$$P = \frac{(V_{RMS})^2}{R} = \frac{(8.165)^2}{4}$$

ex) Find the (RMS) value for ?

$T = 4$

$$v(t) = \begin{cases} 5t, & 0 \le t < 2 \\ 10, & 2 \le t < 4 \end{cases}$$

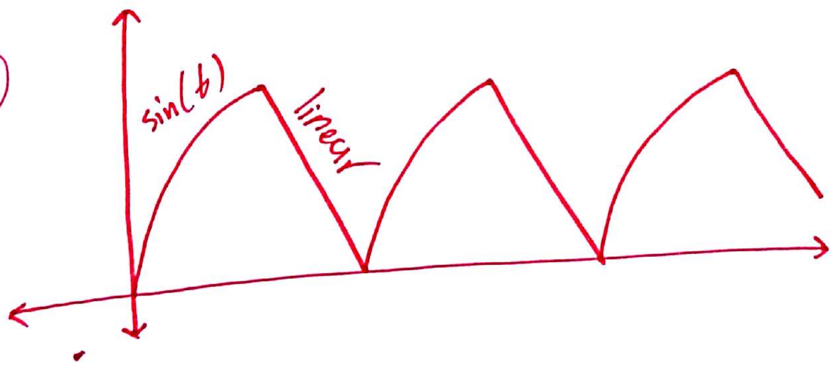


$$V_{RMS} = \sqrt{\frac{1}{4}\left[\int_0^2 25t^2\,dt + \int_2^4 (10)^2\,dt\right]} = 8.165 \text{ V}$$

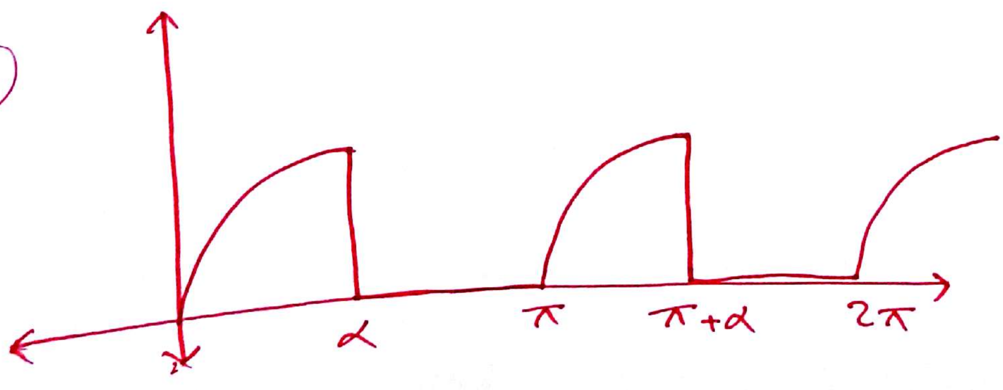
$$V_{avg} = \frac{1}{4}\left[\int_0^2 5t\,dt + \int_2^4 (10)\,dt\right] = \frac{30}{4} = 7.5 \text{ V}$$

ex) Find (RMS) value

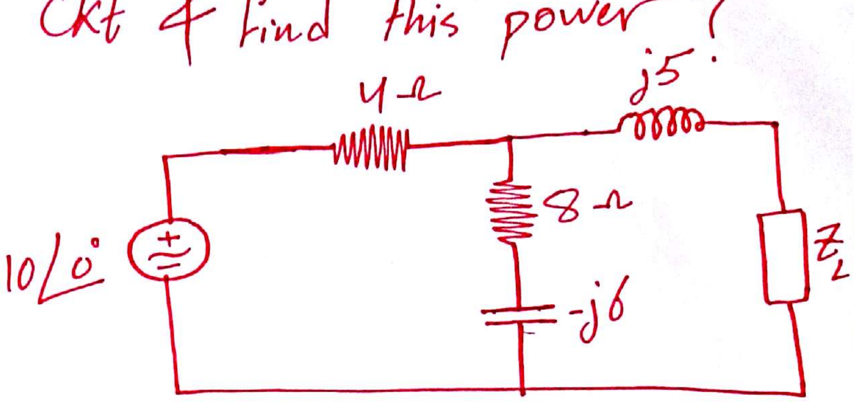
(1)



(2)



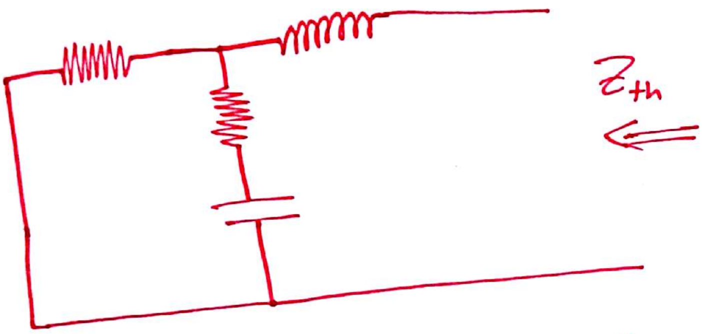
ex) Determine the load impedance (Z_L) that maximize the power drawn from the Ckt & find this power?



Sol:

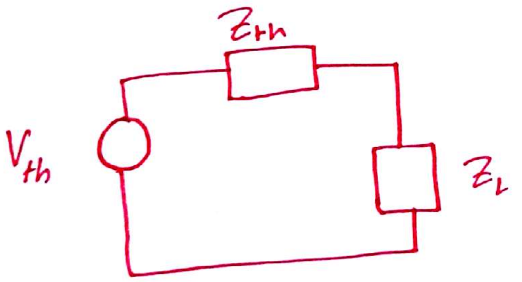
$Z_L = Z_{th}^*$

- Find Z_{th} :-

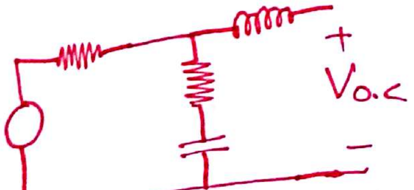


$$Z_{th} = j5 + \frac{4 * (8 - j6)}{12 - j6} = (2.933 + j4.467)\ \Omega$$

$$Z_L = Z_{th}^* = (2.933 - j4.467)\ \Omega$$



$V_{th} = V_{o.c}$



$$V_{o.c} = 10\angle 0° * \frac{(8 - j6)}{(8 + 4 - j6)}$$

$V_{oc} = 7.45\angle -10.3\ V$

$P_{max} = V_{RMS(L)}\, I_{RMS(L)} = \frac{1}{2} V_L I_L$

$P_{max} = I_{RMS}^2 * R$

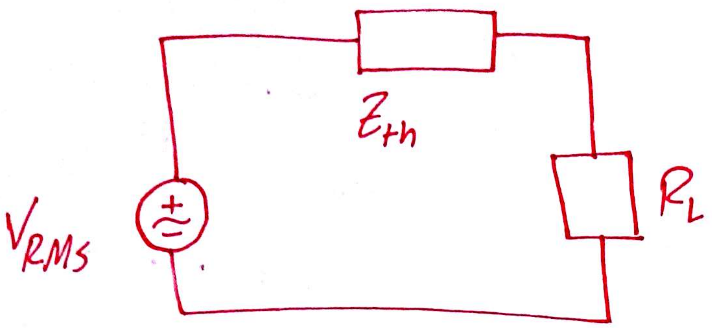
- $I_{RMS} = \dfrac{V_{th(RMS)}}{Z_{th} + Z_L} = \dfrac{V_{th(RMS)}}{2R}$

$$= \frac{7.45\angle -10.3}{2 * 2.933}$$

$$P = I_{RMS}^2 * R = \frac{V_{th(RMS)}^2}{4R^2} * R$$

$$= \frac{V_{thRMS}^2}{4R} = \frac{V_m^2}{8R}$$

- what if $Z_L = R_L$:- ?



$$R_L = |Z_{th}| = \sqrt{(R_{th})^2 + (X_{th})^2}$$