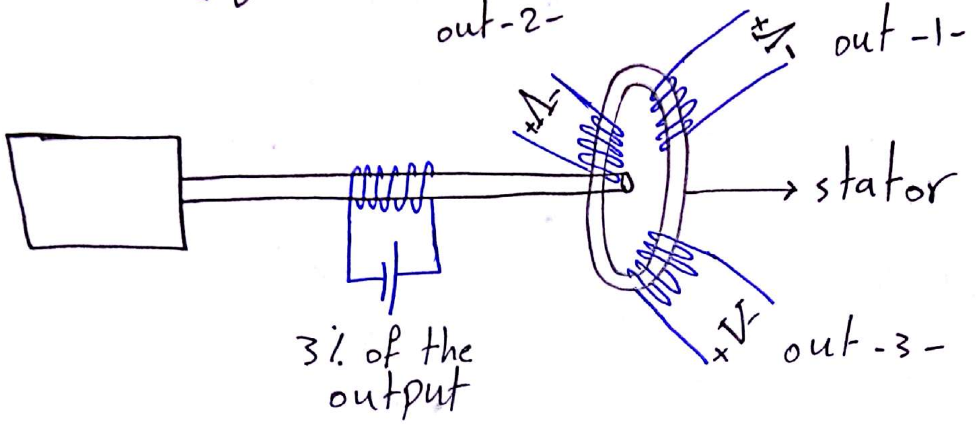
CHp -12- : three - phase circuits.

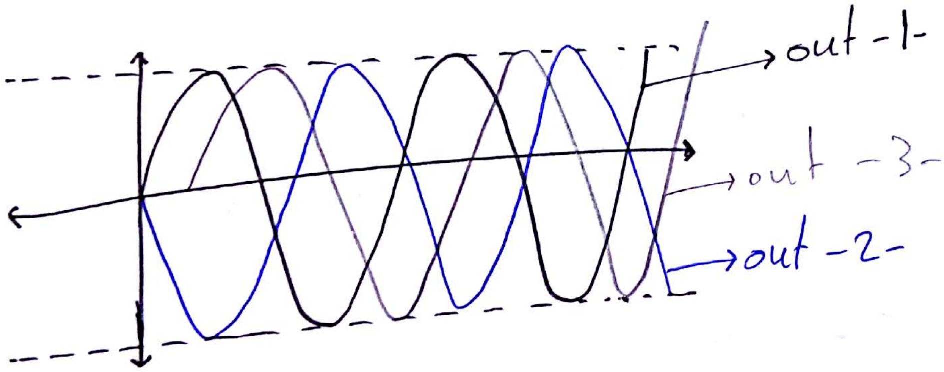
How electricity is being generated ?

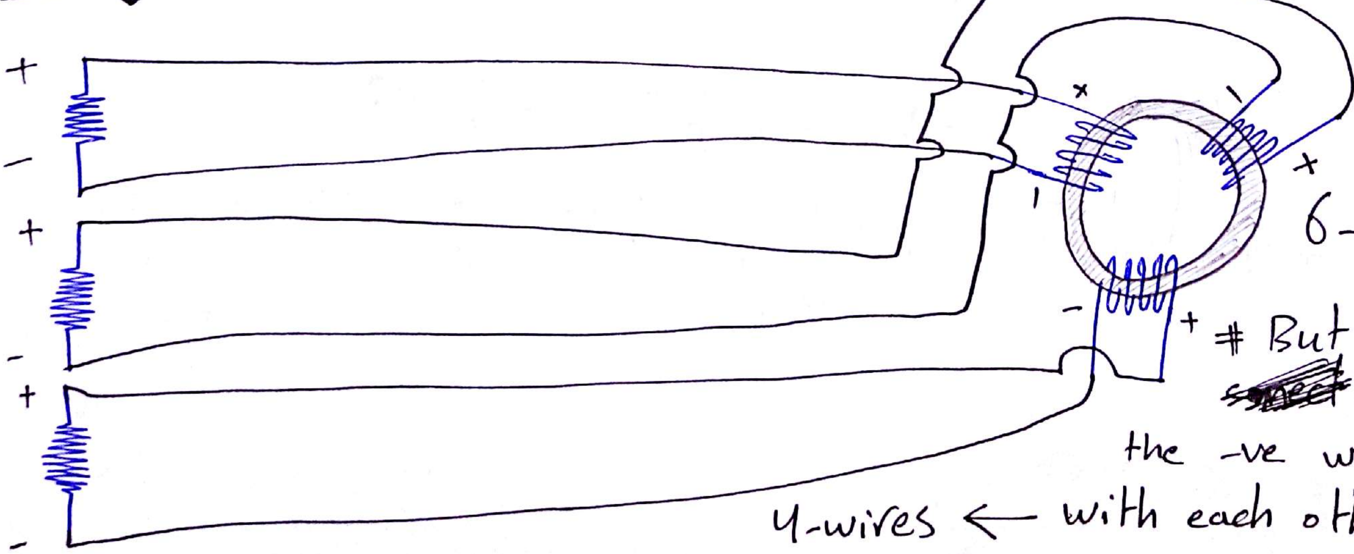
$$V = -N \frac{d\phi}{dt} \rightarrow \text{flux}$$



Amper's law :

$$I = \oint H \cdot dl \rightarrow$$ { اي سلك بمر فيه تيار يولد حوله مجال مغناطيسي . }



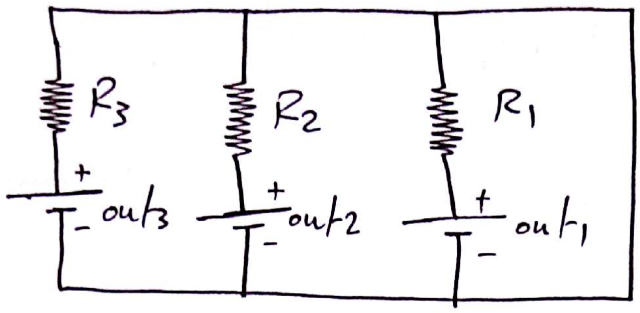


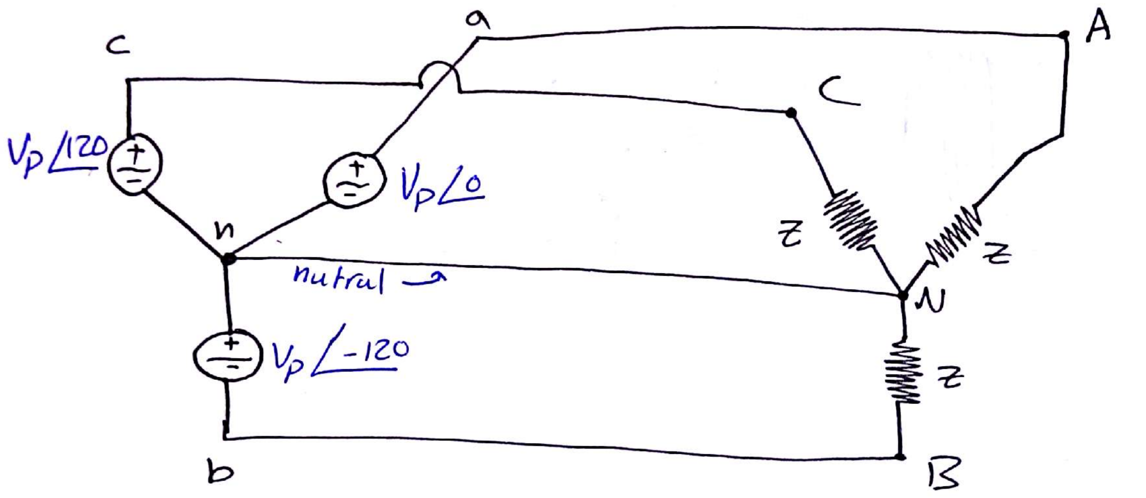
6-wires

But we can ~~connect~~ connect the -ve wires with each others

4-wires ← with each others

common





Y - Y 3 phase-system

$$V_{an} = V_p \angle 0^\circ$$

$$V_{bn} = V_p \angle -120^\circ$$

$$V_{cn} = V_p \angle -240^\circ = V_p \angle +120^\circ$$

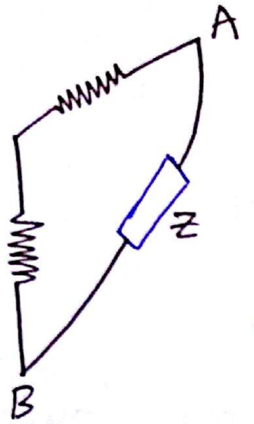
$$\left\{ |V_{an}| = |V_{bn}| = |V_{cn}| \right\}$$

$$* \; V_{an} + V_{bn} + V_{cn} = V_p \angle 0 + V_p \angle -120 + V_p \angle 120$$

$$= \underbrace{V_p}_{\cancel{1}} + V_p \underbrace{\cos(-120)}_{\cancel{-\frac{1}{2}}} + j V_p \underbrace{\sin(-120)}_{\cancel{-\frac{\sqrt{3}}{2}}} + V_p \underbrace{\cos(120)}_{\cancel{-\frac{1}{2}}} + j V_p \underbrace{\sin(120)}_{\cancel{\frac{\sqrt{3}}{2}}}$$

$$= \text{zero}$$

- اذا شبكنا (load) بين (A, B)



$$V_{ab} = V_{AB} = V_{an} - V_{bn}.$$

$$= V_p\angle 0 - V_p\angle -120$$

$$V_{ab} = V_p - V_p\cos(-120) \mp j V_p \sin(-120)$$

$$V_{ab} = V_p - \left(\frac{-1}{2} V_p\right) \mp j V_p \left(\frac{-\sqrt{3}}{2}\right)$$

$$V_{ab} = \frac{+3}{2} V_p + j V_p \frac{\sqrt{3}}{2}$$

$$V_{ab} = \sqrt{3}\left(\frac{\sqrt{3}}{2} V_p + j V_p \frac{1}{2}\right)$$

$$V_{ab} = \sqrt{3}\, V_p \angle 30^\circ$$

- and so on: -

$V_{ca} = V_p \angle 150$, $V_{bc} = \sqrt{3}\, V_p \angle -90^\circ$

* $I_a = I_{aA}$ (from (a) to (A))

$I_b = I_{bB}$

$I_c = I_{cC}$

$I_n = I_{nN}$?? → (الاتجاه)

$V_{an} = V_p \angle 0$

$V_{bn} = V_p \angle -120 = |V_{an}| \angle -120$

$V_{cn} = V_p \angle 120 = |V_{an}| \angle 120$

→ $I_a \rightarrow KVL \rightarrow -V_{an} + I_a Z = 0 \rightarrow I_a = \frac{V_{an}}{Z} = \frac{V_p\angle 0^\circ}{Z} = I_p \angle 0^\circ$

→ $I_b \rightarrow KVL \rightarrow -V_{bn} + I_b Z = 0 \rightarrow I_b = \frac{V_{bn}}{Z} = \frac{V_p\angle -120}{Z} = I_p \angle -120$

→ $I_c \rightarrow KVL \rightarrow -V_{cn} + I_c Z = 0 \rightarrow I_c = \frac{V_{cn}}{Z} = \frac{V_p\angle 120}{Z} = I_p \angle 120$

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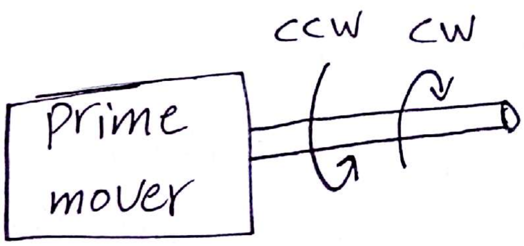
⇒ So we can grounded n and N

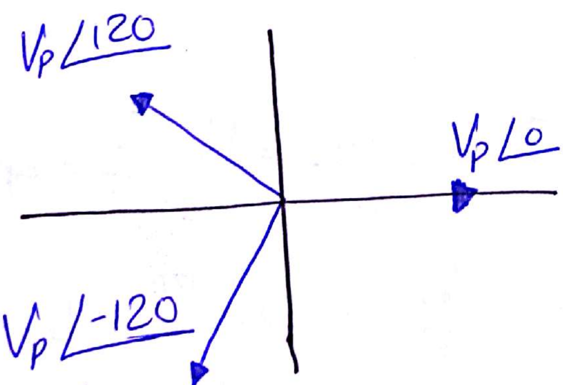
- but we must to transfere the wire that the louds are not equal (un Balanced)

- but at the (Balanced system) → we can ground it.

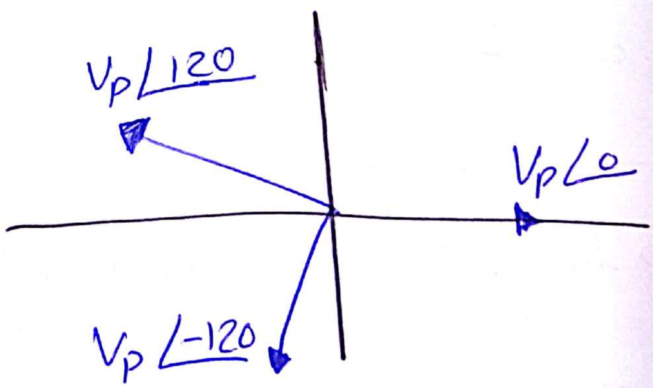
\# I_a , I_b , I_c → line current (خارج من المصدر)

\# I_A , I_B , I_C → phase current (داخل في ال load)





- ccw.
- Negative sequance.
- california.



- cw
- positive sequance
- the rest of world.

I_n (Y) connection :-

$$I_{line} = I_{phase}$$

$V_{ab} = V_{AB} = \sqrt{3}\, V_p \angle +30$
$V_{bc} = V_{BC} = \sqrt{3}\, V_p \angle -90$
$V_{ca} = V_{CA} = \sqrt{3}\, V_p \angle +150$

→ line to line voltage (voltage line).

$V_p \rightarrow \left\{ \begin{matrix} V_{an} \\ V_{bn} \\ V_{cn} \end{matrix} \right\}$ → phasor to nutral voltage → phasor voltage → phase to nutral

proof

$$\Rightarrow \quad V_{ca} = V_{CA} = V_{cn} - V_{an}$$

$$= V_p \angle 120^\circ - V_p \angle 0^\circ$$

$$= V_p(\cos(120) + j\sin(120)) - V_p$$

$$= \frac{-1}{2} V_p + j\frac{\sqrt{3}}{2} V_p - V_p$$

$$= \frac{-3}{2} V_p + j\frac{\sqrt{3}}{2} V_p$$

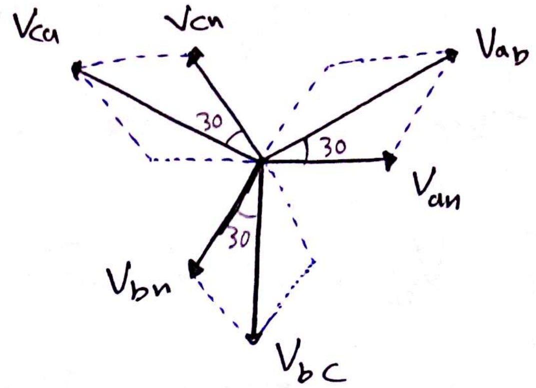
$$= \sqrt{3}\left(\frac{-\sqrt{3}}{2} V_p + j\frac{1}{2} V_p\right)$$

$$= \sqrt{3}\, V_p \angle 150^\circ$$

$V_{ab} = \sqrt{3}\ V_p \angle 30^\circ$

$V_{bc} = \sqrt{3}\ V_p \angle -90$

$V_{ca} = \sqrt{3}\ V_p \angle 150$



- كل (line) يسبق ال (phase)
ال كونتها بمقدار (30°)

- in general :-

In Y - connection loads :-

$$V_{LL} = \sqrt{3}\ V_p \angle +30$$

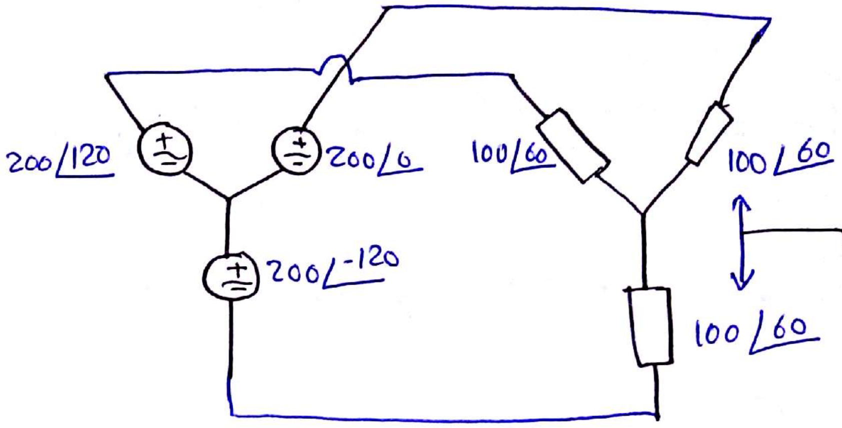
* $V_{ab} = \sqrt{3}\ V_{an} \angle +30$

* $V_{bc} = \sqrt{3}\ V_{bn} \angle +30$

* $V_{ca} = \sqrt{3}\ V_{cn} \angle +30$

ex) for the Balanced ckt shown, find (line phase voltage) and (line & phase current), then find the (total power absorbed By the load).

$$\left\{ \begin{aligned} V_{an} &= 200 \angle 0^\circ \\ V_{bn} &= 200 \angle -120^\circ \\ V_{cn} &= 200 \angle 120^\circ \end{aligned} \right\} \text{phase voltage}$$



نفس ال load (balanced)

$V_{ab} = \sqrt{3}\, V_{an} \angle +30 \; V$

$= \sqrt{3} * 200\angle 0 * \angle +30$

$= 346.4 \angle +30 \; V$

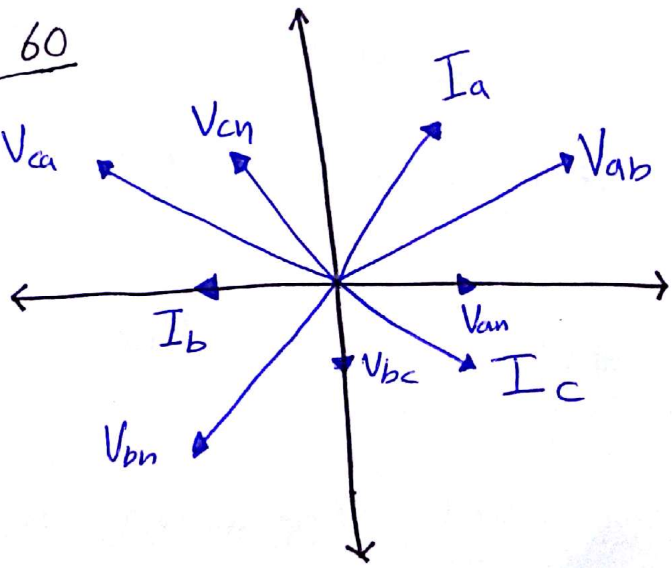
$V_{bc} = 346.4 \angle -90 \; V$

$V_{ca} = 346.4 \angle 150 \; V$

$* \; I_a = \dfrac{V_{an}}{Z} = \dfrac{200\angle 0}{100\angle 60} = 2\angle -60$

$* \; I_b = \dfrac{V_{bn}}{Z} = \dfrac{200\angle -120}{100\angle 60} = 2\angle -180$

$* \; I_c = \dfrac{V_{cn}}{Z} = \dfrac{200\angle 120}{100\angle 60} = 2\angle 60$



$I_{line\,a} = I_a$

$I_{line\,b} = I_b$

$I_{line\,c} = I_c$

$\Bigg\}$ Y - Y

$I_{line} = I_{phase}$

power

$$P_A = |V_{an}||I_a| \cos(\theta_v - \theta_i)$$
$$= 200 * 2 * \cos(0 - -60) = 200\ W$$

$$P_B = 200\ W \rightarrow \cos(\theta_v - \theta_i) = \cos(-120 - -180)$$

$$P_c = 200\ W \rightarrow \cos(120 - 60).$$

$P_{total} = 600\ W \rightarrow$ (مجموعهم) (avg. power).

For Balanced system $\rightarrow \boxed{P_{3\phi} = 3 P_{1\phi}}$

---

inst. power :-

$$V_{an} = 200 \angle 0 \rightarrow V_{an}(t) = 200\sqrt{2} \cos(wt).$$
$$I_a = 2 \angle -60 \rightarrow I_a(t) = 2\sqrt{2} \cos(wt - 60).$$

$$p(t) = V_{an}(t) * I_a(t)$$
$$= 400 \cos(wt - 60) \cos(wt)$$

$$\left.\begin{aligned} &\# P_a(t) = 200 + 400 \cos(2wt - 60) \\ &\# P_b(t) = 200 + 400 \cos(2wt + 60) \\ &\# P_c(t) = 200 + 400 \cos(2wt - 180) \end{aligned}\right\} \text{instantenuous power}$$

But $P(t)_{total} = P_a(t) + P_b(t) + P_c(t)$.

$$= 600 + \left.\begin{array}{l} 400\cos(2wt - 60) \\ + 400\cos(2wt + 60) \\ + 400\cos(2wt - 180) \end{array}\right] \text{Zero.}$$

$$= 600 \text{ W}$$

$\rightarrow P_{inst} = P_{avg}$ in 3ϕ

$P = wT \rightarrow P \rightarrow$ constant. $\{w, T \rightarrow \text{constant}\}$

↳ less vibration and stress on the mechanical system.

---

in 3ϕ:-

$$P_{losses} = I_a^2 R + I_b^2 R + I_c^2 R$$

$$= 12R \text{ W}$$

- let $R = 1$

$$P = 12 \text{ W}$$

in 1ϕ:-

$$P = VI\cos(\theta v - \theta i)$$

$$= I^2 R$$

$$= 36 R \text{ W (losses)}$$

- let $R = 1 \rightarrow P = 36 \text{W}$

the 3ϕ Better than 1ϕ system.

* advantages of 3ϕ:-

1. generator
2. constant torque
3. power high.

(ex) a Balanced 3ϕ system with $\underline{\underline{V_{LL}}} = 300\ V$

↳ By defult

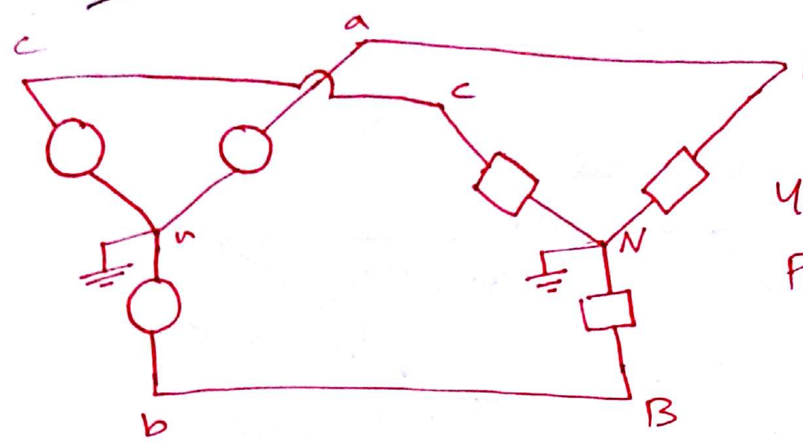
and suppling a balanced Y- connected loud

with (1200 W) @ a leading power Factor = .8

, Find the line current and the per phase

loud impedance ?

ex Sol



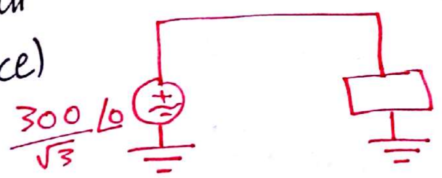
400 W
P.F = .8 leading

$V_{ab} = 300\angle 30$

$\downarrow$

$V_{an} = \frac{300}{\sqrt{3}} \angle 0$ → By defult (refrance)

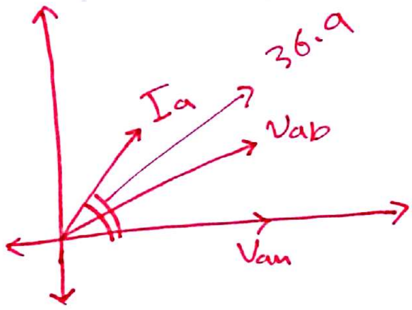
single phase equ ckt:-



$I_{a\,line} = I_{phase}$

$P = V_{an} I_a \; P.F.$

$400 = \frac{300}{\sqrt{3}} * I_a * .8$



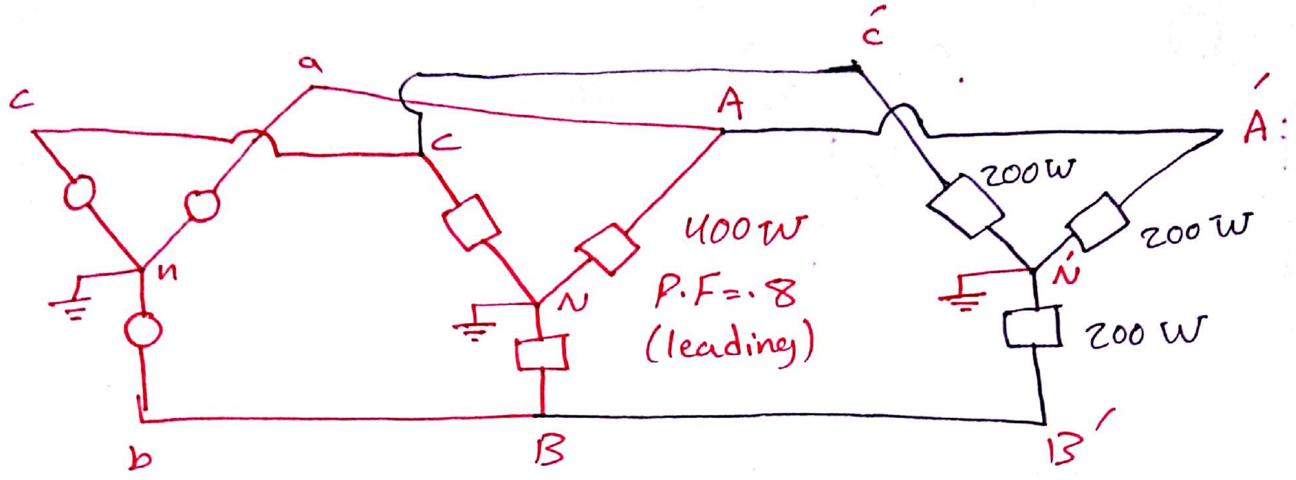
$|I_a| = 2.89$ A

$I_a = 2.89 \angle 36.9$ → $\cos^{-1}(.8)$

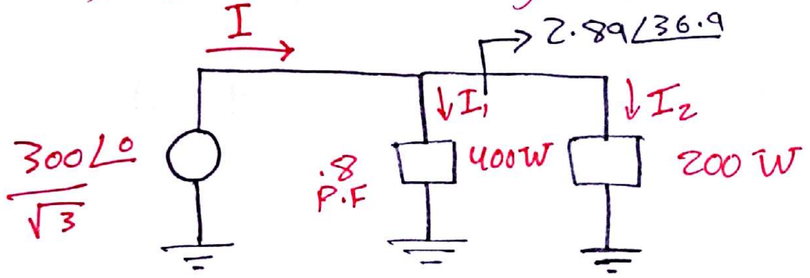
* find the per phase impedance?

$$Z = \frac{V_{an}}{I_a} = \frac{\frac{300}{\sqrt{3}}\angle 0}{2.89\angle 36.9} = 60\angle -36.9° \; \Omega$$

ex) Find the line current if a 600 W load is added in parallel with the previous load? $\boxed{10}$



$\Rightarrow$ Draw the single phase equ ckt :-



$I = I_1 + I_2$

Find I_2 :-

$P = V_{an} \, I_2 \, P.F$

$200 = \frac{300}{\sqrt{3}} * I_2 * 1$

$I_2 = 1.155 \angle 0$

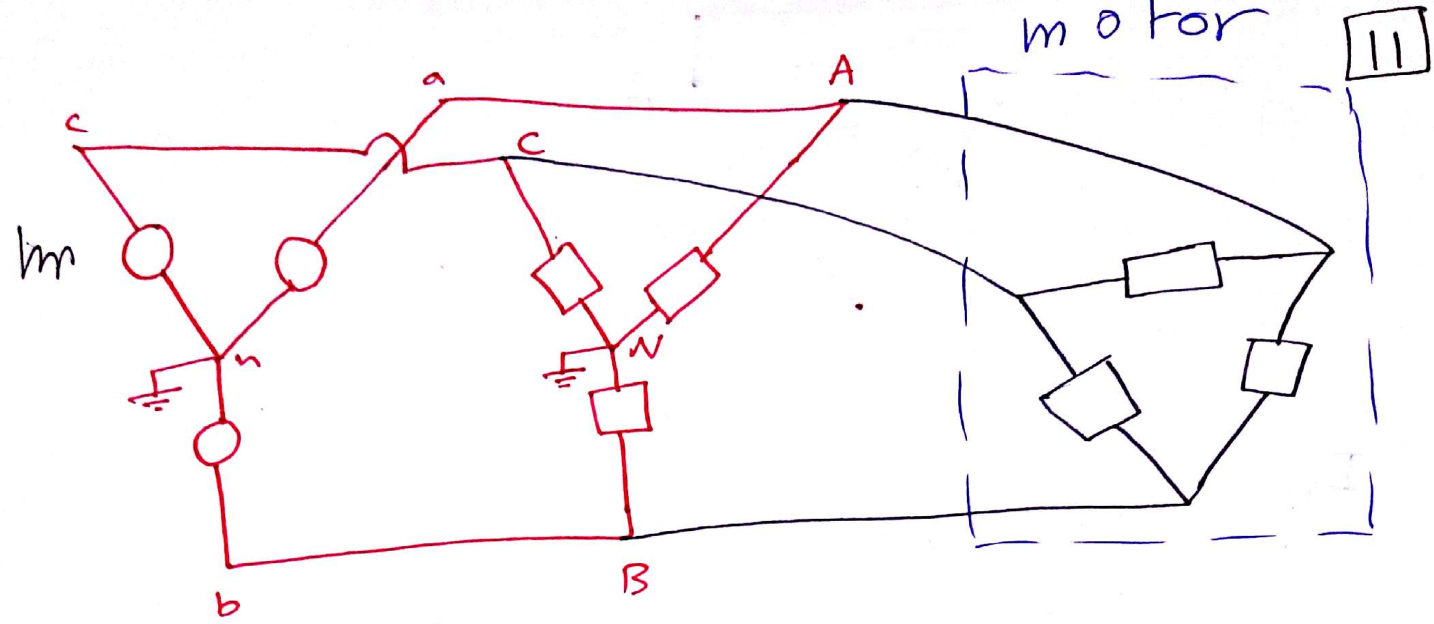
$I_{total} = I_1 + I_2$

$= 2.89 \angle 36.9 + 1.155 \angle 0$

$= 3.87 \angle 26.6$

$P = \frac{300}{\sqrt{3}} * 3.87 * \cos(0 - 26.6) = 600 \text{ W } \{1\phi\}$

$P_{3\phi \text{ source}} = 600 \times 3 = 1800 \text{ W}$



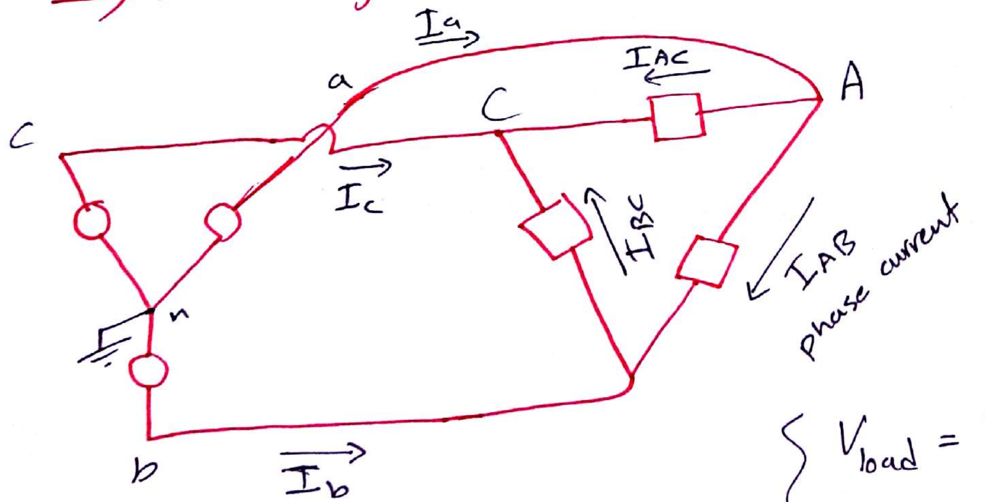
Delta connection → (useful)

- increase the voltage.
- No nutural.

Y source ▷ load :

⇒ the neutral point is not always accessible

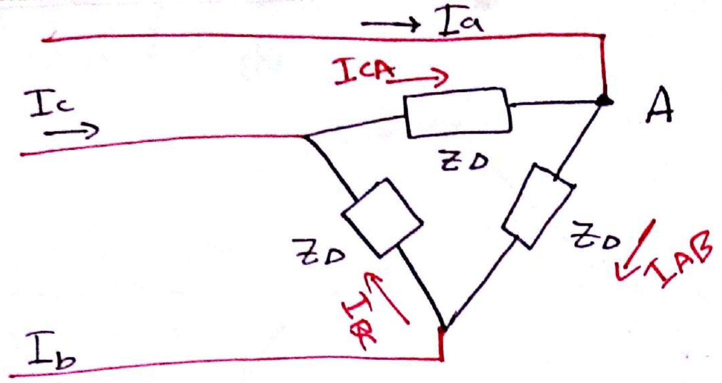
⇒ the voltage across the phase is V_{LL}



$I_a \neq I_{AB}$

the voltage across any load is line-to-line voltage.

$$\begin{cases} V_{load} = V_{LL} = |V_{AB}| = |V_{BC}| = |V_{CA}| \\ V_{AB} = \sqrt{3}\ V_p \angle +30^\circ \\ V_{BC} = \sqrt{3}\ V_p \angle -90^\circ \\ V_{CA} = \sqrt{3}\ V_p \angle 150^\circ \end{cases}$$



$$I_{AB} = \frac{V_{AB}}{Z_D} = I_p \angle 0$$

$$I_a \neq I_{AB}$$

$$I_{BC} = \frac{V_{BC}}{Z_D} = I_p \angle -120$$

$$I_{CA} = \frac{V_{CA}}{Z_D} = I_p \angle 120$$

- apply kcl at (A):-

$$I_a + I_{CA} = I_{AB} \longrightarrow I_a = I_{AB} - I_{CA}$$

$$I_a = I_p \angle 0 - I_p \angle 120$$

$$I_a = I_p - I_p \underbrace{\cos(120)}_{\frac{-1}{2}} - jI_p \underbrace{\sin(120)}_{\frac{\sqrt{3}}{2}}$$

$$I_a = \frac{3}{2} I_p - j\frac{\sqrt{3}}{2} I_p = \sqrt{3} I_p \left(\frac{\sqrt{3}}{2} - \frac{1}{2} j \right)$$

$\longrightarrow$ * $I_a = \sqrt{3}\, I_p \angle -30^\circ$

$\longrightarrow$ * $I_b = \sqrt{3}\, I_p \angle -150^\circ$

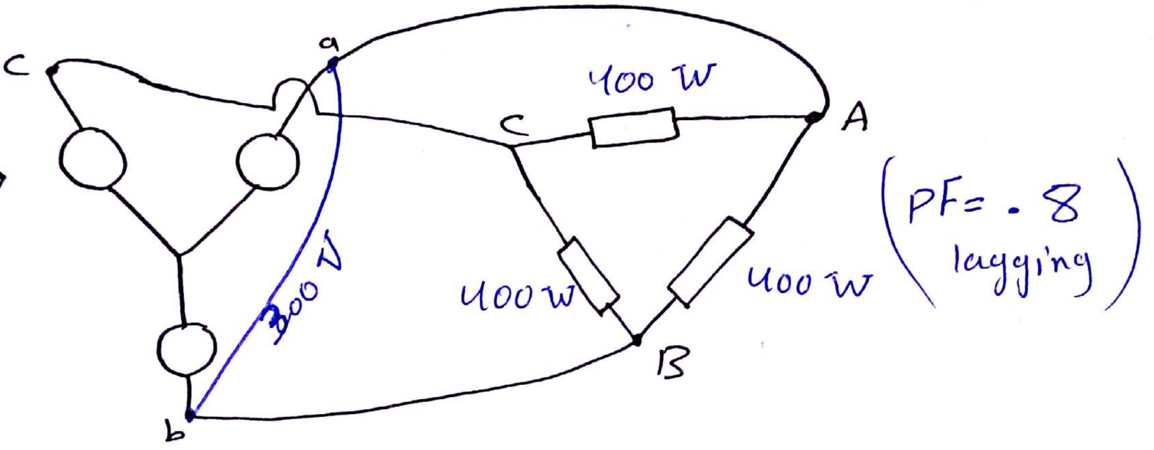
$\longrightarrow$ * $I_c = \sqrt{3}\, I_p \angle 90^\circ$

ex) what is the line current in a (3ϕ) system with line voltage of (300) V and supplies (1200) W to a (Δ-connected) load at a lagging P.F of (0.8) [redacted] $\boxed{13}$

- then find the per phase impedance?

sol

① draw →



$\boxed{2}$ $I\ phase = \frac{V_{AB}}{Z_D} = I_{AB}$

$$I_{AB} = \frac{300}{Z_D} \rightarrow P = V_{AB}\, I_{AB} \cos(\theta)$$

$$400 = 300 * I_{AB} * .8$$

$$I_{AB} = \frac{400}{300 * .8} = 1.6667\,A$$

- take V_{AB} as refrance:

$$V_{AB} = 300\angle 0^\circ \rightarrow I_{AB} = 1.6667\angle \cos^{-1}(.8) = 1.667\angle -36.9 \text{ (lagging)}$$

$$I_a = \sqrt{3}\, I_{AB}\angle -30^\circ = \sqrt{3} * 1.667\angle -36.9\ \angle -30$$
$$= 2.89\angle -66.9\ A$$

$$I_b = 2.89\angle -66.9^\circ - 120^\circ\ A$$

$$I_c = 2.89\angle -66.9^\circ + 120\ A$$

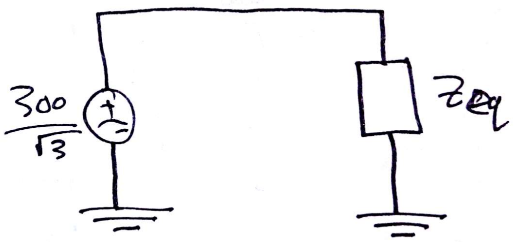
→

$$* Z_\Delta = \frac{V_{AB}}{I_{AB}} = \frac{300\angle 0^\circ}{1.667\angle -36.9} = 180\angle 36.9\ \Omega$$

→ what if the load is (Y) connected?

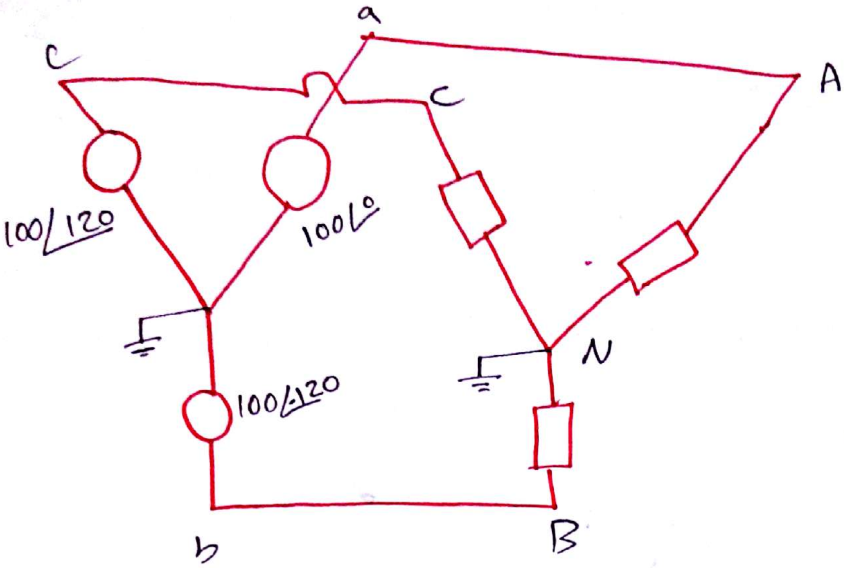
$I_a = ??$

$$P = V_{an}\, I_a * \cos\theta$$



$$400 = \frac{300}{\sqrt{3}} * I_a * .8$$

$$I_a = 2.84\ A$$



- the voltage across any load phase voltage = 100

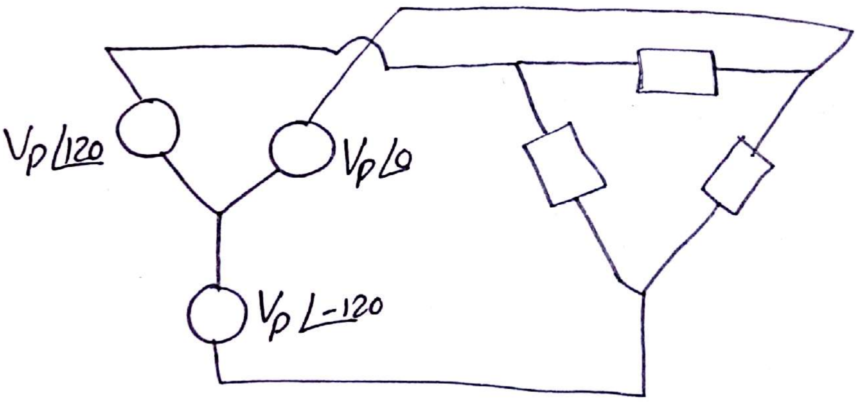
V_{an} , V_{bn} , V_{cn}

- $I_{phase} = I_{line} = \frac{V_{an}}{Z_Y}$

let $V_p = 100\,V$

(Y source → Y load)

(Y source → Δ loud)



* let $V_p = 100\,V$

- the voltage across the loud is :

line voltage $= \sqrt{3} * 100 \angle org + 30^\circ$

$= 173.1 \angle +30 + org$

- $I_{line} \neq I_{phase}$

- $I_{line} = \sqrt{3} * I_{phase} \angle -30$

- $I\ phase = \frac{V_{AB}}{Z_D}$

Power

(Real) $P_{3\phi} = 3 V_p I_p \cos(\theta_v - \theta_i)$

$$= \underbrace{\sqrt{3}\, V_L I_L}\cos(\theta_v - \theta_i)$$

in (Y) connected louds :-

$I_L = I_p$

$V_L = \sqrt{3}\, V_p$

in (Δ) connected louds

$V_L = V_p$

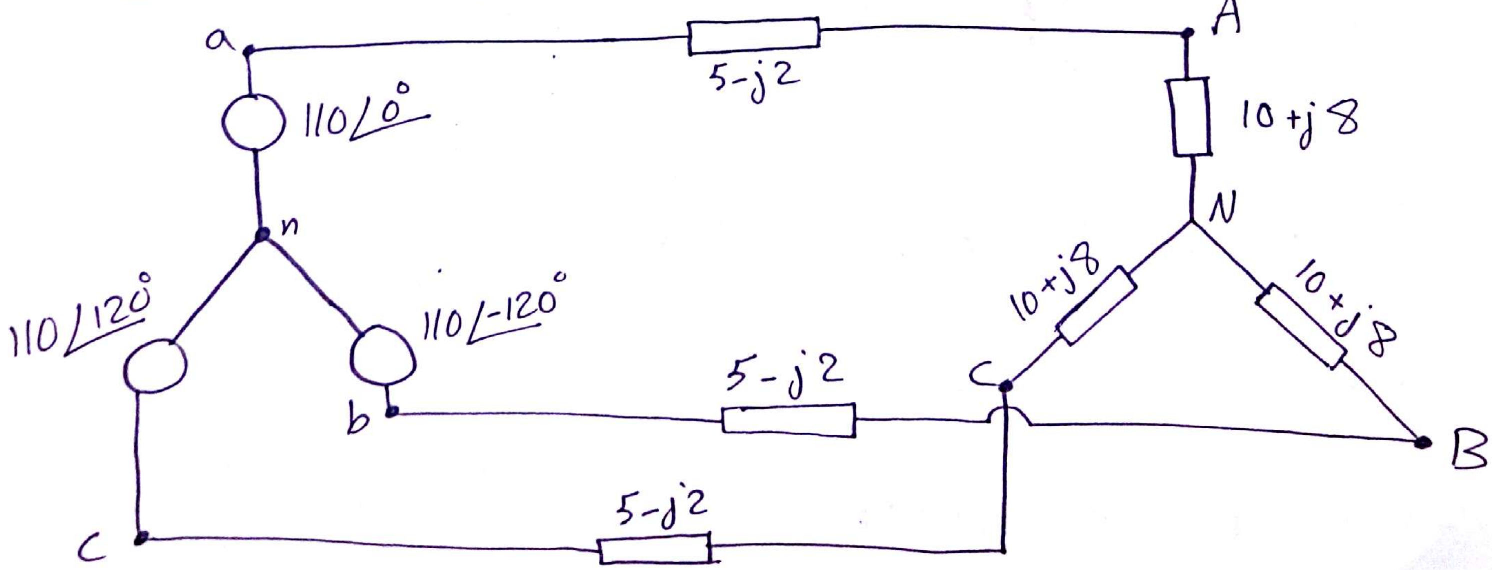
$I_L = \sqrt{3}\, I_p$

$Q_{3\phi} = \sqrt{3}\, V_L I_L \sin(\theta_v - \theta_i)$

$\longrightarrow S = \sqrt{3}\, V_L I_L^*$

---

ex) Find the complex, Real and Reactive power absorbed by the loud for the shown system :-



sol: since there is an impedance in the transmission lines → there is a voltage drop.

→ $V_{an} \neq V_{AN}$

→ $V_{bn} \neq V_{BN}$

→ $V_{cn} \neq V_{CN}$

Reactive power:-

$Q_A = (V_p I_p \sin\theta)_A$

$Q_B = (V_p I_p \sin\theta)_B$

$Q_C = (V_p I_p \sin\theta)_C$

$S_A = (V_p I_p^*)_A = P_A + j\,Q_A$

$S_B = (V_p I_p^*)_B = P_B + j\,Q_B$

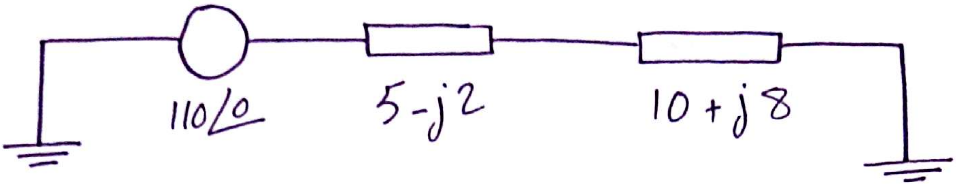
$S_C = (V_p I_p^*)_C = P_C + j\,Q_C$

→ $S_{total\,(3\phi)} = 3 V_p I_p^* = 3\, S_{(1\phi)}$

→ $|S| = |V_p|\,|I_p|$

→ $|S_{3\phi}| = 3\,|V_p|\,|I_p|$ ⟶ حل السؤال في صفحة $\boxed{17}$

→ Draw the single phase equivelant crkt :-



- For phase A :-

$$I_p = I_{line} = \frac{110\angle 0^\circ}{5-j2+10+j8} = 6.8\angle -21.8^\circ \text{ A}$$

(source)

→ $\underset{\substack{\downarrow \\ \text{supplied by} \\ \text{the voltage} \\ \text{source}}}{S_{3\phi}} = 3\,\underset{\substack{\downarrow \\ \text{source} \\ \text{voltage}}}{V_p}\,I_p^* = 3 * 110\angle 0^\circ * 6.8\angle 21.8^\circ$

$= 2247\angle 21.8 \text{ VA}$

$= \underbrace{2084}_{P_{3\phi}} + j\,\underbrace{834.4}_{Q_{3\phi}} \text{ VA}$

(sourse)

→ $P_{3\phi} = 3\,|V_p|\,|I_p| \overset{\cos(\theta)}{=} 3 * 110 * 6.8 * \cos(21.8)$

$= 2084 \text{ W}$

(sourse)

→ $Q_{3\phi} = 3\,|V_p|\,|I_p| \sin\theta = 3 * 110 * 6.8 * \sin(21.8)$

$= 834.4 \text{ VAR}$

same as previous.

(loud)

$$\rightarrow \underset{\substack{\downarrow \\ \text{loud voltage}}}{S_{3\phi} = 3\, V_p}\, I_p^*$$

loud voltage

$$* \; V_p \text{ loud} = V_{P(\text{source})} * \frac{10+j8}{10+j8+5-j2} = 110\angle 0^\circ * \frac{10+j8}{15+j6}$$

$$= 87.3\angle 16.8^\circ$$

$$\rightarrow S_{3\phi} = 3 * 87.3\angle 16.8^\circ * 6.8\angle 21.8^\circ$$

$$S_{3\phi} = \underbrace{1392}_{P_{3\phi}} + j\underbrace{1113}_{Q_{3\phi}} \text{ VA}$$

OR $S_{3\phi} = 3 V_p I_p^*$, $V_{P \text{ loud}} = I_{P \text{ loud}} * Z_{P \text{ loud}}$

$$= 3*(I_P * Z_P) * I_P^*$$

$$= 3 * |I_P|^2 * Z_P = 3 * |6.8|^2 * 10+8j$$

$$= \underbrace{1392}_{P_{3\phi}} + j\underbrace{1113}_{Q_{3\phi}} = 1782\angle 36.6 \quad \#$$

loud

$$\rightarrow P_{3\phi} = 3 V_p I_p \cos\theta = 3 * 110\angle 0^\circ * \left(\frac{10+8j}{15+j6}\right) * 6.8 \cos(\theta$$

$$= 3 * 87.3 * 6.8 * \cos(16.8 + 21.8)$$

$$= 1392 \text{ W}$$

loud

$$\rightarrow Q_{3\phi} = 3 V_p I_p \sin\theta = 3 * 87.3 * 6.8 * \sin(16.8+21.8)$$

$$= 1113 \text{ VAR}$$

→ lost power :-

$* \; P_{loss} = 3|I^2| \underline{\underline{R}} \rightarrow$ (resistance of $\underset{✓}{5} - \underset{X}{j2}$)

$= 3 * (6.8)^2 * 5 = 693.6$

$* \; P_{loss} + P_{loud} = P_{source}$

$693 + 1392 \overset{?}{=} 2087$

$2087 = 2087$ ✓

$* \; Q_{loss} = 3|I^2| X_c$

$= (3) * (6.8)^2 * (-2) = (-) 278$ VAR

↳ because it's a capacitance loud.

(# (Y - Δ) → include in first exam.)