

Spring 017



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BY:

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# Measurements

second  
semester  
(2017)

Dr. Sahban.

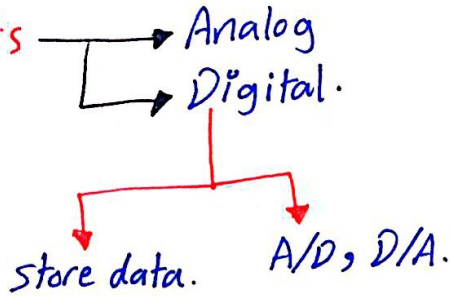
\* Mohammad \*  
Abu Hashia. \*



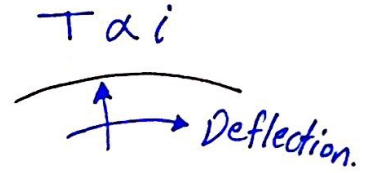
\* Instrument:



\* Instruments



Ammeter.



EN50160 standard  
Distribution code

1 week  
10-min  $\Rightarrow$  95% within voltage limit.  
 $-6\% \leq \Delta V \leq 10\%$

\* Does this instrument provide true value?

true value  $\triangleq$  theoretical value based on models.

\* Characteristics of measurement instruments:

① Accuracy: How close is the reading from the true value.

Ex. Volt meter:

measured value = 10V.

accuracy = 10% (close to the true value)

Range 0-15V

find true value?

$$\text{True value} = 10 \pm (10\% * 15)$$

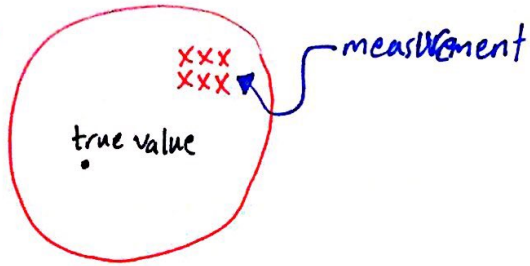
$$= 8.5 \rightarrow 11.5 \text{ volt.}$$

\* Accuracy (expressed inaccuracy)



② Precision:  $\Rightarrow$  Reproducibility.

Ex.



for Ex.  $\Rightarrow$

- \* High Precise.
- \* Low accuracy.

③ Sensitivity: it is the ability to respond to the changes in the measured quantity. (inputs).

$$S = \frac{\Delta \text{output}}{\Delta \text{input}}$$

④ Threshold: minimum input that could be measured.

⑤ Resolution: -.

Precision:

metric.

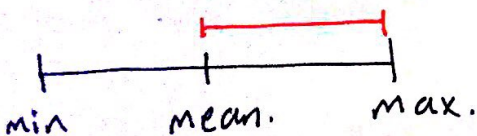
$\sigma \triangleq$  standard deviation.

$$\sigma^2 = E[(x - \bar{x})^2]$$

Expectation.  $\bar{x}$  Mean.

$$\sigma = \sqrt{\frac{\sum_{i=1}^N (x_i - \bar{x})^2}{N}}$$

Reading.  $x_i$  mean "average".  $N$  sample size.



$$|\text{max} - \text{mean}| \rightarrow \text{precision.}$$

we use it to compare between two devices to know which one has better precision.

$\Rightarrow$  the smallest value  $|\text{max} - \text{mean}|$  is higher precision.



Ex. A voltmeter is used to read voltage of 5V. "DC load" (3)

A: "5.03, 4.97, 5, 5.02, 4.99, 4.98, 5.03, 5.02, 5.01, 4.97"

B: "5.2, 5.2, 5.2, 5.2"

⇒ Compare accuracy, precision?

Accuracy:

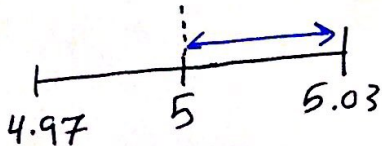
A true = 5V.  
mean = 5V.

B true = 5V.  
average = 5.2V.

A is more accurate than B.

precision:

the precision in B is higher than in A.

A  ⇒ |max - mean| = 0.03.

B  ⇒ |max - mean| = 0.

Note:

reading	10	8	20
freq.	100	20	80

$$\bar{X} = E[X]$$

$$= 10 * \left(\frac{100}{200}\right) + 8 * \left(\frac{20}{200}\right)$$

$$+ 20 * \left(\frac{80}{200}\right)$$

$$= \underline{\underline{13.8}}$$

mean average.

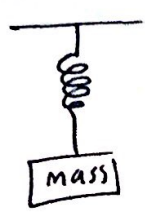
Ex. (Resolution) A digital system (A/D) uses a specific # of bits 12 bits. How much is the resolution? (Full scale = 5V).

12 bits ⇒ # of levels =  $2^{12}$

$$\text{Resolution} \equiv \text{step size} = \frac{5}{2^{12}} = \underline{\underline{1.221 * 10^{-3}}}$$

# Ex. (sensitivity)

measurent system aims to measure the mass

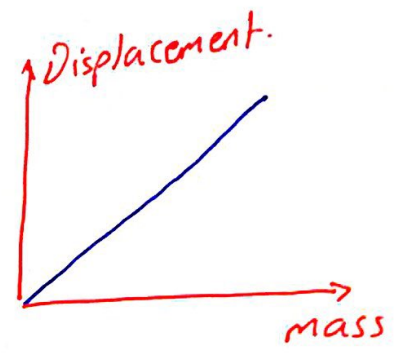


⇒ mass ∝ Displacement.

always like these table it must have operating condition.

input (Kg)	0	50	100
o/p (cm)	0	10	20

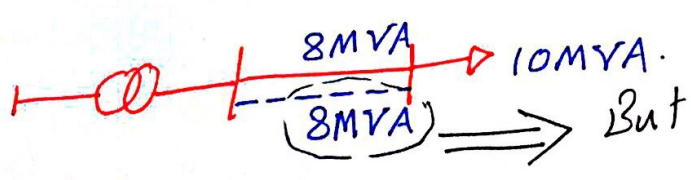
sensitivity ≅ slope =  $\frac{10}{50}$  cm/Kg.



we can't do this.

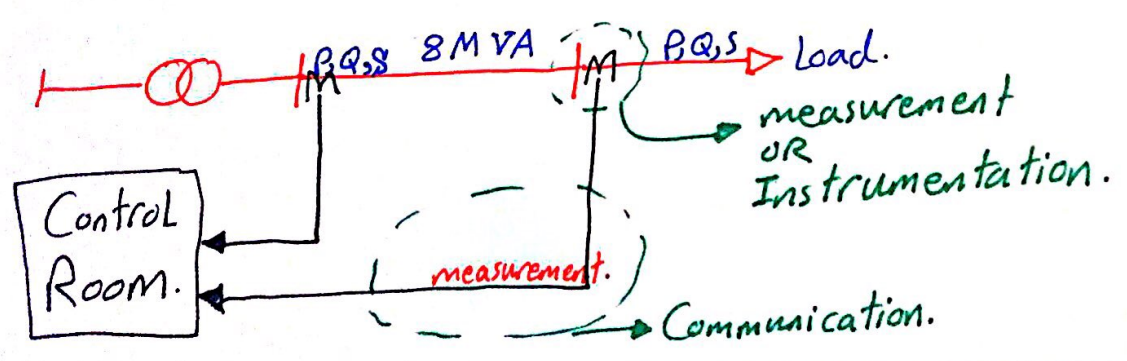
⇒ power-system single line diagram.

⇒ Solution ①: install new line.



But This need time and cost.

⇒ Solution ②: Control load ⇒ "Active Network Management".

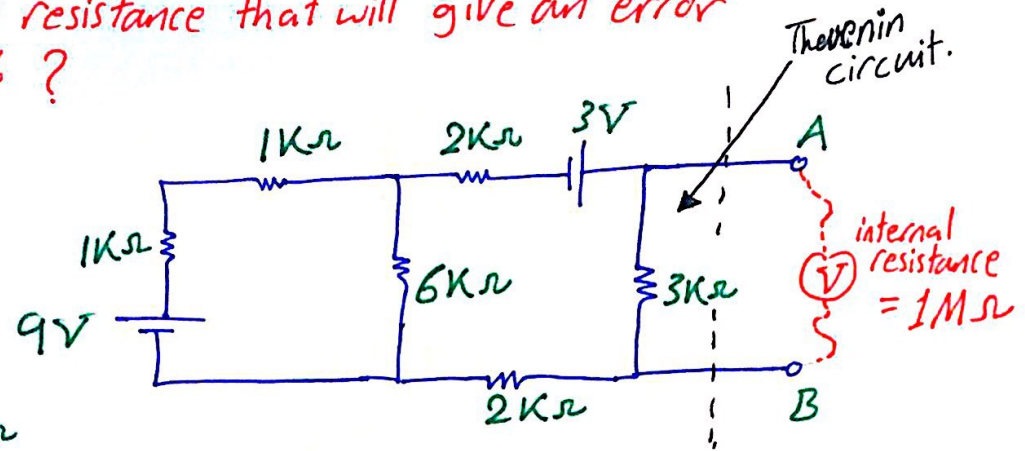




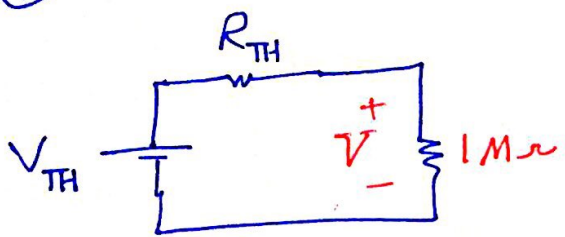
Example:

For the shown circuit:

- ① Find the error in the measurement?
- ② Find the internal resistance that will give an error smaller than 1%?



①  $R_{TH} = 2k\Omega$



$V_{real} = V_{th}$  (when  $R_V \rightarrow \infty$ )

$V_{measured} = \frac{1M}{1M+2K} V_{th}$   
 $= 0.998 V_{th}$

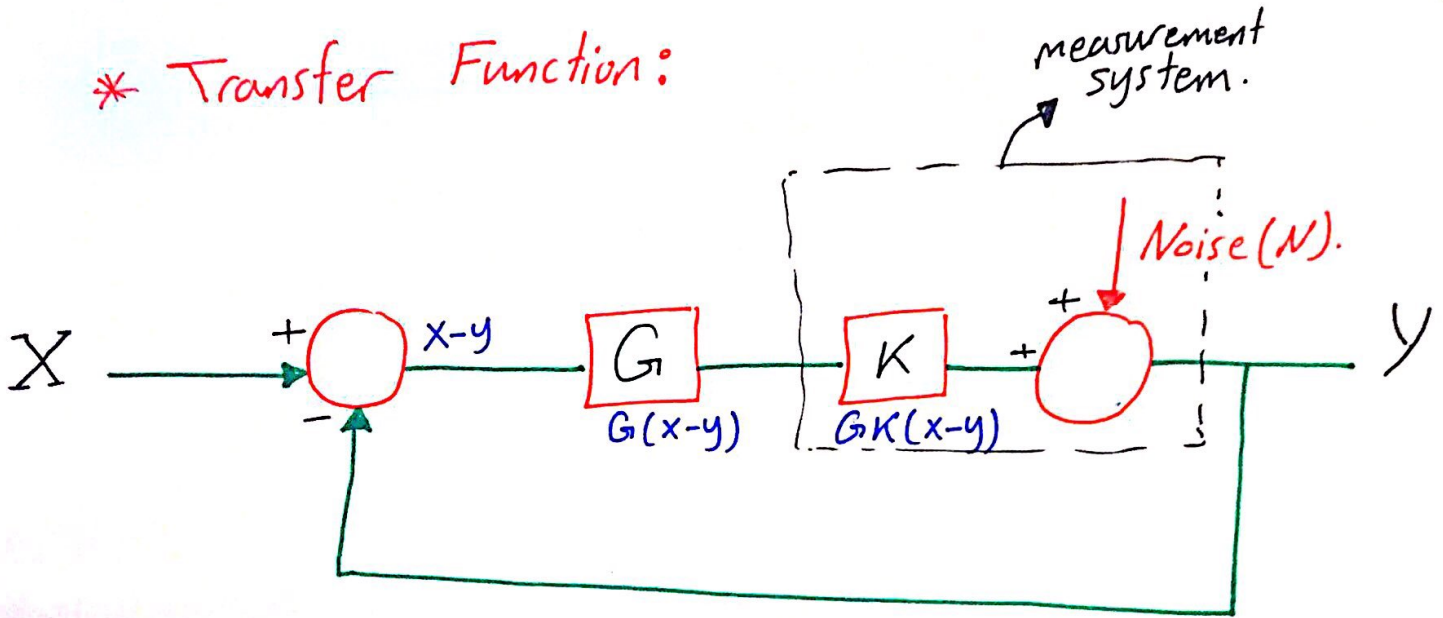
Error =  $\frac{V_{real} - V_{measured}}{V_{real}} * 100\%$   
 $= \frac{V_{th} - 0.998V_{th}}{V_{th}} * 100\% = \boxed{0.2\%}$

② Final Answer:  
 $R_V \geq 198k\Omega$  To achieve error smaller than 1%

$\Rightarrow \left[ 1 - \frac{R_V}{R_V+2K} \right] * 100 \leq 1 \Rightarrow 0.01 R_V \geq 1980$   
 so  $R_V \geq 198k\Omega$



\* Transfer Function:



$$\Rightarrow y = (x-y) GK + N$$

$$\Rightarrow y = \frac{x GK}{1+GK} + \frac{N}{1+GK}$$

if  $G$  is very large:  
 $\Rightarrow$  Eliminate the noise

$$y = x$$

Note:  
 with No feedback  
 $y = xGK + N$   
 "open loop" system

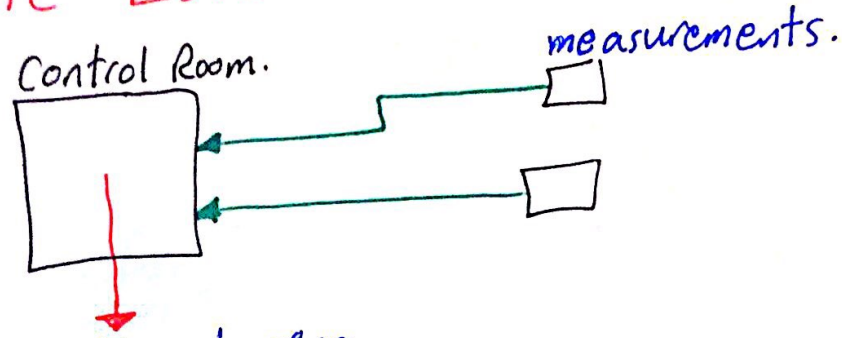
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Note:

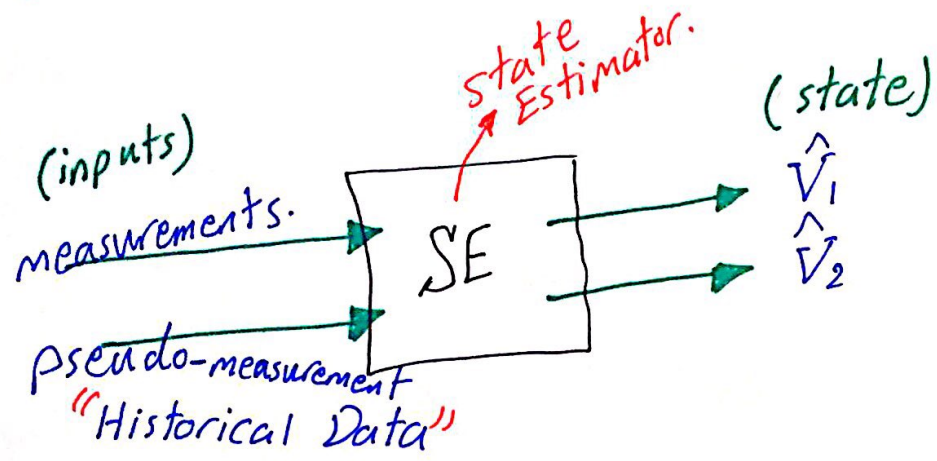
There is a Time Constant for the measurement.

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# \* State Estimation:



- 1) Monitoring.
- 2) Decision-making algorithm.



\*\*

$$\sigma^2 = \frac{1}{n} \sum (x - \bar{x})^2$$

Variance.      sample.      measurement      mean.

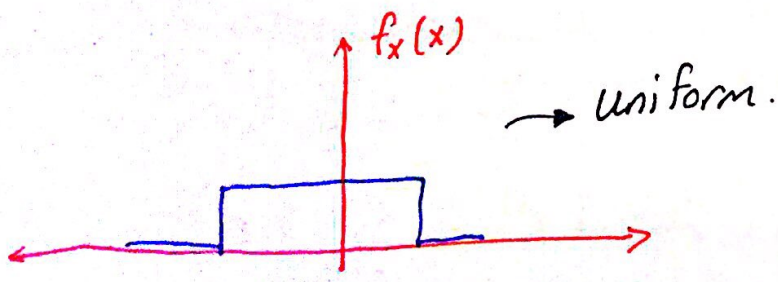
$\sigma \triangleq$  standard deviation.

\* {

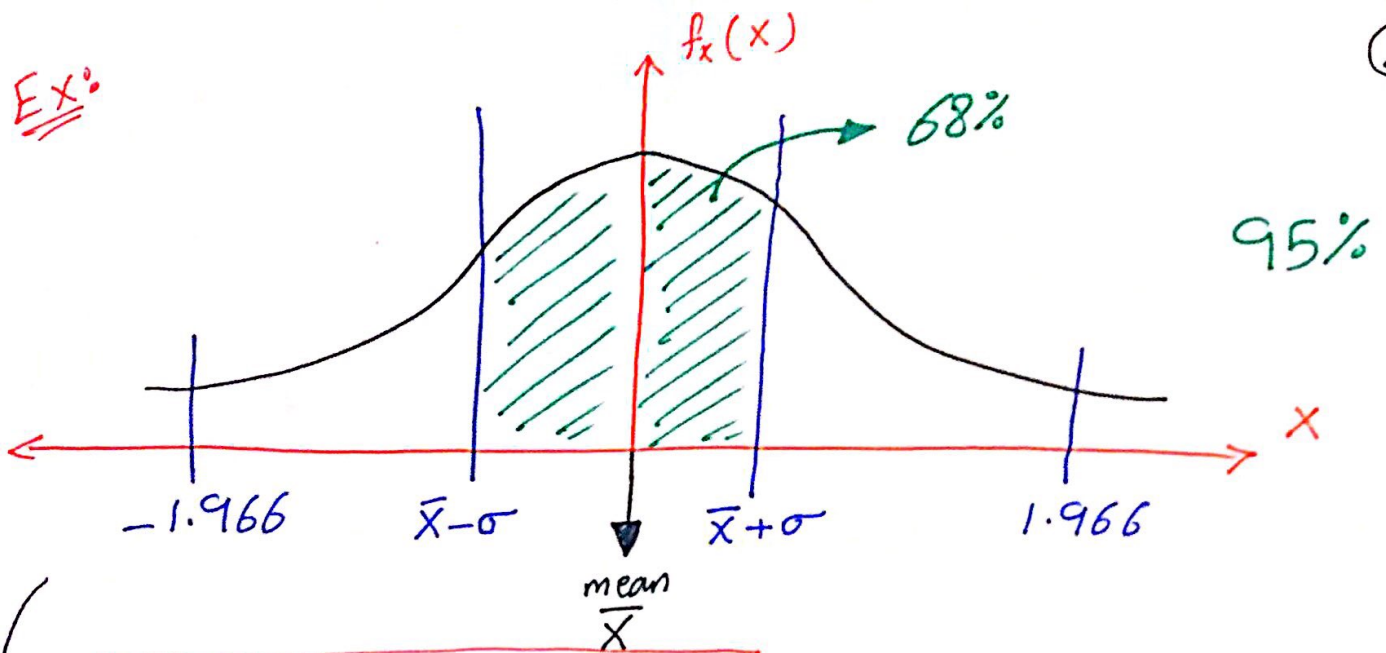
$$\text{mean} = \frac{1}{n} \sum X$$

$$\text{mean} = \sum x_i P_r(x_i)$$

\* }



Ex:



$$f_x(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\bar{x})^2}{2\sigma^2}}$$

$$Pr(\bar{x} - 1.966 \leq x \leq \bar{x} + 1.966) = 95\%$$

$$Pr(\bar{x} - \sigma \leq x \leq \bar{x} + \sigma) = 68\%$$

\* Pdf: "probability density function".

symbol  $\rightarrow f_x(x)$ .

$$\Rightarrow Pr(x_1 \leq x \leq x_2) = \int_{x_1}^{x_2} f_x(x) dx.$$

\* cdf: "cumulative distribution function".

$$\Rightarrow \underline{F_x(x_0) = Pr(x \leq x_0)}$$

$\hookrightarrow$  it is called non decreasing.

$$\Rightarrow \int_{-\infty}^{x_0} f_x(x) dx.$$

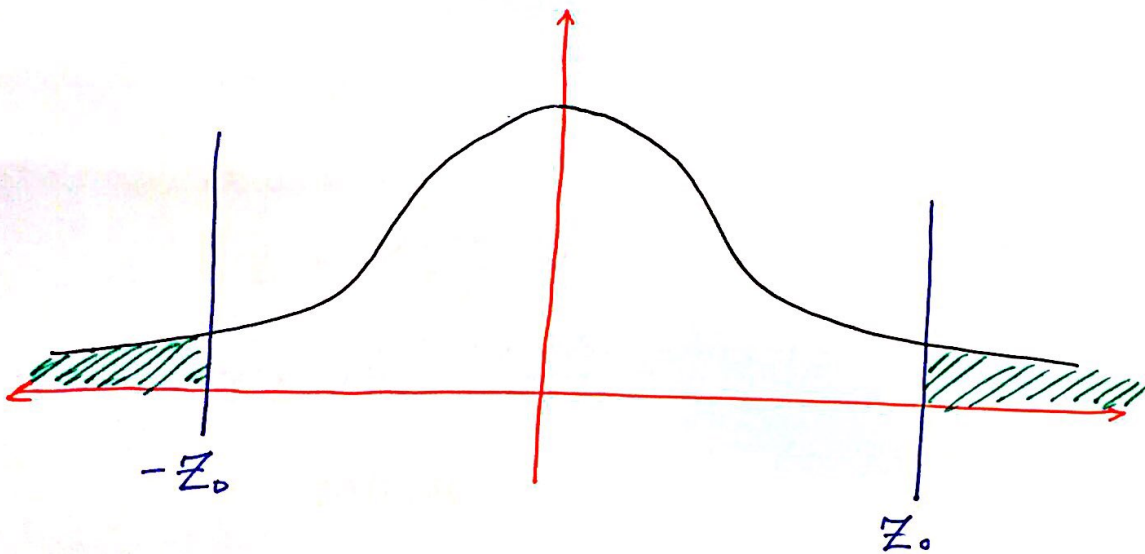


$$F_X(x_0) = Pr(X \leq x_0)$$

$$Z_0 = \frac{x_0 - \text{mean}}{\sigma}$$

from table.

\* How to find  $Z_0$  from table:



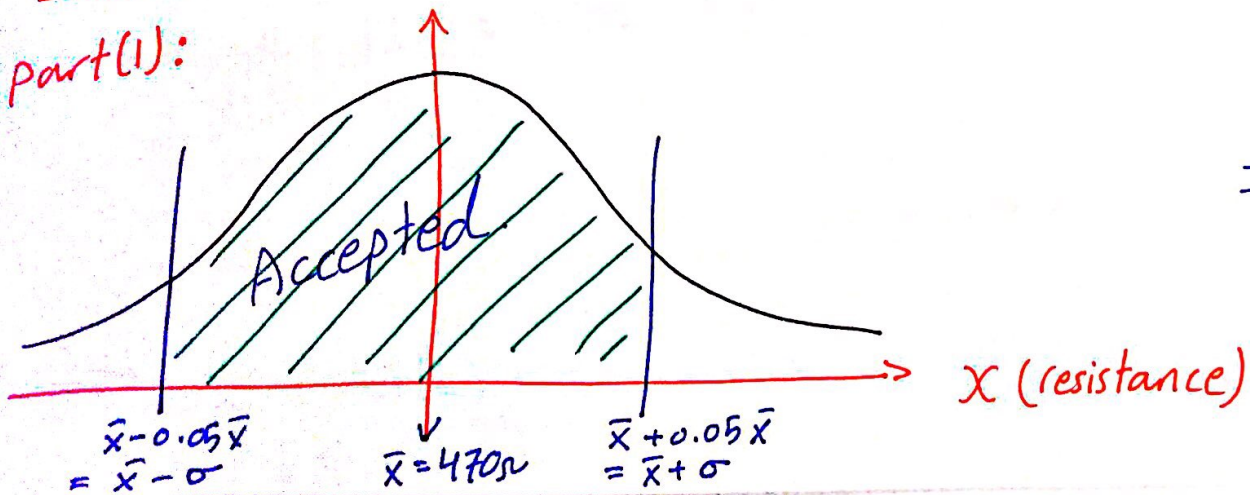
if we want to find  $f(-Z_0)$ :

$$f(-Z_0) = 1 - F(Z_0)$$

Example: slide 21

$$0.05 \bar{x} = 23.5 = \sigma$$

part (1):



⇒ accepted:

$$[\bar{x} - \sigma \leq x \leq \bar{x} + \sigma] = \underline{\underline{68\%}}$$

But assume we don't know that is = 68% !!

$$\begin{aligned} \Rightarrow Pr(\bar{x} - 0.05\bar{x} \leq x \leq \bar{x} + 0.05\bar{x}) \\ = Pr(x \leq \bar{x} + 0.05\bar{x}) - Pr(x \leq \bar{x} - 0.05\bar{x}) \\ = F_x(\bar{x} + 0.05\bar{x}) - F_x(\bar{x} - 0.05\bar{x}) \end{aligned}$$

⇒ Now from table:

$$Z = \frac{x - \text{mean}}{\sigma}$$

$$\Rightarrow F_Z\left(\frac{\bar{x} + 0.05\bar{x} - \bar{x}}{\sigma}\right) - F_Z\left(\frac{\bar{x} - 0.05\bar{x} - \bar{x}}{\sigma}\right)$$

substitute  $\bar{x} = 470, \sigma = 23.5$

$$so \Rightarrow F_Z(1) - F_Z(-1) = 68\% \text{ Acceptance value.}$$

\* For rejection:

$$Pr(\text{rejection}) = 1 - 68\% = \underline{\underline{32\%}}$$

$$\Rightarrow 32\% * 10,000 = \boxed{3174} \text{ resistors will be rejected.}$$

Part(2):

$$\bar{x} = 475 \Omega$$

$$\sigma^2 = \frac{1}{N} \sum (x - \bar{x})^2$$

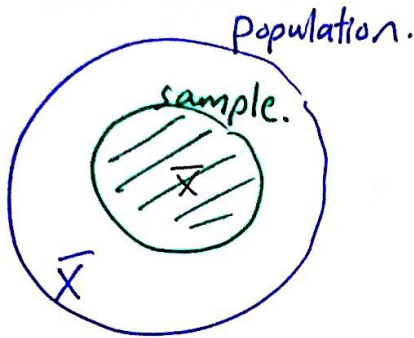


cancel each other.  
so standard deviation doesn't change.



# \* Standard Error of the mean:

(11)



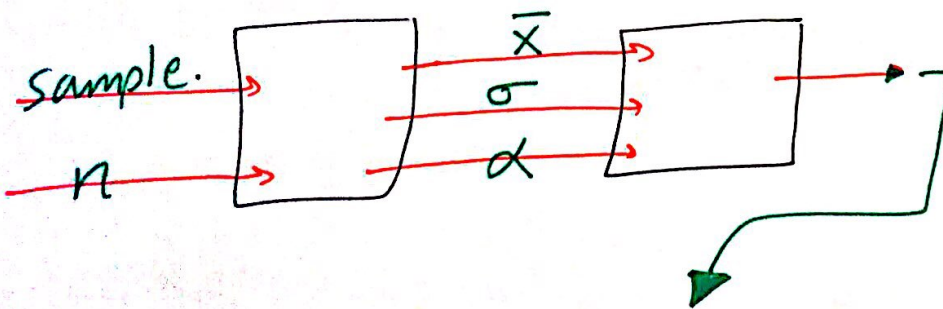
⇒ Confidence Level.

2.2	3.25	68%
2.5	3.5	95%
2.25	3.75	99%

$\alpha \equiv$  standard error of the mean.

$$\alpha = \frac{\sigma}{\sqrt{n}}$$

$X|_{\text{population}} = \alpha \pm X_{\text{mean}}$  under condition confidence level 68%



- $\bar{x} \pm \alpha$  (confidence level 68%)
- $\bar{x} \pm 2\alpha$  (confidence level 95.4%)
- $\bar{x} \pm 1.96\alpha$  (confidence level 95%)



Example: slide (26)

Sample :  $\bar{x} = \frac{\sum x_i}{N} = 35.93 \Omega$

$$\sigma_{n-1} = \sqrt{\frac{1}{N-1} \left[ \sum_{i=1}^N (x_i - \bar{x})^2 \right]}$$

= 0.365

$$\Rightarrow \alpha = \frac{\sigma}{\sqrt{n}} = \frac{0.365}{\sqrt{10}} = 0.115$$

confidence level 95.4%

↳ we have  $\bar{x} \pm 2\alpha$

so mean true value is:

$35.93 \pm 2(0.115)$

\* Aggregation of Errors:

\* Addition:

Assume two measurements with sum = U:

$U = \underbrace{V}_{\substack{\downarrow \\ \text{tolerance} \\ \pm m\%}} + \underbrace{W}_{\substack{\downarrow \\ \text{tolerance} \\ \pm n\%}}$

→  
Ex.

Ex.  $V = 10$  volt.  
 $w = 5$  volt.

$\Rightarrow u = V + w = 15$  volt. (No error).

with error:

Given:  $V = 10 \pm 10\%$  &  $w = 5 \pm 10\%$

for  $U_{max}$ :

$$U_{max} = 10 * 1.1 + 5 * 1.1 = 11 + 5.5 = 16.5 \text{ Volt.}$$

for  $U_{min}$ :

$$U_{min} = 10 * 0.9 + 5 * 0.9 = 13.5 \text{ Volt.}$$

\* Multiplication:

$$u = v \cdot w (1 \pm (m+n))$$

Example: slide (32)

$$F = \frac{(E) Ad}{l} \xrightarrow{\text{constant}}$$

$$F = \Delta A \frac{\partial F}{\partial A} + \Delta d \frac{\partial F}{\partial d} + \Delta l \frac{\partial F}{\partial l}$$

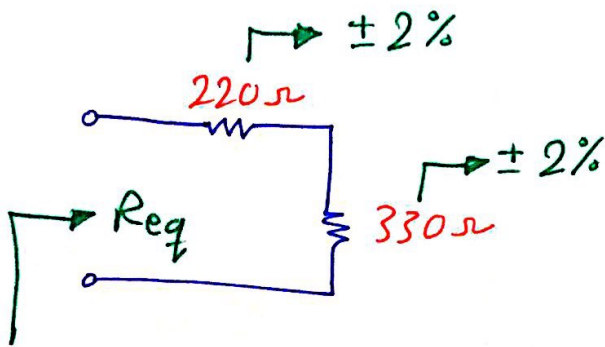
$$\left| \frac{\Delta F}{F} \right|_{min} = \frac{\Delta A}{A} + \frac{\Delta l}{l} + \frac{\Delta d}{d}$$

$$\left| \frac{\Delta F}{F} \right|_{max} = -\frac{\Delta A}{A} - \frac{\Delta l}{l} - \frac{\Delta d}{d}$$



Example: slide (36)

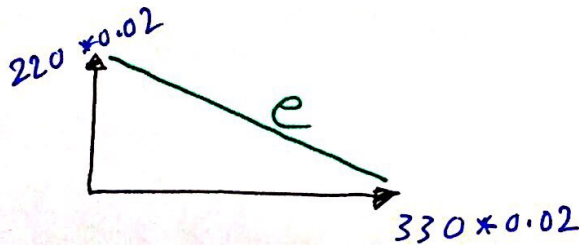
(14)



\* using limiting error:

$$\frac{220 * 0.02}{\quad} \quad \frac{330 * 0.02}{\quad}$$
$$220 (1 + 2\%) + 330 (1 + 2\%)$$
$$= \underline{\underline{561 \Omega}}$$

\* Using Probable error:



probable error:

$$e = \sqrt{(220 * 0.02)^2 + (330 * 0.02)^2}$$
$$= \underline{\underline{7.93 \Omega}}$$

$$\text{for } R_{\text{series}} = 220 + 330 = 550 \Omega$$

as a relative error:

$$\text{Tolerance} = \frac{7.93}{550} * 100\% = \underline{\underline{1.44\%}}$$

$$\text{so } R = 550 \pm (1.44\% * 550) = \underline{\underline{557.92 \Omega}}$$

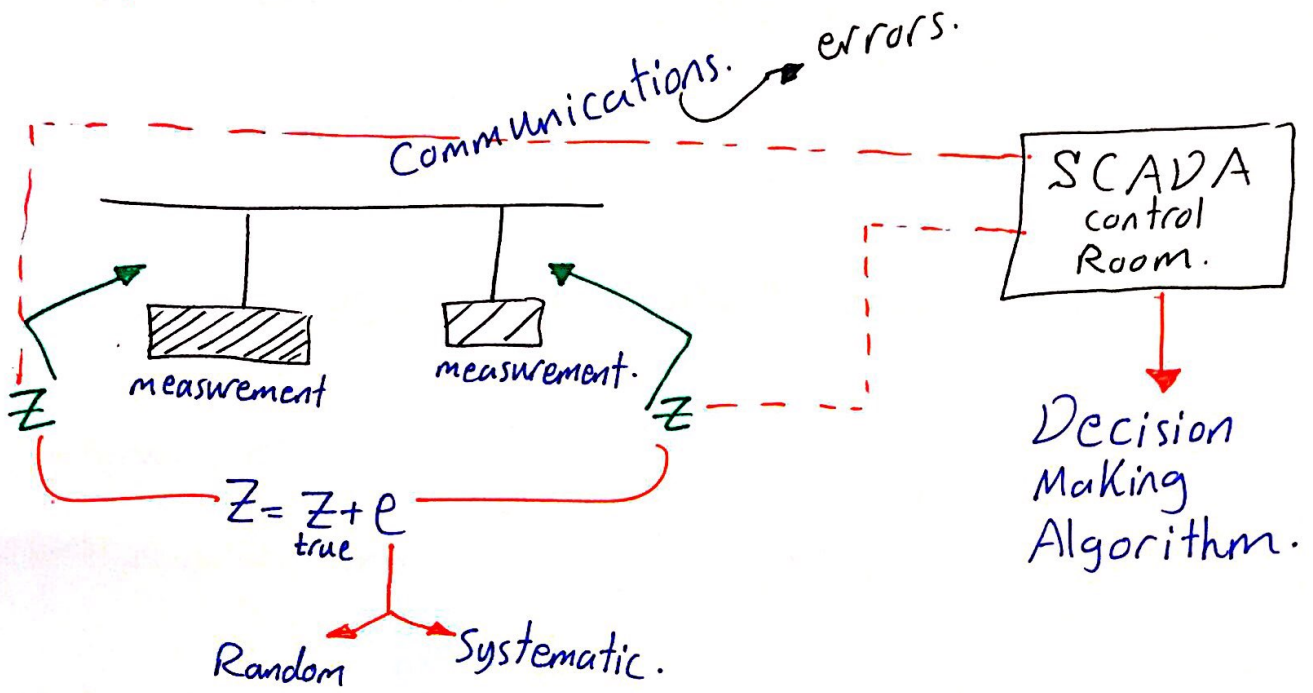
\*\* The value in Probable error best than in limiting error:

since limiting error always take the worst case that both measurements had errors in same time.



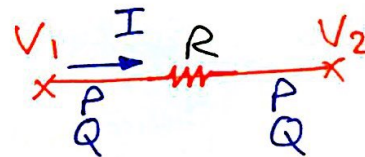
# \*\* State Estimation:

(15)



⇒ Know values of system states (voltages & angles at each node)

↳ from them you find current, P, Q.



[1] \*\* errors in measurements in measurements/comms/SCADA.

[2] \*\* Not enough real measurements.

$$Z_i = h_i(x) + e$$

states [Unknown.]

error [Unknown.]

mathematical model.

measurements [Known.]

\* Random errors (assumptions)

- Gaussian.
- measurements are independent.

\* Weighted Least Square error: (WLSE)

⇒ minimization:

$$\min \sum_{i=1}^m \frac{[z_i - h(x)]^2}{\sigma_i^2}$$

↪ emphasis trusted measurements.

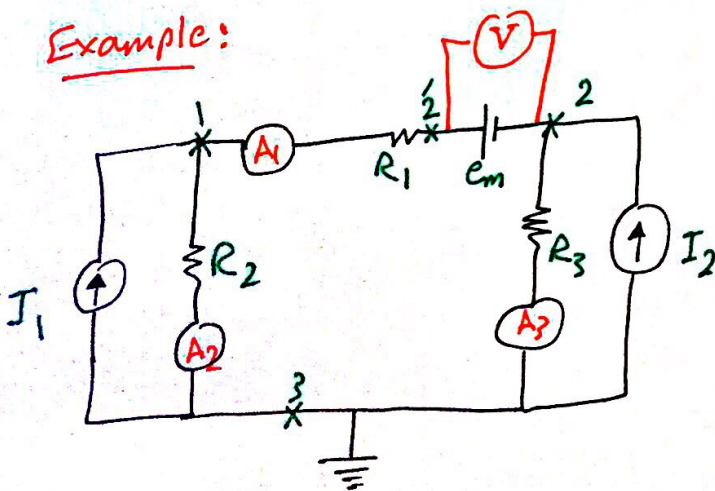
⇒ we solve it by Numerical Analysis

"Newton-Raphson - Method"

\* Pseudo measurements ⇒ Historical Data.

\* Linear Least Squares Estimation:

Example:



$I_1, I_2, e$  are all unknown

Given  $R_1 = R_2 = R_3 = 1 \Omega$

Measurements:

$A_1: i_{12} = 1 A.$

$A_2: i_{31} = -3.2 A.$

$A_3: i_{23} = 0.8 A.$

$V: e = 1.1 \text{ Volt.}$





states  $\begin{bmatrix} v_1 \\ v_2 \\ e_m \end{bmatrix}$  ??

$$Z_i = h(x) \rightarrow \begin{bmatrix} v_1 \\ v_2 \\ e \end{bmatrix}$$

\*  $i_{12} = \frac{v_1 - v_2'}{R_1}$  ;  $v_2' = e_m + v_2$

$$\Rightarrow i_{12} = \frac{v_1 - e_m - v_2}{1} \Rightarrow i_{12} = v_1 - e_m - v_2 = 1 \quad \text{--- [1]}$$

\*  $i_{31} = \frac{v_3 - v_1}{1} = -3.2 = \frac{0 - v_1}{1} \Rightarrow -v_1 = -3.2 \quad \text{--- [2]}$

\*  $i_{23} = \frac{v_2 - v_3}{1} \Rightarrow \frac{v_2 - 0}{1} = 0.8 \Rightarrow v_2 = 0.8 \quad \text{--- [3]}$

\*  $e = 1.1 v \quad \text{--- [4]}$

As a matrix:

$$\begin{bmatrix} 1 & -1 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ e \end{bmatrix} = \begin{bmatrix} 1 \\ -3.2 \\ 0.8 \\ 1.1 \end{bmatrix}$$

$$A \cdot X = Z$$

$\rightarrow$  ??  
since 4 equations & 3 variables.  
 $\Rightarrow$  "Best Solution"

$$\text{Error} = Z - AX$$

$\rightarrow$  minimize WLSE.

$$A \underline{x} = \underline{b} \rightarrow \text{measurements.}$$

$\downarrow$   
 Best estimate.

$\Rightarrow$  minimize error between measurements & values resulting from the model.

$$A_1 = 1A + e$$

$$\Rightarrow e = 1A - (h(x))$$

$\downarrow$   
model.

$$[\text{Error}] = \begin{bmatrix} 1 \\ -3.2 \\ 0.8 \\ 1.1 \end{bmatrix} - \left\{ \begin{bmatrix} 1 & -1 & 1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ e_m \end{bmatrix} \right\} = \begin{bmatrix} 1 - V_1 + V_2 + e_m \\ -3.2 + V_1 \\ 0.8 - V_2 \\ 1.1 - e_m \end{bmatrix}$$

WLSE

$$\Rightarrow \text{Min } (1 - V_1 - V_2 + e_m)^2 + (-3.2 + V_1)^2 + (0.8 - V_2)^2 + (1.1 - e_m)^2$$

$V_1, V_2, e_m$

or use the following relation:

$$\underline{X} = G^{-1} A^T \underline{b}$$

$$\Rightarrow \underline{X} = \begin{bmatrix} 3.125 \\ 0.875 \\ 1.175 \end{bmatrix} \begin{matrix} V_1 \\ V_2 \\ e_m \end{matrix}$$

$\hookrightarrow$  Gain Matrix.  $G = A^T \cdot A$ .

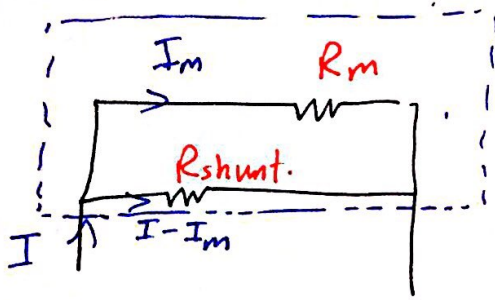
$$\text{* residual: } r = \underline{b} - A\underline{X} = \begin{bmatrix} 1 \\ -3.2 \\ 0.8 \\ 1.1 \end{bmatrix} - \begin{bmatrix} 1 & -1 & 1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3.125 \\ 0.875 \\ 1.175 \end{bmatrix}$$

$$= \underline{\underline{\begin{bmatrix} -0.075 \\ -0.075 \\ -0.075 \end{bmatrix}}}$$



### \* meters:

#### \* Basic DC ammeter:



$$\Rightarrow I_m R_m = R_{shunt} (I - I_m)$$

$$R_{shunt} = \frac{I_m R_m}{I - I_m} = \frac{R_m}{\frac{I}{I_m} - 1}$$

$$\Rightarrow R_{shunt} = \frac{R_m}{n - 1}$$

#### Example: slide ( ) :

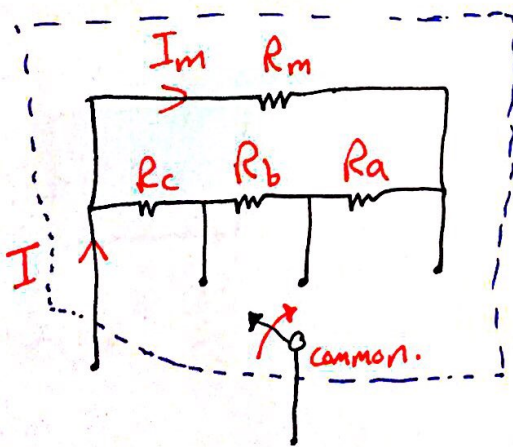
$R_m = 100 \Omega$

$I_m = 1 \text{ mA}$

$I = 0 - 10 \text{ mA}$

$$\Rightarrow R_{shunt} = \frac{100}{\frac{10 \text{ mA}}{1 \text{ mA}} - 1} = \underline{\underline{11.11 \Omega}}$$

#### \* Multiple Range Ammeter:



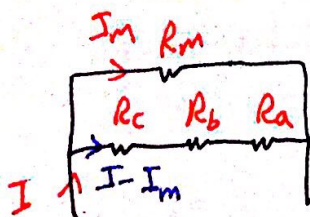
#### Example: slide ( ) :

$I_m = 50 \mu\text{A}$

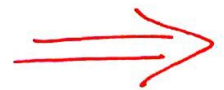
$R_m = 2400 \Omega$

- 3 ranges  $\rightarrow 5 \text{ mA}$
- $\rightarrow 50 \text{ mA}$
- $\rightarrow 500 \text{ mA}$

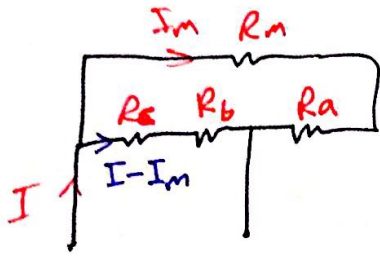
#### \* 5 mA



$$R_a + R_b + R_c = \frac{I_m R_m}{\frac{I}{I_m} - 1} = \frac{50 \mu\text{A} \cdot 2400}{\frac{5 \text{ mA}}{50 \mu\text{A}} - 1}$$

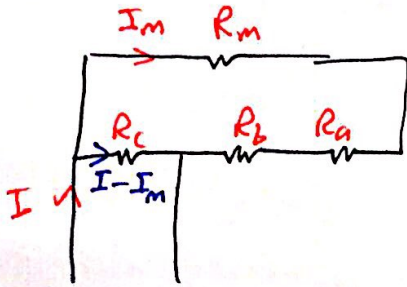


\* 50 mA.



$$\Rightarrow \frac{I_m (R_m + R_a)}{I - I_m} = R_c + R_b \quad \text{--- [2]}$$

\* 500 mA.

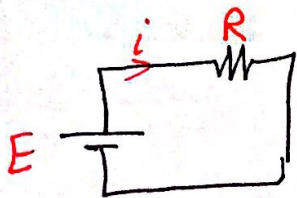


$$\Rightarrow \frac{I_m (R_m + R_a + R_b)}{I - I_m} = R_c \quad \text{--- [3]}$$

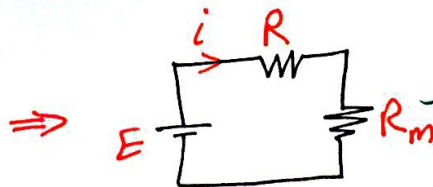
solving 3 equations:

$$\begin{aligned} R_a &= 21.81 \Omega \\ R_b &= 0.94 \Omega \\ R_b &= 2.18 \Omega \end{aligned}$$

\* Ammeter Loading Effect:

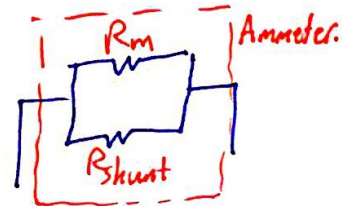


$$i_{\text{expected}} = \frac{E}{R}$$



$$i_{\text{measured}} = \frac{E}{R + R_m}$$

Ammeter Resistance.

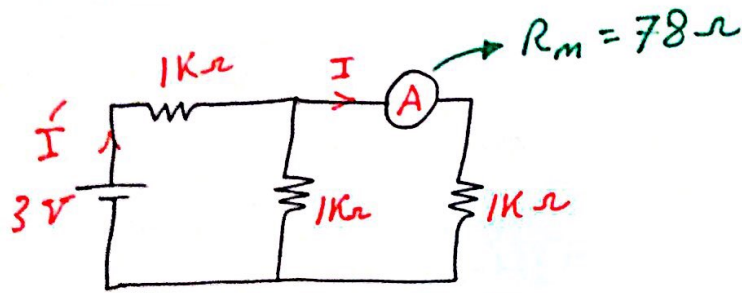


$$\text{Insertion Error} = \frac{I_{\text{expected}} - I_{\text{measured}}}{I_{\text{expected}}} * 100\%$$



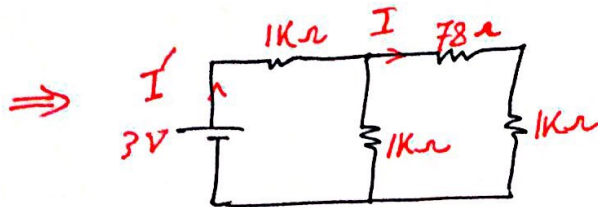
Example: slide ( ):

(21)



$$I'_{\text{expected}} = \frac{3}{1.5k} = 2 \text{ mA}$$

$$I_{\text{expected}} = 1 \text{ mA}$$



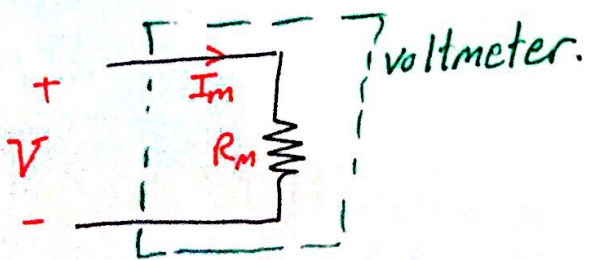
$$R_{\text{eq}} = (1k + 75) \parallel 1k + 1k$$

$$I'_{\text{measured}} = \frac{3}{R_{\text{eq}}} = 1.97 \text{ mA}$$

$$I_{\text{measured}} = I' \frac{1k}{1k + 1k + 78} = 0.95 \text{ mA}$$

$$\text{error \%} = \frac{1 - 0.95}{1} * 100\% = \underline{\underline{5\%}}$$

✳ Basic DC voltmeter:



$$V_{\text{full scale deflection}} = I_m R_m$$

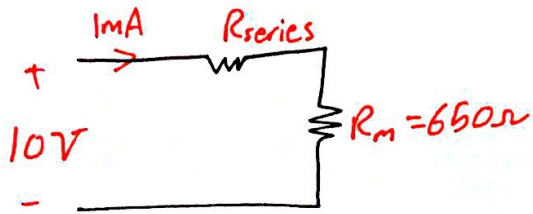
↳ maximum that voltmeter can handle.

\* To extend the  $V_{\text{full scale deflection}}$  we have to add  $R_s$  series with  $R_m$ .

Example.



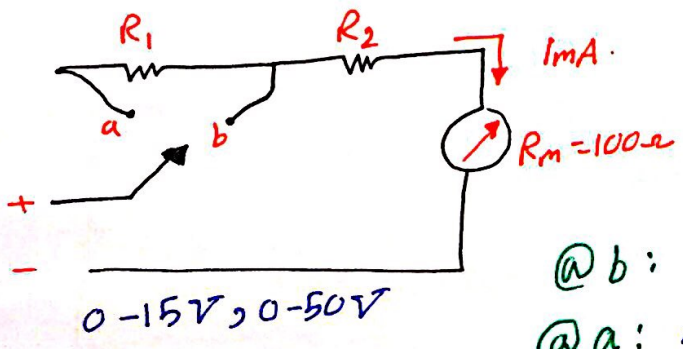
Example: slide ( ) :



$$\Rightarrow -10 + 1m(R_s + R_m) = 0$$

$$R_s = 9.35K\Omega$$

Example: slide ( ) :



Design  $\Rightarrow R_1, R_2 = ?!$

@ b:  $15 = I_m (R_2 + R_m)$  --- [1]

@ a:  $50 = I_m (R_1 + R_2 + R_m)$  --- [2]

solving:  $R_1 = 35K\Omega$   
 $R_2 = 14.9K\Omega$

\* Sensitivity:

$$\text{sensitivity} = \frac{1}{I_{\text{fullscale deflection}}} \left( \frac{\Omega}{V} \right) \Rightarrow S = \frac{1}{I_m}$$

\* for the last example what is the sensitivity?

$$S = \frac{1}{I_m} = \frac{1}{1m} = 1000 \Omega/V$$

Total series resistance	= $15 * 1000 = 15K\Omega$	15V
" " "	= $50 * 1000 = 50K\Omega$	50V

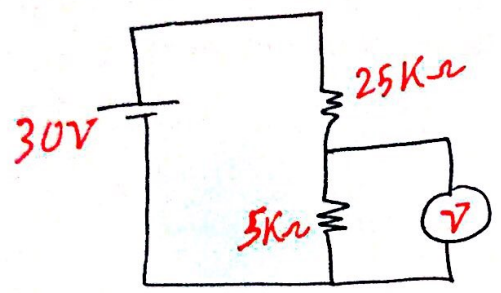
$$15K = R_2 + R_m$$

$$50K = R_1 + R_2 + R_m$$

solving, it will give same results.



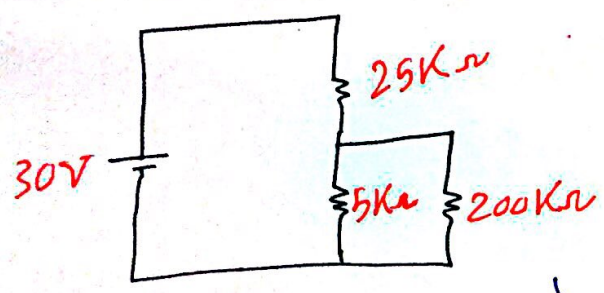
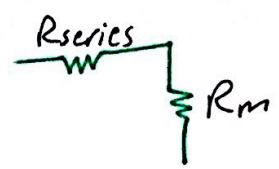
Example: slide (58):



meter A:

$$R_{\text{voltmeter}} = 10 * \frac{1k\Omega}{V} = 10k\Omega$$

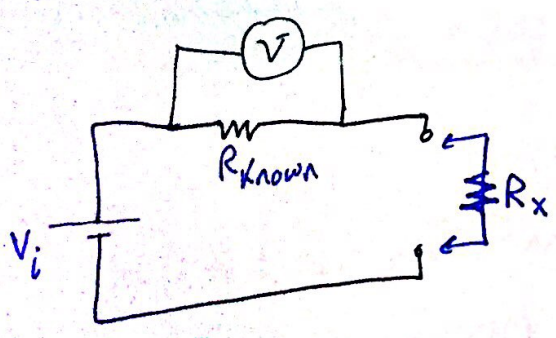
represent  $R_{\text{series}}$  &  $R_m$



meter B:

$$R_v = 10 * \frac{20k\Omega}{V} = 200k\Omega$$

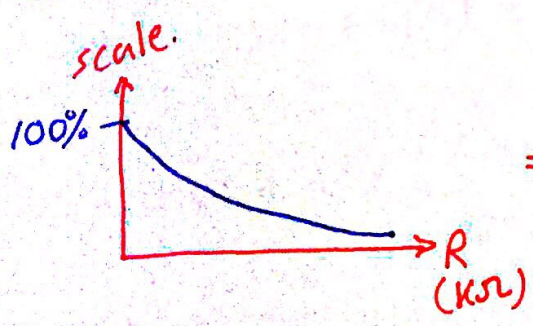
find voltage by voltage division.



when  $R_x = 0$

voltmeter will read maximum voltage. (full scale).

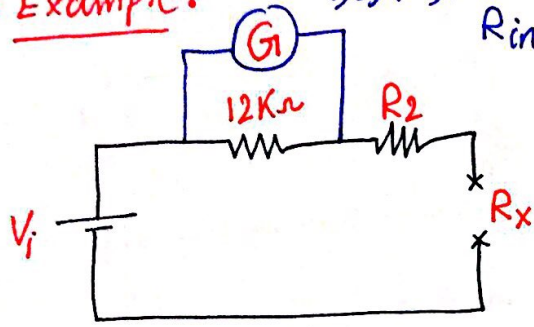
as long as  $R_x$  increasing voltmeter reading decrease.



⇒ This type called:

Reversed - Nonlinear scale.

\* Example: 50μA, Full scale deflection  
R<sub>in</sub> = 2400 Ω



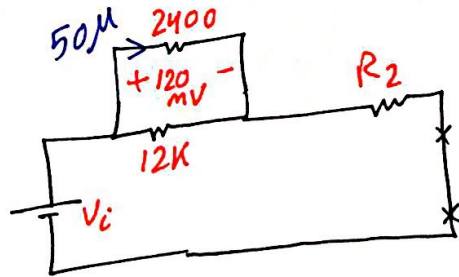
Find V<sub>i</sub>, R<sub>2</sub> ?

- under these conditions:
- Full scale deflection, R<sub>x</sub> = 0
- 20% Full scale deflection, R<sub>x</sub> = 200 KΩ.

⇒  $50\mu A \quad 2400\Omega$

full scale deflection voltage =  $50\mu \times 2400 = 120mV$

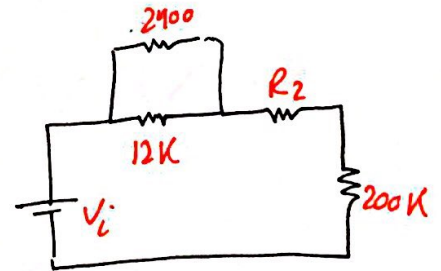
$$120mV = V_i \times \frac{12K // 2400}{(12K // 2400) + R_2}$$



⇒  $240 + 0.12R_2 = 2000V_i$  --- [1]

Now: 20% full scale deflection :

$$20\% (2400 \times 50\mu) = 0.024 \text{ volt.}$$



$$\Rightarrow 0.024 = V_i \frac{(12K // 2400)}{(12K // 2400) + R_2 + 200K}$$

⇒  $4848 + 0.024R_2 = 2000V_i$  --- [2]

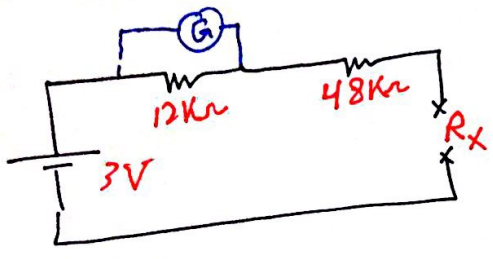
solving [1] & [2] :

$R_2 = 48K\Omega$ ,  $V_i = 3 \text{ volt.}$

continue. ⇒



Now find the deflection when  $\begin{cases} R_x = 450 K\Omega : \\ R_x = 75 K\Omega \sim \\ R_x = 50 K\Omega \sim \end{cases}$



full scale deflection = 120 mV.

$\frac{\% d}{100} * 120m =$   
!?

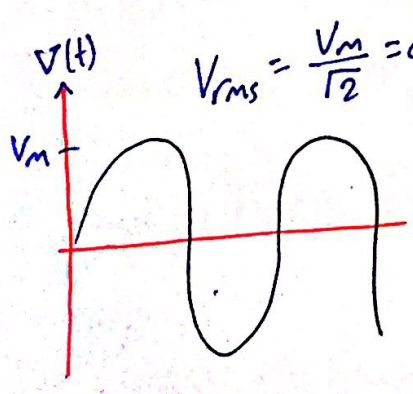
$$3 * \frac{(2400 // 12K)}{(2400 // 12K) + 48K + R_x}$$

75K  
50K

Answers: 450K  $\rightarrow$  10%  
75K  $\rightarrow$  40%  
50K  $\rightarrow$  50%

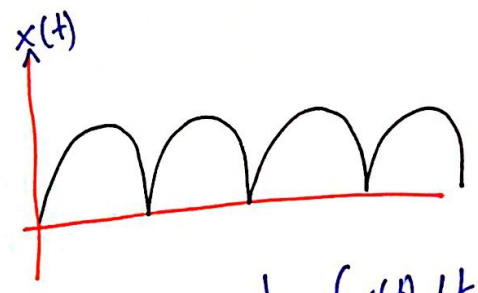
$\Rightarrow$  Test for the slope to know if it is linear or Not.

### \* AC voltmeter:



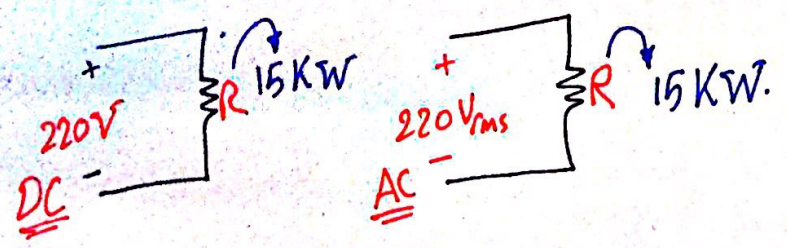
$V_{rms} = \frac{V_m}{\sqrt{2}} = \underline{\underline{0.707 V_m}}$

full wave rectify.



average =  $\frac{1}{T} \int x(t) dt = \underline{\underline{0.636 V_m}}$

Example:



$\Rightarrow$  same rated power in DC & AC.

\* AC signal:  $V_{rms} = 0.707 V_m$ .

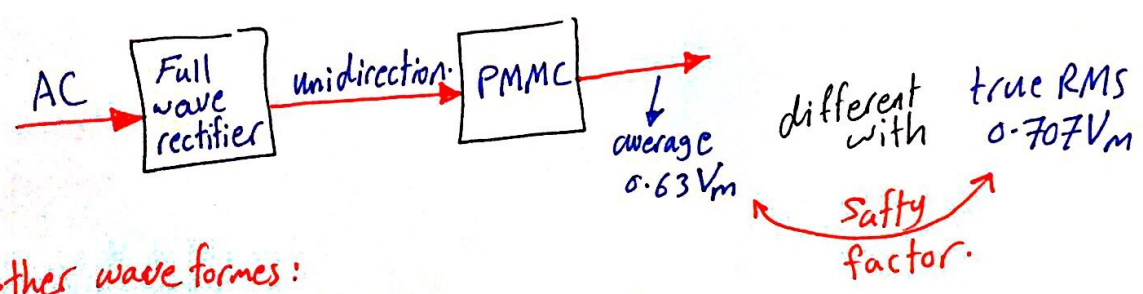
\* Half-wave-Rectifier:  $V_{av} = 0.5 V_m$ .

\* Full-wave-Rectifier:  $V_{av} = 0.636 V_m$ .

\* Average for any signal:  $\frac{1}{T} \int x(t) dt$ .

\* Safe Factor (SF) =  $\frac{V_{rms}}{V_{avg}}$ .

↳ for pure sinusoidal:  $SF = \frac{0.707 V_m}{0.636 V_m}$   
⇒ SF = 1.1



\* for other wave formes:

Correction Factor (CF) =  $\frac{SF_{waveform}}{SF_{sinusoidal}}$

\* if we have: Accuracy = 3% Full scale.  
Full scale = 10 V

⇒ Accuracy =  $10 \times 0.03 = 0.3$  Volt.

⇒ reading = 9V ⇒  $9 \pm 0.3V$   
reading = 1V ⇒  $1 \pm 0.3V$  } better than

the best is to choose the full scale close to the expected readings.

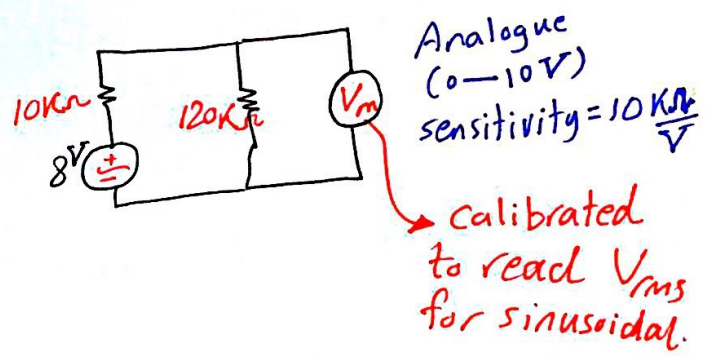


Example: slide (73) : find  $V_{measured}$  8% ?

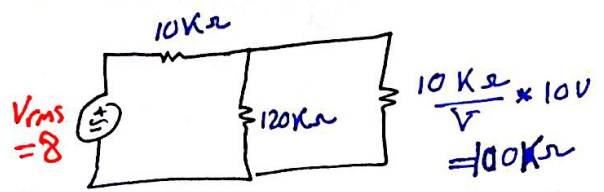
$V_m$  o/c.

$$V_{true} \text{ (ideal) voltage} = 8 * \frac{120K}{120K + 10K}$$

$$= \boxed{7.38 \text{ Volt.}}$$



\*measured value:

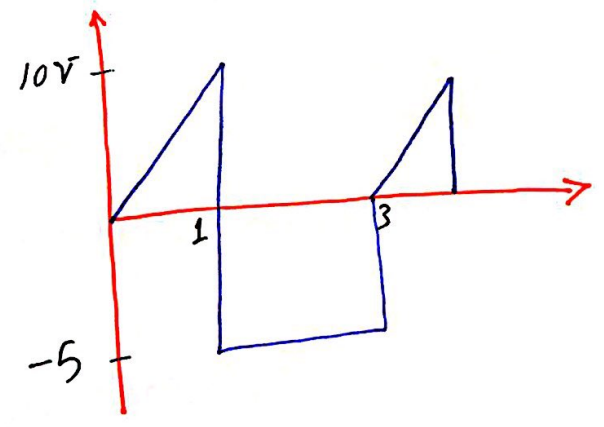
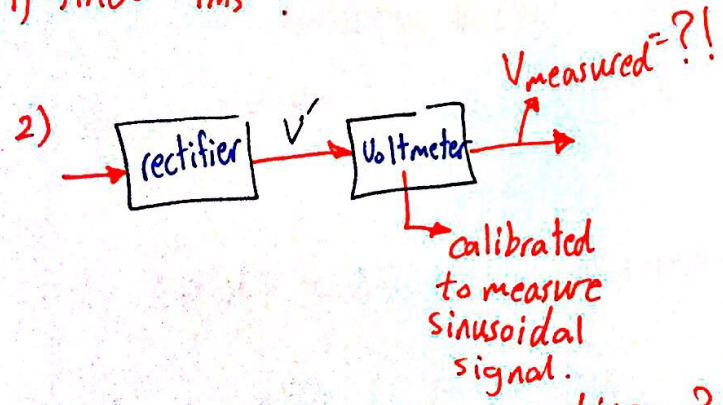


$$\Rightarrow V_{measured} = \frac{8 * (120K // 100K)}{10K + (120K // 100K)} = \boxed{6.76 \text{ volt}}$$

$$\% \text{ error} = \frac{6.76 - 7.38}{7.38} * 100\% = \boxed{-8.4\%}$$

Example: slide (73) :

1) find  $V_{rms}$  ?




3) error in the rms reading ?

solution.  
=>

Solution:

$$1) V_{rms} = \sqrt{\frac{1}{3} \left[ \int_0^1 (10t)^2 dt + \int_1^3 (-5)^2 dt \right]} = 5.27 \text{ volt.}$$

2)   $\Rightarrow V_{average} = \frac{1}{3} \left[ \int_0^1 10t dt + \int_1^3 5 dt \right] = 5 \text{ volt.}$

$$\Rightarrow V_{measured} = 5 * SF = 5 * 1.11 = 5.55 \text{ volt.}$$

we multiply by SF since the voltmeter calibrated to measure sinusoidal signal.

$$3) \%error = \frac{5.55 - 5.27}{5.27} * 100\% = 5.31\%$$

Example :

Need the correction factor given that

$$V_{rms} = 28.87 \text{ V, } V_{average} = 25 \text{ V (rectified signal), } \text{correction factor} = ?!$$

$$28.87 = 25 * 1.11 * \text{Correction factor}$$

$$\Rightarrow CF = 1.04$$

also we could use  $CF = \frac{SF_{waveform}}{SF_{sinusoidal}} \rightarrow \frac{V_{rms}}{V_{avg}}$

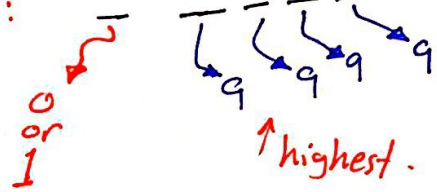


### \* Digital multimeter:

the type could be  $4 \frac{1}{2}$  digit.

depend on quantization

↳ This means:



$$\text{Accuracy} = \pm 0.1\% \pm 1$$

↳ \* LSD. Least significant digit.

### \* For the resolution:

as long as we increase the range  $\Rightarrow$  it will cause the resolution to be worse.

### \*\* $4 \frac{1}{2}$ digit

count 1.9999 (decimal point)

if max voltage  $< 2$ :

$\Rightarrow$  1.9999 (max display)

↳ resolution:

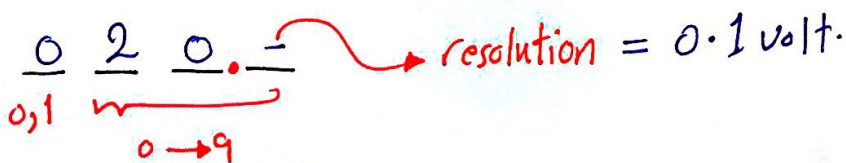
$$\Rightarrow 0.0001 = \underline{\underline{100 \mu V}}$$

if we want to display 2:

$$02.000 \Rightarrow \text{resolution} = 0.001 = \underline{\underline{1 mV}}$$

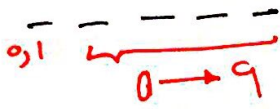
so increasing in range cause a decreasing in resolution.

$3 \frac{1}{2}$  digit reads 20 volt:



Example: slide(81):

4 1/2 digit.



reading: 1.8000

Accuracy:  $(\pm 0.05\% + 1)$

\* Range:  $\begin{cases} \text{max} = 1.8000 * \frac{1.05}{100} = \underline{\underline{1.8009V}} \\ \text{min} = 1.8000 * \frac{0.95}{100} = \underline{\underline{1.7991V}} \end{cases}$

$\pm 0.05\% + 1$

1  $\Rightarrow$  LSD.

LSD = 0.0001

we add it to the max.

we subtract it from the min.

1.8000

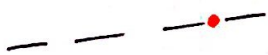
\* expected readings:

$1.8009 + 0.0001 = \underline{\underline{1.8010V}}$

$1.7991 - 0.0001 = \underline{\underline{1.7990V}}$

Example: slide(83):

4 digit:



resolution = 0.1

reading: 1.8V.  
Accuracy:  $\pm 0.1\% + 5$   
Range: 1000V.

$\Rightarrow$  need it:  $(\underline{\underline{< 1000V}})$   
max. display: 999.9

$0.1\% * 1.8$  almost zero.

$\Rightarrow 0.1 * 5 = \underline{\underline{0.5V}}$

range:  $1.8 + 0.5 = 2.3$   
 $1.8 - 0.5 = 1.3$

1.3 -> 2.3 V.

\* End of first Material. \*



# Summary "First"

## \* Accuracy:

accuracy = A%

True = measured ± A% \* range.

\* resolution =  $\frac{\text{full scale}}{2^{\text{\# of bits}}}$

## \* Gaussian Curve:

$$f_x(x) = \frac{e^{-\frac{(x-\bar{x})^2}{2\sigma^2}}}{\sqrt{2\pi\sigma^2}}$$

## \* standard error of mean:

$$\alpha = \frac{\sigma}{\sqrt{n}} \left\{ \begin{array}{l} \bar{x} \pm \alpha \text{ (68\%)} \\ \bar{x} \pm 1.96\alpha \text{ (95\%)} \\ \bar{x} \pm 2\alpha \text{ (95.4\%)} \\ \bar{x} \pm 3\alpha \text{ (99.7\%)} \end{array} \right.$$

## \* Limiting error:

Addition:  $u = v + w$

$\Rightarrow u = v * (1 \pm n) + w * (1 \pm m)$

## Subtraction:

$u = v * (1 \pm n) - w * (1 \pm m)$

## multiplication & division:

$u = v \cdot w \cdot (1 \pm (m+n))$

## \* precision:

1  $\sigma^2 = \frac{\sum (x - \bar{x})^2}{N \text{ or } (N-1)}$

2  $| \text{max} - \text{mean} |$

\* Error =  $\frac{\text{real} - \text{measured}}{\text{real}} * 100\%$

\* pdf:  $P_r(x_1 \leq x \leq x_2) = \int_{x_1}^{x_2} f_x(x) dx$

\* cdf:  $P_r(x \leq x_0) = \int_{-\infty}^{x_0} f_x(x) dx$

$\rightarrow Z_0 = \frac{x_0 - \text{mean}}{\sigma}$

## \* Probable error:

### Addition:

$u = v + w \pm \sqrt{(nv)^2 + (mw)^2}$

### Subtraction:

$u = v - w \pm \sqrt{(nv)^2 + (mw)^2}$

### Division & multiplication:

$u = v \cdot w * (1 \pm \sqrt{m^2 + n^2})$

WLSE:

⇒ How to find X!?

$$z_i = h_i(x) + e$$

$$\min \sum_{i=1}^m \frac{[z_i - h_i(x)]^2}{\sigma_i^2}$$

$$X = G^{-1} A^T b$$

$$G^{-1} = A^T \cdot A$$

\* Residual:  $r = b - A \cdot X$

\* DC ammeter:

$$R_{shunt} = \frac{R_m}{\frac{I}{I_m} - 1}$$

\* Ammeter loading effect:

$$\text{insertion error} = \frac{I_{\text{expected}} - I_{\text{measured}}}{I_{\text{expected}}} * 100\%$$

\* sensitivity:

$$S = \frac{1}{I_m}$$

Total series Resistance  $\equiv$  sensitivity \* full voltage Range.



\* full-rectified signal:



$$V_{avg} = 0.636 V_m$$

for sinusoidal:

$$V_{rms} = 0.707 V_m$$

\* Half-rec:

$$V_{avg} = 0.3 V_m$$

\* Safe factor:  $\rightarrow$  Form factor

$$SF = V_{rms} / V_{avg}$$

$\rightarrow$  for sinusoidal  $SF = 1.11$

\* Error:

$$e = \frac{V_{avg} - V_{rms}}{V_{rms}} * 100\%$$

$$V_{ind} = SF * V_{meas. av.}$$

$$V_{ind} * CF = V_{rms}$$

\* Correction Factor:

$$CF = \frac{SF_{\text{waveform}}}{SF_{\text{sinusoidal}}}$$

$$\text{error} = \frac{1 - CF}{CF}$$

\* Triangular: full-rec:

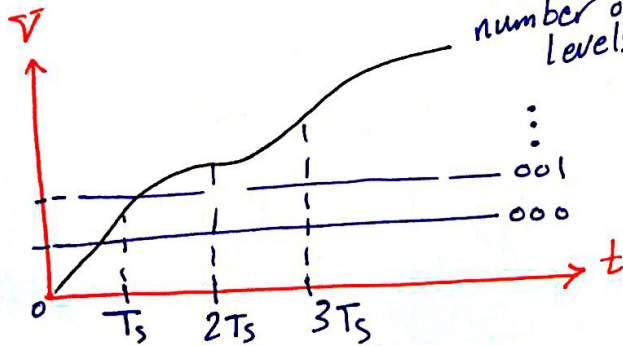
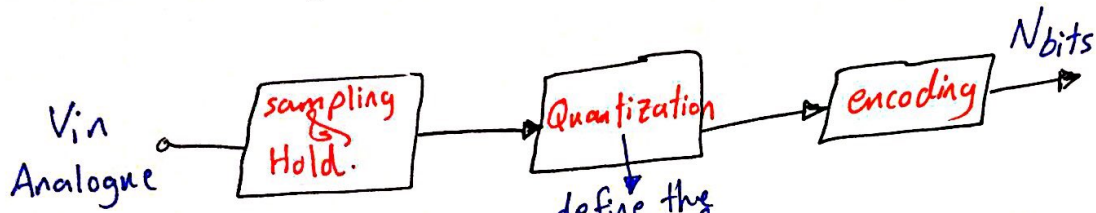
$$V_{avg} = 0.5 V_m$$

$$V_{rms} = 0.577 V_m, SF = 1.155$$



# \* Analogue & Digital:

Basic for digital  $\Rightarrow$  (ADC) analogue to digital convertor.



$T_s \triangleq$  sampling Time.  
 $f_s \triangleq$  sampling frequency.

$$f_s = \frac{1}{T_s} \text{ Hz.}$$

$$\# \text{ of Levels} = 2^N$$

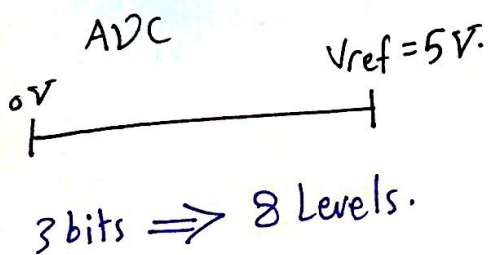
$$\text{step size} = \frac{V_{\text{max}} - V_{\text{min}}}{\# \text{ of levels.}}$$

"Resolution"

Quantization Error =  $\frac{\Delta}{2}$

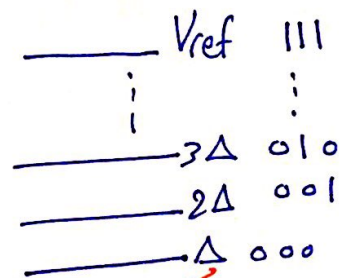
$$V_{in} = \frac{V_{ref}}{2^N} * R \rightarrow \text{digital code.}$$

Example:



# of bits = 3.

$V_{in} = ?$



step size.

$$\Delta = \text{step size} = \frac{V_{ref}}{2^N}$$

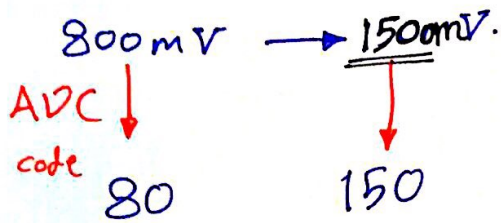
resolution.

Max Quantization error =  $\frac{\Delta}{2}$

$$\text{Quantization error \%} = \frac{\Delta/2}{V_{\text{fullscale}}} * 100\% = \frac{\left(\frac{V_{\text{fullscale}}}{2^N}\right)/2}{V_{\text{fullscale}}} * 100\%$$

$$\text{Quantization error \%} = \frac{1}{2^{N+1}} * 100\%$$

Example: slide (89):

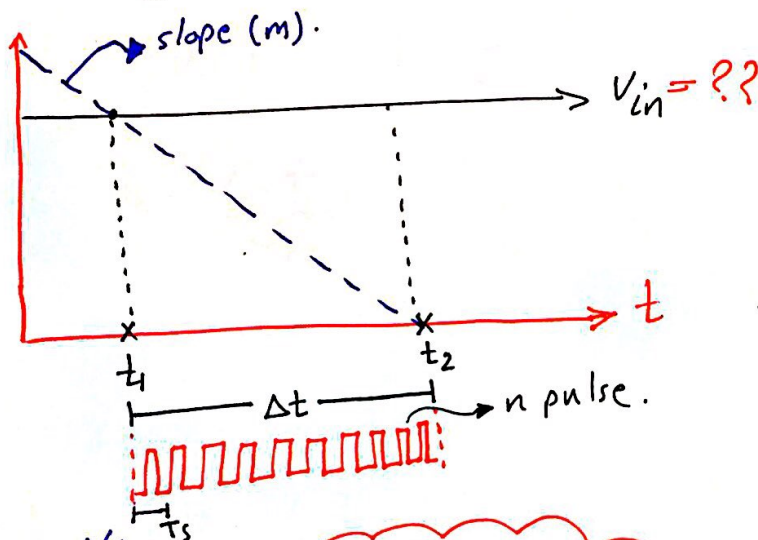


$800mV = 80 \Delta$   
 $\Delta = 10mV$

$\Delta = ??$

**\* Ramp-Type Digital Voltmeter:**

"Voltage to Time Conversion".



$V(t) = V_0 - mt$

@  $t_1$ :

$V_{in} = V_0 - mt_1 \dots \text{[1]}$

@  $t_2$ :

$V_{in} = V_0 - mt_2 = 0 \dots \text{[2]}$

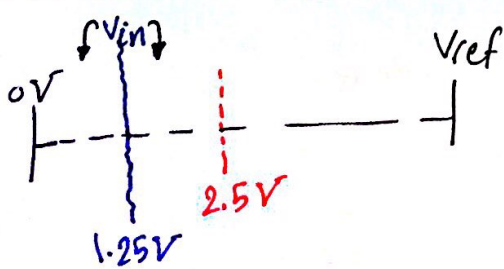
from [1] & [2]:

$V_{in} = m(t_2 - t_1) \rightarrow \Delta t.$

\*  $\Delta t = \frac{V_{in}}{m}$  →  $V_{in} = mnT_s$

\*  $\Delta t = n T_s$

**\* Successive Approximation Digital Voltmeter:**



let  $V_{ref} = 5V$  & # of bits = 3

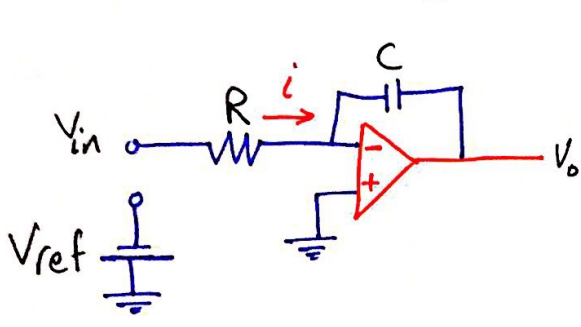
suppose  $V_{in}$  is between 0 & 2.5V.

- \* Compare with 2.5V ⇒ 0
- \* Compare with 1.25V ⇒ 1
- \* Compare with — ⇒ 0

MSB  
010



# # Dual-Slope Digital Voltmeter:



$$i_c = C \frac{dV}{dt}$$

$$\Rightarrow V_c = \frac{1}{C} \int i_c dt = \frac{1}{C} \int \frac{V_{in}}{R} dt$$

$$V_c = \frac{1}{RC} \int V_{in} dt$$

$$V_c + V_o = 0$$

$$\Rightarrow V_o = -\frac{1}{RC} \int V_{in} dt$$

T → defined.

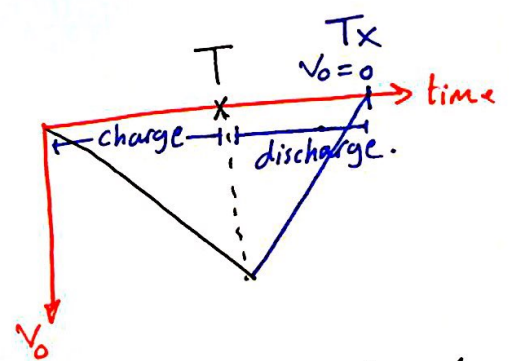
when we connect it to Vref:

$$V_o = -\frac{1}{RC} \int -V_{ref} dt + V_{initial} = 0$$

$$0 = \frac{V_{ref}}{RC} T_x - \frac{V_{in}}{RC} T$$

$$\Rightarrow T_x = \frac{V_{in} T}{V_{ref}}$$

$$\Rightarrow V_{in} = \frac{T_x V_{ref}}{T}$$



→ advantage: doesn't depend on R & C.  
 → disadvantage: slow.

Example: slide (102):

$$V_{ref} = 10V$$

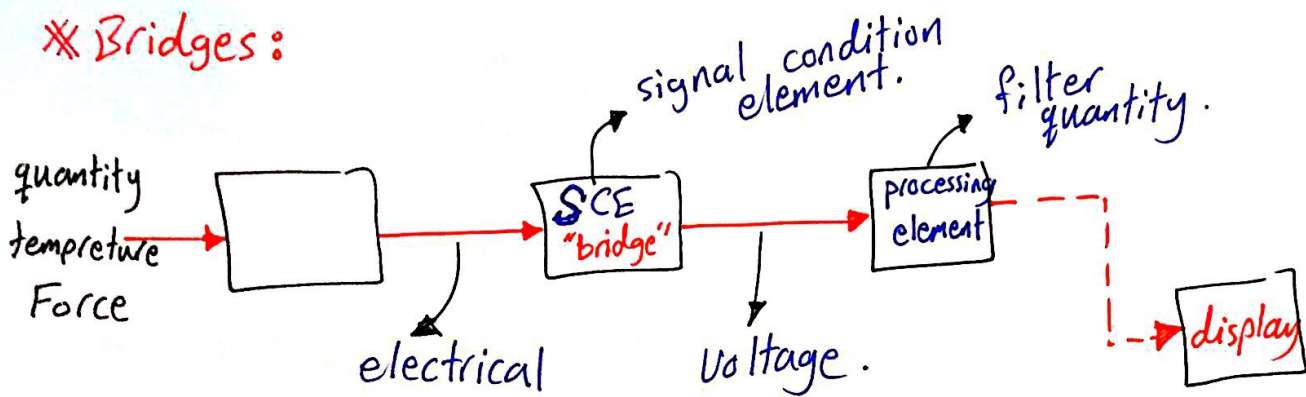
$$T = 10ms$$

$$\Rightarrow T_x = \frac{V_{in} T}{V_{ref}} = \frac{6.8 * 10m}{10} = \underline{\underline{6.8msec.}}$$

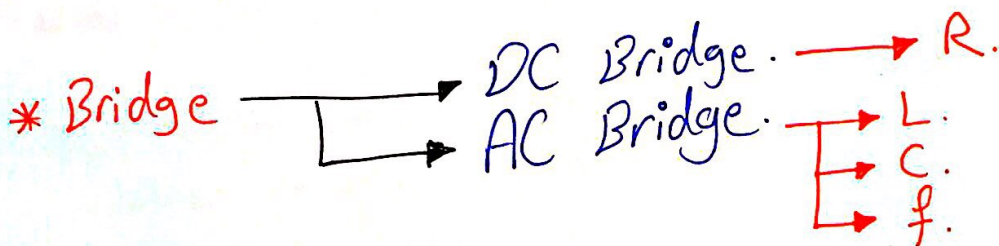
if he asked about Tconversion:

$$T_{conversion} = T + T_x = 10m + 6.8m = \underline{\underline{16.8msec.}}$$

**\* Bridges :**

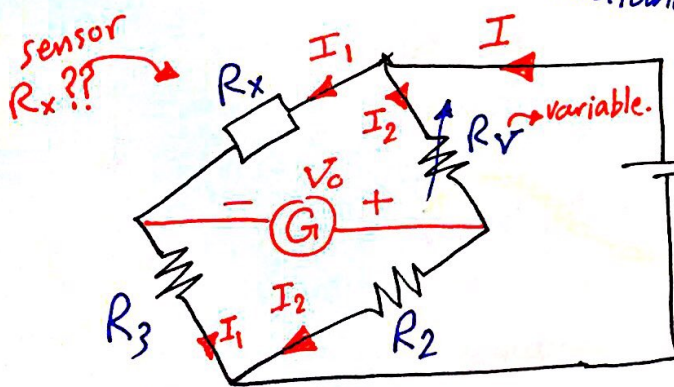


- Resistance.
- Inductor.
- Capacitor.
- frequency.
- phase.



**\* Wheatstone Bridge :**

- Balanced. (Null)
- Unbalanced. (deflection)



Excitation Voltage.

**\* ideal voltmeter:**  
\* Rin → ∞

By using voltage division:

$$V_0 = V_{in} \left( \frac{R_x}{R_x + R_3} \right) - V_{in} \left( \frac{R_v}{R_v + R_2} \right)$$

**\* Null method:**  $R_v \Rightarrow V_0 = 0$  so  $V_{in} \left( \frac{R_x}{R_x + R_3} - \frac{R_v}{R_v + R_2} \right) = 0$

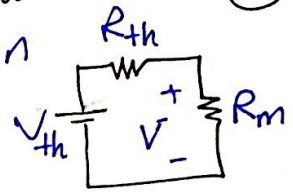
$$\Rightarrow \frac{R_x}{R_x + R_3} = \frac{R_v}{R_v + R_2} \Rightarrow \cancel{R_x R_v} + R_x R_2 = \cancel{R_v R_x} + R_v R_3$$

**$R_x = \frac{R_v R_3}{R_2}$**  "balanced"



\*if the internal resistance of the voltmeter considered  
Obtaining the thevenin circuit would be a solution

(35)



\* Deflection method:

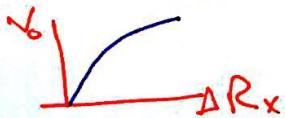
$R_V$  replaced  $\rightarrow R_1$  (Nominal  $R_x$ )

$$V_o = V_{in} \left( \frac{R_x}{R_x + R_3} - \frac{R_1}{R_1 + R_2} \right)$$

$\Rightarrow$  Case:  $R_2 = R_3$   
 $R_x = R_1$  (Nominal)  $\Rightarrow V_o = 0$   
 $R_x = R_1 \pm \Delta R_x \rightarrow V_o +ve$   
 $\rightarrow V_o -ve$

$\rightarrow$  one of the problem of this method:  
that is the relation isn't a linear between  $V_o$  &  $R_x$ .

$$V_o = f(R_x)$$



Example: pressure sensor measures pressure (0-10 bar)

$$R = 120 + \frac{338 \text{ m}\Omega}{\text{bar}}, R_{\text{voltmeter}} \rightarrow \infty$$

max current sensor  $\leq 30 \text{ mA}$   
"R<sub>x</sub>"

Find  $V_o$  when pressure is 10 bar & max  $V_{in}$  is used?

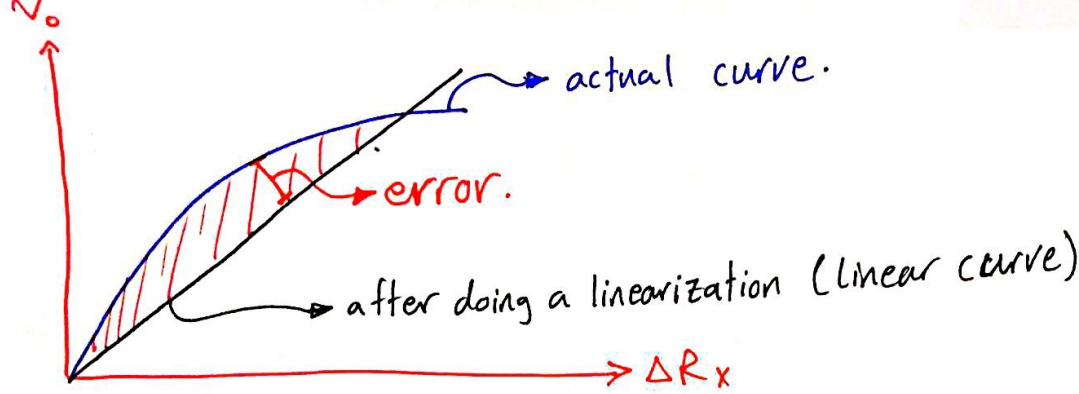
given  $R_1 = R_2 = R_3 = 120 \Omega$

$$I_1 \leq 30 \text{ mA}$$

$$\frac{V_{in}}{R_x + R_3} \leq 30 \text{ mA} \Rightarrow \frac{V_{in}}{120 + 120} \leq 30 \text{ mA} \Rightarrow V_{in} \leq 7.2 \text{ volt.}$$

@ 10 bar:  $R_x = 120 + 0.338 * 10 = 123.38 \Omega$

$$V_o = V_{in} \left( \frac{R_x}{R_x + R_3} - \frac{R_1}{R_1 + R_2} \right) \Rightarrow V_o = 50 \text{ mV.}$$



\* An illustration case:  $\Delta R_x$  is small compared to nominal.

$$V_o = V_{in} \left( \frac{R_x}{R_x + R_3} - \frac{R_1}{R_1 + R_2} \right) \Rightarrow V_o = V_{in} \left( \frac{R_x + \delta R_x}{R_x + \delta R_x + R_3} - \frac{R_1}{R_1 + R_2} \right)$$

$\delta V_o = f(\delta R_x)$       [1]      [2]

subtracting [1] from [2] & approximate:

$\hookrightarrow R_x + \delta R_x \xrightarrow{\text{small}} \approx R_x$   
just for the Dominator.

$$\delta V_o = V_o - V_o$$

$$\Rightarrow \delta V_o = V_{in} \frac{\delta R_x}{R_x + R_3}$$

\* Sensitivity Bridge:  $\frac{\delta V_o}{\delta R_x} = \frac{V_{in}}{R_x + R_3}$

Example:

A resistance thermometer ( $0^\circ\text{C} \rightarrow 50^\circ\text{C}$ ),  $R_{\text{nominal}} = 500 \Omega @ 0^\circ\text{C}$   
&  $\Delta R = \frac{4 \Omega}{1^\circ\text{C}}$ ,  $R_1 = R_2 = R_3 = 500 \Omega$  &  $V_{in} = 10 \text{ volt}$ .

$$\Rightarrow R_x = 400 + 4T$$

$$V_o = V_{in} \left( \frac{R_x}{R_x + R_3} - \frac{R_1}{R_1 + R_2} \right)$$

$$V_o = 10 \left( \frac{125 + T}{250 + T} - 0.5 \right)$$

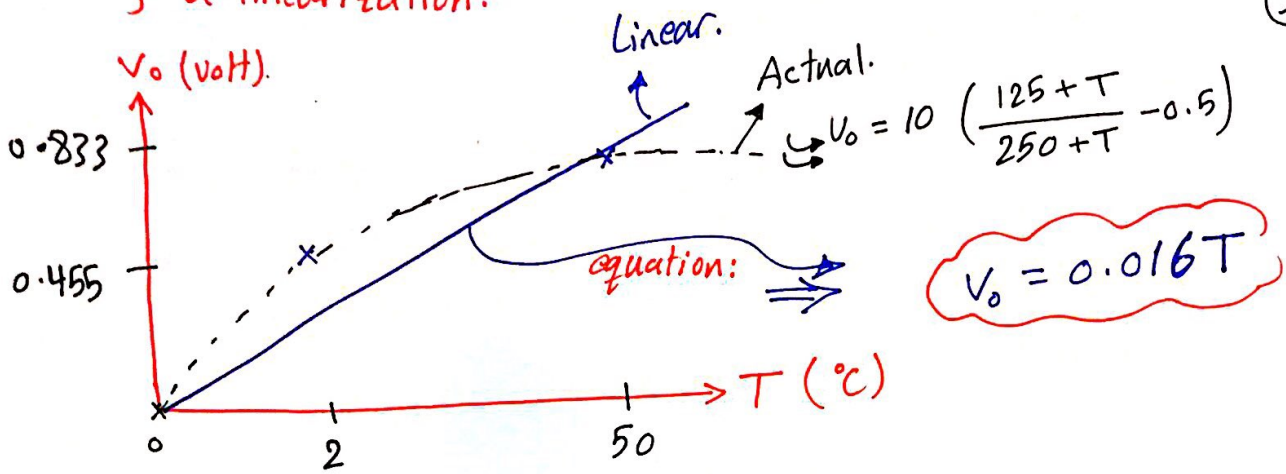
- $\Rightarrow @ T = 0 \Rightarrow V_o = 0$
- $@ T = 2 \Rightarrow V_o = 0.455 \text{ volt}$
- $@ T = 50 \Rightarrow V_o = 0.833 \text{ volt}$

$\Rightarrow$  continue.



doing a linearization:

(37)



\* Measurement errors:

Balanced:

$$R_2 R_3 = R_x R_1 \Rightarrow R_x = \frac{R_2 R_3}{R_1}$$

Tolerance.

$$R_2 = R_2 \pm \Delta R_2$$

$$R_3 = R_3 \pm \Delta R_3$$

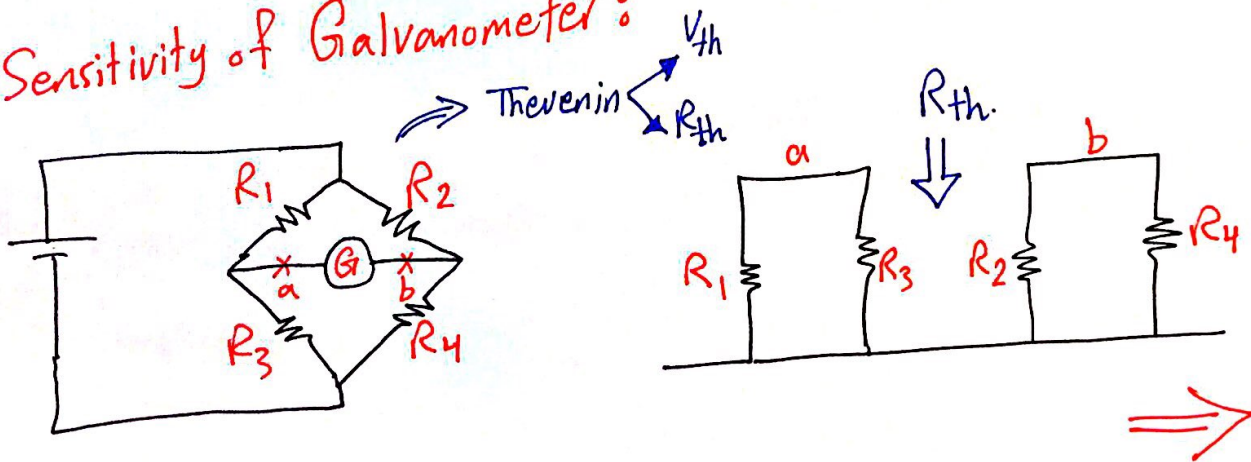
$$R_1 = R_1 \pm \Delta R_1$$

↳ \* Limiting error:

$$R_x = \frac{(R_2 \pm \Delta R_2)(R_3 \pm \Delta R_3)}{(R_1 \pm \Delta R_1)}$$

↳ Do an approximation:  $R_x = \frac{R_3 R_2}{R_1} \left( 1 \pm \frac{\Delta R_1}{R_1} \pm \frac{\Delta R_2}{R_2} \pm \frac{\Delta R_3}{R_3} \right)$

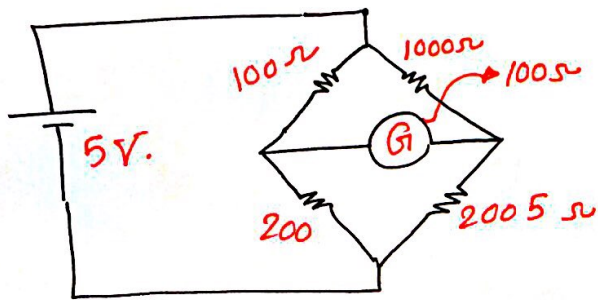
\* Sensitivity of Galvanometer:



$$\Rightarrow R_{th} = (R_1 \parallel R_3) + (R_2 \parallel R_4)$$

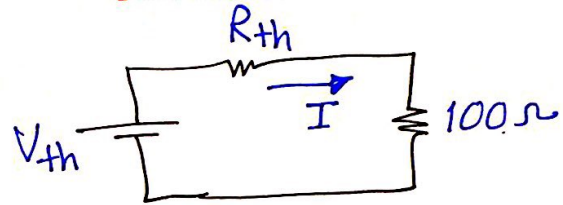
$$V_{th} = V_1 \left( \frac{R_1}{R_1 + R_3} - \frac{R_2}{R_2 + R_4} \right)$$

Example: slide (121):



$$\text{sensitivity} = \frac{10\text{mm}}{1\mu\text{A}}$$

\* Deflection by the Galvanometer:



$$R_{th} = 734\Omega$$

$$V_{th} = 2.77\text{mV}$$

$$I = 3.32\mu\text{A}$$

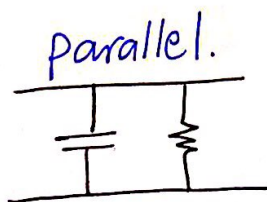
$$\Rightarrow \text{deflection} = I * \text{sensitivity} = 3.32\mu * \frac{10\text{mm}}{1\mu\text{A}}$$

$$= \boxed{33.2\text{mm}}$$

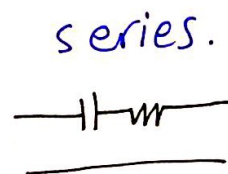
### \* Capacitor:

\* any capacitor has:

$$\underline{\underline{\epsilon, \sigma}}$$



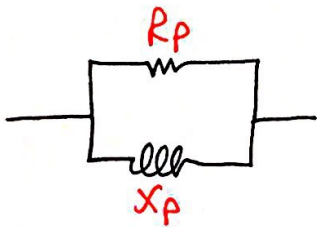
or





\* Quality Factor =  $\frac{\text{Energy stored}}{\text{Average Energy Dissipated}}$

\* Dissipation factor =  $\frac{\text{Energy Lost}}{\text{Energy stored}}$



→ Need it in series:

$$Z_{eq} = R_p // X_p$$

$$= \frac{R_p X_p^2}{R_p^2 + X_p^2} + j \frac{R_p^2 X_p}{R_p^2 + X_p^2}$$

$X_s$

$$Z_{eq} = R_s + j X_s$$



\* Quality factor of inductance:

in series:

$$Q = \frac{|I|^2 * X_L}{|I|^2 * R} = \frac{\omega L}{R}$$

in Parallel:



$$Q = \frac{V^2 / X_L}{V^2 / R} = \frac{R}{\omega L}$$

\* AC Bridge (Balanced Condition):

Balanced:

$$Z_1 Z_4 = Z_2 Z_3$$

↳ magnitude balance.  $Z_1 Z_4 = Z_2 Z_3$

↳ phase balance.  $\angle \theta_1 + \angle \theta_4 = \angle \theta_2 + \angle \theta_3$

# \* Oscilloscope Fundamentals:

10:1 voltage divider Probe.

↳ it is reduce the input signal if it was 10 volt signal, make it 1 volt.

\* Why the internal resistance of oscilloscope very high (in M $\Omega$ )?

To Reduce the high loading effect.

\* Oscilloscope consists R&C, at low frequency C become open circuit.

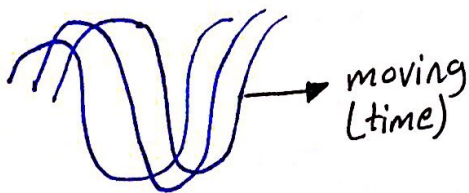
\* For a 1  $\mu$ s/Div & 1 V/Div:

→ max  $V_{pp}$  is 8 volt.

→ max period is 10  $\mu$ sec.

\* Trigger: we use it to have the signal stable on the oscilloscope screen.

\* if the trigger was above the peak voltage then the picture will be unsynchronized.

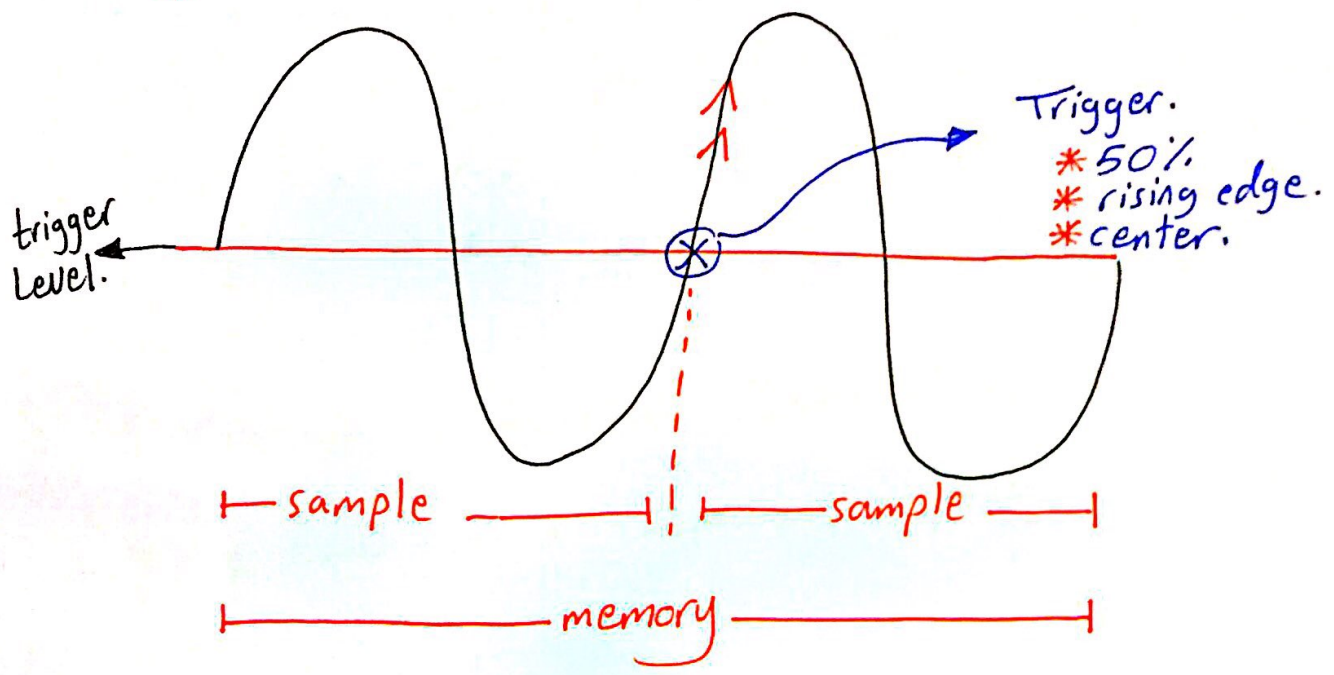


⇒ Trigger Level >  $V_{max}$

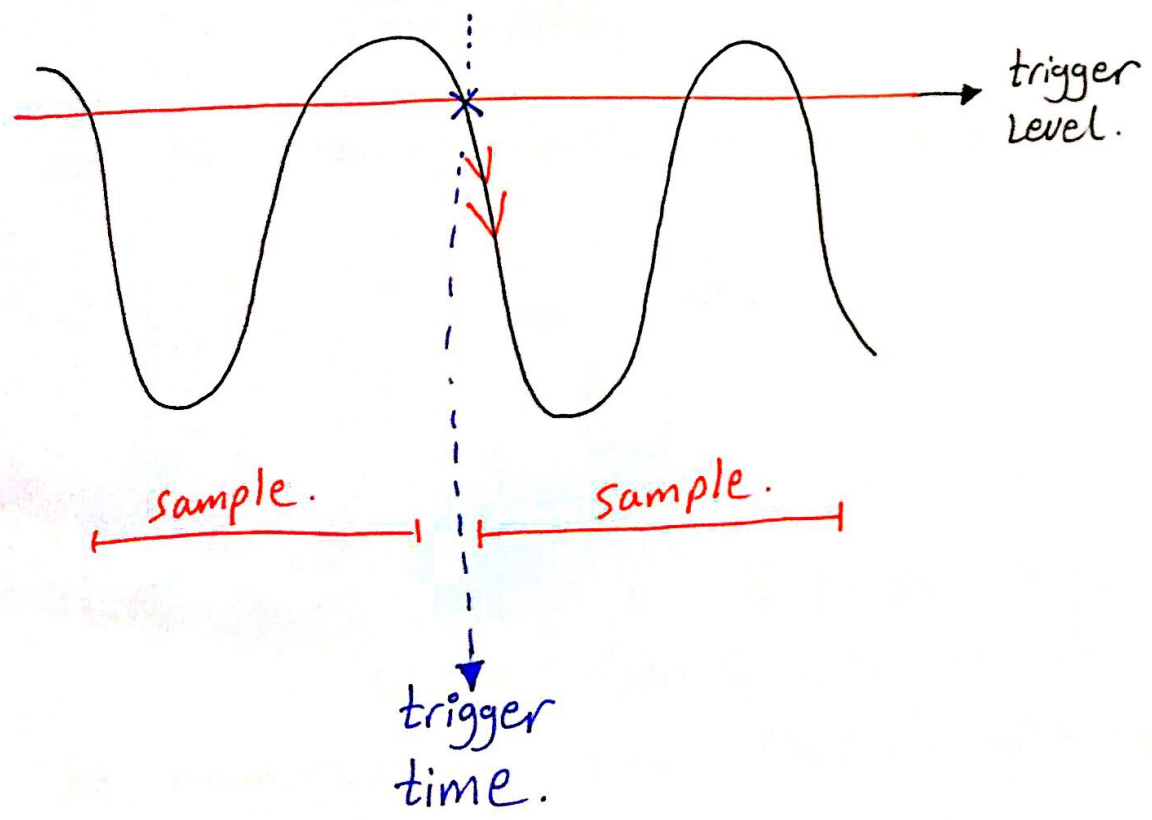
\*\* If the oscilloscope was ANALOGUE, then the picture will be shown starting from the trigger point.



\* Trigger Level in a DIGITAL oscilloscope:



\* in the following we have falling edge:

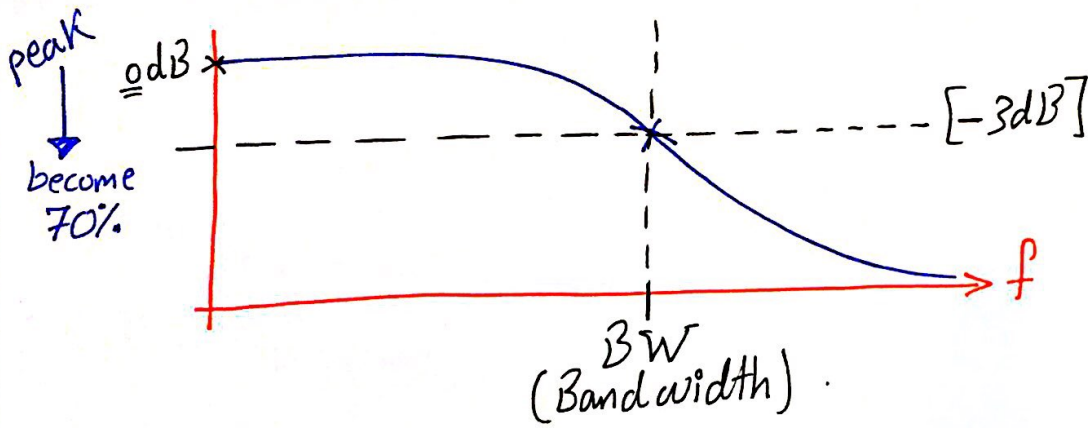


\* Always when determine trigger point we need the following:

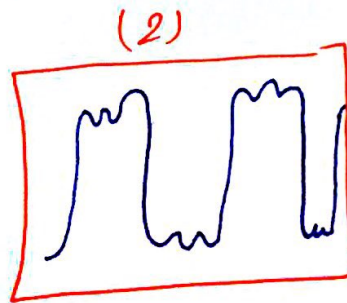
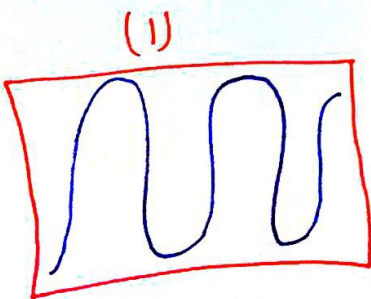
- 1) Determine Trigger Level.
- 2) Centre of the screen.
- 3) Determine if it is falling edge or Rising edge.

### \* Oscilloscope Performance Specifications:

\* for a low pass frequency response:



\* for the two following signals on oscilloscope:

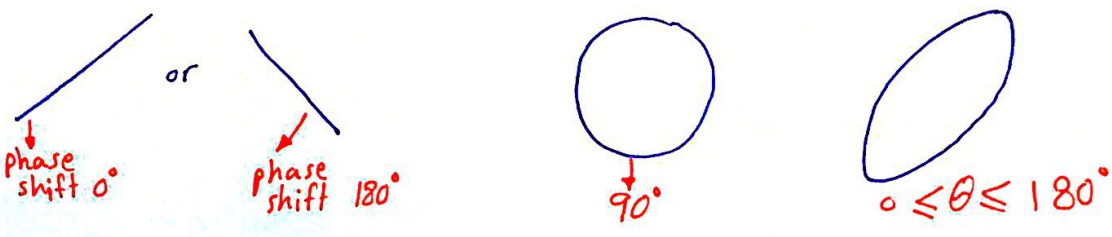


(2) is better than (1) since it is with higher BW. and (2) is more accurate than (1) since you will see the harmonics in the signal so that it is distorted.

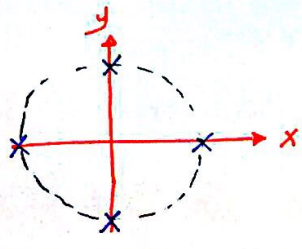


\* Oscilloscope (X-y mode):

if we insert a sinusoidal signal, 3 possible cases in the X-y mode:



Ex.  $x = \sin \omega t, y = \cos \omega t$



\* Zero crossing ( $x=0$ )

$$x = \sin \omega t = 0 \Rightarrow \omega t = 0, \pi$$

$$y(\omega t = 0) = 1 \quad \& \quad y(\omega t = \pi) = -1$$

$$\Downarrow \qquad \qquad \qquad \Downarrow$$

$$(0, 1) \qquad \qquad \qquad (0, -1)$$

\* ( $y=0$ )

$$\cos \omega t = 0 \Rightarrow \omega t = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$x\left(\frac{\pi}{2}\right) = 1 \quad \& \quad x\left(\frac{3\pi}{2}\right) = -1$$

$$\Downarrow \qquad \qquad \qquad \Downarrow$$

$$(1, 0) \qquad \qquad \qquad (-1, 0)$$

\* Max X :

$$\max \sin \omega t \Rightarrow \omega t = \frac{\pi}{2}$$

$$x\left(\frac{\pi}{2}\right) = 1 \quad \Rightarrow (1, 0)$$

$$y\left(\frac{\pi}{2}\right) = 0$$

\* Max y :  $\max \cos(\omega t) \Rightarrow \omega t = 0 \quad x(0) = 0 \Rightarrow (0, 1)$   
 $y(0) = 1$

\* Min X :

$$\min \sin \omega t \Rightarrow \omega t = \frac{3\pi}{2}$$

$$y\left(\frac{3\pi}{2}\right) = 0$$

$$x\left(\frac{3\pi}{2}\right) = -1 \quad (-1, 0)$$

\* Min y :

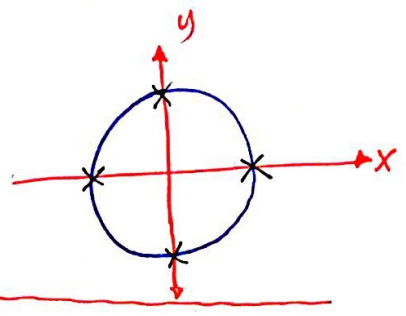
$$\min \cos \omega t \Rightarrow \omega t = \pi$$

$$x(\pi) = 0$$

$$y(\pi) = -1 \quad (0, -1)$$



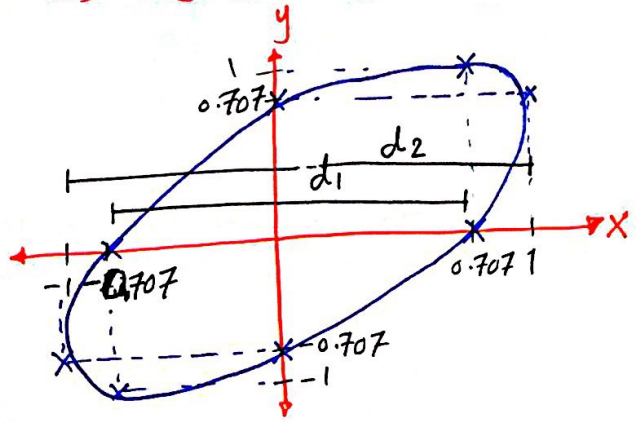
⇒ so we have:  
 $(0,1), (1,0), (0,-1), (-1,0)$



Ex.  $x = \cos(\omega t)$   
 $y = \cos(\omega t - 45^\circ)$

Do the same steps as the previous example.

⇒ The final figure would be:



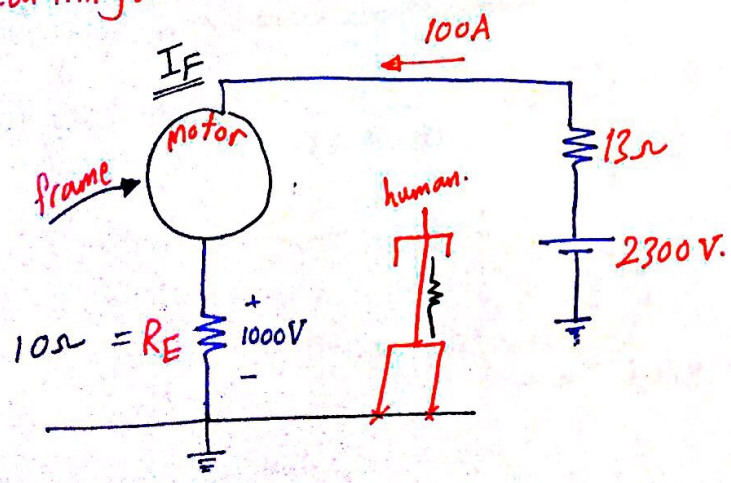
$d_1 \equiv$  Zero crossing.  
 $d_2 \equiv$  peak-to-peak.

From  $d_1$  &  $d_2$  we evaluate the phase shift:

$$\theta = \sin^{-1}\left(\frac{d_1}{d_2}\right)$$

Note: also we could use the y-axis to find the phase-shift with the same way.

**\* Earthing:**



if the human touch the motor after a fault happend we protect him :

- by wearing a safty shoes.
- by using Tire to clear fault.
- by reducing  $R_E$ .

\* The conductor between motor & the earth  
 ⇒ must be able to carry the fault current.



0 → 10 mA (NO Impact)

(45)

150 mA → 200 mA (Dead).

\* Current relation that related to human to has NO impact from the current:

$$I = \frac{116 \text{ mA}}{\sqrt{t}}$$

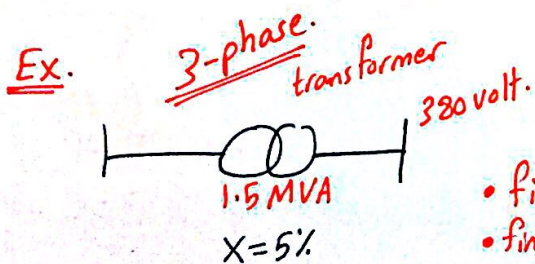
Max Safe Current.

Time to clear fault.

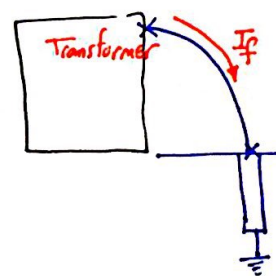
### \* Electrical Shock:

Difference in the voltage & will create a path of the current.

\* The worst path for the current passing through the human, that between human hands (since it will pass through the heart).



- find  $I_f$  ?!
- find cross section ?!



with stand fault current: cross section.

$$I_f = \frac{1}{X\%} * \text{Full Load current.}$$

$$\text{full load current} = \frac{1.5 \text{ M}}{\sqrt{3} * 380} \Rightarrow I_f = 20 * \text{full load}$$
$$I_f \approx 45 \text{ KA}$$

$$\text{cross section (mm}^2\text{)} = 9 * \sqrt{t} * I_f$$

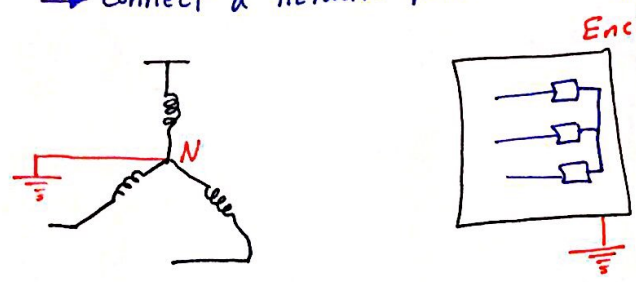
time to clear fault.

assume  $t=0.5 \text{ sec.}$

$$\text{cross section} = 9 * \sqrt{0.5} * 45 \text{ K} = 286 \text{ mm}^2$$

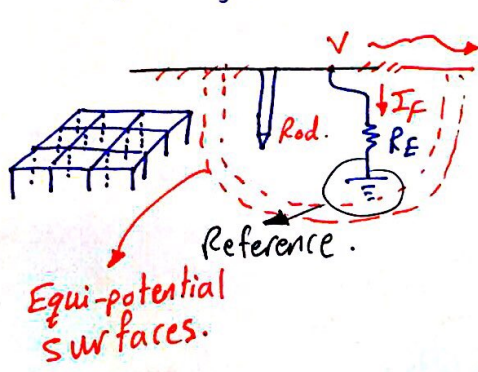
**\* Grounding:**

→ Connect a network point to the ground through an impedance.

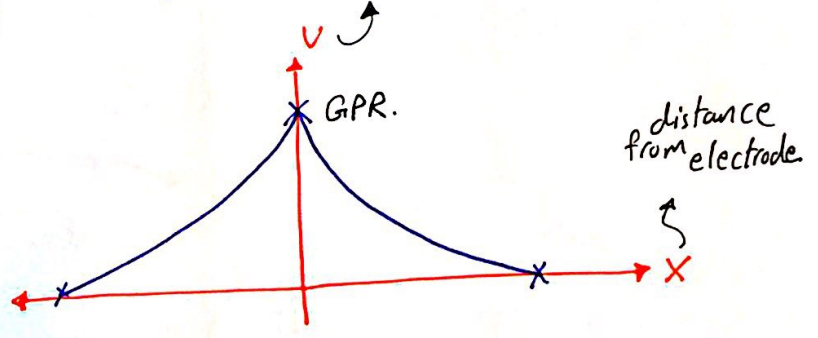


Grounding → Safty [Unenergized element.]  
→ performance of the network.

\* How to do the earthing?  
By using Rod or a mesh of Rods.



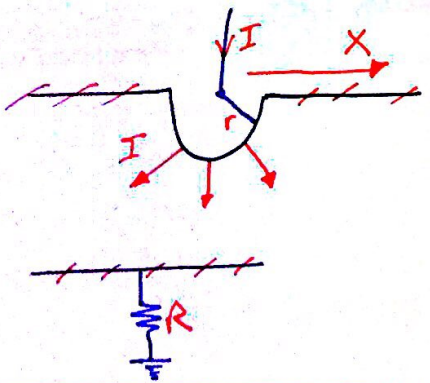
This voltage called: ground potential rise. (GPR)



**\* Safty:**

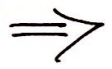
- Insolation (Gloves, boots).
- Grounding → Reduce GPR.
- Equi-potential rise → Reduce voltage difference in the site.

**\* Hemispherical electrode.**

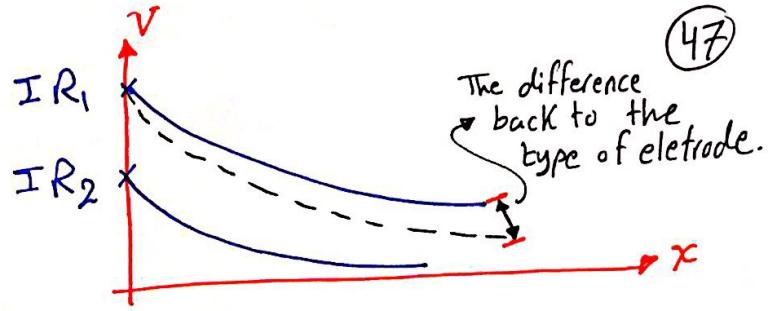
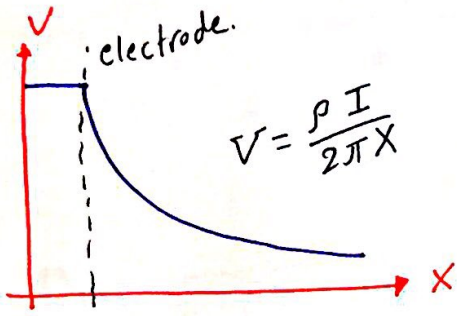


where R: soil Resistivity.

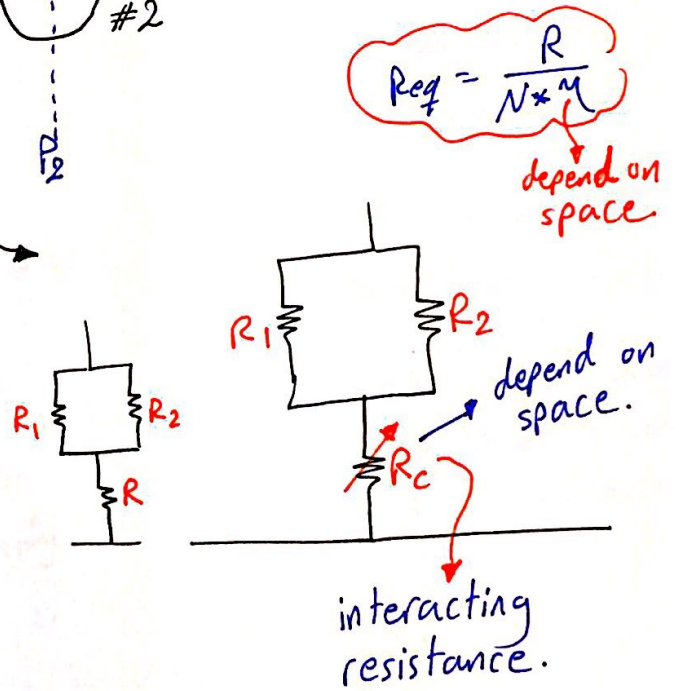
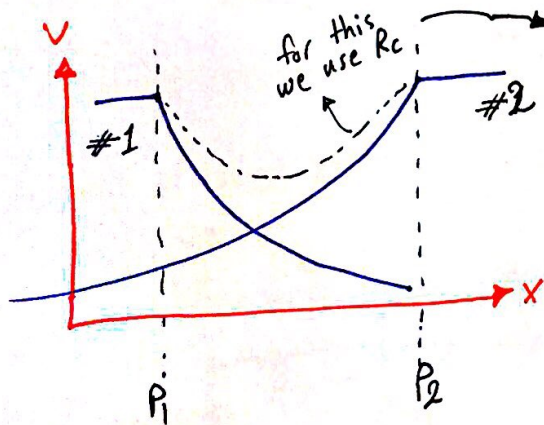
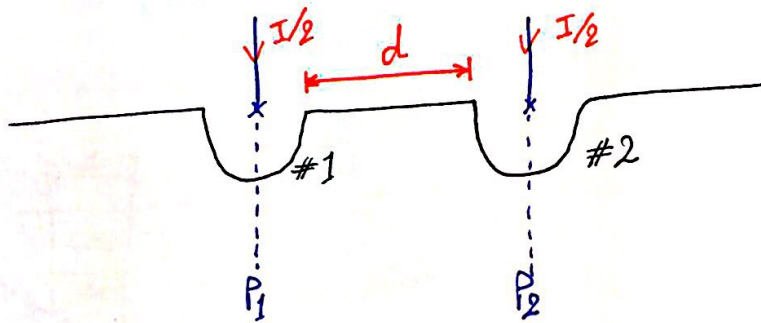
$$R = \frac{\rho}{2\pi r}$$







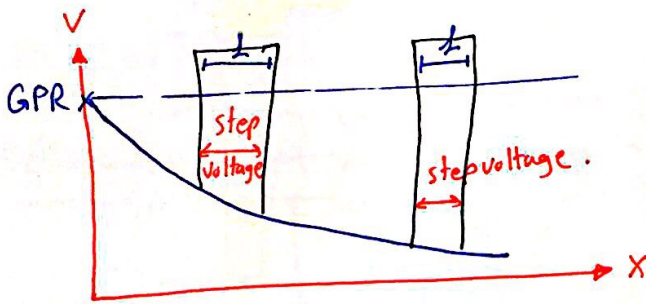
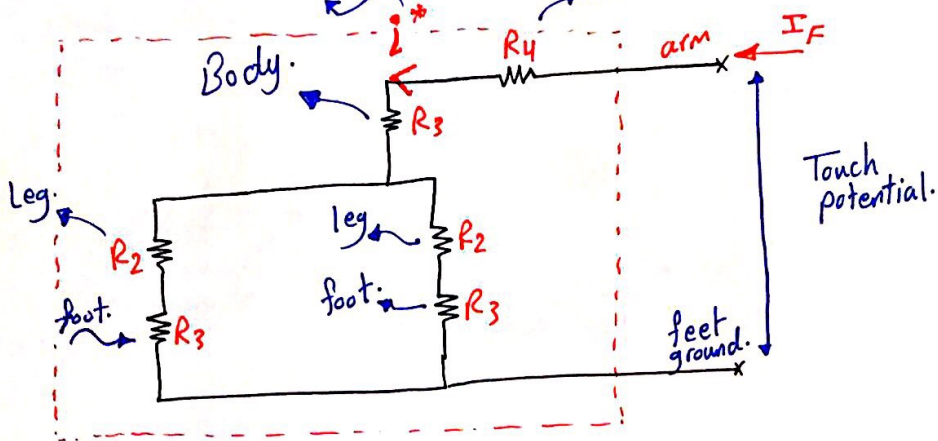
\* Using 2-electrode:



\* Touch & Step voltage:

- Touch: voltage difference between your hands & your feet.
- step voltage: voltage difference between your feet.

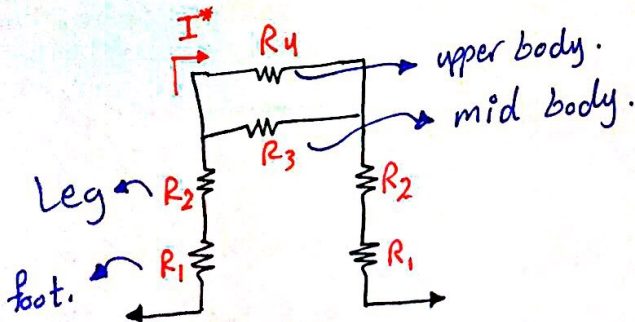
\* model of touch: most important current.



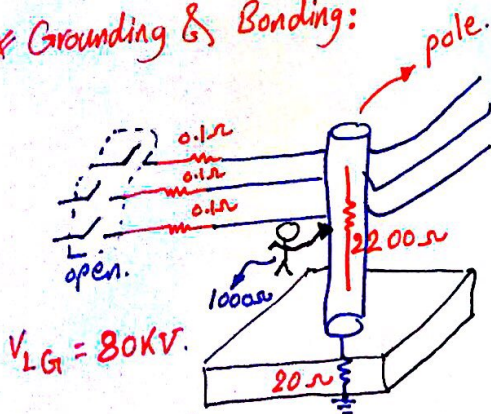
\* IEEE

< 1mA	No sensation.
1-8mA	sensation of shock.
8-15mA	painful shock
15-20mA	loss of muscle control.
20-50mA	muscles failure
50-200mA	heart failure
> 200mA	death & burns.

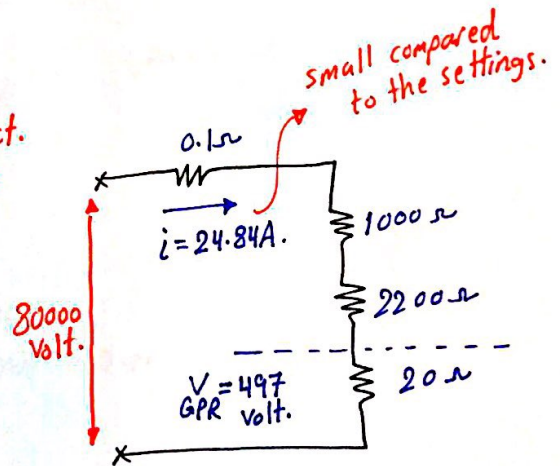
\* model of step voltage:



\* Grounding & Bonding:

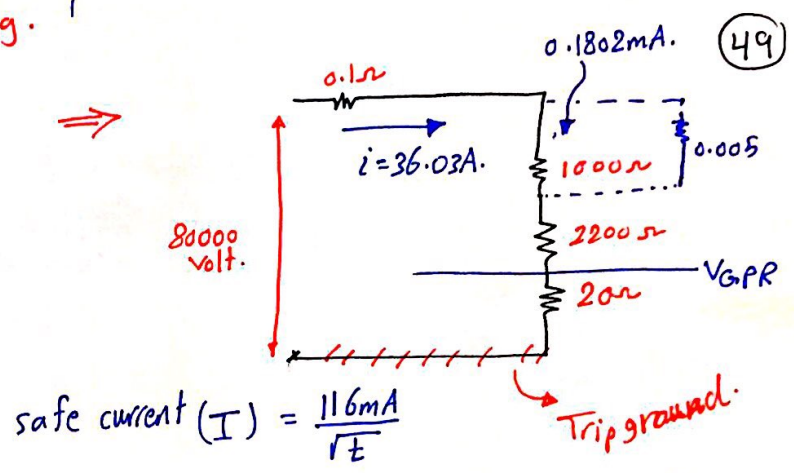
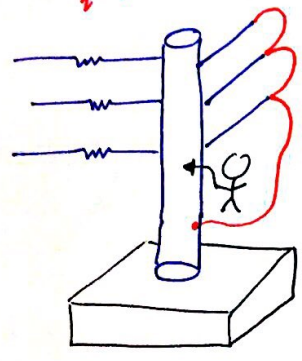


equ. cct.



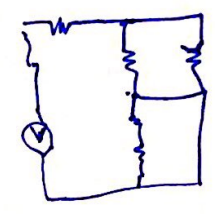
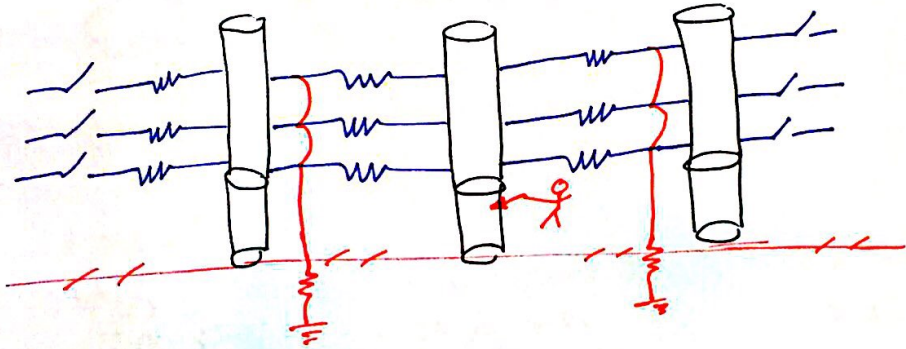


equipotential Bonding.

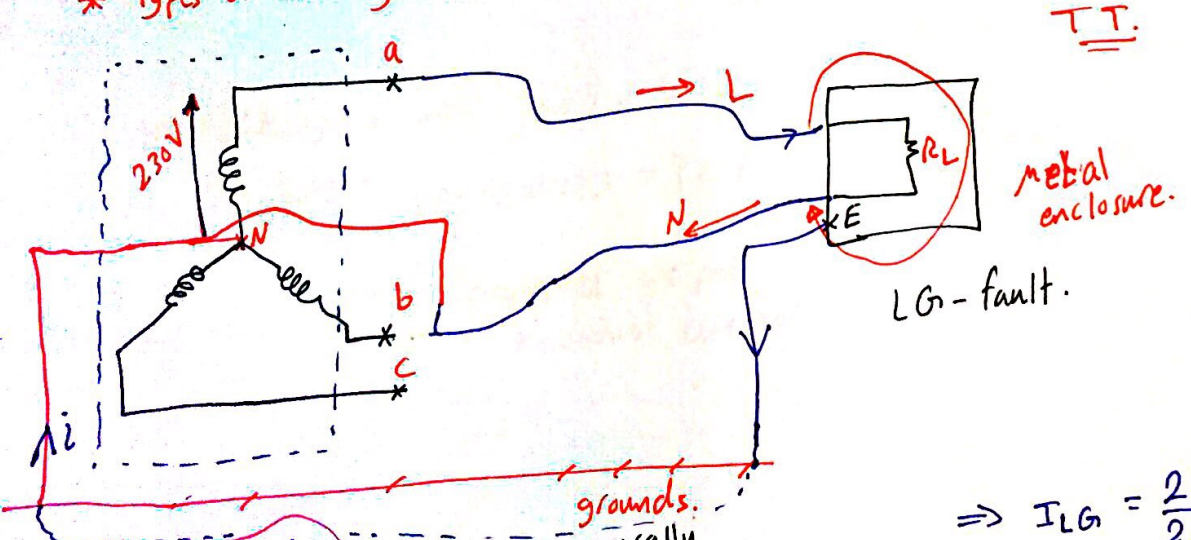


safe current (I) =  $\frac{116mA}{\sqrt{t}}$

brackets grounding.



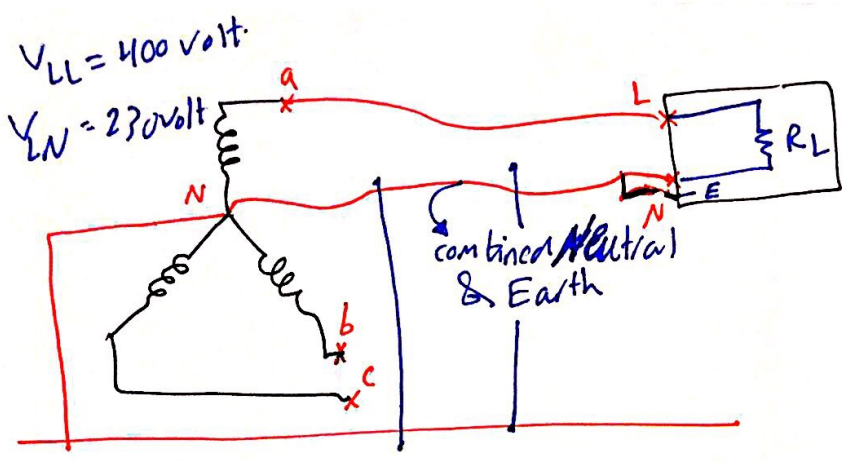
\* Types of earthing systems:



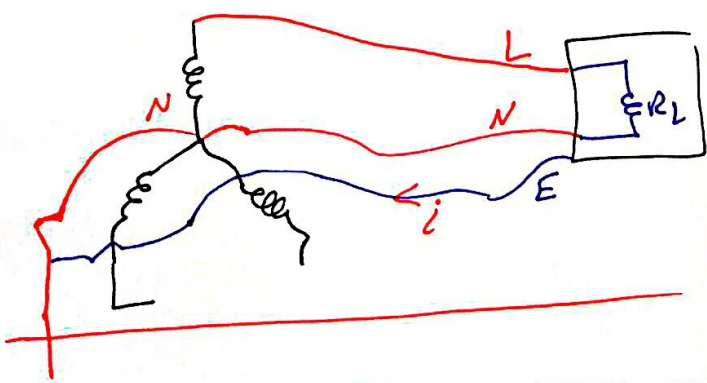
$I_{LG} = \frac{V}{R}$  ground loop resistance.

usually  $I_{LG}$  small. ground loop  $\approx 20 \Omega$

$\Rightarrow I_{LG} = \frac{230}{20} = 11.5A.$



$\Rightarrow$  TN-C-S  
 ↗ separate  
 ↘ combined.  
 PME  
 (protective multiple)  
 earthing.

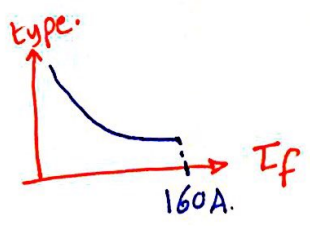


$\Rightarrow$  TN-S ↗ separate.

\* ground loop:

TNS	$\approx 0.8 \Omega$	$\Rightarrow I_{LG} = 230/0.8 = 287 \text{ A}$
TN-C-S	$\approx 0.35 \Omega$	$\Rightarrow I_{LG} = 657 \text{ A}$
TT	$= 21 \Omega$	$\Rightarrow I_{LG} = 11 \text{ A}$
	or $100 \Omega$	$\Rightarrow I_{LG} = 2.3 \text{ A}$

MCB 32A type B  $\Rightarrow 32 * 5 = 160 \text{ A}$ .  
 MCB 32A type C  $\Rightarrow 10 * 32 \text{ A} = 320 \text{ A}$ .

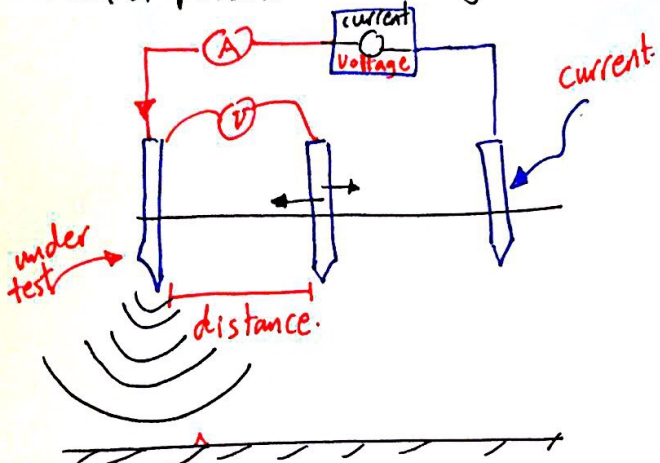


$\Rightarrow$  To be connected we put RCD  
 Residual current Device.

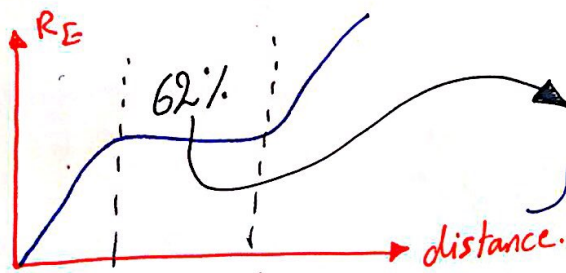
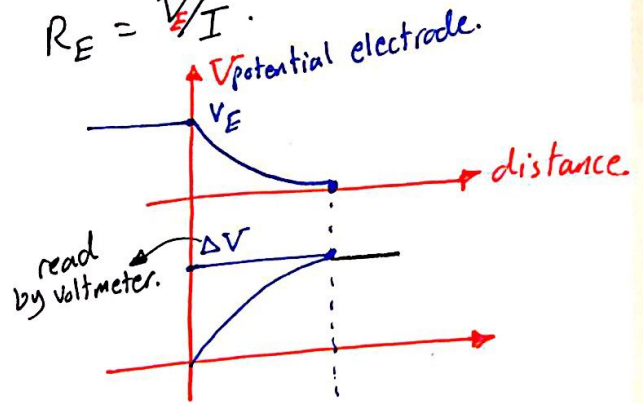


**\* Ground Testing:**

Fall of potential - Testing:



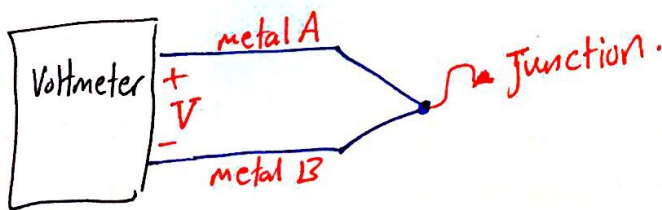
$$R_E = \frac{V}{I}$$



it is the distance between Electrode under Test & potential electrode.

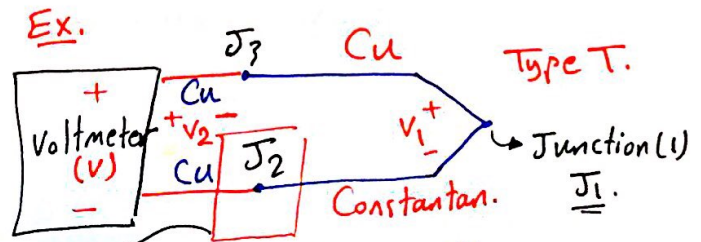
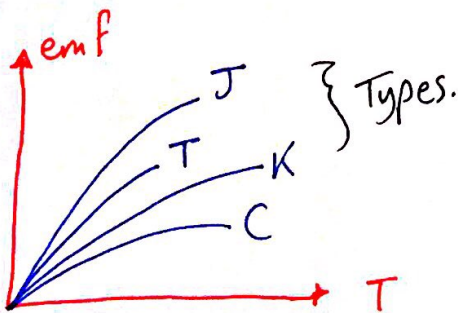
**\* Transducer:**

\* Thermocouple:  $\Rightarrow$  Temperature  $\xrightarrow{\text{convert to}}$  Voltage.



$$V = \alpha T$$

emf.



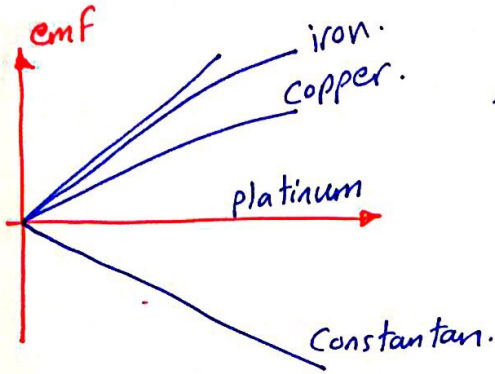
$J_3 \Rightarrow$  Cu with Cu (No emf).  $\xrightarrow{\text{By KVL}}$   
 $J_2 \Rightarrow$  Cu with C (emf).  $\Rightarrow V = V_1 - V_2$

ice  $\left\{ \begin{array}{l} \text{ref} \Rightarrow T=0 \\ V_2=0 \end{array} \right\}$

$\Rightarrow$  Continue.

⇒ \*Reference Junction:

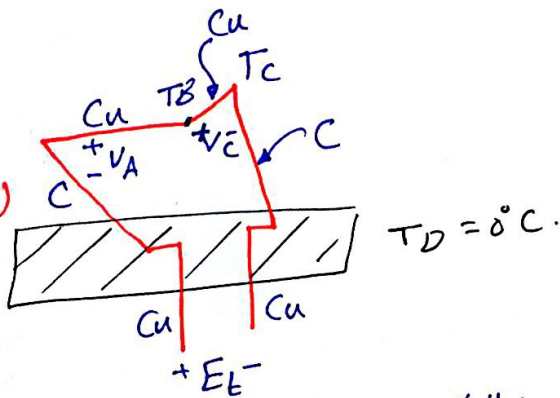
$J_2 \Rightarrow$  reference (ice)  $\Rightarrow 0^\circ\text{C} \Rightarrow V_2 = 0$   
 since  $V_2 = \alpha T$   
 $V = V_1 = \alpha T_{J_1}$



⇒ depend on the junction between the two material.

Example:

- ① find Emf (B,C)
- ② find  $T_A, T_C$ .

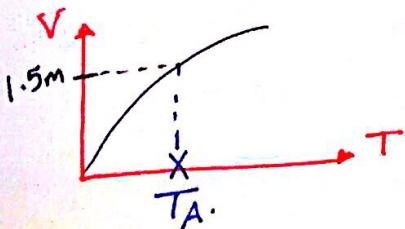


given:  $T_B = 121.1^\circ\text{C}$   
 $V_A = +1.517\text{mV}$   
 $E_T = 2.05\text{mV}$   
 curve Cu-C is given.

solution:

①  $V_2 = 0$  (Cu-Cu Junction). for C: KVL:  $-E_T - V_A + V_C = 0$   
 so  $V_C = E_T + V_A \Rightarrow V_C = 3.567\text{mV}$

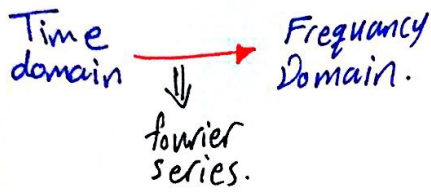
②  $T_A \Rightarrow$  from Curve.



if a table is given for V & T & we didn't find the needed point.  $\Rightarrow$  we do a linearization and find that point.

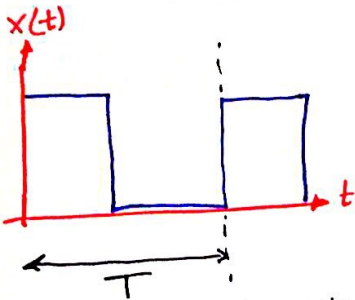


### \* Signal Analysers:



$$\Rightarrow x(t) = a_0 + \sum_{n=1}^N (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)$$

$\omega_0 \equiv$  Fundamental Frequency.



$$f_0 = \frac{1}{T} \text{ Hz.}$$

$f_0$   
 $2f_0$   
 $3f_0$   
 $4f_0$   
 $\vdots$   
 Harmonics.

$a_0 \equiv$  DC signal Average.  
 $a_0 = \frac{1}{T} \int x(t) dt.$

for  $a_n$  &  $b_n$ : harmonic order.

$$a_n = \frac{2}{T} \int x(t) \cos n\omega_0 t dt.$$

$$b_n = \frac{2}{T} \int x(t) \sin n\omega_0 t dt.$$

### \* Power Quality Analysers:

This device can read:  
 Voltage, current, power, Accumulated Energy, magnitude, frequency  
 , THD "Total Harmonics Distortion".  
 It also has a memory.

$$dB = 20 \log \frac{V_2}{V_1}$$

$$= 10 \log \frac{P_2}{P_1}$$

$dB_m \rightarrow mW.$

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End of Material

